

Computer algebra independent integration tests

4-Trig-functions/4.7-Miscellaneous/4.7.3-c+d-x^m-trigⁿ-trig^p

Nasser M. Abbasi

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3.206	$\int x^2 \cos^2(x) \cot^3(x) dx$	967
3.207	$\int x \cos^2(x) \cot^3(x) dx$	973
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3.209	$\int (c+dx)^4 \tan(a+bx) dx$	980

3.210	$\int (c + dx)^3 \tan(a + bx) dx$	984
3.211	$\int (c + dx)^2 \tan(a + bx) dx$	988
3.212	$\int (c + dx) \tan(a + bx) dx$	992
3.213	$\int \frac{\tan(a+bx)}{c+dx} dx$	995
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3.223	$\int (c + dx)^2 \sin^2(a + bx) \tan(a + bx) dx$	1026
3.224	$\int (c + dx) \sin^2(a + bx) \tan(a + bx) dx$	1030
3.225	$\int \frac{\sin^2(a+bx) \tan(a+bx)}{c+dx} dx$	1034
3.226	$\int \frac{\sin^2(a+bx) \tan(a+bx)}{(c+dx)^2} dx$	1037
3.227	$\int (c + dx)^m \csc(a + bx) \sec(a + bx) dx$	1040
3.228	$\int (c + dx)^4 \csc(a + bx) \sec(a + bx) dx$	1042
3.229	$\int (c + dx)^3 \csc(a + bx) \sec(a + bx) dx$	1048
3.230	$\int (c + dx)^2 \csc(a + bx) \sec(a + bx) dx$	1053
3.231	$\int (c + dx) \csc(a + bx) \sec(a + bx) dx$	1057
3.232	$\int \frac{\csc(a+bx) \sec(a+bx)}{c+dx} dx$	1060
3.233	$\int \frac{\csc(a+bx) \sec(a+bx)}{(c+dx)^2} dx$	1062
3.234	$\int (c + dx)^m \csc^2(a + bx) \sec(a + bx) dx$	1064
3.235	$\int (c + dx)^3 \csc^2(a + bx) \sec(a + bx) dx$	1066
3.236	$\int (c + dx)^2 \csc^2(a + bx) \sec(a + bx) dx$	1073
3.237	$\int (c + dx) \csc^2(a + bx) \sec(a + bx) dx$	1079
3.238	$\int \frac{\csc^2(a+bx) \sec(a+bx)}{c+dx} dx$	1083
3.239	$\int \frac{\csc^2(a+bx) \sec(a+bx)}{(c+dx)^2} dx$	1085
3.240	$\int (c + dx)^m \csc^3(a + bx) \sec(a + bx) dx$	1087
3.241	$\int (c + dx)^3 \csc^3(a + bx) \sec(a + bx) dx$	1089
3.242	$\int (c + dx)^2 \csc^3(a + bx) \sec(a + bx) dx$	1099
3.243	$\int (c + dx) \csc^3(a + bx) \sec(a + bx) dx$	1106
3.244	$\int \frac{\csc^3(a+bx) \sec(a+bx)}{c+dx} dx$	1111
3.245	$\int \frac{\csc^3(a+bx) \sec(a+bx)}{(c+dx)^2} dx$	1113
3.246	$\int (c + dx)^m \sec(a + bx) \tan(a + bx) dx$	1115
3.247	$\int (c + dx)^4 \sec(a + bx) \tan(a + bx) dx$	1117
3.248	$\int (c + dx)^3 \sec(a + bx) \tan(a + bx) dx$	1122
3.249	$\int (c + dx)^2 \sec(a + bx) \tan(a + bx) dx$	1126
3.250	$\int (c + dx) \sec(a + bx) \tan(a + bx) dx$	1129
3.251	$\int \frac{\sec(a+bx) \tan(a+bx)}{c+dx} dx$	1132
3.252	$\int \frac{\sec(a+bx) \tan(a+bx)}{(c+dx)^2} dx$	1134
3.253	$\int (c + dx)^m \tan^2(a + bx) dx$	1136
3.254	$\int (c + dx)^3 \tan^2(a + bx) dx$	1138
3.255	$\int (c + dx)^2 \tan^2(a + bx) dx$	1142
3.256	$\int (c + dx) \tan^2(a + bx) dx$	1146
3.257	$\int \frac{\tan^2(a+bx)}{c+dx} dx$	1149
3.258	$\int \frac{\tan^2(a+bx)}{(c+dx)^2} dx$	1151

3.259	$\int (c + dx)^m \sin(a + bx) \tan^2(a + bx) dx$	1153
3.260	$\int (c + dx)^3 \sin(a + bx) \tan^2(a + bx) dx$	1155
3.261	$\int (c + dx)^2 \sin(a + bx) \tan^2(a + bx) dx$	1164
3.262	$\int (c + dx) \sin(a + bx) \tan^2(a + bx) dx$	1168
3.263	$\int \frac{\sin(a+bx) \tan^2(a+bx)}{c+dx} dx$	1173
3.264	$\int \frac{\sin(a+bx) \tan^2(a+bx)}{(c+dx)^2} dx$	1175
3.265	$\int (c + dx)^m \csc(a + bx) \sec^2(a + bx) dx$	1178
3.266	$\int (c + dx)^4 \csc(a + bx) \sec^2(a + bx) dx$	1180
3.267	$\int (c + dx)^3 \csc(a + bx) \sec^2(a + bx) dx$	1189
3.268	$\int (c + dx)^2 \csc(a + bx) \sec^2(a + bx) dx$	1196
3.269	$\int (c + dx) \csc(a + bx) \sec^2(a + bx) dx$	1202
3.270	$\int \frac{\csc(a+bx) \sec^2(a+bx)}{c+dx} dx$	1206
3.271	$\int \frac{\csc(a+bx) \sec^2(a+bx)}{(c+dx)^2} dx$	1208
3.272	$\int (c + dx)^m \csc^2(a + bx) \sec^2(a + bx) dx$	1210
3.273	$\int (c + dx)^3 \csc^2(a + bx) \sec^2(a + bx) dx$	1212
3.274	$\int (c + dx)^2 \csc^2(a + bx) \sec^2(a + bx) dx$	1217
3.275	$\int (c + dx) \csc^2(a + bx) \sec^2(a + bx) dx$	1221
3.276	$\int \frac{\csc^2(a+bx) \sec^2(a+bx)}{c+dx} dx$	1224
3.277	$\int \frac{\csc^2(a+bx) \sec^2(a+bx)}{(c+dx)^2} dx$	1226
3.278	$\int (c + dx)^m \csc^3(a + bx) \sec^2(a + bx) dx$	1228
3.279	$\int (c + dx)^3 \csc^3(a + bx) \sec^2(a + bx) dx$	1230
3.280	$\int (c + dx)^2 \csc^3(a + bx) \sec^2(a + bx) dx$	1242
3.281	$\int (c + dx) \csc^3(a + bx) \sec^2(a + bx) dx$	1251
3.282	$\int \frac{\csc^3(a+bx) \sec^2(a+bx)}{c+dx} dx$	1256
3.283	$\int \frac{\csc^3(a+bx) \sec^2(a+bx)}{(c+dx)^2} dx$	1258
3.284	$\int x^m \csc^3(a + bx) \sec^2(a + bx) dx$	1260
3.285	$\int x^3 \csc^3(a + bx) \sec^2(a + bx) dx$	1262
3.286	$\int x^2 \csc^3(a + bx) \sec^2(a + bx) dx$	1270
3.287	$\int x \csc^3(a + bx) \sec^2(a + bx) dx$	1277
3.288	$\int \frac{\csc^3(a+bx) \sec^2(a+bx)}{x} dx$	1282
3.289	$\int \frac{\csc^3(a+bx) \sec^2(a+bx)}{x^2} dx$	1284
3.290	$\int (c + dx)^m \sec^2(a + bx) \tan(a + bx) dx$	1286
3.291	$\int (c + dx)^4 \sec^2(a + bx) \tan(a + bx) dx$	1288
3.292	$\int (c + dx)^3 \sec^2(a + bx) \tan(a + bx) dx$	1294
3.293	$\int (c + dx)^2 \sec^2(a + bx) \tan(a + bx) dx$	1298
3.294	$\int (c + dx) \sec^2(a + bx) \tan(a + bx) dx$	1303
3.295	$\int \frac{\sec^2(a+bx) \tan(a+bx)}{c+dx} dx$	1306
3.296	$\int \frac{\sec^2(a+bx) \tan(a+bx)}{(c+dx)^2} dx$	1308
3.297	$\int (c + dx)^m \sec(a + bx) \tan^2(a + bx) dx$	1310
3.298	$\int (c + dx)^3 \sec(a + bx) \tan^2(a + bx) dx$	1312
3.299	$\int (c + dx)^2 \sec(a + bx) \tan^2(a + bx) dx$	1319
3.300	$\int (c + dx) \sec(a + bx) \tan^2(a + bx) dx$	1324
3.301	$\int \frac{\sec(a+bx) \tan^2(a+bx)}{c+dx} dx$	1328
3.302	$\int \frac{\sec(a+bx) \tan^2(a+bx)}{(c+dx)^2} dx$	1330
3.303	$\int (c + dx)^m \tan^3(a + bx) dx$	1332
3.304	$\int (c + dx)^3 \tan^3(a + bx) dx$	1334
3.305	$\int (c + dx)^2 \tan^3(a + bx) dx$	1340
3.306	$\int (c + dx) \tan^3(a + bx) dx$	1345
3.307	$\int \frac{\tan^3(a+bx)}{c+dx} dx$	1349

3.308	$\int \frac{\tan^3(a+bx)}{(c+dx)^2} dx$	1352
3.309	$\int (c+dx)^m \csc(a+bx) \sec^3(a+bx) dx$	1355
3.310	$\int (c+dx)^4 \csc(a+bx) \sec^3(a+bx) dx$	1357
3.311	$\int (c+dx)^3 \csc(a+bx) \sec^3(a+bx) dx$	1369
3.312	$\int (c+dx)^2 \csc(a+bx) \sec^3(a+bx) dx$	1378
3.313	$\int (c+dx) \csc(a+bx) \sec^3(a+bx) dx$	1384
3.314	$\int \frac{\csc(a+bx) \sec^3(a+bx)}{c+dx} dx$	1389
3.315	$\int \frac{\csc(a+bx) \sec^3(a+bx)}{(c+dx)^2} dx$	1392
3.316	$\int (c+dx)^m \csc^2(a+bx) \sec^3(a+bx) dx$	1394
3.317	$\int (c+dx)^3 \csc^2(a+bx) \sec^3(a+bx) dx$	1396
3.318	$\int (c+dx)^2 \csc^2(a+bx) \sec^3(a+bx) dx$	1407
3.319	$\int (c+dx) \csc^2(a+bx) \sec^3(a+bx) dx$	1415
3.320	$\int \frac{\csc^2(a+bx) \sec^3(a+bx)}{c+dx} dx$	1420
3.321	$\int \frac{\csc^2(a+bx) \sec^3(a+bx)}{(c+dx)^2} dx$	1422
3.322	$\int (c+dx)^m \csc^3(a+bx) \sec^3(a+bx) dx$	1424
3.323	$\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx) dx$	1426
3.324	$\int (c+dx)^2 \csc^3(a+bx) \sec^3(a+bx) dx$	1435
3.325	$\int (c+dx) \csc^3(a+bx) \sec^3(a+bx) dx$	1441
3.326	$\int \frac{\csc^3(a+bx) \sec^3(a+bx)}{c+dx} dx$	1445
3.327	$\int \frac{\csc^3(a+bx) \sec^3(a+bx)}{(c+dx)^2} dx$	1447
3.328	$\int x \cos^{\frac{5}{2}}(a+bx) \sin(a+bx) dx$	1449
3.329	$\int x \cos^{\frac{3}{2}}(a+bx) \sin(a+bx) dx$	1452
3.330	$\int x \sqrt{\cos(a+bx)} \sin(a+bx) dx$	1455
3.331	$\int \frac{x \sin(a+bx)}{\sqrt{\cos(a+bx)}} dx$	1458
3.332	$\int \frac{x \sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)} dx$	1461
3.333	$\int \frac{x \sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)} dx$	1464
3.334	$\int \frac{x \sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)} dx$	1467
3.335	$\int \frac{x \sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)} dx$	1470
3.336	$\int x \sec^{\frac{2}{7}}(a+bx) \sin(a+bx) dx$	1473
3.337	$\int x \sec^{\frac{2}{5}}(a+bx) \sin(a+bx) dx$	1476
3.338	$\int x \sec^{\frac{2}{3}}(a+bx) \sin(a+bx) dx$	1479
3.339	$\int x \sec^{\frac{2}{5}}(a+bx) \sin(a+bx) dx$	1482
3.340	$\int x \sqrt{\sec(a+bx)} \sin(a+bx) dx$	1485
3.341	$\int \frac{x \sin(a+bx)}{\sqrt{\sec(a+bx)}} dx$	1488
3.342	$\int \frac{x \sin(a+bx)}{\sec^{\frac{2}{3}}(a+bx)} dx$	1491
3.343	$\int \frac{x \sin(a+bx)}{\sec^{\frac{2}{5}}(a+bx)} dx$	1494
3.344	$\int x \cos(a+bx) \sin^{\frac{5}{2}}(a+bx) dx$	1497
3.345	$\int x \cos(a+bx) \sin^{\frac{3}{2}}(a+bx) dx$	1500
3.346	$\int x \cos(a+bx) \sqrt{\sin(a+bx)} dx$	1503
3.347	$\int \frac{x \cos(a+bx)}{\sqrt{\sin(a+bx)}} dx$	1506
3.348	$\int \frac{x \cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)} dx$	1509
3.349	$\int \frac{x \cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)} dx$	1512

3.350	$\int \frac{x \cos(ax+bx)}{\sin^2(ax+bx)} dx$	1515
3.351	$\int \frac{x \cos(ax+bx)}{\sin^2(ax+bx)} dx$	1518
3.352	$\int x \cos(a+bx) \csc^2(a+bx) dx$	1521
3.353	$\int x \cos(a+bx) \csc^2(a+bx) dx$	1524
3.354	$\int x \cos(a+bx) \csc^2(a+bx) dx$	1527
3.355	$\int x \cos(a+bx) \csc^2(a+bx) dx$	1530
3.356	$\int x \cos(a+bx) \sqrt{\csc(a+bx)} dx$	1533
3.357	$\int \frac{x \cos(ax+bx)}{\sqrt{\csc(ax+bx)}} dx$	1536
3.358	$\int \frac{x \cos(ax+bx)}{\csc^2(ax+bx)} dx$	1539
3.359	$\int \frac{x \cos(ax+bx)}{\csc^2(ax+bx)} dx$	1542
3.360	$\int x \csc(x) \sin(3x) dx$	1545
3.361	$\int (c+dx)^4 \csc(x) \sin(3x) dx$	1548
3.362	$\int (c+dx)^3 \csc(x) \sin(3x) dx$	1551
3.363	$\int (c+dx)^2 \csc(x) \sin(3x) dx$	1554
3.364	$\int (c+dx) \csc(x) \sin(3x) dx$	1557
3.365	$\int \frac{\csc(x) \sin(3x)}{c+dx} dx$	1560
3.366	$\int \frac{\csc(x) \sin(3x)}{(c+dx)^2} dx$	1563
3.367	$\int \frac{\csc(x) \sin(3x)}{(c+dx)^3} dx$	1567
3.368	$\int (c+dx)^4 \csc(a+bx) \sin(3a+3bx) dx$	1571
3.369	$\int (c+dx)^3 \csc(a+bx) \sin(3a+3bx) dx$	1577
3.370	$\int (c+dx)^2 \csc(a+bx) \sin(3a+3bx) dx$	1582
3.371	$\int (c+dx) \csc(a+bx) \sin(3a+3bx) dx$	1586
3.372	$\int \frac{\csc(a+bx) \sin(3a+3bx)}{c+dx} dx$	1589
3.373	$\int \frac{\csc(a+bx) \sin(3a+3bx)}{(c+dx)^2} dx$	1593
3.374	$\int \frac{\csc(a+bx) \sin(3a+3bx)}{(c+dx)^3} dx$	1596
3.375	$\int \frac{\csc(a+bx) \sin(3a+3bx)}{(c+dx)^4} dx$	1604
3.376	$\int (c+dx)^3 \csc^2(a+bx) \sin(3a+3bx) dx$	1608
3.377	$\int (c+dx)^2 \csc^2(a+bx) \sin(3a+3bx) dx$	1613
3.378	$\int (c+dx) \csc^2(a+bx) \sin(3a+3bx) dx$	1617
3.379	$\int \frac{\csc^2(a+bx) \sin(3a+3bx)}{c+dx} dx$	1621
3.380	$\int \frac{\csc^2(a+bx) \sin(3a+3bx)}{(c+dx)^2} dx$	1624
3.381	$\int \frac{\csc^2(a+bx) \sin(3a+3bx)}{(c+dx)^3} dx$	1627
3.382	$\int (c+dx)^4 \sec(a+bx) \sin(3a+3bx) dx$	1630
3.383	$\int (c+dx)^3 \sec(a+bx) \sin(3a+3bx) dx$	1637
3.384	$\int (c+dx)^2 \sec(a+bx) \sin(3a+3bx) dx$	1643
3.385	$\int (c+dx) \sec(a+bx) \sin(3a+3bx) dx$	1648
3.386	$\int \frac{\sec(a+bx) \sin(3a+3bx)}{c+dx} dx$	1652
3.387	$\int \frac{\sec(a+bx) \sin(3a+3bx)}{(c+dx)^2} dx$	1655
3.388	$\int \frac{\sec(a+bx) \sin(3a+3bx)}{(c+dx)^3} dx$	1658
3.389	$\int (c+dx)^3 \sec^2(a+bx) \sin(3a+3bx) dx$	1661
3.390	$\int (c+dx)^2 \sec^2(a+bx) \sin(3a+3bx) dx$	1670
3.391	$\int (c+dx) \sec^2(a+bx) \sin(3a+3bx) dx$	1674
3.392	$\int \frac{\sec^2(a+bx) \sin(3a+3bx)}{c+dx} dx$	1680
3.393	$\int \frac{\sec^2(a+bx) \sin(3a+3bx)}{(c+dx)^2} dx$	1683
3.394	$\int \frac{\sec^2(a+bx) \sin(3a+3bx)}{(c+dx)^3} dx$	1686

3.395	$\int x \cos(2x) \sec(x) dx$	1689
3.396	$\int x \cos(2x) \sec^2(x) dx$	1692
3.397	$\int x \cos(2x) \sec^3(x) dx$	1695
4	Listing of Grading functions	1699

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [397]. This is test number [137].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (397)	% 0. (0)
Mathematica	% 100. (397)	% 0. (0)
Maple	% 90.43 (359)	% 9.57 (38)
Maxima	% 73.3 (291)	% 26.7 (106)
Fricas	% 91.94 (365)	% 8.06 (32)
Sympy	% 25.69 (102)	% 74.31 (295)
Giac	% 55.92 (222)	% 44.08 (175)

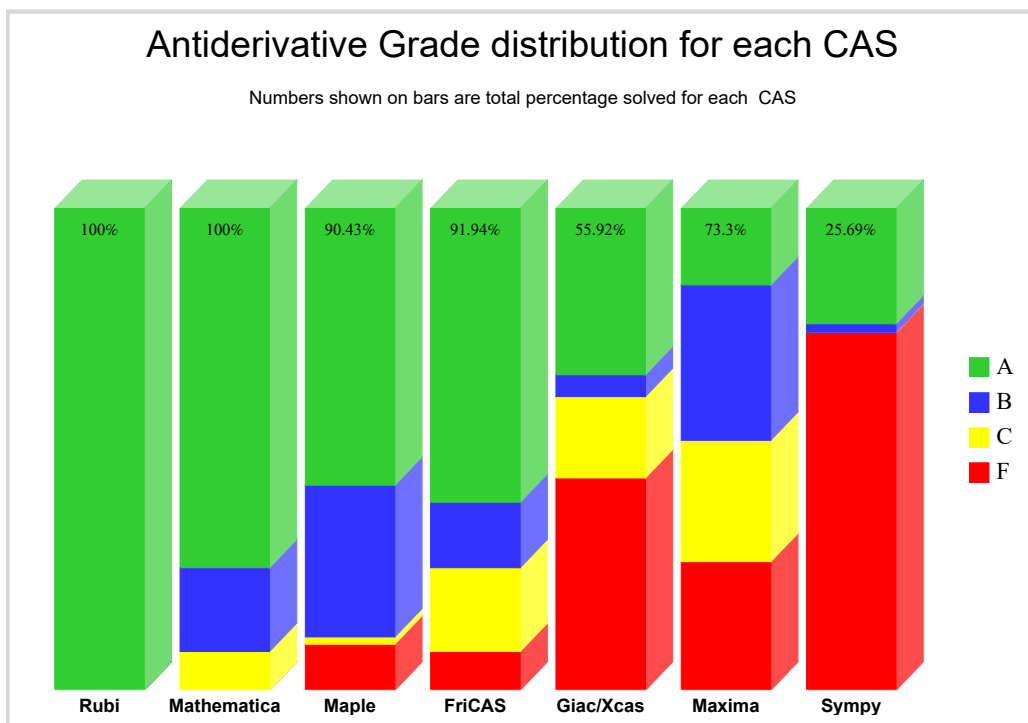
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

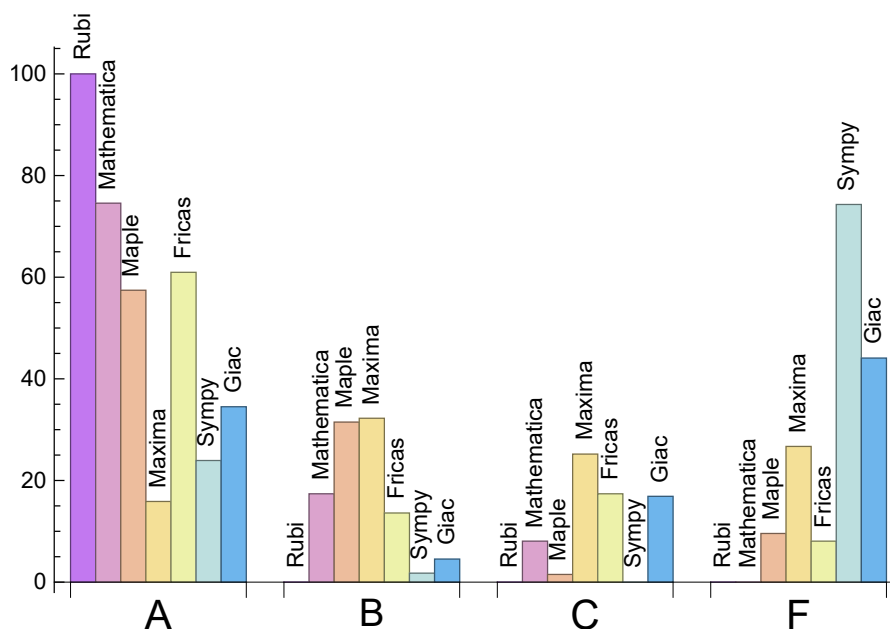
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	74.56	17.38	8.06	0.
Maple	57.43	31.49	1.51	9.57
Maxima	15.87	32.24	25.19	26.7
Fricas	60.96	13.6	17.38	8.06
Sympy	23.93	1.76	0.	74.31
Giac	34.51	4.53	16.88	44.08

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.26	149.21	0.77	115.	1.
Mathematica	4.79	277.11	1.3	132.	0.92
Maple	0.36	344.87	1.81	242.	1.41
Maxima	2.2	1482.18	7.27	706.	4.6
Fricas	0.48	1006.59	5.1	599.	3.08
Sympy	10.68	208.78	1.5	0.	0.
Giac	0.86	1058.52	7.85	155.	1.53

1.4 list of integrals that has no closed form antiderivative

{31, 36, 37, 38, 43, 44, 45, 50, 51, 97, 102, 103, 104, 109, 110, 111, 116, 117, 163, 168, 169, 170, 175, 176, 177, 182, 183, 208, 213, 214, 215, 219, 220, 221, 225, 226, 227, 232, 233, 234, 238, 239, 240, 244, 245, 246, 251, 252, 253, 257, 258, 259, 263, 264, 265, 270, 271, 272, 276, 277, 278, 282, 283, 284, 288, 289, 290, 295, 296, 297, 301, 302, 303, 307, 308, 309, 314, 315, 316, 320, 321, 322, 326, 327, 379, 380, 381, 386, 387, 388, 392, 393, 394}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {32, 33, 34, 35, 39, 41, 46, 47, 58, 63, 99, 105, 106, 107, 130, 131, 134, 135, 164, 165, 166, 171, 173, 178, 179, 180, 181, 190, 195, 222, 223, 236, 237, 241, 242, 249, 254, 255, 261, 267, 268, 280, 285, 291, 292, 300, 304, 305, 306, 310, 311, 312, 318, 319, 331, 376, 382, 383, 384, 385, 389, 390}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
```

```
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via `sagemath`) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in->

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

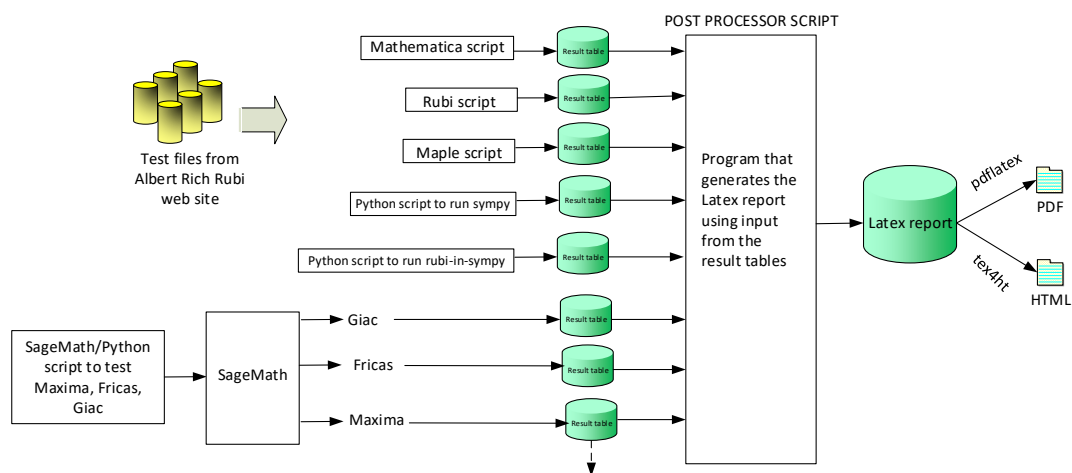
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer. the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 36, 37, 38, 39, 43, 44, 45, 49, 50, 51, 52, 53, 54, 55, 56, 57, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 109, 110, 111, 113, 116, 117, 124, 125, 126, 127, 128, 129, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 167, 168, 169, 170, 174, 175, 176, 177, 182, 183, 184, 185, 186, 187, 188, 189, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 219, 220, 221, 224, 225, 226, 227, 229, 230, 231, 232, 233, 234, 238, 239, 240, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 256, 257, 258, 259, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 275, 276, 277, 278, 279, 282, 283, 284, 285, 288, 289, 290, 293, 294, 295, 296, 297, 298, 301, 302, 303, 307, 308, 309, 313, 314, 315, 316, 317, 320, 321, 322, 323, 326, 327, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 341, 343, 344, 346, 348, 349, 350, 351, 352, 353, 354, 355, 357, 359, 360, 361, 362, 363, 364,

365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 386, 387, 388, 391, 392, 393, 394, 395, 396 }

B grade: { 32, 33, 34, 35, 40, 41, 42, 46, 47, 98, 105, 106, 107, 112, 114, 115, 164, 165, 166, 171, 172, 173, 178, 179, 180, 181, 216, 218, 222, 223, 228, 235, 236, 241, 242, 250, 254, 255, 260, 261, 273, 274, 280, 281, 286, 287, 291, 292, 299, 300, 304, 305, 306, 310, 311, 312, 318, 324, 325, 331, 340, 342, 382, 383, 384, 385, 389, 390, 397 }

C grade: { 48, 58, 59, 60, 61, 62, 63, 108, 118, 119, 120, 121, 122, 123, 130, 131, 132, 133, 134, 135, 190, 191, 192, 193, 194, 195, 237, 319, 345, 347, 356, 358 }

F grade: { }

2.1.3 Maple

A grade: { 5, 6, 7, 8, 9, 10, 11, 12, 17, 18, 19, 20, 21, 26, 27, 28, 29, 30, 31, 36, 37, 38, 42, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 102, 103, 104, 108, 109, 110, 111, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 149, 150, 151, 152, 153, 159, 160, 161, 162, 163, 168, 169, 170, 175, 176, 177, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 213, 214, 215, 219, 220, 221, 224, 225, 226, 227, 232, 233, 234, 237, 238, 239, 240, 244, 245, 246, 250, 251, 252, 253, 256, 257, 258, 259, 263, 264, 265, 269, 270, 271, 272, 276, 277, 278, 281, 282, 283, 284, 287, 288, 289, 290, 293, 294, 295, 296, 297, 301, 302, 303, 306, 307, 308, 309, 314, 315, 316, 320, 321, 322, 326, 327, 360, 362, 363, 364, 365, 366, 367, 371, 372, 373, 374, 375, 379, 380, 381, 385, 386, 387, 388, 391, 392, 393, 394, 395, 396 }

B grade: { 2, 3, 4, 14, 15, 16, 23, 24, 25, 32, 33, 34, 35, 39, 40, 41, 46, 47, 71, 72, 73, 80, 81, 82, 83, 89, 90, 91, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 137, 138, 139, 146, 147, 148, 155, 156, 157, 158, 164, 165, 166, 167, 171, 172, 173, 178, 179, 180, 181, 204, 209, 210, 211, 212, 216, 217, 218, 222, 223, 228, 229, 230, 231, 235, 236, 241, 242, 243, 247, 248, 249, 254, 255, 260, 261, 266, 267, 268, 273, 274, 275, 279, 280, 286, 291, 292, 298, 299, 300, 304, 305, 310, 311, 312, 313, 317, 318, 319, 323, 324, 325, 361, 368, 369, 370, 376, 377, 378, 382, 383, 384, 389, 390, 397 }

C grade: { 174, 262, 331, 340, 347, 356 }

F grade: { 1, 13, 22, 70, 79, 88, 136, 145, 154, 285, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 357, 358, 359 }

2.1.4 Maxima

A grade: { 5, 17, 26, 31, 36, 37, 38, 45, 74, 92, 97, 102, 104, 111, 116, 117, 140, 149, 158, 163, 168, 170, 177, 208, 213, 214, 215, 221, 224, 227, 232, 233, 234, 240, 246, 253, 259, 265, 272, 278, 284, 290, 297, 303, 307, 308, 309, 314, 316, 322, 360, 361, 362, 363, 364, 368, 369, 370, 371, 379, 384, 386, 387 }

B grade: { 2, 3, 4, 14, 15, 16, 23, 24, 25, 32, 33, 34, 35, 39, 40, 41, 42, 46, 47, 48, 49, 71, 72, 73, 80, 81, 82, 83, 89, 90, 91, 98, 99, 100, 101, 105, 106, 107, 108, 112, 113, 114, 115, 137, 138, 139, 146, 147, 148, 155, 156, 157, 164, 165, 166, 167, 172, 173, 174, 178, 179, 180, 181, 205, 206, 207, 209, 210, 211, 212, 216, 217, 222, 223, 228, 229, 230, 231, 235, 236, 241, 242, 243, 247, 248, 250, 254, 255, 256, 260, 262, 266, 267, 268, 269, 273, 274, 275, 279, 280, 281, 285, 286, 287, 291, 292, 293, 294, 298, 299, 304, 305, 306, 310, 311, 312, 313, 317, 318, 323, 324, 325, 376, 377, 382, 383, 391, 396 }

C grade: { 6, 7, 8, 9, 10, 11, 12, 18, 19, 20, 21, 27, 28, 29, 30, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 75, 76, 77, 78, 84, 85, 86, 87, 93, 94, 95, 96, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 365, 366, 367, 372, 373, 374, 375 }

F grade: { 1, 13, 22, 43, 44, 50, 51, 70, 79, 88, 103, 109, 110, 136, 145, 154, 169, 171, 175, 176, 182, 183, 202, 203, 204, 218, 219, 220, 225, 226, 237, 238, 239, 244, 245, 249, 251, 252, 257, 258, 261, 263, 264, 270, 271, 276, 277, 282, 283, 288, 289, 295, 296, 300, 301, 302, 315, 319, 320, 321, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, }

349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 378, 380, 381, 385, 388, 389, 390, 392, 393, 394, 395, 397 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 31, 36, 37, 38, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 97, 102, 103, 104, 109, 110, 111, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 154, 157, 158, 159, 160, 161, 163, 168, 169, 170, 174, 175, 176, 177, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 208, 213, 214, 215, 219, 220, 221, 225, 226, 227, 232, 233, 234, 238, 239, 240, 244, 245, 246, 251, 252, 253, 256, 257, 258, 259, 262, 263, 264, 265, 270, 271, 272, 276, 277, 278, 282, 283, 284, 288, 289, 290, 293, 294, 295, 296, 297, 301, 302, 303, 306, 307, 308, 309, 314, 315, 316, 320, 321, 322, 326, 327, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 386, 387, 388, 391, 392, 393, 394, 396 }

B grade: { 8, 9, 21, 30, 35, 41, 42, 47, 78, 80, 81, 82, 86, 87, 96, 101, 107, 108, 115, 144, 153, 155, 156, 162, 167, 173, 181, 203, 207, 212, 218, 224, 231, 237, 243, 249, 250, 255, 261, 269, 274, 275, 281, 287, 292, 300, 313, 319, 325, 378, 385, 390, 395, 397 }

C grade: { 32, 33, 34, 39, 40, 46, 98, 99, 100, 105, 106, 112, 113, 114, 164, 165, 166, 171, 172, 178, 179, 180, 202, 205, 206, 209, 210, 211, 216, 217, 222, 223, 228, 229, 230, 235, 236, 241, 242, 247, 248, 254, 260, 266, 267, 268, 273, 279, 280, 285, 286, 291, 298, 299, 304, 305, 310, 311, 312, 317, 318, 323, 324, 376, 377, 382, 383, 384, 389 }

F grade: { 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359 }

2.1.6 Sympy

A grade: { 2, 3, 4, 5, 10, 11, 12, 14, 15, 16, 17, 23, 24, 25, 26, 36, 37, 43, 44, 50, 51, 71, 72, 73, 74, 80, 81, 82, 83, 89, 90, 91, 92, 102, 103, 104, 108, 109, 110, 111, 116, 117, 137, 138, 139, 140, 146, 147, 148, 149, 155, 156, 157, 158, 168, 169, 175, 176, 182, 183, 213, 214, 219, 220, 232, 233, 238, 239, 244, 245, 246, 251, 252, 253, 256, 257, 258, 263, 264, 270, 271, 276, 277, 282, 295, 296, 301, 302, 303, 307, 308, 314, 315, 360, 364 }

B grade: { 53, 54, 55, 56, 362, 363, 396 }

C grade: { }

F grade: { 1, 6, 7, 8, 9, 13, 18, 19, 20, 21, 22, 27, 28, 29, 30, 31, 32, 33, 34, 35, 38, 39, 40, 41, 42, 45, 46, 47, 48, 49, 52, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 75, 76, 77, 78, 79, 84, 85, 86, 87, 88, 93, 94, 95, 96, 97, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 141, 142, 143, 144, 145, 150, 151, 152, 153, 154, 159, 160, 161, 162, 163, 164, 165, 166, 167, 170, 171, 172, 173, 174, 177, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 215, 216, 217, 218, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 234, 235, 236, 237, 240, 241, 242, 243, 247, 248, 249, 250, 254, 255, 259, 260, 261, 262, 265, 266, 267, 268, 269, 272, 273, 274, 275, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 297, 298, 299, 300, 304, 305, 306, 309, 310, 311, 312, 313, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 361, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397 }

2.1.7 Giac

A grade: { 2, 3, 4, 5, 10, 11, 12, 14, 15, 16, 17, 23, 24, 25, 26, 31, 36, 37, 38, 43, 44, 45, 50, 51, 71, 72, 73, 74, 80, 81, 82, 83, 89, 90, 91, 92, 97, 102, 103, 104, 109, 110, 111, 116, 117, 137, 138, 139, 140, 146, 147, 148, 149, 155, 156, 157, 158, 163, 168, 169, 170, 175, 176, 177, 182, 183, 208, 213, 214, 215,

219, 220, 221, 225, 226, 227, 232, 233, 234, 238, 239, 240, 244, 245, 246, 251, 252, 253, 257, 258, 259, 263, 264, 265, 270, 271, 272, 276, 277, 278, 283, 284, 288, 289, 290, 295, 296, 297, 301, 302, 303, 307, 308, 309, 314, 315, 316, 321, 322, 326, 327, 360, 361, 362, 363, 364, 365, 366, 379, 380, 381, 386, 387, 388, 392, 393, 394 }

B grade: { 42, 48, 49, 108, 174, 204, 250, 256, 262, 293, 294, 367, 368, 369, 370, 371, 391, 396 }

C grade: { 6, 8, 9, 18, 27, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 75, 84, 86, 87, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 141, 159, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 372, 374 }

F grade: { 1, 7, 13, 19, 20, 21, 22, 28, 29, 30, 32, 33, 34, 35, 39, 40, 41, 46, 47, 70, 76, 77, 78, 79, 85, 88, 93, 94, 95, 96, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 136, 142, 143, 144, 145, 150, 151, 152, 153, 154, 160, 161, 162, 164, 165, 166, 167, 171, 172, 173, 178, 179, 180, 181, 202, 203, 205, 206, 207, 209, 210, 211, 212, 216, 217, 218, 222, 223, 224, 228, 229, 230, 231, 235, 236, 237, 241, 242, 243, 247, 248, 249, 254, 255, 260, 261, 266, 267, 268, 269, 273, 274, 275, 279, 280, 281, 282, 285, 286, 287, 291, 292, 298, 299, 300, 304, 305, 306, 310, 311, 312, 313, 317, 318, 319, 320, 323, 324, 325, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 373, 375, 376, 377, 378, 382, 383, 384, 385, 389, 390, 395, 397 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	138	0	0	246	0	0
normalized size	1	1.	1.01	0.	0.	1.8	0.	0.
time (sec)	N/A	0.156	0.085	0.209	0.	0.516	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	86	853	791	518	502	244
normalized size	1	1.	0.55	5.47	5.07	3.32	3.22	1.56
time (sec)	N/A	0.107	0.513	0.034	1.221	0.492	7.636	1.093

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	71	466	462	348	342	163
normalized size	1	1.	0.59	3.88	3.85	2.9	2.85	1.36
time (sec)	N/A	0.083	0.3	0.015	1.132	0.483	3.107	1.145

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	50	215	231	203	175	99
normalized size	1	1.	0.56	2.42	2.6	2.28	1.97	1.11
time (sec)	N/A	0.054	0.233	0.015	1.156	0.474	1.374	1.122

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	34	74	88	108	80	51
normalized size	1	1.	0.68	1.48	1.76	2.16	1.6	1.02
time (sec)	N/A	0.026	0.101	0.014	1.121	0.464	0.599	1.142

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	60	84	190	213	0	768
normalized size	1	1.	0.92	1.29	2.92	3.28	0.	11.82
time (sec)	N/A	0.139	0.131	0.026	1.38	0.47	0.	1.208

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	80	124	221	333	0	0
normalized size	1	1.	0.94	1.46	2.6	3.92	0.	0.
time (sec)	N/A	0.149	0.316	0.02	1.451	0.493	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	102	162	269	525	0	7287
normalized size	1	1.	0.89	1.42	2.36	4.61	0.	63.92
time (sec)	N/A	0.175	1.091	0.02	1.741	0.513	0.	1.605

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	164	200	336	703	0	10249
normalized size	1	1.	1.14	1.39	2.33	4.88	0.	71.17
time (sec)	N/A	0.198	0.661	0.021	2.113	0.533	0.	1.809

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	18	31	5	8
normalized size	1	1.	1.	0.88	2.25	3.88	0.62	1.
time (sec)	N/A	0.029	0.006	0.026	1.219	0.452	1.075	1.117

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	20	95	22	26
normalized size	1	1.	1.	0.94	1.25	5.94	1.38	1.62
time (sec)	N/A	0.046	0.006	0.026	1.208	0.459	2.16	1.121

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	20	96	24	35
normalized size	1	1.	1.	0.9	0.69	3.31	0.83	1.21
time (sec)	N/A	0.059	0.008	0.029	1.265	0.469	1.596	1.134

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	237	0	0	471	0	0
normalized size	1	1.	0.86	0.	0.	1.71	0.	0.
time (sec)	N/A	0.33	0.723	0.335	0.	0.554	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	385	835	1188	759	646	473
normalized size	1	1.	1.88	4.07	5.8	3.7	3.15	2.31
time (sec)	N/A	0.2	1.514	0.058	1.358	0.521	11.338	1.129

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	121	447	674	491	391	312
normalized size	1	1.	0.8	2.96	4.46	3.25	2.59	2.07
time (sec)	N/A	0.135	0.992	0.019	1.118	0.502	5.268	1.166

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	93	204	324	296	216	185
normalized size	1	1.	0.9	1.98	3.15	2.87	2.1	1.8
time (sec)	N/A	0.077	0.614	0.016	1.11	0.48	2.62	1.141

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	44	71	115	149	85	93
normalized size	1	1.	0.86	1.39	2.25	2.92	1.67	1.82
time (sec)	N/A	0.033	0.182	0.018	1.132	0.472	1.178	1.102

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	102	166	370	404	0	8180
normalized size	1	1.	0.84	1.37	3.06	3.34	0.	67.6
time (sec)	N/A	0.27	0.335	0.017	1.478	0.483	0.	1.737

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	139	242	408	593	0	0
normalized size	1	1.	0.83	1.44	2.43	3.53	0.	0.
time (sec)	N/A	0.301	1.435	0.023	1.771	0.541	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	183	311	455	914	0	0
normalized size	1	1.	0.83	1.41	2.06	4.14	0.	0.
time (sec)	N/A	0.358	2.238	0.021	2.273	0.575	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	298	384	522	1243	0	0
normalized size	1	1.	1.1	1.42	1.93	4.6	0.	0.
time (sec)	N/A	0.42	1.752	0.024	2.808	0.637	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	246	0	0	486	0	0
normalized size	1	1.	0.91	0.	0.	1.79	0.	0.
time (sec)	N/A	0.331	0.3	0.265	0.	0.551	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	158	1143	1305	936	976	487
normalized size	1	1.	0.61	4.4	5.02	3.6	3.75	1.87
time (sec)	N/A	0.241	1.81	0.063	1.313	0.54	18.541	1.085

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	135	594	741	621	634	325
normalized size	1	1.	0.69	3.03	3.78	3.17	3.23	1.66
time (sec)	N/A	0.165	0.97	0.019	1.213	0.518	10.49	1.133

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	91	260	355	359	350	196
normalized size	1	1.	0.68	1.94	2.65	2.68	2.61	1.46
time (sec)	N/A	0.092	0.524	0.02	1.186	0.486	5.116	1.139

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	75	85	124	192	160	101
normalized size	1	1.	1.04	1.18	1.72	2.67	2.22	1.4
time (sec)	N/A	0.045	0.116	0.021	1.093	0.474	2.371	1.114

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	110	178	370	421	0	8162
normalized size	1	1.	0.85	1.38	2.87	3.26	0.	63.27
time (sec)	N/A	0.232	0.416	0.021	1.437	0.48	0.	1.72

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	151	256	406	621	0	0
normalized size	1	1.	0.84	1.43	2.27	3.47	0.	0.
time (sec)	N/A	0.28	1.296	0.023	1.778	0.554	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	199	329	454	976	0	0
normalized size	1	1.	0.87	1.44	1.98	4.26	0.	0.
time (sec)	N/A	0.344	2.826	0.024	2.171	0.617	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	316	404	521	1293	0	0
normalized size	1	1.	1.1	1.41	1.82	4.51	0.	0.
time (sec)	N/A	0.389	2.406	0.024	3.091	0.685	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	2.539	0.234	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	799	1150	1704	2938	0	0
normalized size	1	1.	5.29	7.62	11.28	19.46	0.	0.
time (sec)	N/A	0.22	5.769	0.329	1.978	0.694	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	560	783	1008	2056	0	0
normalized size	1	1.	4.41	6.17	7.94	16.19	0.	0.
time (sec)	N/A	0.192	2.76	0.296	1.701	0.627	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	356	468	545	1330	0	0
normalized size	1	1.	3.83	5.03	5.86	14.3	0.	0.
time (sec)	N/A	0.166	1.425	0.27	1.526	0.567	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	188	215	255	721	0	0
normalized size	1	1.	2.89	3.31	3.92	11.09	0.	0.
time (sec)	N/A	0.096	5.18	0.248	1.532	0.547	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	3.585	0.244	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	7.065	0.279	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.204	1.242	0.155	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	208	208	308	716	3974	2569	0	0
normalized size	1	1.	1.48	3.44	19.11	12.35	0.	0.
time (sec)	N/A	0.172	1.375	0.28	2.5	0.7	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	311	433	2390	1740	0	0
normalized size	1	1.	2.13	2.97	16.37	11.92	0.	0.
time (sec)	N/A	0.116	1.187	0.256	1.817	0.605	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	234	212	751	1006	0	0
normalized size	1	1.	2.6	2.36	8.34	11.18	0.	0.
time (sec)	N/A	0.062	2.051	0.205	1.634	0.559	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	131	52	350	181	0	1081
normalized size	1	1.	4.37	1.73	11.67	6.03	0.	36.03
time (sec)	N/A	0.02	0.058	0.024	1.092	0.488	0.	1.527

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	16.76	0.295	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.152	20.204	0.411	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.217	3.292	0.162	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	137	137	504	716	6129	2562	0	0
normalized size	1	1.	3.68	5.23	44.74	18.7	0.	0.
time (sec)	N/A	0.256	6.602	0.188	2.477	0.695	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	277	409	1411	1445	0	0
normalized size	1	1.	2.41	3.56	12.27	12.57	0.	0.
time (sec)	N/A	0.174	6.412	0.164	2.279	0.61	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	94	95	1526	231	0	4701
normalized size	1	1.	1.74	1.76	28.26	4.28	0.	87.06
time (sec)	N/A	0.065	0.918	0.029	1.346	0.504	0.	2.577

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	48	61	387	103	0	710
normalized size	1	1.	1.37	1.74	11.06	2.94	0.	20.29
time (sec)	N/A	0.031	0.074	0.029	1.166	0.457	0.	1.209

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.137	11.553	0.445	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.166	11.151	0.595	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	179	234	910	540	0	1327
normalized size	1	1.	0.91	1.19	4.64	2.76	0.	6.77
time (sec)	N/A	0.448	2.37	0.037	1.956	0.551	0.	1.318

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	157	187	869	413	665	732
normalized size	1	1.	0.93	1.11	5.17	2.46	3.96	4.36
time (sec)	N/A	0.295	0.886	0.025	1.888	0.532	130.886	1.23

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	134	142	788	308	389	316
normalized size	1	1.	0.94	1.	5.55	2.17	2.74	2.23
time (sec)	N/A	0.231	0.269	0.023	2.06	0.521	5.903	1.169

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	134	142	788	308	389	316
normalized size	1	1.	0.94	1.	5.55	2.17	2.74	2.23
time (sec)	N/A	0.221	0.026	0.023	1.978	0.52	5.846	1.158

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	157	187	869	413	665	732
normalized size	1	1.	0.93	1.11	5.17	2.46	3.96	4.36
time (sec)	N/A	0.275	0.042	0.026	1.941	0.531	132.766	1.261

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	179	234	910	540	0	1327
normalized size	1	1.	0.91	1.19	4.64	2.76	0.	6.77
time (sec)	N/A	0.334	0.081	0.024	2.034	0.546	0.	1.315

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	406	406	1171	474	1866	918	0	2722
normalized size	1	1.	2.88	1.17	4.6	2.26	0.	6.7
time (sec)	N/A	1.137	15.523	0.04	2.503	0.643	0.	1.589

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	677	386	1787	756	0	1517
normalized size	1	1.	1.92	1.09	5.06	2.14	0.	4.3
time (sec)	N/A	0.684	9.307	0.036	2.536	0.612	0.	1.37

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	280	294	1648	639	0	662
normalized size	1	1.	0.92	0.97	5.42	2.1	0.	2.18
time (sec)	N/A	0.47	6.566	0.036	2.312	0.583	0.	1.244

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	280	294	1648	639	0	662
normalized size	1	1.	0.92	0.97	5.42	2.1	0.	2.18
time (sec)	N/A	0.468	6.489	0.035	2.431	0.586	0.	1.28

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	677	386	1787	756	0	1517
normalized size	1	1.	1.92	1.09	5.06	2.14	0.	4.3
time (sec)	N/A	0.571	9.306	0.04	2.375	0.607	0.	1.374

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	406	406	1171	474	1866	918	0	2722
normalized size	1	1.	2.88	1.17	4.6	2.26	0.	6.7
time (sec)	N/A	0.668	14.663	0.036	2.456	0.638	0.	1.562

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	550	470	1874	1011	0	2681
normalized size	1	1.	1.35	1.15	4.6	2.48	0.	6.59
time (sec)	N/A	1.051	15.219	0.036	2.361	0.693	0.	1.861

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	393	376	1804	802	0	1485
normalized size	1	1.	1.12	1.07	5.14	2.28	0.	4.23
time (sec)	N/A	0.674	3.344	0.039	2.318	0.647	0.	1.558

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	264	286	1655	630	0	643
normalized size	1	1.	0.88	0.96	5.54	2.11	0.	2.15
time (sec)	N/A	0.499	0.874	0.034	2.308	0.607	0.	1.337

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	264	286	1655	630	0	643
normalized size	1	1.	0.88	0.96	5.54	2.11	0.	2.15
time (sec)	N/A	0.46	0.326	0.035	2.447	0.608	0.	1.364

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	393	376	1804	802	0	1485
normalized size	1	1.	1.12	1.07	5.14	2.28	0.	4.23
time (sec)	N/A	0.566	3.129	0.033	2.471	0.646	0.	1.539

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	550	470	1874	1011	0	2681
normalized size	1	1.	1.35	1.15	4.6	2.48	0.	6.59
time (sec)	N/A	0.697	11.788	0.033	2.415	0.687	0.	1.799

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	250	0	0	460	0	0
normalized size	1	1.	0.94	0.	0.	1.72	0.	0.
time (sec)	N/A	0.283	0.476	0.239	0.	0.547	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	150	835	1200	636	646	473
normalized size	1	1.	0.73	4.07	5.85	3.1	3.15	2.31
time (sec)	N/A	0.203	1.623	0.043	1.395	0.526	10.47	1.205

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	127	447	682	404	391	312
normalized size	1	1.	0.84	2.96	4.52	2.68	2.59	2.07
time (sec)	N/A	0.133	0.961	0.018	1.175	0.497	5.385	1.203

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	86	204	328	236	216	185
normalized size	1	1.	0.83	1.98	3.18	2.29	2.1	1.8
time (sec)	N/A	0.079	0.51	0.018	1.208	0.483	2.627	1.153

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	71	71	116	112	85	93
normalized size	1	1.	1.39	1.39	2.27	2.2	1.67	1.82
time (sec)	N/A	0.034	0.153	0.018	1.117	0.474	1.135	1.204

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	100	167	369	404	0	8477
normalized size	1	1.	0.83	1.38	3.05	3.34	0.	70.06
time (sec)	N/A	0.224	0.304	0.02	1.383	0.482	0.	1.771

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	139	240	405	586	0	0
normalized size	1	1.	0.83	1.43	2.41	3.49	0.	0.
time (sec)	N/A	0.266	1.135	0.022	1.725	0.538	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	181	313	452	907	0	0
normalized size	1	1.	0.82	1.42	2.05	4.1	0.	0.
time (sec)	N/A	0.324	2.615	0.02	2.171	0.59	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	300	381	520	1231	0	0
normalized size	1	1.	1.11	1.41	1.93	4.56	0.	0.
time (sec)	N/A	0.377	1.933	0.023	2.787	0.626	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	213	0	0	342	0	0
normalized size	1	1.	1.31	0.	0.	2.11	0.	0.
time (sec)	N/A	0.205	1.104	0.209	0.	0.527	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	132	1915	992	986	1209	302
normalized size	1	1.	1.01	14.62	7.57	7.53	9.23	2.31
time (sec)	N/A	0.164	1.367	0.076	1.371	0.531	17.601	1.16

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	106	1074	597	647	813	207
normalized size	1	1.	1.01	10.23	5.69	6.16	7.74	1.97
time (sec)	N/A	0.13	0.685	0.022	1.304	0.514	9.976	1.14

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	77	519	313	398	484	127
normalized size	1	1.	0.97	6.57	3.96	5.04	6.13	1.61
time (sec)	N/A	0.123	0.438	0.02	1.228	0.501	5.12	1.084

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	54	194	130	204	231	65
normalized size	1	1.	1.02	3.66	2.45	3.85	4.36	1.23
time (sec)	N/A	0.054	0.298	0.02	1.256	0.482	2.355	1.087

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	65	105	216	239	0	903
normalized size	1	1.	0.83	1.35	2.77	3.06	0.	11.58
time (sec)	N/A	0.14	0.165	0.026	1.456	0.487	0.	1.197

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	81	156	231	346	0	0
normalized size	1	1.	0.78	1.5	2.22	3.33	0.	0.
time (sec)	N/A	0.169	0.463	0.025	1.469	0.55	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	105	193	278	582	0	7560
normalized size	1	1.	0.83	1.52	2.19	4.58	0.	59.53
time (sec)	N/A	0.198	0.883	0.024	1.834	0.564	0.	1.641

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	123	230	346	880	0	11486
normalized size	1	1.	0.78	1.46	2.19	5.57	0.	72.7
time (sec)	N/A	0.228	1.754	0.026	2.338	0.601	0.	1.87

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	376	0	0	709	0	0
normalized size	1	1.	0.92	0.	0.	1.74	0.	0.
time (sec)	N/A	0.403	0.67	0.334	0.	0.57	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	238	1812	1808	1121	1098	717
normalized size	1	1.	0.72	5.49	5.48	3.4	3.33	2.17
time (sec)	N/A	0.391	3.416	0.087	1.428	0.611	79.215	1.131

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	369	992	1034	706	690	474
normalized size	1	1.	1.42	3.83	3.99	2.73	2.66	1.83
time (sec)	N/A	0.279	1.599	0.024	1.385	0.538	23.666	1.112

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	127	466	506	406	382	282
normalized size	1	1.	0.69	2.53	2.75	2.21	2.08	1.53
time (sec)	N/A	0.197	0.923	0.026	1.251	0.506	20.576	1.129

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	94	163	188	194	163	143
normalized size	1	1.	0.86	1.5	1.72	1.78	1.5	1.31
time (sec)	N/A	0.097	0.324	0.023	1.235	0.49	9.738	1.204

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	154	253	549	612	0	0
normalized size	1	1.	0.83	1.37	2.97	3.31	0.	0.
time (sec)	N/A	0.339	0.523	0.024	1.727	0.509	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	213	365	591	883	0	0
normalized size	1	1.	0.83	1.42	2.3	3.44	0.	0.
time (sec)	N/A	0.416	1.562	0.027	2.122	0.64	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	338	279	475	639	1358	0	0
normalized size	1	1.	0.83	1.41	1.89	4.02	0.	0.
time (sec)	N/A	0.505	4.217	0.027	3.01	0.76	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	413	413	457	580	706	1841	0	0
normalized size	1	1.	1.11	1.4	1.71	4.46	0.	0.
time (sec)	N/A	0.59	3.174	0.028	4.008	0.815	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	143	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	6.433	0.297	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	837	1295	2076	3281	0	0
normalized size	1	1.	2.51	3.89	6.23	9.85	0.	0.
time (sec)	N/A	0.284	1.371	0.354	2.329	0.849	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	254	254	330	847	1241	2280	0	0
normalized size	1	1.	1.3	3.33	4.89	8.98	0.	0.
time (sec)	N/A	0.198	0.983	0.218	1.991	0.742	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	221	479	684	1462	0	0
normalized size	1	1.	1.29	2.8	4.	8.55	0.	0.
time (sec)	N/A	0.138	0.874	0.184	1.47	0.638	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	176	203	269	787	0	0
normalized size	1	1.	1.87	2.16	2.86	8.37	0.	0.
time (sec)	N/A	0.062	0.175	0.164	1.654	0.607	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	69	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	8.493	0.302	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	4.125	0.63	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	1.173	0.16	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	155	155	795	913	4359	2034	0	0
normalized size	1	1.	5.13	5.89	28.12	13.12	0.	0.
time (sec)	N/A	0.229	6.727	0.213	3.246	0.619	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	374	573	2626	1439	0	0
normalized size	1	1.	2.94	4.51	20.68	11.33	0.	0.
time (sec)	N/A	0.197	6.445	0.194	2.369	0.565	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	198	297	872	950	0	0
normalized size	1	1.	2.04	3.06	8.99	9.79	0.	0.
time (sec)	N/A	0.13	6.318	0.152	2.282	0.543	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	82	49	394	242	100	1856
normalized size	1	1.	2.	1.2	9.61	5.9	2.44	45.27
time (sec)	N/A	0.026	0.451	0.041	1.683	0.49	1.291	1.607

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	4.647	0.332	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	2.418	0.447	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	10.297	0.18	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	966	1673	9385	6342	0	0
normalized size	1	1.	2.32	4.02	22.56	15.25	0.	0.
time (sec)	N/A	0.502	8.439	0.381	18.885	1.049	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	528	1056	5227	4091	0	0
normalized size	1	1.	1.71	3.43	16.97	13.28	0.	0.
time (sec)	N/A	0.344	5.051	0.342	6.897	0.809	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	471	546	2607	2379	0	0
normalized size	1	1.	2.63	3.05	14.56	13.29	0.	0.
time (sec)	N/A	0.223	7.595	0.281	2.404	0.688	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	260	246	1037	1191	0	0
normalized size	1	1.	2.41	2.28	9.6	11.03	0.	0.
time (sec)	N/A	0.107	1.836	0.139	1.909	0.576	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	36.712	2.23	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	42.607	3.468	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	406	406	1168	476	1874	856	0	2724
normalized size	1	1.	2.88	1.17	4.62	2.11	0.	6.71
time (sec)	N/A	0.667	16.39	0.033	2.533	0.653	0.	1.521

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	676	384	1793	716	0	1513
normalized size	1	1.	1.92	1.09	5.08	2.03	0.	4.29
time (sec)	N/A	0.526	9.261	0.031	2.521	0.624	0.	1.354

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	278	296	1656	613	0	659
normalized size	1	1.	0.91	0.97	5.45	2.02	0.	2.17
time (sec)	N/A	0.416	6.64	0.03	2.401	0.591	0.	1.227

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	278	296	1656	613	0	659
normalized size	1	1.	0.91	0.97	5.45	2.02	0.	2.17
time (sec)	N/A	0.419	6.643	0.03	2.215	0.592	0.	1.257

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	676	384	1793	716	0	1513
normalized size	1	1.	1.92	1.09	5.08	2.03	0.	4.29
time (sec)	N/A	0.528	9.215	0.029	2.497	0.611	0.	1.39

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	406	406	1168	476	1874	856	0	2724
normalized size	1	1.	2.88	1.17	4.62	2.11	0.	6.71
time (sec)	N/A	0.627	16.242	0.029	2.42	0.672	0.	1.532

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	206	251	922	840	0	1467
normalized size	1	1.	0.9	1.1	4.04	3.68	0.	6.43
time (sec)	N/A	0.398	3.657	0.037	2.088	0.652	0.	1.686

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	187	206	886	621	0	805
normalized size	1	1.	0.94	1.03	4.43	3.1	0.	4.02
time (sec)	N/A	0.329	3.155	0.036	1.947	0.597	0.	1.458

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	161	159	817	447	0	347
normalized size	1	1.	0.93	0.91	4.7	2.57	0.	1.99
time (sec)	N/A	0.27	0.885	0.036	1.936	0.569	0.	1.29

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	161	159	817	447	0	347
normalized size	1	1.	0.93	0.91	4.7	2.57	0.	1.99
time (sec)	N/A	0.249	0.137	0.036	2.013	0.572	0.	1.3

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	187	206	886	621	0	805
normalized size	1	1.	0.94	1.03	4.43	3.1	0.	4.02
time (sec)	N/A	0.318	1.694	0.036	2.16	0.606	0.	1.439

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	206	251	922	840	0	1467
normalized size	1	1.	0.9	1.1	4.04	3.68	0.	6.43
time (sec)	N/A	0.378	2.734	0.034	2.138	0.649	0.	1.639

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	615	615	3348	719	2943	1337	0	4084
normalized size	1	1.	5.44	1.17	4.79	2.17	0.	6.64
time (sec)	N/A	1.153	25.825	0.043	3.005	0.843	0.	2.235

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	534	534	1041	580	2790	1118	0	2269
normalized size	1	1.	1.95	1.09	5.22	2.09	0.	4.25
time (sec)	N/A	0.879	12.284	0.042	2.901	0.78	0.	1.807

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	459	432	447	2531	952	0	988
normalized size	1	1.	0.94	0.97	5.51	2.07	0.	2.15
time (sec)	N/A	0.671	7.401	0.041	2.849	0.712	0.	1.425

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	459	432	447	2531	952	0	988
normalized size	1	1.	0.94	0.97	5.51	2.07	0.	2.15
time (sec)	N/A	0.658	7.346	0.042	2.759	0.721	0.	1.455

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	534	534	1041	580	2790	1118	0	2269
normalized size	1	1.	1.95	1.09	5.22	2.09	0.	4.25
time (sec)	N/A	0.801	12.183	0.038	2.936	0.767	0.	1.815

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	615	615	3348	719	2943	1337	0	4084
normalized size	1	1.	5.44	1.17	4.79	2.17	0.	6.64
time (sec)	N/A	0.953	25.314	0.043	2.95	0.832	0.	2.218

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	245	0	0	487	0	0
normalized size	1	1.	0.9	0.	0.	1.78	0.	0.
time (sec)	N/A	0.288	0.248	0.266	0.	0.544	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	158	1150	1305	802	976	487
normalized size	1	1.	0.61	4.42	5.02	3.08	3.75	1.87
time (sec)	N/A	0.234	1.883	0.049	1.394	0.536	21.918	1.11

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	135	594	741	518	634	325
normalized size	1	1.	0.69	3.03	3.78	2.64	3.23	1.66
time (sec)	N/A	0.161	1.012	0.021	1.268	0.506	11.69	1.145

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	89	260	355	294	350	196
normalized size	1	1.	0.66	1.94	2.65	2.19	2.61	1.46
time (sec)	N/A	0.087	0.486	0.019	1.155	0.493	13.536	1.105

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	75	85	124	147	160	101
normalized size	1	1.	1.04	1.18	1.72	2.04	2.22	1.4
time (sec)	N/A	0.047	0.152	0.02	1.112	0.488	2.812	1.115

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	110	178	370	421	0	8162
normalized size	1	1.	0.85	1.38	2.87	3.26	0.	63.27
time (sec)	N/A	0.212	0.344	0.021	1.487	0.481	0.	1.755

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	151	256	406	597	0	0
normalized size	1	1.	0.84	1.43	2.27	3.34	0.	0.
time (sec)	N/A	0.268	1.673	0.023	1.741	0.558	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	197	329	454	918	0	0
normalized size	1	1.	0.85	1.42	1.97	3.97	0.	0.
time (sec)	N/A	0.329	3.792	0.023	2.254	0.625	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	316	404	521	1256	0	0
normalized size	1	1.	1.1	1.41	1.82	4.38	0.	0.
time (sec)	N/A	0.451	2.55	0.024	2.91	0.683	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	419	419	409	0	0	726	0	0
normalized size	1	1.	0.98	0.	0.	1.73	0.	0.
time (sec)	N/A	0.435	0.606	0.331	0.	0.577	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	563	1842	1808	1245	1098	717
normalized size	1	1.	1.71	5.58	5.48	3.77	3.33	2.17
time (sec)	N/A	0.368	3.757	0.052	1.489	0.568	48.01	1.147

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	195	1016	1034	798	690	474
normalized size	1	1.	0.75	3.92	3.99	3.08	2.66	1.83
time (sec)	N/A	0.273	2.217	0.021	1.268	0.54	53.576	1.118

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	252	484	506	470	382	282
normalized size	1	1.	1.37	2.63	2.75	2.55	2.08	1.53
time (sec)	N/A	0.19	1.011	0.02	1.177	0.5	11.542	1.114

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	110	175	188	235	163	143
normalized size	1	1.	1.01	1.61	1.72	2.16	1.5	1.31
time (sec)	N/A	0.094	0.365	0.023	1.053	0.488	5.467	1.116

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	154	252	551	612	0	0
normalized size	1	1.	0.83	1.36	2.98	3.31	0.	0.
time (sec)	N/A	0.28	0.521	0.023	1.627	0.497	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	212	367	593	867	0	0
normalized size	1	1.	0.82	1.43	2.31	3.37	0.	0.
time (sec)	N/A	0.345	2.139	0.026	1.968	0.632	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	338	283	473	640	1312	0	0
normalized size	1	1.	0.84	1.4	1.89	3.88	0.	0.
time (sec)	N/A	0.438	3.296	0.026	2.603	0.724	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	413	413	451	583	707	1814	0	0
normalized size	1	1.	1.09	1.41	1.71	4.39	0.	0.
time (sec)	N/A	0.538	3.48	0.024	3.975	0.845	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	255	0	0	487	0	0
normalized size	1	1.	0.89	0.	0.	1.71	0.	0.
time (sec)	N/A	0.317	3.412	0.264	0.	0.557	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	153	2061	1395	1153	1334	485
normalized size	1	1.	0.66	8.85	5.99	4.95	5.73	2.08
time (sec)	N/A	0.266	1.553	0.074	1.448	0.562	46.242	1.131

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	132	1100	813	747	867	325
normalized size	1	1.	0.73	6.08	4.49	4.13	4.79	1.8
time (sec)	N/A	0.219	2.39	0.027	1.334	0.533	60.503	1.132

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	91	498	409	447	471	196
normalized size	1	1.	0.71	3.86	3.17	3.47	3.65	1.52
time (sec)	N/A	0.144	0.545	0.024	1.238	0.504	22.629	1.145

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	63	176	161	221	201	101
normalized size	1	1.	0.82	2.29	2.09	2.87	2.61	1.31
time (sec)	N/A	0.074	0.223	0.022	1.18	0.486	11.178	1.122

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	110	178	370	421	0	8162
normalized size	1	1.	0.85	1.38	2.87	3.26	0.	63.27
time (sec)	N/A	0.246	0.309	0.024	1.535	0.491	0.	1.783

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	189	256	406	632	0	0
normalized size	1	1.	1.06	1.43	2.27	3.53	0.	0.
time (sec)	N/A	0.297	0.981	0.028	1.761	0.597	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	239	329	454	1010	0	0
normalized size	1	1.	1.02	1.4	1.93	4.3	0.	0.
time (sec)	N/A	0.353	1.091	0.028	2.301	0.706	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	554	404	521	1419	0	0
normalized size	1	1.	1.93	1.41	1.82	4.94	0.	0.
time (sec)	N/A	0.419	5.131	0.028	3.221	0.762	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	151	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.172	7.752	0.192	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	307	307	2828	1492	2207	3460	0	0
normalized size	1	1.	9.21	4.86	7.19	11.27	0.	0.
time (sec)	N/A	0.34	6.522	0.49	2.635	0.922	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	246	246	1918	1001	1305	2437	0	0
normalized size	1	1.	7.8	4.07	5.3	9.91	0.	0.
time (sec)	N/A	0.278	6.402	0.438	2.02	0.825	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	181	181	564	590	705	1538	0	0
normalized size	1	1.	3.12	3.26	3.9	8.5	0.	0.
time (sec)	N/A	0.227	2.988	0.47	1.678	0.689	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	131	271	300	844	0	0
normalized size	1	1.	1.15	2.38	2.63	7.4	0.	0.
time (sec)	N/A	0.128	0.353	0.383	1.536	0.602	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.144	0.794	0.355	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.17	2.499	0.63	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	153	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.239	3.96	0.164	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	798	1056	0	3009	0	0
normalized size	1	1.	2.67	3.53	0.	10.06	0.	0.
time (sec)	N/A	0.293	1.722	0.209	0.	0.887	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	539	649	14876	2018	0	0
normalized size	1	1.	2.5	3.	68.87	9.34	0.	0.
time (sec)	N/A	0.222	1.464	0.189	6.757	0.726	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	310	332	4435	1172	0	0
normalized size	1	1.	2.23	2.39	31.91	8.43	0.	0.
time (sec)	N/A	0.149	3.956	0.16	5.39	0.611	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	104	124	2849	269	0	2655
normalized size	1	1.	1.79	2.14	49.12	4.64	0.	45.78
time (sec)	N/A	0.063	0.707	0.119	1.184	0.521	0.	2.221

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	74	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.214	3.705	0.346	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	92	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.256	4.008	0.492	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	5.536	0.178	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	302	302	1534	1868	9600	3915	0	0
normalized size	1	1.	5.08	6.19	31.79	12.96	0.	0.
time (sec)	N/A	0.462	7.107	0.403	19.409	0.707	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	256	256	994	1194	5335	2637	0	0
normalized size	1	1.	3.88	4.66	20.84	10.3	0.	0.
time (sec)	N/A	0.368	6.881	0.363	6.142	0.621	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	168	168	540	635	2654	1581	0	0
normalized size	1	1.	3.21	3.78	15.8	9.41	0.	0.
time (sec)	N/A	0.266	6.672	0.313	2.509	0.564	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	240	281	1133	859	0	0
normalized size	1	1.	2.2	2.58	10.39	7.88	0.	0.
time (sec)	N/A	0.128	6.144	0.161	1.696	0.537	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	8.051	2.391	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	9.165	3.544	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	550	470	1879	944	0	2680
normalized size	1	1.	1.35	1.15	4.62	2.32	0.	6.58
time (sec)	N/A	0.797	13.652	0.035	2.311	0.698	0.	1.775

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	393	376	1806	749	0	1485
normalized size	1	1.	1.12	1.07	5.15	2.13	0.	4.23
time (sec)	N/A	0.556	3.297	0.035	2.256	0.675	0.	1.533

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	264	286	1661	599	0	643
normalized size	1	1.	0.88	0.96	5.56	2.	0.	2.15
time (sec)	N/A	0.447	0.835	0.032	2.184	0.61	0.	1.338

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	264	286	1661	599	0	643
normalized size	1	1.	0.88	0.96	5.56	2.	0.	2.15
time (sec)	N/A	0.455	0.391	0.033	2.36	0.614	0.	1.324

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	393	376	1806	749	0	1485
normalized size	1	1.	1.12	1.07	5.15	2.13	0.	4.23
time (sec)	N/A	0.572	1.619	0.033	2.276	0.64	0.	1.537

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	550	470	1879	944	0	2680
normalized size	1	1.	1.35	1.15	4.62	2.32	0.	6.58
time (sec)	N/A	0.672	10.917	0.034	2.242	0.694	0.	1.741

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	615	615	1795	716	2940	1397	0	4077
normalized size	1	1.	2.92	1.16	4.78	2.27	0.	6.63
time (sec)	N/A	1.145	23.492	0.051	2.648	0.839	0.	2.167

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	534	534	1043	583	2790	1162	0	2271
normalized size	1	1.	1.95	1.09	5.22	2.18	0.	4.25
time (sec)	N/A	0.838	12.585	0.051	2.617	0.766	0.	1.78

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	459	435	444	2531	973	0	988
normalized size	1	1.	0.95	0.97	5.51	2.12	0.	2.15
time (sec)	N/A	0.693	7.043	0.049	2.535	0.716	0.	1.442

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	459	435	444	2531	973	0	988
normalized size	1	1.	0.95	0.97	5.51	2.12	0.	2.15
time (sec)	N/A	0.672	7.044	0.05	2.515	0.719	0.	1.421

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	534	534	1043	583	2790	1162	0	2271
normalized size	1	1.	1.95	1.09	5.22	2.18	0.	4.25
time (sec)	N/A	0.858	12.406	0.052	2.625	0.769	0.	1.812

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	615	615	1795	716	2940	1397	0	4077
normalized size	1	1.	2.92	1.16	4.78	2.27	0.	6.63
time (sec)	N/A	1.015	23.409	0.051	2.696	0.83	0.	2.129

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	550	477	1916	1089	0	2677
normalized size	1	1.	1.35	1.17	4.71	2.68	0.	6.58
time (sec)	N/A	0.897	5.347	0.043	2.248	0.767	0.	2.7

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	391	383	1829	821	0	1481
normalized size	1	1.	1.11	1.09	5.21	2.34	0.	4.22
time (sec)	N/A	0.628	3.086	0.041	2.221	0.706	0.	2.076

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	264	293	1681	632	0	643
normalized size	1	1.	0.88	0.98	5.62	2.11	0.	2.15
time (sec)	N/A	0.458	1.385	0.041	2.194	0.646	0.	1.555

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	264	293	1681	632	0	643
normalized size	1	1.	0.88	0.98	5.62	2.11	0.	2.15
time (sec)	N/A	0.448	0.613	0.037	2.169	0.652	0.	1.561

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	391	383	1829	821	0	1481
normalized size	1	1.	1.11	1.09	5.21	2.34	0.	4.22
time (sec)	N/A	0.559	0.216	0.039	2.218	0.72	0.	2.078

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	550	477	1916	1089	0	2677
normalized size	1	1.	1.35	1.17	4.71	2.68	0.	6.58
time (sec)	N/A	0.669	3.002	0.04	2.202	0.765	0.	2.743

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	104	150	0	851	0	0
normalized size	1	1.	0.93	1.34	0.	7.6	0.	0.
time (sec)	N/A	0.185	0.176	0.106	0.	0.595	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	72	112	0	551	0	0
normalized size	1	1.	0.87	1.35	0.	6.64	0.	0.
time (sec)	N/A	0.17	0.1	0.106	0.	0.558	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	76	0	138	0	278
normalized size	1	1.	1.	2.3	0.	4.18	0.	8.42
time (sec)	N/A	0.055	0.025	0.093	0.	0.505	0.	1.18

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	159	240	5022	1566	0	0
normalized size	1	1.	0.88	1.33	27.9	8.7	0.	0.
time (sec)	N/A	0.401	0.416	0.157	3.474	0.644	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	108	170	3856	1204	0	0
normalized size	1	1.	1.02	1.6	36.38	11.36	0.	0.
time (sec)	N/A	0.278	0.324	0.224	2.631	0.613	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	62	109	2349	640	0	0
normalized size	1	1.	0.85	1.49	32.18	8.77	0.	0.
time (sec)	N/A	0.164	0.108	0.126	1.881	0.582	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	2.517	0.2	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	157	616	1069	3355	0	0
normalized size	1	1.	0.99	3.9	6.77	21.23	0.	0.
time (sec)	N/A	0.21	0.069	0.315	2.058	0.742	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	126	423	662	2367	0	0
normalized size	1	1.	0.95	3.2	5.02	17.93	0.	0.
time (sec)	N/A	0.183	0.087	0.289	1.857	0.664	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	100	257	378	1534	0	0
normalized size	1	1.	1.04	2.68	3.94	15.98	0.	0.
time (sec)	N/A	0.153	0.041	0.277	1.851	0.597	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	70	123	154	837	0	0
normalized size	1	1.	1.06	1.86	2.33	12.68	0.	0.
time (sec)	N/A	0.093	0.015	0.252	1.726	0.572	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	3.57	0.222	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	5.364	0.235	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	147	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.136	6.586	0.323	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	557	901	1247	2591	0	0
normalized size	1	1.	2.03	3.28	4.53	9.42	0.	0.
time (sec)	N/A	0.215	1.546	0.418	2.243	0.776	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	315	512	689	1666	0	0
normalized size	1	1.	1.69	2.75	3.7	8.96	0.	0.
time (sec)	N/A	0.146	0.883	0.361	1.989	0.677	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	213	221	0	905	0	0
normalized size	1	1.	2.07	2.15	0.	8.79	0.	0.
time (sec)	N/A	0.068	0.424	0.227	0.	0.595	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	68	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	6.078	0.34	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	86	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.139	7.24	0.607	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	151	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.171	7.772	0.274	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	251	251	1720	641	925	2753	0	0
normalized size	1	1.	6.85	2.55	3.69	10.97	0.	0.
time (sec)	N/A	0.298	7.147	0.285	2.097	0.843	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	184	184	518	379	512	1746	0	0
normalized size	1	1.	2.82	2.06	2.78	9.49	0.	0.
time (sec)	N/A	0.228	6.459	0.353	1.835	0.706	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	134	179	196	965	0	0
normalized size	1	1.	1.17	1.56	1.7	8.39	0.	0.
time (sec)	N/A	0.128	0.304	0.355	1.77	0.614	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.151	0.779	0.488	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.176	2.503	0.812	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.183	5.942	0.141	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	578	1242	2402	6280	0	0
normalized size	1	1.	2.34	5.03	9.72	25.43	0.	0.
time (sec)	N/A	0.229	1.379	0.329	2.506	1.04	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	350	816	1435	4410	0	0
normalized size	1	1.	1.78	4.14	7.28	22.39	0.	0.
time (sec)	N/A	0.166	1.016	0.313	2.201	0.865	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	213	469	797	2850	0	0
normalized size	1	1.	1.68	3.69	6.28	22.44	0.	0.
time (sec)	N/A	0.117	0.628	0.297	1.975	0.738	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	141	208	360	1544	0	0
normalized size	1	1.	1.99	2.93	5.07	21.75	0.	0.
time (sec)	N/A	0.056	0.119	0.234	1.95	0.631	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	4.53	0.2	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	5.883	0.197	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.197	9.154	0.163	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	350	739	1158	4374	4446	0	0
normalized size	1	1.	2.11	3.31	12.5	12.7	0.	0.
time (sec)	N/A	0.64	6.417	0.679	4.389	0.951	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	226	226	593	556	2202	2844	0	0
normalized size	1	1.	2.62	2.46	9.74	12.58	0.	0.
time (sec)	N/A	0.384	6.291	0.449	2.355	0.764	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	517	235	0	1215	0	0
normalized size	1	1.	3.95	1.79	0.	9.27	0.	0.
time (sec)	N/A	0.135	2.947	0.299	0.	0.607	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.16	11.7	2.356	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.19	11.908	2.649	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.237	11.017	0.178	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	325	325	1477	1223	6939	8092	0	0
normalized size	1	1.	4.54	3.76	21.35	24.9	0.	0.
time (sec)	N/A	0.823	6.993	0.421	7.781	1.261	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	872	632	3405	4868	0	0
normalized size	1	1.	4.34	3.14	16.94	24.22	0.	0.
time (sec)	N/A	0.443	6.757	0.369	2.945	0.915	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	210	270	1397	2457	0	0
normalized size	1	1.	1.49	1.91	9.91	17.43	0.	0.
time (sec)	N/A	0.139	0.958	0.173	2.219	0.722	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.147	14.386	3.076	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.192	16.889	4.781	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.142	0.93	0.162	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	428	767	3974	2930	0	0
normalized size	1	1.	1.89	3.38	17.51	12.91	0.	0.
time (sec)	N/A	0.186	1.24	0.315	3.043	0.785	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	256	463	2395	1993	0	0
normalized size	1	1.	1.61	2.91	15.06	12.53	0.	0.
time (sec)	N/A	0.126	0.855	0.274	2.261	0.669	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	174	227	0	1166	0	0
normalized size	1	1.	1.79	2.34	0.	12.02	0.	0.
time (sec)	N/A	0.068	1.713	0.135	0.	0.596	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	93	49	350	163	0	2075
normalized size	1	1.	3.21	1.69	12.07	5.62	0.	71.55
time (sec)	N/A	0.019	0.047	0.022	1.5	0.493	0.	1.685

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	11.262	0.268	0.	0.	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	19.338	0.32	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	3.14	0.166	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	128	128	424	348	1840	913	0	0
normalized size	1	1.	3.31	2.72	14.38	7.13	0.	0.
time (sec)	N/A	0.21	6.564	0.197	1.93	0.503	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	276	191	563	522	0	0
normalized size	1	1.	2.88	1.99	5.86	5.44	0.	0.
time (sec)	N/A	0.141	6.381	0.162	1.845	0.49	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	76	47	320	131	65	301
normalized size	1	1.	1.9	1.18	8.	3.28	1.62	7.52
time (sec)	N/A	0.028	0.265	0.044	1.467	0.472	0.373	1.403

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	3.957	0.264	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	5.266	0.324	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	149	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.157	5.097	0.17	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	532	677	14923	2237	0	0
normalized size	1	1.	2.33	2.97	65.45	9.81	0.	0.
time (sec)	N/A	0.211	1.61	0.296	6.936	0.745	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	362	345	0	1316	0	0
normalized size	1	1.	2.5	2.38	0.	9.08	0.	0.
time (sec)	N/A	0.131	3.118	0.158	0.	0.637	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	107	123	2866	251	0	3729
normalized size	1	1.	1.91	2.2	51.18	4.48	0.	66.59
time (sec)	N/A	0.054	0.316	0.129	1.763	0.512	0.	2.529

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.15	3.818	0.299	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.181	4.183	0.378	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.248	8.947	0.16	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	469	469	694	1866	7688	6263	0	0
normalized size	1	1.	1.48	3.98	16.39	13.35	0.	0.
time (sec)	N/A	0.795	3.63	0.758	9.458	1.358	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	343	343	473	1152	4327	4352	0	0
normalized size	1	1.	1.38	3.36	12.62	12.69	0.	0.
time (sec)	N/A	0.573	1.439	0.651	4.02	1.005	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	219	219	317	568	2157	2773	0	0
normalized size	1	1.	1.45	2.59	9.85	12.66	0.	0.
time (sec)	N/A	0.377	2.588	0.448	2.265	0.776	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	122	212	160	1085	1046	0	0
normalized size	1	1.08	1.88	1.42	9.6	9.26	0.	0.
time (sec)	N/A	0.13	0.504	0.29	2.046	0.581	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.153	9.847	2.332	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.18	10.095	2.663	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.19	2.785	0.173	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	285	687	3179	4263	0	0
normalized size	1	1.	2.42	5.82	26.94	36.13	0.	0.
time (sec)	N/A	0.28	2.231	0.336	2.355	0.898	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	277	351	1049	2573	0	0
normalized size	1	1.	3.15	3.99	11.92	29.24	0.	0.
time (sec)	N/A	0.191	1.759	0.332	2.129	0.746	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	182	416	197	0	0
normalized size	1	1.	0.91	5.2	11.89	5.63	0.	0.
time (sec)	N/A	0.059	0.204	0.074	1.501	0.5	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	7.335	0.518	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	7.586	0.811	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.215	11.762	0.21	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	601	601	907	1613	10858	7588	0	0
normalized size	1	1.	1.51	2.68	18.07	12.63	0.	0.
time (sec)	N/A	2.313	8.316	0.727	18.615	1.347	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	305	305	889	802	5157	4454	0	0
normalized size	1	1.	2.91	2.63	16.91	14.6	0.	0.
time (sec)	N/A	0.865	7.923	0.508	5.042	0.958	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	174	520	267	2029	1652	0	0
normalized size	1	1.13	3.38	1.73	13.18	10.73	0.	0.
time (sec)	N/A	0.191	5.329	0.352	2.806	0.634	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.201	23.06	2.755	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.224	31.86	4.368	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.924	11.334	0.102	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	387	387	672	0	5385	4625	0	0
normalized size	1	1.	1.74	0.	13.91	11.95	0.	0.
time (sec)	N/A	0.96	7.193	1.401	3.74	0.911	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	613	429	2996	3302	0	0
normalized size	1	1.	2.61	1.83	12.75	14.05	0.	0.
time (sec)	N/A	0.537	6.663	0.402	2.521	0.77	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	282	182	1598	1459	0	0
normalized size	1	1.	2.24	1.44	12.68	11.58	0.	0.
time (sec)	N/A	0.168	2.563	0.299	2.344	0.608	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.5	49.516	0.784	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.513	20.98	0.884	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.152	2.416	0.17	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	418	489	4641	2211	0	0
normalized size	1	1.	3.01	3.52	33.39	15.91	0.	0.
time (sec)	N/A	0.258	6.585	0.193	2.318	0.777	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	286	301	900	1374	0	0
normalized size	1	1.	2.49	2.62	7.83	11.95	0.	0.
time (sec)	N/A	0.174	6.395	0.158	2.241	0.691	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	66	95	1334	208	0	6040
normalized size	1	1.	1.2	1.73	24.25	3.78	0.	109.82
time (sec)	N/A	0.062	0.524	0.029	1.575	0.508	0.	2.833

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	48	61	382	95	0	771
normalized size	1	1.	1.37	1.74	10.91	2.71	0.	22.03
time (sec)	N/A	0.032	0.062	0.027	0.977	0.465	0.	1.237

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	7.371	0.367	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	10.436	0.382	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	8.989	0.174	0.	0.	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	530	1127	5168	3214	0	0
normalized size	1	1.	1.57	3.34	15.34	9.54	0.	0.
time (sec)	N/A	0.408	3.635	0.355	6.806	0.945	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	526	584	2556	2033	0	0
normalized size	1	1.	2.73	3.03	13.24	10.53	0.	0.
time (sec)	N/A	0.271	7.225	0.31	2.898	0.747	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	117	117	555	267	0	1169	0	0
normalized size	1	1.	4.74	2.28	0.	9.99	0.	0.
time (sec)	N/A	0.129	6.522	0.162	0.	0.624	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	26.7	1.455	0.	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	29.522	2.21	0.	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	5.543	0.197	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	259	259	803	720	3247	1443	0	0
normalized size	1	1.	3.1	2.78	12.54	5.57	0.	0.
time (sec)	N/A	0.356	6.923	0.368	4.481	0.537	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	169	169	454	400	1655	883	0	0
normalized size	1	1.	2.69	2.37	9.79	5.22	0.	0.
time (sec)	N/A	0.222	6.64	0.321	2.442	0.518	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	240	183	701	447	0	0
normalized size	1	1.	2.22	1.69	6.49	4.14	0.	0.
time (sec)	N/A	0.117	6.154	0.159	2.042	0.493	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	6.457	1.853	0.	0.	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	7.034	2.576	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.269	9.561	0.185	0.	0.	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	399	399	2090	1729	11840	8008	0	0
normalized size	1	1.	5.24	4.33	29.67	20.07	0.	0.
time (sec)	N/A	0.971	7.508	0.483	23.457	1.968	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	325	325	1486	1115	6639	5605	0	0
normalized size	1	1.	4.57	3.43	20.43	17.25	0.	0.
time (sec)	N/A	0.64	6.926	0.4	7.39	1.418	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	875	614	3297	3629	0	0
normalized size	1	1.	4.35	3.05	16.4	18.05	0.	0.
time (sec)	N/A	0.414	6.742	0.339	2.849	0.963	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	141	212	270	1397	2072	0	0
normalized size	1	1.01	1.53	1.94	10.05	14.91	0.	0.
time (sec)	N/A	0.135	0.565	0.158	2.224	0.752	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.189	9.068	3.113	0.	0.	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.209	6.709	4.859	0.	0.	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.233	11.962	0.215	0.	0.	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	486	486	819	1629	10843	5646	0	0
normalized size	1	1.	1.69	3.35	22.31	11.62	0.	0.
time (sec)	N/A	1.206	7.913	0.738	20.687	1.453	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	341	341	889	770	5154	3596	0	0
normalized size	1	1.	2.61	2.26	15.11	10.55	0.	0.
time (sec)	N/A	0.648	7.562	0.499	5.443	0.973	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	162	182	669	344	0	1644	0	0
normalized size	1	1.12	4.13	2.12	0.	10.15	0.	0.
time (sec)	N/A	0.196	6.577	0.355	0.	0.696	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.187	21.19	2.754	0.	0.	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.197	26.433	4.312	0.	0.	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.265	10.558	0.187	0.	0.	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	582	1329	7574	9802	0	0
normalized size	1	1.	1.83	4.18	23.82	30.82	0.	0.
time (sec)	N/A	0.321	9.182	0.295	11.693	1.741	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	381	716	3676	5801	0	0
normalized size	1	1.	2.01	3.77	19.35	30.53	0.	0.
time (sec)	N/A	0.213	8.167	0.239	3.668	1.117	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	236	325	1455	3087	0	0
normalized size	1	1.	2.15	2.95	13.23	28.06	0.	0.
time (sec)	N/A	0.106	2.211	0.209	2.468	0.797	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	29.331	2.741	0.	0.	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	33.482	4.84	0.	0.	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	73	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.335	0.095	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.39	0.087	0.	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	52	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.152	0.092	0.	0.	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	33	33	181	310	0	0	0	0
normalized size	1	1.	5.48	9.39	0.	0.	0.	0.
time (sec)	N/A	0.026	1.771	0.212	0.	0.	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.165	0.089	0.	0.	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	54	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.198	0.087	0.	0.	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	53	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.24	0.09	0.	0.	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	65	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.276	0.09	0.	0.	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	65	0	0	0	0	0
normalized size	1	1.	0.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.287	0.101	0.	0.	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	61	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.243	0.095	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	54	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.195	0.096	0.	0.	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	42	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.141	0.101	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	132	310	0	0	0	0
normalized size	1	1.	2.49	5.85	0.	0.	0.	0.
time (sec)	N/A	0.037	2.211	0.184	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	63	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.248	0.097	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	212	0	0	0	0	0
normalized size	1	1.	2.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	8.098	0.097	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	89	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.339	0.099	0.	0.	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	67	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.546	0.104	0.	0.	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	108	0	0	0	0	0
normalized size	1	1.	1.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.926	0.092	0.	0.	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	56	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.189	0.095	0.	0.	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	86	308	0	0	0	0
normalized size	1	1.	2.26	8.11	0.	0.	0.	0.
time (sec)	N/A	0.022	1.186	0.204	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	37	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.181	0.098	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	56	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.184	0.096	0.	0.	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	57	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.233	0.092	0.	0.	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	73	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.299	0.096	0.	0.	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	73	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.272	0.116	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	65	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.298	0.11	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	56	0	0	0	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.184	0.104	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	46	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.148	0.109	0.	0.	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	106	308	0	0	0	0
normalized size	1	1.	1.83	5.31	0.	0.	0.	0.
time (sec)	N/A	0.032	0.73	0.212	0.	0.	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	65	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.245	0.102	0.	0.	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	114	0	0	0	0	0
normalized size	1	1.	1.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.998	0.103	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	93	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.42	0.099	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	22	26	24	54	37	24
normalized size	1	1.	0.71	0.84	0.77	1.74	1.19	0.77
time (sec)	N/A	0.041	0.015	0.048	0.99	0.518	6.234	1.143

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	154	260	197	419	0	225
normalized size	1	1.	1.18	1.98	1.5	3.2	0.	1.72
time (sec)	N/A	0.188	0.219	0.066	1.049	0.515	0.	1.136

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	109	179	136	278	289	151
normalized size	1	1.	0.95	1.56	1.18	2.42	2.51	1.31
time (sec)	N/A	0.141	0.157	0.051	1.028	0.496	146.651	1.144

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	60	107	81	159	155	86
normalized size	1	1.	0.82	1.47	1.11	2.18	2.12	1.18
time (sec)	N/A	0.103	0.11	0.054	0.995	0.513	21.014	1.128

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	34	52	36	78	56	36
normalized size	1	1.	0.83	1.27	0.88	1.9	1.37	0.88
time (sec)	N/A	0.056	0.02	0.048	1.022	0.492	14.303	1.099

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	49	58	128	182	0	69
normalized size	1	1.	0.86	1.02	2.25	3.19	0.	1.21
time (sec)	N/A	0.252	0.063	0.051	1.234	0.505	0.	1.121

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	61	82	437	251	0	150
normalized size	1	1.	0.78	1.05	5.6	3.22	0.	1.92
time (sec)	N/A	0.238	0.138	0.054	1.225	0.514	0.	1.12

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	77	104	489	392	0	271
normalized size	1	1.	0.78	1.05	4.94	3.96	0.	2.74
time (sec)	N/A	0.329	0.234	0.053	1.372	0.53	0.	1.114

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	128	1000	329	579	0	6323
normalized size	1	1.	0.65	5.05	1.66	2.92	0.	31.93
time (sec)	N/A	0.252	0.693	0.049	1.2	0.565	0.	1.743

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	105	580	234	389	0	4238
normalized size	1	1.	0.61	3.39	1.37	2.27	0.	24.78
time (sec)	N/A	0.184	0.415	0.043	1.129	0.505	0.	1.464

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	73	294	146	240	0	2538
normalized size	1	1.	0.65	2.62	1.3	2.14	0.	22.66
time (sec)	N/A	0.136	0.394	0.041	1.073	0.498	0.	1.309

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	46	119	74	132	0	1242
normalized size	1	1.	0.7	1.8	1.12	2.	0.	18.82
time (sec)	N/A	0.068	0.135	0.037	1.064	0.484	0.	1.206

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	63	116	158	228	0	1509
normalized size	1	1.	0.89	1.63	2.23	3.21	0.	21.25
time (sec)	N/A	0.281	0.153	0.037	1.286	0.489	0.	1.298

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	81	169	159	321	0	0
normalized size	1	1.	0.79	1.66	1.56	3.15	0.	0.
time (sec)	N/A	0.276	0.545	0.041	1.329	0.519	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	104	207	176	516	0	12712
normalized size	1	1.	0.76	1.52	1.29	3.79	0.	93.47
time (sec)	N/A	0.374	0.999	0.038	1.45	0.531	0.	2.116

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	125	243	190	748	0	0
normalized size	1	1.	0.61	1.19	0.93	3.65	0.	0.
time (sec)	N/A	0.38	1.127	0.039	1.511	0.556	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	255	255	459	849	813	2314	0	0
normalized size	1	1.	1.8	3.33	3.19	9.07	0.	0.
time (sec)	N/A	0.347	1.572	0.196	1.809	0.756	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	223	481	552	1480	0	0
normalized size	1	1.	1.3	2.8	3.21	8.6	0.	0.
time (sec)	N/A	0.229	1.156	0.259	1.56	0.638	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	171	205	0	814	0	0
normalized size	1	1.	1.8	2.16	0.	8.57	0.	0.
time (sec)	N/A	0.109	0.341	0.249	0.	0.584	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	71	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.21	6.232	0.349	0.	0.	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	91	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.283	6.787	0.7	0.	0.	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	114	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.328	7.249	0.71	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	2482	956	819	3866	0	0
normalized size	1	1.	8.3	3.2	2.74	12.93	0.	0.
time (sec)	N/A	0.503	6.742	0.255	1.679	1.002	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	242	242	1719	639	597	2703	0	0
normalized size	1	1.	7.1	2.64	2.47	11.17	0.	0.
time (sec)	N/A	0.446	6.608	0.214	1.5	0.866	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	173	173	516	377	406	1728	0	0
normalized size	1	1.	2.98	2.18	2.35	9.99	0.	0.
time (sec)	N/A	0.328	6.541	0.243	1.399	0.719	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	254	177	0	937	0	0
normalized size	1	1.	2.37	1.65	0.	8.76	0.	0.
time (sec)	N/A	0.18	5.824	0.324	0.	0.622	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	79	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.299	3.157	0.371	0.	0.	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.342	4.256	0.663	0.	0.	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	128	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.398	5.894	0.504	0.	0.	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	230	230	547	677	0	2240	0	0
normalized size	1	1.	2.38	2.94	0.	9.74	0.	0.
time (sec)	N/A	0.33	2.335	0.302	0.	0.755	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	364	345	0	1320	0	0
normalized size	1	1.	2.48	2.35	0.	8.98	0.	0.
time (sec)	N/A	0.212	3.737	0.308	0.	0.638	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	105	87	4496	252	0	3883
normalized size	1	1.	1.84	1.53	78.88	4.42	0.	68.12
time (sec)	N/A	0.091	0.454	0.046	1.853	0.514	0.	2.289

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	77	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.266	13.558	0.388	0.	0.	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	97	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.344	16.554	0.5	0.	0.	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	120	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.436	18.928	0.996	0.	0.	0.	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	77	81	0	405	0	0
normalized size	1	1.	1.35	1.42	0.	7.11	0.	0.
time (sec)	N/A	0.067	0.031	0.192	0.	0.529	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	150	73	144	159
normalized size	1	1.	1.	1.07	10.71	5.21	10.29	11.36
time (sec)	N/A	0.033	0.023	0.042	1.568	0.486	53.066	1.161

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	146	102	0	500	0	0
normalized size	1	1.	2.18	1.52	0.	7.46	0.	0.
time (sec)	N/A	0.137	0.274	0.223	0.	0.539	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [205] had the largest ratio of [1.25]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.	20	0.2
2	A	5	4	1.	20	0.2
3	A	5	5	1.	20	0.25
4	A	3	2	1.	20	0.1
5	A	3	3	1.	18	0.167
6	A	5	5	1.	20	0.25
7	A	6	6	1.	20	0.3
8	A	7	6	1.	20	0.3
9	A	8	6	1.	20	0.3
10	A	3	3	1.	8	0.375
11	A	4	4	1.	8	0.5
12	A	5	4	1.	8	0.5
13	A	8	3	1.	22	0.136
14	A	9	5	1.	22	0.227
15	A	7	5	1.	22	0.227
16	A	4	4	1.	22	0.182
17	A	3	2	1.	20	0.1
18	A	8	4	1.	22	0.182
19	A	10	5	1.	22	0.227
20	A	12	5	1.	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	14	5	1.	22	0.227
22	A	8	3	1.	22	0.136
23	A	9	4	1.	22	0.182
24	A	9	5	1.	22	0.227
25	A	4	2	1.	22	0.091
26	A	4	3	1.	20	0.15
27	A	8	4	1.	22	0.182
28	A	10	5	1.	22	0.227
29	A	12	5	1.	22	0.227
30	A	14	5	1.	22	0.227
31	A	0	0	0.	0	0.
32	A	7	6	1.	14	0.429
33	A	6	6	1.	14	0.429
34	A	5	5	1.	14	0.357
35	A	4	4	1.	12	0.333
36	A	0	0	0.	0	0.
37	A	0	0	0.	0	0.
38	A	0	0	0.	0	0.
39	A	10	6	1.	20	0.3
40	A	8	5	1.	20	0.25
41	A	6	4	1.	20	0.2
42	A	2	2	1.	18	0.111
43	A	0	0	0.	0	0.
44	A	0	0	0.	0	0.
45	A	0	0	0.	0	0.
46	A	7	7	1.	22	0.318
47	A	6	6	1.	22	0.273
48	A	3	3	1.	22	0.136
49	A	3	3	1.	20	0.15
50	A	0	0	0.	0	0.
51	A	0	0	0.	0	0.
52	A	10	8	1.	22	0.364
53	A	9	8	1.	22	0.364
54	A	8	8	1.	22	0.364
55	A	8	8	1.	22	0.364
56	A	9	8	1.	22	0.364
57	A	10	8	1.	22	0.364
58	A	18	7	1.	24	0.292
59	A	16	7	1.	24	0.292
60	A	14	7	1.	24	0.292
61	A	14	7	1.	24	0.292
62	A	16	7	1.	24	0.292
63	A	18	7	1.	24	0.292
64	A	18	7	1.	24	0.292

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
65	A	16	7	1.	24	0.292
66	A	14	7	1.	24	0.292
67	A	14	7	1.	24	0.292
68	A	16	7	1.	24	0.292
69	A	18	7	1.	24	0.292
70	A	8	3	1.	22	0.136
71	A	9	5	1.	22	0.227
72	A	7	5	1.	22	0.227
73	A	4	4	1.	22	0.182
74	A	3	2	1.	20	0.1
75	A	8	4	1.	22	0.182
76	A	10	5	1.	22	0.227
77	A	12	5	1.	22	0.227
78	A	14	5	1.	22	0.227
79	A	5	3	1.	24	0.125
80	A	7	3	1.	24	0.125
81	A	6	3	1.	24	0.125
82	A	5	3	1.	24	0.125
83	A	4	3	1.	22	0.136
84	A	5	4	1.	24	0.167
85	A	6	5	1.	24	0.208
86	A	7	5	1.	24	0.208
87	A	8	5	1.	24	0.208
88	A	11	3	1.	24	0.125
89	A	17	3	1.	24	0.125
90	A	14	3	1.	24	0.125
91	A	11	3	1.	24	0.125
92	A	8	3	1.	22	0.136
93	A	11	4	1.	24	0.167
94	A	14	5	1.	24	0.208
95	A	17	5	1.	24	0.208
96	A	20	5	1.	24	0.208
97	A	0	0	0.	0	0.
98	A	17	8	1.	20	0.4
99	A	14	8	1.	20	0.4
100	A	11	7	1.	20	0.35
101	A	8	6	1.	18	0.333
102	A	0	0	0.	0	0.
103	A	0	0	0.	0	0.
104	A	0	0	0.	0	0.
105	A	8	8	1.	16	0.5
106	A	7	7	1.	16	0.438
107	A	6	6	1.	16	0.375
108	A	3	2	1.	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
109	A	0	0	0.	0	0.
110	A	0	0	0.	0	0.
111	A	0	0	0.	0	0.
112	A	31	7	1.	22	0.318
113	A	25	9	1.	22	0.409
114	A	17	7	1.	22	0.318
115	A	12	5	1.	20	0.25
116	A	0	0	0.	0	0.
117	A	0	0	0.	0	0.
118	A	18	7	1.	24	0.292
119	A	16	7	1.	24	0.292
120	A	14	7	1.	24	0.292
121	A	14	7	1.	24	0.292
122	A	16	7	1.	24	0.292
123	A	18	7	1.	24	0.292
124	A	10	7	1.	26	0.269
125	A	9	7	1.	26	0.269
126	A	8	7	1.	26	0.269
127	A	8	7	1.	26	0.269
128	A	9	7	1.	26	0.269
129	A	10	7	1.	26	0.269
130	A	26	7	1.	26	0.269
131	A	23	7	1.	26	0.269
132	A	20	7	1.	26	0.269
133	A	20	7	1.	26	0.269
134	A	23	7	1.	26	0.269
135	A	26	7	1.	26	0.269
136	A	8	3	1.	22	0.136
137	A	9	4	1.	22	0.182
138	A	9	5	1.	22	0.227
139	A	4	2	1.	22	0.091
140	A	4	3	1.	20	0.15
141	A	8	4	1.	22	0.182
142	A	10	5	1.	22	0.227
143	A	12	5	1.	22	0.227
144	A	14	5	1.	22	0.227
145	A	11	3	1.	24	0.125
146	A	17	3	1.	24	0.125
147	A	14	3	1.	24	0.125
148	A	11	3	1.	24	0.125
149	A	8	3	1.	22	0.136
150	A	11	4	1.	24	0.167
151	A	14	5	1.	24	0.208
152	A	17	5	1.	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
153	A	20	5	1.	24	0.208
154	A	8	3	1.	24	0.125
155	A	12	3	1.	24	0.125
156	A	10	3	1.	24	0.125
157	A	8	3	1.	24	0.125
158	A	6	3	1.	22	0.136
159	A	8	4	1.	24	0.167
160	A	10	5	1.	24	0.208
161	A	12	5	1.	24	0.208
162	A	14	5	1.	24	0.208
163	A	0	0	0.	0	0.
164	A	13	11	1.	22	0.5
165	A	12	12	1.	22	0.546
166	A	9	8	1.	22	0.364
167	A	8	8	1.	20	0.4
168	A	0	0	0.	0	0.
169	A	0	0	0.	0	0.
170	A	0	0	0.	0	0.
171	A	16	9	1.	22	0.409
172	A	13	8	1.	22	0.364
173	A	10	7	1.	22	0.318
174	A	5	5	1.	20	0.25
175	A	0	0	0.	0	0.
176	A	0	0	0.	0	0.
177	A	0	0	0.	0	0.
178	A	15	8	1.	16	0.5
179	A	13	10	1.	16	0.625
180	A	9	7	1.	16	0.438
181	A	7	7	1.	14	0.5
182	A	0	0	0.	0	0.
183	A	0	0	0.	0	0.
184	A	18	7	1.	24	0.292
185	A	16	7	1.	24	0.292
186	A	14	7	1.	24	0.292
187	A	14	7	1.	24	0.292
188	A	16	7	1.	24	0.292
189	A	18	7	1.	24	0.292
190	A	26	7	1.	26	0.269
191	A	23	7	1.	26	0.269
192	A	20	7	1.	26	0.269
193	A	20	7	1.	26	0.269
194	A	23	7	1.	26	0.269
195	A	26	7	1.	26	0.269
196	A	18	7	1.	26	0.269

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
197	A	16	7	1.	26	0.269
198	A	14	7	1.	26	0.269
199	A	14	7	1.	26	0.269
200	A	16	7	1.	26	0.269
201	A	18	7	1.	26	0.269
202	A	12	10	1.	12	0.833
203	A	11	10	1.	12	0.833
204	A	6	5	1.	10	0.5
205	A	26	15	1.	12	1.25
206	A	19	11	1.	12	0.917
207	A	16	10	1.	10	1.
208	A	0	0	0.	0	0.
209	A	7	6	1.	14	0.429
210	A	6	6	1.	14	0.429
211	A	5	5	1.	14	0.357
212	A	4	4	1.	12	0.333
213	A	0	0	0.	0	0.
214	A	0	0	0.	0	0.
215	A	0	0	0.	0	0.
216	A	14	8	1.	20	0.4
217	A	11	7	1.	20	0.35
218	A	8	6	1.	18	0.333
219	A	0	0	0.	0	0.
220	A	0	0	0.	0	0.
221	A	0	0	0.	0	0.
222	A	12	12	1.	22	0.546
223	A	9	8	1.	22	0.364
224	A	8	8	1.	20	0.4
225	A	0	0	0.	0	0.
226	A	0	0	0.	0	0.
227	A	0	0	0.	0	0.
228	A	12	6	1.	20	0.3
229	A	10	6	1.	20	0.3
230	A	8	5	1.	20	0.25
231	A	6	4	1.	18	0.222
232	A	0	0	0.	0	0.
233	A	0	0	0.	0	0.
234	A	0	0	0.	0	0.
235	A	23	14	1.	22	0.636
236	A	19	15	1.	22	0.682
237	A	10	10	1.	20	0.5
238	A	0	0	0.	0	0.
239	A	0	0	0.	0	0.
240	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
241	A	22	18	1.	22	0.818
242	A	17	13	1.	22	0.591
243	A	11	10	1.	20	0.5
244	A	0	0	0.	0	0.
245	A	0	0	0.	0	0.
246	A	0	0	0.	0	0.
247	A	10	6	1.	20	0.3
248	A	8	5	1.	20	0.25
249	A	6	4	1.	20	0.2
250	A	2	2	1.	18	0.111
251	A	0	0	0.	0	0.
252	A	0	0	0.	0	0.
253	A	0	0	0.	0	0.
254	A	7	7	1.	16	0.438
255	A	6	6	1.	16	0.375
256	A	3	2	1.	14	0.143
257	A	0	0	0.	0	0.
258	A	0	0	0.	0	0.
259	A	0	0	0.	0	0.
260	A	13	8	1.	22	0.364
261	A	10	7	1.	22	0.318
262	A	5	5	1.	20	0.25
263	A	0	0	0.	0	0.
264	A	0	0	0.	0	0.
265	A	0	0	0.	0	0.
266	A	27	14	1.	22	0.636
267	A	23	14	1.	22	0.636
268	A	19	15	1.	22	0.682
269	A	10	10	1.08	20	0.5
270	A	0	0	0.	0	0.
271	A	0	0	0.	0	0.
272	A	0	0	0.	0	0.
273	A	7	7	1.	24	0.292
274	A	6	6	1.	24	0.25
275	A	3	3	1.	22	0.136
276	A	0	0	0.	0	0.
277	A	0	0	0.	0	0.
278	A	0	0	0.	0	0.
279	A	64	24	1.	24	1.
280	A	36	22	1.	24	0.917
281	A	13	12	1.13	22	0.546
282	A	0	0	0.	0	0.
283	A	0	0	0.	0	0.
284	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
285	A	40	18	1.	20	0.9
286	A	29	19	1.	20	0.95
287	A	13	12	1.	18	0.667
288	A	0	0	0.	0	0.
289	A	0	0	0.	0	0.
290	A	0	0	0.	0	0.
291	A	7	7	1.	22	0.318
292	A	6	6	1.	22	0.273
293	A	3	3	1.	22	0.136
294	A	3	3	1.	20	0.15
295	A	0	0	0.	0	0.
296	A	0	0	0.	0	0.
297	A	0	0	0.	0	0.
298	A	25	9	1.	22	0.409
299	A	17	7	1.	22	0.318
300	A	12	5	1.	20	0.25
301	A	0	0	0.	0	0.
302	A	0	0	0.	0	0.
303	A	0	0	0.	0	0.
304	A	13	10	1.	16	0.625
305	A	9	7	1.	16	0.438
306	A	7	7	1.	14	0.5
307	A	0	0	0.	0	0.
308	A	0	0	0.	0	0.
309	A	0	0	0.	0	0.
310	A	25	16	1.	22	0.727
311	A	22	18	1.	22	0.818
312	A	17	13	1.	22	0.591
313	A	11	10	1.01	20	0.5
314	A	0	0	0.	0	0.
315	A	0	0	0.	0	0.
316	A	0	0	0.	0	0.
317	A	44	19	1.	24	0.792
318	A	31	19	1.	24	0.792
319	A	13	12	1.12	22	0.546
320	A	0	0	0.	0	0.
321	A	0	0	0.	0	0.
322	A	0	0	0.	0	0.
323	A	16	9	1.	24	0.375
324	A	10	7	1.	24	0.292
325	A	7	5	1.	22	0.227
326	A	0	0	0.	0	0.
327	A	0	0	0.	0	0.
328	A	4	3	1.	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	3	3	1.	18	0.167
330	A	3	3	1.	18	0.167
331	A	2	2	1.	18	0.111
332	A	2	2	1.	18	0.111
333	A	3	3	1.	18	0.167
334	A	3	3	1.	18	0.167
335	A	4	3	1.	18	0.167
336	A	5	4	1.	18	0.222
337	A	4	4	1.	18	0.222
338	A	4	4	1.	18	0.222
339	A	3	3	1.	18	0.167
340	A	3	3	1.	18	0.167
341	A	4	4	1.	18	0.222
342	A	4	4	1.	18	0.222
343	A	5	4	1.	18	0.222
344	A	4	3	1.	18	0.167
345	A	3	3	1.	18	0.167
346	A	3	3	1.	18	0.167
347	A	2	2	1.	18	0.111
348	A	2	2	1.	18	0.111
349	A	3	3	1.	18	0.167
350	A	3	3	1.	18	0.167
351	A	4	3	1.	18	0.167
352	A	5	4	1.	18	0.222
353	A	4	4	1.	18	0.222
354	A	4	4	1.	18	0.222
355	A	3	3	1.	18	0.167
356	A	3	3	1.	18	0.167
357	A	4	4	1.	18	0.222
358	A	4	4	1.	18	0.222
359	A	5	4	1.	18	0.222
360	A	6	3	1.	8	0.375
361	A	14	5	1.	14	0.357
362	A	10	4	1.	14	0.286
363	A	10	5	1.	14	0.357
364	A	6	2	1.	12	0.167
365	A	12	5	1.	14	0.357
366	A	12	6	1.	14	0.429
367	A	16	7	1.	14	0.5
368	A	14	5	1.	23	0.217
369	A	10	4	1.	23	0.174
370	A	10	5	1.	23	0.217
371	A	6	2	1.	21	0.095
372	A	12	5	1.	23	0.217

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
373	A	12	6	1.	23	0.261
374	A	16	7	1.	23	0.304
375	A	16	8	1.	23	0.348
376	A	20	9	1.	25	0.36
377	A	16	8	1.	25	0.32
378	A	12	7	1.	23	0.304
379	A	0	0	0.	0	0.
380	A	0	0	0.	0	0.
381	A	0	0	0.	0	0.
382	A	20	12	1.	23	0.522
383	A	19	13	1.	23	0.565
384	A	14	9	1.	23	0.391
385	A	13	9	1.	21	0.429
386	A	0	0	0.	0	0.
387	A	0	0	0.	0	0.
388	A	0	0	0.	0	0.
389	A	19	9	1.	25	0.36
390	A	15	8	1.	25	0.32
391	A	9	6	1.	23	0.261
392	A	0	0	0.	0	0.
393	A	0	0	0.	0	0.
394	A	0	0	0.	0	0.
395	A	12	7	1.	8	0.875
396	A	5	4	1.	10	0.4
397	A	19	6	1.	10	0.6

Chapter 3

Listing of integrals

3.1 $\int (c + dx)^m \cos(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=137

$$\frac{2^{-m-3} e^{2i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{2ib(c+dx)}{d}\right)}{b} - \frac{2^{-m-3} e^{-2i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, \frac{2ib(c+dx)}{d}\right)}{b}$$

[Out] $-\left(\frac{2^{(-3-m)} E^{((2I)(a-(b*c)/d)} (c+dx)^m \Gamma[1+m, ((-2I)*b*(c+d*x))/d]}{(b*((-I)*b*(c+d*x))/d)^m}\right) - \left(\frac{2^{(-3-m)} (c+dx)^m \Gamma[1+m, ((2I)*b*(c+d*x))/d]}{(b E^{((2I)(a-(b*c)/d)} ((I*b*(c+d*x))/d))^m}\right)$

Rubi [A] time = 0.155546, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {4406, 12, 3308, 2181}

$$\frac{2^{-m-3} e^{2i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{2ib(c+dx)}{d}\right)}{b} - \frac{2^{-m-3} e^{-2i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, \frac{2ib(c+dx)}{d}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^m * Cos[a + b*x] * Sin[a + b*x], x]

[Out] $-\left(\frac{2^{(-3-m)} E^{((2I)(a-(b*c)/d)} (c+dx)^m \Gamma[1+m, ((-2I)*b*(c+d*x))/d]}{(b*((-I)*b*(c+d*x))/d)^m}\right) - \left(\frac{2^{(-3-m)} (c+dx)^m \Gamma[1+m, ((2I)*b*(c+d*x))/d]}{(b E^{((2I)(a-(b*c)/d)} ((I*b*(c+d*x))/d))^m}\right)$

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n * Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^m \cos(a + bx) \sin(a + bx) dx &= \int \frac{1}{2} (c + dx)^m \sin(2a + 2bx) dx \\ &= \frac{1}{2} \int (c + dx)^m \sin(2a + 2bx) dx \\ &= \frac{1}{4} i \int e^{-i(2a+2bx)} (c + dx)^m dx - \frac{1}{4} i \int e^{i(2a+2bx)} (c + dx)^m dx \\ &= -\frac{2^{-3-m} e^{2i\left(a-\frac{bc}{d}\right)} (c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, -\frac{2ib(c+dx)}{d}\right)}{b} - \frac{2^{-3-m} e^{-2i\left(a-\frac{bc}{d}\right)} (c + dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, \frac{2ib(c+dx)}{d}\right)}{b} \end{aligned}$$

Mathematica [A] time = 0.0851013, size = 138, normalized size = 1.01

$$\frac{2^{-m-3} e^{-\frac{2i(ad+bc)}{d}} (c + dx)^m \left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m} \left(e^{Aia} \left(\frac{ib(c+dx)}{d}\right)^m \Gamma\left(m + 1, -\frac{2ib(c+dx)}{d}\right) + e^{\frac{4ibc}{d}} \left(-\frac{ib(c+dx)}{d}\right)^m \Gamma\left(m + 1, \frac{2ib(c+dx)}{d}\right) \right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m * Cos[a + b*x] * Sin[a + b*x], x]
```

```
[Out] -((2^(-3 - m)*(c + d*x)^m*(E^((4*I)*a))*((I*b*(c + d*x))/d)^m*Gamma[1 + m, (-2*I)*b*(c + d*x))/d] + E^(((4*I)*b*c)/d)*((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d])/ (b * E^(((2*I)*(b*c + a*d))/d) * ((b^2*(c + d*x)^2)/d^2)^m))
```

Maple [F] time = 0.209, size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")

[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a), x)

Fricas [A] time = 0.515529, size = 246, normalized size = 1.8

$$\frac{e^{\left(\frac{dm \log\left(\frac{2ib}{d}\right) - 2i bc + 2i ad}{d}\right)} \Gamma\left(m + 1, \frac{2i b dx + 2i bc}{d}\right) + e^{\left(\frac{dm \log\left(-\frac{2ib}{d}\right) + 2i bc - 2i ad}{d}\right)} \Gamma\left(m + 1, \frac{-2i b dx - 2i bc}{d}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")

[Out] $-1/8 * (e^{-(d*m*\log(2*I*b/d) - 2*I*b*c + 2*I*a*d)/d} * \text{gamma}(m + 1, (2*I*b*d*x + 2*I*b*c)/d) + e^{-(d*m*\log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d} * \text{gamma}(m + 1, (-2*I*b*d*x - 2*I*b*c)/d)) / b$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**m*cos(b*x+a)*sin(b*x+a),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a), x)

3.2 $\int (c + dx)^4 \cos(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=156

$$-\frac{3d^2(c + dx)^2 \sin^2(a + bx)}{2b^3} - \frac{3d^3(c + dx) \sin(a + bx) \cos(a + bx)}{2b^4} + \frac{d(c + dx)^3 \sin(a + bx) \cos(a + bx)}{b^2} + \frac{3d^4 \sin^2(a + bx)}{4b^5}$$

[Out] $(3*c*d^3*x)/(2*b^3) + (3*d^4*x^2)/(4*b^3) - (c + d*x)^4/(4*b) - (3*d^3*(c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b^4) + (d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/b^2 + (3*d^4*Sin[a + b*x]^2)/(4*b^5) - (3*d^2*(c + d*x)^2*Sin[a + b*x]^2)/(2*b^3) + ((c + d*x)^4*Sin[a + b*x]^2)/(2*b)$

Rubi [A] time = 0.106998, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {4404, 3311, 32, 3310}

$$-\frac{3d^2(c + dx)^2 \sin^2(a + bx)}{2b^3} - \frac{3d^3(c + dx) \sin(a + bx) \cos(a + bx)}{2b^4} + \frac{d(c + dx)^3 \sin(a + bx) \cos(a + bx)}{b^2} + \frac{3d^4 \sin^2(a + bx)}{4b^5}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x],x]

[Out] $(3*c*d^3*x)/(2*b^3) + (3*d^4*x^2)/(4*b^3) - (c + d*x)^4/(4*b) - (3*d^3*(c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b^4) + (d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/b^2 + (3*d^4*Sin[a + b*x]^2)/(4*b^5) - (3*d^2*(c + d*x)^2*Sin[a + b*x]^2)/(2*b^3) + ((c + d*x)^4*Sin[a + b*x]^2)/(2*b)$

Rule 4404

Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 3311

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 3310

Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rubi steps

$$\begin{aligned}
\int (c + dx)^4 \cos(a + bx) \sin(a + bx) dx &= \frac{(c + dx)^4 \sin^2(a + bx)}{2b} - \frac{(2d) \int (c + dx)^3 \sin^2(a + bx) dx}{b} \\
&= \frac{d(c + dx)^3 \cos(a + bx) \sin(a + bx)}{b^2} - \frac{3d^2(c + dx)^2 \sin^2(a + bx)}{2b^3} + \frac{(c + dx)^4 \sin^2(a + bx)}{2b^4} \\
&= -\frac{(c + dx)^4}{4b} - \frac{3d^3(c + dx) \cos(a + bx) \sin(a + bx)}{2b^4} + \frac{d(c + dx)^3 \cos(a + bx) \sin(a + bx)}{b^2} \\
&= \frac{3cd^3x}{2b^3} + \frac{3d^4x^2}{4b^3} - \frac{(c + dx)^4}{4b} - \frac{3d^3(c + dx) \cos(a + bx) \sin(a + bx)}{2b^4} + \frac{d(c + dx)^3 \cos(a + bx) \sin(a + bx)}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.512725, size = 86, normalized size = 0.55

$$\frac{4bd(c + dx) \sin(2(a + bx)) (2b^2(c + dx)^2 - 3d^2) - 2 \cos(2(a + bx)) (-6b^2d^2(c + dx)^2 + 2b^4(c + dx)^4 + 3d^4)}{16b^5}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x],x]

[Out] (-2*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)] + 4*b*d*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)]/(16*b^5)

Maple [B] time = 0.034, size = 853, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x)

[Out] 1/b*(1/b^4*d^4*(-1/2*(b*x+a)^4*cos(b*x+a)^2+2*(b*x+a)^3*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/2*(b*x+a)^2*cos(b*x+a)^2-3*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)^2+3/4*sin(b*x+a)^2-3/4*(b*x+a)^4)-4/b^4*a*d^4*(-1/2*(b*x+a)^3*cos(b*x+a)^2+3/2*(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)*cos(b*x+a)^2-3/8*cos(b*x+a)*sin(b*x+a)-3/8*b*x-3/8*a-1/2*(b*x+a)^3)+4/b^3*c*d^3*(-1/2*(b*x+a)^3*cos(b*x+a)^2+3/2*(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)*cos(b*x+a)^2-3/8*cos(b*x+a)*sin(b*x+a)-3/8*b*x-3/8*a-1/2*(b*x+a)^3)+6/b^4*a^2*d^4*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)-12/b^3*a*c*d^3*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+6/b^2*c^2*d^2*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)-4/b^4*a^3*d^4*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)+12/b^3*a^2*c*d^3*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)-12/b^2*a*c^2*d^2*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)+4/b*c^3*d*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)-1/2/b^4*a^4*d^4*cos(b*x+a)^2+2/b^3*a^3*c*d^3*cos(b*x+a)^2-3/b^2*a^2*c^2*d^2*cos(b*x+a)^2+2/b*a*c^3*d*cos(b*x+a)^2-1/2*c^4*cos(b*x+a)^2)

Maxima [B] time = 1.22132, size = 791, normalized size = 5.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")

[Out] $-1/8*(4*c^4*\cos(b*x + a)^2 - 16*a*c^3*d*\cos(b*x + a)^2/b + 24*a^2*c^2*d^2*\cos(b*x + a)^2/b^2 - 16*a^3*c*d^3*\cos(b*x + a)^2/b^3 + 4*a^4*d^4*\cos(b*x + a)^2/b^4 + 4*(2*(b*x + a)*\cos(2*b*x + 2*a) - \sin(2*b*x + 2*a))*c^3*d/b - 12*(2*(b*x + a)*\cos(2*b*x + 2*a) - \sin(2*b*x + 2*a))*a*c^2*d^2/b^2 + 12*(2*(b*x + a)*\cos(2*b*x + 2*a) - \sin(2*b*x + 2*a))*a^2*c*d^3/b^3 - 4*(2*(b*x + a)*\cos(2*b*x + 2*a) - \sin(2*b*x + 2*a))*a^3*d^4/b^4 + 6*((2*(b*x + a)^2 - 1)*\cos(2*b*x + 2*a) - 2*(b*x + a)*\sin(2*b*x + 2*a))*c^2*d^2/b^2 - 12*((2*(b*x + a)^2 - 1)*\cos(2*b*x + 2*a) - 2*(b*x + a)*\sin(2*b*x + 2*a))*a*c*d^3/b^3 + 6*((2*(b*x + a)^2 - 1)*\cos(2*b*x + 2*a) - 2*(b*x + a)*\sin(2*b*x + 2*a))*a^2*d^4/b^4 + 2*(2*(2*(b*x + a)^3 - 3*b*x - 3*a)*\cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*\sin(2*b*x + 2*a))*c*d^3/b^3 - 2*(2*(2*(b*x + a)^3 - 3*b*x - 3*a)*\cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*\sin(2*b*x + 2*a))*a*d^4/b^4 + ((2*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*\cos(2*b*x + 2*a) - 2*(2*(b*x + a)^3 - 3*b*x - 3*a)*\sin(2*b*x + 2*a))*d^4/b^4)/b$

Fricas [A] time = 0.49188, size = 518, normalized size = 3.32

$\frac{b^4 d^4 x^4 + 4 b^4 c d^3 x^3 + 3 (2 b^4 c^2 d^2 - b^2 d^4) x^2 - (2 b^4 d^4 x^4 + 8 b^4 c d^3 x^3 + 2 b^4 c^4 - 6 b^2 c^2 d^2 + 3 d^4 + 6 (2 b^4 c^2 d^2 - b^2 d^4) x^2 + 4$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")

[Out] $1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 3*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 - (2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*\cos(b*x + a)^2 + 2*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*x)*\cos(b*x + a)*\sin(b*x + a) + 2*(2*b^4*c^3*d - 3*b^2*c*d^3)*x/b^5$

Sympy [A] time = 7.63633, size = 502, normalized size = 3.22

$\left\{ \frac{c^4 \sin^2(a+bx)}{2b} + \frac{c^3 dx \sin^2(a+bx)}{b} - \frac{c^3 dx \cos^2(a+bx)}{b} + \frac{3c^2 d^2 x^2 \sin^2(a+bx)}{2b} - \frac{3c^2 d^2 x^2 \cos^2(a+bx)}{2b} + \frac{cd^3 x^3 \sin^2(a+bx)}{b} - \frac{cd^3 x^3 \cos^2(a+bx)}{b} + \frac{d^4 x^4 \sin^2(a+bx)}{5} \right\} \sin(a) \cos(a)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**4*cos(b*x+a)*sin(b*x+a),x)

[Out] Piecewise(((c**4*sin(a + b*x)**2/(2*b) + c**3*d*x*sin(a + b*x)**2/b - c**3*d*x*cos(a + b*x)**2/b + 3*c**2*d**2*x**2*sin(a + b*x)**2/(2*b) - 3*c**2*d**2*x**2*cos(a + b*x)**2/(2*b) + c*d**3*x**3*sin(a + b*x)**2/b - c*d**3*x**3*cos(a + b*x)**2/b + d**4*x**4*sin(a + b*x)**2/(4*b) - d**4*x**4*cos(a + b*x)**2/(4*b) + c**3*d*sin(a + b*x)*cos(a + b*x)/b**2 + 3*c**2*d**2*x*sin(a + b

```
*x)*cos(a + b*x)/b**2 + 3*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)/b**2 + d**4
*x**3*sin(a + b*x)*cos(a + b*x)/b**2 - 3*c**2*d**2*sin(a + b*x)**2/(2*b**3)
- 3*c*d**3*x*sin(a + b*x)**2/(2*b**3) + 3*c*d**3*x*cos(a + b*x)**2/(2*b**3)
) - 3*d**4*x**2*sin(a + b*x)**2/(4*b**3) + 3*d**4*x**2*cos(a + b*x)**2/(4*b
**3) - 3*c*d**3*sin(a + b*x)*cos(a + b*x)/(2*b**4) - 3*d**4*x*sin(a + b*x)*
cos(a + b*x)/(2*b**4) + 3*d**4*sin(a + b*x)**2/(4*b**5), Ne(b, 0)), ((c**4*x
+ 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*co
s(a), True))
```

Giac [A] time = 1.09295, size = 244, normalized size = 1.56

$$\frac{(2b^4d^4x^4 + 8b^4cd^3x^3 + 12b^4c^2d^2x^2 + 8b^4c^3dx + 2b^4c^4 - 6b^2d^4x^2 - 12b^2cd^3x - 6b^2c^2d^2 + 3d^4)\cos(2bx + 2a)}{8b^5} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x
+ 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3*x - 6*b^2*c^2*d^2 + 3*d^4)*cos(2
*b*x + 2*a)/b^5 + 1/4*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x +
2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

3.3 $\int (c + dx)^3 \cos(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=120

$$-\frac{3d^2(c + dx) \sin^2(a + bx)}{4b^3} + \frac{3d(c + dx)^2 \sin(a + bx) \cos(a + bx)}{4b^2} - \frac{3d^3 \sin(a + bx) \cos(a + bx)}{8b^4} + \frac{(c + dx)^3 \sin^2(a + bx)}{2b}$$

[Out] $(3*d^3*x)/(8*b^3) - (c + d*x)^3/(4*b) - (3*d^3*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/(8*b^4) + (3*d*(c + d*x)^2*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/(4*b^2) - (3*d^2*(c + d*x)*\text{Sin}[a + b*x]^2)/(4*b^3) + ((c + d*x)^3*\text{Sin}[a + b*x]^2)/(2*b)$

Rubi [A] time = 0.0834477, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {4404, 3311, 32, 2635, 8}

$$-\frac{3d^2(c + dx) \sin^2(a + bx)}{4b^3} + \frac{3d(c + dx)^2 \sin(a + bx) \cos(a + bx)}{4b^2} - \frac{3d^3 \sin(a + bx) \cos(a + bx)}{8b^4} + \frac{(c + dx)^3 \sin^2(a + bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x],x]

[Out] $(3*d^3*x)/(8*b^3) - (c + d*x)^3/(4*b) - (3*d^3*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/(8*b^4) + (3*d*(c + d*x)^2*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/(4*b^2) - (3*d^2*(c + d*x)*\text{Sin}[a + b*x]^2)/(4*b^3) + ((c + d*x)^3*\text{Sin}[a + b*x]^2)/(2*b)$

Rule 4404

Int[Cos[(a_.) + (b_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.)*Sin[(a_.) + (b_.)*(x_.)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 3311

Int[((c_.) + (d_.)*(x_.))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rule 32

Int[((a_.) + (b_.)*(x_.))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)]^(n_.), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 \cos(a + bx) \sin(a + bx) dx &= \frac{(c + dx)^3 \sin^2(a + bx)}{2b} - \frac{(3d) \int (c + dx)^2 \sin^2(a + bx) dx}{2b} \\
&= \frac{3d(c + dx)^2 \cos(a + bx) \sin(a + bx)}{4b^2} - \frac{3d^2(c + dx) \sin^2(a + bx)}{4b^3} + \frac{(c + dx)^3 \sin^2(a + bx)}{2b} \\
&= -\frac{(c + dx)^3}{4b} - \frac{3d^3 \cos(a + bx) \sin(a + bx)}{8b^4} + \frac{3d(c + dx)^2 \cos(a + bx) \sin(a + bx)}{4b^2} \\
&= \frac{3d^3 x}{8b^3} - \frac{(c + dx)^3}{4b} - \frac{3d^3 \cos(a + bx) \sin(a + bx)}{8b^4} + \frac{3d(c + dx)^2 \cos(a + bx) \sin(a + bx)}{4b^2}
\end{aligned}$$

Mathematica [A] time = 0.300051, size = 71, normalized size = 0.59

$$\frac{3d \sin(2(a + bx)) (2b^2(c + dx)^2 - d^2) - 2b(c + dx) \cos(2(a + bx)) (2b^2(c + dx)^2 - 3d^2)}{16b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x],x]

[Out] (-2*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 3*d*(-d^2 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)]/(16*b^4)

Maple [B] time = 0.015, size = 466, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x)

[Out] 1/b*(1/b^3*d^3*(-1/2*(b*x+a)^3*cos(b*x+a)^2+3/2*(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)*cos(b*x+a)^2-3/8*cos(b*x+a)*sin(b*x+a)-3/8*b*x-3/8*a-1/2*(b*x+a)^3)-3/b^3*a*d^3*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+3/b^2*c*d^2*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+3/b^3*a^2*d^3*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)-6/b^2*a*c*d^2*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)+3/b*c^2*d*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)+1/2/b^3*a^3*d^3*cos(b*x+a)^2-3/2/b^2*a^2*c*d^2*cos(b*x+a)^2+3/2/b*a*c^2*d*cos(b*x+a)^2-1/2*c^3*cos(b*x+a)^2)

Maxima [B] time = 1.13191, size = 462, normalized size = 3.85

$$\frac{8c^3 \cos(bx + a)^2 - \frac{24ac^2d \cos(bx+a)^2}{b} + \frac{24a^2cd^2 \cos(bx+a)^2}{b^2} - \frac{8a^3d^3 \cos(bx+a)^2}{b^3} + \frac{6(2(bx+a) \cos(2bx+2a) - \sin(2bx+2a))c^2d}{b} - \frac{12(2(bx+a) \cos(2bx+2a) - \sin(2bx+2a))cd}{b}}{16b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")

```
[Out] -1/16*(8*c^3*cos(b*x + a)^2 - 24*a*c^2*d*cos(b*x + a)^2/b + 24*a^2*c*d^2*cos(b*x + a)^2/b^2 - 8*a^3*d^3*cos(b*x + a)^2/b^3 + 6*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*c^2*d/b - 12*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a*c*d^2/b^2 + 6*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a^2*d^3/b^3 + 6*((2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*c*d^2/b^2 - 6*((2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*a*d^3/b^3 + (2*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*d^3/b^3)/b
```

Fricas [A] time = 0.483148, size = 348, normalized size = 2.9

$$\frac{2b^3d^3x^3 + 6b^3cd^2x^2 - 2(2b^3d^3x^3 + 6b^3cd^2x^2 + 2b^3c^3 - 3bcd^2 + 3(2b^3c^2d - bd^3)x)\cos(bx + a)^2 + 3(2b^2d^3x^2 + 4b^2cd^2x + 2b^2c^2d - d^3)\cos(bx + a)\sin(bx + a) + 3(2b^3c^2d - bd^3)x/b^4}{8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 3*(2*b^3*c^2*d - b*d^3)*x)/b^4
```

Sympy [A] time = 3.10669, size = 342, normalized size = 2.85

$$\left\{ \frac{c^3 \sin^2(a+bx)}{2b} + \frac{3c^2 dx \sin^2(a+bx)}{4b} - \frac{3c^2 dx \cos^2(a+bx)}{4b} + \frac{3cd^2 x^2 \sin^2(a+bx)}{4b} - \frac{3cd^2 x^2 \cos^2(a+bx)}{4b} + \frac{d^3 x^3 \sin^2(a+bx)}{4b} - \frac{d^3 x^3 \cos^2(a+bx)}{4b} + \frac{3c^2 d \sin(a) \cos(a)}{4} \right\} \left(c^3 x + \frac{3c^2 dx^2}{2} + cd^2 x^3 + \frac{d^3 x^4}{4} \right) \sin(a) \cos(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Piecewise((c**3*sin(a + b*x)**2/(2*b) + 3*c**2*d*x*sin(a + b*x)**2/(4*b) - 3*c**2*d*x*cos(a + b*x)**2/(4*b) + 3*c*d**2*x**2*sin(a + b*x)**2/(4*b) - 3*c*d**2*x**2*cos(a + b*x)**2/(4*b) + d**3*x**3*sin(a + b*x)**2/(4*b) - d**3*x**3*cos(a + b*x)**2/(4*b) + 3*c**2*d*sin(a + b*x)*cos(a + b*x)/(4*b**2) + 3*c*d**2*x*sin(a + b*x)*cos(a + b*x)/(2*b**2) + 3*d**3*x**2*sin(a + b*x)*cos(a + b*x)/(4*b**2) - 3*c*d**2*sin(a + b*x)**2/(4*b**3) - 3*d**3*x*sin(a + b*x)**2/(8*b**3) + 3*d**3*x*cos(a + b*x)**2/(8*b**3) - 3*d**3*sin(a + b*x)*cos(a + b*x)/(8*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)*cos(a), True))
```

Giac [A] time = 1.14494, size = 163, normalized size = 1.36

$$\frac{(2b^3d^3x^3 + 6b^3cd^2x^2 + 6b^3c^2dx + 2b^3c^3 - 3bd^3x - 3bcd^2)\cos(2bx + 2a)}{8b^4} + \frac{3(2b^2d^3x^2 + 4b^2cd^2x + 2b^2c^2d - d^3)\sin(2bx + 2a)}{16b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(2*b*x + 2*a)/b^4 + 3/16*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*sin(2*b*x + 2*a)/b^4
```

3.4 $\int (c + dx)^2 \cos(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=89

$$\frac{d(c + dx) \sin(a + bx) \cos(a + bx)}{2b^2} - \frac{d^2 \sin^2(a + bx)}{4b^3} + \frac{(c + dx)^2 \sin^2(a + bx)}{2b} - \frac{cdx}{2b} - \frac{d^2 x^2}{4b}$$

[Out] $-(c*d*x)/(2*b) - (d^2*x^2)/(4*b) + (d*(c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b^2) - (d^2*Sin[a + b*x]^2)/(4*b^3) + ((c + d*x)^2*Sin[a + b*x]^2)/(2*b)$

Rubi [A] time = 0.0541044, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {4404, 3310}

$$\frac{d(c + dx) \sin(a + bx) \cos(a + bx)}{2b^2} - \frac{d^2 \sin^2(a + bx)}{4b^3} + \frac{(c + dx)^2 \sin^2(a + bx)}{2b} - \frac{cdx}{2b} - \frac{d^2 x^2}{4b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x],x]

[Out] $-(c*d*x)/(2*b) - (d^2*x^2)/(4*b) + (d*(c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b^2) - (d^2*Sin[a + b*x]^2)/(4*b^3) + ((c + d*x)^2*Sin[a + b*x]^2)/(2*b)$

Rule 4404

Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 3310

Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \cos(a + bx) \sin(a + bx) dx &= \frac{(c + dx)^2 \sin^2(a + bx)}{2b} - \frac{d \int (c + dx) \sin^2(a + bx) dx}{b} \\ &= \frac{d(c + dx) \cos(a + bx) \sin(a + bx)}{2b^2} - \frac{d^2 \sin^2(a + bx)}{4b^3} + \frac{(c + dx)^2 \sin^2(a + bx)}{2b} - \frac{cdx}{2b} - \frac{d^2 x^2}{4b} \\ &= -\frac{cdx}{2b} - \frac{d^2 x^2}{4b} + \frac{d(c + dx) \cos(a + bx) \sin(a + bx)}{2b^2} - \frac{d^2 \sin^2(a + bx)}{4b^3} + \frac{(c + dx)^2 \sin^2(a + bx)}{2b} \end{aligned}$$

Mathematica [A] time = 0.233082, size = 50, normalized size = 0.56

$$\frac{\cos(2(a + bx)) (d^2 - 2b^2(c + dx)^2) + 2bd(c + dx) \sin(2(a + bx))}{8b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x], x]

[Out] ((d^2 - 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 2*b*d*(c + d*x)*Sin[2*(a + b*x)])/(8*b^3)

Maple [B] time = 0.015, size = 215, normalized size = 2.4

$$\frac{1}{b} \left(\frac{d^2}{b^2} \left(-\frac{(bx+a)^2 (\cos(bx+a))^2}{2} + (bx+a) \left(\frac{\cos(bx+a) \sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right) - \frac{(bx+a)^2}{4} - \frac{(\sin(bx+a))^2}{4} \right) - 2 \frac{a}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*cos(b*x+a)*sin(b*x+a), x)

[Out] 1/b*(1/b^2*d^2*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)-2/b^2*a*d^2*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)+2/b*c*d*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)-1/2/b^2*a^2*d^2*cos(b*x+a)^2+1/b*a*c*d*cos(b*x+a)^2-1/2*c^2*cos(b*x+a)^2)

Maxima [B] time = 1.1559, size = 231, normalized size = 2.6

$$\frac{4c^2 \cos^2(bx+a) - \frac{8acd \cos(bx+a)^2}{b} + \frac{4a^2d^2 \cos(bx+a)^2}{b^2} + \frac{2(2(bx+a) \cos(2bx+2a) - \sin(2bx+2a))cd}{b} - \frac{2(2(bx+a) \cos(2bx+2a) - \sin(2bx+2a))}{b^2}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a), x, algorithm="maxima")

[Out] -1/8*(4*c^2*cos(b*x + a)^2 - 8*a*c*d*cos(b*x + a)^2/b + 4*a^2*d^2*cos(b*x + a)^2/b^2 + 2*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*c*d/b - 2*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a*d^2/b^2 + ((2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*d^2/b^2)/b

Fricas [A] time = 0.474177, size = 203, normalized size = 2.28

$$\frac{b^2d^2x^2 + 2b^2cdx - (2b^2d^2x^2 + 4b^2cdx + 2b^2c^2 - d^2) \cos(bx+a)^2 + 2(bd^2x + bcd) \cos(bx+a) \sin(bx+a)}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a), x, algorithm="fricas")

[Out] 1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(b*x + a)^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/b^3

Sympy [A] time = 1.37364, size = 175, normalized size = 1.97

$$\left\{ \frac{c^2 \sin^2(a+bx)}{2b} + \frac{cdx \sin^2(a+bx)}{2b} - \frac{cdx \cos^2(a+bx)}{2b} + \frac{d^2x^2 \sin^2(a+bx)}{4b} - \frac{d^2x^2 \cos^2(a+bx)}{4b} + \frac{cd \sin(a+bx) \cos(a+bx)}{2b^2} + \frac{d^2x \sin(a+bx) \cos(a+bx)}{2b^2} \right\} \left(c^2x + cdx^2 + \frac{d^2x^3}{3} \right) \sin(a) \cos(a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*cos(b*x+a)*sin(b*x+a),x)

[Out] Piecewise((c**2*sin(a + b*x)**2/(2*b) + c*d*x*sin(a + b*x)**2/(2*b) - c*d*x*cos(a + b*x)**2/(2*b) + d**2*x**2*sin(a + b*x)**2/(4*b) - d**2*x**2*cos(a + b*x)**2/(4*b) + c*d*sin(a + b*x)*cos(a + b*x)/(2*b**2) + d**2*x*sin(a + b*x)*cos(a + b*x)/(2*b**2) - d**2*sin(a + b*x)**2/(4*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)*cos(a), True))

Giac [A] time = 1.12213, size = 99, normalized size = 1.11

$$-\frac{(2b^2d^2x^2 + 4b^2cdx + 2b^2c^2 - d^2)\cos(2bx + 2a)}{8b^3} + \frac{(bd^2x + bcd)\sin(2bx + 2a)}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")

[Out] -1/8*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a)/b^3 + 1/4*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a)/b^3

3.5 $\int (c + dx) \cos(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=50

$$\frac{d \sin(a + bx) \cos(a + bx)}{4b^2} + \frac{(c + dx) \sin^2(a + bx)}{2b} - \frac{dx}{4b}$$

[Out] $-(d*x)/(4*b) + (d*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/(4*b^2) + ((c + d*x)*\text{Sin}[a + b*x]^2)/(2*b)$

Rubi [A] time = 0.0259538, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4404, 2635, 8}

$$\frac{d \sin(a + bx) \cos(a + bx)}{4b^2} + \frac{(c + dx) \sin^2(a + bx)}{2b} - \frac{dx}{4b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)*\text{Cos}[a + b*x]*\text{Sin}[a + b*x], x]$

[Out] $-(d*x)/(4*b) + (d*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/(4*b^2) + ((c + d*x)*\text{Sin}[a + b*x]^2)/(2*b)$

Rule 4404

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] :> \text{Simp}[(c + d*x)^m*\text{Sin}[a + b*x]^{(n + 1)}/(b*(n + 1)), x] - \text{Dist}[(d*m)/(b*(n + 1)), \text{Int}[(c + d*x)^{(m - 1)}*\text{Sin}[a + b*x]^{(n + 1)}, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 2635

$\text{Int}[(b_.)*\text{sin}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] :> -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{(n - 1)})/(d*n), x] + \text{Dist}[(b^2*(n - 1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n - 2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

$\text{Int}[a_, x_Symbol] :> \text{Simp}[a*x, x] /;$ FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int (c + dx) \cos(a + bx) \sin(a + bx) dx &= \frac{(c + dx) \sin^2(a + bx)}{2b} - \frac{d \int \sin^2(a + bx) dx}{2b} \\ &= \frac{d \cos(a + bx) \sin(a + bx)}{4b^2} + \frac{(c + dx) \sin^2(a + bx)}{2b} - \frac{d \int 1 dx}{4b} \\ &= -\frac{dx}{4b} + \frac{d \cos(a + bx) \sin(a + bx)}{4b^2} + \frac{(c + dx) \sin^2(a + bx)}{2b} \end{aligned}$$

Mathematica [A] time = 0.100934, size = 34, normalized size = 0.68

$$\frac{d \sin(2(a + bx)) - 2b(c + dx) \cos(2(a + bx))}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Cos[a + b*x]*Sin[a + b*x],x]

[Out] (-2*b*(c + d*x)*Cos[2*(a + b*x)] + d*Sin[2*(a + b*x)])/(8*b^2)

Maple [A] time = 0.014, size = 74, normalized size = 1.5

$$\frac{1}{b} \left(\frac{d}{b} \left(-\frac{(bx+a)(\cos(bx+a))^2}{2} + \frac{\cos(bx+a)\sin(bx+a)}{4} + \frac{bx}{4} + \frac{a}{4} \right) + \frac{ad(\cos(bx+a))^2}{2b} - \frac{c(\cos(bx+a))^2}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*cos(b*x+a)*sin(b*x+a),x)

[Out] 1/b*(d/b*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)+1/2/b*a*d*cos(b*x+a)^2-1/2*c*cos(b*x+a)^2)

Maxima [A] time = 1.1213, size = 88, normalized size = 1.76

$$\frac{4c \cos(bx+a)^2 - \frac{4ad \cos(bx+a)^2}{b} + \frac{(2(bx+a)\cos(2bx+2a) - \sin(2bx+2a))d}{b}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")

[Out] -1/8*(4*c*cos(b*x + a)^2 - 4*a*d*cos(b*x + a)^2/b + (2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*d/b)/b

Fricas [A] time = 0.464452, size = 108, normalized size = 2.16

$$\frac{bdx - 2(bdx + bc)\cos(bx+a)^2 + d\cos(bx+a)\sin(bx+a)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")

[Out] 1/4*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a))/b^2

Sympy [A] time = 0.599258, size = 80, normalized size = 1.6

$$\begin{cases} \frac{c \sin^2(a+bx)}{2b} + \frac{dx \sin^2(a+bx)}{4b} - \frac{dx \cos^2(a+bx)}{4b} + \frac{d \sin(a+bx) \cos(a+bx)}{4b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2} \right) \sin(a) \cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Piecewise((c*sin(a + b*x)**2/(2*b) + d*x*sin(a + b*x)**2/(4*b) - d*x*cos(a + b*x)**2/(4*b) + d*sin(a + b*x)*cos(a + b*x)/(4*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*sin(a)*cos(a), True))
```

Giac [A] time = 1.14172, size = 51, normalized size = 1.02

$$-\frac{(bdx + bc) \cos(2bx + 2a)}{4b^2} + \frac{d \sin(2bx + 2a)}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(b*d*x + b*c)*cos(2*b*x + 2*a)/b^2 + 1/8*d*sin(2*b*x + 2*a)/b^2
```

$$3.6 \quad \int \frac{\cos(a+bx) \sin(a+bx)}{c+dx} dx$$

Optimal. Leaf size=65

$$\frac{\sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{2d} + \frac{\cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{2d}$$

[Out] (CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d) + (Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)

Rubi [A] time = 0.139341, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {4406, 12, 3303, 3299, 3302}

$$\frac{\sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{2d} + \frac{\cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{2d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x), x]

[Out] (CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d) + (Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx} dx &= \int \frac{\sin(2a+2bx)}{2(c+dx)} dx \\
&= \frac{1}{2} \int \frac{\sin(2a+2bx)}{c+dx} dx \\
&= \frac{1}{2} \cos\left(2a - \frac{2bc}{d}\right) \int \frac{\sin\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx + \frac{1}{2} \sin\left(2a - \frac{2bc}{d}\right) \int \frac{\cos\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx \\
&= \frac{\text{Ci}\left(\frac{2bc}{d} + 2bx\right) \sin\left(2a - \frac{2bc}{d}\right)}{2d} + \frac{\cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.130577, size = 60, normalized size = 0.92

$$\frac{\sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right) + \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x), x]

[Out] (CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d] + Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)

Maple [A] time = 0.026, size = 84, normalized size = 1.3

$$\frac{1}{2d} \text{Si}\left(2bx + 2a + 2\frac{-ad+bc}{d}\right) \cos\left(2\frac{-ad+bc}{d}\right) - \frac{1}{2d} \text{Ci}\left(2bx + 2a + 2\frac{-ad+bc}{d}\right) \sin\left(2\frac{-ad+bc}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)*sin(b*x+a)/(d*x+c), x)

[Out] 1/2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-1/2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d

Maxima [C] time = 1.38025, size = 190, normalized size = 2.92

$$\frac{b\left(i E_1\left(\frac{2i bc+2i (bx+a)d-2i ad}{d}\right) - i E_1\left(-\frac{2i bc+2i (bx+a)d-2i ad}{d}\right)\right) \cos\left(-\frac{2(bc-ad)}{d}\right) + b\left(E_1\left(\frac{2i bc+2i (bx+a)d-2i ad}{d}\right) + E_1\left(-\frac{2i bc+2i (bx+a)d-2i ad}{d}\right)\right) \sin\left(-\frac{2(bc-ad)}{d}\right)}{4bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c), x, algorithm="maxima")

[Out] -1/4*(b*(I*exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) - I*exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c - a*d)/d) + b*(exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) + exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2*(b*c - a*d)/d)/(b*d)

Fricas [A] time = 0.470119, size = 213, normalized size = 3.28

$$\frac{\left(\operatorname{Ci}\left(\frac{2(bdx+bc)}{d}\right) + \operatorname{Ci}\left(-\frac{2(bdx+bc)}{d}\right)\right) \sin\left(-\frac{2(bc-ad)}{d}\right) + 2 \cos\left(-\frac{2(bc-ad)}{d}\right) \operatorname{Si}\left(\frac{2(bdx+bc)}{d}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="fricas")

[Out] 1/4*((cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d) + 2*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d)/d

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a + bx) \cos(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x)

[Out] Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x), x)

Giac [C] time = 1.20794, size = 768, normalized size = 11.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="giac")

[Out] 1/4*(imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d) + 2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d) - 2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 - imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 + imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - 2*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2 + 4*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) - 4*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 8*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d) - imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 + imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a) - 2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + imag_part(cos_integral(2*b*x + 2*b*c/d)) - imag_part(cos_integral(-2*b*x - 2*b*c/d)) + 2*sin_integral(2*(b*d*x + b*c)/d))/(d*tan(a)^2*tan(b*c/d)^2 + d*tan(a)^2 + d*tan(b*c/d)^2 + d)

$$3.7 \quad \int \frac{\cos(a+bx) \sin(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=85

$$\frac{b \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{d^2} - \frac{b \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^2} - \frac{\sin(2a + 2bx)}{2d(c + dx)}$$

[Out] (b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2 - Sin[2*a + 2*b*x]/(2*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^2

Rubi [A] time = 0.148834, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {4406, 12, 3297, 3303, 3299, 3302}

$$\frac{b \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{d^2} - \frac{b \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^2} - \frac{\sin(2a + 2bx)}{2d(c + dx)}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^2,x]

[Out] (b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2 - Sin[2*a + 2*b*x]/(2*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^2

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2} dx &= \int \frac{\sin(2a+2bx)}{2(c+dx)^2} dx \\ &= \frac{1}{2} \int \frac{\sin(2a+2bx)}{(c+dx)^2} dx \\ &= -\frac{\sin(2a+2bx)}{2d(c+dx)} + \frac{b \int \frac{\cos(2a+2bx)}{c+dx} dx}{d} \\ &= -\frac{\sin(2a+2bx)}{2d(c+dx)} + \frac{\left(b \cos\left(2a - \frac{2bc}{d}\right)\right) \int \frac{\cos\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx}{d} - \frac{\left(b \sin\left(2a - \frac{2bc}{d}\right)\right) \int \frac{\sin\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx}{d} \\ &= \frac{b \cos\left(2a - \frac{2bc}{d}\right) \text{Ci}\left(\frac{2bc}{d} + 2bx\right)}{d^2} - \frac{\sin(2a+2bx)}{2d(c+dx)} - \frac{b \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^2} \end{aligned}$$

Mathematica [A] time = 0.316456, size = 80, normalized size = 0.94

$$\frac{2b \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) - 2b \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2b(c+dx)}{d}\right) - \frac{d \sin(2(a+bx))}{c+dx}}{2d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^2, x]

[Out] (2*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - (d*Sin[2*(a + b*x)]/(c + d*x) - 2*b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/(2*d^2)

Maple [A] time = 0.02, size = 124, normalized size = 1.5

$$\frac{b}{4} \left(-2 \frac{\sin(2bx+2a)}{((bx+a)d - ad + bc)d} + 2 \frac{1}{d} \left(2 \frac{1}{d} \text{Si}\left(2bx+2a+2\frac{-ad+bc}{d}\right) \sin\left(2\frac{-ad+bc}{d}\right) + 2 \frac{1}{d} \text{Ci}\left(2bx+2a+2\frac{-ad+bc}{d}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2, x)

[Out] 1/4*b*(-2*sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)/d+2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)

Maxima [C] time = 1.45052, size = 221, normalized size = 2.6

$$\frac{b^2 \left(i E_2 \left(\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) - i E_2 \left(-\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) \right) \cos \left(-\frac{2(bc-ad)}{d} \right) + b^2 \left(E_2 \left(\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) + E_2 \left(-\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) \right) \sin \left(-\frac{2(bc-ad)}{d} \right)}{4 (bcd + (bx+a)d^2 - ad^2)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")

[Out] $-1/4*(b^2*(I*\exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) - I*\exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*\cos(-2*(b*c - a*d)/d) + b^2*(\exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) + \exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*\sin(-2*(b*c - a*d)/d)/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)$

Fricas [A] time = 0.493468, size = 333, normalized size = 3.92

$$\frac{2 d \cos (bx + a) \sin (bx + a) + 2 (bdx + bc) \sin \left(-\frac{2(bc-ad)}{d} \right) \operatorname{Si} \left(\frac{2(bdx+bc)}{d} \right) - (bdx + bc) \operatorname{Ci} \left(\frac{2(bdx+bc)}{d} \right) + (bdx + bc) \operatorname{Ci} \left(-\frac{2(bc-ad)}{d} \right)}{2 (d^3 x + cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")

[Out] $-1/2*(2*d*\cos(b*x + a)*\sin(b*x + a) + 2*(b*d*x + b*c)*\sin(-2*(b*c - a*d)/d)*\sin_integral(2*(b*d*x + b*c)/d) - ((b*d*x + b*c)*\cos_integral(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*\cos_integral(-2*(b*d*x + b*c)/d))*\cos(-2*(b*c - a*d)/d)/(d^3*x + c*d^2)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a + bx) \cos(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)**2,x)

[Out] Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx + a) \sin(bx + a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(cos(b*x + a)*sin(b*x + a)/(d*x + c)^2, x)

$$3.8 \quad \int \frac{\cos(a+bx) \sin(a+bx)}{(c+dx)^3} dx$$

Optimal. Leaf size=114

$$\frac{b^2 \sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{d^3} - \frac{b^2 \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^3} - \frac{b \cos(2a + 2bx)}{2d^2(c + dx)} - \frac{\sin(2a + 2bx)}{4d(c + dx)^2}$$

[Out] $-(b \cos[2a + 2bx]) / (2d^2(c + dx)) - (b^2 \text{CosIntegral}[(2bc)/d + 2bx] \sin[2a - (2bc)/d]) / d^3 - \sin[2a + 2bx] / (4d^2(c + dx)) - (b^2 \cos[2a - (2bc)/d] \text{SinIntegral}[(2bc)/d + 2bx]) / d^3$

Rubi [A] time = 0.174515, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {4406, 12, 3297, 3303, 3299, 3302}

$$\frac{b^2 \sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{d^3} - \frac{b^2 \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^3} - \frac{b \cos(2a + 2bx)}{2d^2(c + dx)} - \frac{\sin(2a + 2bx)}{4d(c + dx)^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\cos[a + bx] \sin[a + bx]) / (c + dx)^3, x]$

[Out] $-(b \cos[2a + 2bx]) / (2d^2(c + dx)) - (b^2 \text{CosIntegral}[(2bc)/d + 2bx] \sin[2a - (2bc)/d]) / d^3 - \sin[2a + 2bx] / (4d^2(c + dx)) - (b^2 \cos[2a - (2bc)/d] \text{SinIntegral}[(2bc)/d + 2bx]) / d^3$

Rule 4406

$\text{Int}[\cos[(a_.) + (b_.) \cdot (x_)]^{(p_.)} \cdot ((c_.) + (d_.) \cdot (x_))^{(m_.)} \sin[(a_.) + (b_.) \cdot (x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + dx)^m, \sin[a + bx]]^{n \cdot \cos[a + bx]^p}, x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 12

$\text{Int}[(a_.) \cdot (u_.), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$ FreeQ[a, x] && !MatchQ[u, (b_.) \cdot (v_)] /; FreeQ[b, x]

Rule 3297

$\text{Int}[((c_.) + (d_.) \cdot (x_))^{(m_.)} \sin[(e_.) + (f_.) \cdot (x_)], x_Symbol] \rightarrow \text{Simp}[(c + dx)^{(m+1)} \sin[e + fx] / (d \cdot (m+1)), x] - \text{Dist}[f / (d \cdot (m+1)), \text{Int}[(c + dx)^{(m+1)} \cos[e + fx], x], x] /;$ FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

$\text{Int}[\sin[(e_.) + (f_.) \cdot (x_)] / ((c_.) + (d_.) \cdot (x_)), x_Symbol] \rightarrow \text{Dist}[\cos[(d \cdot e - c \cdot f) / d], \text{Int}[\sin[(c \cdot f) / d + f \cdot x] / (c + d \cdot x), x], x] + \text{Dist}[\sin[(d \cdot e - c \cdot f) / d], \text{Int}[\cos[(c \cdot f) / d + f \cdot x] / (c + d \cdot x), x], x] /;$ FreeQ[{c, d, e, f}, x] && NeQ[d \cdot e - c \cdot f, 0]

Rule 3299

`Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]`

Rule 3302

`Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]`

Rubi steps

$$\begin{aligned}
 \int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3} dx &= \int \frac{\sin(2a+2bx)}{2(c+dx)^3} dx \\
 &= \frac{1}{2} \int \frac{\sin(2a+2bx)}{(c+dx)^3} dx \\
 &= -\frac{\sin(2a+2bx)}{4d(c+dx)^2} + \frac{b \int \frac{\cos(2a+2bx)}{(c+dx)^2} dx}{2d} \\
 &= -\frac{b \cos(2a+2bx)}{2d^2(c+dx)} - \frac{\sin(2a+2bx)}{4d(c+dx)^2} - \frac{b^2 \int \frac{\sin(2a+2bx)}{c+dx} dx}{d^2} \\
 &= -\frac{b \cos(2a+2bx)}{2d^2(c+dx)} - \frac{\sin(2a+2bx)}{4d(c+dx)^2} - \frac{\left(b^2 \cos\left(2a - \frac{2bc}{d}\right)\right) \int \frac{\sin\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx}{d^2} - \frac{\left(b^2 \sin\left(2a - \frac{2bc}{d}\right)\right) \int \frac{\cos\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx}{d^2} \\
 &= -\frac{b \cos(2a+2bx)}{2d^2(c+dx)} - \frac{b^2 \operatorname{Ci}\left(\frac{2bc}{d} + 2bx\right) \sin\left(2a - \frac{2bc}{d}\right)}{d^3} - \frac{\sin(2a+2bx)}{4d(c+dx)^2} - \frac{b^2 \cos\left(2a - \frac{2bc}{d}\right) \operatorname{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^3}
 \end{aligned}$$

Mathematica [A] time = 1.09059, size = 102, normalized size = 0.89

$$\frac{4b^2 \sin\left(2a - \frac{2bc}{d}\right) \operatorname{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) + 4b^2 \cos\left(2a - \frac{2bc}{d}\right) \operatorname{Si}\left(\frac{2b(c+dx)}{d}\right) + \frac{d(2b(c+dx)\cos(2(a+bx)) + d\sin(2(a+bx)))}{(c+dx)^2}}{4d^3}$$

Antiderivative was successfully verified.

`[In] Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^3, x]`

`[Out] -(4*b^2*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*(2*b*(c + d*x)*Cos[2*(a + b*x)] + d*Sin[2*(a + b*x)]))/(c + d*x)^2 + 4*b^2*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/(4*d^3)`

Maple [A] time = 0.02, size = 162, normalized size = 1.4

$$\frac{b^2}{4} \left(-\frac{\sin(2bx+2a)}{((bx+a)d-ad+bc)^2 d} + \frac{1}{d} \left(-2 \frac{\cos(2bx+2a)}{((bx+a)d-ad+bc)d} - 2 \frac{1}{d} \left(2 \frac{1}{d} \operatorname{Si}\left(2bx+2a+2\frac{-ad+bc}{d}\right) \cos\left(2\frac{-ad+bc}{d}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3, x)`

`[Out] 1/4*b^2*(-sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)^2/d+(-2*cos(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)`

Maxima [C] time = 1.74066, size = 269, normalized size = 2.36

$$\frac{b^3 \left(i E_3 \left(\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) - i E_3 \left(-\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) \right) \cos \left(-\frac{2(bc-ad)}{d} \right) + b^3 \left(E_3 \left(\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) + E_3 \left(-\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) \right) \sin \left(-\frac{2(bc-ad)}{d} \right)}{4 \left(b^2 c^2 d - 2 abcd^2 + (bx+a)^2 d^3 + a^2 d^3 + 2 (bcd^2 - ad^3) (bx+a) \right) b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")

[Out] -1/4*(b^3*(I*exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) - I*exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c - a*d)/d) + b^3*(exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) + exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2*(b*c - a*d)/d)/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)

Fricas [B] time = 0.513249, size = 525, normalized size = 4.61

$$\frac{bd^2x - d^2 \cos(bx+a) \sin(bx+a) + bcd - 2(bd^2x + bcd) \cos(bx+a)^2 - 2(b^2d^2x^2 + 2b^2cdx + b^2c^2) \cos\left(-\frac{2(bc-ad)}{d}\right) \operatorname{Si}\left(\frac{2(bc-ad)}{d}\right)}{2(d^5x^2 + 2cd^4x + c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")

[Out] 1/2*(b*d^2*x - d^2*cos(b*x + a)*sin(b*x + a) + b*c*d - 2*(b*d^2*x + b*c*d)*cos(b*x + a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-2*(b*d*x + b*c)/d)*sin(-2*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a + bx) \cos(a + bx)}{(c + dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)**3,x)

[Out] Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x)**3, x)

Giac [C] time = 1.60525, size = 7287, normalized size = 63.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/2*(b^2*d^2*x^2*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a) \\ &)^2*\tan(b*c/d)^2 - b^2*d^2*x^2*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d)^2 + 2*b^2*d^2*x^2*\sin_integral(2*(b*d*x + b*c) \\ & /d)*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d)^2 + 2*b^2*d^2*x^2*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d) + 2*b^2*d^2*x^2*\text{real_part} \\ & (\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d) - 2*b^2*d^2*x^2*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a)*\tan(b*c/d)^2 \\ & - 2*b^2*d^2*x^2*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a)*\tan(b*c/d)^2 + 2*b^2*c*d*x*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d)^2 \\ & - 2*b^2*c*d*x*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d)^2 + 4*b^2*c*d*x*\sin_integral(2*(b*d*x + b*c)/d)*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d)^2 \\ & - b^2*d^2*x^2*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a)^2 + b^2*d^2*x^2*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a)^2 \\ & - 2*b^2*d^2*x^2*\sin_integral(2*(b*d*x + b*c)/d)*\tan(b*x)^2*\tan(a)^2 + 4*b^2*d^2*x^2*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a)*\tan(b*c/d) \\ & - 4*b^2*d^2*x^2*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a)*\tan(b*c/d) + 8*b^2*d^2*x^2*\sin_integral(2*(b*d*x + b*c)/d)*\tan(b*x)^2*\tan(a)*\tan(b*c/d) \\ & + 4*b^2*c*d*x*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d) + 4*b^2*c*d*x*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d) \\ & - b^2*d^2*x^2*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(b*c/d)^2 + b^2*d^2*x^2*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(b*c/d)^2 \\ & - 2*b^2*d^2*x^2*\sin_integral(2*(b*d*x + b*c)/d)*\tan(b*x)^2*\tan(b*c/d)^2 - 4*b^2*c*d*x*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a)*\tan(b*c/d)^2 \\ & - 4*b^2*c*d*x*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a)*\tan(b*c/d)^2 + b^2*d^2*x^2*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(a)^2*\tan(b*c/d)^2 \\ & - b^2*d^2*x^2*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(a)^2*\tan(b*c/d)^2 + 2*b^2*d^2*x^2*\sin_integral(2*(b*d*x + b*c)/d)*\tan(a)^2*\tan(b*c/d)^2 + b^2*c^2*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d)^2 \\ & - b^2*c^2*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d)^2 + 2*b^2*c^2*\sin_integral(2*(b*d*x + b*c)/d)*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d)^2 + 2*b^2*d^2*x^2*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a) \\ & + 2*b^2*d^2*x^2*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a) - 2*b^2*c*d*x*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a)^2 + 2*b^2*c*d*x*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a)^2 \\ & - 4*b^2*c*d*x*\sin_integral(2*(b*d*x + b*c)/d)*\tan(b*x)^2*\tan(a)^2 - 2*b^2*d^2*x^2*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(b*c/d) - 2*b^2*d^2*x^2*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(b*c/d) \\ & + 8*b^2*c*d*x*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a)*\tan(b*c/d) - 8*b^2*c*d*x*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a)*\tan(b*c/d) \\ & + 16*b^2*c*d*x*\sin_integral(2*(b*d*x + b*c)/d)*\tan(b*x)^2*\tan(a)*\tan(b*c/d) + 2*b^2*d^2*x^2*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(a)^2*\tan(b*c/d) + 2*b^2*d^2*x^2*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(a)^2*\tan(b*c/d) \\ & + 2*b^2*c^2*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d) + 2*b^2*c^2*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d) - 2*b^2*c*d*x*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(b*c/d)^2 \\ & + 2*b^2*c*d*x*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(b*c/d)^2 - 4*b^2*c*d*x*\sin_integral(2*(b*d*x + b*c)/d)*\tan(b*x)^2*\tan(b*c/d)^2 - 2*b^2*d^2*x^2*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(a)*\tan(b*c/d)^2 \\ & - 2*b^2*d^2*x^2*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(a)*\tan(b*c/d)^2 - 2*b^2*c^2*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a)*\tan(b*c/d)^2 + 2*b^2*c*d*x*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(a)^2*\tan(b*c/d)^2 \\ & - 2*b^2*c*d*x*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(a)^2*\tan(b*c/d)^2 + 4*b^2*c*d*x*\sin_integral(2*(b*d*x + b*c)/d)*\tan(a)^2*\tan(b*c/d)^2 + b*d^2*x*\tan(\end{aligned}$$

$$\begin{aligned}
& (2*(b*d*x + b*c)/d)*\tan(b*x)^2 + 4*b^2*c*d*x*\text{real_part}(\cos_integral(2*b*x + \\
& 2*b*c/d))*\tan(a) + 4*b^2*c*d*x*\text{real_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(a) - b^2*c^2*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(a)^2 + b^2*c^2 \\
& *\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(a)^2 - 2*b^2*c^2*\sin_integral(2*(b*d*x + b*c)/d)*\tan(a)^2 + b*c*d*\tan(b*x)^2*\tan(a)^2 - 4*b^2*c*d*x*\text{rea} \\
& l_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(b*c/d) - 4*b^2*c*d*x*\text{real_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(b*c/d) + 4*b^2*c^2*\text{imag_part}(\cos_integral \\
& (2*b*x + 2*b*c/d))*\tan(a)*\tan(b*c/d) - 4*b^2*c^2*\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(a)*\tan(b*c/d) + 8*b^2*c^2*\sin_integral(2*(b*d*x + b*c)/ \\
& d)*\tan(a)*\tan(b*c/d) - b^2*c^2*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan \\
& (b*c/d)^2 + b^2*c^2*\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(b*c/d)^2 - 2*b^2*c^2*\sin_integral(2*(b*d*x + b*c)/d)*\tan(b*c/d)^2 - b*c*d*\tan(b*x)^2 \\
& *\tan(b*c/d)^2 - 4*b*c*d*\tan(b*x)*\tan(a)*\tan(b*c/d)^2 - d^2*\tan(b*x)^2*\tan(a) \\
& *\tan(b*c/d)^2 - b*c*d*\tan(a)^2*\tan(b*c/d)^2 - d^2*\tan(b*x)*\tan(a)^2*\tan(b* \\
& c/d)^2 + 2*b^2*c*d*x*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) - 2*b^2*c*d*x \\
& *\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d)) + 4*b^2*c*d*x*\sin_integral(2*(b* \\
& d*x + b*c)/d) - b*d^2*x*\tan(b*x)^2 + 2*b^2*c^2*\text{real_part}(\cos_integral(2*b*x \\
& + 2*b*c/d))*\tan(a) + 2*b^2*c^2*\text{real_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan \\
& (a) - 4*b*d^2*x*\tan(b*x)*\tan(a) - b*d^2*x*\tan(a)^2 - 2*b^2*c^2*\text{real_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(b*c/d) - 2*b^2*c^2*\text{real_part}(\cos_integra \\
& l(-2*b*x - 2*b*c/d))*\tan(b*c/d) + b*d^2*x*\tan(b*c/d)^2 + b^2*c^2*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) - b^2*c^2*\text{imag_part}(\cos_integral(-2*b*x - 2* \\
& b*c/d)) + 2*b^2*c^2*\sin_integral(2*(b*d*x + b*c)/d) - b*c*d*\tan(b*x)^2 - 4* \\
& b*c*d*\tan(b*x)*\tan(a) - d^2*\tan(b*x)^2*\tan(a) - b*c*d*\tan(a)^2 - d^2*\tan(b* \\
& x)*\tan(a)^2 + b*c*d*\tan(b*c/d)^2 + d^2*\tan(b*x)*\tan(b*c/d)^2 + d^2*\tan(a)*\tan \\
& (b*c/d)^2 + b*d^2*x + b*c*d + d^2*\tan(b*x) + d^2*\tan(a))/(d^5*x^2*\tan(b*x) \\
&)^2*\tan(a)^2*\tan(b*c/d)^2 + 2*c*d^4*x*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d)^2 + d^5 \\
& *x^2*\tan(b*x)^2*\tan(a)^2 + d^5*x^2*\tan(b*x)^2*\tan(b*c/d)^2 + d^5*x^2*\tan(a) \\
&)^2*\tan(b*c/d)^2 + c^2*d^3*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d)^2 + 2*c*d^4*x*\tan \\
& (b*x)^2*\tan(a)^2 + 2*c*d^4*x*\tan(b*x)^2*\tan(b*c/d)^2 + 2*c*d^4*x*\tan(a)^2*\tan \\
& (b*c/d)^2 + d^5*x^2*\tan(b*x)^2 + d^5*x^2*\tan(a)^2 + c^2*d^3*\tan(b*x)^2*\tan \\
& (a)^2 + d^5*x^2*\tan(b*c/d)^2 + c^2*d^3*\tan(b*x)^2*\tan(b*c/d)^2 + c^2*d^3*\tan \\
& (a)^2*\tan(b*c/d)^2 + 2*c*d^4*x*\tan(b*x)^2 + 2*c*d^4*x*\tan(a)^2 + 2*c*d^4*x \\
& *x*\tan(b*c/d)^2 + d^5*x^2 + c^2*d^3*\tan(b*x)^2 + c^2*d^3*\tan(a)^2 + c^2*d^3*\tan \\
& (b*c/d)^2 + 2*c*d^4*x + c^2*d^3)
\end{aligned}$$

3.9 $\int \frac{\cos(a+bx) \sin(a+bx)}{(c+dx)^4} dx$

Optimal. Leaf size=144

$$-\frac{2b^3 \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} + \frac{2b^3 \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} + \frac{b^2 \sin(2a + 2bx)}{3d^3(c + dx)} - \frac{b \cos(2a + 2bx)}{6d^2(c + dx)^2}$$

[Out] $-(b \cdot \text{Cos}[2a + 2bx]) / (6d^2(c + dx)^2) - (2b^3 \cdot \text{Cos}[2a - (2bc)/d] \cdot \text{CosIntegral}[(2bc)/d + 2bx]) / (3d^4) - \text{Sin}[2a + 2bx] / (6d(c + dx)^3) + (b^2 \cdot \text{Sin}[2a + 2bx]) / (3d^3(c + dx)) + (2b^3 \cdot \text{Sin}[2a - (2bc)/d] \cdot \text{SinIntegral}[(2bc)/d + 2bx]) / (3d^4)$

Rubi [A] time = 0.197655, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {4406, 12, 3297, 3303, 3299, 3302}

$$-\frac{2b^3 \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} + \frac{2b^3 \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} + \frac{b^2 \sin(2a + 2bx)}{3d^3(c + dx)} - \frac{b \cos(2a + 2bx)}{6d^2(c + dx)^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[a + bx] \cdot \text{Sin}[a + bx]) / (c + dx)^4, x]$

[Out] $-(b \cdot \text{Cos}[2a + 2bx]) / (6d^2(c + dx)^2) - (2b^3 \cdot \text{Cos}[2a - (2bc)/d] \cdot \text{CosIntegral}[(2bc)/d + 2bx]) / (3d^4) - \text{Sin}[2a + 2bx] / (6d(c + dx)^3) + (b^2 \cdot \text{Sin}[2a + 2bx]) / (3d^3(c + dx)) + (2b^3 \cdot \text{Sin}[2a - (2bc)/d] \cdot \text{SinIntegral}[(2bc)/d + 2bx]) / (3d^4)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)(x_.)]^{(p_.)} \cdot ((c_.) + (d_.)(x_.))^{(m_.)} \cdot \text{Sin}[(a_.) + (b_.)(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + dx)^m, \text{Sin}[a + bx]^n \cdot \text{Cos}[a + bx]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 12

$\text{Int}[(a_)(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$ FreeQ[a, x] && !MatchQ[u, (b_)(v_)] /; FreeQ[b, x]

Rule 3297

$\text{Int}[(c_.) + (d_.)(x_.)]^{(m_.)} \cdot \text{sin}[(e_.) + (f_.)(x_.)], x_Symbol] \rightarrow \text{Simp}[(c + dx)^{(m+1)} \cdot \text{Sin}[e + fx] / (d(m+1)), x] - \text{Dist}[f / (d(m+1)), \text{Int}[(c + dx)^{(m+1)} \cdot \text{Cos}[e + fx], x], x] /;$ FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

$\text{Int}[\text{sin}[(e_.) + (f_.)(x_.)] / ((c_.) + (d_.)(x_.)), x_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d \cdot e - c \cdot f) / d], \text{Int}[\text{Sin}[(c \cdot f) / d + f \cdot x] / (c + d \cdot x), x], x] + \text{Dist}[\text{Sin}[(d \cdot e - c \cdot f) / d], \text{Int}[\text{Cos}[(c \cdot f) / d + f \cdot x] / (c + d \cdot x), x], x] /;$ FreeQ[{c, d, e, f}, x] && NeQ[d \cdot e - c \cdot f, 0]

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4} dx &= \int \frac{\sin(2a+2bx)}{2(c+dx)^4} dx \\
&= \frac{1}{2} \int \frac{\sin(2a+2bx)}{(c+dx)^4} dx \\
&= -\frac{\sin(2a+2bx)}{6d(c+dx)^3} + \frac{b \int \frac{\cos(2a+2bx)}{(c+dx)^3} dx}{3d} \\
&= -\frac{b \cos(2a+2bx)}{6d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{6d(c+dx)^3} - \frac{b^2 \int \frac{\sin(2a+2bx)}{(c+dx)^2} dx}{3d^2} \\
&= -\frac{b \cos(2a+2bx)}{6d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{6d(c+dx)^3} + \frac{b^2 \sin(2a+2bx)}{3d^3(c+dx)} - \frac{(2b^3) \int \frac{\cos(2a+2bx)}{c+dx} dx}{3d^3} \\
&= -\frac{b \cos(2a+2bx)}{6d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{6d(c+dx)^3} + \frac{b^2 \sin(2a+2bx)}{3d^3(c+dx)} - \frac{\left(2b^3 \cos\left(2a - \frac{2bc}{d}\right)\right) \int \frac{\cos\left(\frac{2b}{d}x\right)}{c+dx} dx}{3d^3} \\
&= -\frac{b \cos(2a+2bx)}{6d^2(c+dx)^2} - \frac{2b^3 \cos\left(2a - \frac{2bc}{d}\right) \text{Ci}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} - \frac{\sin(2a+2bx)}{6d(c+dx)^3} + \frac{b^2 \sin(2a+2bx)}{3d^3(c+dx)}
\end{aligned}$$

Mathematica [A] time = 0.660525, size = 164, normalized size = 1.14

$$\frac{-4b^3(c+dx)^3 \left(\cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) - \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2b(c+dx)}{d}\right) \right) - d \cos(2bx) \left(\sin(2a) (d^2 - 2b^2(c+dx)) \right)}{6d^4(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^4, x]
```

```
[Out] (-d*cos[2*b*x]*(b*d*(c + d*x)*Cos[2*a] + (d^2 - 2*b^2*(c + d*x)^2)*Sin[2*a
])) + d*((-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*a] + b*d*(c + d*x)*Sin[2*a])*Sin[
2*b*x] - 4*b^3*(c + d*x)^3*(Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x)
)/d] - Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d]))/(6*d^4*(c + d*
x)^3)
```

Maple [A] time = 0.021, size = 200, normalized size = 1.4

$$\frac{b^3}{4} \left(-\frac{2 \sin(2bx+2a)}{3((bx+a)d - ad + bc)^3 d} + \frac{2}{3d} \left(-\frac{\cos(2bx+2a)}{((bx+a)d - ad + bc)^2 d} - \frac{1}{d} \left(-2 \frac{\sin(2bx+2a)}{((bx+a)d - ad + bc)d} + 2 \frac{1}{d} \text{Si}\left(2 \frac{bx+a}{d}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4, x)
```

[Out] $\frac{1}{4}b^3 \left(\frac{-2/3 \sin(2bx+2a)}{(bx+a)d - ad + bc} \right)^3 / d + \frac{2/3 (-\cos(2bx+2a))}{(bx+a)d - ad + bc} \left(\frac{-2 \sin(2bx+2a)}{(bx+a)d - ad + bc} \right) / d + 2 \left(\frac{2 \operatorname{Si}(2bx+2a+2(-ad+bc)/d) \sin(2(-ad+bc)/d)}{d} \right) / d + 2 \operatorname{Ci}(2bx+2a+2(-ad+bc)/d) \cos(2(-ad+bc)/d) / d$

Maxima [C] time = 2.11298, size = 336, normalized size = 2.33

$$\frac{b^4 \left(i E_4 \left(\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) - i E_4 \left(-\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) \right) \cos \left(-\frac{2(bc-ad)}{d} \right) + b^4 \left(E_4 \left(\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) + E_4 \left(-\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) \right) \sin \left(-\frac{2(bc-ad)}{d} \right)}{4 \left(b^3 c^3 d - 3 ab^2 c^2 d^2 + 3 a^2 b c d^3 + (bx+a)^3 d^4 - a^3 d^4 + 3 (bcd^3 - ad^4)(bx+a)^2 + 3 (b^2 c^2 d^2 - 2 abcd^3 + a^2 d^4) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x, algorithm="maxima")

[Out] $-\frac{1}{4}b^4 \left(I \exp_{\text{integral}}_e(4, (2I*bc + 2I*(bx+a)d - 2I*ad)/d) - I \exp_{\text{integral}}_e(4, -(2I*bc + 2I*(bx+a)d - 2I*ad)/d) \right) \cos(-2*(bc - ad)/d) + b^4 \left(\exp_{\text{integral}}_e(4, (2I*bc + 2I*(bx+a)d - 2I*ad)/d) + \exp_{\text{integral}}_e(4, -(2I*bc + 2I*(bx+a)d - 2I*ad)/d) \right) \sin(-2*(bc - ad)/d) / \left((b^3 c^3 d - 3 a^2 b c d^2 + 3 a^2 b c d^3 + (bx+a)^3 d^4 - a^3 d^4 + 3 (bcd^3 - ad^4)(bx+a)^2 + 3 (b^2 c^2 d^2 - 2 abcd^3 + a^2 d^4) \right) * (bx+a) \right)$

Fricas [B] time = 0.53332, size = 703, normalized size = 4.88

$$bd^3x + bcd^2 - 2(bd^3x + bcd^2) \cos(bx+a)^2 + 2(2b^2d^3x^2 + 4b^2cd^2x + 2b^2c^2d - d^3) \cos(bx+a) \sin(bx+a) + 4(b^3d^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x, algorithm="fricas")

[Out] $\frac{1}{6} \left(b^3 d^3 x^3 + b^2 c d^2 x^2 - 2(b^3 d^3 x^3 + b^2 c d^2 x^2) \cos(bx+a)^2 + 2(2b^2 d^3 x^2 + 4b^2 c d^2 x + 2b^2 c^2 d - d^3) \cos(bx+a) \sin(bx+a) + 4(b^3 d^3 x^3 + 3b^2 c d^2 x^2 + 3b^2 c^2 d x + b^3 c^2) \sin(-2*(bc - ad)/d) \sin_{\text{integral}}(2*(b*d*x + b*c)/d) - 2((b^3 d^3 x^3 + 3b^2 c d^2 x^2 + 3b^2 c^2 d x + b^3 c^2) \cos_{\text{integral}}(2*(b*d*x + b*c)/d) + (b^3 d^3 x^3 + 3b^2 c d^2 x^2 + 3b^2 c^2 d x + b^3 c^2) \cos_{\text{integral}}(-2*(b*d*x + b*c)/d) \right) \cos(-2*(bc - ad)/d) / (d^7 x^3 + 3c^2 d^6 x^2 + 3c^2 d^5 x + c^3 d^4)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a + bx) \cos(a + bx)}{(c + dx)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)**4,x)

[Out] Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x)**4, x)

Giac [C] time = 1.80927, size = 10249, normalized size = 71.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/6*(2*b^3*d^3*x^3*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan \\ & (a)^2*\tan(b*c/d)^2 + 2*b^3*d^3*x^3*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d) \\ &)*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d)^2 - 4*b^3*d^3*x^3*\text{imag_part}(\text{cos_integral}(2 \\ & *b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d) + 4*b^3*d^3*x^3*\text{imag_part}(c \\ & os_integral(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d) - 8*b^3*d^3*x \\ & ^3*\text{sin_integral}(2*(b*d*x + b*c)/d)*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d) + 4*b^3*d \\ & ^3*x^3*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a)*\tan(b*c/d \\ &)^2 - 4*b^3*d^3*x^3*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan \\ & (a)*\tan(b*c/d)^2 + 8*b^3*d^3*x^3*\text{sin_integral}(2*(b*d*x + b*c)/d)*\tan(b*x)^ \\ & 2*\tan(a)*\tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*\text{real_part}(\text{cos_integral}(2*b*x + 2*b* \\ & c/d))*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*\text{real_part}(\text{cos_inte \\ & gral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d)^2 - 2*b^3*d^3*x^3*re \\ & al_part(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a)^2 - 2*b^3*d^3*x^3* \\ & real_part(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a)^2 + 8*b^3*d^3*x \\ & ^3*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a)*\tan(b*c/d) + \\ & 8*b^3*d^3*x^3*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a)*\tan \\ & (b*c/d) - 12*b^3*c*d^2*x^2*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b \\ & *x)^2*\tan(a)^2*\tan(b*c/d) + 12*b^3*c*d^2*x^2*\text{imag_part}(\text{cos_integral}(-2*b*x \\ & - 2*b*c/d))*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d) - 24*b^3*c*d^2*x^2*\text{sin_integral} \\ & (2*(b*d*x + b*c)/d)*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d) - 2*b^3*d^3*x^3*\text{real_part} \\ & (\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(b*c/d)^2 - 2*b^3*d^3*x^3*rea \\ & l_part(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(b*c/d)^2 + 12*b^3*c*d \\ & ^2*x^2*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a)*\tan(b*c/d \\ &)^2 - 12*b^3*c*d^2*x^2*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2 \\ & *\tan(a)*\tan(b*c/d)^2 + 24*b^3*c*d^2*x^2*\text{sin_integral}(2*(b*d*x + b*c)/d)*\tan \\ & (b*x)^2*\tan(a)*\tan(b*c/d)^2 + 2*b^3*d^3*x^3*\text{real_part}(\text{cos_integral}(2*b*x + \\ & 2*b*c/d))*\tan(a)^2*\tan(b*c/d)^2 + 2*b^3*d^3*x^3*\text{real_part}(\text{cos_integral}(-2*b \\ & *x - 2*b*c/d))*\tan(a)^2*\tan(b*c/d)^2 + 6*b^3*c^2*d*x*\text{real_part}(\text{cos_integral} \\ & (2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d)^2 + 6*b^3*c^2*d*x*\text{real_pa} \\ & rt(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d)^2 - 4*b^3 \\ & *d^3*x^3*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a) + 4*b^3 \\ & *d^3*x^3*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a) - 8*b^ \\ & 3*d^3*x^3*\text{sin_integral}(2*(b*d*x + b*c)/d)*\tan(b*x)^2*\tan(a) - 6*b^3*c*d^2*x \\ & ^2*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a)^2 - 6*b^3*c*d \\ & ^2*x^2*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a)^2 + 4*b^ \\ & 3*d^3*x^3*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(b*c/d) - \\ & 4*b^3*d^3*x^3*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(b*c/ \\ & d) + 8*b^3*d^3*x^3*\text{sin_integral}(2*(b*d*x + b*c)/d)*\tan(b*x)^2*\tan(b*c/d) + \\ & 24*b^3*c*d^2*x^2*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a) \\ & *\tan(b*c/d) + 24*b^3*c*d^2*x^2*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan \\ & (b*x)^2*\tan(a)*\tan(b*c/d) - 4*b^3*d^3*x^3*\text{imag_part}(\text{cos_integral}(2*b*x + 2 \\ & *b*c/d))*\tan(a)^2*\tan(b*c/d) + 4*b^3*d^3*x^3*\text{imag_part}(\text{cos_integral}(-2*b*x \\ & - 2*b*c/d))*\tan(a)^2*\tan(b*c/d) - 8*b^3*d^3*x^3*\text{sin_integral}(2*(b*d*x + b*c \\ &)/d)*\tan(a)^2*\tan(b*c/d) - 12*b^3*c^2*d*x*\text{imag_part}(\text{cos_integral}(2*b*x + 2* \\ & b*c/d))*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d) + 12*b^3*c^2*d*x*\text{imag_part}(\text{cos_integ} \\ & ral(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d) - 24*b^3*c^2*d*x*\text{sin_} \\ & integral(2*(b*d*x + b*c)/d)*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d) - 6*b^3*c*d^2*x^ \\ & 2*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(b*c/d)^2 - 6*b^3*c \\ & *d^2*x^2*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(b*c/d)^2 \\ & + 4*b^3*d^3*x^3*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(a)*\tan(b*c/d) \end{aligned}$$

$$\begin{aligned}
&^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + \\
&12*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 12*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 24*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^3*c^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^3*c^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 12*b^3*c*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 12*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 24*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - 2*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 - 2*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - 6*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 6*b^3*c^2*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 12*b^3*c*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) - 12*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d) + 24*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d) + 8*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) + 8*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 24*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 24*b^3*c^2*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 12*b^3*c*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d) + 12*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d) - 24*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d) - 4*b^3*c^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 4*b^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 8*b^3*c^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - 2*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 6*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 6*b^3*c^2*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 12*b^3*c*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 12*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 24*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 4*b^2*d^3*x^2*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 4*b^3*c^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 4*b^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 8*b^3*c^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 6*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c^2*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 4*b^2*d^3*x^2*tan(b*x)*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a) - 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(a) - 12*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 12*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 24*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - 6*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 - 6*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - 2*b^3*c^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 2*b^3*c^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*ta
\end{aligned}$$

$$\begin{aligned}
& n(b*c/d) + 8*b^3*d^3*x^3*\sin_integral(2*(b*d*x + b*c)/d)*\tan(b*c/d) + 12*b^3*c^2*d*x*imag_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(b*c/d) - \\
& 12*b^3*c^2*d*x*imag_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(b*c/d) + 24*b^3*c^2*d*x*\sin_integral(2*(b*d*x + b*c)/d)*\tan(b*x)^2*\tan(b*c/d) \\
& + 24*b^3*c*d^2*x^2*\real_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(a)*\tan(b*c/d) + 24*b^3*c*d^2*x^2*\real_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(a)*\tan(b*c/d) \\
& + 8*b^3*c^3*\real_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a)*\tan(b*c/d) + 8*b^3*c^3*\real_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a)*\tan(b*c/d) \\
& - 12*b^3*c^2*d*x*imag_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(a)^2*\tan(b*c/d) + 12*b^3*c^2*d*x*imag_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(a)^2*\tan(b*c/d) \\
& - 24*b^3*c^2*d*x*\sin_integral(2*(b*d*x + b*c)/d)*\tan(a)^2*\tan(b*c/d) - 6*b^3*c*d^2*x^2*\real_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(b*c/d)^2 \\
& - 6*b^3*c*d^2*x^2*\real_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(b*c/d)^2 - 2*b^3*c^3*\real_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(b*c/d)^2 \\
& - 2*b^3*c^3*\real_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(b*c/d)^2 + 12*b^3*c^2*d*x*imag_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(a)*\tan(b*c/d)^2 \\
& - 12*b^3*c^2*d*x*imag_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(a)*\tan(b*c/d)^2 + 24*b^3*c^2*d*x*\sin_integral(2*(b*d*x + b*c)/d)*\tan(a)*\tan(b*c/d)^2 \\
& + 8*b^2*c*d^2*x*\tan(b*x)^2*\tan(a)*\tan(b*c/d)^2 + 2*b^3*c^3*\real_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(a)^2*\tan(b*c/d)^2 + 2*b^3*c^3*\real_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(a)^2*\tan(b*c/d)^2 \\
& + 8*b^2*c*d^2*x*\tan(b*x)*\tan(a)^2*\tan(b*c/d)^2 + b*d^3*x*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d)^2 + 2*b^3*d^3*x^3*\real_part(\cos_integral(2*b*x + 2*b*c/d)) \\
& + 2*b^3*d^3*x^3*\real_part(\cos_integral(-2*b*x - 2*b*c/d)) + 6*b^3*c^2*d*x*\real_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(b*x)^2 + 6*b^3*c^2*d*x*\real_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(b*x)^2 \\
& - 12*b^3*c*d^2*x^2*imag_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(a) + 12*b^3*c*d^2*x^2*imag_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(a) - 24*b^3*c*d^2*x^2*\sin_integral(2*(b*d*x + b*c)/d)*\tan(a) \\
& + 4*b^2*d^3*x^2*\tan(b*x)^2*\tan(a) - 4*b^3*c^3*imag_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(a) + 4*b^3*c^3*imag_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(a) \\
& - 8*b^3*c^3*\sin_integral(2*(b*d*x + b*c)/d)*\tan(b*x)^2*\tan(a) - 6*b^3*c^2*d*x*\real_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(a)^2 - 6*b^3*c^2*d*x*\real_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(a)^2 \\
& + 4*b^2*d^3*x^2*\tan(b*x)*\tan(a)^2 + 12*b^3*c*d^2*x^2*imag_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(b*c/d) - 12*b^3*c*d^2*x^2*imag_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(b*c/d) \\
& + 24*b^3*c*d^2*x^2*\sin_integral(2*(b*d*x + b*c)/d)*\tan(b*c/d) + 4*b^3*c^3*imag_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(b*c/d) - 4*b^3*c^3*imag_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(b*c/d) \\
& + 8*b^3*c^3*\sin_integral(2*(b*d*x + b*c)/d)*\tan(b*x)^2*\tan(b*c/d) + 24*b^3*c^2*d*x*\real_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(a)*\tan(b*c/d) + 24*b^3*c^2*d*x*\real_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(a)*\tan(b*c/d) \\
& - 4*b^3*c^3*imag_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(a)^2*\tan(b*c/d) + 4*b^3*c^3*imag_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(a)^2*\tan(b*c/d) - 8*b^3*c^3*\sin_integral(2*(b*d*x + b*c)/d)*\tan(a)^2*\tan(b*c/d) \\
& - 6*b^3*c^2*d*x*\real_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(b*c/d)^2 - 6*b^3*c^2*d*x*\real_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(b*c/d)^2 - 4*b^2*d^3*x^2*\tan(b*x)*\tan(b*c/d)^2 \\
& - 4*b^2*d^3*x^2*\tan(a)*\tan(b*c/d)^2 + 4*b^3*c^3*imag_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(a)*\tan(b*c/d)^2 - 4*b^3*c^3*imag_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(a)*\tan(b*c/d)^2 \\
& + 8*b^3*c^3*\sin_integral(2*(b*d*x + b*c)/d)*\tan(a)*\tan(b*c/d)^2 + 4*b^2*c^2*d*\tan(b*x)^2*\tan(a)*\tan(b*c/d)^2 + 4*b^2*c^2*d*\tan(b*x)*\tan(a)^2*\tan(b*c/d)^2 + b*c*d^2*\tan(b*x)^2*\tan(a)^2*\tan(b*c/d)^2 \\
& + 6*b^3*c*d^2*x^2*\real_part(\cos_integral(2*b*x + 2*b*c/d)) + 6*b^3*c*d^2*x^2*\real_part(\cos_integral(-2*b*x - 2*b*c/d)) + 2*b^3*c^3*\real_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(b*x)^2 \\
& + 2*b^3*c^3*\real_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(b*x)^2 - 12*b^3*c^2*d*x*imag_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(a) + 12*b^3*c^2*d*x*imag_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(a) \\
& - 24*b^3*c^2*d*x*\sin_integral(2*(b*d*x + b*c)/d)*\tan(a) + 8*b^2*c*d^2*x*\tan(b*x)^2*\tan(a) - 2*b^3*c^3*\real_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(a)^2 - 2*b^3*c^3*\real_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(a)^2
\end{aligned}$$

$$\begin{aligned}
& 2*b*x - 2*b*c/d)) * \tan(a)^2 + 8*b^2*c*d^2*x * \tan(b*x) * \tan(a)^2 + b*d^3*x * \tan(b*x)^2 * \tan(a)^2 + 12*b^3*c^2*d*x * \operatorname{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(b*c/d) - 12*b^3*c^2*d*x * \operatorname{imag_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(b*c/d) + 24*b^3*c^2*d*x * \sin_integral(2*(b*d*x + b*c)/d) * \tan(b*c/d) + 8*b^3*c^3 * \operatorname{real_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(a) * \tan(b*c/d) + 8*b^3*c^3 * \operatorname{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(a) * \tan(b*c/d) - 2*b^3*c^3 * \operatorname{real_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(b*c/d)^2 - 2*b^3*c^3 * \operatorname{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(b*c/d)^2 - 8*b^2*c*d^2*x * \tan(b*x) * \tan(b*c/d)^2 - b*d^3*x * \tan(b*x)^2 * \tan(b*c/d)^2 - 8*b^2*c*d^2*x * \tan(a) * \tan(b*c/d)^2 - 4*b*d^3*x * \tan(b*x) * \tan(a) * \tan(b*c/d)^2 - b*d^3*x * \tan(a)^2 * \tan(b*c/d)^2 + 6*b^3*c^2*d*x * \operatorname{real_part}(\cos_integral(2*b*x + 2*b*c/d)) + 6*b^3*c^2*d*x * \operatorname{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) - 4*b^2*d^3*x^2 * \tan(b*x) - 4*b^2*d^3*x^2 * \tan(a) - 4*b^3*c^3 * \operatorname{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(a) + 4*b^3*c^3 * \operatorname{imag_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(a) - 8*b^3*c^3 * \sin_integral(2*(b*d*x + b*c)/d) * \tan(a) + 4*b^2*c^2*d * \tan(b*x)^2 * \tan(a) + 4*b^2*c^2*d * \tan(b*x) * \tan(a)^2 + b*c*d^2 * \tan(b*x)^2 * \tan(a)^2 + 4*b^3*c^3 * \operatorname{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(b*c/d) - 4*b^3*c^3 * \operatorname{imag_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(b*c/d) + 8*b^3*c^3 * \sin_integral(2*(b*d*x + b*c)/d) * \tan(b*c/d) - 4*b^2*c^2*d * \tan(b*x) * \tan(b*c/d)^2 - b*c*d^2 * \tan(b*x)^2 * \tan(b*c/d)^2 - 4*b^2*c^2*d * \tan(a) * \tan(b*c/d)^2 - 4*b*c*d^2 * \tan(b*x) * \tan(a) * \tan(b*c/d)^2 - 2*d^3 * \tan(b*x)^2 * \tan(a) * \tan(b*c/d)^2 - b*c*d^2 * \tan(a)^2 * \tan(b*c/d)^2 - 2*d^3 * \tan(b*x) * \tan(a)^2 * \tan(b*c/d)^2 + 2*b^3*c^3 * \operatorname{real_part}(\cos_integral(2*b*x + 2*b*c/d)) + 2*b^3*c^3 * \operatorname{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) - 8*b^2*c*d^2*x * \tan(b*x) - b*d^3*x * \tan(b*x)^2 - 8*b^2*c*d^2*x * \tan(a) - 4*b*d^3*x * \tan(b*x) * \tan(a) - b*d^3*x * \tan(a)^2 + b*d^3*x * \tan(b*c/d)^2 - 4*b^2*c^2*d * \tan(b*x) - b*c*d^2 * \tan(b*x)^2 - 4*b^2*c^2*d * \tan(a) - 4*b*c*d^2 * \tan(b*x) * \tan(a) - 2*d^3 * \tan(b*x)^2 * \tan(a) - b*c*d^2 * \tan(a)^2 - 2*d^3 * \tan(b*x) * \tan(a)^2 + b*c*d^2 * \tan(b*c/d)^2 + 2*d^3 * \tan(b*x) * \tan(b*c/d)^2 + 2*d^3 * \tan(a) * \tan(b*c/d)^2 + b*d^3*x + b*c*d^2 + 2*d^3 * \tan(b*x) + 2*d^3 * \tan(a)) / (d^7*x^3 * \tan(b*x)^2 * \tan(a)^2 * \tan(b*c/d)^2 + 3*c*d^6*x^2 * \tan(b*x)^2 * \tan(a)^2 * \tan(b*c/d)^2 + d^7*x^3 * \tan(b*x)^2 * \tan(a)^2 + d^7*x^3 * \tan(b*x) * \tan(b*c/d)^2 + d^7*x^3 * \tan(a)^2 * \tan(b*c/d)^2 + 3*c^2*d^5*x * \tan(b*x)^2 * \tan(a)^2 * \tan(b*c/d)^2 + 3*c*d^6*x^2 * \tan(b*x)^2 * \tan(a)^2 + 3*c*d^6*x^2 * \tan(b*x) * \tan(b*c/d)^2 + 3*c*d^6*x^2 * \tan(a)^2 * \tan(b*c/d)^2 + c^3*d^4 * \tan(b*x)^2 * \tan(a)^2 * \tan(b*c/d)^2 + d^7*x^3 * \tan(b*x)^2 + d^7*x^3 * \tan(a)^2 + 3*c^2*d^5*x * \tan(b*x)^2 * \tan(a)^2 + d^7*x^3 * \tan(b*c/d)^2 + 3*c^2*d^5*x * \tan(b*x) * \tan(b*c/d)^2 + 3*c^2*d^5*x * \tan(a)^2 * \tan(b*c/d)^2 + 3*c*d^6*x^2 * \tan(b*x)^2 + 3*c*d^6*x^2 * \tan(a)^2 + c^3*d^4 * \tan(b*x)^2 * \tan(a)^2 + 3*c*d^6*x^2 * \tan(b*c/d)^2 + c^3*d^4 * \tan(b*x)^2 * \tan(b*c/d)^2 + c^3*d^4 * \tan(a)^2 * \tan(b*c/d)^2 + d^7*x^3 + 3*c^2*d^5*x * \tan(b*x)^2 + 3*c^2*d^5*x * \tan(a)^2 + 3*c^2*d^5*x * \tan(b*c/d)^2 + 3*c*d^6*x^2 + c^3*d^4 * \tan(b*x)^2 + c^3*d^4 * \tan(a)^2 + c^3*d^4 * \tan(b*c/d)^2 + 3*c^2*d^5*x + c^3*d^4)
\end{aligned}$$

$$3.10 \quad \int \frac{\cos(x) \sin(x)}{x} dx$$

Optimal. Leaf size=8

$$\frac{\text{Si}(2x)}{2}$$

[Out] SinIntegral[2*x]/2

Rubi [A] time = 0.0285319, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4406, 12, 3299}

$$\frac{\text{Si}(2x)}{2}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]*Sin[x])/x,x]

[Out] SinIntegral[2*x]/2

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cos(x) \sin(x)}{x} dx &= \int \frac{\sin(2x)}{2x} dx \\ &= \frac{1}{2} \int \frac{\sin(2x)}{x} dx \\ &= \frac{\text{Si}(2x)}{2} \end{aligned}$$

Mathematica [A] time = 0.0058604, size = 8, normalized size = 1.

$$\frac{\text{Si}(2x)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]*Sin[x])/x,x]

[Out] SinIntegral[2*x]/2

Maple [A] time = 0.026, size = 7, normalized size = 0.9

$$\frac{\text{Si}(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sin(x)/x,x)

[Out] 1/2*Si(2*x)

Maxima [C] time = 1.21864, size = 18, normalized size = 2.25

$$-\frac{1}{4}i \text{Ei}(2ix) + \frac{1}{4}i \text{Ei}(-2ix)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)/x,x, algorithm="maxima")

[Out] -1/4*I*Ei(2*I*x) + 1/4*I*Ei(-2*I*x)

Fricas [A] time = 0.451929, size = 31, normalized size = 3.88

$$\frac{1}{2} \text{Si}(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)/x,x, algorithm="fricas")

[Out] 1/2*sin_integral(2*x)

Sympy [A] time = 1.07542, size = 5, normalized size = 0.62

$$\frac{\text{Si}(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)/x,x)

[Out] Si(2*x)/2

Giac [A] time = 1.11652, size = 8, normalized size = 1.

$$\frac{1}{2} \operatorname{Si}(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x,x, algorithm="giac")
```

```
[Out] 1/2*sin_integral(2*x)
```

3.11 $\int \frac{\cos(x) \sin(x)}{x^2} dx$

Optimal. Leaf size=16

$$\text{CosIntegral}(2x) - \frac{\sin(2x)}{2x}$$

[Out] CosIntegral[2*x] - Sin[2*x]/(2*x)

Rubi [A] time = 0.0457541, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {4406, 12, 3297, 3302}

$$\text{CosIntegral}(2x) - \frac{\sin(2x)}{2x}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]*Sin[x])/x^2,x]

[Out] CosIntegral[2*x] - Sin[2*x]/(2*x)

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]]^n*Cos[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{\cos(x) \sin(x)}{x^2} dx &= \int \frac{\sin(2x)}{2x^2} dx \\
 &= \frac{1}{2} \int \frac{\sin(2x)}{x^2} dx \\
 &= -\frac{\sin(2x)}{2x} + \int \frac{\cos(2x)}{x} dx \\
 &= \text{Ci}(2x) - \frac{\sin(2x)}{2x}
 \end{aligned}$$

Mathematica [A] time = 0.0060185, size = 16, normalized size = 1.

$$\text{CosIntegral}(2x) - \frac{\sin(2x)}{2x}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]*Sin[x])/x^2,x]

[Out] CosIntegral[2*x] - Sin[2*x]/(2*x)

Maple [A] time = 0.026, size = 15, normalized size = 0.9

$$\text{Ci}(2x) - \frac{\sin(2x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sin(x)/x^2,x)

[Out] Ci(2*x)-1/2*sin(2*x)/x

Maxima [C] time = 1.20794, size = 20, normalized size = 1.25

$$\frac{1}{2} \Gamma(-1, 2ix) + \frac{1}{2} \Gamma(-1, -2ix)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)/x^2,x, algorithm="maxima")

[Out] 1/2*gamma(-1, 2*I*x) + 1/2*gamma(-1, -2*I*x)

Fricas [A] time = 0.459234, size = 95, normalized size = 5.94

$$\frac{x \text{Ci}(2x) + x \text{Ci}(-2x) - 2 \cos(x) \sin(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)/x^2,x, algorithm="fricas")

[Out] $1/2*(x*\cos_integral(2*x) + x*\cos_integral(-2*x) - 2*\cos(x)*\sin(x))/x$

Sympy [A] time = 2.15956, size = 22, normalized size = 1.38

$$-\log(x) + \frac{\log(x^2)}{2} + \text{Ci}(2x) - \frac{\sin(2x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*sin(x)/x**2,x)`

[Out] $-\log(x) + \log(x**2)/2 + \text{Ci}(2*x) - \sin(2*x)/(2*x)$

Giac [A] time = 1.12137, size = 26, normalized size = 1.62

$$\frac{2x \text{Ci}(2x) - \sin(2x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*sin(x)/x^2,x, algorithm="giac")`

[Out] $1/2*(2*x*\cos_integral(2*x) - \sin(2*x))/x$

$$3.12 \quad \int \frac{\cos(x) \sin(x)}{x^3} dx$$

Optimal. Leaf size=29

$$-\text{Si}(2x) - \frac{\sin(2x)}{4x^2} - \frac{\cos(2x)}{2x}$$

[Out] $-\text{Cos}[2*x]/(2*x) - \text{Sin}[2*x]/(4*x^2) - \text{SinIntegral}[2*x]$

Rubi [A] time = 0.0589954, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {4406, 12, 3297, 3299}

$$-\text{Si}(2x) - \frac{\sin(2x)}{4x^2} - \frac{\cos(2x)}{2x}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[x]*\text{Sin}[x])/x^3, x]$

[Out] $-\text{Cos}[2*x]/(2*x) - \text{Sin}[2*x]/(4*x^2) - \text{SinIntegral}[2*x]$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 12

$\text{Int}[(a_)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$ FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 3297

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m + 1)}*\text{Sin}[e + f*x]/(d*(m + 1)), x] - \text{Dist}[f/(d*(m + 1)), \text{Int}[(c + d*x)^{(m + 1)}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3299

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] \rightarrow \text{Simp}[\text{SinIntegral}[e + f*x]/d, x] /;$ FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x) \sin(x)}{x^3} dx &= \int \frac{\sin(2x)}{2x^3} dx \\
&= \frac{1}{2} \int \frac{\sin(2x)}{x^3} dx \\
&= -\frac{\sin(2x)}{4x^2} + \frac{1}{2} \int \frac{\cos(2x)}{x^2} dx \\
&= -\frac{\cos(2x)}{2x} - \frac{\sin(2x)}{4x^2} - \int \frac{\sin(2x)}{x} dx \\
&= -\frac{\cos(2x)}{2x} - \frac{\sin(2x)}{4x^2} - \text{Si}(2x)
\end{aligned}$$

Mathematica [A] time = 0.0075177, size = 29, normalized size = 1.

$$-\text{Si}(2x) - \frac{\sin(2x)}{4x^2} - \frac{\cos(2x)}{2x}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]*Sin[x])/x^3,x]

[Out] -Cos[2*x]/(2*x) - Sin[2*x]/(4*x^2) - SinIntegral[2*x]

Maple [A] time = 0.029, size = 26, normalized size = 0.9

$$-\frac{\cos(2x)}{2x} - \text{Si}(2x) - \frac{\sin(2x)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sin(x)/x^3,x)

[Out] -1/2*cos(2*x)/x-Si(2*x)-1/4*sin(2*x)/x^2

Maxima [C] time = 1.26484, size = 20, normalized size = 0.69

$$i\Gamma(-2, 2ix) - i\Gamma(-2, -2ix)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)/x^3,x, algorithm="maxima")

[Out] I*gamma(-2, 2*I*x) - I*gamma(-2, -2*I*x)

Fricas [A] time = 0.469186, size = 96, normalized size = 3.31

$$\frac{2x \cos(x)^2 + 2x^2 \text{Si}(2x) + \cos(x) \sin(x) - x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)/x^3,x, algorithm="fricas")

[Out] $-1/2*(2*x*\cos(x)^2 + 2*x^2*\sin_integral(2*x) + \cos(x)*\sin(x) - x)/x^2$

Sympy [A] time = 1.59581, size = 24, normalized size = 0.83

$$-\operatorname{Si}(2x) - \frac{\cos(2x)}{2x} - \frac{\sin(2x)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)/x**3,x)

[Out] $-\operatorname{Si}(2*x) - \cos(2*x)/(2*x) - \sin(2*x)/(4*x**2)$

Giac [A] time = 1.13422, size = 35, normalized size = 1.21

$$-\frac{4x^2 \operatorname{Si}(2x) + 2x \cos(2x) + \sin(2x)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)/x^3,x, algorithm="giac")

[Out] $-1/4*(4*x^2*\sin_integral(2*x) + 2*x*\cos(2*x) + \sin(2*x))/x^2$

3.13 $\int (c + dx)^m \cos(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=275

$$\frac{i e^{i\left(a - \frac{bc}{d}\right)} (c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, -\frac{ib(c+dx)}{d}\right)}{8b} + \frac{i 3^{-m-1} e^{3i\left(a - \frac{bc}{d}\right)} (c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, -\frac{3ib(c+dx)}{d}\right)}{8b}$$

```
[Out] ((-I/8)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])
/(b*(((I)*b*(c + d*x))/d)^m) + ((I/8)*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d
*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + ((I/8)*3^(-1 - m)*
E^((3*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(
b*(((I)*b*(c + d*x))/d)^m) - ((I/8)*3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((
3*I)*b*(c + d*x))/d])/(b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.32994, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4406, 3307, 2181}

$$\frac{i e^{i\left(a - \frac{bc}{d}\right)} (c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, -\frac{ib(c+dx)}{d}\right)}{8b} + \frac{i 3^{-m-1} e^{3i\left(a - \frac{bc}{d}\right)} (c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, -\frac{3ib(c+dx)}{d}\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m * Cos[a + b*x] * Sin[a + b*x]^2, x]
```

```
[Out] ((-I/8)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])
/(b*(((I)*b*(c + d*x))/d)^m) + ((I/8)*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d
*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + ((I/8)*3^(-1 - m)*
E^((3*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(
b*(((I)*b*(c + d*x))/d)^m) - ((I/8)*3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((
3*I)*b*(c + d*x))/d])/(b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n * Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m * E^(I*k*Pi) * E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:= -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^m \cos(a+bx) \sin^2(a+bx) dx &= \int \left(\frac{1}{4}(c+dx)^m \cos(a+bx) - \frac{1}{4}(c+dx)^m \cos(3a+3bx) \right) dx \\
&= \frac{1}{4} \int (c+dx)^m \cos(a+bx) dx - \frac{1}{4} \int (c+dx)^m \cos(3a+3bx) dx \\
&= \frac{1}{8} \int e^{-i(a+bx)}(c+dx)^m dx + \frac{1}{8} \int e^{i(a+bx)}(c+dx)^m dx - \frac{1}{8} \int e^{-i(3a+3bx)}(c+dx)^m dx \\
&\quad - \frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{ib(c+dx)}{d}\right)}{8b} + \frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, \frac{ib(c+dx)}{d}\right)}{8b}
\end{aligned}$$

Mathematica [A] time = 0.722672, size = 237, normalized size = 0.86

$$\frac{ie^{-\frac{3i(ad+bc)}{d}}(c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \left(3^{-m} \left(e^{\frac{6ibc}{d}} \Gamma\left(m+1, \frac{3ib(c+dx)}{d}\right) - e^{6ia} \left(\frac{ib(c+dx)}{d}\right)^{2m} \left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m} \Gamma\left(m+1, \frac{ib(c+dx)}{d}\right)\right)}{24b}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^m * Cos[a + b*x] * Sin[a + b*x]^2, x]

[Out] $((-I/24)*(c + d*x)^m*((3*E^{((2*I)*(2*a + (b*c)/d)})*\Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(((-I)*b*(c + d*x))/d)^m + (-3*E^{((2*I)*a + ((4*I)*b*c)/d})*\Gamma[m + 1, (I*b*(c + d*x))/d] + (-((E^{((6*I)*a)}*((I*b*(c + d*x))/d)^{(2*m)})*\Gamma[m + 1, ((-3*I)*b*(c + d*x))/d])/((b^2*(c + d*x)^2/d^2)^m) + E^{((6*I)*b*c)/d}*\Gamma[1 + m, ((3*I)*b*(c + d*x))/d])/3^m)/((I*b*(c + d*x))/d)^m)/b*E^{(((3*I)*(b*c + a*d))/d)}$

Maple [F] time = 0.335, size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) (\sin(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x)

[Out] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) \sin(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")

[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^2, x)

Fricas [A] time = 0.553723, size = 471, normalized size = 1.71

$$\frac{-ie^{\left(-\frac{dm \log\left(\frac{3ib}{d}\right) - 3ibc + 3iad}{d}\right)} \Gamma\left(m+1, \frac{3ibdx + 3ibc}{d}\right) + 3ie^{\left(-\frac{dm \log\left(\frac{ib}{d}\right) - ibc + iad}{d}\right)} \Gamma\left(m+1, \frac{ibdx + ibc}{d}\right) - 3ie^{\left(-\frac{dm \log\left(-\frac{ib}{d}\right) + ibc - iad}{d}\right)} \Gamma\left(m+1, \frac{-ibdx - ibc}{d}\right)}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")

[Out] 1/24*(-I*e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d)*gamma(m + 1, (3*I*b*d*x + 3*I*b*c)/d) + 3*I*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x + I*b*c)/d) - 3*I*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma(m + 1, (-I*b*d*x - I*b*c)/d) + I*e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d)*gamma(m + 1, (-3*I*b*d*x - 3*I*b*c)/d))/b

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**m*cos(b*x+a)*sin(b*x+a)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) \sin(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^2, x)

3.14 $\int (c + dx)^4 \cos(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=205

$$\frac{4d^2(c + dx)^2 \sin^3(a + bx)}{9b^3} - \frac{8d^2(c + dx)^2 \sin(a + bx)}{3b^3} - \frac{160d^3(c + dx) \cos(a + bx)}{27b^4} - \frac{8d^3(c + dx) \sin^2(a + bx) \cos(a + bx)}{27b^4}$$

```
[Out] (-160*d^3*(c + d*x)*Cos[a + b*x])/(27*b^4) + (8*d*(c + d*x)^3*Cos[a + b*x])
/(9*b^2) + (160*d^4*Sin[a + b*x])/(27*b^5) - (8*d^2*(c + d*x)^2*Sin[a + b*x
])/(3*b^3) - (8*d^3*(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2)/(27*b^4) + (4*d*
(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^2)/(9*b^2) + (8*d^4*Sin[a + b*x]^3)/(
81*b^5) - (4*d^2*(c + d*x)^2*Sin[a + b*x]^3)/(9*b^3) + ((c + d*x)^4*Sin[a +
b*x]^3)/(3*b)
```

Rubi [A] time = 0.199716, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4404, 3311, 3296, 2637, 3310}

$$\frac{4d^2(c + dx)^2 \sin^3(a + bx)}{9b^3} - \frac{8d^2(c + dx)^2 \sin(a + bx)}{3b^3} - \frac{160d^3(c + dx) \cos(a + bx)}{27b^4} - \frac{8d^3(c + dx) \sin^2(a + bx) \cos(a + bx)}{27b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (-160*d^3*(c + d*x)*Cos[a + b*x])/(27*b^4) + (8*d*(c + d*x)^3*Cos[a + b*x])
/(9*b^2) + (160*d^4*Sin[a + b*x])/(27*b^5) - (8*d^2*(c + d*x)^2*Sin[a + b*x
])/(3*b^3) - (8*d^3*(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2)/(27*b^4) + (4*d*
(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^2)/(9*b^2) + (8*d^4*Sin[a + b*x]^3)/(
81*b^5) - (4*d^2*(c + d*x)^2*Sin[a + b*x]^3)/(9*b^3) + ((c + d*x)^4*Sin[a +
b*x]^3)/(3*b)
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbo
l] := Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
Simp[(d*(b*Sine + f*x))^n/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sine + f*x)^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sine + f*x)^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^4 \cos(a + bx) \sin^2(a + bx) dx &= \frac{(c + dx)^4 \sin^3(a + bx)}{3b} - \frac{(4d) \int (c + dx)^3 \sin^3(a + bx) dx}{3b} \\
&= \frac{4d(c + dx)^3 \cos(a + bx) \sin^2(a + bx)}{9b^2} - \frac{4d^2(c + dx)^2 \sin^3(a + bx)}{9b^3} + \frac{(c + dx)^4 \sin^3(a + bx)}{3b} \\
&= \frac{8d(c + dx)^3 \cos(a + bx)}{9b^2} - \frac{8d^3(c + dx) \cos(a + bx) \sin^2(a + bx)}{27b^4} + \frac{4d(c + dx)^3 \cos(a + bx)}{9b^2} \\
&= -\frac{16d^3(c + dx) \cos(a + bx)}{27b^4} + \frac{8d(c + dx)^3 \cos(a + bx)}{9b^2} - \frac{8d^2(c + dx)^2 \sin(a + bx)}{3b^3} \\
&= -\frac{160d^3(c + dx) \cos(a + bx)}{27b^4} + \frac{8d(c + dx)^3 \cos(a + bx)}{9b^2} + \frac{16d^4 \sin(a + bx)}{27b^5} - \frac{8d^2(c + dx)^2 \sin(a + bx)}{3b^3} \\
&= -\frac{160d^3(c + dx) \cos(a + bx)}{27b^4} + \frac{8d(c + dx)^3 \cos(a + bx)}{9b^2} + \frac{160d^4 \sin(a + bx)}{27b^5} - \frac{8d^2(c + dx)^2 \sin(a + bx)}{3b^3}
\end{aligned}$$

Mathematica [A] time = 1.5138, size = 385, normalized size = 1.88

$$486b^4c^2d^2x^2 \sin(a + bx) - 162b^4c^2d^2x^2 \sin(3(a + bx)) - 972b^2c^2d^2 \sin(a + bx) + 36b^2c^2d^2 \sin(3(a + bx)) + 324b^4c^3dx \sin(a + bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (324*b*d*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 12*b*d*(c + d*x)*(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 81*b^4*c^4*Sin[a + b*x] - 972*b^2*c^2*d^2*Sin[a + b*x] + 1944*d^4*Sin[a + b*x] + 324*b^4*c^3*d*x*Sin[a + b*x] - 1944*b^2*c*d^3*x*Sin[a + b*x] + 486*b^4*c^2*d^2*x^2*Sin[a + b*x] - 972*b^2*d^4*x^2*Sin[a + b*x] + 324*b^4*c*d^3*x^3*Sin[a + b*x] + 81*b^4*d^4*x^4*Sin[a + b*x] - 27*b^4*c^4*Sin[3*(a + b*x)] + 36*b^2*c^2*d^2*Sin[3*(a + b*x)] - 8*d^4*Sin[3*(a + b*x)] - 108*b^4*c^3*d*x*Sin[3*(a + b*x)] + 72*b^2*c*d^3*x*Sin[3*(a + b*x)] - 162*b^4*c^2*d^2*x^2*Sin[3*(a + b*x)] + 36*b^2*d^4*x^2*Sin[3*(a + b*x)] - 108*b^4*c*d^3*x^3*Sin[3*(a + b*x)] - 27*b^4*d^4*x^4*Sin[3*(a + b*x)])/(324*b^5)
```

Maple [B] time = 0.058, size = 835, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^4*d^4*(1/3*(b*x+a)^4*sin(b*x+a)^3+4/9*(b*x+a)^3*(2+sin(b*x+a)^2)*cos(b*x+a)-8/3*(b*x+a)^2*sin(b*x+a)+160/27*sin(b*x+a)-16/3*(b*x+a)*cos(b*x+a)-4/9*(b*x+a)^2*sin(b*x+a)^3-8/27*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+8/81*sin(b*x+a)^3)-4/b^4*a*d^4*(1/3*(b*x+a)^3*sin(b*x+a)^3+1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)-4/3*cos(b*x+a)-4/3*(b*x+a)*sin(b*x+a)-2/9*(b*x+a)*sin(b*x+a)^3-2/27*(2+sin(b*x+a)^2)*cos(b*x+a))+4/b^3*c*d^3*(1/3*(b*x+a)^3*sin(b*x+a)^3+1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)-4/3*cos(b*x+a)-4/3*(b*x+a)*sin(b*x+a)-2/9*(b*x+a)*sin(b*x+a)^3-2/27*(2+sin(b*x+a)^2)*cos(b*x+a))+6/b^4*a^2*d^4*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))-12/b^3*a*c*d^3*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))+6/b^2*c^2*d^2*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))-4/b^4*a^3*d^4*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))+12/b^3*a^2*c*d^3*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))-12/b^2*a*c^2*d^2*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))+4/b*c^3*d*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))+1/3/b^4*a^4*d^4*sin(b*x+a)^3-4/3/b^3*a^3*c*d^3*sin(b*x+a)^3+2/b^2*a^2*c^2*d^2*sin(b*x+a)^3-4/3/b*a*c^3*d*sin(b*x+a)^3+1/3*c^4*sin(b*x+a)^3)
```

Maxima [B] time = 1.35791, size = 1188, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/324*(108*c^4*sin(b*x + a)^3 - 432*a*c^3*d*sin(b*x + a)^3/b + 648*a^2*c^2*d^2*sin(b*x + a)^3/b^2 - 432*a^3*c*d^3*sin(b*x + a)^3/b^3 + 108*a^4*d^4*sin(b*x + a)^3/b^4 - 36*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*c^3*d/b + 108*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*a*c^2*d^2/b^2 - 108*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*a^2*c*d^3/b^3 + 36*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*a^3*d^4/b^4 - 18*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))*c^2*d^2/b^2 + 36*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))*a*c*d^3/b^3 - 18*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))*a^2*d^4/b^4 - 12*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a) + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) - 27*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*c*d^3/b^3 + 12*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a) + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) - 27*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*a*d^4/b^4 - (12*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) - 324*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) + (27*(b*x + a)^4 - 36*(b*x + a)^2 + 8)*sin(3*b*x + 3*a) - 81*((b*x + a)^4 - 12*(b*x + a)^2 + 24)*sin(b*x + a))*d^4/b^4)/b
```

Fricas [A] time = 0.520872, size = 759, normalized size = 3.7

$$\frac{12(3b^3d^4x^3 + 9b^3cd^3x^2 + 3b^3c^3d - 2bcd^3 + (9b^3c^2d^2 - 2bd^4)x) \cos(bx + a)^3 - 36(3b^3d^4x^3 + 9b^3cd^3x^2 + 3b^3c^3d - 12b^3c^2d^2 - 2bd^4)x \sin(bx + a)^2}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")

[Out]
$$\frac{-1/81*(12*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d - 2*b*c*d^3 + (9*b^3*c^2*d^2 - 2*b*d^4)*x)*\cos(b*x + a)^3 - 36*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d - 14*b*c*d^3 + (9*b^3*c^2*d^2 - 14*b*d^4)*x)*\cos(b*x + a)^2 - (27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^2*d^2 - 252*b^2*c^2*d^2 + 488*d^4 + 18*(9*b^4*c^2*d^2 - 14*b^2*d^4)*x^2 - (27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^2*d^2 - 36*b^2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 36*(3*b^4*c^3*d - 2*b^2*c*d^3)*x)*\cos(b*x + a)^2 + 36*(3*b^4*c^3*d - 14*b^2*c*d^3)*x*\sin(b*x + a))/b^5$$

Sympy [A] time = 11.3384, size = 646, normalized size = 3.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**4*cos(b*x+a)*sin(b*x+a)**2,x)

[Out]
$$\text{Piecewise}((c**4*\sin(a + b*x)**3/(3*b) + 4*c**3*d*x*\sin(a + b*x)**3/(3*b) + 2*c**2*d**2*x**2*\sin(a + b*x)**3/b + 4*c*d**3*x**3*\sin(a + b*x)**3/(3*b) + d**4*x**4*\sin(a + b*x)**3/(3*b) + 4*c**3*d*\sin(a + b*x)**2*\cos(a + b*x)/(3*b**2) + 8*c**3*d*\cos(a + b*x)**3/(9*b**2) + 4*c**2*d**2*x*\sin(a + b*x)**2*\cos(a + b*x)/b**2 + 8*c**2*d**2*x*\cos(a + b*x)**3/(3*b**2) + 4*c*d**3*x**2*\sin(a + b*x)**2*\cos(a + b*x)/b**2 + 8*c*d**3*x**2*\cos(a + b*x)**3/(3*b**2) + 4*d**4*x**3*\sin(a + b*x)**2*\cos(a + b*x)/(3*b**2) + 8*d**4*x**3*\cos(a + b*x)**3/(9*b**2) - 28*c**2*d**2*\sin(a + b*x)**3/(9*b**3) - 8*c**2*d**2*\sin(a + b*x)*\cos(a + b*x)**2/(3*b**3) - 56*c*d**3*x*\sin(a + b*x)**3/(9*b**3) - 16*c*d**3*x*\sin(a + b*x)*\cos(a + b*x)**2/(3*b**3) - 28*d**4*x**2*\sin(a + b*x)**3/(9*b**3) - 8*d**4*x**2*\sin(a + b*x)*\cos(a + b*x)**2/(3*b**3) - 56*c*d**3*\sin(a + b*x)**2*\cos(a + b*x)/(9*b**4) - 160*c*d**3*\cos(a + b*x)**3/(27*b**4) - 56*d**4*x*\sin(a + b*x)**2*\cos(a + b*x)/(9*b**4) - 160*d**4*x*\cos(a + b*x)**3/(27*b**4) + 488*d**4*\sin(a + b*x)**3/(81*b**5) + 160*d**4*\sin(a + b*x)*\cos(a + b*x)**2/(27*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*\sin(a)**2*\cos(a), True))$$

Giac [A] time = 1.12878, size = 473, normalized size = 2.31

$$\frac{(3b^3d^4x^3 + 9b^3cd^3x^2 + 9b^3c^2d^2x + 3b^3c^3d - 2bd^4x - 2bcd^3) \cos(3bx + 3a)}{27b^5} + \frac{(b^3d^4x^3 + 3b^3cd^3x^2 + 3b^3c^2d^2x + b^3c^3d - 2bd^4x - 2bcd^3) \sin(3bx + 3a)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")

[Out]
$$-1/27*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*\cos(3*b*x + 3*a)/b^5 + (b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*\sin(3*b*x + 3*a)/b^5$$

$$\begin{aligned}
& + 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*\cos(b*x + a)/b^5 - 1 \\
& /324*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*c^3*d*x \\
& + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8*d^4)*\sin(3*b*x + 3*a)/b^5 \\
& + 1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 \\
& - 24*b^2*c*d^3*x - 12*b^2*c^2*d^2 + 24*d^4)*\sin(b*x + a)/b^5
\end{aligned}$$

3.15 $\int (c + dx)^3 \cos(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=151

$$-\frac{2d^2(c + dx) \sin^3(a + bx)}{9b^3} - \frac{4d^2(c + dx) \sin(a + bx)}{3b^3} + \frac{2d(c + dx)^2 \cos(a + bx)}{3b^2} + \frac{d(c + dx)^2 \sin^2(a + bx) \cos(a + bx)}{3b^2} + \dots$$

[Out] $(-14*d^3*\text{Cos}[a + b*x])/(9*b^4) + (2*d*(c + d*x)^2*\text{Cos}[a + b*x])/(3*b^2) + (2*d^3*\text{Cos}[a + b*x]^3)/(27*b^4) - (4*d^2*(c + d*x)*\text{Sin}[a + b*x])/(3*b^3) + (d*(c + d*x)^2*\text{Cos}[a + b*x]*\text{Sin}[a + b*x]^2)/(3*b^2) - (2*d^2*(c + d*x)*\text{Sin}[a + b*x]^3)/(9*b^3) + ((c + d*x)^3*\text{Sin}[a + b*x]^3)/(3*b)$

Rubi [A] time = 0.134578, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4404, 3311, 3296, 2638, 2633}

$$-\frac{2d^2(c + dx) \sin^3(a + bx)}{9b^3} - \frac{4d^2(c + dx) \sin(a + bx)}{3b^3} + \frac{2d(c + dx)^2 \cos(a + bx)}{3b^2} + \frac{d(c + dx)^2 \sin^2(a + bx) \cos(a + bx)}{3b^2} + \dots$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^3*\text{Cos}[a + b*x]*\text{Sin}[a + b*x]^2, x]$

[Out] $(-14*d^3*\text{Cos}[a + b*x])/(9*b^4) + (2*d*(c + d*x)^2*\text{Cos}[a + b*x])/(3*b^2) + (2*d^3*\text{Cos}[a + b*x]^3)/(27*b^4) - (4*d^2*(c + d*x)*\text{Sin}[a + b*x])/(3*b^3) + (d*(c + d*x)^2*\text{Cos}[a + b*x]*\text{Sin}[a + b*x]^2)/(3*b^2) - (2*d^2*(c + d*x)*\text{Sin}[a + b*x]^3)/(9*b^3) + ((c + d*x)^3*\text{Sin}[a + b*x]^3)/(3*b)$

Rule 4404

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] := \text{Simp}[(c + d*x)^m*\text{Sin}[a + b*x]^{(n + 1)}/(b*(n + 1)), x] - \text{Dist}[(d*m)/(b*(n + 1)), \text{Int}[(c + d*x)^{(m - 1)}*\text{Sin}[a + b*x]^{(n + 1)}, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 3311

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)*((b_.)*\text{sin}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] := \text{Simp}[(d*m*(c + d*x)^{(m - 1)}*(b*\text{Sin}[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n - 1))/n, \text{Int}[(c + d*x)^m*(b*\text{Sin}[e + f*x])^{(n - 2)}, x], x] - \text{Dist}[(d^2*m*(m - 1))/(f^2*n^2), \text{Int}[(c + d*x)^{(m - 2)}*(b*\text{Sin}[e + f*x])^n, x], x] - \text{Simp}[(b*(c + d*x)^m*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(n - 1)})/(f*n), x]) /;$ FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rule 3296

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)*\text{sin}[(e_.) + (f_.)*(x_.)]}, x_Symbol] := -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m - 1)}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

$\text{Int}[\text{sin}[(c_.) + (d_.)*(x_.)], x_Symbol] := -\text{Simp}[\text{Cos}[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^3 \cos(a + bx) \sin^2(a + bx) dx &= \frac{(c + dx)^3 \sin^3(a + bx)}{3b} - \frac{d \int (c + dx)^2 \sin^3(a + bx) dx}{b} \\ &= \frac{d(c + dx)^2 \cos(a + bx) \sin^2(a + bx)}{3b^2} - \frac{2d^2(c + dx) \sin^3(a + bx)}{9b^3} + \frac{(c + dx)^3 \sin^3(a + bx)}{3b^3} \\ &= \frac{2d(c + dx)^2 \cos(a + bx)}{3b^2} + \frac{d(c + dx)^2 \cos(a + bx) \sin^2(a + bx)}{3b^2} - \frac{2d^2(c + dx) \sin^3(a + bx)}{9b^3} \\ &= -\frac{2d^3 \cos(a + bx)}{9b^4} + \frac{2d(c + dx)^2 \cos(a + bx)}{3b^2} + \frac{2d^3 \cos^3(a + bx)}{27b^4} - \frac{4d^2(c + dx) \sin^3(a + bx)}{9b^3} \\ &= -\frac{14d^3 \cos(a + bx)}{9b^4} + \frac{2d(c + dx)^2 \cos(a + bx)}{3b^2} + \frac{2d^3 \cos^3(a + bx)}{27b^4} - \frac{4d^2(c + dx) \sin^3(a + bx)}{9b^3} \end{aligned}$$

Mathematica [A] time = 0.991882, size = 121, normalized size = 0.8

$$\frac{-81d \cos(a + bx) (b^2(c + dx)^2 - 2d^2) + d \cos(3(a + bx)) (9b^2(c + dx)^2 - 2d^2) + 6b(c + dx) \sin(a + bx) (\cos(2(a + bx)) - 1)}{108b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] -(-81*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + d*(-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 6*b*(c + d*x)*(26*d^2 - 3*b^2*(c + d*x)^2 + (-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/(108*b^4)
```

Maple [B] time = 0.019, size = 447, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^3*d^3*(1/3*(b*x+a)^3*sin(b*x+a)^3+1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)-4/3*cos(b*x+a)-4/3*(b*x+a)*sin(b*x+a)-2/9*(b*x+a)*sin(b*x+a)^3-2/27*(2+sin(b*x+a)^2)*cos(b*x+a))-3/b^3*a*d^3*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))+3/b^2*c*d^2*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))+3/b^3*a^2*d^3*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))-6/b^2*a*c*d^2*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))+3/b*c^2*d*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))-1/3/b^3*a^3*d^3*sin(b*x+a)^3+1/b^2*a^2*c*d^2*sin(b*x+a)^3-1/b*a*c^2*d*sin(b*x+a)^3+1/3*c^3*sin(b*x+a)^3)
```

Maxima [B] time = 1.11837, size = 674, normalized size = 4.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")

[Out] $\frac{1}{108}*(36*c^3*\sin(b*x + a)^3 - 108*a*c^2*d*\sin(b*x + a)^3/b + 108*a^2*c*d^2*\sin(b*x + a)^3/b^2 - 36*a^3*d^3*\sin(b*x + a)^3/b^3 - 9*(3*(b*x + a)*\sin(3*b*x + 3*a) - 9*(b*x + a)*\sin(b*x + a) + \cos(3*b*x + 3*a) - 9*\cos(b*x + a))*c^2*d/b + 18*(3*(b*x + a)*\sin(3*b*x + 3*a) - 9*(b*x + a)*\sin(b*x + a) + \cos(3*b*x + 3*a) - 9*\cos(b*x + a))*a*c*d^2/b^2 - 9*(3*(b*x + a)*\sin(3*b*x + 3*a) - 9*(b*x + a)*\sin(b*x + a) + \cos(3*b*x + 3*a) - 9*\cos(b*x + a))*a^2*d^3/b^3 - 3*(6*(b*x + a)*\cos(3*b*x + 3*a) - 54*(b*x + a)*\cos(b*x + a) + (9*(b*x + a)^2 - 2)*\sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*\sin(b*x + a))*c*d^2/b^2 + 3*(6*(b*x + a)*\cos(3*b*x + 3*a) - 54*(b*x + a)*\cos(b*x + a) + (9*(b*x + a)^2 - 2)*\sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*\sin(b*x + a))*a*d^3/b^3 - ((9*(b*x + a)^2 - 2)*\cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*\cos(b*x + a) + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*\sin(3*b*x + 3*a) - 27*((b*x + a)^3 - 6*b*x - 6*a)*\sin(b*x + a))*d^3/b^3)/b$

Fricas [A] time = 0.502087, size = 491, normalized size = 3.25

$$\frac{(9b^2d^3x^2 + 18b^2cd^2x + 9b^2c^2d - 2d^3)\cos(bx + a)^3 - 3(9b^2d^3x^2 + 18b^2cd^2x + 9b^2c^2d - 14d^3)\cos(bx + a) - 3(3b^3d^3x^3 + 9b^3cd^2x^2 + 3b^3c^2d - 14b^3cd^2 - (3b^3d^3x^3 + 9b^3cd^2x^2 + 3b^3c^2d - 2b^3cd^2 + (9b^3c^2d - 2b^3d^3)x)\cos(bx + a)^2 + (9b^3c^2d - 14b^3d^3)x)\sin(bx + a))/b^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")

[Out] $-1/27*((9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*\cos(b*x + a)^3 - 3*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 14*d^3)*\cos(b*x + a) - 3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^2*d - 14*b^3*c*d^2 - (3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^2*d - 2*b^3*c*d^2 + (9*b^3*c^2*d - 2*b^3*d^3)*x)*\cos(b*x + a)^2 + (9*b^3*c^2*d - 14*b^3*d^3)*x)\sin(b*x + a))/b^4$

Sympy [A] time = 5.26756, size = 391, normalized size = 2.59

$$\left\{ \frac{c^3 \sin^3(a+bx)}{3b} + \frac{c^2 dx \sin^3(a+bx)}{b} + \frac{cd^2 x^2 \sin^3(a+bx)}{b} + \frac{d^3 x^3 \sin^3(a+bx)}{3b} + \frac{c^2 d \sin^2(a+bx) \cos(a+bx)}{b^2} + \frac{2c^2 d \cos^3(a+bx)}{3b^2} + \frac{2cd^2 x \sin^2(a+bx) \cos(a+bx)}{b^2} \right\} \left(c^3 x + \frac{3c^2 dx^2}{2} + cd^2 x^3 + \frac{d^3 x^4}{4} \right) \sin^2(a) \cos(a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*cos(b*x+a)*sin(b*x+a)**2,x)

[Out] Piecewise((c**3*sin(a + b*x)**3/(3*b) + c**2*d*x*sin(a + b*x)**3/b + c*d**2*x**2*sin(a + b*x)**3/b + d**3*x**3*sin(a + b*x)**3/(3*b) + c**2*d*sin(a + b*x)**2*cos(a + b*x)/b**2 + 2*c**2*d*cos(a + b*x)**3/(3*b**2) + 2*c*d**2*x*sin(a + b*x)**2*cos(a + b*x)/b**2 + 4*c*d**2*x*cos(a + b*x)**3/(3*b**2) + d**3*x**2*sin(a + b*x)**2*cos(a + b*x)/b**2 + 2*d**3*x**2*cos(a + b*x)**3/(3*b**2) - 14*c*d**2*sin(a + b*x)**3/(9*b**3) - 4*c*d**2*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 14*d**3*x*sin(a + b*x)**3/(9*b**3) - 4*d**3*x*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 14*d**3*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 40*d**3*cos(a + b*x)**3/(27*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**2*cos(a), True))

Giac [A] time = 1.16585, size = 312, normalized size = 2.07

$$\frac{(9b^2d^3x^2 + 18b^2cd^2x + 9b^2c^2d - 2d^3)\cos(3bx + 3a)}{108b^4} + \frac{3(b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3)\cos(bx + a)}{4b^4} - \frac{(3b^3d^3x^2 + 9b^3cd^2x + 3b^3c^2d - 2d^3)\sin(bx + a)}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")

[Out] -1/108*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(3*b*x + 3*a)/b^4 + 3/4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a)/b^4 - 1/36*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^3*x - 2*b*c*d^2)*sin(3*b*x + 3*a)/b^4 + 1/4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(b*x + a)/b^4

3.16 $\int (c + dx)^2 \cos(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=103

$$\frac{4d(c + dx) \cos(a + bx)}{9b^2} + \frac{2d(c + dx) \sin^2(a + bx) \cos(a + bx)}{9b^2} - \frac{2d^2 \sin^3(a + bx)}{27b^3} - \frac{4d^2 \sin(a + bx)}{9b^3} + \frac{(c + dx)^2 \sin^3(a + bx)}{3b}$$

[Out] (4*d*(c + d*x)*Cos[a + b*x])/(9*b^2) - (4*d^2*Sin[a + b*x])/(9*b^3) + (2*d*(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2)/(9*b^2) - (2*d^2*Sin[a + b*x]^3)/(27*b^3) + ((c + d*x)^2*Sin[a + b*x]^3)/(3*b)

Rubi [A] time = 0.0772006, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4404, 3310, 3296, 2637}

$$\frac{4d(c + dx) \cos(a + bx)}{9b^2} + \frac{2d(c + dx) \sin^2(a + bx) \cos(a + bx)}{9b^2} - \frac{2d^2 \sin^3(a + bx)}{27b^3} - \frac{4d^2 \sin(a + bx)}{9b^3} + \frac{(c + dx)^2 \sin^3(a + bx)}{3b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2,x]

[Out] (4*d*(c + d*x)*Cos[a + b*x])/(9*b^2) - (4*d^2*Sin[a + b*x])/(9*b^3) + (2*d*(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2)/(9*b^2) - (2*d^2*Sin[a + b*x]^3)/(27*b^3) + ((c + d*x)^2*Sin[a + b*x]^3)/(3*b)

Rule 4404

Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 3310

Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (c + dx)^2 \cos(a + bx) \sin^2(a + bx) dx &= \frac{(c + dx)^2 \sin^3(a + bx)}{3b} - \frac{(2d) \int (c + dx) \sin^3(a + bx) dx}{3b} \\
&= \frac{2d(c + dx) \cos(a + bx) \sin^2(a + bx)}{9b^2} - \frac{2d^2 \sin^3(a + bx)}{27b^3} + \frac{(c + dx)^2 \sin^3(a + bx)}{3b} \\
&= \frac{4d(c + dx) \cos(a + bx)}{9b^2} + \frac{2d(c + dx) \cos(a + bx) \sin^2(a + bx)}{9b^2} - \frac{2d^2 \sin^3(a + bx)}{27b^3} \\
&= \frac{4d(c + dx) \cos(a + bx)}{9b^2} - \frac{4d^2 \sin(a + bx)}{9b^3} + \frac{2d(c + dx) \cos(a + bx) \sin^2(a + bx)}{9b^2}
\end{aligned}$$

Mathematica [A] time = 0.6145, size = 93, normalized size = 0.9

$$\frac{-2 \sin(a + bx) (\cos(2(a + bx)) (9b^2(c + dx)^2 - 2d^2) - 9b^2(c + dx)^2 + 26d^2) + 54bd(c + dx) \cos(a + bx) - 6bd(c + dx)}{108b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2,x]

[Out] (54*b*d*(c + d*x)*Cos[a + b*x] - 6*b*d*(c + d*x)*Cos[3*(a + b*x)] - 2*(26*d^2 - 9*b^2*(c + d*x)^2 + (-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x]/(108*b^3)

Maple [B] time = 0.016, size = 204, normalized size = 2.

$$\frac{1}{b} \left(\frac{d^2}{b^2} \left(\frac{(bx + a)^2 (\sin(bx + a))^3}{3} + \frac{(2bx + 2a) (2 + (\sin(bx + a))^2) \cos(bx + a)}{9} - \frac{2 (\sin(bx + a))^3}{27} - \frac{4 \sin(bx + a)}{9} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x)

[Out] 1/b*(1/b^2*d^2*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))-2/b^2*a*d^2*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))+2/b*c*d*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))+1/3/b^2*a^2*d^2*sin(b*x+a)^3-2/3/b*a*c*d*sin(b*x+a)^3+1/3*c^2*sin(b*x+a)^3)

Maxima [B] time = 1.10982, size = 324, normalized size = 3.15

$$\frac{36c^2 \sin^3(bx + a) - \frac{72acd \sin^3(bx+a)}{b} + \frac{36a^2d^2 \sin^3(bx+a)}{b^2} - \frac{6(3(bx+a) \sin(3bx+3a) - 9(bx+a) \sin(bx+a) + \cos(3bx+3a) - 9 \cos(bx+a))cd}{b} + \frac{6(3(bx+a) \sin(3bx+3a) - 9(bx+a) \sin(bx+a) + \cos(3bx+3a) - 9 \cos(bx+a))d^2}{b^2}}{108}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")

[Out] 1/108*(36*c^2*sin(b*x + a)^3 - 72*a*c*d*sin(b*x + a)^3/b + 36*a^2*d^2*sin(b*x + a)^3/b^2 - 6*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*c*d/b + 6*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*d^2/b^2)

$$\begin{aligned} & - 9*(b*x + a)*\sin(b*x + a) + \cos(3*b*x + 3*a) - 9*\cos(b*x + a))*a*d^2/b^2 \\ & - (6*(b*x + a)*\cos(3*b*x + 3*a) - 54*(b*x + a)*\cos(b*x + a) + (9*(b*x + a) \\ & ^2 - 2)*\sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*\sin(b*x + a))*d^2/b^2)/b \end{aligned}$$

Fricas [A] time = 0.479784, size = 296, normalized size = 2.87

$$\frac{6(bd^2x + bcd)\cos(bx + a)^3 - 18(bd^2x + bcd)\cos(bx + a) - (9b^2d^2x^2 + 18b^2cdx + 9b^2c^2 - (9b^2d^2x^2 + 18b^2cdx + 9b^2c^2 - 2)d^2)\sin(3bx + 3a) - 27((b^2x^2 + 2bx + a)^2 - 2)\sin(bx + a)}{27b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")

[Out] -1/27*(6*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 18*(b*d^2*x + b*c*d)*cos(b*x + a) - (9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - (9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - 14*d^2)*sin(b*x + a))/b^3

Sympy [A] time = 2.62016, size = 216, normalized size = 2.1

$$\left\{ \begin{array}{l} \frac{c^2 \sin^3(a+bx)}{3b} + \frac{2cdx \sin^3(a+bx)}{3b} + \frac{d^2x^2 \sin^3(a+bx)}{3b} + \frac{2cd \sin^2(a+bx) \cos(a+bx)}{3b^2} + \frac{4cd \cos^3(a+bx)}{9b^2} + \frac{2d^2x \sin^2(a+bx) \cos(a+bx)}{3b^2} + \frac{4d^2x \cos^3(a+bx)}{9b^2} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3} \right) \sin^2(a) \cos(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*cos(b*x+a)*sin(b*x+a)**2,x)

[Out] Piecewise(((c**2*sin(a + b*x)**3/(3*b) + 2*c*d*x*sin(a + b*x)**3/(3*b) + d**2*x**2*sin(a + b*x)**3/(3*b) + 2*c*d*sin(a + b*x)**2*cos(a + b*x)/(3*b**2) + 4*c*d*cos(a + b*x)**3/(9*b**2) + 2*d**2*x*sin(a + b*x)**2*cos(a + b*x)/(3*b**2) + 4*d**2*x*cos(a + b*x)**3/(9*b**2) - 14*d**2*sin(a + b*x)**3/(27*b**3) - 4*d**2*sin(a + b*x)*cos(a + b*x)**2/(9*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**2*cos(a), True))

Giac [A] time = 1.1409, size = 185, normalized size = 1.8

$$-\frac{(bd^2x + bcd)\cos(3bx + 3a)}{18b^3} + \frac{(bd^2x + bcd)\cos(bx + a)}{2b^3} - \frac{(9b^2d^2x^2 + 18b^2cdx + 9b^2c^2 - 2d^2)\sin(3bx + 3a)}{108b^3} + \frac{(b^2x^2 + 2bx + a)^2 \sin(bx + a)}{27b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")

[Out] -1/18*(b*d^2*x + b*c*d)*cos(3*b*x + 3*a)/b^3 + 1/2*(b*d^2*x + b*c*d)*cos(b*x + a)/b^3 - 1/108*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*sin(3*b*x + 3*a)/b^3 + 1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b*x + a)/b^3

3.17 $\int (c + dx) \cos(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=51

$$-\frac{d \cos^3(a + bx)}{9b^2} + \frac{d \cos(a + bx)}{3b^2} + \frac{(c + dx) \sin^3(a + bx)}{3b}$$

[Out] (d*Cos[a + b*x])/(3*b^2) - (d*Cos[a + b*x]^3)/(9*b^2) + ((c + d*x)*Sin[a + b*x]^3)/(3*b)

Rubi [A] time = 0.0332234, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {4404, 2633}

$$-\frac{d \cos^3(a + bx)}{9b^2} + \frac{d \cos(a + bx)}{3b^2} + \frac{(c + dx) \sin^3(a + bx)}{3b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2,x]

[Out] (d*Cos[a + b*x])/(3*b^2) - (d*Cos[a + b*x]^3)/(9*b^2) + ((c + d*x)*Sin[a + b*x]^3)/(3*b)

Rule 4404

Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*SIN[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^(n - 1)/2], x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rubi steps

$$\begin{aligned} \int (c + dx) \cos(a + bx) \sin^2(a + bx) dx &= \frac{(c + dx) \sin^3(a + bx)}{3b} - \frac{d \int \sin^3(a + bx) dx}{3b} \\ &= \frac{(c + dx) \sin^3(a + bx)}{3b} + \frac{d \text{Subst}\left(\int (1 - x^2) dx, x, \cos(a + bx)\right)}{3b^2} \\ &= \frac{d \cos(a + bx)}{3b^2} - \frac{d \cos^3(a + bx)}{9b^2} + \frac{(c + dx) \sin^3(a + bx)}{3b} \end{aligned}$$

Mathematica [A] time = 0.181565, size = 44, normalized size = 0.86

$$\frac{12b(c + dx) \sin^3(a + bx) + 9d \cos(a + bx) - d \cos(3(a + bx))}{36b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2,x]

[Out] (9*d*Cos[a + b*x] - d*Cos[3*(a + b*x)] + 12*b*(c + d*x)*Sin[a + b*x]^3)/(36*b^2)

Maple [A] time = 0.018, size = 71, normalized size = 1.4

$$\frac{1}{b} \left(\frac{d}{b} \left(\frac{(bx+a)(\sin(bx+a))^3}{3} + \frac{(2+(\sin(bx+a))^2)\cos(bx+a)}{9} \right) - \frac{ad(\sin(bx+a))^3}{3b} + \frac{c(\sin(bx+a))^3}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x)

[Out] 1/b*(d/b*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))-1/3/b*a*d*sin(b*x+a)^3+1/3*c*sin(b*x+a)^3)

Maxima [A] time = 1.13164, size = 115, normalized size = 2.25

$$\frac{12c \sin(bx+a)^3 - \frac{12ad \sin(bx+a)^3}{b} - \frac{(3(bx+a)\sin(3bx+3a)-9(bx+a)\sin(bx+a)+\cos(3bx+3a)-9\cos(bx+a))d}{b}}{36b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")

[Out] 1/36*(12*c*sin(b*x + a)^3 - 12*a*d*sin(b*x + a)^3/b - (3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*d/b)/b

Fricas [A] time = 0.471744, size = 149, normalized size = 2.92

$$\frac{d \cos(bx+a)^3 - 3d \cos(bx+a) - 3(bdx - (bdx + bc)\cos(bx+a)^2 + bc)\sin(bx+a)}{9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")

[Out] -1/9*(d*cos(b*x + a)^3 - 3*d*cos(b*x + a) - 3*(b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*sin(b*x + a))/b^2

Sympy [A] time = 1.17803, size = 85, normalized size = 1.67

$$\begin{cases} \frac{c \sin^3(a+bx)}{3b} + \frac{dx \sin^3(a+bx)}{3b} + \frac{d \sin^2(a+bx) \cos(a+bx)}{3b^2} + \frac{2d \cos^3(a+bx)}{9b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2} \right) \sin^2(a) \cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c*sin(a + b*x)**3/(3*b) + d*x*sin(a + b*x)**3/(3*b) + d*sin(a +
b*x)**2*cos(a + b*x)/(3*b**2) + 2*d*cos(a + b*x)**3/(9*b**2), Ne(b, 0)), ((
c*x + d*x**2/2)*sin(a)**2*cos(a), True))
```

Giac [A] time = 1.10169, size = 93, normalized size = 1.82

$$-\frac{d \cos(3bx + 3a)}{36b^2} + \frac{d \cos(bx + a)}{4b^2} - \frac{(bdx + bc) \sin(3bx + 3a)}{12b^2} + \frac{(bdx + bc) \sin(bx + a)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/36*d*cos(3*b*x + 3*a)/b^2 + 1/4*d*cos(b*x + a)/b^2 - 1/12*(b*d*x + b*c)*
sin(3*b*x + 3*a)/b^2 + 1/4*(b*d*x + b*c)*sin(b*x + a)/b^2
```

3.18 $\int \frac{\cos(a+bx) \sin^2(a+bx)}{c+dx} dx$

Optimal. Leaf size=121

$$\frac{\cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{4d} - \frac{\cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{4d} - \frac{\sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{4d} + \frac{\sin\left(3a - \frac{3bc}{d}\right) \text{Si}\left(\frac{3bc}{d} + 3bx\right)}{4d}$$

[Out] (Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d) - (Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d) - (Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(4*d) + (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(4*d)

Rubi [A] time = 0.26993, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4406, 3303, 3299, 3302}

$$\frac{\cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{4d} - \frac{\cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{4d} - \frac{\sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{4d} + \frac{\sin\left(3a - \frac{3bc}{d}\right) \text{Si}\left(\frac{3bc}{d} + 3bx\right)}{4d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x), x]

[Out] (Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d) - (Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d) - (Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(4*d) + (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(4*d)

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx} dx &= \int \left(\frac{\cos(a+bx)}{4(c+dx)} - \frac{\cos(3a+3bx)}{4(c+dx)} \right) dx \\
&= \frac{1}{4} \int \frac{\cos(a+bx)}{c+dx} dx - \frac{1}{4} \int \frac{\cos(3a+3bx)}{c+dx} dx \\
&= - \left(\frac{1}{4} \cos\left(3a - \frac{3bc}{d}\right) \int \frac{\cos\left(\frac{3bc}{d} + 3bx\right)}{c+dx} dx \right) + \frac{1}{4} \cos\left(a - \frac{bc}{d}\right) \int \frac{\cos\left(\frac{bc}{d} + bx\right)}{c+dx} dx + \dots \\
&= \frac{\cos\left(a - \frac{bc}{d}\right) \text{Ci}\left(\frac{bc}{d} + bx\right)}{4d} - \frac{\cos\left(3a - \frac{3bc}{d}\right) \text{Ci}\left(\frac{3bc}{d} + 3bx\right)}{4d} - \frac{\sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{4d} + \dots
\end{aligned}$$

Mathematica [A] time = 0.335325, size = 102, normalized size = 0.84

$$\frac{\cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(b\left(\frac{c}{d} + x\right)\right) - \cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right) - \sin\left(a - \frac{bc}{d}\right) \text{Si}\left(b\left(\frac{c}{d} + x\right)\right) + \sin\left(3a - \frac{3bc}{d}\right) \text{Si}\left(\frac{3b(c+dx)}{d}\right)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x),x]

[Out] (Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] - Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*(c + d*x))/d] - Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d)

Maple [A] time = 0.017, size = 166, normalized size = 1.4

$$\frac{1}{b} \left(\frac{1}{4} \left(\frac{1}{d} \text{Si}\left(bx + a + \frac{-ad + bc}{d}\right) \sin\left(\frac{-ad + bc}{d}\right) + \frac{1}{d} \text{Ci}\left(bx + a + \frac{-ad + bc}{d}\right) \cos\left(\frac{-ad + bc}{d}\right) \right) - \frac{b}{12} \left(3 \frac{1}{d} \text{Si}\left(3bx + 3a + \frac{-3ad + 3bc}{d}\right) \sin\left(\frac{-3ad + 3bc}{d}\right) + \frac{3}{d} \text{Ci}\left(3bx + 3a + \frac{-3ad + 3bc}{d}\right) \cos\left(\frac{-3ad + 3bc}{d}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x)

[Out] 1/b*(1/4*b*(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)-1/12*b*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d)

Maxima [C] time = 1.47769, size = 370, normalized size = 3.06

$$\frac{b \left(E_1 \left(\frac{ibc+i(bx+a)d-id}{d} \right) + E_1 \left(-\frac{ibc+i(bx+a)d-id}{d} \right) \right) \cos\left(-\frac{bc-ad}{d}\right) - b \left(E_1 \left(\frac{3ibc+3i(bx+a)d-3iad}{d} \right) + E_1 \left(-\frac{3ibc+3i(bx+a)d-3iad}{d} \right) \right) \cos\left(-\frac{3(bc-ad)}{d}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")

[Out] -1/8*(b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b*(exp_integral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(1, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d) + b*(-I*exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b*(3*I*exp_integral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(1, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d) + b*(-I*exp_integral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(1, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d)

```
integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e(1, -(I*
b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(I*exp_integral_e(
1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_integral_e(1, -(3*I*b*c
+ 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d))/(b*d)
```

Fricas [A] time = 0.482637, size = 404, normalized size = 3.34

$$\frac{\left(\operatorname{Ci}\left(\frac{bdx+bc}{d}\right) + \operatorname{Ci}\left(-\frac{bdx+bc}{d}\right)\right) \cos\left(-\frac{bc-ad}{d}\right) - \left(\operatorname{Ci}\left(\frac{3(bdx+bc)}{d}\right) + \operatorname{Ci}\left(-\frac{3(bdx+bc)}{d}\right)\right) \cos\left(-\frac{3(bc-ad)}{d}\right) + 2 \sin\left(-\frac{3(bc-ad)}{d}\right) \operatorname{Si}\left(\frac{3(bdx+bc)}{d}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/8*((cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*cos(-
(b*c - a*d)/d) - (cos_integral(3*(b*d*x + b*c)/d) + cos_integral(-3*(b*d*x
+ b*c)/d))*cos(-3*(b*c - a*d)/d) + 2*sin(-3*(b*c - a*d)/d)*sin_integral(3*(
b*d*x + b*c)/d) - 2*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a + bx) \cos(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x), x)
```

Giac [C] time = 1.73713, size = 8180, normalized size = 67.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] -1/8*(real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(b*x + b*c/d))*tan(
3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_int
egral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(b*x + b*c/d))
*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(co
s_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/
2*b*c/d) + 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(
3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_part(cos_int
egral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b
*c/d)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3
/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2
```


$$\begin{aligned}
& 3*b*c/d)) * \tan(3/2*a)^2 * \tan(3/2*b*c/d) * \tan(1/2*b*c/d)^2 - 4 * \sin_integral(3* \\
& (b*d*x + b*c)/d) * \tan(3/2*a)^2 * \tan(3/2*b*c/d) * \tan(1/2*b*c/d)^2 + 2 * \text{imag_part} \\
& (\cos_integral(3*b*x + 3*b*c/d)) * \tan(1/2*a)^2 * \tan(3/2*b*c/d) * \tan(1/2*b*c/d)^2 \\
& - 2 * \text{imag_part}(\cos_integral(-3*b*x - 3*b*c/d)) * \tan(1/2*a)^2 * \tan(3/2*b*c/d) \\
& * \tan(1/2*b*c/d)^2 + 4 * \sin_integral(3*(b*d*x + b*c)/d) * \tan(1/2*a)^2 * \tan(3/2* \\
& b*c/d) * \tan(1/2*b*c/d)^2 + 2 * \text{imag_part}(\cos_integral(3*b*x + 3*b*c/d)) * \tan(3/ \\
& 2*a) * \tan(3/2*b*c/d)^2 * \tan(1/2*b*c/d)^2 - 2 * \text{imag_part}(\cos_integral(-3*b*x - \\
& 3*b*c/d)) * \tan(3/2*a) * \tan(3/2*b*c/d)^2 * \tan(1/2*b*c/d)^2 + 4 * \sin_integral(3*(\\
& b*d*x + b*c)/d) * \tan(3/2*a) * \tan(3/2*b*c/d)^2 * \tan(1/2*b*c/d)^2 - 2 * \text{imag_part} \\
& (\cos_integral(b*x + b*c/d)) * \tan(1/2*a) * \tan(3/2*b*c/d)^2 * \tan(1/2*b*c/d)^2 + 2 \\
& * \text{imag_part}(\cos_integral(-b*x - b*c/d)) * \tan(1/2*a) * \tan(3/2*b*c/d)^2 * \tan(1/2* \\
& b*c/d)^2 - 4 * \sin_integral((b*d*x + b*c)/d) * \tan(1/2*a) * \tan(3/2*b*c/d)^2 * \tan(\\
& 1/2*b*c/d)^2 - \text{real_part}(\cos_integral(3*b*x + 3*b*c/d)) * \tan(3/2*a)^2 * \tan(1/ \\
& 2*a)^2 + \text{real_part}(\cos_integral(b*x + b*c/d)) * \tan(3/2*a)^2 * \tan(1/2*a)^2 + \text{r} \\
& \text{eal_part}(\cos_integral(-b*x - b*c/d)) * \tan(3/2*a)^2 * \tan(1/2*a)^2 - \text{real_part} \\
& (\cos_integral(-3*b*x - 3*b*c/d)) * \tan(3/2*a)^2 * \tan(1/2*a)^2 + 4 * \text{real_part}(\cos \\
& _integral(3*b*x + 3*b*c/d)) * \tan(3/2*a) * \tan(1/2*a)^2 * \tan(3/2*b*c/d) + 4 * \text{real} \\
& _part(\cos_integral(-3*b*x - 3*b*c/d)) * \tan(3/2*a) * \tan(1/2*a)^2 * \tan(3/2*b*c/d) \\
&) + \text{real_part}(\cos_integral(3*b*x + 3*b*c/d)) * \tan(3/2*a)^2 * \tan(3/2*b*c/d)^2 \\
& - \text{real_part}(\cos_integral(b*x + b*c/d)) * \tan(3/2*a)^2 * \tan(3/2*b*c/d)^2 - \text{real} \\
& _part(\cos_integral(-b*x - b*c/d)) * \tan(3/2*a)^2 * \tan(3/2*b*c/d)^2 + \text{real_part} \\
& (\cos_integral(-3*b*x - 3*b*c/d)) * \tan(3/2*a)^2 * \tan(3/2*b*c/d)^2 - \text{real_part} \\
& (\cos_integral(3*b*x + 3*b*c/d)) * \tan(1/2*a)^2 * \tan(3/2*b*c/d)^2 + \text{real_part}(\cos_inte \\
& gral(b*x + b*c/d)) * \tan(1/2*a)^2 * \tan(3/2*b*c/d)^2 + \text{real_part}(\cos_inte \\
& gral(-b*x - b*c/d)) * \tan(1/2*a)^2 * \tan(3/2*b*c/d)^2 - \text{real_part}(\cos_integral(\\
& -3*b*x - 3*b*c/d)) * \tan(1/2*a)^2 * \tan(3/2*b*c/d)^2 - 4 * \text{real_part}(\cos_integral \\
& (b*x + b*c/d)) * \tan(3/2*a)^2 * \tan(1/2*a) * \tan(1/2*b*c/d) - 4 * \text{real_part}(\cos_int \\
& egral(-b*x - b*c/d)) * \tan(3/2*a)^2 * \tan(1/2*a) * \tan(1/2*b*c/d) - 4 * \text{real_part}(c \\
& os_integral(b*x + b*c/d)) * \tan(1/2*a) * \tan(3/2*b*c/d)^2 * \tan(1/2*b*c/d) - 4 * \text{re} \\
& \text{al_part}(\cos_integral(-b*x - b*c/d)) * \tan(1/2*a) * \tan(3/2*b*c/d)^2 * \tan(1/2*b*c \\
& /d) - \text{real_part}(\cos_integral(3*b*x + 3*b*c/d)) * \tan(3/2*a)^2 * \tan(1/2*b*c/d)^2 \\
& + \text{real_part}(\cos_integral(b*x + b*c/d)) * \tan(3/2*a)^2 * \tan(1/2*b*c/d)^2 + \text{re} \\
& \text{al_part}(\cos_integral(-b*x - b*c/d)) * \tan(3/2*a)^2 * \tan(1/2*b*c/d)^2 - \text{real_pa} \\
& \text{rt}(\cos_integral(-3*b*x - 3*b*c/d)) * \tan(3/2*a)^2 * \tan(1/2*b*c/d)^2 + \text{real_par} \\
& \text{t}(\cos_integral(3*b*x + 3*b*c/d)) * \tan(1/2*a)^2 * \tan(1/2*b*c/d)^2 - \text{real_part} \\
& (\cos_integral(b*x + b*c/d)) * \tan(1/2*a)^2 * \tan(1/2*b*c/d)^2 - \text{real_part}(\cos_in \\
& tegral(-b*x - b*c/d)) * \tan(1/2*a)^2 * \tan(1/2*b*c/d)^2 + \text{real_part}(\cos_integra \\
& l(-3*b*x - 3*b*c/d)) * \tan(1/2*a)^2 * \tan(1/2*b*c/d)^2 + 4 * \text{real_part}(\cos_integr \\
& al(3*b*x + 3*b*c/d)) * \tan(3/2*a) * \tan(3/2*b*c/d) * \tan(1/2*b*c/d)^2 + 4 * \text{real_pa} \\
& \text{rt}(\cos_integral(-3*b*x - 3*b*c/d)) * \tan(3/2*a) * \tan(3/2*b*c/d) * \tan(1/2*b*c/d) \\
& ^2 - \text{real_part}(\cos_integral(3*b*x + 3*b*c/d)) * \tan(3/2*b*c/d)^2 * \tan(1/2*b*c/d) \\
& ^2 + \text{real_part}(\cos_integral(b*x + b*c/d)) * \tan(3/2*b*c/d)^2 * \tan(1/2*b*c/d) \\
& ^2 + \text{real_part}(\cos_integral(-b*x - b*c/d)) * \tan(3/2*b*c/d)^2 * \tan(1/2*b*c/d)^2 \\
& - \text{real_part}(\cos_integral(-3*b*x - 3*b*c/d)) * \tan(3/2*b*c/d)^2 * \tan(1/2*b*c/d) \\
& ^2 + 2 * \text{imag_part}(\cos_integral(b*x + b*c/d)) * \tan(3/2*a)^2 * \tan(1/2*a) - 2 * \text{i} \\
& \text{mag_part}(\cos_integral(-b*x - b*c/d)) * \tan(3/2*a)^2 * \tan(1/2*a) + 4 * \sin_integr \\
& al((b*d*x + b*c)/d) * \tan(3/2*a)^2 * \tan(1/2*a) - 2 * \text{imag_part}(\cos_integral(3*b* \\
& x + 3*b*c/d)) * \tan(3/2*a) * \tan(1/2*a)^2 + 2 * \text{imag_part}(\cos_integral(-3*b*x - 3 \\
& *b*c/d)) * \tan(3/2*a) * \tan(1/2*a)^2 - 4 * \sin_integral(3*(b*d*x + b*c)/d) * \tan(3/ \\
& 2*a) * \tan(1/2*a)^2 - 2 * \text{imag_part}(\cos_integral(3*b*x + 3*b*c/d)) * \tan(3/2*a)^2 \\
& * \tan(3/2*b*c/d) + 2 * \text{imag_part}(\cos_integral(-3*b*x - 3*b*c/d)) * \tan(3/2*a)^2 * \\
& \tan(3/2*b*c/d) - 4 * \sin_integral(3*(b*d*x + b*c)/d) * \tan(3/2*a)^2 * \tan(3/2*b*c \\
& /d) + 2 * \text{imag_part}(\cos_integral(3*b*x + 3*b*c/d)) * \tan(1/2*a)^2 * \tan(3/2*b*c/d) \\
&) - 2 * \text{imag_part}(\cos_integral(-3*b*x - 3*b*c/d)) * \tan(1/2*a)^2 * \tan(3/2*b*c/d) \\
& + 4 * \sin_integral(3*(b*d*x + b*c)/d) * \tan(1/2*a)^2 * \tan(3/2*b*c/d) + 2 * \text{imag_p} \\
& \text{art}(\cos_integral(3*b*x + 3*b*c/d)) * \tan(3/2*a) * \tan(3/2*b*c/d)^2 - 2 * \text{imag_par} \\
& \text{t}(\cos_integral(-3*b*x - 3*b*c/d)) * \tan(3/2*a) * \tan(3/2*b*c/d)^2 + 4 * \sin_integ \\
& ral(3*(b*d*x + b*c)/d) * \tan(3/2*a) * \tan(3/2*b*c/d)^2 + 2 * \text{imag_part}(\cos_integr \\
& al(b*x + b*c/d)) * \tan(1/2*a) * \tan(3/2*b*c/d)^2 - 2 * \text{imag_part}(\cos_integral(-b*
\end{aligned}$$

$$\begin{aligned}
& x - b*c/d)) * \tan(1/2*a) * \tan(3/2*b*c/d)^2 + 4 * \sin_integral((b*d*x + b*c)/d) * \tan(1/2*a) * \tan(3/2*b*c/d)^2 - 2 * \operatorname{imag_part}(\cos_integral(b*x + b*c/d)) * \tan(3/2*a)^2 * \tan(1/2*b*c/d) + 2 * \operatorname{imag_part}(\cos_integral(-b*x - b*c/d)) * \tan(3/2*a)^2 * \tan(1/2*b*c/d) - 4 * \sin_integral((b*d*x + b*c)/d) * \tan(3/2*a)^2 * \tan(1/2*b*c/d) + 2 * \operatorname{imag_part}(\cos_integral(b*x + b*c/d)) * \tan(1/2*a)^2 * \tan(1/2*b*c/d) - 2 * \operatorname{imag_part}(\cos_integral(-b*x - b*c/d)) * \tan(1/2*a)^2 * \tan(1/2*b*c/d) + 4 * \sin_integral((b*d*x + b*c)/d) * \tan(1/2*a)^2 * \tan(1/2*b*c/d) - 2 * \operatorname{imag_part}(\cos_integral(b*x + b*c/d)) * \tan(3/2*b*c/d)^2 * \tan(1/2*b*c/d) + 2 * \operatorname{imag_part}(\cos_integral(-b*x - b*c/d)) * \tan(3/2*b*c/d)^2 * \tan(1/2*b*c/d) - 4 * \sin_integral((b*d*x + b*c)/d) * \tan(3/2*b*c/d)^2 * \tan(1/2*b*c/d) - 2 * \operatorname{imag_part}(\cos_integral(3*b*x + 3*b*c/d)) * \tan(3/2*a) * \tan(1/2*b*c/d)^2 + 2 * \operatorname{imag_part}(\cos_integral(-3*b*x - 3*b*c/d)) * \tan(3/2*a) * \tan(1/2*b*c/d)^2 - 4 * \sin_integral(3*(b*d*x + b*c)/d) * \tan(3/2*a) * \tan(1/2*b*c/d)^2 - 2 * \operatorname{imag_part}(\cos_integral(b*x + b*c/d)) * \tan(1/2*a) * \tan(1/2*b*c/d)^2 + 2 * \operatorname{imag_part}(\cos_integral(-b*x - b*c/d)) * \tan(1/2*a) * \tan(1/2*b*c/d)^2 - 4 * \sin_integral((b*d*x + b*c)/d) * \tan(1/2*a) * \tan(1/2*b*c/d)^2 + 2 * \operatorname{imag_part}(\cos_integral(3*b*x + 3*b*c/d)) * \tan(3/2*b*c/d) * \tan(1/2*b*c/d)^2 - 2 * \operatorname{imag_part}(\cos_integral(-3*b*x - 3*b*c/d)) * \tan(3/2*b*c/d) * \tan(1/2*b*c/d)^2 + 4 * \sin_integral(3*(b*d*x + b*c)/d) * \tan(3/2*b*c/d) * \tan(1/2*b*c/d)^2 - \operatorname{real_part}(\cos_integral(3*b*x + 3*b*c/d)) * \tan(3/2*a)^2 - \operatorname{real_part}(\cos_integral(b*x + b*c/d)) * \tan(3/2*a)^2 - \operatorname{real_part}(\cos_integral(-b*x - b*c/d)) * \tan(3/2*a)^2 - \operatorname{real_part}(\cos_integral(-3*b*x - 3*b*c/d)) * \tan(3/2*a)^2 + \operatorname{real_part}(\cos_integral(3*b*x + 3*b*c/d)) * \tan(1/2*a)^2 + \operatorname{real_part}(\cos_integral(b*x + b*c/d)) * \tan(1/2*a)^2 + \operatorname{real_part}(\cos_integral(-b*x - b*c/d)) * \tan(1/2*a)^2 + \operatorname{real_part}(\cos_integral(-3*b*x - 3*b*c/d)) * \tan(1/2*a)^2 + 4 * \operatorname{real_part}(\cos_integral(3*b*x + 3*b*c/d)) * \tan(3/2*a) * \tan(3/2*b*c/d) + 4 * \operatorname{real_part}(\cos_integral(-3*b*x - 3*b*c/d)) * \tan(3/2*a) * \tan(3/2*b*c/d) - \operatorname{real_part}(\cos_integral(3*b*x + 3*b*c/d)) * \tan(3/2*b*c/d)^2 - \operatorname{real_part}(\cos_integral(b*x + b*c/d)) * \tan(3/2*b*c/d)^2 - \operatorname{real_part}(\cos_integral(-b*x - b*c/d)) * \tan(3/2*b*c/d)^2 - \operatorname{real_part}(\cos_integral(-3*b*x - 3*b*c/d)) * \tan(3/2*b*c/d)^2 - 4 * \operatorname{real_part}(\cos_integral(b*x + b*c/d)) * \tan(1/2*a) * \tan(1/2*b*c/d) - 4 * \operatorname{real_part}(\cos_integral(-b*x - b*c/d)) * \tan(1/2*a) * \tan(1/2*b*c/d) + \operatorname{real_part}(\cos_integral(3*b*x + 3*b*c/d)) * \tan(1/2*b*c/d)^2 + \operatorname{real_part}(\cos_integral(b*x + b*c/d)) * \tan(1/2*b*c/d)^2 + \operatorname{real_part}(\cos_integral(-b*x - b*c/d)) * \tan(1/2*b*c/d)^2 + \operatorname{real_part}(\cos_integral(-3*b*x - 3*b*c/d)) * \tan(1/2*b*c/d)^2 - 2 * \operatorname{imag_part}(\cos_integral(3*b*x + 3*b*c/d)) * \tan(3/2*a) + 2 * \operatorname{imag_part}(\cos_integral(-3*b*x - 3*b*c/d)) * \tan(3/2*a) - 4 * \sin_integral(3*(b*d*x + b*c)/d) * \tan(3/2*a) + 2 * \operatorname{imag_part}(\cos_integral(b*x + b*c/d)) * \tan(1/2*a) - 2 * \operatorname{imag_part}(\cos_integral(-b*x - b*c/d)) * \tan(1/2*a) + 4 * \sin_integral((b*d*x + b*c)/d) * \tan(1/2*a) + 2 * \operatorname{imag_part}(\cos_integral(3*b*x + 3*b*c/d)) * \tan(3/2*b*c/d) - 2 * \operatorname{imag_part}(\cos_integral(-3*b*x - 3*b*c/d)) * \tan(3/2*b*c/d) + 4 * \sin_integral(3*(b*d*x + b*c)/d) * \tan(3/2*b*c/d) - 2 * \operatorname{imag_part}(\cos_integral(b*x + b*c/d)) * \tan(1/2*b*c/d) + 2 * \operatorname{imag_part}(\cos_integral(-b*x - b*c/d)) * \tan(1/2*b*c/d) - 4 * \sin_integral((b*d*x + b*c)/d) * \tan(1/2*b*c/d) + \operatorname{real_part}(\cos_integral(3*b*x + 3*b*c/d)) - \operatorname{real_part}(\cos_integral(b*x + b*c/d)) - \operatorname{real_part}(\cos_integral(-b*x - b*c/d)) + \operatorname{real_part}(\cos_integral(-3*b*x - 3*b*c/d)) / (d * \tan(3/2*a)^2 * \tan(1/2*a)^2 * \tan(3/2*b*c/d)^2 * \tan(1/2*b*c/d)^2 + d * \tan(3/2*a)^2 * \tan(1/2*a)^2 * \tan(3/2*b*c/d)^2 + d * \tan(3/2*a)^2 * \tan(1/2*a)^2 * \tan(1/2*b*c/d)^2 + d * \tan(3/2*a)^2 * \tan(3/2*b*c/d)^2 * \tan(1/2*b*c/d)^2 + d * \tan(1/2*a)^2 * \tan(3/2*b*c/d)^2 * \tan(1/2*b*c/d)^2 + d * \tan(3/2*a)^2 * \tan(1/2*a)^2 + d * \tan(3/2*a)^2 * \tan(3/2*b*c/d)^2 + d * \tan(1/2*a)^2 * \tan(3/2*b*c/d)^2 + d * \tan(3/2*a)^2 * \tan(1/2*b*c/d)^2 + d * \tan(1/2*a)^2 * \tan(1/2*b*c/d)^2 + d * \tan(3/2*a)^2 + d * \tan(1/2*a)^2 + d * \tan(3/2*b*c/d)^2 + d * \tan(1/2*b*c/d)^2 + d)
\end{aligned}$$

$$3.19 \quad \int \frac{\cos(a+bx) \sin^2(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=168

$$\frac{3b \sin\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{4d^2} - \frac{b \sin\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{4d^2} - \frac{b \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{4d^2} + \frac{3b \cos\left(a - \frac{bc}{d}\right) \text{Ci}\left(\frac{bc}{d} + bx\right)}{4d^2}$$

[Out] -Cos[a + b*x]/(4*d*(c + d*x)) + Cos[3*a + 3*b*x]/(4*d*(c + d*x)) + (3*b*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(4*d^2) - (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(4*d^2) - (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(4*d^2) + (3*b*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(4*d^2)

Rubi [A] time = 0.301291, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{3b \sin\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{4d^2} - \frac{b \sin\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{4d^2} - \frac{b \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{4d^2} + \frac{3b \cos\left(a - \frac{bc}{d}\right) \text{Ci}\left(\frac{bc}{d} + bx\right)}{4d^2}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^2,x]

[Out] -Cos[a + b*x]/(4*d*(c + d*x)) + Cos[3*a + 3*b*x]/(4*d*(c + d*x)) + (3*b*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(4*d^2) - (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(4*d^2) - (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(4*d^2) + (3*b*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(4*d^2)

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2} dx &= \int \left(\frac{\cos(a+bx)}{4(c+dx)^2} - \frac{\cos(3a+3bx)}{4(c+dx)^2} \right) dx \\ &= \frac{1}{4} \int \frac{\cos(a+bx)}{(c+dx)^2} dx - \frac{1}{4} \int \frac{\cos(3a+3bx)}{(c+dx)^2} dx \\ &= -\frac{\cos(a+bx)}{4d(c+dx)} + \frac{\cos(3a+3bx)}{4d(c+dx)} - \frac{b \int \frac{\sin(a+bx)}{c+dx} dx}{4d} + \frac{(3b) \int \frac{\sin(3a+3bx)}{c+dx} dx}{4d} \\ &= -\frac{\cos(a+bx)}{4d(c+dx)} + \frac{\cos(3a+3bx)}{4d(c+dx)} + \frac{\left(3b \cos\left(3a - \frac{3bc}{d}\right)\right) \int \frac{\sin\left(\frac{3bc}{d} + 3bx\right)}{c+dx} dx}{4d} - \frac{\left(b \cos\left(a - \frac{bc}{d}\right)\right) \int \frac{\sin\left(\frac{bc}{d} + bx\right)}{c+dx} dx}{4d} \\ &= -\frac{\cos(a+bx)}{4d(c+dx)} + \frac{\cos(3a+3bx)}{4d(c+dx)} + \frac{3b \operatorname{Ci}\left(\frac{3bc}{d} + 3bx\right) \sin\left(3a - \frac{3bc}{d}\right)}{4d^2} - \frac{b \operatorname{Ci}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{4d^2} \end{aligned}$$

Mathematica [A] time = 1.43532, size = 139, normalized size = 0.83

$$\frac{-3b \sin\left(3a - \frac{3bc}{d}\right) \operatorname{CosIntegral}\left(\frac{3b(c+dx)}{d}\right) + b \sin\left(a - \frac{bc}{d}\right) \operatorname{CosIntegral}\left(b\left(\frac{c}{d} + x\right)\right) + b \cos\left(a - \frac{bc}{d}\right) \operatorname{Si}\left(b\left(\frac{c}{d} + x\right)\right) - \frac{3b \operatorname{Ci}\left(\frac{3bc}{d} + 3bx\right) \sin\left(3a - \frac{3bc}{d}\right) - b \operatorname{Ci}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{4d^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^2, x]

[Out] -((d*cos[a + b*x])/(c + d*x) - (d*cos[3*(a + b*x)])/(c + d*x) - 3*b*cosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + b*cosIntegral[b*(c/d + x)]*Sin[a - (b*c)/d] + b*cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 3*b*cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d^2)

Maple [A] time = 0.023, size = 242, normalized size = 1.4

$$\frac{1}{b} \left(\frac{b^2}{4} \left(-\frac{\cos(bx+a)}{((bx+a)d - ad + bc)d} - \frac{1}{d} \left(\frac{1}{d} \operatorname{Si}\left(bx+a + \frac{-ad+bc}{d}\right) \cos\left(\frac{-ad+bc}{d}\right) - \frac{1}{d} \operatorname{Ci}\left(bx+a + \frac{-ad+bc}{d}\right) \sin\left(\frac{-ad+bc}{d}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2, x)

[Out] 1/b*(1/4*b^2*(-cos(b*x+a)/((b*x+a)*d-a*d+b*c)/d-(Si(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)-1/12*b^2*(-3*cos(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)/d-3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d)

Maxima [C] time = 1.77077, size = 408, normalized size = 2.43

$$8192 b^2 \left(E_2 \left(\frac{ibc+i(bx+a)d-id}{d} \right) + E_2 \left(-\frac{ibc+i(bx+a)d-id}{d} \right) \right) \cos \left(-\frac{bc-ad}{d} \right) - 8192 b^2 \left(E_2 \left(\frac{3ibc+3i(bx+a)d-3iad}{d} \right) + E_2 \left(-\frac{3ibc+3i(bx+a)d-3iad}{d} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")

[Out] -1/65536*(8192*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - 8192*b^2*(exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d) + b^2*(-8192*I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 8192*I*exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^2*(8192*I*exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - 8192*I*exp_integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d)/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)

Fricas [A] time = 0.54075, size = 593, normalized size = 3.53

$$8 d \cos (b x+a)^3+6(b d x+b c) \cos \left(-\frac{3(b c-a d)}{d}\right) \operatorname{Si}\left(\frac{3(b d x+b c)}{d}\right)-2(b d x+b c) \cos \left(-\frac{b c-a d}{d}\right) \operatorname{Si}\left(\frac{b d x+b c}{d}\right)-8 d \cos (b x+a)-\left(\frac{b d x+b c}{d}\right)^2 \sin \left(-\frac{3(b c-a d)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")

[Out] 1/8*(8*d*cos(b*x + a)^3 + 6*(b*d*x + b*c)*cos(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - 2*(b*d*x + b*c)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) - 8*d*cos(b*x + a) - ((b*d*x + b*c)*cos_integral((b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/d) + 3*((b*d*x + b*c)*cos_integral(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d)/(d^3*x + c*d^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a + bx) \cos(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c)**2,x)

[Out] Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx + a) \sin(bx + a)^2}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*sin(b*x + a)^2/(d*x + c)^2, x)
```

$$3.20 \quad \int \frac{\cos(a+bx) \sin^2(a+bx)}{(c+dx)^3} dx$$

Optimal. Leaf size=221

$$-\frac{b^2 \cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{8d^3} + \frac{9b^2 \cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{8d^3} + \frac{b^2 \sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{8d^3} - 9$$

[Out] $-\text{Cos}[a + b*x]/(8*d*(c + d*x)^2) + \text{Cos}[3*a + 3*b*x]/(8*d*(c + d*x)^2) - (b^2 * \text{Cos}[a - (b*c)/d] * \text{CosIntegral}[(b*c)/d + b*x])/(8*d^3) + (9*b^2 * \text{Cos}[3*a - (3*b*c)/d] * \text{CosIntegral}[(3*b*c)/d + 3*b*x])/(8*d^3) + (b * \text{Sin}[a + b*x])/(8*d^2 * (c + d*x)) - (3*b * \text{Sin}[3*a + 3*b*x])/(8*d^2 * (c + d*x)) + (b^2 * \text{Sin}[a - (b*c)/d] * \text{SinIntegral}[(b*c)/d + b*x])/(8*d^3) - (9*b^2 * \text{Sin}[3*a - (3*b*c)/d] * \text{SinIntegral}[(3*b*c)/d + 3*b*x])/(8*d^3)$

Rubi [A] time = 0.358156, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^2 \cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{8d^3} + \frac{9b^2 \cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{8d^3} + \frac{b^2 \sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{8d^3} - 9$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[a + b*x] * \text{Sin}[a + b*x]^2)/(c + d*x)^3, x]$

[Out] $-\text{Cos}[a + b*x]/(8*d*(c + d*x)^2) + \text{Cos}[3*a + 3*b*x]/(8*d*(c + d*x)^2) - (b^2 * \text{Cos}[a - (b*c)/d] * \text{CosIntegral}[(b*c)/d + b*x])/(8*d^3) + (9*b^2 * \text{Cos}[3*a - (3*b*c)/d] * \text{CosIntegral}[(3*b*c)/d + 3*b*x])/(8*d^3) + (b * \text{Sin}[a + b*x])/(8*d^2 * (c + d*x)) - (3*b * \text{Sin}[3*a + 3*b*x])/(8*d^2 * (c + d*x)) + (b^2 * \text{Sin}[a - (b*c)/d] * \text{SinIntegral}[(b*c)/d + b*x])/(8*d^3) - (9*b^2 * \text{Sin}[3*a - (3*b*c)/d] * \text{SinIntegral}[(3*b*c)/d + 3*b*x])/(8*d^3)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)} * ((c_.) + (d_.)*(x_.))^{(m_.)} * \text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n * \text{Cos}[a + b*x]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3297

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)} * \text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m + 1)} * \text{Sin}[e + f*x]/(d*(m + 1)), x] - \text{Dist}[f/(d*(m + 1)), \text{Int}[(c + d*x)^{(m + 1)} * \text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/(c + d*x), x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/(c + d*x), x], x] /;$ FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3} dx &= \int \left(\frac{\cos(a+bx)}{4(c+dx)^3} - \frac{\cos(3a+3bx)}{4(c+dx)^3} \right) dx \\ &= \frac{1}{4} \int \frac{\cos(a+bx)}{(c+dx)^3} dx - \frac{1}{4} \int \frac{\cos(3a+3bx)}{(c+dx)^3} dx \\ &= -\frac{\cos(a+bx)}{8d(c+dx)^2} + \frac{\cos(3a+3bx)}{8d(c+dx)^2} - \frac{b \int \frac{\sin(a+bx)}{(c+dx)^2} dx}{8d} + \frac{(3b) \int \frac{\sin(3a+3bx)}{(c+dx)^2} dx}{8d} \\ &= -\frac{\cos(a+bx)}{8d(c+dx)^2} + \frac{\cos(3a+3bx)}{8d(c+dx)^2} + \frac{b \sin(a+bx)}{8d^2(c+dx)} - \frac{3b \sin(3a+3bx)}{8d^2(c+dx)} - \frac{b^2 \int \frac{\cos(a+bx)}{c+dx} dx}{8d^2} \\ &= -\frac{\cos(a+bx)}{8d(c+dx)^2} + \frac{\cos(3a+3bx)}{8d(c+dx)^2} + \frac{b \sin(a+bx)}{8d^2(c+dx)} - \frac{3b \sin(3a+3bx)}{8d^2(c+dx)} + \frac{(9b^2 \cos(3a - \frac{3bc}{d}))}{8d^3} \\ &= -\frac{\cos(a+bx)}{8d(c+dx)^2} + \frac{\cos(3a+3bx)}{8d(c+dx)^2} - \frac{b^2 \cos\left(a - \frac{bc}{d}\right) \text{Ci}\left(\frac{bc}{d} + bx\right)}{8d^3} + \frac{9b^2 \cos\left(3a - \frac{3bc}{d}\right) \text{Ci}\left(\frac{3bc}{d} + 3bx\right)}{8d^3} \end{aligned}$$

Mathematica [A] time = 2.2384, size = 183, normalized size = 0.83

$$\frac{b^2 \left(-\cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(b\left(\frac{c}{d} + x\right)\right) + 9b^2 \cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right) + b^2 \sin\left(a - \frac{bc}{d}\right) \text{Si}\left(b\left(\frac{c}{d} + x\right)\right) \right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^3, x]
```

```
[Out] (-b^2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)]) + 9*b^2*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*(c + d*x))/d] + (d*(-(d*Cos[a + b*x]) + b*(c + d*x)*Sin[a + b*x]))/(c + d*x)^2 + (d*(d*Cos[3*(a + b*x)] - 3*b*(c + d*x)*Sin[3*(a + b*x)]))/(c + d*x)^2 + b^2*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 9*b^2*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d]/(8*d^3)
```

Maple [A] time = 0.021, size = 311, normalized size = 1.4

$$\frac{1}{b} \left(\frac{b^3}{4} \left(-\frac{\cos(bx+a)}{2((bx+a)d - ad + bc)^2 d} - \frac{1}{2d} \left(-\frac{\sin(bx+a)}{((bx+a)d - ad + bc)d} + \frac{1}{d} \left(\frac{1}{d} \text{Si}\left(bx+a + \frac{-ad+bc}{d}\right) \sin\left(\frac{-ad+bc}{d}\right) \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3, x)
```

```
[Out] 1/b*(1/4*b^3*(-1/2*cos(b*x+a)/((b*x+a)*d-a*d+b*c)^2/d-1/2*(-sin(b*x+a)/((b*x+a)*d-a*d+b*c)/d+(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*
```

$d+bc)/d)*\cos((-a*d+bc)/d)/d)-1/12*b^3*(-3/2*\cos(3*b*x+3*a)/((b*x+a)*d-a*d+bc)^2/d-3/2*(-3*\sin(3*b*x+3*a)/((b*x+a)*d-a*d+bc)/d+3*(3*Si(3*b*x+3*a+3*(-a*d+bc)/d)*\sin(3*(-a*d+bc)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+bc)/d)*\cos(3*(-a*d+bc)/d)/d)/d)$

Maxima [C] time = 2.27331, size = 455, normalized size = 2.06

$$8192 b^3 \left(E_3 \left(\frac{ibc+i(bx+a)d-iad}{d} \right) + E_3 \left(-\frac{ibc+i(bx+a)d-iad}{d} \right) \right) \cos \left(-\frac{bc-ad}{d} \right) - 8192 b^3 \left(E_3 \left(\frac{3ibc+3i(bx+a)d-3iad}{d} \right) + E_3 \left(-\frac{3ibc+3i(bx+a)d-3iad}{d} \right) \right)$$

655

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")

[Out] $-1/65536*(8192*b^3*(\exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + \exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*\cos(-(b*c - a*d)/d) - 8192*b^3*(\exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + \exp_integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*\cos(-3*(b*c - a*d)/d) + b^3*(-8192*I*\exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 8192*I*\exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*\sin(-(b*c - a*d)/d) + b^3*(8192*I*\exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - 8192*I*\exp_integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*\sin(-3*(b*c - a*d)/d)/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)$

Fricas [A] time = 0.574991, size = 914, normalized size = 4.14

$$8d^2 \cos(bx + a)^3 - 8d^2 \cos(bx + a) - 18(b^2d^2x^2 + 2b^2cdx + b^2c^2) \sin\left(-\frac{3(bc-ad)}{d}\right) Si\left(\frac{3(bdx+bc)}{d}\right) + 2(b^2d^2x^2 + 2b^2cdx + b^2c^2) \cos\left(-\frac{3(bc-ad)}{d}\right) Si\left(\frac{3(bdx+bc)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")

[Out] $1/16*(8*d^2*\cos(b*x + a)^3 - 8*d^2*\cos(b*x + a) - 18*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\sin(-3*(b*c - a*d)/d)*\sin_integral(3*(b*d*x + b*c)/d) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\sin(-(b*c - a*d)/d)*\sin_integral((b*d*x + b*c)/d) - ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos_integral((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos_integral(-(b*d*x + b*c)/d))*\cos(-(b*c - a*d)/d) + 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos_integral(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos_integral(-3*(b*d*x + b*c)/d))*\cos(-3*(b*c - a*d)/d) + 8*(b*d^2*x + b*c*d - 3*(b*d^2*x + b*c*d)*\cos(b*x + a)^2*\sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a + bx) \cos(a + bx)}{(c + dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.21 $\int \frac{\cos(a+bx) \sin^2(a+bx)}{(c+dx)^4} dx$

Optimal. Leaf size=270

$$-\frac{9b^3 \sin\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{8d^4} + \frac{b^3 \sin\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{24d^4} + \frac{b^3 \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{24d^4} - 9$$

[Out] $-\text{Cos}[a + b*x]/(12*d*(c + d*x)^3) + (b^2*\text{Cos}[a + b*x])/(24*d^3*(c + d*x)) + \text{Cos}[3*a + 3*b*x]/(12*d*(c + d*x)^3) - (3*b^2*\text{Cos}[3*a + 3*b*x])/(8*d^3*(c + d*x)) - (9*b^3*\text{CosIntegral}[(3*b*c)/d + 3*b*x]*\text{Sin}[3*a - (3*b*c)/d])/(8*d^4) + (b^3*\text{CosIntegral}[(b*c)/d + b*x]*\text{Sin}[a - (b*c)/d])/(24*d^4) + (b*\text{Sin}[a + b*x])/(24*d^2*(c + d*x)^2) - (b*\text{Sin}[3*a + 3*b*x])/(8*d^2*(c + d*x)^2) + (b^3*\text{Cos}[a - (b*c)/d]*\text{SinIntegral}[(b*c)/d + b*x])/(24*d^4) - (9*b^3*\text{Cos}[3*a - (3*b*c)/d]*\text{SinIntegral}[(3*b*c)/d + 3*b*x])/(8*d^4)$

Rubi [A] time = 0.419572, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{9b^3 \sin\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{8d^4} + \frac{b^3 \sin\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{24d^4} + \frac{b^3 \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{24d^4} - 9$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[a + b*x]*\text{Sin}[a + b*x]^2)/(c + d*x)^4, x]$

[Out] $-\text{Cos}[a + b*x]/(12*d*(c + d*x)^3) + (b^2*\text{Cos}[a + b*x])/(24*d^3*(c + d*x)) + \text{Cos}[3*a + 3*b*x]/(12*d*(c + d*x)^3) - (3*b^2*\text{Cos}[3*a + 3*b*x])/(8*d^3*(c + d*x)) - (9*b^3*\text{CosIntegral}[(3*b*c)/d + 3*b*x]*\text{Sin}[3*a - (3*b*c)/d])/(8*d^4) + (b^3*\text{CosIntegral}[(b*c)/d + b*x]*\text{Sin}[a - (b*c)/d])/(24*d^4) + (b*\text{Sin}[a + b*x])/(24*d^2*(c + d*x)^2) - (b*\text{Sin}[3*a + 3*b*x])/(8*d^2*(c + d*x)^2) + (b^3*\text{Cos}[a - (b*c)/d]*\text{SinIntegral}[(b*c)/d + b*x])/(24*d^4) - (9*b^3*\text{Cos}[3*a - (3*b*c)/d]*\text{SinIntegral}[(3*b*c)/d + 3*b*x])/(8*d^4)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_)]^{(p_.)*((c_.) + (d_.)*(x_))^{(m_.)*\text{Sin}[(a_.) + (b_.)*(x_)]^{(n_.)}, x_Symbol] := \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3297

$\text{Int}[(c_.) + (d_.)*(x_)]^{(m_.)*\text{sin}[(e_.) + (f_.)*(x_)], x_Symbol] := \text{Simp}[(c + d*x)^{(m + 1)*\text{Sin}[e + f*x]/(d*(m + 1)), x] - \text{Dist}[f/(d*(m + 1)), \text{Int}[(c + d*x)^{(m + 1)*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/(c + d*x), x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/(c + d*x), x], x] /;$ FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4} dx &= \int \left(\frac{\cos(a+bx)}{4(c+dx)^4} - \frac{\cos(3a+3bx)}{4(c+dx)^4} \right) dx \\
 &= \frac{1}{4} \int \frac{\cos(a+bx)}{(c+dx)^4} dx - \frac{1}{4} \int \frac{\cos(3a+3bx)}{(c+dx)^4} dx \\
 &= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} - \frac{b \int \frac{\sin(a+bx)}{(c+dx)^3} dx}{12d} + \frac{b \int \frac{\sin(3a+3bx)}{(c+dx)^3} dx}{4d} \\
 &= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} + \frac{b \sin(a+bx)}{24d^2(c+dx)^2} - \frac{b \sin(3a+3bx)}{8d^2(c+dx)^2} - \frac{b^2 \int \frac{\cos(a+bx)}{(c+dx)^2} dx}{24d^2} \\
 &= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{b^2 \cos(a+bx)}{24d^3(c+dx)} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} - \frac{3b^2 \cos(3a+3bx)}{8d^3(c+dx)} + \frac{b \sin(a+bx)}{24d^2(c+dx)} \\
 &= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{b^2 \cos(a+bx)}{24d^3(c+dx)} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} - \frac{3b^2 \cos(3a+3bx)}{8d^3(c+dx)} + \frac{b \sin(a+bx)}{24d^2(c+dx)} \\
 &= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{b^2 \cos(a+bx)}{24d^3(c+dx)} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} - \frac{3b^2 \cos(3a+3bx)}{8d^3(c+dx)} - \frac{9b^3 \text{Ci}\left(\frac{3bc}{d}\right)}{24d^2(c+dx)}
 \end{aligned}$$

Mathematica [A] time = 1.75246, size = 298, normalized size = 1.1

$$b^3(c+dx)^3 \left(\sin\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(b\left(\frac{c}{d} + x\right)\right) + \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(b\left(\frac{c}{d} + x\right)\right) \right) - 27b^3(c+dx)^3 \left(\sin\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(b\left(\frac{c}{d} + x\right)\right) + \cos\left(3a - \frac{3bc}{d}\right) \text{Si}\left(b\left(\frac{c}{d} + x\right)\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^4, x]

[Out] (d*Cos[b*x]*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*d*(c + d*x)*Sin[a]) - d*Cos[3*b*x]*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*x)*Sin[3*a]) + d*(b*d*(c + d*x)*Cos[a] - (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x] - d*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)*Sin[3*a])*Sin[3*b*x] + b^3*(c + d*x)^3*(CosIntegral[b*(c/d + x)]*Sin[a - (b*c)/d] + Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)]) - 27*b^3*(c + d*x)^3*(CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(24*d^4*(c + d*x)^3)

Maple [A] time = 0.024, size = 384, normalized size = 1.4

$$\frac{1}{b} \left(\frac{b^4}{4} \left(-\frac{\cos(bx+a)}{3((bx+a)d - ad + bc)^3 d} - \frac{1}{3d} \left(-\frac{\sin(bx+a)}{2((bx+a)d - ad + bc)^2 d} + \frac{1}{2d} \left(-\frac{\cos(bx+a)}{((bx+a)d - ad + bc)d} - \frac{1}{d} \left(\frac{1}{d} \text{Si}\left(\frac{3bc}{d}\right) \right) \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x)`

[Out] $\frac{1}{b} \left(\frac{1}{4} b^4 \left(-\frac{1}{3} \cos(bx+a) / ((bx+a)d - a*d + b*c)^3 / d - \frac{1}{3} \left(-\frac{1}{2} \sin(bx+a) / ((bx+a)d - a*d + b*c)^2 / d + \frac{1}{2} \left(-\cos(bx+a) / ((bx+a)d - a*d + b*c) / d - \text{Si}(bx+a + (-a*d + b*c)/d) \right) \cos((-a*d + b*c)/d) / d - \text{Ci}(bx+a + (-a*d + b*c)/d) \right) \sin((-a*d + b*c)/d) / d \right) / d - \frac{1}{12} b^4 \left(-\cos(3bx+3a) / ((bx+a)d - a*d + b*c)^3 / d - \left(-\frac{3}{2} \sin(3bx+3a) / ((bx+a)d - a*d + b*c)^2 / d + \frac{3}{2} \left(-3 \cos(3bx+3a) / ((bx+a)d - a*d + b*c) / d - 3 \left(3 \text{Si}(3bx+3a + 3(-a*d + b*c)/d) \right) \cos(3(-a*d + b*c)/d) / d - 3 \text{Ci}(3bx+3a + 3(-a*d + b*c)/d) \right) \sin(3(-a*d + b*c)/d) / d \right) / d \right) / d \right)$

Maxima [C] time = 2.80831, size = 522, normalized size = 1.93

$$\frac{8192 b^4 \left(E_4 \left(\frac{ibc+i(bx+a)d-iad}{d} \right) + E_4 \left(-\frac{ibc+i(bx+a)d-iad}{d} \right) \right) \cos \left(-\frac{bc-ad}{d} \right) - 8192 b^4 \left(E_4 \left(\frac{3ibc+3i(bx+a)d-3iad}{d} \right) + E_4 \left(-\frac{3ibc+3i(bx+a)d-3iad}{d} \right) \right)}{65536 (b^3 c^3 d - 3 a b^2 c^2 d^2 + 3 a^2 b c d^3 - a^3 d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")`

[Out] $-\frac{1}{65536} (8192 b^4 (\exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + \exp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d)) \cos(-(b*c - a*d)/d) - 8192 b^4 (\exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + \exp_integral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d)) \cos(-3*(b*c - a*d)/d) + b^4 (-8192 I \exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 8192 I \exp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d)) \sin(-(b*c - a*d)/d) + b^4 (8192 I \exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - 8192 I \exp_integral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d)) \sin(-3*(b*c - a*d)/d) / ((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)$

Fricas [B] time = 0.637271, size = 1243, normalized size = 4.6

$$\frac{8 (9 b^2 d^3 x^2 + 18 b^2 c d^2 x + 9 b^2 c^2 d - 2 d^3) \cos(bx + a)^3 + 54 (b^3 d^3 x^3 + 3 b^3 c d^2 x^2 + 3 b^3 c^2 d x + b^3 c^3) \cos \left(-\frac{3(bc-ad)}{d} \right) \text{Si} \left(\frac{3(bc-ad)}{d} \right)}{65536 (b^3 c^3 d - 3 a b^2 c^2 d^2 + 3 a^2 b c d^3 - a^3 d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")`

[Out] $-\frac{1}{48} (8 (9 b^2 d^3 x^2 + 18 b^2 c d^2 x + 9 b^2 c^2 d - 2 d^3) \cos(bx + a)^3 + 54 (b^3 d^3 x^3 + 3 b^3 c d^2 x^2 + 3 b^3 c^2 d x + b^3 c^3) \cos(-3*(b*c - a*d)/d) \sin_integral(3*(b*d*x + b*c)/d) - 2 (b^3 d^3 x^3 + 3 b^3 c d^2 x^2 + 3 b^3 c^2 d x + b^3 c^3) \cos(-(b*c - a*d)/d) \sin_integral((b*d*x + b*c)/d) - 8 (7 b^2 d^3 x^2 + 14 b^2 c d^2 x + 7 b^2 c^2 d - 2 d^3) \cos(bx + a) - 8 (b^2 d^3 x + b^2 c d^2 - 3 (b^2 d^3 x + b^2 c d^2)) \cos(bx + a)^2 \sin(bx + a) - ((b^3 d^3 x^3 + 3 b^3 c d^2 x^2 + 3 b^3 c^2 d x + b^3 c^3) \cos_integral((b*d*x + b*c)/d) + (b^3 d^3 x^3 + 3 b^3 c d^2 x^2 + 3 b^3 c^2 d x + b^3 c^3) \cos_integral(-(b*d*x + b*c)/d)) \sin(-(b*c - a*d)/d) + 27 ((b^3 d^3 x^3 + 3 b^3 c d^2 x^2 + 3 b^3 c^2 d x + b^3 c^3) \cos_integral(3*(b*d*x + b*c)/d) - 27 (b^3 d^3 x^3 + 3 b^3 c d^2 x^2 + 3 b^3 c^2 d x + b^3 c^3) \cos_integral(-(b*d*x + b*c)/d))) / ((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4) + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)$

)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integr
 al(-3*(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c
 ^2*d^5*x + c^3*d^4)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a + bx) \cos(a + bx)}{(c + dx)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c)**4,x)

[Out] Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x)**4, x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")

[Out] Exception raised: NotImplementedError

3.22 $\int (c + dx)^m \cos(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=271

$$\frac{2^{-m-4} e^{2i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{2ib(c+dx)}{d}\right)}{b} + \frac{2^{-2(m+3)} e^{4i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{4ib(c+dx)}{d}\right)}{b}$$

[Out] $-\left(\frac{2^{-4-m} E^{(2I)(a-(b*c)/d)} (c+d*x)^m \Gamma[1+m, ((-2I)*b*(c+d*x))/d]}{b \left(\frac{(-I)*b*(c+d*x)}{d}\right)^m}\right) - \left(\frac{2^{-4-m} (c+d*x)^m \Gamma[1+m, ((2I)*b*(c+d*x))/d]}{b E^{(2I)(a-(b*c)/d)} \left(\frac{I*b*(c+d*x)}{d}\right)^m}\right) + \left(\frac{E^{(4I)(a-(b*c)/d)} (c+d*x)^m \Gamma[1+m, ((-4I)*b*(c+d*x))/d]}{2^{2*(3+m)} b \left(\frac{(-I)*b*(c+d*x)}{d}\right)^m}\right) + \left(\frac{(c+d*x)^m \Gamma[1+m, ((4I)*b*(c+d*x))/d]}{2^{2*(3+m)} b E^{(4I)(a-(b*c)/d)} \left(\frac{I*b*(c+d*x)}{d}\right)^m}\right)$

Rubi [A] time = 0.33105, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4406, 3308, 2181}

$$\frac{2^{-m-4} e^{2i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{2ib(c+dx)}{d}\right)}{b} + \frac{2^{-2(m+3)} e^{4i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{4ib(c+dx)}{d}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^m * Cos[a + b*x] * Sin[a + b*x]^3, x]

[Out] $-\left(\frac{2^{-4-m} E^{(2I)(a-(b*c)/d)} (c+d*x)^m \Gamma[1+m, ((-2I)*b*(c+d*x))/d]}{b \left(\frac{(-I)*b*(c+d*x)}{d}\right)^m}\right) - \left(\frac{2^{-4-m} (c+d*x)^m \Gamma[1+m, ((2I)*b*(c+d*x))/d]}{b E^{(2I)(a-(b*c)/d)} \left(\frac{I*b*(c+d*x)}{d}\right)^m}\right) + \left(\frac{E^{(4I)(a-(b*c)/d)} (c+d*x)^m \Gamma[1+m, ((-4I)*b*(c+d*x))/d]}{2^{2*(3+m)} b \left(\frac{(-I)*b*(c+d*x)}{d}\right)^m}\right) + \left(\frac{(c+d*x)^m \Gamma[1+m, ((4I)*b*(c+d*x))/d]}{2^{2*(3+m)} b E^{(4I)(a-(b*c)/d)} \left(\frac{I*b*(c+d*x)}{d}\right)^m}\right)$

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_.)]^(p_.)*((c_.) + (d_.)*(x_.))^(m_.)*Sin[(a_.) + (b_.)*(x_.)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n * Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3308

Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] :> Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m * E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 2181

Int[(F_)^(g_.)*((e_.) + (f_.)*(x_.))*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] :> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-(f*g*Log[F])/d)]*(c + d*x)]/(d*(-(f*g*Log[F])/d)^(IntPart[m] + 1)*(-(f*g*Log[F])*(c + d*x)/d)^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int (c + dx)^m \cos(a + bx) \sin^3(a + bx) dx &= \int \left(\frac{1}{4}(c + dx)^m \sin(2a + 2bx) - \frac{1}{8}(c + dx)^m \sin(4a + 4bx) \right) dx \\
&= -\left(\frac{1}{8} \int (c + dx)^m \sin(4a + 4bx) dx \right) + \frac{1}{4} \int (c + dx)^m \sin(2a + 2bx) dx \\
&= -\left(\frac{1}{16} i \int e^{-i(4a+4bx)} (c + dx)^m dx \right) + \frac{1}{16} i \int e^{i(4a+4bx)} (c + dx)^m dx + \frac{1}{8} i \int e^{-i(4a+4bx)} (c + dx)^m dx \\
&= -\frac{2^{-4-m} e^{2i\left(a-\frac{bc}{d}\right)} (c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, -\frac{2ib(c+dx)}{d}\right)}{b} - \frac{2^{-4-m} e^{-2i\left(a-\frac{bc}{d}\right)} (c + dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, \frac{2ib(c+dx)}{d}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.299893, size = 246, normalized size = 0.91

$$4^{-m-3} e^{-\frac{4i(ad+bc)}{d}} (c + dx)^m \left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m} \left(-2^{m+2} e^{2i\left(a+\frac{3bc}{d}\right)} \left(-\frac{ib(c+dx)}{d}\right)^m \Gamma\left(m+1, \frac{2ib(c+dx)}{d}\right) - 2^{m+2} e^{2i\left(3a+\frac{bc}{d}\right)} \left(\frac{ib(c+dx)}{d}\right)^m \Gamma\left(m+1, -\frac{2ib(c+dx)}{d}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^m * Cos[a + b*x] * Sin[a + b*x]^3, x]

[Out] (4^(-3 - m) * (c + d*x)^m * (-2^(2 + m) * E^((2*I)*(3*a + (b*c)/d)) * ((I*b*(c + d*x))/d)^m * Gamma[1 + m, ((-2*I)*b*(c + d*x))/d] - 2^(2 + m) * E^((2*I)*(a + (3*b*c)/d)) * (((-I)*b*(c + d*x))/d)^m * Gamma[1 + m, ((2*I)*b*(c + d*x))/d] + E^((8*I)*a) * ((I*b*(c + d*x))/d)^m * Gamma[1 + m, ((-4*I)*b*(c + d*x))/d] + E^(((8*I)*b*c)/d) * (((-I)*b*(c + d*x))/d)^m * Gamma[1 + m, ((4*I)*b*(c + d*x))/d]) / (b * E^(((4*I)*(b*c + a*d))/d) * ((b^2*(c + d*x)^2)/d^2)^m)

Maple [F] time = 0.265, size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) (\sin(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x)

[Out] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) \sin(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")

[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^3, x)

Fricas [A] time = 0.551049, size = 486, normalized size = 1.79

$$\frac{e^{\left(-\frac{dm \log\left(\frac{4ib}{d}\right) - 4ibc + 4iad}{d}\right)} \Gamma\left(m+1, \frac{4ibdx + 4ibc}{d}\right) - 4e^{\left(-\frac{dm \log\left(\frac{2ib}{d}\right) - 2ibc + 2iad}{d}\right)} \Gamma\left(m+1, \frac{2ibdx + 2ibc}{d}\right) - 4e^{\left(-\frac{dm \log\left(-\frac{2ib}{d}\right) + 2ibc - 2iad}{d}\right)} \Gamma\left(m+1, \frac{-2ibdx - 2ibc}{d}\right) + e^{\left(-\frac{dm \log(-4ib/d) + 4ibc - 4iad}{d}\right)} \Gamma\left(m+1, \frac{-4ibdx - 4ibc}{d}\right)}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")

[Out] 1/64*(e^(-(d*m*log(4*I*b/d) - 4*I*b*c + 4*I*a*d)/d)*gamma(m + 1, (4*I*b*d*x + 4*I*b*c)/d) - 4*e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a*d)/d)*gamma(m + 1, (2*I*b*d*x + 2*I*b*c)/d) - 4*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d)*gamma(m + 1, (-2*I*b*d*x - 2*I*b*c)/d) + e^(-(d*m*log(-4*I*b/d) + 4*I*b*c - 4*I*a*d)/d)*gamma(m + 1, (-4*I*b*d*x - 4*I*b*c)/d))/b

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**m*cos(b*x+a)*sin(b*x+a)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) \sin(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")

[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^3, x)

3.23 $\int (c + dx)^4 \cos(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=260

$$\frac{3d^2(c + dx)^2 \sin^4(a + bx)}{16b^3} - \frac{9d^2(c + dx)^2 \sin^2(a + bx)}{16b^3} - \frac{3d^3(c + dx) \sin^3(a + bx) \cos(a + bx)}{32b^4} - \frac{45d^3(c + dx) \sin(a + bx) \cos^2(a + bx)}{64b^4}$$

[Out] $(45*c*d^3*x)/(64*b^3) + (45*d^4*x^2)/(128*b^3) - (3*(c + d*x)^4)/(32*b) - (45*d^3*(c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(64*b^4) + (3*d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/(8*b^2) + (45*d^4*Sin[a + b*x]^2)/(128*b^5) - (9*d^2*(c + d*x)^2*Sin[a + b*x]^2)/(16*b^3) - (3*d^3*(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^3)/(32*b^4) + (d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^3)/(4*b^2) + (3*d^4*Sin[a + b*x]^4)/(128*b^5) - (3*d^2*(c + d*x)^2*Sin[a + b*x]^4)/(16*b^3) + ((c + d*x)^4*Sin[a + b*x]^4)/(4*b)$

Rubi [A] time = 0.241104, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4404, 3311, 32, 3310}

$$\frac{3d^2(c + dx)^2 \sin^4(a + bx)}{16b^3} - \frac{9d^2(c + dx)^2 \sin^2(a + bx)}{16b^3} - \frac{3d^3(c + dx) \sin^3(a + bx) \cos(a + bx)}{32b^4} - \frac{45d^3(c + dx) \sin(a + bx) \cos^2(a + bx)}{64b^4}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^3,x]

[Out] $(45*c*d^3*x)/(64*b^3) + (45*d^4*x^2)/(128*b^3) - (3*(c + d*x)^4)/(32*b) - (45*d^3*(c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(64*b^4) + (3*d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/(8*b^2) + (45*d^4*Sin[a + b*x]^2)/(128*b^5) - (9*d^2*(c + d*x)^2*Sin[a + b*x]^2)/(16*b^3) - (3*d^3*(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^3)/(32*b^4) + (d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^3)/(4*b^2) + (3*d^4*Sin[a + b*x]^4)/(128*b^5) - (3*d^2*(c + d*x)^2*Sin[a + b*x]^4)/(16*b^3) + ((c + d*x)^4*Sin[a + b*x]^4)/(4*b)$

Rule 4404

Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 3311

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^4 \cos(a + bx) \sin^3(a + bx) dx &= \frac{(c + dx)^4 \sin^4(a + bx)}{4b} - \frac{d \int (c + dx)^3 \sin^4(a + bx) dx}{b} \\
&= \frac{d(c + dx)^3 \cos(a + bx) \sin^3(a + bx)}{4b^2} - \frac{3d^2(c + dx)^2 \sin^4(a + bx)}{16b^3} + \frac{(c + dx)^4 \sin^4(a + bx)}{4b} \\
&= \frac{3d(c + dx)^3 \cos(a + bx) \sin(a + bx)}{8b^2} - \frac{9d^2(c + dx)^2 \sin^2(a + bx)}{16b^3} - \frac{3d^3(c + dx)}{16b^3} \\
&= -\frac{3(c + dx)^4}{32b} - \frac{45d^3(c + dx) \cos(a + bx) \sin(a + bx)}{64b^4} + \frac{3d(c + dx)^3 \cos(a + bx)}{8b^2} \\
&= \frac{45cd^3x}{64b^3} + \frac{45d^4x^2}{128b^3} - \frac{3(c + dx)^4}{32b} - \frac{45d^3(c + dx) \cos(a + bx) \sin(a + bx)}{64b^4} + \frac{3d(c + dx)^3 \cos(a + bx)}{8b^2}
\end{aligned}$$

Mathematica [A] time = 1.81049, size = 158, normalized size = 0.61

$$\frac{-64 \cos(2(a + bx)) \left(-6b^2d^2(c + dx)^2 + 2b^4(c + dx)^4 + 3d^4 \right) + \cos(4(a + bx)) \left(-24b^2d^2(c + dx)^2 + 32b^4(c + dx)^4 + 3d^4 \right) - 1024b^5}{1024b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-64*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)] +
(3*d^4 - 24*b^2*d^2*(c + d*x)^2 + 32*b^4*(c + d*x)^4)*Cos[4*(a + b*x)] - 8
*b*d*(c + d*x)*(-16*(-3*d^2 + 2*b^2*(c + d*x)^2) + (-3*d^2 + 8*b^2*(c + d*x
)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)]/(1024*b^5)
```

Maple [B] time = 0.063, size = 1143, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^4*d^4*(1/4*(b*x+a)^4*sin(b*x+a)^4-(b*x+a)^3*(-1/4*(sin(b*x+a)^3+3/
2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/16*(b*x+a)^2*sin(b*x+a)^4+3/8*(b*
x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+27/128*(
b*x+a)^2+3/128*sin(b*x+a)^4+45/128*sin(b*x+a)^2+9/16*(b*x+a)^2*cos(b*x+a)^2
-9/8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+9/32*(b*x+a)^4)-4/b^
4*a*d^4*(1/4*(b*x+a)^3*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*s
in(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)*sin(b*x+a)^4-3/128*(sin(b
*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-27/256*b*x-27/256*a+9/32*(b*x+a)*cos(b*x
+a)^2-9/64*cos(b*x+a)*sin(b*x+a)+3/16*(b*x+a)^3)+4/b^3*c*d^3*(1/4*(b*x+a)^3
*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+
3/8*b*x+3/8*a)-3/32*(b*x+a)*sin(b*x+a)^4-3/128*(sin(b*x+a)^3+3/2*sin(b*x+a)
```

```

) * cos(b*x+a) - 27/256*b*x - 27/256*a + 9/32*(b*x+a) * cos(b*x+a)^2 - 9/64*cos(b*x+a) *
sin(b*x+a) + 3/16*(b*x+a)^3 + 6/b^4*a^2*d^4*(1/4*(b*x+a)^2*sin(b*x+a)^4 - 1/2*(b
*x+a)*(-1/4*(sin(b*x+a)^3 + 3/2*sin(b*x+a)) * cos(b*x+a) + 3/8*b*x + 3/8*a) + 3/32*(b
*x+a)^2 - 1/32*sin(b*x+a)^4 - 3/32*sin(b*x+a)^2) - 12/b^3*a*c*d^3*(1/4*(b*x+a)^2*
sin(b*x+a)^4 - 1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3 + 3/2*sin(b*x+a)) * cos(b*x+a) + 3/8
*b*x + 3/8*a) + 3/32*(b*x+a)^2 - 1/32*sin(b*x+a)^4 - 3/32*sin(b*x+a)^2) + 6/b^2*c^2*d
^2*(1/4*(b*x+a)^2*sin(b*x+a)^4 - 1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3 + 3/2*sin(b*x+
a)) * cos(b*x+a) + 3/8*b*x + 3/8*a) + 3/32*(b*x+a)^2 - 1/32*sin(b*x+a)^4 - 3/32*sin(b*x
+a)^2) - 4/b^4*a^3*d^4*(1/4*(b*x+a)*sin(b*x+a)^4 + 1/16*(sin(b*x+a)^3 + 3/2*sin(b
*x+a)) * cos(b*x+a) - 3/32*b*x - 3/32*a) + 12/b^3*a^2*c*d^3*(1/4*(b*x+a)*sin(b*x+a)
^4 + 1/16*(sin(b*x+a)^3 + 3/2*sin(b*x+a)) * cos(b*x+a) - 3/32*b*x - 3/32*a) - 12/b^2*a*
c^2*d^2*(1/4*(b*x+a)*sin(b*x+a)^4 + 1/16*(sin(b*x+a)^3 + 3/2*sin(b*x+a)) * cos(b
*x+a) - 3/32*b*x - 3/32*a) + 4/b*c^3*d*(1/4*(b*x+a)*sin(b*x+a)^4 + 1/16*(sin(b*x+a)^
3 + 3/2*sin(b*x+a)) * cos(b*x+a) - 3/32*b*x - 3/32*a) + 1/4/b^4*a^4*d^4*sin(b*x+a)^4 -
1/b^3*a^3*c*d^3*sin(b*x+a)^4 + 3/2/b^2*a^2*c^2*d^2*sin(b*x+a)^4 - 1/b*a*c^3*d*s
in(b*x+a)^4 + 1/4*c^4*sin(b*x+a)^4)

```

Maxima [B] time = 1.313, size = 1305, normalized size = 5.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```

[Out] 1/1024*(256*c^4*sin(b*x + a)^4 - 1024*a*c^3*d*sin(b*x + a)^4/b + 1536*a^2*c
^2*d^2*sin(b*x + a)^4/b^2 - 1024*a^3*c*d^3*sin(b*x + a)^4/b^3 + 256*a^4*d^4
*sin(b*x + a)^4/b^4 + 32*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2
*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*c^3*d/b - 96*(4*(b*x +
a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8
*sin(2*b*x + 2*a))*a*c^2*d^2/b^2 + 96*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b
*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*a^2*c*d^3
/b^3 - 32*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - s
in(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*a^3*d^4/b^4 + 24*((8*(b*x + a)^2 - 1)
*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*s
in(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 - 48*((8*(b*x
+ a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*
(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/b^3 + 2
4*((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x
+ 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*a^2*
d^4/b^4 + 4*(4*(8*(b*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) - 64*(2*(b*x
+ a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x +
4*a) + 96*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3 - 4*(4*(8*(b*x +
a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) - 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos
(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) + 96*(2*(b*x + a)^2
- 1)*sin(2*b*x + 2*a))*a*d^4/b^4 + ((32*(b*x + a)^4 - 24*(b*x + a)^2 + 3)*c
os(4*b*x + 4*a) - 64*(2*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*cos(2*b*x + 2*a) -
4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*b*x + 4*a) + 128*(2*(b*x + a)^3 - 3*
b*x - 3*a)*sin(2*b*x + 2*a))*d^4/b^4)/b

```

Fricas [A] time = 0.540208, size = 936, normalized size = 3.6

$$20b^4d^4x^4 + 80b^4cd^3x^3 + (32b^4d^4x^4 + 128b^4cd^3x^3 + 32b^4c^4 - 24b^2c^2d^2 + 3d^4 + 24(8b^4c^2d^2 - b^2d^4)x^2 + 16(8b^4c^3d^2 - b^2c^2d^4)x + 16(8b^4c^4 - b^2c^2d^4))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")

[Out] $\frac{1}{128} \cdot (20 \cdot b^4 \cdot d^4 \cdot x^4 + 80 \cdot b^4 \cdot c \cdot d^3 \cdot x^3 + (32 \cdot b^4 \cdot d^4 \cdot x^4 + 128 \cdot b^4 \cdot c \cdot d^3 \cdot x^3 + 32 \cdot b^4 \cdot c^4 - 24 \cdot b^2 \cdot c^2 \cdot d^2 + 3 \cdot d^4 + 24 \cdot (8 \cdot b^4 \cdot c^2 \cdot d^2 - b^2 \cdot d^4)) \cdot x^2 + 16 \cdot (8 \cdot b^4 \cdot c^3 \cdot d - 3 \cdot b^2 \cdot c \cdot d^3) \cdot x) \cdot \cos(b \cdot x + a)^4 + 3 \cdot (40 \cdot b^4 \cdot c^2 \cdot d^2 - 17 \cdot b^2 \cdot d^4) \cdot x^2 - (64 \cdot b^4 \cdot d^4 \cdot x^4 + 256 \cdot b^4 \cdot c \cdot d^3 \cdot x^3 + 64 \cdot b^4 \cdot c^4 - 120 \cdot b^2 \cdot c^2 \cdot d^2 + 51 \cdot d^4 + 24 \cdot (16 \cdot b^4 \cdot c^2 \cdot d^2 - 5 \cdot b^2 \cdot d^4)) \cdot x^2 + 16 \cdot (16 \cdot b^4 \cdot c^3 \cdot d - 15 \cdot b^2 \cdot c \cdot d^3) \cdot x) \cdot \cos(b \cdot x + a)^2 + 2 \cdot (40 \cdot b^4 \cdot c^3 \cdot d - 51 \cdot b^2 \cdot c \cdot d^3) \cdot x - 2 \cdot (2 \cdot (8 \cdot b^3 \cdot d^4 \cdot x^3 + 24 \cdot b^3 \cdot c \cdot d^3 \cdot x^2 + 8 \cdot b^3 \cdot c^3 \cdot d - 3 \cdot b \cdot c \cdot d^3 + 3 \cdot (8 \cdot b^3 \cdot c^2 \cdot d^2 - b \cdot d^4)) \cdot x) \cdot \cos(b \cdot x + a)^3 - (40 \cdot b^3 \cdot d^4 \cdot x^3 + 120 \cdot b^3 \cdot c \cdot d^3 \cdot x^2 + 4 \cdot 0 \cdot b^3 \cdot c^3 \cdot d - 51 \cdot b \cdot c \cdot d^3 + 3 \cdot (40 \cdot b^3 \cdot c^2 \cdot d^2 - 17 \cdot b \cdot d^4)) \cdot x) \cdot \cos(b \cdot x + a) \cdot \sin(b \cdot x + a) / b^5$

Sympy [A] time = 18.5406, size = 976, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**4*cos(b*x+a)*sin(b*x+a)**3,x)

[Out] Piecewise((-c**4*sin(a + b*x)**2*cos(a + b*x)**2/(2*b) - c**4*cos(a + b*x)**4/(4*b) + 5*c**3*d*x*sin(a + b*x)**4/(8*b) - 3*c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(4*b) - 3*c**3*d*x*cos(a + b*x)**4/(8*b) + 15*c**2*d**2*x**2*sin(a + b*x)**4/(16*b) - 9*c**2*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(8*b) - 9*c**2*d**2*x**2*cos(a + b*x)**4/(16*b) + 5*c*d**3*x**3*sin(a + b*x)**4/(8*b) - 3*c*d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**2/(4*b) - 3*c*d**3*x**3*cos(a + b*x)**4/(8*b) + 5*d**4*x**4*sin(a + b*x)**4/(32*b) - 3*d**4*x**4*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 3*d**4*x**4*cos(a + b*x)**4/(32*b) + 5*c**3*d*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 3*c**3*d*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 15*c**2*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 9*c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 15*c*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 9*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 5*d**4*x**3*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 3*d**4*x**3*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 15*c**2*d**2*sin(a + b*x)**2*cos(a + b*x)**2/(16*b**3) + 3*c**2*d**2*cos(a + b*x)**4/(4*b**3) - 51*c*d**3*x*sin(a + b*x)**4/(64*b**3) + 9*c*d**3*x*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**3) + 45*c*d**3*x*cos(a + b*x)**4/(64*b**3) - 51*d**4*x**2*sin(a + b*x)**4/(128*b**3) + 9*d**4*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(64*b**3) + 45*d**4*x**2*cos(a + b*x)**4/(128*b**3) - 51*c*d**3*sin(a + b*x)**3*cos(a + b*x)/(64*b**4) - 45*c*d**3*sin(a + b*x)*cos(a + b*x)**3/(64*b**4) - 51*d**4*x*sin(a + b*x)**3*cos(a + b*x)/(64*b**4) - 45*d**4*x*sin(a + b*x)*cos(a + b*x)**3/(64*b**4) - 51*d**4*sin(a + b*x)**2*cos(a + b*x)**2/(128*b**5) - 3*d**4*cos(a + b*x)**4/(8*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**3*cos(a), True))

Giac [A] time = 1.08493, size = 487, normalized size = 1.87

$$\frac{(32 b^4 d^4 x^4 + 128 b^4 c d^3 x^3 + 192 b^4 c^2 d^2 x^2 + 128 b^4 c^3 d x + 32 b^4 c^4 - 24 b^2 d^4 x^2 - 48 b^2 c d^3 x - 24 b^2 c^2 d^2 + 3 d^4) \cos(4 b x + a)}{1024 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")


```
[Out] 1/1024*(32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 192*b^4*c^2*d^2*x^2 + 128*b^4*c^3*d*x + 32*b^4*c^4 - 24*b^2*d^4*x^2 - 48*b^2*c*d^3*x - 24*b^2*c^2*d^2 + 3*d^4)*cos(4*b*x + 4*a)/b^5 - 1/16*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3*x - 6*b^2*c^2*d^2 + 3*d^4)*cos(2*b*x + 2*a)/b^5 - 1/256*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 24*b^3*c^2*d^2*x + 8*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(4*b*x + 4*a)/b^5 + 1/8*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x + 2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

3.24 $\int (c + dx)^3 \cos(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=196

$$-\frac{3d^2(c + dx) \sin^4(a + bx)}{32b^3} - \frac{9d^2(c + dx) \sin^2(a + bx)}{32b^3} + \frac{3d(c + dx)^2 \sin^3(a + bx) \cos(a + bx)}{16b^2} + \frac{9d(c + dx)^2 \sin(a + bx) \cos(a + bx)}{32b^2}$$

[Out] $(45*d^3*x)/(256*b^3) - (3*(c + d*x)^3)/(32*b) - (45*d^3*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/(256*b^4) + (9*d*(c + d*x)^2*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/(32*b^2) - (9*d^2*(c + d*x)*\text{Sin}[a + b*x]^2)/(32*b^3) - (3*d^3*\text{Cos}[a + b*x]*\text{Sin}[a + b*x]^3)/(128*b^4) + (3*d*(c + d*x)^2*\text{Cos}[a + b*x]*\text{Sin}[a + b*x]^3)/(16*b^2) - (3*d^2*(c + d*x)*\text{Sin}[a + b*x]^4)/(32*b^3) + ((c + d*x)^3*\text{Sin}[a + b*x]^4)/(4*b)$

Rubi [A] time = 0.165402, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4404, 3311, 32, 2635, 8}

$$-\frac{3d^2(c + dx) \sin^4(a + bx)}{32b^3} - \frac{9d^2(c + dx) \sin^2(a + bx)}{32b^3} + \frac{3d(c + dx)^2 \sin^3(a + bx) \cos(a + bx)}{16b^2} + \frac{9d(c + dx)^2 \sin(a + bx) \cos(a + bx)}{32b^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^3,x]

[Out] $(45*d^3*x)/(256*b^3) - (3*(c + d*x)^3)/(32*b) - (45*d^3*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/(256*b^4) + (9*d*(c + d*x)^2*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/(32*b^2) - (9*d^2*(c + d*x)*\text{Sin}[a + b*x]^2)/(32*b^3) - (3*d^3*\text{Cos}[a + b*x]*\text{Sin}[a + b*x]^3)/(128*b^4) + (3*d*(c + d*x)^2*\text{Cos}[a + b*x]*\text{Sin}[a + b*x]^3)/(16*b^2) - (3*d^2*(c + d*x)*\text{Sin}[a + b*x]^4)/(32*b^3) + ((c + d*x)^3*\text{Sin}[a + b*x]^4)/(4*b)$

Rule 4404

Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 3311

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_.), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 1), x], x]

$+ d*x])^{(n - 2), x], x] /; \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 8

$\text{Int}[a_, x_Symbol] \ :> \ \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rubi steps

$$\begin{aligned} \int (c + dx)^3 \cos(a + bx) \sin^3(a + bx) dx &= \frac{(c + dx)^3 \sin^4(a + bx)}{4b} - \frac{(3d) \int (c + dx)^2 \sin^4(a + bx) dx}{4b} \\ &= \frac{3d(c + dx)^2 \cos(a + bx) \sin^3(a + bx)}{16b^2} - \frac{3d^2(c + dx) \sin^4(a + bx)}{32b^3} + \frac{(c + dx)^3}{32b^3} \\ &= \frac{9d(c + dx)^2 \cos(a + bx) \sin(a + bx)}{32b^2} - \frac{9d^2(c + dx) \sin^2(a + bx)}{32b^3} - \frac{3d^3 \cos(a + bx)}{32b^3} \\ &= -\frac{3(c + dx)^3}{32b} - \frac{45d^3 \cos(a + bx) \sin(a + bx)}{256b^4} + \frac{9d(c + dx)^2 \cos(a + bx) \sin(a + bx)}{32b^2} \\ &= \frac{45d^3 x}{256b^3} - \frac{3(c + dx)^3}{32b} - \frac{45d^3 \cos(a + bx) \sin(a + bx)}{256b^4} + \frac{9d(c + dx)^2 \cos(a + bx) \sin(a + bx)}{32b^2} \end{aligned}$$

Mathematica [A] time = 0.969691, size = 135, normalized size = 0.69

$$\frac{-64b(c + dx) \cos(2(a + bx)) (2b^2(c + dx)^2 - 3d^2) + 4b(c + dx) \cos(4(a + bx)) (8b^2(c + dx)^2 - 3d^2) - 6d \sin(2(a + bx))}{1024b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^3,x]

[Out] (-64*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 4*b*(c + d*x)*(-3*d^2 + 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] - 6*d*(-16*(-d^2 + 2*b^2*(c + d*x)^2) + (-d^2 + 8*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)]/(1024*b^4)

Maple [B] time = 0.019, size = 594, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x)

[Out] $1/b*(1/b^3*d^3*(1/4*(b*x+a)^3*\sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(\sin(b*x+a)^{3+3/2*\sin(b*x+a)}*\cos(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)*\sin(b*x+a)^4-3/128*(\sin(b*x+a)^{3+3/2*\sin(b*x+a)}*\cos(b*x+a)-27/256*b*x-27/256*a+9/32*(b*x+a)*\cos(b*x+a)^2-9/64*\cos(b*x+a)*\sin(b*x+a)+3/16*(b*x+a)^3)-3/b^3*a*d^3*(1/4*(b*x+a)^2*\sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(\sin(b*x+a)^{3+3/2*\sin(b*x+a)}*\cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)^2-1/32*\sin(b*x+a)^4-3/32*\sin(b*x+a)^2)+3/b^2*c*d^2*(1/4*(b*x+a)^2*\sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(\sin(b*x+a)^{3+3/2*\sin(b*x+a)}*\cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)^2-1/32*\sin(b*x+a)^4-3/32*\sin(b*x+a)^2)+3/b^3*a^2*d^3*(1/4*(b*x+a)*\sin(b*x+a)^4+1/16*(\sin(b*x+a)^{3+3/2*\sin(b*x+a)}*\cos(b*x+a)-3/32*b*x-3/32*a)-6/b^2*a*c*d^2*(1/4*(b*x+a)*\sin(b*x$

$$+a)^4 + 1/16 * (\sin(b*x+a)^3 + 3/2 * \sin(b*x+a)) * \cos(b*x+a) - 3/32 * b*x - 3/32 * a + 3/b * c^2 * d * (1/4 * (b*x+a) * \sin(b*x+a)^4 + 1/16 * (\sin(b*x+a)^3 + 3/2 * \sin(b*x+a)) * \cos(b*x+a) - 3/32 * b*x - 3/32 * a) - 1/4 / b^3 * a^3 * d^3 * \sin(b*x+a)^4 + 3/4 / b^2 * a^2 * c * d^2 * \sin(b*x+a)^4 - 3/4 / b * a * c^2 * d * \sin(b*x+a)^4 + 1/4 * c^3 * \sin(b*x+a)^4$$

Maxima [B] time = 1.21312, size = 741, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")

[Out] $1/1024 * (256 * c^3 * \sin(b*x + a)^4 - 768 * a * c^2 * d * \sin(b*x + a)^4 / b + 768 * a^2 * c * d^2 * \sin(b*x + a)^4 / b^2 - 256 * a^3 * d^3 * \sin(b*x + a)^4 / b^3 + 24 * (4 * (b*x + a) * \cos(4 * b*x + 4 * a) - 16 * (b*x + a) * \cos(2 * b*x + 2 * a) - \sin(4 * b*x + 4 * a) + 8 * \sin(2 * b*x + 2 * a)) * c^2 * d / b - 48 * (4 * (b*x + a) * \cos(4 * b*x + 4 * a) - 16 * (b*x + a) * \cos(2 * b*x + 2 * a) - \sin(4 * b*x + 4 * a) + 8 * \sin(2 * b*x + 2 * a)) * a * c * d^2 / b^2 + 24 * (4 * (b*x + a) * \cos(4 * b*x + 4 * a) - 16 * (b*x + a) * \cos(2 * b*x + 2 * a) - \sin(4 * b*x + 4 * a) + 8 * \sin(2 * b*x + 2 * a)) * a^2 * d^3 / b^3 + 12 * ((8 * (b*x + a)^2 - 1) * \cos(4 * b*x + 4 * a) - 16 * (2 * (b*x + a)^2 - 1) * \cos(2 * b*x + 2 * a) - 4 * (b*x + a) * \sin(4 * b*x + 4 * a) + 32 * (b*x + a) * \sin(2 * b*x + 2 * a)) * c * d^2 / b^2 - 12 * ((8 * (b*x + a)^2 - 1) * \cos(4 * b*x + 4 * a) - 16 * (2 * (b*x + a)^2 - 1) * \cos(2 * b*x + 2 * a) - 4 * (b*x + a) * \sin(4 * b*x + 4 * a) + 32 * (b*x + a) * \sin(2 * b*x + 2 * a)) * a * d^3 / b^3 + (4 * (8 * (b*x + a)^3 - 3 * b*x - 3 * a) * \cos(4 * b*x + 4 * a) - 64 * (2 * (b*x + a)^3 - 3 * b*x - 3 * a) * \cos(2 * b*x + 2 * a) - 3 * (8 * (b*x + a)^2 - 1) * \sin(4 * b*x + 4 * a) + 96 * (2 * (b*x + a)^2 - 1) * \sin(2 * b*x + 2 * a)) * d^3 / b^3) / b$

Fricas [A] time = 0.517854, size = 621, normalized size = 3.17

$$40 b^3 d^3 x^3 + 120 b^3 c d^2 x^2 + 8 (8 b^3 d^3 x^3 + 24 b^3 c d^2 x^2 + 8 b^3 c^3 - 3 b c d^2 + 3 (8 b^3 c^2 d - b d^3) x) \cos(bx + a)^4 - 8 (16 b^3 d^3 x^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")

[Out] $1/256 * (40 * b^3 * d^3 * x^3 + 120 * b^3 * c * d^2 * x^2 + 8 * (8 * b^3 * d^3 * x^3 + 24 * b^3 * c * d^2 * x^2 + 8 * b^3 * c^3 - 3 * b * c * d^2 + 3 * (8 * b^3 * c^2 * d - b * d^3) * x) * \cos(b*x + a)^4 - 8 * (16 * b^3 * d^3 * x^3 + 48 * b^3 * c * d^2 * x^2 + 16 * b^3 * c^3 - 15 * b * c * d^2 + 3 * (16 * b^3 * c^2 * d - 5 * b * d^3) * x) * \cos(b*x + a)^2 + 3 * (40 * b^3 * c^2 * d - 17 * b * d^3) * x - 3 * (2 * (8 * b^2 * d^3 * x^2 + 16 * b^2 * c * d^2 * x + 8 * b^2 * c^2 * d - d^3) * \cos(b*x + a)^3 - (40 * b^2 * d^3 * x^2 + 80 * b^2 * c * d^2 * x + 40 * b^2 * c^2 * d - 17 * d^3) * \cos(b*x + a)) * \sin(b*x + a)) / b^4$

Sympy [A] time = 10.4897, size = 634, normalized size = 3.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*cos(b*x+a)*sin(b*x+a)**3,x)

```
[Out] Piecewise((-c**3*sin(a + b*x)**2*cos(a + b*x)**2/(2*b) - c**3*cos(a + b*x)*
**4/(4*b) + 15*c**2*d*x*sin(a + b*x)**4/(32*b) - 9*c**2*d*x*sin(a + b*x)**2*
cos(a + b*x)**2/(16*b) - 9*c**2*d*x*cos(a + b*x)**4/(32*b) + 15*c*d**2*x**2
*sin(a + b*x)**4/(32*b) - 9*c*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(16
*b) - 9*c*d**2*x**2*cos(a + b*x)**4/(32*b) + 5*d**3*x**3*sin(a + b*x)**4/(3
2*b) - 3*d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 3*d**3*x**3*cos
(a + b*x)**4/(32*b) + 15*c**2*d*sin(a + b*x)**3*cos(a + b*x)/(32*b**2) + 9*
c**2*d*sin(a + b*x)*cos(a + b*x)**3/(32*b**2) + 15*c*d**2*x*sin(a + b*x)**3
*cos(a + b*x)/(16*b**2) + 9*c*d**2*x*sin(a + b*x)*cos(a + b*x)**3/(16*b**2)
+ 15*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)/(32*b**2) + 9*d**3*x**2*sin(a
+ b*x)*cos(a + b*x)**3/(32*b**2) + 15*c*d**2*sin(a + b*x)**2*cos(a + b*x)**
2/(32*b**3) + 3*c*d**2*cos(a + b*x)**4/(8*b**3) - 51*d**3*x*sin(a + b*x)**4
/(256*b**3) + 9*d**3*x*sin(a + b*x)**2*cos(a + b*x)**2/(128*b**3) + 45*d**3
*x*cos(a + b*x)**4/(256*b**3) - 51*d**3*sin(a + b*x)**3*cos(a + b*x)/(256*b
**4) - 45*d**3*sin(a + b*x)*cos(a + b*x)**3/(256*b**4), Ne(b, 0)), ((c**3*x
+ 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**3*cos(a), True))
```

Giac [A] time = 1.13294, size = 325, normalized size = 1.66

$$\frac{(8b^3d^3x^3 + 24b^3cd^2x^2 + 24b^3c^2dx + 8b^3c^3 - 3bd^3x - 3bcd^2) \cos(4bx + 4a)}{256b^4} - \frac{(2b^3d^3x^3 + 6b^3cd^2x^2 + 6b^3c^2dx + 2b^3c^3 - 3bd^3x - 3bcd^2) \sin(4bx + 4a)}{256b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/256*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 3*b*
d^3*x - 3*b*c*d^2)*cos(4*b*x + 4*a)/b^4 - 1/16*(2*b^3*d^3*x^3 + 6*b^3*c*d^2
*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(2*b*x + 2*a)/
b^4 - 3/1024*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*sin(4*b*x
+ 4*a)/b^4 + 3/32*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*sin(
2*b*x + 2*a)/b^4
```

3.25 $\int (c + dx)^2 \cos(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=134

$$\frac{d(c + dx) \sin^3(a + bx) \cos(a + bx)}{8b^2} + \frac{3d(c + dx) \sin(a + bx) \cos(a + bx)}{16b^2} - \frac{d^2 \sin^4(a + bx)}{32b^3} - \frac{3d^2 \sin^2(a + bx)}{32b^3} + \frac{(c + dx)^2}{8b^2}$$

[Out] $(-3*c*d*x)/(16*b) - (3*d^2*x^2)/(32*b) + (3*d*(c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(16*b^2) - (3*d^2*Sin[a + b*x]^2)/(32*b^3) + (d*(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^3)/(8*b^2) - (d^2*Sin[a + b*x]^4)/(32*b^3) + ((c + d*x)^2*Sin[a + b*x]^4)/(4*b)$

Rubi [A] time = 0.0924045, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4404, 3310}

$$\frac{d(c + dx) \sin^3(a + bx) \cos(a + bx)}{8b^2} + \frac{3d(c + dx) \sin(a + bx) \cos(a + bx)}{16b^2} - \frac{d^2 \sin^4(a + bx)}{32b^3} - \frac{3d^2 \sin^2(a + bx)}{32b^3} + \frac{(c + dx)^2}{8b^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^3,x]

[Out] $(-3*c*d*x)/(16*b) - (3*d^2*x^2)/(32*b) + (3*d*(c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(16*b^2) - (3*d^2*Sin[a + b*x]^2)/(32*b^3) + (d*(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^3)/(8*b^2) - (d^2*Sin[a + b*x]^4)/(32*b^3) + ((c + d*x)^2*Sin[a + b*x]^4)/(4*b)$

Rule 4404

Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 3310

Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \cos(a + bx) \sin^3(a + bx) dx &= \frac{(c + dx)^2 \sin^4(a + bx)}{4b} - \frac{d \int (c + dx) \sin^4(a + bx) dx}{2b} \\ &= \frac{d(c + dx) \cos(a + bx) \sin^3(a + bx)}{8b^2} - \frac{d^2 \sin^4(a + bx)}{32b^3} + \frac{(c + dx)^2 \sin^4(a + bx)}{4b} \\ &= \frac{3d(c + dx) \cos(a + bx) \sin(a + bx)}{16b^2} - \frac{3d^2 \sin^2(a + bx)}{32b^3} + \frac{d(c + dx) \cos(a + bx)}{8b^2} \\ &= -\frac{3cdx}{16b} - \frac{3d^2x^2}{32b} + \frac{3d(c + dx) \cos(a + bx) \sin(a + bx)}{16b^2} - \frac{3d^2 \sin^2(a + bx)}{32b^3} + \frac{d(c + dx) \cos(a + bx)}{8b^2} \end{aligned}$$

Mathematica [A] time = 0.524018, size = 91, normalized size = 0.68

$$\frac{-16 \cos(2(a + bx)) (2b^2(c + dx)^2 - d^2) + \cos(4(a + bx)) (8b^2(c + dx)^2 - d^2) - 4bd(c + dx)(\sin(4(a + bx)) - 8 \sin(2(a + bx)))}{256b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^3,x]

[Out] (-16*(-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + (-d^2 + 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] - 4*b*d*(c + d*x)*(-8*Sin[2*(a + b*x)] + Sin[4*(a + b*x)]))/(256*b^3)

Maple [B] time = 0.02, size = 260, normalized size = 1.9

$$\frac{1}{b} \left(\frac{d^2}{b^2} \left(\frac{(bx + a)^2 (\sin(bx + a))^4}{4} - \frac{bx + a}{2} \left(-\frac{\cos(bx + a)}{4} \left((\sin(bx + a))^3 + \frac{3 \sin(bx + a)}{2} \right) + \frac{3bx}{8} + \frac{3a}{8} \right) + \frac{3(bx + a)}{32} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x)

[Out] 1/b*(1/b^2*d^2*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)^2-1/32*sin(b*x+a)^4-3/32*sin(b*x+a)^2)-2/b^2*a*d^2*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)+2/b*c*d*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)+1/4/b^2*a^2*d^2*sin(b*x+a)^4-1/2/b*a*c*d*sin(b*x+a)^4+1/4*c^2*sin(b*x+a)^4)

Maxima [B] time = 1.1864, size = 355, normalized size = 2.65

$$\frac{64c^2 \sin(bx + a)^4 - \frac{128acd \sin(bx+a)^4}{b} + \frac{64a^2d^2 \sin(bx+a)^4}{b^2} + \frac{4(4(bx+a) \cos(4bx+4a) - 16(bx+a) \cos(2bx+2a) - \sin(4bx+4a) + 8 \sin(2bx+2a))}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")

[Out] 1/256*(64*c^2*sin(b*x + a)^4 - 128*a*c*d*sin(b*x + a)^4/b + 64*a^2*d^2*sin(b*x + a)^4/b^2 + 4*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*c*d/b - 4*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*a*d^2/b^2 + ((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*d^2/b^2)/b

Fricas [A] time = 0.486331, size = 359, normalized size = 2.68

$$\frac{5b^2d^2x^2 + 10b^2cdx + (8b^2d^2x^2 + 16b^2cdx + 8b^2c^2 - d^2) \cos(bx + a)^4 - (16b^2d^2x^2 + 32b^2cdx + 16b^2c^2 - 5d^2) \cos(2bx + 2a)}{32b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")

[Out] $\frac{1}{32}*(5*b^2*d^2*x^2 + 10*b^2*c*d*x + (8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*\cos(b*x + a)^4 - (16*b^2*d^2*x^2 + 32*b^2*c*d*x + 16*b^2*c^2 - 5*d^2)*\cos(b*x + a)^2 - 2*(2*(b*d^2*x + b*c*d)*\cos(b*x + a)^3 - 5*(b*d^2*x + b*c*d)*\cos(b*x + a))*\sin(b*x + a))/b^3$

Sympy [A] time = 5.11579, size = 350, normalized size = 2.61

$$\left\{ \begin{array}{l} -\frac{c^2 \sin^2(a+bx) \cos^2(a+bx)}{2b} - \frac{c^2 \cos^4(a+bx)}{4b} + \frac{5cdx \sin^4(a+bx)}{16b} - \frac{3cdx \sin^2(a+bx) \cos^2(a+bx)}{8b} - \frac{3cdx \cos^4(a+bx)}{16b} + \frac{5d^2x^2 \sin^4(a+bx)}{32b} - \frac{3d^2x^2 \sin^2(a+bx) \cos^2(a+bx)}{64b} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3} \right) \sin^3(a) \cos(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*cos(b*x+a)*sin(b*x+a)**3,x)

[Out] Piecewise((-c**2*sin(a + b*x)**2*cos(a + b*x)**2/(2*b) - c**2*cos(a + b*x)**4/(4*b) + 5*c*d*x*sin(a + b*x)**4/(16*b) - 3*c*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(8*b) - 3*c*d*x*cos(a + b*x)**4/(16*b) + 5*d**2*x**2*sin(a + b*x)**4/(32*b) - 3*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 3*d**2*x**2*cos(a + b*x)**4/(32*b) + 5*c*d*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 3*c*d*sin(a + b*x)*cos(a + b*x)**3/(16*b**2) + 5*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 3*d**2*x*sin(a + b*x)*cos(a + b*x)**3/(16*b**2) + 5*d**2*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**3) + d**2*cos(a + b*x)**4/(8*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**3*cos(a), True))

Giac [A] time = 1.13904, size = 196, normalized size = 1.46

$$\frac{(8b^2d^2x^2 + 16b^2cdx + 8b^2c^2 - d^2) \cos(4bx + 4a)}{256b^3} - \frac{(2b^2d^2x^2 + 4b^2cdx + 2b^2c^2 - d^2) \cos(2bx + 2a)}{16b^3} - \frac{(bd^2x + bcd) \sin(2bx + 2a)}{64b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")

[Out] $\frac{1}{256}*(8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*\cos(4*b*x + 4*a)/b^3 - \frac{1}{16}*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*\cos(2*b*x + 2*a)/b^3 - \frac{1}{64}*(b*d^2*x + b*c*d)*\sin(4*b*x + 4*a)/b^3 + \frac{1}{8}*(b*d^2*x + b*c*d)*\sin(2*b*x + 2*a)/b^3$

3.26 $\int (c + dx) \cos(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=72

$$\frac{d \sin^3(a + bx) \cos(a + bx)}{16b^2} + \frac{3d \sin(a + bx) \cos(a + bx)}{32b^2} + \frac{(c + dx) \sin^4(a + bx)}{4b} - \frac{3dx}{32b}$$

[Out] $(-3*d*x)/(32*b) + (3*d*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/(32*b^2) + (d*\text{Cos}[a + b*x]*\text{Sin}[a + b*x]^3)/(16*b^2) + ((c + d*x)*\text{Sin}[a + b*x]^4)/(4*b)$

Rubi [A] time = 0.04547, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {4404, 2635, 8}

$$\frac{d \sin^3(a + bx) \cos(a + bx)}{16b^2} + \frac{3d \sin(a + bx) \cos(a + bx)}{32b^2} + \frac{(c + dx) \sin^4(a + bx)}{4b} - \frac{3dx}{32b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)*\text{Cos}[a + b*x]*\text{Sin}[a + b*x]^3, x]$

[Out] $(-3*d*x)/(32*b) + (3*d*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/(32*b^2) + (d*\text{Cos}[a + b*x]*\text{Sin}[a + b*x]^3)/(16*b^2) + ((c + d*x)*\text{Sin}[a + b*x]^4)/(4*b)$

Rule 4404

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] :> \text{Simp}[(c + d*x)^m*\text{Sin}[a + b*x]^{(n + 1)} / (b*(n + 1)), x] - \text{Dist}[(d*m) / (b*(n + 1)), \text{Int}[(c + d*x)^{(m - 1)}*\text{Sin}[a + b*x]^{(n + 1)}, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 2635

$\text{Int}[(b_.)*\text{sin}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] :> -\text{Simp}[(b*\text{Cos}[c + d*x]*\text{Sin}[c + d*x]^{(n - 1)}) / (d*n), x] + \text{Dist}[(b^2*(n - 1)) / n, \text{Int}[(b*\text{Sin}[c + d*x]^{(n - 2)})], x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

$\text{Int}[a_, x_Symbol] :> \text{Simp}[a*x, x] /;$ FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int (c + dx) \cos(a + bx) \sin^3(a + bx) dx &= \frac{(c + dx) \sin^4(a + bx)}{4b} - \frac{d \int \sin^4(a + bx) dx}{4b} \\ &= \frac{d \cos(a + bx) \sin^3(a + bx)}{16b^2} + \frac{(c + dx) \sin^4(a + bx)}{4b} - \frac{(3d) \int \sin^2(a + bx) dx}{16b} \\ &= \frac{3d \cos(a + bx) \sin(a + bx)}{32b^2} + \frac{d \cos(a + bx) \sin^3(a + bx)}{16b^2} + \frac{(c + dx) \sin^4(a + bx)}{4b} \\ &= -\frac{3dx}{32b} + \frac{3d \cos(a + bx) \sin(a + bx)}{32b^2} + \frac{d \cos(a + bx) \sin^3(a + bx)}{16b^2} + \frac{(c + dx) \sin^4(a + bx)}{4b} \end{aligned}$$

Mathematica [A] time = 0.116165, size = 75, normalized size = 1.04

$$\frac{d(\sin(2(a+bx)) - 2bx \cos(2(a+bx)))}{16b^2} - \frac{d(\sin(4(a+bx)) - 4bx \cos(4(a+bx)))}{128b^2} + \frac{c \sin^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^3,x]

[Out] (c*Sin[a + b*x]^4)/(4*b) + (d*(-2*b*x*Cos[2*(a + b*x)] + Sin[2*(a + b*x)])) / (16*b^2) - (d*(-4*b*x*Cos[4*(a + b*x)] + Sin[4*(a + b*x)])) / (128*b^2)

Maple [A] time = 0.021, size = 85, normalized size = 1.2

$$\frac{1}{b} \left(\frac{d}{b} \left(\frac{(bx+a)(\sin(bx+a))^4}{4} + \frac{\cos(bx+a)}{16} \left((\sin(bx+a))^3 + \frac{3 \sin(bx+a)}{2} \right) - \frac{3bx}{32} - \frac{3a}{32} \right) - \frac{ad(\sin(bx+a))^4}{4b} + \frac{c(\sin(bx+a))^4}{4b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x)

[Out] 1/b*(d/b*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)-1/4/b*a*d*sin(b*x+a)^4+1/4*c*sin(b*x+a)^4)

Maxima [A] time = 1.09336, size = 124, normalized size = 1.72

$$\frac{32c \sin(bx+a)^4 - \frac{32ad \sin(bx+a)^4}{b} + \frac{(4(bx+a) \cos(4bx+4a) - 16(bx+a) \cos(2bx+2a) - \sin(4bx+4a) + 8 \sin(2bx+2a))d}{b}}{128b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")

[Out] 1/128*(32*c*sin(b*x + a)^4 - 32*a*d*sin(b*x + a)^4/b + (4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*d/b)/b

Fricas [A] time = 0.473922, size = 192, normalized size = 2.67

$$\frac{8(bdx + bc) \cos(bx + a)^4 + 5bdx - 16(bdx + bc) \cos(bx + a)^2 - (2d \cos(bx + a)^3 - 5d \cos(bx + a)) \sin(bx + a)}{32b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")

[Out] 1/32*(8*(b*d*x + b*c)*cos(b*x + a)^4 + 5*b*d*x - 16*(b*d*x + b*c)*cos(b*x + a)^2 - (2*d*cos(b*x + a)^3 - 5*d*cos(b*x + a))*sin(b*x + a))/b^2

Sympy [A] time = 2.37126, size = 160, normalized size = 2.22

$$\left\{ \begin{array}{l} -\frac{c \sin^2(a+bx) \cos^2(a+bx)}{2b} - \frac{c \cos^4(a+bx)}{4b} + \frac{5dx \sin^4(a+bx)}{32b} - \frac{3dx \sin^2(a+bx) \cos^2(a+bx)}{16b} - \frac{3dx \cos^4(a+bx)}{32b} + \frac{5d \sin^3(a+bx) \cos(a+bx)}{32b^2} + \frac{3d \sin(a+bx) \cos^3(a+bx)}{32b^2} \\ \left(cx + \frac{dx^2}{2} \right) \sin^3(a) \cos(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)**3,x)

[Out] Piecewise((-c*sin(a + b*x)**2*cos(a + b*x)**2/(2*b) - c*cos(a + b*x)**4/(4*b) + 5*d*x*sin(a + b*x)**4/(32*b) - 3*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 3*d*x*cos(a + b*x)**4/(32*b) + 5*d*sin(a + b*x)**3*cos(a + b*x)/(32*b**2) + 3*d*sin(a + b*x)*cos(a + b*x)**3/(32*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*sin(a)**3*cos(a), True))

Giac [A] time = 1.11426, size = 101, normalized size = 1.4

$$\frac{(bdx + bc) \cos(4bx + 4a)}{32b^2} - \frac{(bdx + bc) \cos(2bx + 2a)}{8b^2} - \frac{d \sin(4bx + 4a)}{128b^2} + \frac{d \sin(2bx + 2a)}{16b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")

[Out] 1/32*(b*d*x + b*c)*cos(4*b*x + 4*a)/b^2 - 1/8*(b*d*x + b*c)*cos(2*b*x + 2*a)/b^2 - 1/128*d*sin(4*b*x + 4*a)/b^2 + 1/16*d*sin(2*b*x + 2*a)/b^2

$$3.27 \quad \int \frac{\cos(a+bx) \sin^3(a+bx)}{c+dx} dx$$

Optimal. Leaf size=129

$$\frac{\sin\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{8d} + \frac{\sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{4d} + \frac{\cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{4d} - \frac{\cos\left(4a - \frac{4bc}{d}\right) \text{Si}\left(\frac{4bc}{d} + 4bx\right)}{8d}$$

[Out] -(CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(8*d) + (CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(4*d) + (Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(4*d) - (Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(8*d)

Rubi [A] time = 0.232436, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4406, 3303, 3299, 3302}

$$\frac{\sin\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{8d} + \frac{\sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{4d} + \frac{\cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{4d} - \frac{\cos\left(4a - \frac{4bc}{d}\right) \text{Si}\left(\frac{4bc}{d} + 4bx\right)}{8d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x),x]

[Out] -(CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(8*d) + (CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(4*d) + (Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(4*d) - (Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(8*d)

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx} dx &= \int \left(\frac{\sin(2a+2bx)}{4(c+dx)} - \frac{\sin(4a+4bx)}{8(c+dx)} \right) dx \\
&= -\left(\frac{1}{8} \int \frac{\sin(4a+4bx)}{c+dx} dx \right) + \frac{1}{4} \int \frac{\sin(2a+2bx)}{c+dx} dx \\
&= -\left(\frac{1}{8} \cos\left(4a - \frac{4bc}{d}\right) \int \frac{\sin\left(\frac{4bc}{d} + 4bx\right)}{c+dx} dx \right) + \frac{1}{4} \cos\left(2a - \frac{2bc}{d}\right) \int \frac{\sin\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx \\
&= -\frac{\text{Ci}\left(\frac{4bc}{d} + 4bx\right)\sin\left(4a - \frac{4bc}{d}\right)}{8d} + \frac{\text{Ci}\left(\frac{2bc}{d} + 2bx\right)\sin\left(2a - \frac{2bc}{d}\right)}{4d} + \frac{\cos\left(2a - \frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d} + 2bx\right)}{4d}
\end{aligned}$$

Mathematica [A] time = 0.416103, size = 110, normalized size = 0.85

$$\frac{\sin\left(4a - \frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right) - 2\sin\left(2a - \frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) - 2\cos\left(2a - \frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) + \cos\left(2a - \frac{2bc}{d}\right)\text{Si}\left(\frac{4b(c+dx)}{d}\right)}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x), x]

[Out] -(CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] - 2*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - 2*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(8*d)

Maple [A] time = 0.021, size = 178, normalized size = 1.4

$$\frac{1}{b} \left(\frac{b}{8} \left(2 \frac{1}{d} \text{Si} \left(2bx + 2a + 2 \frac{-ad+bc}{d} \right) \cos \left(2 \frac{-ad+bc}{d} \right) - 2 \frac{1}{d} \text{Ci} \left(2bx + 2a + 2 \frac{-ad+bc}{d} \right) \sin \left(2 \frac{-ad+bc}{d} \right) \right) - \frac{b}{32} \left(4 \frac{1}{d} \text{Si} \left(4bx + 4a + 4 \frac{-ad+bc}{d} \right) \cos \left(4 \frac{-ad+bc}{d} \right) - 4 \frac{1}{d} \text{Ci} \left(4bx + 4a + 4 \frac{-ad+bc}{d} \right) \sin \left(4 \frac{-ad+bc}{d} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c), x)

[Out] 1/b*(1/8*b*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d-1/32*b*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d))

Maxima [C] time = 1.43695, size = 370, normalized size = 2.87

$$b \left(-2i E_1 \left(\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) + 2i E_1 \left(-\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) \right) \cos \left(-\frac{2(bc-ad)}{d} \right) + b \left(i E_1 \left(\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) - i E_1 \left(-\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) \right) \cos \left(-\frac{4(bc-ad)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c), x, algorithm="maxima")

[Out] 1/16*(b*(-2*I*exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) + 2*I*exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*

$$c - a*d)/d) + b*(I*\exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) - I*\exp_integral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*\cos(-4*(b*c - a*d)/d) - 2*b*(\exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) + \exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*\sin(-2*(b*c - a*d)/d) + b*(\exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) + \exp_integral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*\sin(-4*(b*c - a*d)/d))/(b*d)$$

Fricas [A] time = 0.480233, size = 421, normalized size = 3.26

$$\frac{2 \left(\operatorname{Ci} \left(\frac{2(bdx+bc)}{d} \right) + \operatorname{Ci} \left(-\frac{2(bdx+bc)}{d} \right) \right) \sin \left(-\frac{2(bc-ad)}{d} \right) - \left(\operatorname{Ci} \left(\frac{4(bdx+bc)}{d} \right) + \operatorname{Ci} \left(-\frac{4(bdx+bc)}{d} \right) \right) \sin \left(-\frac{4(bc-ad)}{d} \right) - 2 \cos \left(-\frac{4(bc-ad)}{d} \right)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")

[Out] 1/16*(2*(cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d) - (cos_integral(4*(b*d*x + b*c)/d) + cos_integral(-4*(b*d*x + b*c)/d))*sin(-4*(b*c - a*d)/d) - 2*cos(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + 4*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d))/d

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3(a + bx) \cos(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c),x)

[Out] Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x), x)

Giac [C] time = 1.72027, size = 8162, normalized size = 63.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")

[Out] -1/16*(imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 2*real_part(cos_integr

$$\begin{aligned}
& s_integral(-4*b*x - 4*b*c/d)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 2*real_p \\
& art(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2 \\
& *real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2*tan(b*c/ \\
& d)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(2*b*c/d)^2*tan \\
& (b*c/d)^2 + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(2*b*c/d) \\
& ^2*tan(b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a) \\
& ^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2 - 2*im \\
& ag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2 + imag_part(cos \\
& _integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2 - 2*sin_integral(4*(b*d*x \\
& + b*c)/d)*tan(2*a)^2*tan(a)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^ \\
& 2*tan(a)^2 + 4*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2*t \\
& an(2*b*c/d) - 4*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2 \\
& *tan(2*b*c/d) + 8*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(a)^2*tan(2*b \\
& *c/d) + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 \\
& - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 2* \\
& imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 - imag_ \\
& part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 2*sin_inte \\
& gral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2 - 4*sin_integral(2*(b*d*x \\
& + b*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c \\
& /d))*tan(a)^2*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*t \\
& an(a)^2*tan(2*b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a) \\
& ^2*tan(2*b*c/d)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(\\
& 2*b*c/d)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2 + 4* \\
& sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2 - 8*imag_part(cos_i \\
& ntegral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(b*c/d) + 8*imag_part(cos_in \\
& tegral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)*tan(b*c/d) - 16*sin_integral(2* \\
& (b*d*x + b*c)/d)*tan(2*a)^2*tan(a)*tan(b*c/d) - 8*imag_part(cos_integral(2* \\
& b*x + 2*b*c/d))*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) + 8*imag_part(cos_integral \\
& (-2*b*x - 2*b*c/d))*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - 16*sin_integral(2*(b \\
& *d*x + b*c)/d)*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - imag_part(cos_integral(4* \\
& b*x + 4*b*c/d))*tan(2*a)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + \\
& 2*b*c/d))*tan(2*a)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2*b*c \\
& /d))*tan(2*a)^2*tan(b*c/d)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/d))*ta \\
& n(2*a)^2*tan(b*c/d)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(b* \\
& c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(b*c/d)^2 + imag_p \\
& art(cos_integral(4*b*x + 4*b*c/d))*tan(a)^2*tan(b*c/d)^2 - 2*imag_part(cos_ \\
& integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral \\
& (-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x - \\
& 4*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(a) \\
& ^2*tan(b*c/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + \\
& 4*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)*tan(b*c/d \\
&)^2 - 4*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)*tan \\
& (b*c/d)^2 + 8*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d)*tan(b*c \\
& /d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^ \\
& 2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2 \\
& - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2 + \\
& imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2* \\
& sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*sin_integra \\
& l(2*(b*d*x + b*c)/d)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*real_part(cos_integral \\
& (2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a) - 4*real_part(cos_integral(-2*b*x - 2* \\
& b*c/d))*tan(2*a)^2*tan(a) + 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(\\
& 2*a)*tan(a)^2 + 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a) \\
& ^2 + 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 2 \\
& *real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) - 2*real \\
& _part(cos_integral(4*b*x + 4*b*c/d))*tan(a)^2*tan(2*b*c/d) - 2*real_part(co \\
& s_integral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2*b*c/d) - 2*real_part(cos_integ \\
& ral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 2*real_part(cos_integral(-4 \\
& *b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 4*real_part(cos_integral(2*b*x + \\
& 2*b*c/d))*tan(a)*tan(2*b*c/d)^2 - 4*real_part(cos_integral(-2*b*x - 2*b*c/
\end{aligned}$$

$$\begin{aligned}
& d)) * \tan(a) * \tan(2*b*c/d)^2 + 4*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d)) * \tan(2*a)^2 * \tan(b*c/d) + 4*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d)) * \tan(2*a)^2 * \tan(b*c/d) - 4*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d)) * \tan(a)^2 * \tan(b*c/d) \\
& - 4*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d)) * \tan(a)^2 * \tan(b*c/d) + 4*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d)) * \tan(2*b*c/d)^2 * \tan(b*c/d) + 4*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d)) * \tan(2*b*c/d)^2 * \tan(b*c/d) + 2*\text{real_part}(\text{cos_integral}(4*b*x + 4*b*c/d)) * \tan(2*a) * \tan(b*c/d)^2 + 2*\text{real_part}(\text{cos_integral}(-4*b*x - 4*b*c/d)) * \tan(2*a) * \tan(b*c/d)^2 + 4*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d)) * \tan(a) * \tan(b*c/d)^2 + 4*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d)) * \tan(a) * \tan(b*c/d)^2 - 2*\text{real_part}(\text{cos_integral}(4*b*x + 4*b*c/d)) * \tan(2*b*c/d) * \tan(b*c/d)^2 - 2*\text{real_part}(\text{cos_integral}(-4*b*x - 4*b*c/d)) * \tan(2*b*c/d) * \tan(b*c/d)^2 - \text{imag_part}(\text{cos_integral}(4*b*x + 4*b*c/d)) * \tan(2*a)^2 - 2*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d)) * \tan(2*a)^2 + 2*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d)) * \tan(2*a)^2 + \text{imag_part}(\text{cos_integral}(-4*b*x - 4*b*c/d)) * \tan(2*a)^2 - 2*\text{sin_integral}(4*(b*d*x + b*c)/d) * \tan(2*a)^2 - 4*\text{sin_integral}(2*(b*d*x + b*c)/d) * \tan(2*a)^2 + \text{imag_part}(\text{cos_integral}(4*b*x + 4*b*c/d)) * \tan(a)^2 + 2*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d)) * \tan(a)^2 - 2*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d)) * \tan(a)^2 - \text{imag_part}(\text{cos_integral}(-4*b*x - 4*b*c/d)) * \tan(a)^2 + 2*\text{sin_integral}(4*(b*d*x + b*c)/d) * \tan(a)^2 + 4*\text{sin_integral}(2*(b*d*x + b*c)/d) * \tan(a)^2 + 4*\text{imag_part}(\text{cos_integral}(4*b*x + 4*b*c/d)) * \tan(2*a) * \tan(2*b*c/d) - 4*\text{imag_part}(\text{cos_integral}(-4*b*x - 4*b*c/d)) * \tan(2*a) * \tan(2*b*c/d) + 8*\text{sin_integral}(4*(b*d*x + b*c)/d) * \tan(2*a) * \tan(2*b*c/d) - \text{imag_part}(\text{cos_integral}(4*b*x + 4*b*c/d)) * \tan(2*b*c/d)^2 - 2*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d)) * \tan(2*b*c/d)^2 + 2*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d)) * \tan(2*b*c/d)^2 + \text{imag_part}(\text{cos_integral}(-4*b*x - 4*b*c/d)) * \tan(2*b*c/d)^2 - 2*\text{sin_integral}(4*(b*d*x + b*c)/d) * \tan(2*b*c/d)^2 - 4*\text{sin_integral}(2*(b*d*x + b*c)/d) * \tan(2*b*c/d)^2 - 8*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d)) * \tan(a) * \tan(b*c/d) + 8*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d)) * \tan(a) * \tan(b*c/d) - 16*\text{sin_integral}(2*(b*d*x + b*c)/d) * \tan(a) * \tan(b*c/d) + \text{imag_part}(\text{cos_integral}(4*b*x + 4*b*c/d)) * \tan(b*c/d)^2 + 2*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d)) * \tan(b*c/d)^2 - 2*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d)) * \tan(b*c/d)^2 - \text{imag_part}(\text{cos_integral}(-4*b*x - 4*b*c/d)) * \tan(b*c/d)^2 + 2*\text{sin_integral}(4*(b*d*x + b*c)/d) * \tan(b*c/d)^2 + 4*\text{sin_integral}(2*(b*d*x + b*c)/d) * \tan(b*c/d)^2 + 2*\text{real_part}(\text{cos_integral}(4*b*x + 4*b*c/d)) * \tan(2*a) + 2*\text{real_part}(\text{cos_integral}(-4*b*x - 4*b*c/d)) * \tan(2*a) - 4*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d)) * \tan(a) - 4*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d)) * \tan(a) - 2*\text{real_part}(\text{cos_integral}(4*b*x + 4*b*c/d)) * \tan(2*b*c/d) - 2*\text{real_part}(\text{cos_integral}(-4*b*x - 4*b*c/d)) * \tan(2*b*c/d) + 4*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d)) * \tan(b*c/d) + 4*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d)) * \tan(b*c/d) + \text{imag_part}(\text{cos_integral}(4*b*x + 4*b*c/d)) - 2*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d)) + 2*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d)) - \text{imag_part}(\text{cos_integral}(-4*b*x - 4*b*c/d)) + 2*\text{sin_integral}(4*(b*d*x + b*c)/d) - 4*\text{sin_integral}(2*(b*d*x + b*c)/d)) / (d*\tan(2*a)^2 * \tan(a)^2 * \tan(2*b*c/d)^2 * \tan(b*c/d)^2 + d*\tan(2*a)^2 * \tan(a)^2 * \tan(2*b*c/d)^2 + d*\tan(2*a)^2 * \tan(a)^2 * \tan(b*c/d)^2 + d*\tan(2*a)^2 * \tan(2*b*c/d)^2 * \tan(b*c/d)^2 + d*\tan(2*a)^2 * \tan(a)^2 * \tan(2*b*c/d)^2 * \tan(b*c/d)^2 + d*\tan(2*a)^2 * \tan(a)^2 * \tan(2*b*c/d)^2 + d*\tan(2*a)^2 * \tan(b*c/d)^2 + d*\tan(a)^2 * \tan(2*b*c/d)^2 * \tan(b*c/d)^2 + d*\tan(2*a)^2 * \tan(a)^2 * \tan(2*b*c/d)^2 + d*\tan(2*a)^2 * \tan(a)^2 * \tan(b*c/d)^2 + d*\tan(2*b*c/d)^2 * \tan(b*c/d)^2 + d*\tan(2*a)^2 * \tan(a)^2 * \tan(2*b*c/d)^2 + d*\tan(2*b*c/d)^2 * \tan(b*c/d)^2 + d)
\end{aligned}$$

$$3.28 \quad \int \frac{\cos(a+bx) \sin^3(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=179

$$\frac{b \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{2d^2} - \frac{b \cos\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{2d^2} - \frac{b \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{2d^2}$$

[Out] (b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) - (b*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(2*d^2) - Sin[2*a + 2*b*x]/(4*d*(c + d*x)) + Sin[4*a + 4*b*x]/(8*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) + (b*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(2*d^2)

Rubi [A] time = 0.280256, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{2d^2} - \frac{b \cos\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{2d^2} - \frac{b \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{2d^2}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^2,x]

[Out] (b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) - (b*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(2*d^2) - Sin[2*a + 2*b*x]/(4*d*(c + d*x)) + Sin[4*a + 4*b*x]/(8*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) + (b*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(2*d^2)

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2} dx &= \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^2} - \frac{\sin(4a+4bx)}{8(c+dx)^2} \right) dx \\ &= -\left(\frac{1}{8} \int \frac{\sin(4a+4bx)}{(c+dx)^2} dx \right) + \frac{1}{4} \int \frac{\sin(2a+2bx)}{(c+dx)^2} dx \\ &= -\frac{\sin(2a+2bx)}{4d(c+dx)} + \frac{\sin(4a+4bx)}{8d(c+dx)} + \frac{b \int \frac{\cos(2a+2bx)}{c+dx} dx}{2d} - \frac{b \int \frac{\cos(4a+4bx)}{c+dx} dx}{2d} \\ &= -\frac{\sin(2a+2bx)}{4d(c+dx)} + \frac{\sin(4a+4bx)}{8d(c+dx)} - \frac{\left(b \cos\left(4a - \frac{4bc}{d}\right) \right) \int \frac{\cos\left(\frac{4bc}{d} + 4bx\right)}{c+dx} dx}{2d} + \frac{\left(b \cos\left(2a - \frac{2bc}{d}\right) \right) \text{Ci}\left(\frac{2bc}{d} + 2bx\right)}{2d^2} \\ &\quad - \frac{b \cos\left(4a - \frac{4bc}{d}\right) \text{Ci}\left(\frac{4bc}{d} + 4bx\right)}{2d^2} - \frac{\sin(2a+2bx)}{4d(c+dx)} + \end{aligned}$$

Mathematica [A] time = 1.29574, size = 151, normalized size = 0.84

$$\frac{4b \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) - 4b \cos\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right) - 4b \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2b(c+dx)}{d}\right) + 4b \sin\left(4a - \frac{4bc}{d}\right) \text{Si}\left(\frac{4b(c+dx)}{d}\right)}{8d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^2,x]

[Out] (4*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - 4*b*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] - (2*d*Sin[2*(a + b*x)]/(c + d*x) + (d*Sin[4*(a + b*x)]/(c + d*x) - 4*b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 4*b*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(8*d^2)

Maple [A] time = 0.023, size = 256, normalized size = 1.4

$$\frac{1}{b} \left(\frac{b^2}{8} \left(-2 \frac{\sin(2bx + 2a)}{((bx+a)d - ad + bc)d} + 2 \frac{1}{d} \left(2 \frac{1}{d} \text{Si}\left(2bx + 2a + 2 \frac{-ad + bc}{d}\right) \sin\left(2 \frac{-ad + bc}{d}\right) + 2 \frac{1}{d} \text{Ci}\left(2bx + 2a + 2 \frac{-ad + bc}{d}\right) \cos\left(2 \frac{-ad + bc}{d}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x)

[Out] 1/b*(1/8*b^2*(-2*sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)/d+2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)-1/32*b^2*(-4*sin(4*b*x+4*a)/((b*x+a)*d-a*d+b*c)/d+4*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d)/d)

Maxima [C] time = 1.77806, size = 406, normalized size = 2.27

$$b^2 \left(-2i E_2 \left(\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) + 2i E_2 \left(-\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) \right) \cos \left(-\frac{2(bc-ad)}{d} \right) + b^2 \left(i E_2 \left(\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) - i E_2 \left(-\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) \right) \sin \left(-\frac{4(bc-ad)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")

[Out] 1/16*(b^2*(-2*I*exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) + 2*I*exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c - a*d)/d) + b^2*(I*exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) - I*exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*(b*c - a*d)/d) - 2*b^2*(exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) + exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2*(b*c - a*d)/d) + b^2*(exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) + exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*sin(-4*(b*c - a*d)/d)/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)

Fricas [A] time = 0.553995, size = 621, normalized size = 3.47

$$2(bdx + bc) \sin \left(-\frac{4(bc-ad)}{d} \right) \text{Si} \left(\frac{4(bdx+bc)}{d} \right) - 2(bdx + bc) \sin \left(-\frac{2(bc-ad)}{d} \right) \text{Si} \left(\frac{2(bdx+bc)}{d} \right) + \left((bdx + bc) \text{Ci} \left(\frac{2(bdx+bc)}{d} \right) + (bdx + bc) \text{Ci} \left(\frac{4(bdx+bc)}{d} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")

[Out] 1/4*(2*(b*d*x + b*c)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) - 2*(b*d*x + b*c)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) + ((b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d) - ((b*d*x + b*c)*cos_integral(4*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-4*(b*d*x + b*c)/d))*cos(-4*(b*c - a*d)/d) + 4*(d*cos(b*x + a)^3 - d*cos(b*x + a))*sin(b*x + a)/(d^3*x + c*d^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3(a + bx) \cos(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c)**2,x)

[Out] Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx + a) \sin(bx + a)^3}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*sin(b*x + a)^3/(d*x + c)^2, x)
```

$$3.29 \quad \int \frac{\cos(a+bx) \sin^3(a+bx)}{(c+dx)^3} dx$$

Optimal. Leaf size=229

$$\frac{b^2 \sin\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{d^3} - \frac{b^2 \sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{2d^3} - \frac{b^2 \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{2d^3}$$

[Out] $-(b \cos[2a + 2bx])/(4d^2(c + dx)) + (b \cos[4a + 4bx])/(4d^2(c + dx)) + (b^2 \text{CosIntegral}[(4bc)/d + 4bx] \sin[4a - (4bc)/d])/d^3 - (b^2 \text{CosIntegral}[(2bc)/d + 2bx] \sin[2a - (2bc)/d])/(2d^3) - \sin[2a + 2bx]/(8d(c + dx)^2) + \sin[4a + 4bx]/(16d(c + dx)^2) - (b^2 \text{Cos}[2a - (2bc)/d] \text{SinIntegral}[(2bc)/d + 2bx])/(2d^3) + (b^2 \text{Cos}[4a - (4bc)/d] \text{SinIntegral}[(4bc)/d + 4bx])/d^3$

Rubi [A] time = 0.34384, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b^2 \sin\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{d^3} - \frac{b^2 \sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{2d^3} - \frac{b^2 \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{2d^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\cos[a + bx] \sin[a + bx]^3)/(c + dx)^3, x]$

[Out] $-(b \cos[2a + 2bx])/(4d^2(c + dx)) + (b \cos[4a + 4bx])/(4d^2(c + dx)) + (b^2 \text{CosIntegral}[(4bc)/d + 4bx] \sin[4a - (4bc)/d])/d^3 - (b^2 \text{CosIntegral}[(2bc)/d + 2bx] \sin[2a - (2bc)/d])/(2d^3) - \sin[2a + 2bx]/(8d(c + dx)^2) + \sin[4a + 4bx]/(16d(c + dx)^2) - (b^2 \text{Cos}[2a - (2bc)/d] \text{SinIntegral}[(2bc)/d + 2bx])/(2d^3) + (b^2 \text{Cos}[4a - (4bc)/d] \text{SinIntegral}[(4bc)/d + 4bx])/d^3$

Rule 4406

$\text{Int}[\cos[(a_.) + (b_.)(x_)]^{(p_.)}((c_.) + (d_.)(x_))^{(m_.)} \sin[(a_.) + (b_.)(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + dx)^m, \sin[a + bx]]^n \cos[a + bx]^p, x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3297

$\text{Int}[(c_.) + (d_.)(x_)]^{(m_.)} \sin[(e_.) + (f_.)(x_)], x_Symbol] \rightarrow \text{Simp}[(c + dx)^{(m + 1)} \sin[e + fx]/(d(m + 1)), x] - \text{Dist}[f/(d(m + 1)), \text{Int}[(c + dx)^{(m + 1)} \cos[e + fx], x], x] /;$ FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

$\text{Int}[\sin[(e_.) + (f_.)(x_)]/((c_.) + (d_.)(x_)), x_Symbol] \rightarrow \text{Dist}[\cos[(d * e - c * f)/d], \text{Int}[\sin[(c * f)/d + f * x]/(c + d * x), x], x] + \text{Dist}[\sin[(d * e - c * f)/d], \text{Int}[\cos[(c * f)/d + f * x]/(c + d * x), x], x] /;$ FreeQ[{c, d, e, f}, x] && NeQ[d * e - c * f, 0]

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3} dx &= \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^3} - \frac{\sin(4a+4bx)}{8(c+dx)^3} \right) dx \\ &= -\left(\frac{1}{8} \int \frac{\sin(4a+4bx)}{(c+dx)^3} dx \right) + \frac{1}{4} \int \frac{\sin(2a+2bx)}{(c+dx)^3} dx \\ &= -\frac{\sin(2a+2bx)}{8d(c+dx)^2} + \frac{\sin(4a+4bx)}{16d(c+dx)^2} + \frac{b \int \frac{\cos(2a+2bx)}{(c+dx)^2} dx}{4d} - \frac{b \int \frac{\cos(4a+4bx)}{(c+dx)^2} dx}{4d} \\ &= -\frac{b \cos(2a+2bx)}{4d^2(c+dx)} + \frac{b \cos(4a+4bx)}{4d^2(c+dx)} - \frac{\sin(2a+2bx)}{8d(c+dx)^2} + \frac{\sin(4a+4bx)}{16d(c+dx)^2} - \frac{b^2 \int \frac{\sin(2a+bx)}{c+dx} dx}{2d^2} \\ &= -\frac{b \cos(2a+2bx)}{4d^2(c+dx)} + \frac{b \cos(4a+4bx)}{4d^2(c+dx)} - \frac{\sin(2a+2bx)}{8d(c+dx)^2} + \frac{\sin(4a+4bx)}{16d(c+dx)^2} + \frac{b^2 \cos\left(4a - \frac{2bc}{d}\right)}{2d^2} \\ &= -\frac{b \cos(2a+2bx)}{4d^2(c+dx)} + \frac{b \cos(4a+4bx)}{4d^2(c+dx)} + \frac{b^2 \text{Ci}\left(\frac{4bc}{d} + 4bx\right) \sin\left(4a - \frac{4bc}{d}\right)}{d^3} - \frac{b^2 \text{Ci}\left(\frac{2bc}{d}\right)}{2d^2} \end{aligned}$$

Mathematica [A] time = 2.82636, size = 199, normalized size = 0.87

$$\frac{-2 \left(4b^2 \sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) + 4b^2 \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2b(c+dx)}{d}\right) + \frac{d(2b(c+dx)\cos(2(a+bx))+d\sin(2(a+bx)))}{(c+dx)^2} \right)}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] (16*b^2*CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + (d*(4*b*(c + d*x)*Cos[4*(a + b*x)] + d*Sin[4*(a + b*x)]))/(c + d*x)^2 - 2*(4*b^2*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*(2*b*(c + d*x)*Cos[2*(a + b*x)] + d*Sin[2*(a + b*x)]))/(c + d*x)^2 + 4*b^2*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 16*b^2*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d]/(16*d^3)
```

Maple [A] time = 0.024, size = 329, normalized size = 1.4

$$\frac{1}{b} \left(\frac{b^3}{8} \left(-\frac{\sin(2bx+2a)}{((bx+a)d - ad + bc)^2 d} + \frac{1}{d} \left(-2 \frac{\cos(2bx+2a)}{((bx+a)d - ad + bc)d} - 2 \frac{1}{d} \left(2 \frac{1}{d} \text{Si}\left(2bx+2a+2\frac{-ad+bc}{d}\right) \cos\left(2\frac{-ad+bc}{d}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x)
```

[Out] $\frac{1}{b} \left(\frac{1}{8} b^3 \frac{-\sin(2bx+2a)}{((bx+a)d-ad+bc)^2/d} + \frac{-2\cos(2bx+2a)}{((bx+a)d-ad+bc)/d} - 2 \frac{2\operatorname{Si}(2bx+2a+2(-ad+bc)/d) \cos(2(-ad+bc)/d)}{d} - 2 \frac{2\operatorname{Ci}(2bx+2a+2(-ad+bc)/d) \sin(2(-ad+bc)/d)}{d} - \frac{1}{32} b^3 \frac{-2\sin(4bx+4a)}{((bx+a)d-ad+bc)^2/d} + 2 \frac{-4\cos(4bx+4a)}{((bx+a)d-ad+bc)/d} - 4 \frac{4\operatorname{Si}(4bx+4a+4(-ad+bc)/d) \cos(4(-ad+bc)/d)}{d} - 4 \frac{4\operatorname{Ci}(4bx+4a+4(-ad+bc)/d) \sin(4(-ad+bc)/d)}{d} \right)$

Maxima [C] time = 2.17134, size = 454, normalized size = 1.98

$$\frac{b^3 \left(-2i E_3 \left(\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) + 2i E_3 \left(-\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) \right) \cos \left(-\frac{2(bc-ad)}{d} \right) + b^3 \left(i E_3 \left(\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) - i E_3 \left(-\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) \right) \sin \left(-\frac{4(bc-ad)}{d} \right)}{16(b^2 c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")`

[Out] $\frac{1}{16} b^3 \left(-2I \exp_{\text{integral}_e}(3, (2I*bc + 2I*(bx+a)d - 2I*a*d)/d) + 2I \exp_{\text{integral}_e}(3, -(2I*bc + 2I*(bx+a)d - 2I*a*d)/d) \right) \cos(-2*(bc-a*d)/d) + b^3 \left(I \exp_{\text{integral}_e}(3, (4I*bc + 4I*(bx+a)d - 4I*a*d)/d) - I \exp_{\text{integral}_e}(3, -(4I*bc + 4I*(bx+a)d - 4I*a*d)/d) \right) \cos(-4*(bc-a*d)/d) - 2*b^3 \left(\exp_{\text{integral}_e}(3, (2I*bc + 2I*(bx+a)d - 2I*a*d)/d) + \exp_{\text{integral}_e}(3, -(2I*bc + 2I*(bx+a)d - 2I*a*d)/d) \right) \sin(-2*(bc-a*d)/d) + b^3 \left(\exp_{\text{integral}_e}(3, (4I*bc + 4I*(bx+a)d - 4I*a*d)/d) + \exp_{\text{integral}_e}(3, -(4I*bc + 4I*(bx+a)d - 4I*a*d)/d) \right) \sin(-4*(bc-a*d)/d) \right) / ((b^2*c^2*d - 2*a*b*c*d^2 + (bx+a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(bx+a))*b)$

Fricas [A] time = 0.617182, size = 976, normalized size = 4.26

$$\frac{8(bd^2x + bcd) \cos(bx+a)^4 + 2bd^2x + 2bcd - 10(bd^2x + bcd) \cos(bx+a)^2 + 4(b^2d^2x^2 + 2b^2cdx + b^2c^2) \cos\left(-\frac{4(bc-ad)}{d}\right)}{16(b^2 c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")`

[Out] $\frac{1}{4} (8*(b*d^2*x + b*c*d) \cos(bx+a)^4 + 2*b*d^2*x + 2*b*c*d - 10*(b*d^2*x + b*c*d) \cos(bx+a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2) \cos(-4*(bc-a*d)/d) \sin_{\text{integral}}(4*(b*d*x + b*c)/d) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2) \cos(-2*(bc-a*d)/d) \sin_{\text{integral}}(2*(b*d*x + b*c)/d) + 2*(d^2 \cos(bx+a)^3 - d^2 \cos(bx+a)) \sin(bx+a) - ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2) \cos_{\text{integral}}(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2) \cos_{\text{integral}}(-2*(b*d*x + b*c)/d)) \sin(-2*(bc-a*d)/d) + 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2) \cos_{\text{integral}}(4*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2) \cos_{\text{integral}}(-4*(b*d*x + b*c)/d)) \sin(-4*(bc-a*d)/d)) / (d^5*x^2 + 2*c*d^4*x + c^2*d^3)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3(a+bx) \cos(a+bx)}{(c+dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.30 \quad \int \frac{\cos(a+bx) \sin^3(a+bx)}{(c+dx)^4} dx$$

Optimal. Leaf size=287

$$-\frac{b^3 \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} + \frac{4b^3 \cos\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{3d^4} + \frac{b^3 \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{3d^4}$$

[Out] $-(b \cdot \text{Cos}[2a + 2bx]) / (12d^2(c + dx)^2) + (b \cdot \text{Cos}[4a + 4bx]) / (12d^2(c + dx)^2) - (b^3 \cdot \text{Cos}[2a - (2bc)/d] \cdot \text{CosIntegral}[(2bc)/d + 2bx]) / (3d^4) + (4b^3 \cdot \text{Cos}[4a - (4bc)/d] \cdot \text{CosIntegral}[(4bc)/d + 4bx]) / (3d^4) - \text{Sin}[2a + 2bx] / (12d(c + dx)^3) + (b^2 \cdot \text{Sin}[2a + 2bx]) / (6d^3(c + dx)) + \text{Sin}[4a + 4bx] / (24d(c + dx)^3) - (b^2 \cdot \text{Sin}[4a + 4bx]) / (3d^3(c + dx)) + (b^3 \cdot \text{Sin}[2a - (2bc)/d] \cdot \text{SinIntegral}[(2bc)/d + 2bx]) / (3d^4) - (4b^3 \cdot \text{Sin}[4a - (4bc)/d] \cdot \text{SinIntegral}[(4bc)/d + 4bx]) / (3d^4)$

Rubi [A] time = 0.389443, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^3 \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} + \frac{4b^3 \cos\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{3d^4} + \frac{b^3 \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{3d^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[a + bx] \cdot \text{Sin}[a + bx]^3) / (c + dx)^4, x]$

[Out] $-(b \cdot \text{Cos}[2a + 2bx]) / (12d^2(c + dx)^2) + (b \cdot \text{Cos}[4a + 4bx]) / (12d^2(c + dx)^2) - (b^3 \cdot \text{Cos}[2a - (2bc)/d] \cdot \text{CosIntegral}[(2bc)/d + 2bx]) / (3d^4) + (4b^3 \cdot \text{Cos}[4a - (4bc)/d] \cdot \text{CosIntegral}[(4bc)/d + 4bx]) / (3d^4) - \text{Sin}[2a + 2bx] / (12d(c + dx)^3) + (b^2 \cdot \text{Sin}[2a + 2bx]) / (6d^3(c + dx)) + \text{Sin}[4a + 4bx] / (24d(c + dx)^3) - (b^2 \cdot \text{Sin}[4a + 4bx]) / (3d^3(c + dx)) + (b^3 \cdot \text{Sin}[2a - (2bc)/d] \cdot \text{SinIntegral}[(2bc)/d + 2bx]) / (3d^4) - (4b^3 \cdot \text{Sin}[4a - (4bc)/d] \cdot \text{SinIntegral}[(4bc)/d + 4bx]) / (3d^4)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)(x_)]^{(p_.)} \cdot ((c_.) + (d_.)(x_))^{(m_.)} \cdot \text{Sin}[(a_.) + (b_.)(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + dx)^m, \text{Sin}[a + bx]^n \cdot \text{Cos}[a + bx]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3297

$\text{Int}[(c_.) + (d_.)(x_)]^{(m_.)} \cdot \text{sin}[(e_.) + (f_.)(x_)], x_Symbol] \rightarrow \text{Simp}[(c + dx)^{(m + 1)} \cdot \text{Sin}[e + fx] / (d(m + 1)), x] - \text{Dist}[f / (d(m + 1)), \text{Int}[(c + dx)^{(m + 1)} \cdot \text{Cos}[e + fx], x], x] /;$ FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

$\text{Int}[\text{sin}[(e_.) + (f_.)(x_)] / ((c_.) + (d_.)(x_)), x_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d \cdot e - c \cdot f) / d], \text{Int}[\text{Sin}[(c \cdot f) / d + f \cdot x] / (c + d \cdot x), x], x] + \text{Dist}[\text{Sin}[(d \cdot e - c \cdot f) / d], \text{Int}[\text{Cos}[(c \cdot f) / d + f \cdot x] / (c + d \cdot x), x], x] /;$ FreeQ[{c, d, e, f}, x] && NeQ[d \cdot e - c \cdot f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4} dx &= \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^4} - \frac{\sin(4a+4bx)}{8(c+dx)^4} \right) dx \\
 &= -\left(\frac{1}{8} \int \frac{\sin(4a+4bx)}{(c+dx)^4} dx \right) + \frac{1}{4} \int \frac{\sin(2a+2bx)}{(c+dx)^4} dx \\
 &= -\frac{\sin(2a+2bx)}{12d(c+dx)^3} + \frac{\sin(4a+4bx)}{24d(c+dx)^3} + \frac{b \int \frac{\cos(2a+2bx)}{(c+dx)^3} dx}{6d} - \frac{b \int \frac{\cos(4a+4bx)}{(c+dx)^3} dx}{6d} \\
 &= -\frac{b \cos(2a+2bx)}{12d^2(c+dx)^2} + \frac{b \cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{12d(c+dx)^3} + \frac{\sin(4a+4bx)}{24d(c+dx)^3} - \frac{b^2 \int \frac{\sin(2a+2bx)}{(c+dx)^2} dx}{6d^2} \\
 &= -\frac{b \cos(2a+2bx)}{12d^2(c+dx)^2} + \frac{b \cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{12d(c+dx)^3} + \frac{b^2 \sin(2a+2bx)}{6d^3(c+dx)} + \frac{\sin(4a+4bx)}{24d(c+dx)^3} \\
 &= -\frac{b \cos(2a+2bx)}{12d^2(c+dx)^2} + \frac{b \cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{12d(c+dx)^3} + \frac{b^2 \sin(2a+2bx)}{6d^3(c+dx)} + \frac{\sin(4a+4bx)}{24d(c+dx)^3} \\
 &= -\frac{b \cos(2a+2bx)}{12d^2(c+dx)^2} + \frac{b \cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{b^3 \cos\left(2a - \frac{2bc}{d}\right) \text{Ci}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} + \frac{4b^3 \cos\left(4a - \frac{4bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{3d^4}
 \end{aligned}$$

Mathematica [A] time = 2.40638, size = 316, normalized size = 1.1

$$-8b^3(c+dx)^3 \left(\cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) - \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2b(c+dx)}{d}\right) \right) + 32b^3(c+dx)^3 \left(\cos\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right) - \sin\left(4a - \frac{4bc}{d}\right) \text{Si}\left(\frac{4b(c+dx)}{d}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^4, x]

[Out] (-2*d*Cos[2*b*x]*(b*d*(c + d*x)*Cos[2*a] + (d^2 - 2*b^2*(c + d*x)^2)*Sin[2*a]) + d*Cos[4*b*x]*(2*b*d*(c + d*x)*Cos[4*a] + (d^2 - 8*b^2*(c + d*x)^2)*Sin[4*a]) + 2*d*((-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*a] + b*d*(c + d*x)*Sin[2*a])*Sin[2*b*x] - d*((-d^2 + 8*b^2*(c + d*x)^2)*Cos[4*a] + 2*b*d*(c + d*x)*Sin[4*a])*Sin[4*b*x] - 8*b^3*(c + d*x)^3*(Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d]) + 32*b^3*(c + d*x)^3*(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] - Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(24*d^4*(c + d*x)^3)

Maple [A] time = 0.024, size = 404, normalized size = 1.4

$$\frac{1}{b} \left(\frac{b^4}{8} \left(-\frac{2 \sin(2bx+2a)}{3((bx+a)d - ad + bc)^3 d} + \frac{2}{3d} \left(-\frac{\cos(2bx+2a)}{((bx+a)d - ad + bc)^2 d} - \frac{1}{d} \left(-2 \frac{\sin(2bx+2a)}{((bx+a)d - ad + bc)d} + 2 \frac{1}{d} \left(2 \frac{1}{d} \text{Si}\left(2 \frac{b(c+dx)}{d}\right) \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x)`

[Out] $\frac{1}{b} \left(\frac{1}{8} b^4 \left(-\frac{2}{3} \sin(2bx+2a) / ((bx+a)d - ad + bc)^3 / d + \frac{2}{3} (-\cos(2bx+2a) / ((bx+a)d - ad + bc)^2 / d - (-2 \sin(2bx+2a) / ((bx+a)d - ad + bc) / d + 2(2 \operatorname{Si}(2bx+2a+2(-ad+bc)/d) \sin(2(-ad+bc)/d) / d + 2 \operatorname{Ci}(2bx+2a+2(-ad+bc)/d) \cos(2(-ad+bc)/d) / d) / d) / d) - \frac{1}{32} b^4 \left(-\frac{4}{3} \sin(4bx+4a) / ((bx+a)d - ad + bc)^3 / d + \frac{4}{3} (-2 \cos(4bx+4a) / ((bx+a)d - ad + bc)^2 / d - 2(-4 \sin(4bx+4a) / ((bx+a)d - ad + bc) / d + 4(4 \operatorname{Si}(4bx+4a+4(-ad+bc)/d) \sin(4(-ad+bc)/d) / d + 4 \operatorname{Ci}(4bx+4a+4(-ad+bc)/d) \cos(4(-ad+bc)/d) / d) / d) / d) \right) \right)$

Maxima [C] time = 3.09143, size = 521, normalized size = 1.82

$$\frac{b^4 \left(-2i E_4 \left(\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) + 2i E_4 \left(-\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) \right) \cos \left(-\frac{2(bc-ad)}{d} \right) + b^4 \left(i E_4 \left(\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) - i E_4 \left(-\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) \right) \sin \left(-\frac{2(bc-ad)}{d} \right)}{16 (b^3 c^3 d - 3 a b^2 c^2 d^2 + 3 a^2 b c d^3 + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="maxima")`

[Out] $\frac{1}{16} b^4 \left(-2 \operatorname{I} \exp_{\text{integral}_e}(4, (2 \operatorname{I} b^* c + 2 \operatorname{I} (b^* x + a) d - 2 \operatorname{I} a^* d) / d) + 2 \operatorname{I} \exp_{\text{integral}_e}(4, -(2 \operatorname{I} b^* c + 2 \operatorname{I} (b^* x + a) d - 2 \operatorname{I} a^* d) / d) \right) \cos(-2(b^* c - a^* d) / d) + b^4 \left(\operatorname{I} \exp_{\text{integral}_e}(4, (4 \operatorname{I} b^* c + 4 \operatorname{I} (b^* x + a) d - 4 \operatorname{I} a^* d) / d) - \operatorname{I} \exp_{\text{integral}_e}(4, -(4 \operatorname{I} b^* c + 4 \operatorname{I} (b^* x + a) d - 4 \operatorname{I} a^* d) / d) \right) \cos(-4(b^* c - a^* d) / d) - 2 b^4 \left(\exp_{\text{integral}_e}(4, (2 \operatorname{I} b^* c + 2 \operatorname{I} (b^* x + a) d - 2 \operatorname{I} a^* d) / d) + \exp_{\text{integral}_e}(4, -(2 \operatorname{I} b^* c + 2 \operatorname{I} (b^* x + a) d - 2 \operatorname{I} a^* d) / d) \right) \sin(-2(b^* c - a^* d) / d) + b^4 \left(\exp_{\text{integral}_e}(4, (4 \operatorname{I} b^* c + 4 \operatorname{I} (b^* x + a) d - 4 \operatorname{I} a^* d) / d) + \exp_{\text{integral}_e}(4, -(4 \operatorname{I} b^* c + 4 \operatorname{I} (b^* x + a) d - 4 \operatorname{I} a^* d) / d) \right) \sin(-4(b^* c - a^* d) / d) / ((b^3 c^3 d - 3 a^* b^2 c^2 d^2 + 3 a^2 b^* c d^3 + (b^* x + a)^3 d^4 - a^3 d^4 + 3(b^* c d^3 - a^* d^4)(b^* x + a)^2 + 3(b^2 c^2 d^2 - 2 a^* b^* c d^3 + a^2 d^4)(b^* x + a)) b)$

Fricas [B] time = 0.684968, size = 1293, normalized size = 4.51

$$bd^3x + 4(bd^3x + bcd^2) \cos(bx+a)^4 + bcd^2 - 5(bd^3x + bcd^2) \cos(bx+a)^2 - 8(b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + b^3c^3) \sin(bx+a)^4 + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="fricas")`

[Out] $\frac{1}{6} (b^* d^3 x^3 + 4(b^* d^3 x^2 + b^* c d^2) \cos(b^* x + a)^4 + b^* c d^2 - 5(b^* d^3 x^2 + b^* c d^2) \cos(b^* x + a)^2 - 8(b^3 d^3 x^3 + 3b^3 c d^2 x^2 + 3b^3 c^2 d x + b^3 c^3) \sin(-4(b^* c - a^* d) / d) \sin_{\text{integral}}(4(b^* d x + b^* c) / d) + 2(b^3 d^3 x^3 + 3b^3 c d^2 x^2 + 3b^3 c^2 d x + b^3 c^3) \sin(-2(b^* c - a^* d) / d) \sin_{\text{integral}}(2(b^* d x + b^* c) / d) - ((b^3 d^3 x^3 + 3b^3 c d^2 x^2 + 3b^3 c^2 d x + b^3 c^3) \cos_{\text{integral}}(2(b^* d x + b^* c) / d) + (b^3 d^3 x^3 + 3b^3 c d^2 x^2 + 3b^3 c^2 d x + b^3 c^3) \cos_{\text{integral}}(-2(b^* d x + b^* c) / d)) \cos(-2(b^* c - a^* d) / d) + 4((b^3 d^3 x^3 + 3b^3 c d^2 x^2 + 3b^3 c^2 d x + b^3 c^3) \cos_{\text{integral}}(4(b^* d x + b^* c) / d) + (b^3 d^3 x^3 + 3b^3 c d^2 x^2 + 3b^3 c^2 d x + b^3 c^3) \cos_{\text{integral}}(-4(b^* d x + b^* c) / d)) \sin(-4(b^* c - a^* d) / d) / ((b^3 c^3 d - 3 a^* b^2 c^2 d^2 + 3 a^2 b^* c d^3 + (b^* x + a)^3 d^4 - a^3 d^4 + 3(b^* c d^3 - a^* d^4)(b^* x + a)^2 + 3(b^2 c^2 d^2 - 2 a^* b^* c d^3 + a^2 d^4)(b^* x + a)) b)$

$$\frac{\begin{aligned} &^3c^2d^2x + b^3c^3 \end{aligned} \cos_integral(-4*(b*d*x + b*c)/d) * \cos(-4*(b*c - a*d)/d) - 2*((8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3) * \cos(b*x + a)^3 - (5*b^2*d^3*x^2 + 10*b^2*c*d^2*x + 5*b^2*c^2*d - d^3) * \cos(b*x + a)) * \sin(b*x + a)}{(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c)**4,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")

[Out] Exception raised: NotImplementedError

3.31 $\int (c + dx)^m \cot(a + bx) dx$

Optimal. Leaf size=16

Unintegrable(cot(a + bx)(c + dx)^m, x)

[Out] Unintegrable[(c + d*x)^m*Cot[a + b*x], x]

Rubi [A] time = 0.0182283, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \cot(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Cot[a + b*x], x]

[Out] Defer[Int] [(c + d*x)^m*Cot[a + b*x], x]

Rubi steps

$$\int (c + dx)^m \cot(a + bx) dx = \int (c + dx)^m \cot(a + bx) dx$$

Mathematica [A] time = 2.53905, size = 0, normalized size = 0.

$$\int (c + dx)^m \cot(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Cot[a + b*x], x]

[Out] Integrate[(c + d*x)^m*Cot[a + b*x], x]

Maple [A] time = 0.234, size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) \csc(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a), x)

[Out] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) \csc(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx + c)^m \cos(bx + a) \csc(bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cos(b*x + a)*csc(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) \csc(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a), x)
```

3.32 $\int (c + dx)^4 \cot(a + bx) dx$

Optimal. Leaf size=151

$$\frac{3d^2(c + dx)^2 \text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{b^3} + \frac{3id^3(c + dx) \text{PolyLog}\left(4, e^{2i(a+bx)}\right)}{b^4} - \frac{2id(c + dx)^3 \text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^2} - \frac{3d^4 \text{PolyLog}\left(5, e^{2i(a+bx)}\right)}{b^5}$$

[Out] $((-I/5)*(c + d*x)^5)/d + ((c + d*x)^4*\text{Log}[1 - E^{((2*I)*(a + b*x))}])/b - ((2*I)*d*(c + d*x)^3*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^2 + (3*d^2*(c + d*x)^2*\text{PolyLog}[3, E^{((2*I)*(a + b*x))}])/b^3 + (((3*I)*d^3*(c + d*x)*\text{PolyLog}[4, E^{((2*I)*(a + b*x))}])/b^4 - (3*d^4*\text{PolyLog}[5, E^{((2*I)*(a + b*x))}])/(2*b^5)$

Rubi [A] time = 0.220252, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3717, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c + dx)^2 \text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{b^3} + \frac{3id^3(c + dx) \text{PolyLog}\left(4, e^{2i(a+bx)}\right)}{b^4} - \frac{2id(c + dx)^3 \text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^2} - \frac{3d^4 \text{PolyLog}\left(5, e^{2i(a+bx)}\right)}{b^5}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^4*Cot[a + b*x], x]

[Out] $((-I/5)*(c + d*x)^5)/d + ((c + d*x)^4*\text{Log}[1 - E^{((2*I)*(a + b*x))}])/b - ((2*I)*d*(c + d*x)^3*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^2 + (3*d^2*(c + d*x)^2*\text{PolyLog}[3, E^{((2*I)*(a + b*x))}])/b^3 + (((3*I)*d^3*(c + d*x)*\text{PolyLog}[4, E^{((2*I)*(a + b*x))}])/b^4 - (3*d^4*\text{PolyLog}[5, E^{((2*I)*(a + b*x))}])/(2*b^5)$

Rule 3717

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[(c + d*x)^m * E^(2*I*k*Pi) * E^(2*I*(e + f*x))]/(1 + E^(2*I*k*Pi) * E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] := Simp[((c + d*x)^m * Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/ (b*f*g*n * Log[F]), x] - Dist[(d*m)/(b*f*g*n * Log[F]), Int[(c + d*x)^(m - 1) * Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*(f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[(f + g*x)^m * PolyLog[2, -(e*(F^(c*(a + b*x)))^n)]/(b*c*n * Log[F]), x] + Dist[(g*m)/(b*c*n * Log[F]), Int[(f + g*x)^(m - 1) * PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 6609

Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(p_.)], x_Symbol] := Simp[(e + f*x)^m * PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]/(b*c*p * Log[F]), x] - Dist[(f*m)/(b*c*p * Log[F]), Int[(e + f*x)^(m - 1) * PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,

d, e, f, n, p}, x] && GtQ[m, 0]

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^4 \cot(a + bx) dx &= -\frac{i(c + dx)^5}{5d} - 2i \int \frac{e^{2i(a+bx)}(c + dx)^4}{1 - e^{2i(a+bx)}} dx \\ &= -\frac{i(c + dx)^5}{5d} + \frac{(c + dx)^4 \log(1 - e^{2i(a+bx)})}{b} - \frac{(4d) \int (c + dx)^3 \log(1 - e^{2i(a+bx)}) dx}{b} \\ &= -\frac{i(c + dx)^5}{5d} + \frac{(c + dx)^4 \log(1 - e^{2i(a+bx)})}{b} - \frac{2id(c + dx)^3 \text{Li}_2(e^{2i(a+bx)})}{b^2} + \frac{(6id^2) \int (c + dx)^2 \log(1 - e^{2i(a+bx)}) dx}{b^2} \\ &= -\frac{i(c + dx)^5}{5d} + \frac{(c + dx)^4 \log(1 - e^{2i(a+bx)})}{b} - \frac{2id(c + dx)^3 \text{Li}_2(e^{2i(a+bx)})}{b^2} + \frac{3d^2(c + dx)^2 \text{Li}_2(e^{2i(a+bx)})}{b^3} \\ &= -\frac{i(c + dx)^5}{5d} + \frac{(c + dx)^4 \log(1 - e^{2i(a+bx)})}{b} - \frac{2id(c + dx)^3 \text{Li}_2(e^{2i(a+bx)})}{b^2} + \frac{3d^2(c + dx)^2 \text{Li}_2(e^{2i(a+bx)})}{b^3} \\ &= -\frac{i(c + dx)^5}{5d} + \frac{(c + dx)^4 \log(1 - e^{2i(a+bx)})}{b} - \frac{2id(c + dx)^3 \text{Li}_2(e^{2i(a+bx)})}{b^2} + \frac{3d^2(c + dx)^2 \text{Li}_2(e^{2i(a+bx)})}{b^3} \\ &= -\frac{i(c + dx)^5}{5d} + \frac{(c + dx)^4 \log(1 - e^{2i(a+bx)})}{b} - \frac{2id(c + dx)^3 \text{Li}_2(e^{2i(a+bx)})}{b^2} + \frac{3d^2(c + dx)^2 \text{Li}_2(e^{2i(a+bx)})}{b^3} \end{aligned}$$

Mathematica [B] time = 5.76902, size = 799, normalized size = 5.29

$$\frac{1}{5}id^4x^5 + icd^3x^4 + \frac{d^4 \log(1 - e^{-i(a+bx)})x^4}{b} + \frac{d^4 \log(1 + e^{-i(a+bx)})x^4}{b} + 2ic^2d^2x^3 + \frac{4cd^3 \log(1 - e^{-i(a+bx)})x^3}{b} + \frac{4cd^3 \log(1 + e^{-i(a+bx)})x^3}{b}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^4*Cot[a + b*x], x]

[Out] ((2*I)*c^3*d*Pi*x)/b + (2*I)*c^2*d^2*x^3 + I*c*d^3*x^4 + (I/5)*d^4*x^5 - ((4*I)*c^3*d*x*ArcTan[Tan[a]])/b + 2*c^3*d*x^2*Cot[a] + (2*c^3*d*Pi*Log[1 + E^((-2*I)*b*x)]/b^2 + (6*c^2*d^2*x^2*Log[1 - E^((-I)*(a + b*x))])/b + (4*c*d^3*x^3*Log[1 - E^((-I)*(a + b*x))])/b + (d^4*x^4*Log[1 - E^((-I)*(a + b*x))])/b + (6*c^2*d^2*x^2*Log[1 + E^((-I)*(a + b*x))])/b + (4*c*d^3*x^3*Log[1 + E^((-I)*(a + b*x))])/b + (d^4*x^4*Log[1 + E^((-I)*(a + b*x))])/b + (4*c^3*d*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))]/b + (4*c^3*d*ArcTan[Tan[a]]*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))]/b^2 - (2*c^3*d*Pi*Log[Cos[b*x]]/b^2 + (c^4*Log[Sin[a + b*x]]/b - (4*c^3*d*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]])/b^2 + ((4*I)*d^2*x*(3*c^2 + 3*c*d*x + d^2*x^2)*PolyLog[2, -E^((-I)*(a + b*x))])/b^2 + ((4*I)*d^2*x*(3*c^2 + 3*c*d*x + d^2*x^2)*PolyLog[2, E^((-I)*(a + b*x))])/b^2 - ((2*I)*c^3*d*PolyLog[2, E^((2*I)*(b*x + Arc

$$\begin{aligned} & \text{Tan}[\text{Tan}[a]])])/b^2 + (12*c^2*d^2*\text{PolyLog}[3, -E^{((-I)*(a + b*x))}])/b^3 + (2 \\ & 4*c*d^3*x*\text{PolyLog}[3, -E^{((-I)*(a + b*x))}])/b^3 + (12*d^4*x^2*\text{PolyLog}[3, -E^{ \\ & ((-I)*(a + b*x))}])/b^3 + (12*c^2*d^2*\text{PolyLog}[3, E^{((-I)*(a + b*x))}])/b^3 + \\ & (24*c*d^3*x*\text{PolyLog}[3, E^{((-I)*(a + b*x))}])/b^3 + (12*d^4*x^2*\text{PolyLog}[3, E^{ \\ & ((-I)*(a + b*x))}])/b^3 - ((24*I)*c*d^3*\text{PolyLog}[4, -E^{((-I)*(a + b*x))}])/b^4 \\ & - ((24*I)*d^4*x*\text{PolyLog}[4, -E^{((-I)*(a + b*x))}])/b^4 - ((24*I)*c*d^3*\text{PolyL} \\ & \text{og}[4, E^{((-I)*(a + b*x))}])/b^4 - ((24*I)*d^4*x*\text{PolyLog}[4, E^{((-I)*(a + b*x)} \\ &)])/b^4 - (24*d^4*\text{PolyLog}[5, -E^{((-I)*(a + b*x))}])/b^5 - (24*d^4*\text{PolyLog}[5, \\ & E^{((-I)*(a + b*x))}])/b^5 - 2*c^3*d*E^{(I*\text{ArcTan}[\text{Tan}[a]])}*x^2*\text{Cot}[a]*\text{Sqrt}[\text{Se} \\ & c[a]^2] \end{aligned}$$

Maple [B] time = 0.329, size = 1150, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x)`

[Out]
$$\begin{aligned} & -1/5*I*d^4*x^5+12/b^3*d^4*\text{polylog}(3, \exp(I*(b*x+a)))*x^2+1/b^5*d^4*a^4*\ln(\exp(I*(b*x+a))-1) \\ & -2/b^5*d^4*a^4*\ln(\exp(I*(b*x+a)))+12/b^3*c^2*d^2*\text{polylog}(3, \exp(I*(b*x+a)))+12/b^3*c^2*d^2*\text{polylog}(3, -\exp(I*(b*x+a))) \\ & -12*I/b^2*c^2*d^2*\text{polylog}(2, -\exp(I*(b*x+a)))*x-12*I/b^2*c^2*d^2*\text{polylog}(2, \exp(I*(b*x+a)))*x-8*I/b^2*c^3*d^3*a^3*x \\ & -12*I/b^2*c^3*d^3*\text{polylog}(2, -\exp(I*(b*x+a)))*x^2+12*I/b^2*c^2*d^2*a^2*x-8*I/b^3*c*d^3*a^3*x-12*I/b^2*c^3*d^3*\text{polylog}(2, \exp(I*(b*x+a)))*x^2+I*c^4*x \\ & -2*I*c^3*d^3*x^2-I*c^3*d^3*x^4-2*I*c^2*d^2*x^3+8/5*I/b^5*d^4*a^5+12/b^3*d^4*\text{polylog}(3, -\exp(I*(b*x+a)))*x^2+6/b^3*c^2*d^2*a^2*\ln(\exp(I*(b*x+a))-1)+24/b^3*c^2*d^3*\text{polylog}(3, -\exp(I*(b*x+a)))*x+4/b^3*c^3*d^3*\ln(\exp(I*(b*x+a))+1)*x+4/b^3*c^3*d^3*\ln(1-\exp(I*(b*x+a)))*x+4/b^2*c^3*d^3*\ln(1-\exp(I*(b*x+a)))*a-4*I/b^2*d^4*\text{polylog}(2, \exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*\text{polylog}(4, \exp(I*(b*x+a)))*x-4*I/b^2*d^4*\text{polylog}(2, -\exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*\text{polylog}(4, -\exp(I*(b*x+a)))*x+24*I/b^4*c*d^3*\text{polylog}(4, \exp(I*(b*x+a)))+24*I/b^4*c*d^3*\text{polylog}(4, -\exp(I*(b*x+a)))+2*I/b^4*d^4*a^4*x+8*I/b^3*c^2*d^2*a^3-6*I/b^4*c*d^3*a^4-4*I/b^2*c^3*d^3*a^2-4*I/b^2*c^3*d^3*\text{polylog}(2, \exp(I*(b*x+a)))+1/b*d^4*\ln(1-\exp(I*(b*x+a)))*x^4-1/b^5*d^4*\ln(1-\exp(I*(b*x+a)))*a^4+1/b*d^4*\ln(\exp(I*(b*x+a))+1)*x^4+24/b^3*c^3*d^3*\text{polylog}(3, \exp(I*(b*x+a)))*x+6/b^3*c^2*d^2*\ln(\exp(I*(b*x+a))+1)*x^2+1/b*c^4*\ln(\exp(I*(b*x+a))+1)+1/b*c^4*\ln(\exp(I*(b*x+a))-1)-2/b*c^4*\ln(\exp(I*(b*x+a)))-24*d^4*\text{polylog}(5, -\exp(I*(b*x+a)))/b^5-24*d^4*\text{polylog}(5, \exp(I*(b*x+a)))/b^5-4*I/b^2*c^3*d^3*\text{polylog}(2, -\exp(I*(b*x+a)))+6/b^3*c^2*d^2*\ln(1-\exp(I*(b*x+a)))*x^2-6/b^3*c^2*d^2*\ln(1-\exp(I*(b*x+a)))*a^2+8/b^4*c*d^3*a^3*\ln(\exp(I*(b*x+a)))-4/b^2*c^3*d^3*a^3*\ln(\exp(I*(b*x+a))-1)+8/b^2*c^3*d^3*a^3*\ln(\exp(I*(b*x+a)))-4/b^4*c*d^3*a^3*\ln(\exp(I*(b*x+a))-1)-12/b^3*c^2*d^2*a^2*\ln(\exp(I*(b*x+a)))+4/b*c^3*d^3*\ln(\exp(I*(b*x+a))+1)*x^3+4/b*c^3*d^3*\ln(1-\exp(I*(b*x+a)))*x^3+4/b^4*c^3*d^3*\ln(1-\exp(I*(b*x+a)))*a^3 \end{aligned}$$

Maxima [B] time = 1.97812, size = 1704, normalized size = 11.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")`

[Out]
$$\begin{aligned} & 1/10*(10*c^4*\log(\sin(b*x + a)) - 40*a*c^3*d*\log(\sin(b*x + a))/b + 60*a^2*c^2*d^2*\log(\sin(b*x + a))/b^2 - 40*a^3*c*d^3*\log(\sin(b*x + a))/b^3 + 10*a^4*d \end{aligned}$$

$$\begin{aligned} &^4 \log(\sin(b*x + a))/b^4 + (-2*I*(b*x + a)^5*d^4 + (-10*I*b*c*d^3 + 10*I*a*d^4)*(b*x + a)^4 - 240*d^4*\text{polylog}(5, -e^{(I*b*x + I*a)}) - 240*d^4*\text{polylog}(5, e^{(I*b*x + I*a)}) + (-20*I*b^2*c^2*d^2 + 40*I*a*b*c*d^3 - 20*I*a^2*d^4)*(b*x + a)^3 + (-20*I*b^3*c^3*d + 60*I*a*b^2*c^2*d^2 - 60*I*a^2*b*c*d^3 + 20*I*a^3*d^4)*(b*x + a)^2 + (10*I*(b*x + a)^4*d^4 + (40*I*b*c*d^3 - 40*I*a*d^4)*(b*x + a)^3 + (60*I*b^2*c^2*d^2 - 120*I*a*b*c*d^3 + 60*I*a^2*d^4)*(b*x + a)^2 + (40*I*b^3*c^3*d - 120*I*a*b^2*c^2*d^2 + 120*I*a^2*b*c*d^3 - 40*I*a^3*d^4)*(b*x + a))*\text{arctan2}(\sin(b*x + a), \cos(b*x + a) + 1) + (-10*I*(b*x + a)^4*d^4 + (-40*I*b*c*d^3 + 40*I*a*d^4)*(b*x + a)^3 + (-60*I*b^2*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*a^2*d^4)*(b*x + a)^2 + (-40*I*b^3*c^3*d + 120*I*a*b^2*c^2*d^2 - 120*I*a^2*b*c*d^3 + 40*I*a^3*d^4)*(b*x + a))*\text{arctan2}(\sin(b*x + a), -\cos(b*x + a) + 1) + (-40*I*b^3*c^3*d + 120*I*a*b^2*c^2*d^2 - 120*I*a^2*b*c*d^3 - 40*I*(b*x + a)^3*d^4 + 40*I*a^3*d^4 + (-120*I*b*c*d^3 + 120*I*a*d^4)*(b*x + a)^2 + (-120*I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 - 120*I*a^2*d^4)*(b*x + a))*\text{dilog}(-e^{(I*b*x + I*a)}) + (-40*I*b^3*c^3*d + 120*I*a*b^2*c^2*d^2 - 120*I*a^2*b*c*d^3 - 40*I*(b*x + a)^3*d^4 + 40*I*a^3*d^4 + (-120*I*b*c*d^3 + 120*I*a*d^4)*(b*x + a)^2 + (-120*I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 - 120*I*a^2*d^4)*(b*x + a))*\text{dilog}(e^{(I*b*x + I*a)}) + 5*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) + 5*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) + (240*I*b*c*d^3 + 240*I*(b*x + a)*d^4 - 240*I*a*d^4)*\text{polylog}(4, -e^{(I*b*x + I*a)}) + (240*I*b*c*d^3 + 240*I*(b*x + a)*d^4 - 240*I*a*d^4)*\text{polylog}(4, e^{(I*b*x + I*a)}) + 120*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*\text{polylog}(3, -e^{(I*b*x + I*a)}) + 120*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*\text{polylog}(3, e^{(I*b*x + I*a)})/b^4)/b \end{aligned}$$

Fricas [C] time = 0.694346, size = 2938, normalized size = 19.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} &-1/2*(24*d^4*\text{polylog}(5, \cos(b*x + a) + I*\sin(b*x + a)) + 24*d^4*\text{polylog}(5, \cos(b*x + a) - I*\sin(b*x + a)) + 24*d^4*\text{polylog}(5, -\cos(b*x + a) + I*\sin(b*x + a)) + 24*d^4*\text{polylog}(5, -\cos(b*x + a) - I*\sin(b*x + a)) - (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*\text{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*\text{dilog}(\cos(b*x + a) - I*\sin(b*x + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*\text{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) - (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*\text{dilog}(-\cos(b*x + a) - I*\sin(b*x + a)) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*\log(-1/2*\cos(b*x + a) + 1/2*I*\sin(b*x + a) + 1/2) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*\log(-1/2*\cos(b*x + a) - 1/2*I*\sin(b*x + a) + 1/2) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4 \end{aligned}$$

```
*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*
b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) - I*sin(b*x + a) +
1) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, cos(b*x + a) + I*sin(b*x + a
)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, cos(b*x + a) - I*sin(b*x + a
)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, -cos(b*x + a) + I*sin(b*x +
a)) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, -cos(b*x + a) - I*sin(b*x +
a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, cos(b*x + a
) + I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylo
g(3, cos(b*x + a) - I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2
*c^2*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*
b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/b^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^4 \cos(bx + a) \csc(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)*csc(b*x + a), x)
```

3.33 $\int (c + dx)^3 \cot(a + bx) dx$

Optimal. Leaf size=127

$$\frac{3d^2(c + dx)\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3} - \frac{3id(c + dx)^2\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2} + \frac{3id^3\text{PolyLog}\left(4, e^{2i(a+bx)}\right)}{4b^4} + \frac{(c + dx)^3 \log(1)}{b}$$

[Out] $((-I/4)*(c + d*x)^4)/d + ((c + d*x)^3*\text{Log}[1 - E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d*(c + d*x)^2*\text{PolyLog}[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*\text{PolyLog}[3, E^((2*I)*(a + b*x))])/(2*b^3) + (((3*I)/4)*d^3*\text{PolyLog}[4, E^((2*I)*(a + b*x))])/b^4$

Rubi [A] time = 0.192486, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3717, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c + dx)\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3} - \frac{3id(c + dx)^2\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2} + \frac{3id^3\text{PolyLog}\left(4, e^{2i(a+bx)}\right)}{4b^4} + \frac{(c + dx)^3 \log(1)}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^3*Cot[a + b*x], x]

[Out] $((-I/4)*(c + d*x)^4)/d + ((c + d*x)^3*\text{Log}[1 - E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d*(c + d*x)^2*\text{PolyLog}[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*\text{PolyLog}[3, E^((2*I)*(a + b*x))])/(2*b^3) + (((3*I)/4)*d^3*\text{PolyLog}[4, E^((2*I)*(a + b*x))])/b^4$

Rule 3717

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m * E^(2*I*k*Pi) * E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi) * E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] := Simp[((c + d*x)^m * Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n * Log[F]), x] - Dist[(d*m)/(b*f*g*n * Log[F]), Int[(c + d*x)^(m - 1) * Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m * PolyLog[2, -(e*(F^(c*(a + b*x)))^n)])/b*c*n * Log[F], x] + Dist[(g*m)/(b*c*n * Log[F]), Int[(f + g*x)^(m - 1) * PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 6609

Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m * PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/b*c*p * Log[F], x] - Dist[(f*m)/(b*c*p * Log[F]), Int[(e + f*x)^(m - 1) * PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,

d, e, f, n, p}, x] && GtQ[m, 0]

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^3 \cot(a + bx) dx &= -\frac{i(c + dx)^4}{4d} - 2i \int \frac{e^{2i(a+bx)}(c + dx)^3}{1 - e^{2i(a+bx)}} dx \\ &= -\frac{i(c + dx)^4}{4d} + \frac{(c + dx)^3 \log(1 - e^{2i(a+bx)})}{b} - \frac{(3d) \int (c + dx)^2 \log(1 - e^{2i(a+bx)}) dx}{b} \\ &= -\frac{i(c + dx)^4}{4d} + \frac{(c + dx)^3 \log(1 - e^{2i(a+bx)})}{b} - \frac{3id(c + dx)^2 \text{Li}_2(e^{2i(a+bx)})}{2b^2} + \frac{(3id^2) \int (c + dx) \text{Li}_2(e^{2i(a+bx)}) dx}{b^2} \\ &= -\frac{i(c + dx)^4}{4d} + \frac{(c + dx)^3 \log(1 - e^{2i(a+bx)})}{b} - \frac{3id(c + dx)^2 \text{Li}_2(e^{2i(a+bx)})}{2b^2} + \frac{3d^2(c + dx) \text{Li}_3(e^{2i(a+bx)})}{2b^3} \\ &= -\frac{i(c + dx)^4}{4d} + \frac{(c + dx)^3 \log(1 - e^{2i(a+bx)})}{b} - \frac{3id(c + dx)^2 \text{Li}_2(e^{2i(a+bx)})}{2b^2} + \frac{3d^2(c + dx) \text{Li}_3(e^{2i(a+bx)})}{2b^3} \\ &= -\frac{i(c + dx)^4}{4d} + \frac{(c + dx)^3 \log(1 - e^{2i(a+bx)})}{b} - \frac{3id(c + dx)^2 \text{Li}_2(e^{2i(a+bx)})}{2b^2} + \frac{3d^2(c + dx) \text{Li}_3(e^{2i(a+bx)})}{2b^3} \end{aligned}$$

Mathematica [B] time = 2.75977, size = 560, normalized size = 4.41

$$-6ib^2c^2d \text{PolyLog}\left(2, e^{2i(\tan^{-1}(\tan(a))+bx)}\right) + 12ib^2d^2x(2c + dx) \text{PolyLog}\left(2, -e^{-i(a+bx)}\right) + 12ib^2d^2x(2c + dx) \text{PolyLog}\left(2, e^{-i(a+bx)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x], x]
```

```
[Out] ((6*I)*b^3*c^2*d*Pi*x + (4*I)*b^4*c*d^2*x^3 + I*b^4*d^3*x^4 - (12*I)*b^3*c^2*d*x*ArcTan[Tan[a]] + 6*b^4*c^2*d*x^2*Cot[a] + 6*b^2*c^2*d*Pi*Log[1 + E^((-2*I)*b*x)] + 12*b^3*c*d^2*x^2*Log[1 - E^((-I)*(a + b*x))] + 4*b^3*d^3*x^3*Log[1 - E^((-I)*(a + b*x))] + 12*b^3*c*d^2*x^2*Log[1 + E^((-I)*(a + b*x))] + 4*b^3*d^3*x^3*Log[1 + E^((-I)*(a + b*x))] + 12*b^3*c^2*d*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]])] + 12*b^2*c^2*d*ArcTan[Tan[a]]*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]])] - 6*b^2*c^2*d*Pi*Log[Cos[b*x]] + 4*b^3*c^3*Log[Sin[a + b*x]] - 12*b^2*c^2*d*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + (12*I)*b^2*d^2*x*(2*c + d*x)*PolyLog[2, -E^((-I)*(a + b*x))] + (12*I)*b^2*d^2*x*(2*c + d*x)*PolyLog[2, E^((-I)*(a + b*x))] - (6*I)*b^2*c^2*d*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]])] + 24*b*c*d^2*PolyLog[3, -E^((-I)*(a + b*x))] + 24*b*d^3*x*PolyLog[3, -E^((-I)*(a + b*x))] + 24*b*c*d^2*PolyLog[3, E^((-I)*(a + b*x))] + 24*b*d^3*x*PolyLog[3, E^((-I)*(a + b*x))] - (24*I)*d^3*PolyLog[4, -E^((-I)*(a + b*x))] - (24*I)*d^3*PolyLog[4, E^((-I)*(a + b*x))]
```

] - 6*b^4*c^2*d*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^2]/(4*b^4)

Maple [B] time = 0.296, size = 783, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*cos(b*x+a)*csc(b*x+a), x)

[Out] 6/b^3*d^3*polylog(3, exp(I*(b*x+a)))*x-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+6/b^3*d^3*polylog(3, -exp(I*(b*x+a)))*x-3/2*I/b^4*d^3*a^4+6*I/b^4*d^3*polylog(4, -exp(I*(b*x+a)))+2/b^4*d^3*a^3*ln(exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3, -exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3, exp(I*(b*x+a)))-1/4*I*d^3*x^4+6*I/b^2*c*d^2*a^2*x-6*I/b^2*c*d^2*polylog(2, -exp(I*(b*x+a)))*x-6*I/b^2*c*d^2*polylog(2, exp(I*(b*x+a)))*x-6*I/b*c^2*d*a*x-I*c*d^2*x^3-3/2*I*c^2*d*x^2+6*I*d^3*polylog(4, exp(I*(b*x+a)))/b^4+3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-2/b*c^3*ln(exp(I*(b*x+a)))+1/b*c^3*ln(exp(I*(b*x+a))-1)+1/b*c^3*ln(exp(I*(b*x+a))+1)-3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2+3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)+6/b^2*c^2*d*a*ln(exp(I*(b*x+a)))-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)-6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a)))-3*I/b^2*d^3*polylog(2, -exp(I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog(2, exp(I*(b*x+a)))*x^2-3*I/b^2*c^2*d*polylog(2, exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2, -exp(I*(b*x+a)))+4*I/b^3*c*d^2*a^3-2*I/b^3*d^3*a^3*x-3*I/b^2*c^2*d*a^2+I*c^3*x

Maxima [B] time = 1.70128, size = 1008, normalized size = 7.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a), x, algorithm="maxima")

[Out] 1/4*(4*c^3*log(sin(b*x + a)) - 12*a*c^2*d*log(sin(b*x + a))/b + 12*a^2*c*d^2*log(sin(b*x + a))/b^2 - 4*a^3*d^3*log(sin(b*x + a))/b^3 + (-I*(b*x + a)^4*d^3 + (-4*I*b*c*d^2 + 4*I*a*d^3)*(b*x + a)^3 + 24*I*d^3*polylog(4, -e^(I*b*x + I*a)) + 24*I*d^3*polylog(4, e^(I*b*x + I*a)) + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*a^2*d^3)*(b*x + a)^2 + (4*I*(b*x + a)^3*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a)^2 + (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (-4*I*(b*x + a)^3*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 12*I*a^2*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*dilog(-e^(I*b*x + I*a)) + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 12*I*a^2*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*dilog(e^(I*b*x + I*a)) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, -e^(I*b*x + I*a))

+ 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, e^(I*b*x + I*a)))/b^3)/b

Fricas [C] time = 0.626955, size = 2056, normalized size = 16.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")

[Out] 1/2*(6*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4, cos(b*x + a) - I*sin(b*x + a)) - 6*I*d^3*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) + 6*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/b^4

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*cos(b*x+a)*csc(b*x+a),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \cos(bx + a) \csc(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^3*cos(b*x + a)*csc(b*x + a), x)

3.34 $\int (c + dx)^2 \cot(a + bx) dx$

Optimal. Leaf size=93

$$\frac{id(c + dx)\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^2} + \frac{d^2\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3} + \frac{(c + dx)^2 \log\left(1 - e^{2i(a+bx)}\right)}{b} - \frac{i(c + dx)^3}{3d}$$

[Out] $((-I/3)*(c + d*x)^3)/d + ((c + d*x)^2*\text{Log}[1 - E^((2*I)*(a + b*x))])/b - (I*d*(c + d*x)*\text{PolyLog}[2, E^((2*I)*(a + b*x))])/b^2 + (d^2*\text{PolyLog}[3, E^((2*I)*(a + b*x))])/(2*b^3)$

Rubi [A] time = 0.166433, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3717, 2190, 2531, 2282, 6589}

$$\frac{id(c + dx)\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^2} + \frac{d^2\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3} + \frac{(c + dx)^2 \log\left(1 - e^{2i(a+bx)}\right)}{b} - \frac{i(c + dx)^3}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^2*\text{Cot}[a + b*x], x]$

[Out] $((-I/3)*(c + d*x)^3)/d + ((c + d*x)^2*\text{Log}[1 - E^((2*I)*(a + b*x))])/b - (I*d*(c + d*x)*\text{PolyLog}[2, E^((2*I)*(a + b*x))])/b^2 + (d^2*\text{PolyLog}[3, E^((2*I)*(a + b*x))])/(2*b^3)$

Rule 3717

$\text{Int}[(c + d*x)^m*\tan[e + \text{Pi}*k + (f + g*x)], x]$ Symbol
 $\text{Int}[(c + d*x)^m*\tan[e + \text{Pi}*k + (f + g*x)], x] \text{ :> } \text{Simp}[(I*(c + d*x)^{(m+1)})/(d*(m+1)), x] - \text{Dist}[2*I, \text{Int}[(c + d*x)^m*\text{E}^{(2*I*k*\text{Pi})}*\text{E}^{(2*I*(e + f*x))}/(1 + \text{E}^{(2*I*k*\text{Pi})}*\text{E}^{(2*I*(e + f*x))}), x], x] /;$ FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 2190

$\text{Int}[(F + (g + (e + f*x))^n)^m*(c + d*x)^m, x]$ Symbol
 $\text{Int}[(F + (g + (e + f*x))^n)^m*(c + d*x)^m, x] \text{ :> } \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F + (g + (e + f*x))^n)/a)]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + (b*(F + (g + (e + f*x))^n)/a)], x], x] /;$ FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

$\text{Int}[\text{Log}[1 + (e + (f + g*x))^n]*(c + d*x)^m, x]$ Symbol
 $\text{Int}[\text{Log}[1 + (e + (f + g*x))^n]*(c + d*x)^m, x] \text{ :> } -\text{Simp}[(f + g*x)^m*\text{PolyLog}[2, -(e*(F + (c*(a + b*x))^n))] / (b*c*n*\text{Log}[F]), x] + \text{Dist}[(g*m)/(b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^{(m-1)}*\text{PolyLog}[2, -(e*(F + (c*(a + b*x))^n))], x], x] /;$ FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 2282

$\text{Int}[u, x]$ Symbol
 $\text{Int}[u, x] \text{ :> } \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /;$ FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^n)^m] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*(a_ + (b_)*x))* (F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \cot(a + bx) dx &= -\frac{i(c + dx)^3}{3d} - 2i \int \frac{e^{2i(a+bx)}(c + dx)^2}{1 - e^{2i(a+bx)}} dx \\ &= -\frac{i(c + dx)^3}{3d} + \frac{(c + dx)^2 \log(1 - e^{2i(a+bx)})}{b} - \frac{(2d) \int (c + dx) \log(1 - e^{2i(a+bx)}) dx}{b} \\ &= -\frac{i(c + dx)^3}{3d} + \frac{(c + dx)^2 \log(1 - e^{2i(a+bx)})}{b} - \frac{id(c + dx) \text{Li}_2(e^{2i(a+bx)})}{b^2} + \frac{(id^2) \int \text{Li}_2(e^{2i(a+bx)}) dx}{b^2} \\ &= -\frac{i(c + dx)^3}{3d} + \frac{(c + dx)^2 \log(1 - e^{2i(a+bx)})}{b} - \frac{id(c + dx) \text{Li}_2(e^{2i(a+bx)})}{b^2} + \frac{d^2 \text{Subst}\left(\int \frac{\text{Li}_2(x)}{x} dx\right)}{2b^3} \\ &= -\frac{i(c + dx)^3}{3d} + \frac{(c + dx)^2 \log(1 - e^{2i(a+bx)})}{b} - \frac{id(c + dx) \text{Li}_2(e^{2i(a+bx)})}{b^2} + \frac{d^2 \text{Li}_3(e^{2i(a+bx)})}{2b^3} \end{aligned}$$

Mathematica [B] time = 1.42455, size = 356, normalized size = 3.83

$$-3ibcd \text{PolyLog}\left(2, e^{2i(\tan^{-1}(\tan(a))+bx)}\right) + 6ibd^2x \text{PolyLog}\left(2, -e^{-i(a+bx)}\right) + 6ibd^2x \text{PolyLog}\left(2, e^{-i(a+bx)}\right) + 6d^2 \text{PolyLog}\left(3, e^{2i(a+bx)}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^2*Cot[a + b*x], x]

[Out] ((3*I)*b^2*c*d*Pi*x + I*b^3*d^2*x^3 - (6*I)*b^2*c*d*x*ArcTan[Tan[a]] + 3*b^3*c*d*x^2*Cot[a] + 3*b*c*d*Pi*Log[1 + E^((-2*I)*b*x)] + 3*b^2*d^2*x^2*Log[1 - E^((-I)*(a + b*x))] + 3*b^2*d^2*x^2*Log[1 + E^((-I)*(a + b*x))] + 6*b^2*c*d*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + 6*b*c*d*ArcTan[Tan[a]]*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] - 3*b*c*d*Pi*Log[Cos[b*x]] + 3*b^2*c^2*Log[Sin[a + b*x]] - 6*b*c*d*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + (6*I)*b*d^2*x*PolyLog[2, -E^((-I)*(a + b*x))] + (6*I)*b*d^2*x*PolyLog[2, E^((-I)*(a + b*x))] - (3*I)*b*c*d*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))] + 6*d^2*PolyLog[3, -E^((-I)*(a + b*x))] + 6*d^2*PolyLog[3, E^((-I)*(a + b*x))] - 3*b^3*c*d*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^2])/(3*b^3)

Maple [B] time = 0.27, size = 468, normalized size = 5.

$$-2 \frac{a^2 d^2 \ln(e^{i(bx+a)})}{b^3} + \frac{a^2 d^2 \ln(e^{i(bx+a)} - 1)}{b^3} - \frac{d^2 \ln(1 - e^{i(bx+a)}) a^2}{b^3} + \frac{d^2 \ln(e^{i(bx+a)} + 1) x^2}{b} + \frac{d^2 \ln(1 - e^{i(bx+a)}) x^2}{b} + \frac{4i}{3} \frac{a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*cos(b*x+a)*csc(b*x+a), x)

[Out] -2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))+1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2+4/3*I/b^3*d^2*a^3-I*c*d*x^2-4*I/b*c*d*a*x+2*I/b^2*d^2*a

$$\begin{aligned} &^2*x-2*I/b^2*c*d*polylog(2, \exp(I*(b*x+a)))-2*I/b^2*c*d*polylog(2, -\exp(I*(b*x+a))) \\ &-2*I/b^2*c*d*a^2-2*I/b^2*d^2*polylog(2, \exp(I*(b*x+a)))*x-2*I/b^2*d^2*polylog(2, -\exp(I*(b*x+a))) \\ &*x+I*c^2*x+2/b*c*d*\ln(\exp(I*(b*x+a))+1)*x+2/b*c*d*\ln(1-\exp(I*(b*x+a)))*x+2/b^2*c*d*\ln(1-\exp(I*(b*x+a))) \\ &*a-2/b^2*c*d*a*\ln(\exp(I*(b*x+a))-1)+4/b^2*c*d*a*\ln(\exp(I*(b*x+a)))+2*d^2*polylog(3, -\exp(I*(b*x+a))) \\ &)/b^3+2*d^2*polylog(3, \exp(I*(b*x+a)))/b^3+1/b*c^2*\ln(\exp(I*(b*x+a))-1)-2/b*c^2*\ln(\exp(I*(b*x+a))) \\ &)+1/b*c^2*\ln(\exp(I*(b*x+a))+1)-1/3*I*d^2*x^3 \end{aligned}$$

Maxima [B] time = 1.52594, size = 545, normalized size = 5.86

$$6c^2 \log(\sin(bx+a)) - \frac{12acd \log(\sin(bx+a))}{b} + \frac{6a^2d^2 \log(\sin(bx+a))}{b^2} + \frac{-2i(bx+a)^3d^2 + (-6ibcd + 6i ad^2)(bx+a)^2 + 12d^2 \text{Li}_3(-e^{i(bx+a)}) + 12d^2 \text{Li}_3(e^{i(bx+a)})}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")

[Out] $\frac{1}{6}(6c^2 \log(\sin(bx+a)) - 12ac d \log(\sin(bx+a))/b + 6a^2d^2 \log(\sin(bx+a))/b^2 + (-2I(bx+a)^3d^2 + (-6Ib cd + 6I ad^2)(bx+a)^2 + 12d^2 \text{polylog}(3, -e^{i(bx+a)}) + 12d^2 \text{polylog}(3, e^{i(bx+a)}) + (6I(bx+a)^2d^2 + (12Ib cd - 12I ad^2)(bx+a)) \arctan(2(\sin(bx+a), \cos(bx+a) + 1) + (-6I(bx+a)^2d^2 + (-12Ib cd + 12I ad^2)(bx+a)) \arctan(2(\sin(bx+a), -\cos(bx+a) + 1) + (-12Ib cd - 12I(bx+a)d^2 + 12I ad^2) \text{dilog}(-e^{i(bx+a)}) + (-12Ib cd - 12I(bx+a)d^2 + 12I ad^2) \text{dilog}(e^{i(bx+a)}) + 3((bx+a)^2d^2 + 2(bcd - ad^2)(bx+a)) \log(\cos(bx+a)^2 + \sin(bx+a)^2 + 2\cos(bx+a) + 1) + 3((bx+a)^2d^2 + 2(bcd - ad^2)(bx+a)) \log(\cos(bx+a)^2 + \sin(bx+a)^2 - 2\cos(bx+a) + 1))/b^2)/b$

Fricas [C] time = 0.566539, size = 1330, normalized size = 14.3

$$2d^2 \text{polylog}(3, \cos(bx+a) + i \sin(bx+a)) + 2d^2 \text{polylog}(3, \cos(bx+a) - i \sin(bx+a)) + 2d^2 \text{polylog}(3, -\cos(bx+a) + i \sin(bx+a)) + 2d^2 \text{polylog}(3, -\cos(bx+a) - i \sin(bx+a))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")

[Out] $\frac{1}{2}(2d^2 \text{polylog}(3, \cos(bx+a) + I \sin(bx+a)) + 2d^2 \text{polylog}(3, \cos(bx+a) - I \sin(bx+a)) + 2d^2 \text{polylog}(3, -\cos(bx+a) + I \sin(bx+a)) + 2d^2 \text{polylog}(3, -\cos(bx+a) - I \sin(bx+a)) + (-2Ib d^2x - 2Ib cd) \text{dilog}(\cos(bx+a) + I \sin(bx+a)) + (2Ib d^2x + 2Ib cd) \text{dilog}(\cos(bx+a) - I \sin(bx+a)) + (2Ib d^2x + 2Ib cd) \text{dilog}(-\cos(bx+a) + I \sin(bx+a)) + (-2Ib d^2x - 2Ib cd) \text{dilog}(-\cos(bx+a) - I \sin(bx+a)) + (b^2d^2x^2 + 2b^2cdx + b^2c^2) \log(\cos(bx+a) + I \sin(bx+a) + 1) + (b^2d^2x^2 + 2b^2cdx + b^2c^2) \log(\cos(bx+a) - I \sin(bx+a) + 1) + (b^2c^2 - 2ab cd + a^2d^2) \log(-1/2 \cos(bx+a) + 1/2 I \sin(bx+a) + 1/2) + (b^2c^2 - 2ab cd + a^2d^2) \log(-1/2 \cos(bx+a) - 1/2 I \sin(bx+a) + 1/2) + (b^2d^2x^2 + 2b^2cdx + 2ab cd - a^2d^2) \log(-\cos(bx+a) + I \sin(bx+a) + 1) + (b^2d^2x^2 + 2b^2cdx + 2ab cd - a^2d^2) \log(-\cos(bx+a) - I \sin(bx+a) + 1))/b^3$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \cos(a + bx) \csc(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*cos(b*x+a)*csc(b*x+a),x)

[Out] Integral((c + d*x)**2*cos(a + b*x)*csc(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \cos(bx + a) \csc(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^2*cos(b*x + a)*csc(b*x + a), x)

3.35 $\int (c + dx) \cot(a + bx) dx$

Optimal. Leaf size=65

$$-\frac{id\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2} + \frac{(c + dx) \log\left(1 - e^{2i(a+bx)}\right)}{b} - \frac{i(c + dx)^2}{2d}$$

[Out] $((-I/2)*(c + d*x)^2)/d + ((c + d*x)*\text{Log}[1 - E^{((2*I)*(a + b*x))}])/b - ((I/2)*d*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^2$

Rubi [A] time = 0.0963027, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3717, 2190, 2279, 2391}

$$-\frac{id\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2} + \frac{(c + dx) \log\left(1 - e^{2i(a+bx)}\right)}{b} - \frac{i(c + dx)^2}{2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)*\text{Cot}[a + b*x], x]$

[Out] $((-I/2)*(c + d*x)^2)/d + ((c + d*x)*\text{Log}[1 - E^{((2*I)*(a + b*x))}])/b - ((I/2)*d*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^2$

Rule 3717

$\text{Int}[(c + d*x)^m \tan(e + \pi*k + f*x), x] \rightarrow \text{Simp}[(I*(c + d*x)^{m+1})/(d*(m+1)), x] - \text{Dist}[2*I, \text{Int}[(c + d*x)^m E^{2*I*k*\pi} E^{2*I*(e + f*x)}]/(1 + E^{2*I*k*\pi} E^{2*I*(e + f*x)}), x] /;$ $\text{FreeQ}\{c, d, e, f, x\} \ \&\& \ \text{IntegerQ}[4*k] \ \&\& \ \text{IGtQ}[m, 0]$

Rule 2190

$\text{Int}[(F^{(g*(e + f*x))})^n ((c + d*x)^m)/((a + b*(F^{(g*(e + f*x))})^n)], x] \rightarrow \text{Simp}[(c + d*x)^m \text{Log}[1 + (b*(F^{(g*(e + f*x))})^n)/a]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{m-1} \text{Log}[1 + (b*(F^{(g*(e + f*x))})^n)/a], x], x] /;$ $\text{FreeQ}\{F, a, b, c, d, e, f, g, n, x\} \ \&\& \ \text{IGtQ}[m, 0]$

Rule 2279

$\text{Int}[\text{Log}[a + b*(F^{(e*(c + d*x))})^n], x] \rightarrow \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /;$ $\text{FreeQ}\{F, a, b, c, d, e, n, x\} \ \&\& \ \text{GtQ}[a, 0]$

Rule 2391

$\text{Int}[\text{Log}[(c + d*x)^n]/(c + d*x), x] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c + d*x)^n]/n, x] /;$ $\text{FreeQ}\{c, d, e, n, x\} \ \&\& \ \text{EqQ}[c*d, 1]$

Rubi steps

$$\begin{aligned}
\int (c + dx) \cot(a + bx) dx &= -\frac{i(c + dx)^2}{2d} - 2i \int \frac{e^{2i(a+bx)}(c + dx)}{1 - e^{2i(a+bx)}} dx \\
&= -\frac{i(c + dx)^2}{2d} + \frac{(c + dx) \log(1 - e^{2i(a+bx)})}{b} - \frac{d \int \log(1 - e^{2i(a+bx)}) dx}{b} \\
&= -\frac{i(c + dx)^2}{2d} + \frac{(c + dx) \log(1 - e^{2i(a+bx)})}{b} + \frac{(id) \operatorname{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^{2i(a+bx)}\right)}{2b^2} \\
&= -\frac{i(c + dx)^2}{2d} + \frac{(c + dx) \log(1 - e^{2i(a+bx)})}{b} - \frac{id \operatorname{Li}_2(e^{2i(a+bx)})}{2b^2}
\end{aligned}$$

Mathematica [B] time = 5.18029, size = 188, normalized size = 2.89

$$d \operatorname{csc}(a) \operatorname{sec}(a) \left(\frac{\tan(a) \left(i \operatorname{PolyLog}\left(2, e^{2i(\tan^{-1}(\tan(a)+bx)}\right)\right) + ibx(2 \tan^{-1}(\tan(a)) - \pi) - 2(\tan^{-1}(\tan(a)+bx) \log(1 - e^{2i(\tan^{-1}(\tan(a)+bx)}\right) + 2 \tan^{-1}(\tan(a)))\right)}{\sqrt{\tan^2(a)+1}} \right)$$

$$2b^2 \sqrt{\sec^2(a) (\sin^2(a) + \cos^2(a))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)*Cot[a + b*x], x]

[Out] (d*x^2*Cot[a])/2 + (c*(Log[Cos[a + b*x]] + Log[Tan[a + b*x]]))/b - (d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]])]) + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]]) + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]])])]*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])

Maple [B] time = 0.248, size = 215, normalized size = 3.3

$$-\frac{i}{2} dx^2 + icx + \frac{c \ln(e^{i(bx+a)} - 1)}{b} - 2 \frac{c \ln(e^{i(bx+a)})}{b} + \frac{c \ln(e^{i(bx+a)} + 1)}{b} - \frac{2idax}{b} - \frac{ida^2}{b^2} + \frac{d \ln(1 - e^{i(bx+a)})x}{b} + \frac{d \ln(1 - e^{i(bx+a)})}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*cos(b*x+a)*csc(b*x+a), x)

[Out] -1/2*I*d*x^2+I*c*x+1/b*c*ln(exp(I*(b*x+a))-1)-2/b*c*ln(exp(I*(b*x+a)))+1/b*c*ln(exp(I*(b*x+a))+1)-2*I/b*d*a*x-I/b^2*d*a^2+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(b*x+a)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2+1/b*d*ln(exp(I*(b*x+a))+1)*x-I*d*polylog(2,-exp(I*(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1)+2/b^2*d*a*ln(exp(I*(b*x+a)))

Maxima [B] time = 1.53232, size = 255, normalized size = 3.92

$$-i b^2 dx^2 - 2i b^2 cx - 2i b dx \arctan(\sin(bx + a), -\cos(bx + a) + 1) + 2i bc \arctan(\sin(bx + a), \cos(bx + a) - 1) + (2i b c \arctan(\sin(bx + a), \cos(bx + a) - 1) + 2i b c \arctan(\sin(bx + a), -\cos(bx + a) + 1))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")

[Out] $\frac{1}{2}*(-I*b^2*d*x^2 - 2*I*b^2*c*x - 2*I*b*d*x*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) + 2*I*b*c*\arctan2(\sin(b*x + a), \cos(b*x + a) - 1) + (2*I*b*d*x + 2*I*b*c)*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) - 2*I*d*\operatorname{dilog}(-e^{(I*b*x + I*a)}) - 2*I*d*\operatorname{dilog}(e^{(I*b*x + I*a)}) + (b*d*x + b*c)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) + (b*d*x + b*c)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1))/b^2$

Fricas [B] time = 0.546684, size = 721, normalized size = 11.09

$-i d\operatorname{Li}_2(\cos(bx + a) + i \sin(bx + a)) + i d\operatorname{Li}_2(\cos(bx + a) - i \sin(bx + a)) + i d\operatorname{Li}_2(-\cos(bx + a) + i \sin(bx + a)) -$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")

[Out] $\frac{1}{2}*(-I*d*\operatorname{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) + I*d*\operatorname{dilog}(\cos(b*x + a) - I*\sin(b*x + a)) + I*d*\operatorname{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) - I*d*\operatorname{dilog}(-\cos(b*x + a) - I*\sin(b*x + a)) + (b*d*x + b*c)*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) + (b*d*x + b*c)*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1) + (b*c - a*d)*\log(-1/2*\cos(b*x + a) + 1/2*I*\sin(b*x + a) + 1/2) + (b*c - a*d)*\log(-1/2*\cos(b*x + a) - 1/2*I*\sin(b*x + a) + 1/2) + (b*d*x + a*d)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1) + (b*d*x + a*d)*\log(-\cos(b*x + a) - I*\sin(b*x + a) + 1))/b^2$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \cos(a + bx) \csc(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x)

[Out] Integral((c + d*x)*cos(a + b*x)*csc(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \cos(bx + a) \csc(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)*cos(b*x + a)*csc(b*x + a), x)

$$3.36 \quad \int \frac{\cot(a+bx)}{c+dx} dx$$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\cot(a+bx)}{c+dx}, x\right)$$

[Out] Unintegrable[Cot[a + b*x]/(c + d*x), x]

Rubi [A] time = 0.0208909, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cot(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[Cot[a + b*x]/(c + d*x), x]

[Out] Defer[Int][Cot[a + b*x]/(c + d*x), x]

Rubi steps

$$\int \frac{\cot(a+bx)}{c+dx} dx = \int \frac{\cot(a+bx)}{c+dx} dx$$

Mathematica [A] time = 3.58517, size = 0, normalized size = 0.

$$\int \frac{\cot(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[Cot[a + b*x]/(c + d*x), x]

[Out] Integrate[Cot[a + b*x]/(c + d*x), x]

Maple [A] time = 0.244, size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a) \csc(bx+a)}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)*csc(b*x+a)/(d*x+c), x)

[Out] int(cos(b*x+a)*csc(b*x+a)/(d*x+c), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (b x+a) \csc (b x+a)}{d x+c} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x, algorithm="maxima")

[Out] integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos (b x+a) \csc (b x+a)}{d x+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x, algorithm="fricas")

[Out] integral(cos(b*x + a)*csc(b*x + a)/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (a+b x) \csc (a+b x)}{c+d x} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x)

[Out] Integral(cos(a + b*x)*csc(a + b*x)/(c + d*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (b x+a) \csc (b x+a)}{d x+c} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x, algorithm="giac")

[Out] integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c), x)

$$3.37 \quad \int \frac{\cot(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\cot(a+bx)}{(c+dx)^2}, x\right)$$

[Out] Unintegrable[Cot[a + b*x]/(c + d*x)^2, x]

Rubi [A] time = 0.0207621, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cot(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[Cot[a + b*x]/(c + d*x)^2, x]

[Out] Defer[Int][Cot[a + b*x]/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\cot(a+bx)}{(c+dx)^2} dx = \int \frac{\cot(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 7.06491, size = 0, normalized size = 0.

$$\int \frac{\cot(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Cot[a + b*x]/(c + d*x)^2, x]

[Out] Integrate[Cot[a + b*x]/(c + d*x)^2, x]

Maple [A] time = 0.279, size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)\csc(bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2, x)

[Out] int(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx + a) \csc(bx + a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x, algorithm="maxima")

[Out] integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c)^2, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(bx + a) \csc(bx + a)}{d^2x^2 + 2cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(cos(b*x + a)*csc(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(a + bx) \csc(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)**2,x)

[Out] Integral(cos(a + b*x)*csc(a + b*x)/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx + a) \csc(bx + a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c)^2, x)

3.38 $\int (c + dx)^m \cot(a + bx) \csc(a + bx) dx$

Optimal. Leaf size=22

CannotIntegrate(cot(a + bx) csc(a + bx)(c + dx)^m, x)

[Out] CannotIntegrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x], x]

Rubi [A] time = 0.203924, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \cot(a + bx) \csc(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x], x]

[Out] Defer[Int] [(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x], x]

Rubi steps

$$\int (c + dx)^m \cot(a + bx) \csc(a + bx) dx = \int (c + dx)^m \cot(a + bx) \csc(a + bx) dx$$

Mathematica [A] time = 1.24198, size = 0, normalized size = 0.

$$\int (c + dx)^m \cot(a + bx) \csc(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x], x]

[Out] Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x], x]

Maple [A] time = 0.155, size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) (\csc(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)², x)

[Out] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)², x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) \csc(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left((dx + c)^m \cos (bx + a) \csc (bx + a)^2, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*csc(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos (bx + a) \csc (bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^2, x)
```

3.39 $\int (c + dx)^4 \cot(a + bx) \csc(a + bx) dx$

Optimal. Leaf size=208

$$-\frac{24d^3(c+dx)\text{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^4} + \frac{24d^3(c+dx)\text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^4} + \frac{12id^2(c+dx)^2\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} - \frac{12id^2(c+dx)^2\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3}$$

```
[Out] (-8*d*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b^2 - ((c + d*x)^4*Csc[a + b*x])/b + ((12*I)*d^2*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((12*I)*d^2*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^3 - (24*d^3*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^4 + (24*d^3*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^4 - ((24*I)*d^4*PolyLog[4, -E^(I*(a + b*x))])/b^5 + ((24*I)*d^4*PolyLog[4, E^(I*(a + b*x))])/b^5
```

Rubi [A] time = 0.171711, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {4410, 4183, 2531, 6609, 2282, 6589}

$$-\frac{24d^3(c+dx)\text{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^4} + \frac{24d^3(c+dx)\text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^4} + \frac{12id^2(c+dx)^2\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} - \frac{12id^2(c+dx)^2\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x], x]
```

```
[Out] (-8*d*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b^2 - ((c + d*x)^4*Csc[a + b*x])/b + ((12*I)*d^2*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((12*I)*d^2*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^3 - (24*d^3*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^4 + (24*d^3*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^4 - ((24*I)*d^4*PolyLog[4, -E^(I*(a + b*x))])/b^5 + ((24*I)*d^4*PolyLog[4, E^(I*(a + b*x))])/b^5
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/((b*c*n*Log[F])], x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_.))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*(a_.) + (b_.
)*(x_.)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*(a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^4 \cot(a + bx) \csc(a + bx) dx &= -\frac{(c + dx)^4 \csc(a + bx)}{b} + \frac{(4d) \int (c + dx)^3 \csc(a + bx) dx}{b} \\ &= -\frac{8d(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^4 \csc(a + bx)}{b} - \frac{(12d^2) \int (c + dx)^2 \csc(a + bx) dx}{b^2} \\ &= -\frac{8d(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^4 \csc(a + bx)}{b} + \frac{12id^2(c + dx)^2 \operatorname{Li}_2(e^{i(a+bx)})}{b^3} \\ &= -\frac{8d(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^4 \csc(a + bx)}{b} + \frac{12id^2(c + dx)^2 \operatorname{Li}_2(e^{i(a+bx)})}{b^3} \\ &= -\frac{8d(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^4 \csc(a + bx)}{b} + \frac{12id^2(c + dx)^2 \operatorname{Li}_2(e^{i(a+bx)})}{b^3} \\ &= -\frac{8d(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^4 \csc(a + bx)}{b} + \frac{12id^2(c + dx)^2 \operatorname{Li}_2(e^{i(a+bx)})}{b^3} \end{aligned}$$

Mathematica [A] time = 1.37479, size = 308, normalized size = 1.48

$$8id \left(\frac{3d(b^2(c+dx)^2 \operatorname{PolyLog}(2, -\cos(a+bx) - i \sin(a+bx)) + 2ibd(c+dx) \operatorname{PolyLog}(3, -\cos(a+bx) - i \sin(a+bx)) - 2d^2 \operatorname{PolyLog}(4, -\cos(a+bx) - i \sin(a+bx)))}{b^3} - \dots \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x], x]
```

```
[Out] (-2*b*(c + d*x)^4*Csc[a] + (8*I)*d*((2*I)*(c + d*x)^3*ArcTanh[Cos[a + b*x]
+ I*Sin[a + b*x]] + (3*d*(b^2*(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[
a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]]
- 2*d^2*PolyLog[4, -Cos[a + b*x] - I*Sin[a + b*x]]))/b^3 - (3*d*(b^2*(c + d
*x)^2*PolyLog[2, Cos[a + b*x] + I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyL
og[3, Cos[a + b*x] + I*Sin[a + b*x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Si
n[a + b*x]]))/b^3) + b*(c + d*x)^4*Csc[a/2]*Csc[(a + b*x)/2]*Sin[(b*x)/2] -
b*(c + d*x)^4*Sec[a/2]*Sec[(a + b*x)/2]*Sin[(b*x)/2))/(2*b^2)
```

Maple [B] time = 0.28, size = 716, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x)`

[Out]
$$\begin{aligned} & -24*I*d^3/b^3*c*polylog(2, \exp(I*(b*x+a))) * x + 24*I*d^3/b^3*c*polylog(2, -\exp(I*(b*x+a))) * x \\ & + 4*d^4/b^2*\ln(1-\exp(I*(b*x+a))) * x^3 + 8*d^4/b^5*a^3*\operatorname{arctanh}(\exp(I*(b*x+a))) \\ & + 24*d^4/b^4*polylog(3, \exp(I*(b*x+a))) * x - 24*d^4/b^4*polylog(3, -\exp(I*(b*x+a))) * x \\ & - 24*d^3/b^4*c*polylog(3, -\exp(I*(b*x+a))) + 24*d^3/b^4*c*polylog(3, \exp(I*(b*x+a))) \\ & - 8*d/b^2*c^3*\operatorname{arctanh}(\exp(I*(b*x+a))) + 24*I*d^4*polylog(4, \exp(I*(b*x+a))) \\ & /b^5 - 12*d^2/b^3*c^2*\ln(\exp(I*(b*x+a))+1) * a - 12*d^3/b^2*c*\ln(\exp(I*(b*x+a))+1) * x^2 \\ & + 12*d^3/b^4*c*\ln(\exp(I*(b*x+a))+1) * a^2 + 12*d^3/b^2*c*\ln(1-\exp(I*(b*x+a))) * x^2 \\ & - 12*d^3/b^4*c*\ln(1-\exp(I*(b*x+a))) * a^2 + 12*I*d^2/b^3*c^2*polylog(2, -\exp(I*(b*x+a))) \\ & - 12*I*d^2/b^3*c^2*polylog(2, \exp(I*(b*x+a))) + 12*I*d^4/b^3*polylog(2, -\exp(I*(b*x+a))) * x^2 \\ & - 12*I*d^4/b^3*polylog(2, \exp(I*(b*x+a))) * x^2 - 24*I*d^4*polylog(4, -\exp(I*(b*x+a))) /b^5 \\ & - 2*I*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*x^2+4*c^3*d*x+c^4)*\exp(I*(b*x+a))/b/(\exp(2*I*(b*x+a))-1) \\ & + 4*d^4/b^5*\ln(1-\exp(I*(b*x+a))) * a^3 - 4*d^4/b^2*\ln(\exp(I*(b*x+a))+1) * x^3 - 24*d^3/b^4*c*a^2 \\ & *\operatorname{arctanh}(\exp(I*(b*x+a))) + 24*d^2/b^3*c^2*a*\operatorname{arctanh}(\exp(I*(b*x+a))) - 4*d^4/b^5 \\ & *5*\ln(\exp(I*(b*x+a))+1) * a^3 + 12*d^2/b^2*c^2*\ln(1-\exp(I*(b*x+a))) * x + 12*d^2/b^3 \\ & *c^2*\ln(1-\exp(I*(b*x+a))) * a - 12*d^2/b^2*c^2*\ln(\exp(I*(b*x+a))+1) * x \end{aligned}$$

Maxima [B] time = 2.49972, size = 3974, normalized size = 19.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")`

[Out]
$$\begin{aligned} & -(2*(4*(b*x + a)*\cos(b*x + a)*\sin(2*b*x + 2*a) - 4*(b*x + a)*\cos(2*b*x + 2*a) \\ & *\sin(b*x + a) + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1) \\ & *\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - (\cos(2*b*x + 2*a)^2 \\ & + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 \\ & - 2*\cos(b*x + a) + 1) + 4*(b*x + a)*\sin(b*x + a))*c^3*d/((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 \\ & - 2*\cos(2*b*x + 2*a) + 1)*b) - 6*(4*(b*x + a)*\cos(b*x + a)*\sin(2*b*x + 2*a) - 4*(b*x + a)*\cos(2*b*x + 2*a) \\ & *\sin(b*x + a) + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1) \\ & *\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - (\cos(2*b*x + 2*a)^2 \\ & + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 \\ & - 2*\cos(b*x + a) + 1) + 4*(b*x + a)*\sin(b*x + a))*a*c^2*d^2/((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 \\ & - 2*\cos(2*b*x + 2*a) + 1)*b^2) + 6*(4*(b*x + a)*\cos(b*x + a)*\sin(2*b*x + 2*a) - 4*(b*x + a)*\cos(2*b*x + 2*a) \\ & *\sin(b*x + a) + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1) \\ & *\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 \\ & - 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(2*b*x + 2*a) + 1) \\ & *\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(2*b*x + 2*a) + 1) + 4*(b*x + a)*\sin(b*x + a) \\ &) * a^2 * c * d^3 / ((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1) * b^3) \\ & - 2*(4*(b*x + a)*\cos(b*x + a)*\sin(2*b*x + 2*a) - 4*(b*x + a)*\cos(2*b*x + 2*a) * \sin(b*x + a) \\ & + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1) * \log(\cos(b*x + a)^2 + \sin(b*x + a)^2 \\ & + 2*\cos(b*x + a) + 1) - (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1) * \ln \end{aligned}$$


```

og(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(
b*x + a))*a^3*d^4/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x +
2*a) + 1)*b^4) + c^4/sin(b*x + a) - 4*a*c^3*d/(b*sin(b*x + a)) + 6*a^2*c^2
*d^2/(b^2*sin(b*x + a)) - 4*a^3*c*d^3/(b^3*sin(b*x + a)) + a^4*d^4/(b^4*sin
(b*x + a)) - ((4*(b*x + a)^3*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 12*(b
^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) - 4*((b*x + a)^3*d^4 + 3*(b*c
*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x +
a))*cos(2*b*x + 2*a) - (4*I*(b*x + a)^3*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(
b*x + a)^2 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4)*(b*x + a))*
sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (4*(b*x + a)^3*
d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^
2*d^4)*(b*x + a) - 4*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3
*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (4*I*(
b*x + a)^3*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^2 + (12*I*b^2*c^2*d^
2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin
(b*x + a), -cos(b*x + a) + 1) - 2*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b
*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2)*cos(b*x +
a) - (12*b^2*c^2*d^2 - 24*a*b*c*d^3 + 12*(b*x + a)^2*d^4 + 12*a^2*d^4 + 24*
(b*c*d^3 - a*d^4)*(b*x + a) - 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^
4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (-12*I*b^2
*c^2*d^2 + 24*I*a*b*c*d^3 - 12*I*(b*x + a)^2*d^4 - 12*I*a^2*d^4 + (-24*I*b*
c*d^3 + 24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) +
(12*b^2*c^2*d^2 - 24*a*b*c*d^3 + 12*(b*x + a)^2*d^4 + 12*a^2*d^4 + 24*(b*c*
d^3 - a*d^4)*(b*x + a) - 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 +
a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (12*I*b^2*c^2*d
^2 - 24*I*a*b*c*d^3 + 12*I*(b*x + a)^2*d^4 + 12*I*a^2*d^4 + (24*I*b*c*d^3 -
24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (2*I*(b*
x + a)^3*d^4 + (6*I*b*c*d^3 - 6*I*a*d^4)*(b*x + a)^2 + (6*I*b^2*c^2*d^2 - 1
2*I*a*b*c*d^3 + 6*I*a^2*d^4)*(b*x + a) + (-2*I*(b*x + a)^3*d^4 + (-6*I*b*c*
d^3 + 6*I*a*d^4)*(b*x + a)^2 + (-6*I*b^2*c^2*d^2 + 12*I*a*b*c*d^3 - 6*I*a^2
*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 2*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^
4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2*b*
x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-2*I
*(b*x + a)^3*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x + a)^2 + (-6*I*b^2*c^2*d
^2 + 12*I*a*b*c*d^3 - 6*I*a^2*d^4)*(b*x + a) + (2*I*(b*x + a)^3*d^4 + (6*I*
b*c*d^3 - 6*I*a*d^4)*(b*x + a)^2 + (6*I*b^2*c^2*d^2 - 12*I*a*b*c*d^3 + 6*I*
a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) - 2*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*
d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2
*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 24
*(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a) - d^4)*polylog(4, -e^(I*b*x
+ I*a)) + 24*(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a) - d^4)*polylog
(4, e^(I*b*x + I*a)) - (24*I*b*c*d^3 + 24*I*(b*x + a)*d^4 - 24*I*a*d^4 + (-
24*I*b*c*d^3 - 24*I*(b*x + a)*d^4 + 24*I*a*d^4)*cos(2*b*x + 2*a) + 24*(b*c*
d^3 + (b*x + a)*d^4 - a*d^4)*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x + I*a))
- (-24*I*b*c*d^3 - 24*I*(b*x + a)*d^4 + 24*I*a*d^4 + (24*I*b*c*d^3 + 24*I*
(b*x + a)*d^4 - 24*I*a*d^4)*cos(2*b*x + 2*a) - 24*(b*c*d^3 + (b*x + a)*d^4
- a*d^4)*sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) - (2*I*(b*x + a)^4*d
^4 + (8*I*b*c*d^3 - 8*I*a*d^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c
*d^3 + 12*I*a^2*d^4)*(b*x + a)^2)*sin(b*x + a))/(-I*b^4*cos(2*b*x + 2*a) +
b^4*sin(2*b*x + 2*a) + I*b^4))/b

```

Fricas [C] time = 0.699808, size = 2569, normalized size = 12.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")

```
[Out] -(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*I*d^4*polylog(4, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I*d^4*polylog(4, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - 12*I*d^4*polylog(4, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I*d^4*polylog(4, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - (-6*I*b^2*d^4*x^2 - 12*I*b^2*c*d^3*x - 6*I*b^2*c^2*d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*b^2*d^4*x^2 + 12*I*b^2*c*d^3*x + 6*I*b^2*c^2*d^2)*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - (-6*I*b^2*d^4*x^2 - 12*I*b^2*c*d^3*x - 6*I*b^2*c^2*d^2)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*b^2*d^4*x^2 + 12*I*b^2*c*d^3*x + 6*I*b^2*c^2*d^2)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*log(cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 12*(b*d^4*x + b*c*d^3)*polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 12*(b*d^4*x + b*c*d^3)*polylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 12*(b*d^4*x + b*c*d^3)*polylog(3, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*(b*d^4*x + b*c*d^3)*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a))/(b^5*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*csc(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^4 \cos(bx + a) \csc(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)*csc(b*x + a)^2, x)
```

3.40 $\int (c + dx)^3 \cot(a + bx) \csc(a + bx) dx$

Optimal. Leaf size=146

$$\frac{6id^2(c + dx)\text{PolyLog}(2, -e^{i(a+bx)})}{b^3} - \frac{6id^2(c + dx)\text{PolyLog}(2, e^{i(a+bx)})}{b^3} - \frac{6d^3\text{PolyLog}(3, -e^{i(a+bx)})}{b^4} + \frac{6d^3\text{PolyLog}(3, e^{i(a+bx)})}{b^4}$$

[Out] $(-6*d*(c + d*x)^2*\text{ArcTanh}[E^{(I*(a + b*x))}])/b^2 - ((c + d*x)^3*\text{Csc}[a + b*x])/b + ((6*I)*d^2*(c + d*x)*\text{PolyLog}[2, -E^{(I*(a + b*x))}])/b^3 - ((6*I)*d^2*(c + d*x)*\text{PolyLog}[2, E^{(I*(a + b*x))}])/b^3 - (6*d^3*\text{PolyLog}[3, -E^{(I*(a + b*x))}])/b^4 + (6*d^3*\text{PolyLog}[3, E^{(I*(a + b*x))}])/b^4$

Rubi [A] time = 0.115982, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {4410, 4183, 2531, 2282, 6589}

$$\frac{6id^2(c + dx)\text{PolyLog}(2, -e^{i(a+bx)})}{b^3} - \frac{6id^2(c + dx)\text{PolyLog}(2, e^{i(a+bx)})}{b^3} - \frac{6d^3\text{PolyLog}(3, -e^{i(a+bx)})}{b^4} + \frac{6d^3\text{PolyLog}(3, e^{i(a+bx)})}{b^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^3*\text{Cot}[a + b*x]*\text{Csc}[a + b*x], x]$

[Out] $(-6*d*(c + d*x)^2*\text{ArcTanh}[E^{(I*(a + b*x))}])/b^2 - ((c + d*x)^3*\text{Csc}[a + b*x])/b + ((6*I)*d^2*(c + d*x)*\text{PolyLog}[2, -E^{(I*(a + b*x))}])/b^3 - ((6*I)*d^2*(c + d*x)*\text{PolyLog}[2, E^{(I*(a + b*x))}])/b^3 - (6*d^3*\text{PolyLog}[3, -E^{(I*(a + b*x))}])/b^4 + (6*d^3*\text{PolyLog}[3, E^{(I*(a + b*x))}])/b^4$

Rule 4410

$\text{Int}[\text{Cot}[(a_.) + (b_.)*(x_.)]^{(p_.)*\text{Csc}[(a_.) + (b_.)*(x_.)]^{(n_.)*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] := -\text{Simp}[(c + d*x)^m*\text{Csc}[a + b*x]^n/(b*n), x] + \text{Dist}[(d*m)/(b*n), \text{Int}[(c + d*x)^{(m-1)}*\text{Csc}[a + b*x]^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{EqQ}[p, 1] \&\& \text{GtQ}[m, 0]$

Rule 4183

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] := \text{Simp}[(-2*(c + d*x)^m*\text{ArcTanh}[E^{(I*(e + f*x))}])/f, x] + (-\text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 - E^{(I*(e + f*x))}], x], x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + E^{(I*(e + f*x))}], x], x]) /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0]$

Rule 2531

$\text{Int}[\text{Log}[1 + (e_.)*((F_)^{((c_.)*((a_.) + (b_.)*(x_.)))^{(n_.)}})]*((f_.) + (g_.)*(x_.))^{(m_.)}, x_Symbol] := -\text{Simp}[(f + g*x)^m*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x))))^n)]/(b*c*n*\text{Log}[F]), x] + \text{Dist}[(g*m)/(b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^{(m-1)}*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x))))^n)], x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0]$

Rule 2282

$\text{Int}[u_, x_Symbol] := \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \&\& !\text{MatchQ}[u, (w_)*((a_.)*(v_)^{(n_.)})^{(m_.)} /; \text{FreeQ}\{a, m, n\}, x] \&\& \text{IntegerQ}[m*n] \&\& !\text{MatchQ}[u, E^{((c_.)*((a_.) + (b_.)*x))}]$

(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned} \int (c + dx)^3 \cot(a + bx) \csc(a + bx) dx &= -\frac{(c + dx)^3 \csc(a + bx)}{b} + \frac{(3d) \int (c + dx)^2 \csc(a + bx) dx}{b} \\ &= -\frac{6d(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^3 \csc(a + bx)}{b} - \frac{(6d^2) \int (c + dx) \log(1 - E^{i(a+bx)})}{b^2} \\ &= -\frac{6d(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^3 \csc(a + bx)}{b} + \frac{6id^2(c + dx)\text{Li}_2(-e^{i(a+bx)})}{b^3} \\ &= -\frac{6d(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^3 \csc(a + bx)}{b} + \frac{6id^2(c + dx)\text{Li}_2(-e^{i(a+bx)})}{b^3} \\ &= -\frac{6d(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^3 \csc(a + bx)}{b} + \frac{6id^2(c + dx)\text{Li}_2(-e^{i(a+bx)})}{b^3} \end{aligned}$$

Mathematica [B] time = 1.18697, size = 311, normalized size = 2.13

$$\frac{-6ibd^2(c + dx)\text{PolyLog}(2, -e^{i(a+bx)}) + 6ibd^2(c + dx)\text{PolyLog}(2, e^{i(a+bx)}) + 6d^3\text{PolyLog}(3, -e^{i(a+bx)}) - 6d^3\text{PolyLog}(3, e^{i(a+bx)})}{b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x], x]

[Out] -((b^3*c^3*Csc[a + b*x] + 3*b^3*c^2*d*x*Csc[a + b*x] + 3*b^3*c*d^2*x^2*Csc[a + b*x] + b^3*d^3*x^3*Csc[a + b*x] - 3*b^2*c^2*d*Log[1 - E^(I*(a + b*x))] - 6*b^2*c*d^2*x*Log[1 - E^(I*(a + b*x))] - 3*b^2*d^3*x^2*Log[1 - E^(I*(a + b*x))] + 3*b^2*c^2*d*Log[1 + E^(I*(a + b*x))] + 6*b^2*c*d^2*x*Log[1 + E^(I*(a + b*x))] + 3*b^2*d^3*x^2*Log[1 + E^(I*(a + b*x))] - (6*I)*b*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] + (6*I)*b*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] + 6*d^3*PolyLog[3, -E^(I*(a + b*x))] - 6*d^3*PolyLog[3, E^(I*(a + b*x))])/b^4)

Maple [B] time = 0.256, size = 433, normalized size = 3.

$$\frac{-6id^3\text{polylog}(2, e^{i(bx+a)})x}{b^3} - 6\frac{d^3a^2\text{Artanh}(e^{i(bx+a)})}{b^4} - 6\frac{d^2c\ln(e^{i(bx+a)} + 1)x}{b^2} - 6\frac{d^2c\ln(e^{i(bx+a)} + 1)a}{b^3} - 6\frac{c^2d\text{Artanh}(e^{i(bx+a)})}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x)

[Out] -6*I*d^3/b^3*polylog(2, exp(I*(b*x+a)))*x-6*d^3/b^4*a^2*arctanh(exp(I*(b*x+a)))-6*d^2/b^2*c*ln(exp(I*(b*x+a))+1)*x-6*d^2/b^3*c*ln(exp(I*(b*x+a))+1)*a-6*d/b^2*c^2*arctanh(exp(I*(b*x+a)))+6*d^2/b^2*c*ln(1-exp(I*(b*x+a)))*x+6*d^2

$$\begin{aligned} & /b^3*c*\ln(1-\exp(I*(b*x+a)))*a-3*d^3/b^4*\ln(1-\exp(I*(b*x+a)))*a^2+6*I*d^2/b^4 \\ & 3*c*polylog(2,-\exp(I*(b*x+a)))-3*d^3/b^2*\ln(\exp(I*(b*x+a))+1)*x^2+3*d^3/b^4 \\ & *\ln(\exp(I*(b*x+a))+1)*a^2+12*d^2/b^3*c*a*\operatorname{arctanh}(\exp(I*(b*x+a)))+6*I*d^3/b^4 \\ & 3*polylog(2,-\exp(I*(b*x+a)))*x+3*d^3/b^2*\ln(1-\exp(I*(b*x+a)))*x^2+6*d^3*pol \\ & ylog(3,\exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,-\exp(I*(b*x+a)))/b^4-6*I*d^2/b^3 \\ & *c*polylog(2,\exp(I*(b*x+a)))-2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*\exp(I* \\ & (b*x+a))/b/(\exp(2*I*(b*x+a))-1) \end{aligned}$$

Maxima [B] time = 1.81669, size = 2390, normalized size = 16.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/2*(3*(4*(b*x + a)*\cos(b*x + a)*\sin(2*b*x + 2*a) - 4*(b*x + a)*\cos(2*b*x \\ & + 2*a)*\sin(b*x + a) + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x \\ & + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - (\\ & \cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(b \\ & *x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) + 4*(b*x + a)*\sin(b*x + a) \\ &)*c^2*d/((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1) \\ & *b) - 6*(4*(b*x + a)*\cos(b*x + a)*\sin(2*b*x + 2*a) - 4*(b*x + a)*\cos(2*b*x \\ & + 2*a)*\sin(b*x + a) + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x \\ & + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - (\\ & \cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(b \\ & *x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) + 4*(b*x + a)*\sin(b*x + a) \\ &)*a*c*d^2/((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + \\ & 1)*b^2) + 3*(4*(b*x + a)*\cos(b*x + a)*\sin(2*b*x + 2*a) - 4*(b*x + a)*\cos(2* \\ & b*x + 2*a)*\sin(b*x + a) + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(\\ & 2*b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) \\ & - (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\log(\cos \\ & (b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) + 4*(b*x + a)*\sin(b*x \\ & + a))*a^2*d^3/((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) \\ &) + 1)*b^3) + 2*c^3/\sin(b*x + a) - 6*a*c^2*d/(b*\sin(b*x + a)) + 6*a^2*c*d^2 \\ & /(b^2*\sin(b*x + a)) - 2*a^3*d^3/(b^3*\sin(b*x + a)) - 2*((6*(b*x + a)^2*d^3 \\ & + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3) \\ & *(b*x + a))*\cos(2*b*x + 2*a) - (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I* \\ & a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{arctan2}(\sin(b*x + a), \cos(b*x + a) + 1) \\ & + (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3 \\ & + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - (6*I*(b*x + a)^2*d^3 + \\ & (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{arctan2}(\sin(b*x + \\ & a), -\cos(b*x + a) + 1) - 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a) \\ &)^2*\cos(b*x + a) - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 - 12*(b*c*d^2 \\ & + (b*x + a)*d^3 - a*d^3)*\cos(2*b*x + 2*a) + (-12*I*b*c*d^2 - 12*I*(b*x + a) \\ &)*d^3 + 12*I*a*d^3)*\sin(2*b*x + 2*a))*\operatorname{dilog}(-e^{I*b*x + I*a}) + (12*b*c*d^2 \\ & + 12*(b*x + a)*d^3 - 12*a*d^3 - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\cos(2 \\ & *b*x + 2*a) - (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*\sin(2*b*x + \\ & 2*a))*\operatorname{dilog}(e^{I*b*x + I*a}) - (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a \\ & d^3)*(b*x + a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + \\ & a))*\cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))* \\ & \sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) \\ & - (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a) + (3*I*(b*x \\ & + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - 3*((b \\ & *x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b \\ & x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) - (-12*I*d^3*\cos(2*b*x + 2* \\ & a) + 12*d^3*\sin(2*b*x + 2*a) + 12*I*d^3)*\operatorname{polylog}(3, -e^{I*b*x + I*a}) - (12 \end{aligned}$$

```
*I*d^3*cos(2*b*x + 2*a) - 12*d^3*sin(2*b*x + 2*a) - 12*I*d^3)*polylog(3, e^
(I*b*x + I*a)) - (4*I*(b*x + a)^3*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x +
a)^2)*sin(b*x + a))/(-2*I*b^3*cos(2*b*x + 2*a) + 2*b^3*sin(2*b*x + 2*a) + 2
*I*b^3))/b
```

Fricas [C] time = 0.605109, size = 1740, normalized size = 11.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 6*d^3*p
olylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 6*d^3*polylog(3, co
s(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a)
+ I*sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a) - I*sin(b*x
+ a))*sin(b*x + a) - (-6*I*b*d^3*x - 6*I*b*c*d^2)*dilog(cos(b*x + a) + I*s
in(b*x + a))*sin(b*x + a) - (6*I*b*d^3*x + 6*I*b*c*d^2)*dilog(cos(b*x + a)
- I*sin(b*x + a))*sin(b*x + a) - (-6*I*b*d^3*x - 6*I*b*c*d^2)*dilog(-cos(b*
x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*b*d^3*x + 6*I*b*c*d^2)*dilog(-
cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x
+ b^2*c^2*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2
*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) - I*sin(b*x + a) + 1
)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x + a
) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 3*
(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) + I
*sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*
d^2 - a^2*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a))/(b^4*s
in(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*csc(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \cos(bx + a) \csc(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cos(b*x + a)*csc(b*x + a)^2, x)
```

3.41 $\int (c + dx)^2 \cot(a + bx) \csc(a + bx) dx$

Optimal. Leaf size=90

$$\frac{2id^2 \text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} - \frac{2id^2 \text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3} - \frac{4d(c + dx) \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c + dx)^2 \csc(a + bx)}{b}$$

[Out] $(-4*d*(c + d*x)*\text{ArcTanh}[E^{(I*(a + b*x))}])/b^2 - ((c + d*x)^2*\text{Csc}[a + b*x])/b + ((2*I)*d^2*\text{PolyLog}[2, -E^{(I*(a + b*x))}])/b^3 - ((2*I)*d^2*\text{PolyLog}[2, E^{(I*(a + b*x))}])/b^3$

Rubi [A] time = 0.0622306, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {4410, 4183, 2279, 2391}

$$\frac{2id^2 \text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} - \frac{2id^2 \text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3} - \frac{4d(c + dx) \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c + dx)^2 \csc(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^2*\text{Cot}[a + b*x]*\text{Csc}[a + b*x], x]$

[Out] $(-4*d*(c + d*x)*\text{ArcTanh}[E^{(I*(a + b*x))}])/b^2 - ((c + d*x)^2*\text{Csc}[a + b*x])/b + ((2*I)*d^2*\text{PolyLog}[2, -E^{(I*(a + b*x))}])/b^3 - ((2*I)*d^2*\text{PolyLog}[2, E^{(I*(a + b*x))}])/b^3$

Rule 4410

$\text{Int}[\text{Cot}[(a_.) + (b_.)*(x_.)]^{(p_.)}*\text{Csc}[(a_.) + (b_.)*(x_.)]^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow -\text{Simp}[\frac{(c + d*x)^m*\text{Csc}[a + b*x]^n}{(b*n)}, x] + \text{Dist}[\frac{(d*m)}{(b*n)}, \text{Int}[(c + d*x)^{(m-1)}*\text{Csc}[a + b*x]^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{EqQ}[p, 1] \&\& \text{GtQ}[m, 0]$

Rule 4183

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[\frac{-2*(c + d*x)^m*\text{ArcTanh}[E^{(I*(e + f*x))}]]{f}, x] + (-\text{Dist}[\frac{(d*m)}{f}, \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 - E^{(I*(e + f*x))}], x], x] + \text{Dist}[\frac{(d*m)}{f}, \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + E^{(I*(e + f*x))}], x], x]) /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0]$

Rule 2279

$\text{Int}[\text{Log}[(a_.) + (b_.)*((F_)^{((e_.)*((c_.) + (d_.)*(x_.)))})^{(n_.)}], x_Symbol] \rightarrow \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x] \&\& \text{GtQ}[a, 0]$

Rule 2391

$\text{Int}[\text{Log}[(c_.)*((d_.) + (e_.)*(x_.))^{(n_.)}]/(x_.), x_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \&\& \text{EqQ}[c*d, 1]$

Rubi steps

$$\begin{aligned}
\int (c + dx)^2 \cot(a + bx) \csc(a + bx) dx &= -\frac{(c + dx)^2 \csc(a + bx)}{b} + \frac{(2d) \int (c + dx) \csc(a + bx) dx}{b} \\
&= -\frac{4d(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^2 \csc(a + bx)}{b} - \frac{(2d^2) \int \log(1 - e^{i(a+bx)})}{b^2} \\
&= -\frac{4d(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^2 \csc(a + bx)}{b} + \frac{(2id^2) \operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\right)}{b^3} \\
&= -\frac{4d(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^2 \csc(a + bx)}{b} + \frac{2id^2 \operatorname{Li}_2(-e^{i(a+bx)})}{b^3} - \frac{2id^2}{b^3}
\end{aligned}$$

Mathematica [B] time = 2.0513, size = 234, normalized size = 2.6

$$4d^2 \left(2 \tan^{-1}(\tan(a)) \tanh^{-1}\left(\cos(a) - \sin(a) \tan\left(\frac{bx}{2}\right)\right) + \frac{\sec(a) \left(i \operatorname{PolyLog}\left(2, -e^{i(\tan^{-1}(\tan(a))+bx)}\right) - i \operatorname{PolyLog}\left(2, e^{i(\tan^{-1}(\tan(a))+bx)}\right) \right) + (\tan(a) \dots)}{\sqrt{\sec^2(a)}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x], x]

[Out] (-8*b*c*d*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] - 2*b^2*(c + d*x)^2*Csc[a] + 4*d^2*(2*ArcTan[Tan[a]]*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] + (((b*x + ArcTan[Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[Tan[a]])])) - Log[1 + E^(I*(b*x + ArcTan[Tan[a]])])) + I*PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]])]) - I*PolyLog[2, E^(I*(b*x + ArcTan[Tan[a]])])]*Sec[a])/Sqrt[Sec[a]^2] + b^2*(c + d*x)^2*Csc[a/2]*Csc[(a + b*x)/2]*Sin[(b*x)/2] - b^2*(c + d*x)^2*Sec[a/2]*Sec[(a + b*x)/2]*Sin[(b*x)/2])/(2*b^3)

Maple [B] time = 0.205, size = 212, normalized size = 2.4

$$\frac{-2i(d^2x^2 + 2cdx + c^2)e^{i(bx+a)}}{b(e^{2i(bx+a)} - 1)} - 4 \frac{cd \operatorname{Arctanh}(e^{i(bx+a)})}{b^2} + 2 \frac{d^2 \ln(1 - e^{i(bx+a)})x}{b^2} + 2 \frac{d^2 \ln(1 - e^{i(bx+a)})a}{b^3} - \frac{2id^2 \operatorname{polylog}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2, x)

[Out] -2*I*(d^2*x^2+2*c*d*x+c^2)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)-4*d/b^2*c*arctanh(exp(I*(b*x+a)))+2*d^2/b^2*ln(1-exp(I*(b*x+a)))*x+2*d^2/b^3*ln(1-exp(I*(b*x+a)))*a-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^3-2*d^2/b^2*ln(exp(I*(b*x+a))+1)*x-2*d^2/b^3*ln(exp(I*(b*x+a))+1)*a+2*I*d^2*polylog(2,-exp(I*(b*x+a)))/b^3+4*d^2/b^3*a*arctanh(exp(I*(b*x+a)))

Maxima [B] time = 1.63384, size = 751, normalized size = 8.34

$$(2bd^2x + 2bcd - 2(bd^2x + bcd) \cos(2bx + 2a) - (2ibd^2x + 2ibcd) \sin(2bx + 2a)) \arctan(\sin(bx + a), \cos(bx + a)) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")

[Out] ((2*b*d^2*x + 2*b*c*d - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a) - (2*I*b*d^2*x + 2*I*b*c*d)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (2*b*c*d*cos(2*b*x + 2*a) + 2*I*b*c*d*sin(2*b*x + 2*a) - 2*b*c*d)*arctan2(sin(b*x + a), cos(b*x + a) - 1) - (2*b*d^2*x*cos(2*b*x + 2*a) + 2*I*b*d^2*x*sin(2*b*x + 2*a) - 2*b*d^2*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a) + 2*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(-e^(I*b*x + I*a)) - 2*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(e^(I*b*x + I*a)) - (I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(2*b*x + 2*a) + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(2*b*x + 2*a) - (b*d^2*x + b*c*d)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (2*I*b^2*d^2*x^2 + 4*I*b^2*c*d*x + 2*I*b^2*c^2)*sin(b*x + a))/(-I*b^3*cos(2*b*x + 2*a) + b^3*sin(2*b*x + 2*a) + I*b^3)

Fricas [B] time = 0.558857, size = 1006, normalized size = 11.18

$$b^2 d^2 x^2 + 2 b^2 c d x + b^2 c^2 + i d^2 \operatorname{Li}_2(\cos(bx + a) + i \sin(bx + a)) \sin(bx + a) - i d^2 \operatorname{Li}_2(\cos(bx + a) - i \sin(bx + a)) \sin(bx + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")

[Out] -(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - I*d^2*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + I*d^2*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - I*d^2*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - (b*c*d - a*d^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - (b*c*d - a*d^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - (b*d^2*x + a*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - (b*d^2*x + a*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a))/(b^3*sin(b*x + a))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*cos(b*x+a)*csc(b*x+a)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \cos(bx + a) \csc(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cos(b*x + a)*csc(b*x + a)^2, x)
```

3.42 $\int (c + dx) \cot(a + bx) \csc(a + bx) dx$

Optimal. Leaf size=30

$$-\frac{d \tanh^{-1}(\cos(a + bx))}{b^2} - \frac{(c + dx) \csc(a + bx)}{b}$$

[Out] $-\left(\frac{d \operatorname{ArcTanh}[\cos[a + b x]]}{b^2}\right) - \left(\frac{(c + d x) \operatorname{Csc}[a + b x]}{b}\right)$

Rubi [A] time = 0.0195758, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4410, 3770}

$$-\frac{d \tanh^{-1}(\cos(a + bx))}{b^2} - \frac{(c + dx) \csc(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Cot[a + b*x]*Csc[a + b*x], x]

[Out] $-\left(\frac{d \operatorname{ArcTanh}[\cos[a + b x]]}{b^2}\right) - \left(\frac{(c + d x) \operatorname{Csc}[a + b x]}{b}\right)$

Rule 4410

Int[Cot[(a_.) + (b_.)*(x_.)]^(p_.)*Csc[(a_.) + (b_.)*(x_.)]^(n_.)*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (c + dx) \cot(a + bx) \csc(a + bx) dx &= -\frac{(c + dx) \csc(a + bx)}{b} + \frac{d \int \csc(a + bx) dx}{b} \\ &= -\frac{d \tanh^{-1}(\cos(a + bx))}{b^2} - \frac{(c + dx) \csc(a + bx)}{b} \end{aligned}$$

Mathematica [B] time = 0.0584151, size = 131, normalized size = 4.37

$$\frac{d \log\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{b^2} - \frac{d \log\left(\cos\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{b^2} - \frac{c \csc(a + bx)}{b} - \frac{dx \csc(a)}{b} + \frac{dx \csc\left(\frac{a}{2}\right) \sin\left(\frac{bx}{2}\right) \csc\left(\frac{a}{2} + \frac{bx}{2}\right)}{2b} - \frac{dx \sec\left(\frac{a}{2} + \frac{bx}{2}\right)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Cot[a + b*x]*Csc[a + b*x], x]

[Out] $-\left(\frac{d x \operatorname{Csc}[a]}{b}\right) - \left(\frac{c \operatorname{Csc}[a + b x]}{b}\right) - \left(\frac{d \operatorname{Log}[\cos[a/2 + (b x)/2]]}{b^2}\right) + \left(\frac{d \operatorname{Log}[\sin[a/2 + (b x)/2]]}{b^2}\right) + \left(\frac{d x \operatorname{Csc}[a/2] \operatorname{Csc}[a/2 + (b x)/2] \operatorname{Sin}[b x/2]}{2 b}\right) - \left(\frac{d x \operatorname{Sec}[a/2 + (b x)/2]}{2 b}\right)$

)/2)]/(2*b) - (d*x*Sec[a/2]*Sec[a/2 + (b*x)/2]*Sin[(b*x)/2)]/(2*b)

Maple [A] time = 0.024, size = 52, normalized size = 1.7

$$-\frac{dx}{b \sin(bx+a)} + \frac{d \ln(\csc(bx+a) - \cot(bx+a))}{b^2} - \frac{c}{b \sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x)

[Out] -1/b*d/sin(b*x+a)*x+1/b^2*d*ln(csc(b*x+a)-cot(b*x+a))-1/b*c/sin(b*x+a)

Maxima [B] time = 1.09171, size = 350, normalized size = 11.67

$$\frac{(4(bx+a)\cos(bx+a)\sin(2bx+2a)-4(bx+a)\cos(2bx+2a)\sin(bx+a)+(\cos(2bx+2a)^2+\sin(2bx+2a)^2-2\cos(2bx+2a)+1)\log(\cos(bx+a)^2+\sin(bx+a)^2+2\cos(2bx+2a)^2+\sin(2bx+2a)^2-2\cos(2bx+2a)+1))}{(\cos(2bx+2a)^2+\sin(2bx+2a)^2-2\cos(2bx+2a)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")

[Out] -1/2*((4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a))*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b) + 2*c/sin(b*x + a) - 2*a*d/(b*sin(b*x + a))/b

Fricas [B] time = 0.488275, size = 181, normalized size = 6.03

$$\frac{2 b d x + d \log \left(\frac{1}{2} \cos (b x + a) + \frac{1}{2} \right) \sin (b x + a) - d \log \left(-\frac{1}{2} \cos (b x + a) + \frac{1}{2} \right) \sin (b x + a) + 2 b c}{2 b^2 \sin (b x + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")

[Out] -1/2*(2*b*d*x + d*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - d*log(-1/2*cos(b*x + a) + 1/2)*sin(b*x + a) + 2*b*c)/(b^2*sin(b*x + a))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \cos(a + bx) \csc^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)**2,x)

[Out] Integral((c + d*x)*cos(a + b*x)*csc(a + b*x)**2, x)

Giac [B] time = 1.52748, size = 1081, normalized size = 36.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")

[Out] $\frac{1}{2}*(b*d*x*\tan(1/2*b*x)^2*\tan(1/2*a)^2 + b*c*\tan(1/2*b*x)^2*\tan(1/2*a)^2 + b*d*x*\tan(1/2*b*x)^2 - d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 - 2*\tan(1/2*b*x)*\tan(1/2*a) + 1))*\tan(1/2*b*x)^2*\tan(1/2*a) + d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 + 2*\tan(1/2*b*x)*\tan(1/2*a) + \tan(1/2*a)^2))*\tan(1/2*b*x)^2*\tan(1/2*a) + b*d*x*\tan(1/2*a)^2 - d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 - 2*\tan(1/2*b*x)*\tan(1/2*a) + 1))*\tan(1/2*b*x)*\tan(1/2*a)^2 + d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 + 2*\tan(1/2*b*x)*\tan(1/2*a) + \tan(1/2*a)^2))*\tan(1/2*b*x)*\tan(1/2*a)^2 + b*c*\tan(1/2*b*x)^2 + b*c*\tan(1/2*a)^2 + b*d*x + d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 - 2*\tan(1/2*b*x)*\tan(1/2*a) + 1))*\tan(1/2*b*x) - d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 + 2*\tan(1/2*b*x)*\tan(1/2*a) + \tan(1/2*a)^2))*\tan(1/2*b*x) + d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 - 2*\tan(1/2*b*x)*\tan(1/2*a) + 1))*\tan(1/2*a) - d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 + 2*\tan(1/2*b*x)*\tan(1/2*a) + \tan(1/2*a)^2))*\tan(1/2*a) + b*c)/(b^2*\tan(1/2*b*x)^2*\tan(1/2*a) + b^2*\tan(1/2*b*x)*\tan(1/2*a)^2 - b^2*\tan(1/2*b*x) - b^2*\tan(1/2*a))$

$$3.43 \quad \int \frac{\cot(a+bx) \csc(a+bx)}{c+dx} dx$$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\cot(a+bx) \csc(a+bx)}{c+dx}, x\right)$$

[Out] CannotIntegrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x), x]

Rubi [A] time = 0.114518, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cot(a+bx) \csc(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x), x]

[Out] Defer[Int] [(Cot[a + b*x]*Csc[a + b*x])/(c + d*x), x]

Rubi steps

$$\int \frac{\cot(a+bx) \csc(a+bx)}{c+dx} dx = \int \frac{\cot(a+bx) \csc(a+bx)}{c+dx} dx$$

Mathematica [A] time = 16.7603, size = 0, normalized size = 0.

$$\int \frac{\cot(a+bx) \csc(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x), x]

[Out] Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x), x]

Maple [A] time = 0.295, size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a) (\csc(bx+a))^2}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c), x)

[Out] int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c), x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(bx+a)\csc(bx+a)^2}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x, algorithm="fricas")

[Out] integral(cos(b*x + a)*csc(b*x + a)^2/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)\csc^2(a+bx)}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)**2/(d*x+c),x)

[Out] Integral(cos(a + b*x)*csc(a + b*x)**2/(c + d*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)\csc(bx+a)^2}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x, algorithm="giac")

[Out] integrate(cos(b*x + a)*csc(b*x + a)^2/(d*x + c), x)

$$3.44 \quad \int \frac{\cot(a+bx) \csc(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\cot(a+bx) \csc(a+bx)}{(c+dx)^2}, x\right)$$

[Out] CannotIntegrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x]

Rubi [A] time = 0.152228, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cot(a+bx) \csc(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x]

[Out] Defer[Int] [(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\cot(a+bx) \csc(a+bx)}{(c+dx)^2} dx = \int \frac{\cot(a+bx) \csc(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 20.204, size = 0, normalized size = 0.

$$\int \frac{\cot(a+bx) \csc(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x]

[Out] Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x]

Maple [A] time = 0.411, size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a) (\csc(bx+a))^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2, x)

[Out] int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2, x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(bx+a)\csc(bx+a)^2}{d^2x^2+2cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(cos(b*x + a)*csc(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)\csc^2(a+bx)}{(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)**2/(d*x+c)**2,x)

[Out] Integral(cos(a + b*x)*csc(a + b*x)**2/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)\csc(bx+a)^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(cos(b*x + a)*csc(b*x + a)^2/(d*x + c)^2, x)

3.45 $\int (c + dx)^m \cot(a + bx) \csc^2(a + bx) dx$

Optimal. Leaf size=24

CannotIntegrate(cot(a + bx) csc^2(a + bx)(c + dx)^m, x)

[Out] CannotIntegrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2, x]

Rubi [A] time = 0.21658, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \cot(a + bx) \csc^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2, x]

[Out] Defer[Int] [(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2, x]

Rubi steps

$$\int (c + dx)^m \cot(a + bx) \csc^2(a + bx) dx = \int (c + dx)^m \cot(a + bx) \csc^2(a + bx) dx$$

Mathematica [A] time = 3.29169, size = 0, normalized size = 0.

$$\int (c + dx)^m \cot(a + bx) \csc^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2, x]

[Out] Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2, x]

Maple [A] time = 0.162, size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) (\csc(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3, x)

[Out] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) \csc(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx + c)^m \cos(bx + a) \csc(bx + a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) \csc(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^3, x)
```

3.46 $\int (c + dx)^4 \cot(a + bx) \csc^2(a + bx) dx$

Optimal. Leaf size=137

$$-\frac{6id^3(c + dx)\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^4} + \frac{3d^4\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{b^5} + \frac{6d^2(c + dx)^2 \log\left(1 - e^{2i(a+bx)}\right)}{b^3} - \frac{2d(c + dx)^3 \cot(a + bx)}{b^2}$$

[Out] $((-2*I)*d*(c + d*x)^3)/b^2 - (2*d*(c + d*x)^3*\text{Cot}[a + b*x])/b^2 - ((c + d*x)^4*\text{Csc}[a + b*x]^2)/(2*b) + (6*d^2*(c + d*x)^2*\text{Log}[1 - E^((2*I)*(a + b*x))])/b^3 - ((6*I)*d^3*(c + d*x)*\text{PolyLog}[2, E^((2*I)*(a + b*x))])/b^4 + (3*d^4*\text{PolyLog}[3, E^((2*I)*(a + b*x))])/b^5$

Rubi [A] time = 0.256067, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {4410, 4184, 3717, 2190, 2531, 2282, 6589}

$$-\frac{6id^3(c + dx)\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^4} + \frac{3d^4\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{b^5} + \frac{6d^2(c + dx)^2 \log\left(1 - e^{2i(a+bx)}\right)}{b^3} - \frac{2d(c + dx)^3 \cot(a + bx)}{b^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^4*\text{Cot}[a + b*x]*\text{Csc}[a + b*x]^2, x]$

[Out] $((-2*I)*d*(c + d*x)^3)/b^2 - (2*d*(c + d*x)^3*\text{Cot}[a + b*x])/b^2 - ((c + d*x)^4*\text{Csc}[a + b*x]^2)/(2*b) + (6*d^2*(c + d*x)^2*\text{Log}[1 - E^((2*I)*(a + b*x))])/b^3 - ((6*I)*d^3*(c + d*x)*\text{PolyLog}[2, E^((2*I)*(a + b*x))])/b^4 + (3*d^4*\text{PolyLog}[3, E^((2*I)*(a + b*x))])/b^5$

Rule 4410

$\text{Int}[\text{Cot}[(a_.) + (b_.)*(x_.)]^{(p_.)}*\text{Csc}[(a_.) + (b_.)*(x_.)]^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Csc}[a + b*x]^n/(b*n), x] + \text{Dist}[(d*m)/(b*n), \text{Int}[(c + d*x)^{(m-1)}*\text{Csc}[a + b*x]^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{EqQ}[p, 1] \&\& \text{GtQ}[m, 0]$

Rule 4184

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]^{2*((c_.) + (d_.)*(x_.))^{(m_.)}}, x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cot}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cot}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{GtQ}[m, 0]$

Rule 3717

$\text{Int}[(c + d*x)^m*\tan[(e_.) + \text{Pi}*(k_.) + (f_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[(I*(c + d*x)^{(m+1)})/(d*(m+1)), x] - \text{Dist}[2*I, \text{Int}[(c + d*x)^m*\text{E}^{(2*I*k*Pi)}*\text{E}^{(2*I*(e + f*x))}/(1 + \text{E}^{(2*I*k*Pi)}*\text{E}^{(2*I*(e + f*x))}), x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{IntegerQ}[4*k] \&\& \text{IGtQ}[m, 0]$

Rule 2190

$\text{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_.)))^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}}/((a_.) + (b_.)*((F_)^{((g_.)*((e_.) + (f_.)*(x_.)))^{(n_.)}}), x_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x))))^n)/a]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + (b*(F^{(g*(e + f*x))))^n)/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_ /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^4 \cot(a + bx) \csc^2(a + bx) dx &= -\frac{(c + dx)^4 \csc^2(a + bx)}{2b} + \frac{(2d) \int (c + dx)^3 \csc^2(a + bx) dx}{b} \\
&= -\frac{2d(c + dx)^3 \cot(a + bx)}{b^2} - \frac{(c + dx)^4 \csc^2(a + bx)}{2b} + \frac{(6d^2) \int (c + dx)^2 \cot(a + bx) dx}{b^2} \\
&= -\frac{2id(c + dx)^3}{b^2} - \frac{2d(c + dx)^3 \cot(a + bx)}{b^2} - \frac{(c + dx)^4 \csc^2(a + bx)}{2b} - \frac{(12id^2) \int (c + dx) dx}{b^2} \\
&= -\frac{2id(c + dx)^3}{b^2} - \frac{2d(c + dx)^3 \cot(a + bx)}{b^2} - \frac{(c + dx)^4 \csc^2(a + bx)}{2b} + \frac{6d^2(c + dx)}{b^2} \\
&= -\frac{2id(c + dx)^3}{b^2} - \frac{2d(c + dx)^3 \cot(a + bx)}{b^2} - \frac{(c + dx)^4 \csc^2(a + bx)}{2b} + \frac{6d^2(c + dx)}{b^2} \\
&= -\frac{2id(c + dx)^3}{b^2} - \frac{2d(c + dx)^3 \cot(a + bx)}{b^2} - \frac{(c + dx)^4 \csc^2(a + bx)}{2b} + \frac{6d^2(c + dx)}{b^2} \\
&= -\frac{2id(c + dx)^3}{b^2} - \frac{2d(c + dx)^3 \cot(a + bx)}{b^2} - \frac{(c + dx)^4 \csc^2(a + bx)}{2b} + \frac{6d^2(c + dx)}{b^2}
\end{aligned}$$

Mathematica [B] time = 6.60207, size = 504, normalized size = 3.68

$$\frac{6cd^3 \csc(a) \sec(a) \left(\frac{\tan(a) \left(i \operatorname{PolyLog} \left(2, e^{2i(\tan^{-1}(\tan(a)+bx)} \right) + ibx(2 \tan^{-1}(\tan(a)) - \pi) - 2(\tan^{-1}(\tan(a)+bx) \log \left(1 - e^{2i(\tan^{-1}(\tan(a)+bx)} \right) + 2 \tan^{-1}(\tan(a)) \right)}{\sqrt{\tan^2(a)+1}} \right)}{b^4 \sqrt{\sec^2(a) (\sin^2(a) + \cos^2(a))}} \right)}{b^4 \sqrt{\sec^2(a) (\sin^2(a) + \cos^2(a))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -((c + d*x)^4*Csc[a + b*x]^2)/(2*b) - (d^4*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((
2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3
*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2
```

```

*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))])/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a + b*x))])/E^((2*I)*a)))/b^5 + (6*c^2*d^2*Csc[a]*(-b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a])/(b^3*(Cos[a]^2 + Sin[a]^2)) + (2*Csc[a]*Csc[a + b*x]*(c^3*d*Sin[b*x] + 3*c^2*d^2*x*Sin[b*x] + 3*c*d^3*x^2*Sin[b*x] + d^4*x^3*Sin[b*x]))/b^2 - (6*c*d^3*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x]) - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]])]) + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]]) + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]])])]*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^4*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])

```

Maple [B] time = 0.188, size = 716, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x)
```

```

[Out] 6*d^4/b^5*a^2*ln(exp(I*(b*x+a))-1)-12*d^4/b^5*a^2*ln(exp(I*(b*x+a)))-12*d^2/b^3*c^2*ln(exp(I*(b*x+a)))+6*d^2/b^3*c^2*ln(exp(I*(b*x+a))+1)+6*d^2/b^3*c^2*ln(exp(I*(b*x+a))-1)+6*d^4/b^3*ln(1-exp(I*(b*x+a)))*x^2-6*d^4/b^5*ln(1-exp(I*(b*x+a)))*a^2+6*d^4/b^3*ln(exp(I*(b*x+a))+1)*x^2-4*I*d^4/b^2*x^3+8*I*d^4/b^5*a^3+12*d^4*polylog(3,-exp(I*(b*x+a)))/b^5+12*d^4*polylog(3,exp(I*(b*x+a)))/b^5+2*(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^3*exp(2*I*(b*x+a))+6*b*c^2*d^2*x^2*exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*x+a))-2*I*d^4*x^3*exp(2*I*(b*x+a))+b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp(2*I*(b*x+a))-6*I*c^2*d^2*x*exp(2*I*(b*x+a))+2*I*d^4*x^3-2*I*c^3*d*exp(2*I*(b*x+a))+6*I*c*d^3*x^2+6*I*c^2*d^2*x+2*I*c^3*d)/b^2/(exp(2*I*(b*x+a))-1)^2+12*d^3/b^3*c*ln(1-exp(I*(b*x+a)))*x+12*d^3/b^4*c*ln(1-exp(I*(b*x+a)))*a+12*d^3/b^3*c*ln(exp(I*(b*x+a))+1)*x-12*I*d^4/b^4*polylog(2,exp(I*(b*x+a)))*x-12*I*d^4/b^4*polylog(2,-exp(I*(b*x+a)))*x-12*I*d^3/b^4*c*polylog(2,exp(I*(b*x+a)))-12*I*d^3/b^4*c*polylog(2,-exp(I*(b*x+a)))+12*I*d^4/b^4*a^2*x-12*I*d^3/b^4*c*a^2-12*I*d^3/b^2*c*x^2-12*d^3/b^4*c*a*ln(exp(I*(b*x+a))-1)+24*d^3/b^4*c*a*ln(exp(I*(b*x+a)))-24*I*d^3/b^3*c*a*x

```

Maxima [B] time = 2.47741, size = 6129, normalized size = 44.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

```

[Out] 1/2*(8*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x + a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*c^3*d/((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b) - 24*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x + a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) -

```

$$\begin{aligned}
& \cos(2bx + 2a) + 1) \sin(4bx + 4a) + \sin(2bx + 2a) * a^2 c^2 d^2 / ((2 * \\
& (2 * \cos(2bx + 2a) - 1) * \cos(4bx + 4a) - \cos(4bx + 4a)^2 - 4 * \cos(2bx \\
& + 2a)^2 - \sin(4bx + 4a)^2 + 4 * \sin(4bx + 4a) * \sin(2bx + 2a) - 4 * \sin \\
& (2bx + 2a)^2 + 4 * \cos(2bx + 2a) - 1) * b^2) + 24 * (4 * (bx + a) * \cos(2bx \\
& + 2a)^2 + 4 * (bx + a) * \sin(2bx + 2a)^2 - (2 * (bx + a) * \cos(2bx + 2a) \\
& + \sin(2bx + 2a)) * \cos(4bx + 4a) - 2 * (bx + a) * \cos(2bx + 2a) - (2 * (b \\
& * x + a) * \sin(2bx + 2a) - \cos(2bx + 2a) + 1) * \sin(4bx + 4a) + \sin(2b \\
& * x + 2a)) * a^2 * c * d^3 / ((2 * (2 * \cos(2bx + 2a) - 1) * \cos(4bx + 4a) - \cos(4 * \\
& bx + 4a)^2 - 4 * \cos(2bx + 2a)^2 - \sin(4bx + 4a)^2 + 4 * \sin(4bx + 4 * \\
& a) * \sin(2bx + 2a) - 4 * \sin(2bx + 2a)^2 + 4 * \cos(2bx + 2a) - 1) * b^3) - \\
& 8 * (4 * (bx + a) * \cos(2bx + 2a)^2 + 4 * (bx + a) * \sin(2bx + 2a)^2 - (2 * (b \\
& * x + a) * \cos(2bx + 2a) + \sin(2bx + 2a)) * \cos(4bx + 4a) - 2 * (bx + a) \\
& * \cos(2bx + 2a) - (2 * (bx + a) * \sin(2bx + 2a) - \cos(2bx + 2a) + 1) * \sin \\
& (4bx + 4a) + \sin(2bx + 2a)) * a^3 * d^4 / ((2 * (2 * \cos(2bx + 2a) - 1) * \cos \\
& (4bx + 4a) - \cos(4bx + 4a)^2 - 4 * \cos(2bx + 2a)^2 - \sin(4bx + 4 * \\
& a)^2 + 4 * \sin(4bx + 4a) * \sin(2bx + 2a) - 4 * \sin(2bx + 2a)^2 + 4 * \cos(2 \\
& * bx + 2a) - 1) * b^4) + 6 * (8 * (bx + a)^2 * \cos(2bx + 2a)^2 + 8 * (bx + a)^2 \\
& * \sin(2bx + 2a)^2 - 4 * (bx + a)^2 * \cos(2bx + 2a) - 4 * ((bx + a)^2 * \cos(2 \\
& * bx + 2a) + (bx + a) * \sin(2bx + 2a)) * \cos(4bx + 4a) + (2 * (2 * \cos(2 * b * \\
& x + 2a) - 1) * \cos(4bx + 4a) - \cos(4bx + 4a)^2 - 4 * \cos(2bx + 2a)^2 \\
& - \sin(4bx + 4a)^2 + 4 * \sin(4bx + 4a) * \sin(2bx + 2a) - 4 * \sin(2bx + \\
& 2a)^2 + 4 * \cos(2bx + 2a) - 1) * \log(\cos(bx + a)^2 + \sin(bx + a)^2 + 2 * \cos \\
& (bx + a) + 1) + (2 * (2 * \cos(2bx + 2a) - 1) * \cos(4bx + 4a) - \cos(4bx \\
& + 4a)^2 - 4 * \cos(2bx + 2a)^2 - \sin(4bx + 4a)^2 + 4 * \sin(4bx + 4a) * \sin \\
& (2bx + 2a) - 4 * \sin(2bx + 2a)^2 + 4 * \cos(2bx + 2a) - 1) * \log(\cos(bx \\
& + a)^2 + \sin(bx + a)^2 - 2 * \cos(bx + a) + 1) - 4 * ((bx + a)^2 * \sin(2bx \\
& + 2a) + bx - (bx + a) * \cos(2bx + 2a) + a) * \sin(4bx + 4a) + 4 * (bx + \\
& a) * \sin(2bx + 2a) * a^2 * c^2 * d^2 / ((2 * (2 * \cos(2bx + 2a) - 1) * \cos(4bx + 4a) \\
& - \cos(4bx + 4a)^2 - 4 * \cos(2bx + 2a)^2 - \sin(4bx + 4a)^2 + 4 * \sin(4 * \\
& bx + 4a) * \sin(2bx + 2a) - 4 * \sin(2bx + 2a)^2 + 4 * \cos(2bx + 2a) - 1) \\
& * b^2) - 12 * (8 * (bx + a)^2 * \cos(2bx + 2a)^2 + 8 * (bx + a)^2 * \sin(2bx + 2 \\
& * a)^2 - 4 * (bx + a)^2 * \cos(2bx + 2a) - 4 * ((bx + a)^2 * \cos(2bx + 2a) + \\
& (bx + a) * \sin(2bx + 2a)) * \cos(4bx + 4a) + (2 * (2 * \cos(2bx + 2a) - 1) * \\
& \cos(4bx + 4a) - \cos(4bx + 4a)^2 - 4 * \cos(2bx + 2a)^2 - \sin(4bx + \\
& 4a)^2 + 4 * \sin(4bx + 4a) * \sin(2bx + 2a) - 4 * \sin(2bx + 2a)^2 + 4 * \cos \\
& (2bx + 2a) - 1) * \log(\cos(bx + a)^2 + \sin(bx + a)^2 + 2 * \cos(bx + a) + 1) \\
& + (2 * (2 * \cos(2bx + 2a) - 1) * \cos(4bx + 4a) - \cos(4bx + 4a)^2 - 4 * \cos \\
& (2bx + 2a)^2 - \sin(4bx + 4a)^2 + 4 * \sin(4bx + 4a) * \sin(2bx + 2a) \\
&) - 4 * \sin(2bx + 2a)^2 + 4 * \cos(2bx + 2a) - 1) * \log(\cos(bx + a)^2 + \sin \\
& (bx + a)^2 - 2 * \cos(bx + a) + 1) - 4 * ((bx + a)^2 * \sin(2bx + 2a) + bx - \\
& (bx + a) * \cos(2bx + 2a) + a) * \sin(4bx + 4a) + 4 * (bx + a) * \sin(2bx + \\
& 2a)) * a * c * d^3 / ((2 * (2 * \cos(2bx + 2a) - 1) * \cos(4bx + 4a) - \cos(4 * bx + \\
& 4a)^2 - 4 * \cos(2bx + 2a)^2 - \sin(4bx + 4a)^2 + 4 * \sin(4bx + 4a) * \sin \\
& (2bx + 2a) - 4 * \sin(2bx + 2a)^2 + 4 * \cos(2bx + 2a) - 1) * b^3) + 6 * (8 * \\
& (bx + a)^2 * \cos(2bx + 2a)^2 + 8 * (bx + a)^2 * \sin(2bx + 2a)^2 - 4 * (bx \\
& + a)^2 * \cos(2bx + 2a) - 4 * ((bx + a)^2 * \cos(2bx + 2a) + (bx + a) * \sin(2 \\
& * bx + 2a)) * \cos(4bx + 4a) + (2 * (2 * \cos(2bx + 2a) - 1) * \cos(4bx + 4a) \\
&) - \cos(4bx + 4a)^2 - 4 * \cos(2bx + 2a)^2 - \sin(4bx + 4a)^2 + 4 * \sin(\\
& 4bx + 4a) * \sin(2bx + 2a) - 4 * \sin(2bx + 2a)^2 + 4 * \cos(2bx + 2a) - \\
& 1) * \log(\cos(bx + a)^2 + \sin(bx + a)^2 + 2 * \cos(bx + a) + 1) + (2 * (2 * \cos(2 \\
& * bx + 2a) - 1) * \cos(4bx + 4a) - \cos(4bx + 4a)^2 - 4 * \cos(2bx + 2a) \\
& ^2 - \sin(4bx + 4a)^2 + 4 * \sin(4bx + 4a) * \sin(2bx + 2a) - 4 * \sin(2bx \\
& + 2a)^2 + 4 * \cos(2bx + 2a) - 1) * \log(\cos(bx + a)^2 + \sin(bx + a)^2 - 2 \\
& * \cos(bx + a) + 1) - 4 * ((bx + a)^2 * \sin(2bx + 2a) + bx - (bx + a) * \cos(\\
& 2bx + 2a) + a) * \sin(4bx + 4a) + 4 * (bx + a) * \sin(2bx + 2a)) * a^2 * d^4 / \\
& ((2 * (2 * \cos(2bx + 2a) - 1) * \cos(4bx + 4a) - \cos(4bx + 4a)^2 - 4 * \cos(\\
& 2bx + 2a)^2 - \sin(4bx + 4a)^2 + 4 * \sin(4bx + 4a) * \sin(2bx + 2a) - \\
& 4 * \sin(2bx + 2a)^2 + 4 * \cos(2bx + 2a) - 1) * b^4) - c^4 / \sin(bx + a)^2 + \\
& 4 * a^3 * c^3 * d / (b * \sin(bx + a)^2) - 6 * a^2 * c^2 * d^2 / (b^2 * \sin(bx + a)^2) + 4 * a^3 *
\end{aligned}$$

$$\begin{aligned}
& c*d^3/(b^3*\sin(b*x + a)^2) - a^4*d^4/(b^4*\sin(b*x + a)^2) + 2*((6*(b*x + a) \\
& ^2*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a) + 6*((b*x + a)^2*d^4 + 2*(b*c*d^3 - \\
& a*d^4)*(b*x + a))*\cos(4*b*x + 4*a) - 12*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a \\
& d^4)*(b*x + a))*\cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^4 + (12*I*b*c*d^3 - 1 \\
& 2*I*a*d^4)*(b*x + a))*\sin(4*b*x + 4*a) + (-12*I*(b*x + a)^2*d^4 + (-24*I*b* \\
& c*d^3 + 24*I*a*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), \cos(\\
& b*x + a) + 1) - (6*(b*x + a)^2*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a) + 6*((b \\
& *x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*\cos(4*b*x + 4*a) - 12*((b*x \\
& + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*\cos(2*b*x + 2*a) - (-6*I*(b*x + \\
& a)^2*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a))*\sin(4*b*x + 4*a) - (12* \\
& I*(b*x + a)^2*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a))*\sin(2*b*x + 2*a) \\
&)*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) - 4*((b*x + a)^3*d^4 + 3*(b*c*d^ \\
& 3 - a*d^4)*(b*x + a)^2)*\cos(4*b*x + 4*a) + (-2*I*(b*x + a)^4*d^4 + (-8*I*b* \\
& c*d^3 - 4*(-2*I*a - 1)*d^4)*(b*x + a)^3 + 12*(b*c*d^3 - a*d^4)*(b*x + a)^2) \\
& *\cos(2*b*x + 2*a) - (12*b*c*d^3 + 12*(b*x + a)*d^4 - 12*a*d^4 + 12*(b*c*d^3 \\
& + (b*x + a)*d^4 - a*d^4)*\cos(4*b*x + 4*a) - 24*(b*c*d^3 + (b*x + a)*d^4 - \\
& a*d^4)*\cos(2*b*x + 2*a) - (-12*I*b*c*d^3 - 12*I*(b*x + a)*d^4 + 12*I*a*d^4) \\
& *\sin(4*b*x + 4*a) - (24*I*b*c*d^3 + 24*I*(b*x + a)*d^4 - 24*I*a*d^4)*\sin(2* \\
& b*x + 2*a))*\operatorname{dilog}(-e^{(I*b*x + I*a)}) - (12*b*c*d^3 + 12*(b*x + a)*d^4 - 12*a \\
& *d^4 + 12*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*\cos(4*b*x + 4*a) - 24*(b*c*d^3 \\
& + (b*x + a)*d^4 - a*d^4)*\cos(2*b*x + 2*a) - (-12*I*b*c*d^3 - 12*I*(b*x + a) \\
& *d^4 + 12*I*a*d^4)*\sin(4*b*x + 4*a) - (24*I*b*c*d^3 + 24*I*(b*x + a)*d^4 - \\
& 24*I*a*d^4)*\sin(2*b*x + 2*a))*\operatorname{dilog}(e^{(I*b*x + I*a)}) + (-3*I*(b*x + a)^2*d^ \\
& 4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x + a) + (-3*I*(b*x + a)^2*d^4 + (-6*I*b* \\
& c*d^3 + 6*I*a*d^4)*(b*x + a))*\cos(4*b*x + 4*a) + (6*I*(b*x + a)^2*d^4 + (12 \\
& *I*b*c*d^3 - 12*I*a*d^4)*(b*x + a))*\cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^4 + \\
& 2*(b*c*d^3 - a*d^4)*(b*x + a))*\sin(4*b*x + 4*a) - 6*((b*x + a)^2*d^4 + 2*(\\
& b*c*d^3 - a*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x \\
& + a)^2 + 2*\cos(b*x + a) + 1) + (-3*I*(b*x + a)^2*d^4 + (-6*I*b*c*d^3 + 6*I* \\
& a*d^4)*(b*x + a) + (-3*I*(b*x + a)^2*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x \\
& + a))*\cos(4*b*x + 4*a) + (6*I*(b*x + a)^2*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4) \\
& *(b*x + a))*\cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b \\
& x + a))*\sin(4*b*x + 4*a) - 6*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + \\
& a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) \\
& + 1) + (-12*I*d^4*\cos(4*b*x + 4*a) + 24*I*d^4*\cos(2*b*x + 2*a) + 12*d^4*\sin \\
& (4*b*x + 4*a) - 24*d^4*\sin(2*b*x + 2*a) - 12*I*d^4)*\operatorname{polylog}(3, -e^{(I*b*x + \\
& I*a)}) + (-12*I*d^4*\cos(4*b*x + 4*a) + 24*I*d^4*\cos(2*b*x + 2*a) + 12*d^4*\sin \\
& (4*b*x + 4*a) - 24*d^4*\sin(2*b*x + 2*a) - 12*I*d^4)*\operatorname{polylog}(3, e^{(I*b*x + \\
& I*a)}) + (-4*I*(b*x + a)^3*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2)*\sin \\
& (4*b*x + 4*a) + (2*(b*x + a)^4*d^4 + (8*b*c*d^3 - (8*a - 4*I)*d^4)*(b*x + \\
& a)^3 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^2)*\sin(2*b*x + 2*a))/(-I*b^4* \\
& \cos(4*b*x + 4*a) + 2*I*b^4*\cos(2*b*x + 2*a) + b^4*\sin(4*b*x + 4*a) - 2*b^4* \\
& \sin(2*b*x + 2*a) - I*b^4))/b
\end{aligned}$$

Fricas [C] time = 0.694654, size = 2562, normalized size = 18.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")

[Out] $1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 + 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*\cos(b*x + a)*\sin(b*x + a) + (12*I*b*d^4*x + 12*I*b*c*d^3 + (-12*I*b*d^4*x - 12*I*b*c*d^3)*\cos(b*x + a)^2)*\operatorname{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) + (-12*I*b*d^4*x - 12*I*b*c*d^3 + (12*I*b*d^4*x + 12*I*b*c*d^3)*\cos(b*x + a)^2)*\operatorname{dilog}$

$$\begin{aligned}
& (\cos(b*x + a) - I*\sin(b*x + a)) + (-12*I*b*d^4*x - 12*I*b*c*d^3 + (12*I*b*d^4*x + 12*I*b*c*d^3)*\cos(b*x + a)^2)*\operatorname{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) \\
& + (12*I*b*d^4*x + 12*I*b*c*d^3 + (-12*I*b*d^4*x - 12*I*b*c*d^3)*\cos(b*x + a)^2)*\operatorname{dilog}(-\cos(b*x + a) - I*\sin(b*x + a)) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*\cos(b*x + a)^2) \\
& *\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*\cos(b*x + a)^2)* \\
& \log(\cos(b*x + a) - I*\sin(b*x + a) + 1) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4 - (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*\cos(b*x + a)^2)*\log(-1/2*\cos(b*x + a) + 1/2*I*\sin(b*x + a) + 1/2) \\
& - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4 - (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*\cos(b*x + a)^2)*\log(-1/2*\cos(b*x + a) - 1/2*I*\sin(b*x + a) + 1/2) \\
& - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*\cos(b*x + a)^2)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1) \\
& - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*\cos(b*x + a)^2)*\log(-\cos(b*x + a) - I*\sin(b*x + a) + 1) + 12 \\
& *(d^4*\cos(b*x + a)^2 - d^4)*\operatorname{polylog}(3, \cos(b*x + a) + I*\sin(b*x + a)) + 12*(d^4*\cos(b*x + a)^2 - d^4)*\operatorname{polylog}(3, \cos(b*x + a) - I*\sin(b*x + a)) + 12*(d^4*\cos(b*x + a)^2 - d^4)*\operatorname{polylog}(3, -\cos(b*x + a) + I*\sin(b*x + a)) + 12*(d^4*\cos(b*x + a)^2 - d^4)*\operatorname{polylog}(3, -\cos(b*x + a) - I*\sin(b*x + a)))/(b^5*\cos(b*x + a)^2 - b^5)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**4*cos(b*x+a)*csc(b*x+a)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^4 \cos(bx + a) \csc(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")

[Out] integrate((d*x + c)^4*cos(b*x + a)*csc(b*x + a)^3, x)

3.47 $\int (c + dx)^3 \cot(a + bx) \csc^2(a + bx) dx$

Optimal. Leaf size=115

$$-\frac{3id^3 \text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^4} + \frac{3d^2(c+dx) \log\left(1 - e^{2i(a+bx)}\right)}{b^3} - \frac{3d(c+dx)^2 \cot(a+bx)}{2b^2} - \frac{(c+dx)^3 \csc^2(a+bx)}{2b} - \frac{3id(c+dx)}{2b}$$

[Out] ((((-3*I)/2)*d*(c + d*x)^2)/b^2 - (3*d*(c + d*x)^2*Cot[a + b*x])/(2*b^2) - ((c + d*x)^3*Csc[a + b*x]^2)/(2*b) + (3*d^2*(c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b^3 - (((3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^4

Rubi [A] time = 0.173609, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4410, 4184, 3717, 2190, 2279, 2391}

$$-\frac{3id^3 \text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^4} + \frac{3d^2(c+dx) \log\left(1 - e^{2i(a+bx)}\right)}{b^3} - \frac{3d(c+dx)^2 \cot(a+bx)}{2b^2} - \frac{(c+dx)^3 \csc^2(a+bx)}{2b} - \frac{3id(c+dx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x]^2,x]

[Out] ((((-3*I)/2)*d*(c + d*x)^2)/b^2 - (3*d*(c + d*x)^2*Cot[a + b*x])/(2*b^2) - ((c + d*x)^3*Csc[a + b*x]^2)/(2*b) + (3*d^2*(c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b^3 - (((3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^4

Rule 4410

Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

Rule 4184

Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3717

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^3 \cot(a + bx) \csc^2(a + bx) dx &= -\frac{(c + dx)^3 \csc^2(a + bx)}{2b} + \frac{(3d) \int (c + dx)^2 \csc^2(a + bx) dx}{2b} \\ &= -\frac{3d(c + dx)^2 \cot(a + bx)}{2b^2} - \frac{(c + dx)^3 \csc^2(a + bx)}{2b} + \frac{(3d^2) \int (c + dx) \cot(a + bx) dx}{b^2} \\ &= -\frac{3id(c + dx)^2}{2b^2} - \frac{3d(c + dx)^2 \cot(a + bx)}{2b^2} - \frac{(c + dx)^3 \csc^2(a + bx)}{2b} - \frac{(6id^2) \int (c + dx) \cot(a + bx) dx}{b^2} \\ &= -\frac{3id(c + dx)^2}{2b^2} - \frac{3d(c + dx)^2 \cot(a + bx)}{2b^2} - \frac{(c + dx)^3 \csc^2(a + bx)}{2b} + \frac{3d^2(c + dx)}{b^2} \\ &= -\frac{3id(c + dx)^2}{2b^2} - \frac{3d(c + dx)^2 \cot(a + bx)}{2b^2} - \frac{(c + dx)^3 \csc^2(a + bx)}{2b} + \frac{3d^2(c + dx)}{b^2} \\ &= -\frac{3id(c + dx)^2}{2b^2} - \frac{3d(c + dx)^2 \cot(a + bx)}{2b^2} - \frac{(c + dx)^3 \csc^2(a + bx)}{2b} + \frac{3d^2(c + dx)}{b^2} \end{aligned}$$

Mathematica [B] time = 6.41234, size = 277, normalized size = 2.41

$$3d^3 \csc(a) \sec(a) \frac{\left(\tan(a) \left(i \operatorname{PolyLog}\left(2, e^{2i(\tan^{-1}(\tan(a))+bx)} \right) \right) + ibx(2 \tan^{-1}(\tan(a)) - \pi) - 2(\tan^{-1}(\tan(a))+bx) \log\left(1 - e^{2i(\tan^{-1}(\tan(a))+bx)} \right) \right) + 2 \tan^{-1}(\tan(a)) \right)}{\sqrt{\tan^2(a)+1}}$$

$$2b^4 \sqrt{\sec^2(a) (\sin^2(a) + \cos^2(a))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -((c + d*x)^3*Csc[a + b*x]^2)/(2*b) + (3*c*d^2*Csc[a]*(-(b*x*Cos[a]) + Log[
Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x])*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (
3*Csc[a]*Csc[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3*x^2*Sin[b*
x]))/(2*b^2) - (3*d^3*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x
*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Ta
n[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]])])) + Pi*Log[Cos[b*x]] + 2*Arc
Tan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]]) + I*PolyLog[2, E^((2*I)*(b*x + A
rcTan[Tan[a]])]))*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^4*Sqrt[Sec[a]^2*(Cos[a]
^2 + Sin[a]^2]))
```

Maple [B] time = 0.164, size = 409, normalized size = 3.6

$$\frac{2bd^3x^3e^{2i(bx+a)} - 3id^3x^2e^{2i(bx+a)} + 6bcd^2x^2e^{2i(bx+a)} - 6icd^2xe^{2i(bx+a)} + 6bc^2dxe^{2i(bx+a)} - 3ic^2de^{2i(bx+a)} + 3id^3x^2 + 2c^2}{b^2(e^{2i(bx+a)} - 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d*x+c)^3*\cos(b*x+a)*\csc(b*x+a)^3,x)$

[Out] $(2*b*d^3*x^3*\exp(2*I*(b*x+a))-3*I*d^3*x^2*\exp(2*I*(b*x+a))+6*b*c*d^2*x^2*\exp(2*I*(b*x+a))-6*I*c*d^2*x*\exp(2*I*(b*x+a))+6*b*c^2*d*x*\exp(2*I*(b*x+a))-3*I*c^2*d*\exp(2*I*(b*x+a))+3*I*d^3*x^2+2*b*c^3*\exp(2*I*(b*x+a))+6*I*c*d^2*x+3*I*c^2*d)/b^2/(\exp(2*I*(b*x+a))-1)^2+3*d^2/b^3*c*\ln(\exp(I*(b*x+a))-1)-6*d^2/b^3*c*\ln(\exp(I*(b*x+a)))+3*d^2/b^3*c*\ln(\exp(I*(b*x+a))+1)-3*I*d^3/b^2*x^2-6*I*d^3/b^3*a*x-3*I*d^3/b^4*a^2+3*d^3/b^3*\ln(1-\exp(I*(b*x+a)))*x+3*d^3/b^4*\ln(1-\exp(I*(b*x+a)))*a-3*I*d^3*\text{polylog}(2,\exp(I*(b*x+a)))/b^4+3*d^3/b^3*\ln(\exp(I*(b*x+a))+1)*x-3*I*d^3/b^4*\text{polylog}(2,-\exp(I*(b*x+a)))-3*d^3/b^4*a*\ln(\exp(I*(b*x+a))-1)+6*d^3/b^4*a*\ln(\exp(I*(b*x+a)))$

Maxima [B] time = 2.27893, size = 1411, normalized size = 12.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((d*x+c)^3*\cos(b*x+a)*\csc(b*x+a)^3,x, \text{algorithm}="maxima")$

[Out] $(6*b^2*c^2*d + (6*b*d^3*x + 6*b*c*d^2 + 6*(b*d^3*x + b*c*d^2)*\cos(4*b*x + 4*a) - 12*(b*d^3*x + b*c*d^2)*\cos(2*b*x + 2*a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*\sin(4*b*x + 4*a) + (-12*I*b*d^3*x - 12*I*b*c*d^2)*\sin(2*b*x + 2*a))*\arctan(2(\sin(b*x + a), \cos(b*x + a) + 1) + (6*b*c*d^2*\cos(4*b*x + 4*a) - 12*b*c*d^2*\cos(2*b*x + 2*a) + 6*I*b*c*d^2*\sin(4*b*x + 4*a) - 12*I*b*c*d^2*\sin(2*b*x + 2*a) + 6*b*c*d^2)*\arctan(2(\sin(b*x + a), \cos(b*x + a) - 1) - (6*b*d^3*x*\cos(4*b*x + 4*a) - 12*b*d^3*x*\cos(2*b*x + 2*a) + 6*I*b*d^3*x*\sin(4*b*x + 4*a) - 12*I*b*d^3*x*\sin(2*b*x + 2*a) + 6*b*d^3*x)*\arctan(2(\sin(b*x + a), -\cos(b*x + a) + 1) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x)*\cos(4*b*x + 4*a) + (-4*I*b^3*d^3*x^3 - 4*I*b^3*c^3 - 6*b^2*c^2*d + (-12*I*b^3*c*d^2 + 6*b^2*d^3)*x^2 - 12*(I*b^3*c^2*d - b^2*c*d^2)*x)*\cos(2*b*x + 2*a) - (6*d^3*\cos(4*b*x + 4*a) - 12*d^3*\cos(2*b*x + 2*a) + 6*I*d^3*\sin(4*b*x + 4*a) - 12*I*d^3*\sin(2*b*x + 2*a) + 6*d^3)*\text{dilog}(-e^{(I*b*x + I*a)}) - (6*d^3*\cos(4*b*x + 4*a) - 12*d^3*\cos(2*b*x + 2*a) + 6*I*d^3*\sin(4*b*x + 4*a) - 12*I*d^3*\sin(2*b*x + 2*a) + 6*d^3)*\text{dilog}(e^{(I*b*x + I*a)}) + (-3*I*b*d^3*x - 3*I*b*c*d^2 + (-3*I*b*d^3*x - 3*I*b*c*d^2)*\cos(4*b*x + 4*a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*\cos(2*b*x + 2*a) + 3*(b*d^3*x + b*c*d^2)*\sin(4*b*x + 4*a) - 6*(b*d^3*x + b*c*d^2)*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) + (-3*I*b*d^3*x - 3*I*b*c*d^2 + (-3*I*b*d^3*x - 3*I*b*c*d^2)*\cos(4*b*x + 4*a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*\cos(2*b*x + 2*a) + 3*(b*d^3*x + b*c*d^2)*\sin(4*b*x + 4*a) - 6*(b*d^3*x + b*c*d^2)*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*x)*\sin(4*b*x + 4*a) + (4*b^3*d^3*x^3 + 4*b^3*c^3 - 6*I*b^2*c^2*d + 6*(2*b^3*c*d^2 + I*b^2*d^3)*x^2 + (12*b^3*c^2*d + 12*I*b^2*c*d^2)*x)*\sin(2*b*x + 2*a))/(-2*I*b^4*\cos(4*b*x + 4*a) + 4*I*b^4*\cos(2*b*x + 2*a) + 2*b^4*\sin(4*b*x + 4*a) - 4*b^4*\sin(2*b*x + 2*a) - 2*I*b^4)$

Fricas [B] time = 0.609607, size = 1445, normalized size = 12.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")

[Out] $\frac{1}{2}(b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + b^3c^3 + 3(b^2d^3x^2 + 2b^2cd^2x + b^2c^2d)\cos(bx + a)\sin(bx + a) + (-3Id^3\cos(bx + a)^2 + 3Id^3)\operatorname{dilog}(\cos(bx + a) + I\sin(bx + a)) + (3Id^3\cos(bx + a)^2 - 3Id^3)\operatorname{dilog}(\cos(bx + a) - I\sin(bx + a)) + (3Id^3\cos(bx + a)^2 - 3Id^3)\operatorname{dilog}(-\cos(bx + a) + I\sin(bx + a)) + (-3Id^3\cos(bx + a)^2 + 3Id^3)\operatorname{dilog}(-\cos(bx + a) - I\sin(bx + a)) - 3(bd^3x + bcd^2 - (bd^3x + bcd^2)\cos(bx + a)^2)\log(\cos(bx + a) + I\sin(bx + a) + 1) - 3(bd^3x + bcd^2 - (bd^3x + bcd^2)\cos(bx + a)^2)\log(\cos(bx + a) - I\sin(bx + a) + 1) - 3(bcd^2 - ad^3 - (bcd^2 - ad^3)\cos(bx + a)^2)\log(-\frac{1}{2}\cos(bx + a) + \frac{1}{2}I\sin(bx + a) + \frac{1}{2}) - 3(bcd^2 - ad^3 - (bcd^2 - ad^3)\cos(bx + a)^2)\log(-\frac{1}{2}\cos(bx + a) - \frac{1}{2}I\sin(bx + a) + \frac{1}{2}) - 3(bd^3x + ad^3 - (bd^3x + ad^3)\cos(bx + a)^2)\log(-\cos(bx + a) + I\sin(bx + a) + 1) - 3(bd^3x + ad^3 - (bd^3x + ad^3)\cos(bx + a)^2)\log(-\cos(bx + a) - I\sin(bx + a) + 1))/(b^4\cos(bx + a)^2 - b^4)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*cos(b*x+a)*csc(b*x+a)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \cos(bx + a) \csc(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")

[Out] integrate((d*x + c)^3*cos(b*x + a)*csc(b*x + a)^3, x)

3.48 $\int (c + dx)^2 \cot(a + bx) \csc^2(a + bx) dx$

Optimal. Leaf size=54

$$-\frac{d(c + dx) \cot(a + bx)}{b^2} + \frac{d^2 \log(\sin(a + bx))}{b^3} - \frac{(c + dx)^2 \csc^2(a + bx)}{2b}$$

[Out] $-\frac{((d*(c + d*x))*\text{Cot}[a + b*x])/b^2}{b^2} - \frac{((c + d*x)^2*\text{Csc}[a + b*x]^2)/(2*b)}{b^3} + \frac{(d^2*\text{Log}[\text{Sin}[a + b*x]])/b^3}{b^3}$

Rubi [A] time = 0.0653742, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4410, 4184, 3475}

$$-\frac{d(c + dx) \cot(a + bx)}{b^2} + \frac{d^2 \log(\sin(a + bx))}{b^3} - \frac{(c + dx)^2 \csc^2(a + bx)}{2b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^2*\text{Cot}[a + b*x]*\text{Csc}[a + b*x]^2, x]$

[Out] $-\frac{((d*(c + d*x))*\text{Cot}[a + b*x])/b^2}{b^2} - \frac{((c + d*x)^2*\text{Csc}[a + b*x]^2)/(2*b)}{b^3} + \frac{(d^2*\text{Log}[\text{Sin}[a + b*x]])/b^3}{b^3}$

Rule 4410

$\text{Int}[\text{Cot}[(a_.) + (b_.)*(x_.)]^{(p_.)}*\text{Csc}[(a_.) + (b_.)*(x_.)]^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Csc}[a + b*x]^n/(b*n), x] + \text{Dist}[(d*m)/(b*n), \text{Int}[(c + d*x)^{(m-1)}*\text{Csc}[a + b*x]^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{EqQ}[p, 1] \&\& \text{GtQ}[m, 0]$

Rule 4184

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]^2*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cot}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cot}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{GtQ}[m, 0]$

Rule 3475

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \cot(a + bx) \csc^2(a + bx) dx &= -\frac{(c + dx)^2 \csc^2(a + bx)}{2b} + \frac{d \int (c + dx) \csc^2(a + bx) dx}{b} \\ &= -\frac{d(c + dx) \cot(a + bx)}{b^2} - \frac{(c + dx)^2 \csc^2(a + bx)}{2b} + \frac{d^2 \int \cot(a + bx) dx}{b^2} \\ &= -\frac{d(c + dx) \cot(a + bx)}{b^2} - \frac{(c + dx)^2 \csc^2(a + bx)}{2b} + \frac{d^2 \log(\sin(a + bx))}{b^3} \end{aligned}$$

Mathematica [C] time = 0.918487, size = 94, normalized size = 1.74

$$\frac{-b^2(c + dx)^2 \csc^2(a + bx) + 2bd \csc(a) \sin(bx)(c + dx) \csc(a + bx) - 2id^2 \tan^{-1}(\tan(a + bx)) - 2bd^2 x \cot(a) + d^2 \log(\sin(a + bx))}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] ((2*I)*b*d^2*x - (2*I)*d^2*ArcTan[Tan[a + b*x]] - 2*b*d^2*x*Cot[a] - b^2*(c + d*x)^2*Csc[a + b*x]^2 + d^2*Log[Sin[a + b*x]^2] + 2*b*d*(c + d*x)*Csc[a]*Csc[a + b*x]*Sin[b*x])/(2*b^3)
```

Maple [A] time = 0.029, size = 95, normalized size = 1.8

$$\frac{d^2 x^2}{2b(\sin(bx+a))^2} - \frac{d^2 \cot(bx+a)x}{b^2} + \frac{d^2 \ln(\sin(bx+a))}{b^3} - \frac{cdx}{b(\sin(bx+a))^2} - \frac{cd \cot(bx+a)}{b^2} - \frac{c^2}{2b(\sin(bx+a))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x)
```

```
[Out] -1/2/b*d^2/sin(b*x+a)^2*x^2-1/b^2*d^2*cot(b*x+a)*x+d^2*ln(sin(b*x+a))/b^3-1/b*c*d/sin(b*x+a)^2*x-1/b^2*c*d*cot(b*x+a)-1/2/b*c^2/sin(b*x+a)^2
```

Maxima [B] time = 1.3463, size = 1526, normalized size = 28.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x + a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*c*d/((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b) - 4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x + a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*a*d^2/((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^2) + (8*(b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 - 4*(b*x + a)^2*cos(2*b*x + 2*a) - 4*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*((b*x + a)^2*sin(2*b*x + 2*a) + b*x - (b*x + a)*cos(2*b*x + 2*a) + a)*sin(4*b*x + 4*a) + 4*(b*x + a)*sin(2*b*x + 2*a))*d^2/((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^2)
```

$(x + 2a) - 1) \cdot b^2) - c^2 / \sin(bx + a)^2 + 2ac \cdot d / (b \sin(bx + a)^2) - a^2 \cdot d^2 / (b^2 \sin(bx + a)^2) / b$

Fricas [A] time = 0.504084, size = 231, normalized size = 4.28

$$\frac{b^2 d^2 x^2 + 2 b^2 c d x + b^2 c^2 + 2 (b d^2 x + b c d) \cos (b x + a) \sin (b x + a) + 2 (d^2 \cos (b x + a)^2 - d^2) \log \left(\frac{1}{2} \sin (b x + a) \right)}{2 (b^3 \cos (b x + a)^2 - b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")

[Out] $\frac{1}{2} \cdot (b^2 d^2 x^2 + 2 b^2 c d x + b^2 c^2 + 2 (b d^2 x + b c d) \cos (b x + a) \sin (b x + a) + 2 (d^2 \cos (b x + a)^2 - d^2) \log (1/2 \sin (b x + a))) / (b^3 \cos (b x + a)^2 - b^3)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*cos(b*x+a)*csc(b*x+a)**3,x)

[Out] Timed out

Giac [B] time = 2.5771, size = 4701, normalized size = 87.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/8 \cdot (b^2 d^2 x^2 \tan(1/2 b x)^4 \tan(1/2 a)^4 + 2 b^2 c d x \tan(1/2 b x)^4 \tan(1/2 a)^4 + 2 b^2 d^2 x^2 \tan(1/2 b x)^4 \tan(1/2 a)^2 + 2 b^2 d^2 x^2 \tan(1/2 b x)^2 \tan(1/2 a)^4 + b^2 c^2 \tan(1/2 b x)^4 \tan(1/2 a)^4 + 4 b^2 c d x \tan(1/2 b x)^4 \tan(1/2 a)^2 - 4 b d^2 x \tan(1/2 b x)^4 \tan(1/2 a)^3 + 4 b^2 c d x \tan(1/2 b x)^2 \tan(1/2 a)^4 - 4 b d^2 x \tan(1/2 b x)^3 \tan(1/2 a)^4 + b^2 d^2 x^2 \tan(1/2 b x)^4 + 4 b^2 d^2 x^2 \tan(1/2 b x)^2 \tan(1/2 a)^2 + 2 b^2 c^2 \tan(1/2 b x)^4 \tan(1/2 a)^2 - 4 b c d \tan(1/2 b x)^4 \tan(1/2 a)^3 + b^2 d^2 x^2 \tan(1/2 a)^4 + 2 b^2 c^2 \tan(1/2 b x)^2 \tan(1/2 a)^4 - 4 b c d \tan(1/2 b x)^3 \tan(1/2 a)^4 + 2 b^2 c d x \tan(1/2 b x)^4 + 4 b d^2 x \tan(1/2 b x)^4 \tan(1/2 a) + 8 b^2 c d x \tan(1/2 b x)^2 \tan(1/2 a)^2 + 24 b d^2 x \tan(1/2 b x)^3 \tan(1/2 a)^2 - 4 d^2 \log(16 (\tan(1/2 a)^4 + 2 \tan(1/2 a)^2 + 1) / (\tan(1/2 b x)^8 \tan(1/2 a)^2 + 2 \tan(1/2 b x)^7 \tan(1/2 a)^3 + \tan(1/2 b x)^6 \tan(1/2 a)^4 - 2 \tan(1/2 b x)^7 \tan(1/2 a) - 2 \tan(1/2 b x)^6 \tan(1/2 a)^2 + 2 \tan(1/2 b x)^5 \tan(1/2 a)^3 + 2 \tan(1/2 b x)^4 \tan(1/2 a)^4 + \tan(1/2 b x)^6 - 2 \tan(1/2 b x)^5 \tan(1/2 a) - 6 \tan(1/2 b x)^4 \tan(1/2 a)^2 - 2 \tan(1/2 b x)^3 \tan(1/2 a)^3 + \tan(1/2 b x)^2 \tan(1/2 a)^4 + 2 \tan(1/2 b x)^4 + 2 \tan(1/2 b x)^3 \tan(1/2 a) - 2 \tan(1/2 b x)^2 \tan(1/2 a)^2 - \end{aligned}$$

$$\begin{aligned}
& 2*\tan(1/2*b*x)*\tan(1/2*a)^3 + \tan(1/2*b*x)^2 + 2*\tan(1/2*b*x)*\tan(1/2*a) + \\
& \tan(1/2*a)^2))*\tan(1/2*b*x)^4*\tan(1/2*a)^2 + 24*b*d^2*x*\tan(1/2*b*x)^2*\tan \\
& (1/2*a)^3 - 8*d^2*\log(16*(\tan(1/2*a)^4 + 2*\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^ \\
& 8*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^7*\tan(1/2*a)^3 + \tan(1/2*b*x)^6*\tan(1/2*a)^ \\
& 4 - 2*\tan(1/2*b*x)^7*\tan(1/2*a) - 2*\tan(1/2*b*x)^6*\tan(1/2*a)^2 + 2*\tan(1/2 \\
& *b*x)^5*\tan(1/2*a)^3 + 2*\tan(1/2*b*x)^4*\tan(1/2*a)^4 + \tan(1/2*b*x)^6 - 2*t \\
& \tan(1/2*b*x)^5*\tan(1/2*a) - 6*\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^3 \\
& *\tan(1/2*a)^3 + \tan(1/2*b*x)^2*\tan(1/2*a)^4 + 2*\tan(1/2*b*x)^4 + 2*\tan(1/2* \\
& b*x)^3*\tan(1/2*a) - 2*\tan(1/2*b*x)^2*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)*\tan(1/2* \\
& a)^3 + \tan(1/2*b*x)^2 + 2*\tan(1/2*b*x)*\tan(1/2*a) + \tan(1/2*a)^2))*\tan(1/2* \\
& b*x)^3*\tan(1/2*a)^3 + 2*b^2*c*d*x*\tan(1/2*a)^4 + 4*b*d^2*x*\tan(1/2*b*x)*\tan \\
& (1/2*a)^4 - 4*d^2*\log(16*(\tan(1/2*a)^4 + 2*\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^ \\
& 8*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^7*\tan(1/2*a)^3 + \tan(1/2*b*x)^6*\tan(1/2*a)^ \\
& 4 - 2*\tan(1/2*b*x)^7*\tan(1/2*a) - 2*\tan(1/2*b*x)^6*\tan(1/2*a)^2 + 2*\tan(1/2 \\
& *b*x)^5*\tan(1/2*a)^3 + 2*\tan(1/2*b*x)^4*\tan(1/2*a)^4 + \tan(1/2*b*x)^6 - 2*t \\
& \tan(1/2*b*x)^5*\tan(1/2*a) - 6*\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^3 \\
& *\tan(1/2*a)^3 + \tan(1/2*b*x)^2*\tan(1/2*a)^4 + 2*\tan(1/2*b*x)^4 + 2*\tan(1/2* \\
& b*x)^3*\tan(1/2*a) - 2*\tan(1/2*b*x)^2*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)*\tan(1/2* \\
& a)^3 + \tan(1/2*b*x)^2 + 2*\tan(1/2*b*x)*\tan(1/2*a) + \tan(1/2*a)^2))*\tan(1/2* \\
& b*x)^2*\tan(1/2*a)^4 + 2*b^2*d^2*x^2*\tan(1/2*b*x)^2 + b^2*c^2*\tan(1/2*b*x)^4 \\
& + 4*b*c*d*\tan(1/2*b*x)^4*\tan(1/2*a) + 2*b^2*d^2*x^2*\tan(1/2*a)^2 + 4*b^2*c \\
& ^2*\tan(1/2*b*x)^2*\tan(1/2*a)^2 + 24*b*c*d*\tan(1/2*b*x)^3*\tan(1/2*a)^2 + 24* \\
& b*c*d*\tan(1/2*b*x)^2*\tan(1/2*a)^3 + b^2*c^2*\tan(1/2*a)^4 + 4*b*c*d*\tan(1/2* \\
& b*x)*\tan(1/2*a)^4 + 4*b^2*c*d*x*\tan(1/2*b*x)^2 - 4*b*d^2*x*\tan(1/2*b*x)^3 - \\
& 24*b*d^2*x*\tan(1/2*b*x)^2*\tan(1/2*a) + 8*d^2*\log(16*(\tan(1/2*a)^4 + 2*\tan(\\
& 1/2*a)^2 + 1)/(\tan(1/2*b*x)^8*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^7*\tan(1/2*a)^3 \\
& + \tan(1/2*b*x)^6*\tan(1/2*a)^4 - 2*\tan(1/2*b*x)^7*\tan(1/2*a) - 2*\tan(1/2*b*x \\
&)^6*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^5*\tan(1/2*a)^3 + 2*\tan(1/2*b*x)^4*\tan(1/2 \\
& *a)^4 + \tan(1/2*b*x)^6 - 2*\tan(1/2*b*x)^5*\tan(1/2*a) - 6*\tan(1/2*b*x)^4*\tan \\
& (1/2*a)^2 - 2*\tan(1/2*b*x)^3*\tan(1/2*a)^3 + \tan(1/2*b*x)^2*\tan(1/2*a)^4 + 2 \\
& *\tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^3*\tan(1/2*a) - 2*\tan(1/2*b*x)^2*\tan(1/2*a) \\
& ^2 - 2*\tan(1/2*b*x)*\tan(1/2*a)^3 + \tan(1/2*b*x)^2 + 2*\tan(1/2*b*x)*\tan(1/2* \\
& a) + \tan(1/2*a)^2))*\tan(1/2*b*x)^3*\tan(1/2*a) + 4*b^2*c*d*x*\tan(1/2*a)^2 - \\
& 24*b*d^2*x*\tan(1/2*b*x)*\tan(1/2*a)^2 + 16*d^2*\log(16*(\tan(1/2*a)^4 + 2*\tan(\\
& 1/2*a)^2 + 1)/(\tan(1/2*b*x)^8*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^7*\tan(1/2*a)^3 \\
& + \tan(1/2*b*x)^6*\tan(1/2*a)^4 - 2*\tan(1/2*b*x)^7*\tan(1/2*a) - 2*\tan(1/2*b*x \\
&)^6*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^5*\tan(1/2*a)^3 + 2*\tan(1/2*b*x)^4*\tan(1/2 \\
& *a)^4 + \tan(1/2*b*x)^6 - 2*\tan(1/2*b*x)^5*\tan(1/2*a) - 6*\tan(1/2*b*x)^4*\tan \\
& (1/2*a)^2 - 2*\tan(1/2*b*x)^3*\tan(1/2*a)^3 + \tan(1/2*b*x)^2*\tan(1/2*a)^4 + 2 \\
& *\tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^3*\tan(1/2*a) - 2*\tan(1/2*b*x)^2*\tan(1/2*a) \\
& ^2 - 2*\tan(1/2*b*x)*\tan(1/2*a)^3 + \tan(1/2*b*x)^2 + 2*\tan(1/2*b*x)*\tan(1/2* \\
& a) + \tan(1/2*a)^2))*\tan(1/2*b*x)^2*\tan(1/2*a)^2 - 4*b*d^2*x*\tan(1/2*a)^3 + \\
& 8*d^2*\log(16*(\tan(1/2*a)^4 + 2*\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^8*\tan(1/2*a) \\
& ^2 + 2*\tan(1/2*b*x)^7*\tan(1/2*a)^3 + \tan(1/2*b*x)^6*\tan(1/2*a)^4 - 2*\tan(1/ \\
& 2*b*x)^7*\tan(1/2*a) - 2*\tan(1/2*b*x)^6*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^5*\tan(\\
& 1/2*a)^3 + 2*\tan(1/2*b*x)^4*\tan(1/2*a)^4 + \tan(1/2*b*x)^6 - 2*\tan(1/2*b*x)^ \\
& 5*\tan(1/2*a) - 6*\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^3*\tan(1/2*a)^ \\
& 3 + \tan(1/2*b*x)^2*\tan(1/2*a)^4 + 2*\tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^3*\tan(1 \\
& /2*a) - 2*\tan(1/2*b*x)^2*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)*\tan(1/2*a)^3 + \tan(1 \\
& /2*b*x)^2 + 2*\tan(1/2*b*x)*\tan(1/2*a) + \tan(1/2*a)^2))*\tan(1/2*b*x)*\tan(1/2 \\
& *a)^3 + b^2*d^2*x^2 + 2*b^2*c^2*\tan(1/2*b*x)^2 - 4*b*c*d*\tan(1/2*b*x)^3 - 2 \\
& 4*b*c*d*\tan(1/2*b*x)^2*\tan(1/2*a) + 2*b^2*c^2*\tan(1/2*a)^2 - 24*b*c*d*\tan(1 \\
& /2*b*x)*\tan(1/2*a)^2 - 4*b*c*d*\tan(1/2*a)^3 + 2*b^2*c*d*x + 4*b*d^2*x*\tan(1 \\
& /2*b*x) - 4*d^2*\log(16*(\tan(1/2*a)^4 + 2*\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^8* \\
& \tan(1/2*a)^2 + 2*\tan(1/2*b*x)^7*\tan(1/2*a)^3 + \tan(1/2*b*x)^6*\tan(1/2*a)^4 \\
& - 2*\tan(1/2*b*x)^7*\tan(1/2*a) - 2*\tan(1/2*b*x)^6*\tan(1/2*a)^2 + 2*\tan(1/2*b \\
& *x)^5*\tan(1/2*a)^3 + 2*\tan(1/2*b*x)^4*\tan(1/2*a)^4 + \tan(1/2*b*x)^6 - 2*\tan \\
& (1/2*b*x)^5*\tan(1/2*a) - 6*\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^3*t \\
& \tan(1/2*a)^3 + \tan(1/2*b*x)^2*\tan(1/2*a)^4 + 2*\tan(1/2*b*x)^4 + 2*\tan(1/2*b*
\end{aligned}$$

$$\begin{aligned}
& x)^3 \tan(1/2*a) - 2*\tan(1/2*b*x)^2*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)*\tan(1/2*a) \\
& ^3 + \tan(1/2*b*x)^2 + 2*\tan(1/2*b*x)*\tan(1/2*a) + \tan(1/2*a)^2))*\tan(1/2*b* \\
& x)^2 + 4*b*d^2*x*\tan(1/2*a) - 8*d^2*\log(16*(\tan(1/2*a)^4 + 2*\tan(1/2*a)^2 + \\
& 1)/(\tan(1/2*b*x)^8*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^7*\tan(1/2*a)^3 + \tan(1/2* \\
& b*x)^6*\tan(1/2*a)^4 - 2*\tan(1/2*b*x)^7*\tan(1/2*a) - 2*\tan(1/2*b*x)^6*\tan(1/ \\
& 2*a)^2 + 2*\tan(1/2*b*x)^5*\tan(1/2*a)^3 + 2*\tan(1/2*b*x)^4*\tan(1/2*a)^4 + \tan \\
& (1/2*b*x)^6 - 2*\tan(1/2*b*x)^5*\tan(1/2*a) - 6*\tan(1/2*b*x)^4*\tan(1/2*a)^2 \\
& - 2*\tan(1/2*b*x)^3*\tan(1/2*a)^3 + \tan(1/2*b*x)^2*\tan(1/2*a)^4 + 2*\tan(1/2*b \\
& *x)^4 + 2*\tan(1/2*b*x)^3*\tan(1/2*a) - 2*\tan(1/2*b*x)^2*\tan(1/2*a)^2 - 2*\tan \\
& (1/2*b*x)*\tan(1/2*a)^3 + \tan(1/2*b*x)^2 + 2*\tan(1/2*b*x)*\tan(1/2*a) + \tan(1 \\
& /2*a)^2))*\tan(1/2*b*x)*\tan(1/2*a) - 4*d^2*\log(16*(\tan(1/2*a)^4 + 2*\tan(1/2* \\
& a)^2 + 1)/(\tan(1/2*b*x)^8*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^7*\tan(1/2*a)^3 + \tan \\
& (1/2*b*x)^6*\tan(1/2*a)^4 - 2*\tan(1/2*b*x)^7*\tan(1/2*a) - 2*\tan(1/2*b*x)^6* \\
& \tan(1/2*a)^2 + 2*\tan(1/2*b*x)^5*\tan(1/2*a)^3 + 2*\tan(1/2*b*x)^4*\tan(1/2*a)^ \\
& 4 + \tan(1/2*b*x)^6 - 2*\tan(1/2*b*x)^5*\tan(1/2*a) - 6*\tan(1/2*b*x)^4*\tan(1/2 \\
& *a)^2 - 2*\tan(1/2*b*x)^3*\tan(1/2*a)^3 + \tan(1/2*b*x)^2*\tan(1/2*a)^4 + 2*\tan \\
& (1/2*b*x)^4 + 2*\tan(1/2*b*x)^3*\tan(1/2*a) - 2*\tan(1/2*b*x)^2*\tan(1/2*a)^2 - \\
& 2*\tan(1/2*b*x)*\tan(1/2*a)^3 + \tan(1/2*b*x)^2 + 2*\tan(1/2*b*x)*\tan(1/2*a) + \\
& \tan(1/2*a)^2))*\tan(1/2*a)^2 + b^2*c^2 + 4*b*c*d*\tan(1/2*b*x) + 4*b*c*d*\tan \\
& (1/2*a))/(b^3*\tan(1/2*b*x)^4*\tan(1/2*a)^2 + 2*b^3*\tan(1/2*b*x)^3*\tan(1/2*a) \\
& ^3 + b^3*\tan(1/2*b*x)^2*\tan(1/2*a)^4 - 2*b^3*\tan(1/2*b*x)^3*\tan(1/2*a) - 4* \\
& b^3*\tan(1/2*b*x)^2*\tan(1/2*a)^2 - 2*b^3*\tan(1/2*b*x)*\tan(1/2*a)^3 + b^3*\tan \\
& (1/2*b*x)^2 + 2*b^3*\tan(1/2*b*x)*\tan(1/2*a) + b^3*\tan(1/2*a)^2)
\end{aligned}$$

3.49 $\int (c + dx) \cot(a + bx) \csc^2(a + bx) dx$

Optimal. Leaf size=35

$$-\frac{d \cot(a + bx)}{2b^2} - \frac{(c + dx) \csc^2(a + bx)}{2b}$$

[Out] $-(d*\text{Cot}[a + b*x])/(2*b^2) - ((c + d*x)*\text{Csc}[a + b*x]^2)/(2*b)$

Rubi [A] time = 0.0314137, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {4410, 3767, 8}

$$-\frac{d \cot(a + bx)}{2b^2} - \frac{(c + dx) \csc^2(a + bx)}{2b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)*\text{Cot}[a + b*x]*\text{Csc}[a + b*x]^2, x]$

[Out] $-(d*\text{Cot}[a + b*x])/(2*b^2) - ((c + d*x)*\text{Csc}[a + b*x]^2)/(2*b)$

Rule 4410

$\text{Int}[\text{Cot}[(a_.) + (b_.)*(x_.)]^{(p_.)}*\text{Csc}[(a_.) + (b_.)*(x_.)]^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow -\text{Simp}[((c + d*x)^m*\text{Csc}[a + b*x]^n)/(b*n), x] + \text{Dist}[(d*m)/(b*n), \text{Int}[(c + d*x)^{(m-1)}*\text{Csc}[a + b*x]^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{EqQ}[p, 1] \&\& \text{GtQ}[m, 0]$

Rule 3767

$\text{Int}[\text{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] /; \text{FreeQ}\{c, d\}, x] \&\& \text{IGtQ}[n/2, 0]$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rubi steps

$$\begin{aligned} \int (c + dx) \cot(a + bx) \csc^2(a + bx) dx &= -\frac{(c + dx) \csc^2(a + bx)}{2b} + \frac{d \int \csc^2(a + bx) dx}{2b} \\ &= -\frac{(c + dx) \csc^2(a + bx)}{2b} - \frac{d \text{Subst}(\int 1 dx, x, \cot(a + bx))}{2b^2} \\ &= -\frac{d \cot(a + bx)}{2b^2} - \frac{(c + dx) \csc^2(a + bx)}{2b} \end{aligned}$$

Mathematica [A] time = 0.0738112, size = 48, normalized size = 1.37

$$-\frac{d \cot(a + bx)}{2b^2} - \frac{c \csc^2(a + bx)}{2b} - \frac{dx \csc^2(a + bx)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Cot[a + b*x]*Csc[a + b*x]^2,x]

[Out] $-(d*\text{Cot}[a + b*x])/(2*b^2) - (c*\text{Csc}[a + b*x]^2)/(2*b) - (d*x*\text{Csc}[a + b*x]^2)/(2*b)$

Maple [A] time = 0.029, size = 61, normalized size = 1.7

$$\frac{1}{b} \left(\frac{d}{b} \left(-\frac{bx+a}{2(\sin(bx+a))^2} - \frac{\cot(bx+a)}{2} \right) + \frac{ad}{2b(\sin(bx+a))^2} - \frac{c}{2(\sin(bx+a))^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x)

[Out] $1/b*(d/b*(-1/2*(b*x+a)/\sin(b*x+a)^2-1/2*\cot(b*x+a))+1/2/b*d*a/\sin(b*x+a)^2-1/2*c/\sin(b*x+a)^2)$

Maxima [B] time = 1.16603, size = 387, normalized size = 11.06

$$\frac{2(4(bx+a)\cos(2bx+2a)^2+4(bx+a)\sin(2bx+2a)^2-(2(bx+a)\cos(2bx+2a)+\sin(2bx+2a))\cos(4bx+4a)-2(bx+a)\cos(2bx+2a)-(2(bx+a)\sin(2bx+2a)-\cos(2bx+2a))\sin(4bx+4a)-\cos(4bx+4a)-\sin(4bx+4a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")

[Out] $1/2*(2*(4*(b*x + a)*\cos(2*b*x + 2*a)^2 + 4*(b*x + a)*\sin(2*b*x + 2*a)^2 - (2*(b*x + a)*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a))*\cos(4*b*x + 4*a) - 2*(b*x + a)*\cos(2*b*x + 2*a) - (2*(b*x + a)*\sin(2*b*x + 2*a) - \cos(2*b*x + 2*a) + 1)*\sin(4*b*x + 4*a) + \sin(2*b*x + 2*a))*d/((2*(2*\cos(2*b*x + 2*a) - 1)*\cos(4*b*x + 4*a) - \cos(4*b*x + 4*a))^2 - 4*\cos(2*b*x + 2*a)^2 - \sin(4*b*x + 4*a)^2 + 4*\sin(4*b*x + 4*a)*\sin(2*b*x + 2*a) - 4*\sin(2*b*x + 2*a)^2 + 4*\cos(2*b*x + 2*a) - 1)*b) - c/\sin(b*x + a)^2 + a*d/(b*\sin(b*x + a)^2))/b$

Fricas [A] time = 0.456576, size = 103, normalized size = 2.94

$$\frac{bdx + d \cos(bx + a) \sin(bx + a) + bc}{2(b^2 \cos(bx + a)^2 - b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")

[Out] $1/2*(b*d*x + d*\cos(b*x + a)*\sin(b*x + a) + b*c)/(b^2*\cos(b*x + a)^2 - b^2)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20892, size = 710, normalized size = 20.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4 +
2*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4
+ 2*b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + 2*b
*c*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + b*d*x*tan
(1/2*b*x)^4 + 4*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*d*x*tan(1/2*a)^4 + b
*c*tan(1/2*b*x)^4 + 2*d*tan(1/2*b*x)^4*tan(1/2*a) + 4*b*c*tan(1/2*b*x)^2*tan
(1/2*a)^2 + 12*d*tan(1/2*b*x)^3*tan(1/2*a)^2 + 12*d*tan(1/2*b*x)^2*tan(1/2
*a)^3 + b*c*tan(1/2*a)^4 + 2*d*tan(1/2*b*x)*tan(1/2*a)^4 + 2*b*d*x*tan(1/2*
b*x)^2 + 2*b*d*x*tan(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2 - 2*d*tan(1/2*b*x)^3 -
12*d*tan(1/2*b*x)^2*tan(1/2*a) + 2*b*c*tan(1/2*a)^2 - 12*d*tan(1/2*b*x)*tan
(1/2*a)^2 - 2*d*tan(1/2*a)^3 + b*d*x + b*c + 2*d*tan(1/2*b*x) + 2*d*tan(1/
2*a))/(b^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*tan(1/2*b*x)^3*tan(1/2*a)^3
+ b^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*b^2*tan(1/2*b*x)^3*tan(1/2*a) - 4*b^2
*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^2*tan(1/2*b*x)*tan(1/2*a)^3 + b^2*tan(1/
2*b*x)^2 + 2*b^2*tan(1/2*b*x)*tan(1/2*a) + b^2*tan(1/2*a)^2)
```

$$3.50 \quad \int \frac{\cot(a+bx) \csc^2(a+bx)}{c+dx} dx$$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\cot(a+bx) \csc^2(a+bx)}{c+dx}, x\right)$$

[Out] CannotIntegrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x), x]

Rubi [A] time = 0.136579, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cot(a+bx) \csc^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x), x]

[Out] Defer[Int] [(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x), x]

Rubi steps

$$\int \frac{\cot(a+bx) \csc^2(a+bx)}{c+dx} dx = \int \frac{\cot(a+bx) \csc^2(a+bx)}{c+dx} dx$$

Mathematica [A] time = 11.5526, size = 0, normalized size = 0.

$$\int \frac{\cot(a+bx) \csc^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x), x]

[Out] Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x), x]

Maple [A] time = 0.445, size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a) (\csc(bx+a))^3}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c), x)

[Out] int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c), x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(bx+a)\csc(bx+a)^3}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x, algorithm="fricas")

[Out] integral(cos(b*x + a)*csc(b*x + a)^3/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)\csc^3(a+bx)}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)**3/(d*x+c),x)

[Out] Integral(cos(a + b*x)*csc(a + b*x)**3/(c + d*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)\csc(bx+a)^3}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x, algorithm="giac")

[Out] integrate(cos(b*x + a)*csc(b*x + a)^3/(d*x + c), x)

$$3.51 \quad \int \frac{\cot(a+bx) \csc^2(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\cot(a+bx) \csc^2(a+bx)}{(c+dx)^2}, x\right)$$

[Out] CannotIntegrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2, x]

Rubi [A] time = 0.166483, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cot(a+bx) \csc^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2,x]

[Out] Defer[Int] [(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\cot(a+bx) \csc^2(a+bx)}{(c+dx)^2} dx = \int \frac{\cot(a+bx) \csc^2(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 11.1508, size = 0, normalized size = 0.

$$\int \frac{\cot(a+bx) \csc^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2,x]

[Out] Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2, x]

Maple [A] time = 0.595, size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a) (\csc(bx+a))^3}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x)

[Out] int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(bx+a)\csc(bx+a)^3}{d^2x^2+2cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(cos(b*x + a)*csc(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)\csc^3(a+bx)}{(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)**3/(d*x+c)**2,x)

[Out] Integral(cos(a + b*x)*csc(a + b*x)**3/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)\csc(bx+a)^3}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(cos(b*x + a)*csc(b*x + a)^3/(d*x + c)^2, x)

3.52 $\int (c + dx)^{5/2} \cos(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=196

$$\frac{15\sqrt{\pi}d^{5/2} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{7/2}} + \frac{15\sqrt{\pi}d^{5/2} \sin\left(2a - \frac{2bc}{d}\right) S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}} + \frac{15d^2\sqrt{c+dx} \cos(2a + 2bx)}{64b^3} +$$

[Out] $(15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[2*a + 2*b*x])/(64*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[2*a + 2*b*x])/(4*b) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])]/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]])/ (128*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])]/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}])* \text{Sin}[2*a - (2*b*c)/d])/ (128*b^{(7/2)}) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[2*a + 2*b*x])/(16*b^2)$

Rubi [A] time = 0.447523, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\pi}d^{5/2} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{7/2}} + \frac{15\sqrt{\pi}d^{5/2} \sin\left(2a - \frac{2bc}{d}\right) S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}} + \frac{15d^2\sqrt{c+dx} \cos(2a + 2bx)}{64b^3} +$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^{(5/2)}*\text{Cos}[a + b*x]*\text{Sin}[a + b*x], x]$

[Out] $(15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[2*a + 2*b*x])/(64*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[2*a + 2*b*x])/(4*b) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])]/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]])/ (128*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])]/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}])* \text{Sin}[2*a - (2*b*c)/d])/ (128*b^{(7/2)}) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[2*a + 2*b*x])/(16*b^2)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]]^n*\text{Cos}[a + b*x]^p, x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 12

$\text{Int}[(a_)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$ FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3306

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^(2)], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^(2)], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^{5/2} \cos(a + bx) \sin(a + bx) dx &= \int \frac{1}{2} (c + dx)^{5/2} \sin(2a + 2bx) dx \\
&= \frac{1}{2} \int (c + dx)^{5/2} \sin(2a + 2bx) dx \\
&= -\frac{(c + dx)^{5/2} \cos(2a + 2bx)}{4b} + \frac{(5d) \int (c + dx)^{3/2} \cos(2a + 2bx) dx}{8b} \\
&= -\frac{(c + dx)^{5/2} \cos(2a + 2bx)}{4b} + \frac{5d(c + dx)^{3/2} \sin(2a + 2bx)}{16b^2} - \frac{(15d^2) \int \sqrt{c + dx} \cos(2a + 2bx) dx}{16b^2} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{64b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{4b} + \frac{5d(c + dx)^{3/2} \sin(2a + 2bx)}{16b^2} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{64b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{4b} + \frac{5d(c + dx)^{3/2} \sin(2a + 2bx)}{16b^2} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{64b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{4b} + \frac{5d(c + dx)^{3/2} \sin(2a + 2bx)}{16b^2} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{64b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{4b} - \frac{15d^{5/2} \sqrt{\pi} \cos(2a + 2bx)}{16b^2}
\end{aligned}$$

Mathematica [A] time = 2.36958, size = 179, normalized size = 0.91

$$\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}\left(20bd(c+dx)\sin(2(a+bx))-\cos(2(a+bx))(16b^2(c+dx)^2-15d^2)\right)-15\sqrt{\pi}d^2\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left[\frac{2\sqrt{b/d}\sqrt{c+dx}}{\sqrt{\pi}}\right]+15d^2\sqrt{\pi}\text{FresnelS}\left[\frac{2\sqrt{b/d}\sqrt{c+dx}}{\sqrt{\pi}}\right]}{128b^3\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x], x]
```

```
[Out] (-15*d^2*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + 15*d^2*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]])
```

$\frac{\sin(2a - (2bc)/d) + 2\sqrt{b/d}\sqrt{c+dx}(-((-15d^2 + 16b^2(c+dx)^2)\cos(2(a+bx)) + 20bd(c+dx)\sin(2(a+bx))))}{(128b^3\sqrt{b/d})}$

Maple [A] time = 0.037, size = 234, normalized size = 1.2

$$2\frac{1}{d}\left(-\frac{1}{8}\frac{d(dx+c)^{5/2}}{b}\cos\left(2\frac{(dx+c)b}{d}+2\frac{ad-bc}{d}\right)+\frac{5}{8}\frac{d}{b}\left[\frac{1}{4}\frac{d(dx+c)^{3/2}}{b}\sin\left(2\frac{(dx+c)b}{d}+2\frac{ad-bc}{d}\right)-\frac{3}{4}\frac{d}{b}\left(-1\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x)

[Out] $2/d*(-1/8/b*d*(d*x+c)^{(5/2)}*\cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/8/b*d*(1/4/b*d*(d*x+c)^{(3/2)}*\sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+c)^{(1/2)}*\cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*\text{Pi}^{(1/2)}/(b/d)^{(1/2)}*(\cos(2*(a*d-b*c)/d)*\text{FresnelC}(2/\text{Pi}^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d)-\sin(2*(a*d-b*c)/d)*\text{FresnelS}(2/\text{Pi}^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d))$

Maxima [C] time = 1.95552, size = 910, normalized size = 4.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")

[Out] $\frac{1}{1024}\sqrt{2}*(160\sqrt{2}*(d*x+c)^{(3/2)}*b*d^2*\sqrt{\text{abs}(b)/\text{abs}(d)}*\sin(2*((d*x+c)*b-b*c+a*d)/d)-8*(16\sqrt{2}*(d*x+c)^{(5/2)}*b^2*d*\sqrt{\text{abs}(b)/\text{abs}(d)}-15*\sqrt{2}*\sqrt{d*x+c}*d^3*\sqrt{\text{abs}(b)/\text{abs}(d)})*\cos(2*((d*x+c)*b-b*c+a*d)/d)-((15*\sqrt{\text{pi}}*\cos(1/4*\text{pi}+1/2*\arctan2(0,b)+1/2*\arctan2(0,d/\sqrt{d^2}))+15*\sqrt{\text{pi}}*\cos(-1/4*\text{pi}+1/2*\arctan2(0,b)+1/2*\arctan2(0,d/\sqrt{d^2}))-15*I*\sqrt{\text{pi}}*\sin(1/4*\text{pi}+1/2*\arctan2(0,b)+1/2*\arctan2(0,d/\sqrt{d^2}))+15*I*\sqrt{\text{pi}}*\sin(-1/4*\text{pi}+1/2*\arctan2(0,b)+1/2*\arctan2(0,d/\sqrt{d^2}))*d^3*\cos(-2*(b*c-a*d)/d)-(15*I*\sqrt{\text{pi}}*\cos(1/4*\text{pi}+1/2*\arctan2(0,b)+1/2*\arctan2(0,d/\sqrt{d^2}))+15*I*\sqrt{\text{pi}}*\cos(-1/4*\text{pi}+1/2*\arctan2(0,b)+1/2*\arctan2(0,d/\sqrt{d^2}))+15*\sqrt{\text{pi}}*\sin(1/4*\text{pi}+1/2*\arctan2(0,b)+1/2*\arctan2(0,d/\sqrt{d^2}))-15*\sqrt{\text{pi}}*\sin(-1/4*\text{pi}+1/2*\arctan2(0,b)+1/2*\arctan2(0,d/\sqrt{d^2}))*d^3*\sin(-2*(b*c-a*d)/d))*\text{erf}(\sqrt{d*x+c}*\sqrt{2*I*b/d})-((15*\sqrt{\text{pi}}*\cos(1/4*\text{pi}+1/2*\arctan2(0,b)+1/2*\arctan2(0,d/\sqrt{d^2}))+15*\sqrt{\text{pi}}*\cos(-1/4*\text{pi}+1/2*\arctan2(0,b)+1/2*\arctan2(0,d/\sqrt{d^2}))+15*I*\sqrt{\text{pi}}*\sin(1/4*\text{pi}+1/2*\arctan2(0,b)+1/2*\arctan2(0,d/\sqrt{d^2}))-15*I*\sqrt{\text{pi}}*\sin(-1/4*\text{pi}+1/2*\arctan2(0,b)+1/2*\arctan2(0,d/\sqrt{d^2}))*d^3*\cos(-2*(b*c-a*d)/d)-(-15*I*\sqrt{\text{pi}}*\cos(1/4*\text{pi}+1/2*\arctan2(0,b)+1/2*\arctan2(0,d/\sqrt{d^2}))-15*I*\sqrt{\text{pi}}*\cos(-1/4*\text{pi}+1/2*\arctan2(0,b)+1/2*\arctan2(0,d/\sqrt{d^2}))+15*\sqrt{\text{pi}}*\sin(1/4*\text{pi}+1/2*\arctan2(0,b)+1/2*\arctan2(0,d/\sqrt{d^2}))-15*\sqrt{\text{pi}}*\sin(-1/4*\text{pi}+1/2*\arctan2(0,b)+1/2*\arctan2(0,d/\sqrt{d^2}))*d^3*\sin(-2*(b*c-a*d)/d))*\text{erf}(\sqrt{d*x+c}*\sqrt{-2*I*b/d}))/b^3*d*\sqrt{\text{abs}(b)/\text{abs}(d)}$

Fricas [A] time = 0.550609, size = 540, normalized size = 2.76

$$\frac{15 \pi d^3 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{2(bc-ad)}{d}\right) C\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 15 \pi d^3 \sqrt{\frac{b}{\pi d}} S\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{2(bc-ad)}{d}\right) - 2(16b^3d^2x^2 + 32b^3d^2x + 16b^3c^2 - 15b^3d^2 - 2(16b^3d^2x^2 + 32b^3c^2d^2x + 16b^3c^2 - 15b^3d^2) \cos(bx+a)^2 + 40(b^2d^2x + b^2cd) \cos(bx+a) \sin(bx+a)) \sqrt{dx+c}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")

[Out] -1/128*(15*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c^2 - 15*b*d^2 - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c^2 - 15*b*d^2)*cos(b*x + a)^2 + 40*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)*sin(b*x + a))*sqrt(d*x + c)/b^4

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a),x)

[Out] Timed out

Giac [C] time = 1.31783, size = 1327, normalized size = 6.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")

[Out] -1/256*(16*(sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c))*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c))*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 2*sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b)*c^2 + d^2*((I*sqrt(pi))*(-16*I*b^2*c^2*d + 24*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c))*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 32*I*(d*x + c)^(3/2)*b^2*c*d + 16*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 24*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^3)/d^2 + (I*sqrt(pi))*(-16*I*b^2*c^2*d - 24*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c))*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 32*I*(d*x + c)^(3/2)*b^2*c*d + 16*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 24*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^2 + 8*c*(I*sqrt(pi))*(4*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c))*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)

$$\begin{aligned}
& *b^2) + I*\sqrt{\pi}*(4*I*b*c*d + 3*d^2)*d*\operatorname{erf}(-\sqrt{b*d})*\sqrt{d*x + c)*(-I*b \\
& *d/\sqrt{b^2*d^2} + 1)/d)*e^{((-2*I*b*c + 2*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{ \\
& (b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^{(3/2)}*b*d - 4*I*\sqrt{d*x + c)*b*c* \\
& d - 3*\sqrt{d*x + c)*d^2)*e^{((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^2 - \\
& 2*I*(4*I*(d*x + c)^{(3/2)}*b*d - 4*I*\sqrt{d*x + c)*b*c*d + 3*\sqrt{d*x + c)*d^ \\
& 2)*e^{((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2))/d
\end{aligned}$$

3.53 $\int (c + dx)^{3/2} \cos(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=168

$$\frac{3\sqrt{\pi}d^{3/2} \sin\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{32b^{5/2}} - \frac{3\sqrt{\pi}d^{3/2} \cos\left(2a - \frac{2bc}{d}\right) S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}} + \frac{3d\sqrt{c+dx} \sin(2a + 2bx)}{16b^2}$$

[Out] $-\left((c + d*x)^{(3/2)}*\text{Cos}[2*a + 2*b*x]\right)/(4*b) - (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]))/(32*b^{(5/2)}) - (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]))*\text{Sin}[2*a - (2*b*c)/d]/(32*b^{(5/2)}) + (3*d*\text{Sqrt}[c + d*x]*\text{Sin}[2*a + 2*b*x])/(16*b^2)$

Rubi [A] time = 0.29454, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\pi}d^{3/2} \sin\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{32b^{5/2}} - \frac{3\sqrt{\pi}d^{3/2} \cos\left(2a - \frac{2bc}{d}\right) S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}} + \frac{3d\sqrt{c+dx} \sin(2a + 2bx)}{16b^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^{(3/2)}*\text{Cos}[a + b*x]*\text{Sin}[a + b*x], x]$

[Out] $-\left((c + d*x)^{(3/2)}*\text{Cos}[2*a + 2*b*x]\right)/(4*b) - (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]))/(32*b^{(5/2)}) - (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]))*\text{Sin}[2*a - (2*b*c)/d]/(32*b^{(5/2)}) + (3*d*\text{Sqrt}[c + d*x]*\text{Sin}[2*a + 2*b*x])/(16*b^2)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]]^n*\text{Cos}[a + b*x]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 12

$\text{Int}[(a_.)*(u_.), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$ FreeQ[a, x] && !MatchQ[u, (b_.)*(v_.) /]; FreeQ[b, x]

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3306

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^{3/2} \cos(a + bx) \sin(a + bx) dx &= \int \frac{1}{2} (c + dx)^{3/2} \sin(2a + 2bx) dx \\
&= \frac{1}{2} \int (c + dx)^{3/2} \sin(2a + 2bx) dx \\
&= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{4b} + \frac{(3d) \int \sqrt{c + dx} \cos(2a + 2bx) dx}{8b} \\
&= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{4b} + \frac{3d\sqrt{c + dx} \sin(2a + 2bx)}{16b^2} - \frac{(3d^2) \int \frac{\sin(2a + 2bx)}{\sqrt{c + dx}} dx}{32b^2} \\
&= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{4b} + \frac{3d\sqrt{c + dx} \sin(2a + 2bx)}{16b^2} - \frac{(3d^2 \cos(2a - \frac{2bc}{d}))}{32b^2} \\
&= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{4b} + \frac{3d\sqrt{c + dx} \sin(2a + 2bx)}{16b^2} - \frac{(3d \cos(2a - \frac{2bc}{d}))}{32b^2} \\
&= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{4b} - \frac{3d^{3/2} \sqrt{\pi} \cos(2a - \frac{2bc}{d}) S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}}\right)}{32b^{5/2}} - \frac{3d^{3/2} \sqrt{\pi} C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}}\right)}{32b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.886386, size = 157, normalized size = 0.93

$$\frac{-3\sqrt{\pi}d \sin\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - 3\sqrt{\pi}d \cos\left(2a - \frac{2bc}{d}\right) S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - 2\sqrt{\frac{b}{d}}\sqrt{c+dx}(4b(c+dx) \cos(2(a+bx)))}{32d^2\left(\frac{b}{d}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x], x]
```

```
[Out] (-3*d*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 3*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] - 2*Sqrt[b/d]*Sqrt[c + d*x]*(4*b*(c + d*x)*Cos[2*(a + b*x)])
```


$$- 3*d*\text{Sin}[2*(a + b*x)])/(32*(b/d)^(5/2)*d^2)$$

Maple [A] time = 0.025, size = 187, normalized size = 1.1

$$2 \frac{1}{d} \left(-\frac{1}{8} \frac{d(dx+c)^{3/2}}{b} \cos\left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d}\right) + \frac{3}{8} \frac{d}{b} \left(\frac{1}{4} \frac{d\sqrt{dx+c}}{b} \sin\left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d}\right) - \frac{1}{8} \frac{d\sqrt{\pi}}{b} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x)

[Out] 2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/8/b*d*(1/4/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))

Maxima [C] time = 1.88766, size = 869, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")

[Out] -1/256*sqrt(2)*(32*sqrt(2)*(d*x + c)^(3/2)*b*d*sqrt(abs(b)/abs(d))*cos(2*((d*x + c)*b - b*c + a*d)/d) - 24*sqrt(2)*sqrt(d*x + c)*d^2*sqrt(abs(b)/abs(d))*sin(2*((d*x + c)*b - b*c + a*d)/d) - ((-3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*cos(-2*(b*c - a*d)/d) - (3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - ((3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*cos(-2*(b*c - a*d)/d) - (3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/(b^2*d*sqrt(abs(b)/abs(d)))

Fricas [A] time = 0.531847, size = 413, normalized size = 2.46

$$3 \pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{2(bc-ad)}{d}\right) S\left(2 \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) + 3 \pi d^2 \sqrt{\frac{b}{\pi d}} C\left(2 \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{2(bc-ad)}{d}\right) - 4 \left(2 b^2 dx + 3 b d \cos\left(-\frac{2(bc-ad)}{d}\right)\right)$$

32 b³

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")

[Out] $-1/32*(3*\pi*d^2*\sqrt{b/(pi*d)}*\cos(-2*(b*c - a*d)/d)*\text{fresnel_sin}(2*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) + 3*\pi*d^2*\sqrt{b/(pi*d)}*\text{fresnel_cos}(2*\sqrt{d*x + c}*\sqrt{b/(pi*d)})*\sin(-2*(b*c - a*d)/d) - 4*(2*b^2*d*x + 3*b*d*\cos(b*x + a))*\sin(b*x + a) + 2*b^2*c - 4*(b^2*d*x + b^2*c)*\cos(b*x + a)^2*\sqrt{d*x + c})/b^3$

Sympy [B] time = 130.886, size = 665, normalized size = 3.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a),x)

[Out] $-5*\sqrt{\pi}*\sqrt{d/b}*(c + d*x)**2*\sin(2*a - 2*b*c/d)*\text{fresnelc}(2*\sqrt{b}*\sqrt{c + d*x}/(\sqrt{\pi}*\sqrt{d}))*\gamma(1/4)/(32*d*\gamma(9/4)) + \sqrt{\pi}*\sqrt{d/b}*(c + d*x)**2*\sin(2*a - 2*b*c/d)*\text{fresnelc}(2*b*\sqrt{c + d*x}/(\sqrt{\pi}*\sqrt{d})*\sqrt{b/d}))/2*d - 21*\sqrt{\pi}*\sqrt{d/b}*(c + d*x)**2*\cos(2*a - 2*b*c/d)*\text{fresnels}(2*\sqrt{b}*\sqrt{c + d*x}/(\sqrt{\pi}*\sqrt{d}))*\gamma(3/4)/(32*d*\gamma(11/4)) + \sqrt{\pi}*\sqrt{d/b}*(c + d*x)**2*\cos(2*a - 2*b*c/d)*\text{fresnels}(2*b*\sqrt{c + d*x}/(\sqrt{\pi}*\sqrt{d})*\sqrt{b/d}))/2*d - 15*\sqrt{\pi}*d*\sqrt{d/b}*\sin(2*a - 2*b*c/d)*\text{fresnelc}(2*\sqrt{b}*\sqrt{c + d*x}/(\sqrt{\pi}*\sqrt{d}))*\gamma(1/4)/(512*b**2*\gamma(9/4)) - 63*\sqrt{\pi}*d*\sqrt{d/b}*\cos(2*a - 2*b*c/d)*\text{fresnels}(2*\sqrt{b}*\sqrt{c + d*x}/(\sqrt{\pi}*\sqrt{d}))*\gamma(3/4)/(512*b**2*\gamma(11/4)) + 5*\sqrt{d/b}*(c + d*x)**(3/2)*\sin(2*a - 2*b*c/d)*\sin(2*b*c/d + 2*b*x)*\gamma(1/4)/(64*\sqrt{b}*\sqrt{d}*\gamma(9/4)) - 21*\sqrt{d/b}*(c + d*x)**(3/2)*\cos(2*a - 2*b*c/d)*\cos(2*b*c/d + 2*b*x)*\gamma(3/4)/(64*\sqrt{b}*\sqrt{d}*\gamma(11/4)) + 15*\sqrt{d}*\sqrt{d/b}*\sqrt{c + d*x}*\sin(2*a - 2*b*c/d)*\cos(2*b*c/d + 2*b*x)*\gamma(1/4)/(256*b**(3/2)*\gamma(9/4)) + 63*\sqrt{d}*\sqrt{d/b}*\sqrt{c + d*x}*\sin(2*b*c/d + 2*b*x)*\cos(2*a - 2*b*c/d)*\gamma(3/4)/(256*b**(3/2)*\gamma(11/4))$

Giac [C] time = 1.22975, size = 732, normalized size = 4.36

$$4 \left(\frac{\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right) e^{\left(\frac{2ibc-2iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} + \frac{\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right) e^{\left(\frac{-2ibc+2iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} + \frac{2\sqrt{dx+c}de^{\left(\frac{2i(dx+c)b-2ibc+2iad}{d}\right)}}{b} + \frac{2\sqrt{dx+c}de^{\left(\frac{-2i(dx+c)b+2ibc-2iad}{d}\right)}}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")

[Out] $-1/64*(4*(\sqrt{\pi})*d^2*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((2*I*b*c - 2*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b)} + \sqrt{\pi}*d^2*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-2*I*b*c + 2*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b)} + 2*\sqrt{d*x + c}*d*e^{((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b} + 2*\sqrt{d*x + c}*d*$

$$\begin{aligned}
& e^{\left(\frac{-2I(dx+c)b + 2Ib^2c - 2Ia^2d}{d}\right)/b} * c + I\sqrt{\pi} * (4Ib^2cd - \\
& 3d^2) * d * \operatorname{erf}\left(\frac{-\sqrt{bd} * \sqrt{dx+c} * (Ibd/\sqrt{b^2d^2} + 1)}{d}\right) * e^{\left(\frac{2Ib^2c - 2Ia^2d}{d}\right)/(\sqrt{bd} * (Ibd/\sqrt{b^2d^2} + 1) * b^2)} + I\sqrt{\pi} * \\
& (4Ib^2cd + 3d^2) * d * \operatorname{erf}\left(\frac{-\sqrt{bd} * \sqrt{dx+c} * (-Ibd/\sqrt{b^2d^2} + 1)}{d}\right) * e^{\left(\frac{-2Ib^2c + 2Ia^2d}{d}\right)/(\sqrt{bd} * (-Ibd/\sqrt{b^2d^2} + 1) * b^2)} \\
& - 2I * (4I(dx+c)^{3/2} * bd - 4I\sqrt{dx+c} * b^2cd - 3\sqrt{dx+c} * \\
& d^2) * e^{\left(\frac{2I(dx+c)b - 2Ib^2c + 2Ia^2d}{d}\right)/b^2} - 2I * (4I(dx+c)^{3/2} * bd - 4I\sqrt{dx+c} * b^2cd + 3\sqrt{dx+c} * d^2) * e^{\left(\frac{-2I(dx+c)b + 2Ib^2c - 2Ia^2d}{d}\right)/b^2} / d
\end{aligned}$$

3.54 $\int \sqrt{c + dx} \cos(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=142

$$\frac{\sqrt{\pi}\sqrt{d} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{8b^{3/2}} - \frac{\sqrt{\pi}\sqrt{d} \sin\left(2a - \frac{2bc}{d}\right) S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}} - \frac{\sqrt{c + dx} \cos(2a + 2bx)}{4b}$$

[Out] $-(\text{Sqrt}[c + d*x]*\text{Cos}[2*a + 2*b*x])/(4*b) + (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])]/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]])/ (8*b^(3/2)) - (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])]/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]])* \text{Sin}[2*a - (2*b*c)/d])/ (8*b^(3/2))$

Rubi [A] time = 0.230695, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\pi}\sqrt{d} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{8b^{3/2}} - \frac{\sqrt{\pi}\sqrt{d} \sin\left(2a - \frac{2bc}{d}\right) S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}} - \frac{\sqrt{c + dx} \cos(2a + 2bx)}{4b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[c + d*x]*\text{Cos}[a + b*x]*\text{Sin}[a + b*x], x]$

[Out] $-(\text{Sqrt}[c + d*x]*\text{Cos}[2*a + 2*b*x])/(4*b) + (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])]/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]])/ (8*b^(3/2)) - (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])]/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]])* \text{Sin}[2*a - (2*b*c)/d])/ (8*b^(3/2))$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^(p_.)*((c_.) + (d_.)*(x_.))^(m_.)*\text{Sin}[(a_.) + (b_.)*(x_.)]^(n_.), x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]]^n*\text{Cos}[a + b*x]^p, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[p, 0]$

Rule 12

$\text{Int}[(a_.)*(u_.), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_.)*(v_.)] /; \text{FreeQ}[b, x]$

Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_.))^(m_.)*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^(m - 1)*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{GtQ}[m, 0]$

Rule 3306

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{NeQ}[d*e - c*f, 0]$

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{c+dx} \cos(a+bx) \sin(a+bx) dx &= \int \frac{1}{2} \sqrt{c+dx} \sin(2a+2bx) dx \\
&= \frac{1}{2} \int \sqrt{c+dx} \sin(2a+2bx) dx \\
&= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{4b} + \frac{d \int \frac{\cos(2a+2bx)}{\sqrt{c+dx}} dx}{8b} \\
&= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{4b} + \frac{\left(d \cos\left(2a - \frac{2bc}{d}\right)\right) \int \frac{\cos\left(\frac{2bc}{d} + 2bx\right)}{\sqrt{c+dx}} dx}{8b} - \frac{\left(d \sin\left(2a - \frac{2bc}{d}\right)\right) \int \frac{\sin\left(\frac{2bc}{d} + 2bx\right)}{\sqrt{c+dx}} dx}{8b} \\
&= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{4b} + \frac{\cos\left(2a - \frac{2bc}{d}\right) \text{Subst}\left(\int \cos\left(\frac{2bx^2}{d}\right) dx, x, \sqrt{c+dx}\right)}{4b} - \frac{\sin\left(2a - \frac{2bc}{d}\right) \text{Subst}\left(\int \sin\left(\frac{2bx^2}{d}\right) dx, x, \sqrt{c+dx}\right)}{4b} \\
&= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{4b} + \frac{\sqrt{d}\sqrt{\pi} \cos\left(2a - \frac{2bc}{d}\right) C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}} - \frac{\sqrt{d}\sqrt{\pi} S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.269331, size = 134, normalized size = 0.94

$$\frac{\sqrt{\pi} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - \sqrt{\pi} \sin\left(2a - \frac{2bc}{d}\right) \text{FresnelS}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - 2\sqrt{\frac{b}{d}}\sqrt{c+dx} \cos(2(a+bx))}{8b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x], x]
```

```
[Out] (-2*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(8*b*Sqrt[b/d])
```

Maple [A] time = 0.023, size = 142, normalized size = 1.

$$2 \frac{1}{d} \left(-\frac{1}{8} \frac{d\sqrt{dx+c}}{b} \cos\left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d}\right) + \frac{1}{16} \frac{d\sqrt{\pi}}{b} \left(\cos\left(2 \frac{ad-bc}{d}\right) \text{FresnelC}\left(2 \frac{\sqrt{dx+cb}}{d\sqrt{\pi}} \frac{1}{\sqrt{\frac{b}{d}}}\right) - \sin\left(2 \frac{ad-bc}{d}\right) \text{FresnelS}\left(2 \frac{\sqrt{dx+cb}}{d\sqrt{\pi}} \frac{1}{\sqrt{\frac{b}{d}}}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x)

[Out] 2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/16/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))

Maxima [C] time = 2.06039, size = 788, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")

[Out] -1/64*sqrt(2)*(8*sqrt(2)*sqrt(d*x + c)*d*sqrt(abs(b)/abs(d))*cos(2*((d*x + c)*b - b*c + a*d)/d) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*cos(-2*(b*c - a*d)/d) - (I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*cos(-2*(b*c - a*d)/d) - (-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/(b*d*sqrt(abs(b)/abs(d)))

Fricas [A] time = 0.521148, size = 308, normalized size = 2.17

$$\frac{\pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{2(bc-ad)}{d}\right) C\left(2 \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) - \pi d \sqrt{\frac{b}{\pi d}} S\left(2 \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{2(bc-ad)}{d}\right) - 2\left(2b \cos(bx+a)^2 - b\right) \sqrt{dx+c}}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")

```
[Out] 1/8*(pi*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(2*b*cos(b*x + a)^2 - b)*sqrt(d*x + c)/b^2
```

Sympy [B] time = 5.90256, size = 389, normalized size = 2.74

$$\frac{b^{\frac{3}{2}}\sqrt{\frac{d}{b}}(c+dx)^{\frac{5}{2}}\cos\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\frac{3}{2},\frac{5}{4},\frac{9}{4}\left|-\frac{b^2(c+dx)^2}{d^2}\right.\right)}{4d^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)} - \frac{\sqrt{b}\sqrt{\frac{d}{b}}(c+dx)^{\frac{3}{2}}\sin\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\frac{1}{2},\frac{3}{4},\frac{7}{4}\left|-\frac{b^2(c+dx)^2}{d^2}\right.\right)}{8d^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] -b**(3/2)*sqrt(d/b)*(c + d*x)**(5/2)*cos(2*a - 2*b*c/d)*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (3/2, 7/4, 9/4), -b**2*(c + d*x)**2/d**2)/(4*d**(5/2)*gamma(7/4)*gamma(9/4)) - sqrt(b)*sqrt(d/b)*(c + d*x)**(3/2)*sin(2*a - 2*b*c/d)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -b**2*(c + d*x)**2/d**2)/(8*d**(3/2)*gamma(5/4)*gamma(7/4)) + sqrt(pi)*c*sqrt(d/b)*sin(2*a - 2*b*c/d)*fresnelc(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/(2*d) + sqrt(pi)*c*sqrt(d/b)*cos(2*a - 2*b*c/d)*fresnels(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/(2*d) + sqrt(pi)*x*sqrt(d/b)*sin(2*a - 2*b*c/d)*fresnelc(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/2 + sqrt(pi)*x*sqrt(d/b)*cos(2*a - 2*b*c/d)*fresnels(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/2
```

Giac [C] time = 1.16851, size = 316, normalized size = 2.23

$$\frac{\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\left(\frac{2ibc-2iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} + \frac{\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\left(\frac{-2ibc+2iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} + \frac{2\sqrt{dx+c}e^{\left(\frac{2i(dx+c)b-2ibc+2iad}{d}\right)}}{b} + \frac{2\sqrt{dx+c}e^{\left(\frac{-2i(dx+c)b+2ibc-2iad}{d}\right)}}{b}$$

16d

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*(sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 2*sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b/d
```

3.55 $\int \sqrt{c + dx} \cos(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=142

$$\frac{\sqrt{\pi}\sqrt{d} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{8b^{3/2}} - \frac{\sqrt{\pi}\sqrt{d} \sin\left(2a - \frac{2bc}{d}\right) S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}} - \frac{\sqrt{c + dx} \cos(2a + 2bx)}{4b}$$

[Out] $-(\text{Sqrt}[c + d*x]*\text{Cos}[2*a + 2*b*x])/(4*b) + (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])]/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]])/ (8*b^(3/2)) - (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])]/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]))*\text{Sin}[2*a - (2*b*c)/d])/ (8*b^(3/2))$

Rubi [A] time = 0.22096, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\pi}\sqrt{d} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{8b^{3/2}} - \frac{\sqrt{\pi}\sqrt{d} \sin\left(2a - \frac{2bc}{d}\right) S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}} - \frac{\sqrt{c + dx} \cos(2a + 2bx)}{4b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[c + d*x]*\text{Cos}[a + b*x]*\text{Sin}[a + b*x], x]$

[Out] $-(\text{Sqrt}[c + d*x]*\text{Cos}[2*a + 2*b*x])/(4*b) + (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])]/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]])/ (8*b^(3/2)) - (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])]/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]))*\text{Sin}[2*a - (2*b*c)/d])/ (8*b^(3/2))$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^(p_.)*((c_.) + (d_.)*(x_.))^(m_.)*\text{Sin}[(a_.) + (b_.)*(x_.)]^(n_.), x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^(n)*\text{Cos}[a + b*x]^p, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[p, 0]$

Rule 12

$\text{Int}[(a_.)*(u_.), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_.)*(v_.)] /; \text{FreeQ}[b, x]$

Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_.))^(m_.)*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^(m-1)*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{GtQ}[m, 0]$

Rule 3306

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{NeQ}[d*e - c*f, 0]$

Rule 3305


```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{c+dx} \cos(a+bx) \sin(a+bx) dx &= \int \frac{1}{2} \sqrt{c+dx} \sin(2a+2bx) dx \\
&= \frac{1}{2} \int \sqrt{c+dx} \sin(2a+2bx) dx \\
&= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{4b} + \frac{d \int \frac{\cos(2a+2bx)}{\sqrt{c+dx}} dx}{8b} \\
&= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{4b} + \frac{\left(d \cos\left(2a - \frac{2bc}{d}\right)\right) \int \frac{\cos\left(\frac{2bc}{d} + 2bx\right)}{\sqrt{c+dx}} dx}{8b} - \frac{\left(d \sin\left(2a - \frac{2bc}{d}\right)\right) \int \frac{\sin\left(\frac{2bc}{d} + 2bx\right)}{\sqrt{c+dx}} dx}{8b} \\
&= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{4b} + \frac{\cos\left(2a - \frac{2bc}{d}\right) \text{Subst}\left(\int \cos\left(\frac{2bx^2}{d}\right) dx, x, \sqrt{c+dx}\right)}{4b} - \frac{\sin\left(2a - \frac{2bc}{d}\right) \text{Subst}\left(\int \sin\left(\frac{2bx^2}{d}\right) dx, x, \sqrt{c+dx}\right)}{4b} \\
&= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{4b} + \frac{\sqrt{d}\sqrt{\pi} \cos\left(2a - \frac{2bc}{d}\right) C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}} - \frac{\sqrt{d}\sqrt{\pi} S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.026048, size = 134, normalized size = 0.94

$$\frac{\sqrt{\pi} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - \sqrt{\pi} \sin\left(2a - \frac{2bc}{d}\right) \text{FresnelS}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - 2\sqrt{\frac{b}{d}}\sqrt{c+dx} \cos(2(a+bx))}{8b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x], x]
```

```
[Out] (-2*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(8*b*Sqrt[b/d])
```

Maple [A] time = 0.023, size = 142, normalized size = 1.

$$2 \frac{1}{d} \left(-\frac{1}{8} \frac{d\sqrt{dx+c}}{b} \cos\left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d}\right) + \frac{1}{16} \frac{d\sqrt{\pi}}{b} \left(\cos\left(2 \frac{ad-bc}{d}\right) \operatorname{FresnelC}\left(2 \frac{\sqrt{dx+cb}}{d\sqrt{\pi}} \frac{1}{\sqrt{\frac{b}{d}}}\right) - \sin\left(2 \frac{ad-bc}{d}\right) \operatorname{FresnelS}\left(2 \frac{\sqrt{dx+cb}}{d\sqrt{\pi}} \frac{1}{\sqrt{\frac{b}{d}}}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x)`

[Out] `2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/16/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))`

Maxima [C] time = 1.97827, size = 788, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")`

[Out] `-1/64*sqrt(2)*(8*sqrt(2)*sqrt(d*x + c)*d*sqrt(abs(b)/abs(d))*cos(2*((d*x + c)*b - b*c + a*d)/d) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*cos(-2*(b*c - a*d)/d) - (I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*cos(-2*(b*c - a*d)/d) - (-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/(b*d*sqrt(abs(b)/abs(d)))`

Fricas [A] time = 0.519941, size = 308, normalized size = 2.17

$$\frac{\pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{2(bc-ad)}{d}\right) C\left(2 \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) - \pi d \sqrt{\frac{b}{\pi d}} S\left(2 \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{2(bc-ad)}{d}\right) - 2\left(2b \cos(bx+a)^2 - b\right) \sqrt{dx+c}}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")`

```
[Out] 1/8*(pi*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(2*b*cos(b*x + a)^2 - b)*sqrt(d*x + c)/b^2
```

Sympy [B] time = 5.84613, size = 389, normalized size = 2.74

$$\frac{b^{\frac{3}{2}}\sqrt{\frac{d}{b}}(c+dx)^{\frac{5}{2}}\cos\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\frac{3}{2},\frac{5}{4},\frac{9}{4}\left|-\frac{b^2(c+dx)^2}{d^2}\right.\right)}{4d^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)} - \frac{\sqrt{b}\sqrt{\frac{d}{b}}(c+dx)^{\frac{3}{2}}\sin\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\frac{1}{2},\frac{3}{4},\frac{7}{4}\left|-\frac{b^2(c+dx)^2}{d^2}\right.\right)}{8d^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] -b**(3/2)*sqrt(d/b)*(c + d*x)**(5/2)*cos(2*a - 2*b*c/d)*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (3/2, 7/4, 9/4), -b**2*(c + d*x)**2/d**2)/(4*d**(5/2)*gamma(7/4)*gamma(9/4)) - sqrt(b)*sqrt(d/b)*(c + d*x)**(3/2)*sin(2*a - 2*b*c/d)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -b**2*(c + d*x)**2/d**2)/(8*d**(3/2)*gamma(5/4)*gamma(7/4)) + sqrt(pi)*c*sqrt(d/b)*sin(2*a - 2*b*c/d)*fresnelc(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/(2*d) + sqrt(pi)*c*sqrt(d/b)*cos(2*a - 2*b*c/d)*fresnels(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/(2*d) + sqrt(pi)*x*sqrt(d/b)*sin(2*a - 2*b*c/d)*fresnelc(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/2 + sqrt(pi)*x*sqrt(d/b)*cos(2*a - 2*b*c/d)*fresnels(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/2
```

Giac [C] time = 1.15843, size = 316, normalized size = 2.23

$$\frac{\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\left(\frac{2ibc-2iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} + \frac{\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\left(\frac{-2ibc+2iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} + \frac{2\sqrt{dx+c}e^{\left(\frac{2i(dx+c)b-2ibc+2iad}{d}\right)}}{b} + \frac{2\sqrt{dx+c}e^{\left(\frac{-2i(dx+c)b+2ibc-2iad}{d}\right)}}{b}$$

16d

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*(sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 2*sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b/d
```

3.56 $\int (c + dx)^{3/2} \cos(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=168

$$\frac{3\sqrt{\pi}d^{3/2} \sin\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{32b^{5/2}} - \frac{3\sqrt{\pi}d^{3/2} \cos\left(2a - \frac{2bc}{d}\right) S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}} + \frac{3d\sqrt{c+dx} \sin(2a + 2bx)}{16b^2} - \frac{(c + dx)^{3/2} \cos(2a + 2bx)}{4b} - \frac{(3d^{3/2}\sqrt{\pi} \cos(2a - \frac{2bc}{d}) \text{FresnelS}[\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}])}{32b^{5/2}} - \frac{(3d^{3/2}\sqrt{\pi} \text{FresnelC}[\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}]) \sin(2a - \frac{2bc}{d})}{32b^{5/2}} + \frac{(3d\sqrt{c+dx} \sin(2a + 2bx))}{16b^2}$$

[Out] $-\frac{(c + d*x)^{3/2} \cos(2*a + 2*b*x)}{4*b} - \frac{(3*d^{3/2} \sqrt{\pi} \cos(2*a - (2*b*c)/d) \text{FresnelS}[(2*\sqrt{b}*\sqrt{c + d*x})/(\sqrt{d}*\sqrt{\pi})])}{32*b^{5/2}} - \frac{(3*d^{3/2} \sqrt{\pi} \text{FresnelC}[(2*\sqrt{b}*\sqrt{c + d*x})/(\sqrt{d}*\sqrt{\pi})]) \sin(2*a - (2*b*c)/d)}{32*b^{5/2}} + \frac{(3*d*\sqrt{c + d*x} \sin(2*a + 2*b*x))}{16*b^2}$

Rubi [A] time = 0.275287, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\pi}d^{3/2} \sin\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{32b^{5/2}} - \frac{3\sqrt{\pi}d^{3/2} \cos\left(2a - \frac{2bc}{d}\right) S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}} + \frac{3d\sqrt{c+dx} \sin(2a + 2bx)}{16b^2} - \frac{(c + dx)^{3/2} \cos(2a + 2bx)}{4b} - \frac{(3d^{3/2}\sqrt{\pi} \cos(2a - \frac{2bc}{d}) \text{FresnelS}[\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}])}{32b^{5/2}} - \frac{(3d^{3/2}\sqrt{\pi} \text{FresnelC}[\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}]) \sin(2a - \frac{2bc}{d})}{32b^{5/2}} + \frac{(3d\sqrt{c+dx} \sin(2a + 2bx))}{16b^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x], x]

[Out] $-\frac{(c + d*x)^{3/2} \cos(2*a + 2*b*x)}{4*b} - \frac{(3*d^{3/2} \sqrt{\pi} \cos(2*a - (2*b*c)/d) \text{FresnelS}[(2*\sqrt{b}*\sqrt{c + d*x})/(\sqrt{d}*\sqrt{\pi})])}{32*b^{5/2}} - \frac{(3*d^{3/2} \sqrt{\pi} \text{FresnelC}[(2*\sqrt{b}*\sqrt{c + d*x})/(\sqrt{d}*\sqrt{\pi})]) \sin(2*a - (2*b*c)/d)}{32*b^{5/2}} + \frac{(3*d*\sqrt{c + d*x} \sin(2*a + 2*b*x))}{16*b^2}$

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n * Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 12

Int[(a_.)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_.)*(v_) /; FreeQ[b, x]]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[(c + d*x)^m * Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1) * Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3306

Int[sin[(e_.) + (f_.)*(x_) / Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x] / Sqrt[c + d*x], x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x] / Sqrt[c + d*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d,
Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f},
x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^(2)], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[
Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d,
Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] &&
ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^(2)], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[
Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^{3/2} \cos(a + bx) \sin(a + bx) dx &= \int \frac{1}{2} (c + dx)^{3/2} \sin(2a + 2bx) dx \\
&= \frac{1}{2} \int (c + dx)^{3/2} \sin(2a + 2bx) dx \\
&= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{4b} + \frac{(3d) \int \sqrt{c + dx} \cos(2a + 2bx) dx}{8b} \\
&= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{4b} + \frac{3d\sqrt{c + dx} \sin(2a + 2bx)}{16b^2} - \frac{(3d^2) \int \frac{\sin(2a + 2bx)}{\sqrt{c + dx}} dx}{32b^2} \\
&= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{4b} + \frac{3d\sqrt{c + dx} \sin(2a + 2bx)}{16b^2} - \frac{(3d^2 \cos(2a - \frac{2bc}{d}))}{32b^2} \\
&= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{4b} + \frac{3d\sqrt{c + dx} \sin(2a + 2bx)}{16b^2} - \frac{(3d \cos(2a - \frac{2bc}{d}))}{32b^2} \\
&= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{4b} - \frac{3d^{3/2} \sqrt{\pi} \cos(2a - \frac{2bc}{d}) S(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}})}{32b^{5/2}} - \frac{3d^{3/2}}{32b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0417029, size = 157, normalized size = 0.93

$$\frac{-3\sqrt{\pi}d \sin\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - 3\sqrt{\pi}d \cos\left(2a - \frac{2bc}{d}\right) S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - 2\sqrt{\frac{b}{d}}\sqrt{c+dx}(4b(c+dx) \cos(2(a - \frac{2bc}{d})))}{32d^2 \left(\frac{b}{d}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x], x]
```

```
[Out] (-3*d*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 3*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] - 2*Sqrt[b/d]*Sqrt[c + d*x]*(4*b*(c + d*x)*Cos[2*(a + b*x)])
```

$$- 3*d*\sin[2*(a + b*x)]/(32*(b/d)^(5/2)*d^2)$$

Maple [A] time = 0.026, size = 187, normalized size = 1.1

$$2 \frac{1}{d} \left(-1/8 \frac{d(dx+c)^{3/2}}{b} \cos \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) + 3/8 \frac{d}{b} \left(1/4 \frac{d\sqrt{dx+c}}{b} \sin \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) - 1/8 \frac{d\sqrt{\pi}}{b} \left(\cos \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x)

[Out] 2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/8/b*d*(1/4/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))

Maxima [C] time = 1.94144, size = 869, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")

[Out] -1/256*sqrt(2)*(32*sqrt(2)*(d*x + c)^(3/2)*b*d*sqrt(abs(b)/abs(d))*cos(2*((d*x + c)*b - b*c + a*d)/d) - 24*sqrt(2)*sqrt(d*x + c)*d^2*sqrt(abs(b)/abs(d))*sin(2*((d*x + c)*b - b*c + a*d)/d) - ((-3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*cos(-2*(b*c - a*d)/d) - (3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sin(-2*(b*c - a*d)/d)*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - ((3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*cos(-2*(b*c - a*d)/d) - (3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d))/(b^2*d*sqrt(abs(b)/abs(d)))

Fricas [A] time = 0.530726, size = 413, normalized size = 2.46

$$3 \pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{2(bc-ad)}{d}\right) S\left(2 \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) + 3 \pi d^2 \sqrt{\frac{b}{\pi d}} C\left(2 \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{2(bc-ad)}{d}\right) - 4\left(2 b^2 dx + 3 b d \cos(bx)\right)$$

32 b³

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")

[Out] $-1/32*(3*\pi*d^2*\sqrt{b/(pi*d)}*\cos(-2*(b*c - a*d)/d)*\text{fresnel_sin}(2*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) + 3*\pi*d^2*\sqrt{b/(pi*d)}*\text{fresnel_cos}(2*\sqrt{d*x + c}*\sqrt{b/(pi*d)})*\sin(-2*(b*c - a*d)/d) - 4*(2*b^2*d*x + 3*b*d*\cos(b*x + a))*\sin(b*x + a) + 2*b^2*c - 4*(b^2*d*x + b^2*c)*\cos(b*x + a)^2*\sqrt{d*x + c})/b^3$

Sympy [B] time = 132.766, size = 665, normalized size = 3.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a),x)

[Out] $-5*\sqrt{\pi}*\sqrt{d/b}*(c + d*x)**2*\sin(2*a - 2*b*c/d)*\text{fresnelc}(2*\sqrt{b}*\sqrt{c + d*x}/(\sqrt{\pi}*\sqrt{d}))*\gamma(1/4)/(32*d*\gamma(9/4)) + \sqrt{\pi}*\sqrt{d/b}*(c + d*x)**2*\sin(2*a - 2*b*c/d)*\text{fresnelc}(2*b*\sqrt{c + d*x}/(\sqrt{\pi}*\sqrt{d})*\sqrt{b/d}))/2*d - 21*\sqrt{\pi}*\sqrt{d/b}*(c + d*x)**2*\cos(2*a - 2*b*c/d)*\text{fresnels}(2*\sqrt{b}*\sqrt{c + d*x}/(\sqrt{\pi}*\sqrt{d}))*\gamma(3/4)/(32*d*\gamma(11/4)) + \sqrt{\pi}*\sqrt{d/b}*(c + d*x)**2*\cos(2*a - 2*b*c/d)*\text{fresnels}(2*b*\sqrt{c + d*x}/(\sqrt{\pi}*\sqrt{d})*\sqrt{b/d}))/2*d - 15*\sqrt{\pi}*\sqrt{d/b}*\sin(2*a - 2*b*c/d)*\text{fresnelc}(2*\sqrt{b}*\sqrt{c + d*x}/(\sqrt{\pi}*\sqrt{d}))*\gamma(1/4)/(512*b**2*\gamma(9/4)) - 63*\sqrt{\pi}*\sqrt{d/b}*\cos(2*a - 2*b*c/d)*\text{fresnels}(2*\sqrt{b}*\sqrt{c + d*x}/(\sqrt{\pi}*\sqrt{d}))*\gamma(3/4)/(512*b**2*\gamma(11/4)) + 5*\sqrt{d/b}*(c + d*x)**(3/2)*\sin(2*a - 2*b*c/d)*\sin(2*b*c/d + 2*b*x)*\gamma(1/4)/(64*\sqrt{b}*\sqrt{d}*\gamma(9/4)) - 21*\sqrt{d/b}*(c + d*x)**(3/2)*\cos(2*a - 2*b*c/d)*\cos(2*b*c/d + 2*b*x)*\gamma(3/4)/(64*\sqrt{b}*\sqrt{d}*\gamma(11/4)) + 15*\sqrt{d}*\sqrt{d/b}*\sqrt{c + d*x}*\sin(2*a - 2*b*c/d)*\cos(2*b*c/d + 2*b*x)*\gamma(1/4)/(256*b**(3/2)*\gamma(9/4)) + 63*\sqrt{d}*\sqrt{d/b}*\sqrt{c + d*x}*\sin(2*b*c/d + 2*b*x)*\cos(2*a - 2*b*c/d)*\gamma(3/4)/(256*b**(3/2)*\gamma(11/4))$

Giac [C] time = 1.26103, size = 732, normalized size = 4.36

$$4 \left[\frac{\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right) e^{\left(\frac{2i bc-2i ad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} + \frac{\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right) e^{\left(\frac{-2i bc+2i ad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} + \frac{2\sqrt{dx+c}de^{\left(\frac{2i(dx+c)b-2i bc+2i ad}{d}\right)}}{b} + \frac{2\sqrt{dx+c}de^{\left(\frac{-2i(dx+c)b+2i ad}{d}\right)}}{b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")

[Out] $-1/64*(4*(\sqrt{\pi})*d^2*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((2*I*b*c - 2*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b)} + \sqrt{\pi}*d^2*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-2*I*b*c + 2*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b)} + 2*\sqrt{d*x + c}*d*e^{((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b} + 2*\sqrt{d*x + c}*d*$

$$\begin{aligned}
& e^{((-2I*(d*x + c)*b + 2I*b*c - 2I*a*d)/d)/b} * c + I*\sqrt{\pi}*(4I*b*c*d - \\
& 3*d^2)*d*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((2I \\
& *b*c - 2I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2) + I*\sqrt{\pi}*(\\
& 4I*b*c*d + 3*d^2)*d*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x + c})*(-I*b*d/\sqrt{b^2*d^2} + 1 \\
&)/d)*e^{((-2I*b*c + 2I*a*d)/d)/(\sqrt{b*d})*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2) \\
& - 2I*(4I*(d*x + c)^{3/2}*b*d - 4I*\sqrt{d*x + c}*b*c*d - 3*\sqrt{d*x + c}* \\
& d^2)*e^{((2I*(d*x + c)*b - 2I*b*c + 2I*a*d)/d)/b^2 - 2I*(4I*(d*x + c)^{ \\
& 3/2}*b*d - 4I*\sqrt{d*x + c}*b*c*d + 3*\sqrt{d*x + c}*d^2)*e^{((-2I*(d*x + c \\
&)*b + 2I*b*c - 2I*a*d)/d)/b^2)/d
\end{aligned}$$

3.57 $\int (c + dx)^{5/2} \cos(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=196

$$\frac{15\sqrt{\pi}d^{5/2} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{7/2}} + \frac{15\sqrt{\pi}d^{5/2} \sin\left(2a - \frac{2bc}{d}\right) S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}} + \frac{15d^2\sqrt{c+dx} \cos(2a + 2bx)}{64b^3}$$

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b^3) - ((c + d*x)^(5/2)*Cos[2*a + 2*b*x])/(4*b) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(128*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(128*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(16*b^2)
```

Rubi [A] time = 0.334144, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\pi}d^{5/2} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{7/2}} + \frac{15\sqrt{\pi}d^{5/2} \sin\left(2a - \frac{2bc}{d}\right) S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}} + \frac{15d^2\sqrt{c+dx} \cos(2a + 2bx)}{64b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x], x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b^3) - ((c + d*x)^(5/2)*Cos[2*a + 2*b*x])/(4*b) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(128*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(128*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(16*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_.)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_.)*(v_) /; FreeQ[b, x]]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3304

Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}
 \int (c + dx)^{5/2} \cos(a + bx) \sin(a + bx) dx &= \int \frac{1}{2} (c + dx)^{5/2} \sin(2a + 2bx) dx \\
 &= \frac{1}{2} \int (c + dx)^{5/2} \sin(2a + 2bx) dx \\
 &= -\frac{(c + dx)^{5/2} \cos(2a + 2bx)}{4b} + \frac{(5d) \int (c + dx)^{3/2} \cos(2a + 2bx) dx}{8b} \\
 &= -\frac{(c + dx)^{5/2} \cos(2a + 2bx)}{4b} + \frac{5d(c + dx)^{3/2} \sin(2a + 2bx)}{16b^2} - \frac{(15d^2) \int \sqrt{c + dx} \sin(2a + 2bx) dx}{32b^2} \\
 &= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{64b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{4b} + \frac{5d(c + dx)^{3/2} \sin(2a + 2bx)}{16b^2} \\
 &= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{64b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{4b} + \frac{5d(c + dx)^{3/2} \sin(2a + 2bx)}{16b^2} \\
 &= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{64b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{4b} + \frac{5d(c + dx)^{3/2} \sin(2a + 2bx)}{16b^2} \\
 &= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{64b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{4b} - \frac{15d^{5/2} \sqrt{\pi} \cos\left(2a - \frac{2bc}{d}\right)}{128b^3}
 \end{aligned}$$

Mathematica [A] time = 0.0810628, size = 179, normalized size = 0.91

$$\frac{2\sqrt{\frac{b}{d}}\sqrt{c + dx} \left(20bd(c + dx) \sin(2(a + bx)) - \cos(2(a + bx)) (16b^2(c + dx)^2 - 15d^2)\right) - 15\sqrt{\pi}d^2 \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{c + dx}\right)}{128b^3\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x], x]

[Out] (-15*d^2*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + 15*d^2*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]])

*Sin[2*a - (2*b*c)/d] + 2*Sqrt[b/d]*Sqrt[c + d*x]*(-((-15*d^2 + 16*b^2*(c + d*x)^2)*Cos[2*(a + b*x)]) + 20*b*d*(c + d*x)*Sin[2*(a + b*x)]))/(128*b^3*Sqrt[b/d])

Maple [A] time = 0.024, size = 234, normalized size = 1.2

$$2 \frac{1}{d} \left(-1/8 \frac{d(dx+c)^{5/2}}{b} \cos \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) + 5/8 \frac{d}{b} \left(1/4 \frac{d(dx+c)^{3/2}}{b} \sin \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) - 3/4 \frac{d}{b} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x)

[Out] 2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/8/b*d*(1/4/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))

Maxima [C] time = 2.03432, size = 910, normalized size = 4.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")

[Out] 1/1024*sqrt(2)*(160*sqrt(2)*(d*x + c)^(3/2)*b*d^2*sqrt(abs(b)/abs(d))*sin(2*((d*x + c)*b - b*c + a*d)/d) - 8*(16*sqrt(2)*(d*x + c)^(5/2)*b^2*d*sqrt(abs(b)/abs(d)) - 15*sqrt(2)*sqrt(d*x + c)*d^3*sqrt(abs(b)/abs(d)))*cos(2*((d*x + c)*b - b*c + a*d)/d) - ((15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*cos(-2*(b*c - a*d)/d) - (15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - ((15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*cos(-2*(b*c - a*d)/d) - (-15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/(b^3*d*sqrt(abs(b)/abs(d)))

Fricas [A] time = 0.54649, size = 540, normalized size = 2.76

$$15 \pi d^3 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{2(bc-ad)}{d}\right) C\left(2 \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) - 15 \pi d^3 \sqrt{\frac{b}{\pi d}} S\left(2 \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{2(bc-ad)}{d}\right) - 2(16 b^3 d^2 x^2 + 32 b^3 c d x + 16 b^3 c^2 - 15 b^2 d^2 - 2(16 b^3 d^2 x^2 + 32 b^3 c d x + 16 b^3 c^2 - 15 b^2 d^2) \cos(bx+a)^2 + 40(b^2 d^2 x + b^2 c d) \cos(bx+a) \sin(bx+a)) \sqrt{dx+c} / b^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")

[Out] -1/128*(15*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c^2 - 15*b*d^2 - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c^2 - 15*b*d^2)*cos(b*x + a)^2 + 40*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)*sin(b*x + a))*sqrt(d*x + c)/b^4

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a),x)

[Out] Timed out

Giac [C] time = 1.31464, size = 1327, normalized size = 6.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")

[Out] -1/256*(16*(sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c))*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c))*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 2*sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b)*c^2 + d^2*((I*sqrt(pi))*(-16*I*b^2*c^2*d + 24*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c))*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 32*I*(d*x + c)^(3/2)*b^2*c*d + 16*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 24*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^3)/d^2 + (I*sqrt(pi))*(-16*I*b^2*c^2*d - 24*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c))*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 32*I*(d*x + c)^(3/2)*b^2*c*d + 16*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 24*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^2 + 8*c*(I*sqrt(pi))*(4*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c))*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)

$$\begin{aligned}
& *b^2) + I*\sqrt{\pi}*(4*I*b*c*d + 3*d^2)*d*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x + c})*(-I*b \\
& *d/\sqrt{b^2*d^2 + 1})/d)*e^{((-2*I*b*c + 2*I*a*d)/d)/(\sqrt{b*d})*(-I*b*d/\sqrt{ \\
& (b^2*d^2 + 1)*b^2) - 2*I*(4*I*(d*x + c)^{(3/2)}*b*d - 4*I*\sqrt{d*x + c}*b*c* \\
& d - 3*\sqrt{d*x + c}*d^2)*e^{((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^2 - \\
& 2*I*(4*I*(d*x + c)^{(3/2)}*b*d - 4*I*\sqrt{d*x + c}*b*c*d + 3*\sqrt{d*x + c}*d^ \\
& 2)*e^{((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2))/d
\end{aligned}$$

3.58 $\int (c + dx)^{5/2} \cos(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=406

$$\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}} + \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} + \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a - \frac{bc}{d}\right)}{16b^{7/2}}$$

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(8*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a +
3*b*x])/(72*b^2) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[
b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]
*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])
/(144*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/
2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(
16*b^(7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(16*b^3) + ((c + d*x)^(5/
2)*Sin[a + b*x])/(4*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(144*b^3) -
((c + d*x)^(5/2)*Sin[3*a + 3*b*x])/(12*b)
```

Rubi [A] time = 1.13674, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}} + \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} + \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a - \frac{bc}{d}\right)}{16b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(8*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a +
3*b*x])/(72*b^2) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[
b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]
*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])
/(144*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/
2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(
16*b^(7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(16*b^3) + ((c + d*x)^(5/
2)*Sin[a + b*x])/(4*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(144*b^3) -
((c + d*x)^(5/2)*Sin[3*a + 3*b*x])/(12*b)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]
]^n*Cos[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^{5/2} \cos(a + bx) \sin^2(a + bx) dx &= \int \left(\frac{1}{4}(c + dx)^{5/2} \cos(a + bx) - \frac{1}{4}(c + dx)^{5/2} \cos(3a + 3bx) \right) dx \\
&= \frac{1}{4} \int (c + dx)^{5/2} \cos(a + bx) dx - \frac{1}{4} \int (c + dx)^{5/2} \cos(3a + 3bx) dx \\
&= \frac{(c + dx)^{5/2} \sin(a + bx)}{4b} - \frac{(c + dx)^{5/2} \sin(3a + 3bx)}{12b} + \frac{(5d) \int (c + dx)^{3/2} \sin(a + bx) dx}{24b} \\
&= \frac{5d(c + dx)^{3/2} \cos(a + bx)}{8b^2} - \frac{5d(c + dx)^{3/2} \cos(3a + 3bx)}{72b^2} + \frac{(c + dx)^{5/2} \sin(a + bx)}{4b} \\
&= \frac{5d(c + dx)^{3/2} \cos(a + bx)}{8b^2} - \frac{5d(c + dx)^{3/2} \cos(3a + 3bx)}{72b^2} - \frac{15d^2 \sqrt{c + dx} \sin(a + bx)}{16b^3} \\
&= \frac{5d(c + dx)^{3/2} \cos(a + bx)}{8b^2} - \frac{5d(c + dx)^{3/2} \cos(3a + 3bx)}{72b^2} - \frac{15d^2 \sqrt{c + dx} \sin(a + bx)}{16b^3} \\
&= \frac{5d(c + dx)^{3/2} \cos(a + bx)}{8b^2} - \frac{5d(c + dx)^{3/2} \cos(3a + 3bx)}{72b^2} - \frac{15d^2 \sqrt{c + dx} \sin(a + bx)}{16b^3} \\
&= \frac{5d(c + dx)^{3/2} \cos(a + bx)}{8b^2} - \frac{5d(c + dx)^{3/2} \cos(3a + 3bx)}{72b^2} + \frac{15d^{5/2} \sqrt{\frac{\pi}{2}} \cos\left(a + \frac{3bx}{2}\right)}{16b^3}
\end{aligned}$$

Mathematica [C] time = 15.5226, size = 1171, normalized size = 2.88

$$\frac{ie^{-\frac{i(bc+ad)}{d}} \sqrt{c+dx} \left(\frac{e^{2ia} \Gamma\left(\frac{3}{2}, -\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}} \Gamma\left(\frac{3}{2}, \frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}} \right) c^2}{8b} - \frac{\left(-\sqrt{2\pi} \cos\left(3a - \frac{3bc}{d}\right) S\left(\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right) - \sqrt{2\pi}\right)}{8b}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]

[Out]
$$\begin{aligned} &((-I/8)*c^2*\text{Sqrt}[c + d*x]*((E^{((2*I)*a)}*\text{Gamma}[3/2, ((-I)*b*(c + d*x))/d])/S \\ &\text{qrt}[((-I)*b*(c + d*x))/d] - (E^{((2*I)*b*c)/d}*\text{Gamma}[3/2, (I*b*(c + d*x))/d \\ &])/ \text{Sqrt}[(I*b*(c + d*x))/d]))/(b*E^{(I*(b*c + a*d))/d}) + (c*d*(\text{Sqrt}[b/d]*\text{Sq} \\ &\text{rt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[2/Pi]*\text{Sqrt}[c + d*x]]*(-3*d*\text{Cos}[a - (b*c)/d \\ &] + 2*b*c*\text{Sin}[a - (b*c)/d]) + \text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[\\ &2/Pi]*\text{Sqrt}[c + d*x]]*(2*b*c*\text{Cos}[a - (b*c)/d] + 3*d*\text{Sin}[a - (b*c)/d]) + 2*b* \\ &\text{Sqrt}[c + d*x]*(3*\text{Cos}[a + b*x] + 2*b*x*\text{Sin}[a + b*x])))/(8*b^3) + ((b/d)^(3/2 \\ &)*d^2*(-(\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[2/Pi]*\text{Sqrt}[c + d*x]]*((4*b^2*c^ \\ &2 - 15*d^2)*\text{Cos}[a - (b*c)/d] + 12*b*c*d*\text{Sin}[a - (b*c)/d])) - \text{Sqrt}[2*Pi]*\text{Fre} \\ &\text{snelC}[\text{Sqrt}[b/d]*\text{Sqrt}[2/Pi]*\text{Sqrt}[c + d*x]]*(-12*b*c*d*\text{Cos}[a - (b*c)/d] + (4* \\ &b^2*c^2 - 15*d^2)*\text{Sin}[a - (b*c)/d]) + 2*\text{Sqrt}[b/d]*d*\text{Sqrt}[c + d*x]*(-2*b*(c \\ &- 5*d*x)*\text{Cos}[a + b*x] + d*(-15 + 4*b^2*x^2)*\text{Sin}[a + b*x])))/(32*b^5) - (c^2 \\ &*(-(\text{Sqrt}[2*Pi]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + \\ &d*x]]) - \text{Sqrt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*\text{Sin}[3*a - \\ &(3*b*c)/d] + 2*\text{Sqrt}[3]*\text{Sqrt}[b/d]*\text{Sqrt}[c + d*x]*\text{Sin}[3*(a + b*x)])))/(24*\text{Sqrt}[\\ &3]*b*\text{Sqrt}[b/d]) - (c*d*(\text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]* \\ &\text{Sqrt}[c + d*x]]*(-(d*\text{Cos}[3*a - (3*b*c)/d]) + 2*b*c*\text{Sin}[3*a - (3*b*c)/d]) + \text{S} \\ &\text{qrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*(2*b*c*\text{Cos} \\ &[3*a - (3*b*c)/d] + d*\text{Sin}[3*a - (3*b*c)/d]) + 2*\text{Sqrt}[3]*b*\text{Sqrt}[c + d*x]*(\text{Co} \\ &\text{s}[3*(a + b*x)] + 2*b*x*\text{Sin}[3*(a + b*x)])))/(24*\text{Sqrt}[3]*b^3) - ((b/d)^(3/2)* \\ &d^2*(-(\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*((12*b^2*c^2 \\ &- 5*d^2)*\text{Cos}[3*a - (3*b*c)/d] + 12*b*c*d*\text{Sin}[3*a - (3*b*c)/d])) - \text{Sqrt}[2*P \\ &i]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*(-12*b*c*d*\text{Cos}[3*a - (3*b*c \\ &)/d] + (12*b^2*c^2 - 5*d^2)*\text{Sin}[3*a - (3*b*c)/d]) + 2*\text{Sqrt}[3]*\text{Sqrt}[b/d]*d*\text{S} \\ &\text{qrt}[c + d*x]*(-2*b*(c - 5*d*x)*\text{Cos}[3*(a + b*x)] + d*(-5 + 12*b^2*x^2)*\text{Sin}[3 \\ &*(a + b*x)])))/(288*\text{Sqrt}[3]*b^5) \end{aligned}$$

Maple [A] time = 0.04, size = 474, normalized size = 1.2

$$2 \frac{1}{d} \left(\frac{1}{8} \frac{d(dx+c)^{5/2}}{b} \sin \left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d} \right) - \frac{5}{8} \frac{d}{b} \left(-\frac{1}{2} \frac{d(dx+c)^{3/2}}{b} \cos \left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d} \right) + \frac{3}{2} \frac{d}{b} \left(\frac{1}{2} \frac{d\sqrt{dx+c}}{b} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x)

[Out]
$$\begin{aligned} &2/d*(1/8/b*d*(d*x+c)^(5/2)*\text{sin}(1/d*(d*x+c)*b+(a*d-b*c)/d)-5/8/b*d*(-1/2/b*d \\ &*(d*x+c)^(3/2)*\text{cos}(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2 \\ &)*\text{sin}(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*\text{Pi}^(1/2)/(b/d)^(1/2)*(\text{cos} \\ &((a*d-b*c)/d)*\text{FresnelS}(2^(1/2)/\text{Pi}^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+\text{sin}((\\ &a*d-b*c)/d)*\text{FresnelC}(2^(1/2)/\text{Pi}^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))) -1/2 \\ &4/b*d*(d*x+c)^(5/2)*\text{sin}(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/24/b*d*(-1/6/b*d*(d* \\ &x+c)^(3/2)*\text{cos}(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2)* \\ &\text{sin}(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*\text{Pi}^(1/2)*3^(1/2)/(b/d)^(1 \\ &/2)*(\text{cos}(3*(a*d-b*c)/d)*\text{FresnelS}(2^(1/2)/\text{Pi}^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+ \\ &c)^(1/2)*b/d)+\text{sin}(3*(a*d-b*c)/d)*\text{FresnelC}(2^(1/2)/\text{Pi}^(1/2)*3^(1/2)/(b/d)^(1 \\ &/2)*(d*x+c)^(1/2)*b/d)))) \end{aligned}$$

Maxima [C] time = 2.5027, size = 1866, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/3456\sqrt{3}*(80\sqrt{3}*(d*x + c)^{(3/2)}*b*d^2*\text{abs}(b)*\cos(3*((d*x + c)*b \\ & - b*c + a*d)/d)/\text{abs}(d) - 720\sqrt{3}*(d*x + c)^{(3/2)}*b*d^2*\text{abs}(b)*\cos(((d*x \\ & + c)*b - b*c + a*d)/d)/\text{abs}(d) + ((5*I*\sqrt{\pi})*\cos(1/4*\pi + 1/2*\arctan2(0 \\ & , b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 5*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan \\ & 2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 5*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan \\ & 2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 5*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arcta \\ & n2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^3*\sqrt{\text{abs}(b)/\text{abs}(d)}*\cos(-3*(b* \\ & c - a*d)/d) + (5*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d \\ & / \sqrt{d^2})) + 5*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, \\ & d/\sqrt{d^2})) - 5*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0 \\ & , d/\sqrt{d^2})) + 5*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan \\ & 2(0, d/\sqrt{d^2}))) * d^3*\sqrt{\text{abs}(b)/\text{abs}(d)}*\sin(-3*(b*c - a*d)/d))*\text{erf}(\sqrt{ \\ & d*x + c}*\sqrt{3*I*b/d}) + (\sqrt{3}*(-135*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arcta \\ & n2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 135*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2* \\ & arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 135*\sqrt{\pi}*\sin(1/4*\pi + 1/ \\ & 2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 135*\sqrt{\pi}*\sin(-1/4*\pi + \\ & 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^3*\sqrt{\text{abs}(b)/\text{abs}(d)}* \\ & \cos(-(b*c - a*d)/d) - \sqrt{3}*(135*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) \\ & + 1/2*\arctan2(0, d/\sqrt{d^2})) + 135*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, \\ & b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 135*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2 \\ & (0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 135*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\ar \\ & ctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^3*\sqrt{\text{abs}(b)/\text{abs}(d)}*\sin(-(b \\ & *c - a*d)/d))*\text{erf}(\sqrt{d*x + c}*\sqrt{I*b/d}) + (\sqrt{3}*(135*I*\sqrt{\pi}*\cos \\ & (1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 135*I*\sqrt{\pi} \\ & *\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 135*\sqrt{ \\ & \pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 135*\sqrt{ \\ & \pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^3*s \\ & \sqrt{\text{abs}(b)/\text{abs}(d)}*\cos(-(b*c - a*d)/d) - \sqrt{3}*(135*\sqrt{\pi}*\cos(1/4*\pi + \\ & 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 135*\sqrt{\pi}*\cos(-1/4*\pi \\ & + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 135*I*\sqrt{\pi}*\sin(1 \\ & /4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 135*I*\sqrt{\pi}*s \\ & \sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^3*\sqrt{\text{abs} \\ & (b)/\text{abs}(d)}*\sin(-(b*c - a*d)/d))*\text{erf}(\sqrt{d*x + c}*\sqrt{-I*b/d}) + ((-5*I*s \\ & \sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 5*I \\ & *\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + \\ & 5*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - \\ & 5*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * \\ & d^3*\sqrt{\text{abs}(b)/\text{abs}(d)}*\cos(-3*(b*c - a*d)/d) + (5*\sqrt{\pi}*\cos(1/4*\pi + 1/ \\ & 2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 5*\sqrt{\pi}*\cos(-1/4*\pi + 1 \\ & /2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 5*I*\sqrt{\pi}*\sin(1/4*\pi + \\ & 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 5*I*\sqrt{\pi}*\sin(-1/4*\pi \\ & + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^3*\sqrt{\text{abs}(b)/\text{abs}(d \\ &)}*\sin(-3*(b*c - a*d)/d))*\text{erf}(\sqrt{d*x + c}*\sqrt{-3*I*b/d}) + 8*(12*\sqrt{3} \\ & *(d*x + c)^{(5/2)}*b^2*d*\text{abs}(b)/\text{abs}(d) - 5*\sqrt{3}*\sqrt{d*x + c}*d^3*\text{abs}(b)/a \\ & \text{bs}(d)*\sin(3*((d*x + c)*b - b*c + a*d)/d) - 72*(4*\sqrt{3}*(d*x + c)^{(5/2)}*b \\ & ^2*d*\text{abs}(b)/\text{abs}(d) - 15*\sqrt{3}*\sqrt{d*x + c}*d^3*\text{abs}(b)/\text{abs}(d))*\sin(((d*x \\ & + c)*b - b*c + a*d)/d))*\text{abs}(d)/(b^3*d*\text{abs}(b)) \end{aligned}$$

Fricas [A] time = 0.643364, size = 918, normalized size = 2.26

$$\frac{5\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/864*(5*\sqrt{6}*\pi*d^3*\sqrt{b/(pi*d)}*\cos(-3*(b*c - a*d)/d)*\text{fresnel_sin}(\sqrt{6}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) \\ & - 405*\sqrt{2}*\pi*d^3*\sqrt{b/(pi*d)}*\cos(-3*(b*c - a*d)/d)*\text{fresnel_sin}(\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) \\ & - 405*\sqrt{2}*\pi*d^3*\sqrt{b/(pi*d)}*\text{fresnel_cos}(\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) \\ & * \sin(-3*(b*c - a*d)/d) + 5*\sqrt{6}*\pi*d^3*\sqrt{b/(pi*d)}*\text{fresnel_cos}(\sqrt{6}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) \\ & * \sin(-3*(b*c - a*d)/d) + 24*(10*(b^2*d^2*x + b^2*c*d)*\cos(b*x + a)^3 - 30*(b^2*d^2*x + b^2*c*d)*\cos(b*x + a) \\ & - (12*b^3*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 35*b*d^2 - (12*b^3*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*\cos(b*x + a)^2)*\sin(b*x + a))*\sqrt{d*x + c})/b^4 \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**2,x)

[Out] Timed out

Giac [C] time = 1.58877, size = 2722, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/1728*(12*(I*\sqrt{6}*\sqrt{\pi})*d^2*\text{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x + c})* \\ & (I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((3*I*b*c - 3*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b)} \\ & - 9*I*\sqrt{2}*\sqrt{\pi}*d^2*\text{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c})* \\ & (I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((I*b*c - I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b)} \\ & + 9*I*\sqrt{2}*\sqrt{\pi}*d^2*\text{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c})* \\ & (-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-I*b*c + I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b)} \\ & - I*\sqrt{6}*\sqrt{\pi}*d^2*\text{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x + c})* \\ & (-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-3*I*b*c + 3*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b)} \\ & - 6*I*\sqrt{d*x + c})*d*e^{((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b} \\ & + 18*I*\sqrt{d*x + c})*d*e^{((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b} \\ & - 18*I*\sqrt{d*x + c})*d*e^{((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b} \\ & + 6*I*\sqrt{d*x + c})*d*e^{((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b} \\ & *c^2 - d^2*((\sqrt{6}*\sqrt{\pi})*(-12*I*b^2*c^2*d + 12*b*c*d^2 + 5*I*d^3)*d*\text{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x + c})* \\ & (I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((3*I*b*c - 3*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)} \end{aligned}$$

$$\begin{aligned}
&) * b^3) - 6 * (12 * I * (d * x + c)^{(5/2)} * b^2 * d - 24 * I * (d * x + c)^{(3/2)} * b^2 * c * d + 12 * \\
& I * \sqrt{d * x + c} * b^2 * c^2 * d + 10 * (d * x + c)^{(3/2)} * b * d^2 - 12 * \sqrt{d * x + c} * b * c \\
& * d^2 - 5 * I * \sqrt{d * x + c} * d^3) * e^{((-3 * I * (d * x + c) * b + 3 * I * b * c - 3 * I * a * d) / d) /} \\
& b^3) / d^2 + 27 * (\sqrt{2} * \sqrt{\pi}) * (4 * I * b^2 * c^2 * d - 12 * b * c * d^2 - 15 * I * d^3) * d * e \\
& rf(-1/2 * \sqrt{2} * \sqrt{b * d} * \sqrt{d * x + c} * (I * b * d / \sqrt{b^2 * d^2} + 1) / d) * e^{((I * \\
& b * c - I * a * d) / d) / (\sqrt{b * d} * (I * b * d / \sqrt{b^2 * d^2} + 1) * b^3) - 2 * (-4 * I * (d * x + \\
& c)^{(5/2)} * b^2 * d + 8 * I * (d * x + c)^{(3/2)} * b^2 * c * d - 4 * I * \sqrt{d * x + c} * b^2 * c^2 * d \\
& - 10 * (d * x + c)^{(3/2)} * b * d^2 + 12 * \sqrt{d * x + c} * b * c * d^2 + 15 * I * \sqrt{d * x + c} * \\
& d^3) * e^{((-I * (d * x + c) * b + I * b * c - I * a * d) / d) / b^3) / d^2 + 27 * (\sqrt{2} * \sqrt{\pi}) \\
& * (-4 * I * b^2 * c^2 * d - 12 * b * c * d^2 + 15 * I * d^3) * d * erf(-1/2 * \sqrt{2} * \sqrt{b * d} * \sqrt{ \\
& (d * x + c) * (-I * b * d / \sqrt{b^2 * d^2} + 1) / d) * e^{((-I * b * c + I * a * d) / d) / (\sqrt{b * d} * (\\
& -I * b * d / \sqrt{b^2 * d^2} + 1) * b^3) - 2 * (4 * I * (d * x + c)^{(5/2)} * b^2 * d - 8 * I * (d * x + \\
& c)^{(3/2)} * b^2 * c * d + 4 * I * \sqrt{d * x + c} * b^2 * c^2 * d - 10 * (d * x + c)^{(3/2)} * b * d^2 + \\
& 12 * \sqrt{d * x + c} * b * c * d^2 - 15 * I * \sqrt{d * x + c} * d^3) * e^{((I * (d * x + c) * b - I * b \\
& * c + I * a * d) / d) / b^3) / d^2 + (\sqrt{6} * \sqrt{\pi}) * (12 * I * b^2 * c^2 * d + 12 * b * c * d^2 - \\
& 5 * I * d^3) * d * erf(-1/2 * \sqrt{6} * \sqrt{b * d} * \sqrt{d * x + c} * (-I * b * d / \sqrt{b^2 * d^2} + \\
& 1) / d) * e^{((-3 * I * b * c + 3 * I * a * d) / d) / (\sqrt{b * d} * (-I * b * d / \sqrt{b^2 * d^2} + 1) * b^3 \\
&) - 6 * (-12 * I * (d * x + c)^{(5/2)} * b^2 * d + 24 * I * (d * x + c)^{(3/2)} * b^2 * c * d - 12 * I * \sqrt{ \\
& (d * x + c) * b^2 * c^2 * d + 10 * (d * x + c)^{(3/2)} * b * d^2 - 12 * \sqrt{d * x + c} * b * c * d^2 \\
& + 5 * I * \sqrt{d * x + c} * d^3) * e^{((3 * I * (d * x + c) * b - 3 * I * b * c + 3 * I * a * d) / d) / b^3) /} \\
& d^2) - 12 * (\sqrt{6} * \sqrt{\pi}) * (2 * I * b * c * d - d^2) * d * erf(-1/2 * \sqrt{6} * \sqrt{b * d} * \\
& \sqrt{d * x + c} * (I * b * d / \sqrt{b^2 * d^2} + 1) / d) * e^{((3 * I * b * c - 3 * I * a * d) / d) / (\sqrt{ \\
& (b * d) * (I * b * d / \sqrt{b^2 * d^2} + 1) * b^2) + 9 * \sqrt{2} * \sqrt{\pi}) * (-2 * I * b * c * d + 3 * d^2 \\
&) * d * erf(-1/2 * \sqrt{2} * \sqrt{b * d} * \sqrt{d * x + c} * (I * b * d / \sqrt{b^2 * d^2} + 1) / d) * \\
& e^{((I * b * c - I * a * d) / d) / (\sqrt{b * d} * (I * b * d / \sqrt{b^2 * d^2} + 1) * b^2) + 9 * \sqrt{2} \\
& * \sqrt{\pi}) * (2 * I * b * c * d + 3 * d^2) * d * erf(-1/2 * \sqrt{2} * \sqrt{b * d} * \sqrt{d * x + c} * (- \\
& I * b * d / \sqrt{b^2 * d^2} + 1) / d) * e^{((-I * b * c + I * a * d) / d) / (\sqrt{b * d} * (-I * b * d / \sqrt{ \\
& (b^2 * d^2) + 1) * b^2) + \sqrt{6} * \sqrt{\pi}) * (-2 * I * b * c * d - d^2) * d * erf(-1/2 * \sqrt{6} \\
& * \sqrt{b * d} * \sqrt{d * x + c} * (-I * b * d / \sqrt{b^2 * d^2} + 1) / d) * e^{((-3 * I * b * c + 3 * I * a \\
& * d) / d) / (\sqrt{b * d} * (-I * b * d / \sqrt{b^2 * d^2} + 1) * b^2) - 6 * (-2 * I * (d * x + c)^{(3/2)} \\
& * b * d + 2 * I * \sqrt{d * x + c} * b * c * d + \sqrt{d * x + c} * d^2) * e^{((3 * I * (d * x + c) * b - 3 \\
& * I * b * c + 3 * I * a * d) / d) / b^2 - 18 * (2 * I * (d * x + c)^{(3/2)} * b * d - 2 * I * \sqrt{d * x + c} * \\
& b * c * d - 3 * \sqrt{d * x + c} * d^2) * e^{((I * (d * x + c) * b - I * b * c + I * a * d) / d) / b^2 - 18 \\
& * (-2 * I * (d * x + c)^{(3/2)} * b * d + 2 * I * \sqrt{d * x + c} * b * c * d - 3 * \sqrt{d * x + c} * d^2) \\
& * e^{((-I * (d * x + c) * b + I * b * c - I * a * d) / d) / b^2 - 6 * (2 * I * (d * x + c)^{(3/2)} * b * d - \\
& 2 * I * \sqrt{d * x + c} * b * c * d + \sqrt{d * x + c} * d^2) * e^{((-3 * I * (d * x + c) * b + 3 * I * b * c \\
& - 3 * I * a * d) / d) / b^2) * c) / d
\end{aligned}$$

3.59 $\int (c + dx)^{3/2} \cos(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=353

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} + \frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}} - \frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a - \frac{3bc}{d}\right)}{24b^{5/2}}$$

[Out] $(3*d*\text{Sqrt}[c + d*x]*\text{Cos}[a + b*x])/(8*b^2) - (d*\text{Sqrt}[c + d*x]*\text{Cos}[3*a + 3*b*x])/(24*b^2) - (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a - (b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(8*b^{(5/2)}) + (d^{(3/2)}*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(24*b^{(5/2)}) - (d^{(3/2)}*\text{Sqrt}[\text{Pi}/6]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])*\text{Sin}[3*a - (3*b*c)/d])/(24*b^{(5/2)}) + (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])*\text{Sin}[a - (b*c)/d])/(8*b^{(5/2)}) + ((c + d*x)^{(3/2)}*\text{Sin}[a + b*x])/(4*b) - ((c + d*x)^{(3/2)}*\text{Sin}[3*a + 3*b*x])/(12*b)$

Rubi [A] time = 0.684237, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} + \frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}} - \frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a - \frac{3bc}{d}\right)}{24b^{5/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^{(3/2)}*\text{Cos}[a + b*x]*\text{Sin}[a + b*x]^2, x]$

[Out] $(3*d*\text{Sqrt}[c + d*x]*\text{Cos}[a + b*x])/(8*b^2) - (d*\text{Sqrt}[c + d*x]*\text{Cos}[3*a + 3*b*x])/(24*b^2) - (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a - (b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(8*b^{(5/2)}) + (d^{(3/2)}*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(24*b^{(5/2)}) - (d^{(3/2)}*\text{Sqrt}[\text{Pi}/6]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])*\text{Sin}[3*a - (3*b*c)/d])/(24*b^{(5/2)}) + (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])*\text{Sin}[a - (b*c)/d])/(8*b^{(5/2)}) + ((c + d*x)^{(3/2)}*\text{Sin}[a + b*x])/(4*b) - ((c + d*x)^{(3/2)}*\text{Sin}[3*a + 3*b*x])/(12*b)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3306

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[c*f/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d$

$*e - c*f)/d]$, $\text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /;$ $\text{FreeQ}\{c, d, e, f\}, x] \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{NeQ}[d*e - c*f, 0]$

Rule 3305

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \text{ :> } \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Sin}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /;$ $\text{FreeQ}\{c, d, e, f\}, x] \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

Rule 3351

$\text{Int}[\text{Sin}[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] \text{ :> } \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /;$ $\text{FreeQ}\{d, e, f\}, x]$

Rule 3304

$\text{Int}[\text{sin}[\text{Pi}/2 + (e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \text{ :> } \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Cos}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /;$ $\text{FreeQ}\{c, d, e, f\}, x] \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

Rule 3352

$\text{Int}[\text{Cos}[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] \text{ :> } \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /;$ $\text{FreeQ}\{d, e, f\}, x]$

Rubi steps

$$\begin{aligned} \int (c + dx)^{3/2} \cos(a + bx) \sin^2(a + bx) dx &= \int \left(\frac{1}{4}(c + dx)^{3/2} \cos(a + bx) - \frac{1}{4}(c + dx)^{3/2} \cos(3a + 3bx) \right) dx \\ &= \frac{1}{4} \int (c + dx)^{3/2} \cos(a + bx) dx - \frac{1}{4} \int (c + dx)^{3/2} \cos(3a + 3bx) dx \\ &= \frac{(c + dx)^{3/2} \sin(a + bx)}{4b} - \frac{(c + dx)^{3/2} \sin(3a + 3bx)}{12b} + \frac{d \int \sqrt{c + dx} \sin(3a + 3bx) dx}{8b} \\ &= \frac{3d\sqrt{c + dx} \cos(a + bx)}{8b^2} - \frac{d\sqrt{c + dx} \cos(3a + 3bx)}{24b^2} + \frac{(c + dx)^{3/2} \sin(a + bx)}{4b} \\ &= \frac{3d\sqrt{c + dx} \cos(a + bx)}{8b^2} - \frac{d\sqrt{c + dx} \cos(3a + 3bx)}{24b^2} + \frac{(c + dx)^{3/2} \sin(a + bx)}{4b} \\ &= \frac{3d\sqrt{c + dx} \cos(a + bx)}{8b^2} - \frac{d\sqrt{c + dx} \cos(3a + 3bx)}{24b^2} + \frac{(c + dx)^{3/2} \sin(a + bx)}{4b} \\ &= \frac{3d\sqrt{c + dx} \cos(a + bx)}{8b^2} - \frac{d\sqrt{c + dx} \cos(3a + 3bx)}{24b^2} - \frac{3d^{3/2} \sqrt{\frac{\pi}{2}} \cos\left(a - \frac{bc}{d}\right)}{8b^{5/2}} \end{aligned}$$

Mathematica [C] time = 9.30698, size = 677, normalized size = 1.92

$$\frac{ic\sqrt{c + dx} e^{-\frac{i(ad+bc)}{d}} \left(\frac{e^{2ia} \text{Gamma}\left(\frac{3}{2}, -\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}} \text{Gamma}\left(\frac{3}{2}, \frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}} \right)}{8b} + \frac{d \left(\sqrt{2\pi} \sqrt{\frac{b}{d}} \text{FresnelC} \left(\sqrt{\frac{2}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c + dx} \right) \right) (2bc \sin(a))}{8b^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]

[Out]
$$\begin{aligned} &((-1/8)*c*\sqrt{c + d*x}*(E^{((2*I)*a)}*\Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt \\ &t[((-I)*b*(c + d*x))/d] - (E^{((2*I)*b*c)/d}*\Gamma[3/2, (I*b*(c + d*x))/d]) \\ &/Sqrt[(I*b*(c + d*x))/d])/(b*E^{(I*(b*c + a*d))/d}) + (d*(Sqrt[b/d]*Sqrt[2 \\ &*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)/d] + \\ &2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi \\ &]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*b*Sqrt \\ &[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x]))/(16*b^3) - (c*(-(Sqrt[2*P \\ &i]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) - Sqr \\ &t[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] + \\ &2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(24*Sqrt[3]*b*Sqrt[b/d \\ &]) - (d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] \\ &*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[b/d]*Sqrt[\\ &2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c) \\ &/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(Cos[3*(a + b*x)] \\ &+ 2*b*x*Sin[3*(a + b*x)])))/(48*Sqrt[3]*b^3) \end{aligned}$$

Maple [A] time = 0.036, size = 386, normalized size = 1.1

$$2 \frac{1}{d} \left(\frac{1}{8} \frac{d(dx+c)^{3/2}}{b} \sin\left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d}\right) - \frac{3}{8} \frac{d}{b} \left(-\frac{1}{2} \frac{d\sqrt{dx+c}}{b} \cos\left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d}\right) + \frac{1}{4} \frac{d\sqrt{2}\sqrt{\pi}}{b} \cos\left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x)

[Out]
$$\begin{aligned} &2/d*(1/8/b*d*(d*x+c)^{(3/2)}*\sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/8/b*d*(-1/2/b*d \\ &*(d*x+c)^{(1/2)}*\cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^{(1/2)}*Pi^{(1/2)}/(b/d \\ &)^{(1/2)}*(\cos((a*d-b*c)/d)*FresnelC(2^{(1/2)}/Pi^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/ \\ &2)}*b/d)-\sin((a*d-b*c)/d)*FresnelS(2^{(1/2)}/Pi^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2 \\ &)}*b/d))-1/24/b*d*(d*x+c)^{(3/2)}*\sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/8/b*d*(- \\ &1/6/b*d*(d*x+c)^{(1/2)}*\cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^{(1/2)}*Pi^{ \\ &(1/2)}*3^{(1/2)}/(b/d)^{(1/2)}*(\cos(3*(a*d-b*c)/d)*FresnelC(2^{(1/2)}/Pi^{(1/2)}*3^{(\\ &1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d)-\sin(3*(a*d-b*c)/d)*FresnelS(2^{(1/2)}/Pi^{ \\ &(1/2)}*3^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d))) \end{aligned}$$

Maxima [C] time = 2.53624, size = 1787, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")

[Out]
$$\begin{aligned} &-1/576*\sqrt{3}*(16*\sqrt{3}*(d*x + c)^{(3/2)}*b*d*abs(b)*\sin(3*((d*x + c)*b - \\ &b*c + a*d)/d)/abs(d) - 48*\sqrt{3}*(d*x + c)^{(3/2)}*b*d*abs(b)*\sin(((d*x + c) \\ &*b - b*c + a*d)/d)/abs(d) + 8*\sqrt{3}*\sqrt{d*x + c}*d^2*abs(b)*\cos(3*((d*x \\ &+ c)*b - b*c + a*d)/d)/abs(d) - 72*\sqrt{3}*\sqrt{d*x + c}*d^2*abs(b)*\cos(((d \\ &*x + c)*b - b*c + a*d)/d)/abs(d) - ((\sqrt{pi})*\cos(1/4*pi + 1/2*arctan2(0, b \\ &)) + 1/2*arctan2(0, d/\sqrt{d^2})) + \sqrt{pi}*\cos(-1/4*pi + 1/2*arctan2(0, b) \\ &+ 1/2*arctan2(0, d/\sqrt{d^2})) - I*\sqrt{pi}*\sin(1/4*pi + 1/2*arctan2(0, b) \\ &+ 1/2*arctan2(0, d/\sqrt{d^2})) + I*\sqrt{pi}*\sin(-1/4*pi + 1/2*arctan2(0, b \end{aligned}$$

) + 1/2*arctan2(0, d/sqrt(d^2))) * d^2 * sqrt(abs(b)/abs(d)) * cos(-3*(b*c - a*d)/d) - (I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * sin(-3*(b*c - a*d)/d) * erf(sqrt(d*x + c) * sqrt(3*I*b/d)) + (sqrt(3)*(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * cos(-(b*c - a*d)/d) + sqrt(3)*(-9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * sin(-(b*c - a*d)/d) * erf(sqrt(d*x + c) * sqrt(I*b/d)) + (sqrt(3)*(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * cos(-(b*c - a*d)/d) + sqrt(3)*(9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * sin(-(b*c - a*d)/d) * erf(sqrt(d*x + c) * sqrt(-I*b/d)) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * cos(-3*(b*c - a*d)/d) - (-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * sin(-3*(b*c - a*d)/d) * erf(sqrt(d*x + c) * sqrt(-3*I*b/d))) * abs(d)/(b^2*d*abs(b))

Fricas [A] time = 0.612319, size = 756, normalized size = 2.14

$$\sqrt{6\pi d^2} \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{3(bc-ad)}{d}\right) C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 27\sqrt{2}\pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{bc-ad}{d}\right) C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) + 27\sqrt{2}\pi d^2 \sqrt{\frac{b}{\pi d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")

[Out] 1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) * sin(-(b*c - a*d)/d) - sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) * sin(-3*(b*c - a*d)/d) - 24*(b*d*cos(b*x + a)^3 - 3*b*d*cos(b*x + a) - 2*(b^2*d*x + b^2*c - (b^2*d*x + b^2*c)*cos(b*x + a)^2) * sin(b*x + a) * sqrt(d*x + c))/b^3

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**2,x)

[Out] Timed out

Giac [C] time = 1.37037, size = 1517, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")

[Out]
$$-1/288*(2*(I*\sqrt{6})*\sqrt{\pi}*d^2*\operatorname{erf}(-1/2*\sqrt{6})*\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((3*I*b*c - 3*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b) - 9*I*\sqrt{2}*\sqrt{\pi}*d^2*\operatorname{erf}(-1/2*\sqrt{2})*\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((I*b*c - I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b) + 9*I*\sqrt{2}*\sqrt{\pi}*d^2*\operatorname{erf}(-1/2*\sqrt{2})*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-I*b*c + I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b) - I*\sqrt{6}*\sqrt{\pi}*d^2*\operatorname{erf}(-1/2*\sqrt{6})*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-3*I*b*c + 3*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b) - 6*I*\sqrt{d*x + c}*d*e^{((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*I*\sqrt{d*x + c}*d*e^{(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 18*I*\sqrt{d*x + c}*d*e^{((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*I*\sqrt{d*x + c}*d*e^{((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b}*c - \sqrt{6}*\sqrt{\pi}*(2*I*b*c*d - d^2)*d*\operatorname{erf}(-1/2*\sqrt{6})*\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((3*I*b*c - 3*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2) - 9*\sqrt{2}*\sqrt{\pi}*(-2*I*b*c*d + 3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{2})*\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((I*b*c - I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2) - 9*\sqrt{2}*\sqrt{\pi}*(2*I*b*c*d + 3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{2})*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-I*b*c + I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2) - \sqrt{6}*\sqrt{\pi}*(-2*I*b*c*d - d^2)*d*\operatorname{erf}(-1/2*\sqrt{6})*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-3*I*b*c + 3*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2) + 6*(-2*I*(d*x + c)^(3/2)*b*d + 2*I*\sqrt{d*x + c}*b*c*d + \sqrt{d*x + c}*d^2)*e^{((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2 + 18*(2*I*(d*x + c)^(3/2)*b*d - 2*I*\sqrt{d*x + c}*b*c*d - 3*\sqrt{d*x + c}*d^2)*e^{(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2 + 18*(-2*I*(d*x + c)^(3/2)*b*d + 2*I*\sqrt{d*x + c}*b*c*d - 3*\sqrt{d*x + c}*d^2)*e^{((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2 + 6*(2*I*(d*x + c)^(3/2)*b*d - 2*I*\sqrt{d*x + c}*b*c*d + \sqrt{d*x + c}*d^2)*e^{((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d}$$

3.60 $\int \sqrt{c + dx} \cos(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=304

$$\frac{\sqrt{\frac{\pi}{6}} \sqrt{d} \sin\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \sin\left(a - \frac{bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(a - \frac{bc}{d}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}$$

```
[Out] -(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c +
d*x])/Sqrt[d]]/(4*b^(3/2))) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fre
snelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]/(12*b^(3/2))) + (Sqrt[d]*
Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (
3*b*c)/d]/(12*b^(3/2))) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d]/(4*b^(3/2))) + (Sqrt[c + d*x]*Sin[
a + b*x]/(4*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x]/(12*b))
```

Rubi [A] time = 0.469995, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{6}} \sqrt{d} \sin\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \sin\left(a - \frac{bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(a - \frac{bc}{d}\right) S\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] -(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c +
d*x])/Sqrt[d]]/(4*b^(3/2))) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fre
snelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]/(12*b^(3/2))) + (Sqrt[d]*
Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (
3*b*c)/d]/(12*b^(3/2))) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d]/(4*b^(3/2))) + (Sqrt[c + d*x]*Sin[
a + b*x]/(4*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x]/(12*b))
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d,
Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x]
&& ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d,
Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x]
&& ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{c+dx} \cos(a+bx) \sin^2(a+bx) dx &= \int \left(\frac{1}{4} \sqrt{c+dx} \cos(a+bx) - \frac{1}{4} \sqrt{c+dx} \cos(3a+3bx) \right) dx \\
&= \frac{1}{4} \int \sqrt{c+dx} \cos(a+bx) dx - \frac{1}{4} \int \sqrt{c+dx} \cos(3a+3bx) dx \\
&= \frac{\sqrt{c+dx} \sin(a+bx)}{4b} - \frac{\sqrt{c+dx} \sin(3a+3bx)}{12b} + \frac{d \int \frac{\sin(3a+3bx)}{\sqrt{c+dx}} dx}{24b} - \frac{d \int \frac{\sin(a+bx)}{\sqrt{c+dx}} dx}{8b} \\
&= \frac{\sqrt{c+dx} \sin(a+bx)}{4b} - \frac{\sqrt{c+dx} \sin(3a+3bx)}{12b} + \frac{\left(d \cos\left(3a - \frac{3bc}{d}\right) \right) \int \frac{\sin\left(\frac{3bc}{d} + 3bx\right)}{\sqrt{c+dx}} dx}{24b} \\
&= \frac{\sqrt{c+dx} \sin(a+bx)}{4b} - \frac{\sqrt{c+dx} \sin(3a+3bx)}{12b} + \frac{\cos\left(3a - \frac{3bc}{d}\right) \text{Subst}\left(\int \sin\left(\frac{3bc}{d} + 3bx\right) dx\right)}{12b} \\
&= -\frac{\sqrt{d} \sqrt{\frac{\pi}{2}} \cos\left(a - \frac{bc}{d}\right) S\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{d} \sqrt{\frac{\pi}{6}} \cos\left(3a - \frac{3bc}{d}\right) S\left(\frac{\sqrt{b} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} + \dots
\end{aligned}$$

Mathematica [C] time = 6.56647, size = 280, normalized size = 0.92

$$\frac{-\sqrt{2\pi} \sin\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\sqrt{\frac{6}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c+dx}\right) - \sqrt{2\pi} \cos\left(3a - \frac{3bc}{d}\right) S\left(\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right) + 2\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin(3(a+bx))}{24\sqrt{3b} \sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-I/8)*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[
((-I)*b*(c + d*x))/d] - (E^((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/S
qrt[(I*b*(c + d*x))/d])/(b*E^((I*(b*c + a*d))/d)) - ((Sqrt[2*Pi]*Cos[3*a
- (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*Fre
```

$\text{snelC}[\text{Sqrt}[b/d]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x]]*\text{Sin}[3*a - (3*b*c)/d] + 2*\text{Sqrt}[3]*\text{Sqrt}[b/d]*\text{Sqrt}[c + d*x]*\text{Sin}[3*(a + b*x)]/(24*\text{Sqrt}[3]*b*\text{Sqrt}[b/d])$

Maple [A] time = 0.036, size = 294, normalized size = 1.

$$2 \frac{1}{d} \left(1/8 \frac{d\sqrt{dx+c}}{b} \sin\left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d}\right) - 1/16 \frac{d\sqrt{2}\sqrt{\pi}}{b} \left(\cos\left(\frac{ad-bc}{d}\right) \text{FresnelS}\left(\frac{\sqrt{2}\sqrt{dx+cb}}{\sqrt{\pi d}} \frac{1}{\sqrt{b/d}}\right) + \sin\left(\frac{ad-bc}{d}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x)`

[Out] $2/d*(1/8/b*d*(d*x+c)^{(1/2)}*\sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/16/b*d*2^{(1/2)}*\text{Pi}^{(1/2)}/(b/d)^{(1/2)}*(\cos((a*d-b*c)/d)*\text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d)+\sin((a*d-b*c)/d)*\text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d))-1/24/b*d*(d*x+c)^{(1/2)}*\sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/144/b*d*2^{(1/2)}*\text{Pi}^{(1/2)}*3^{(1/2)}/(b/d)^{(1/2)}*(\cos(3*(a*d-b*c)/d)*\text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}*3^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d)+\sin(3*(a*d-b*c)/d)*\text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)}*3^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d))$

Maxima [C] time = 2.31223, size = 1648, normalized size = 5.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")`

[Out] $-1/288*\text{sqrt}(3)*(8*\text{sqrt}(3)*\text{sqrt}(d*x + c)*d*\text{abs}(b)*\sin(3*((d*x + c)*b - b*c + a*d)/d)/\text{abs}(d) - 24*\text{sqrt}(3)*\text{sqrt}(d*x + c)*d*\text{abs}(b)*\sin(((d*x + c)*b - b*c + a*d)/d)/\text{abs}(d) + ((-I*\text{sqrt}(\text{pi})*\cos(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) - I*\text{sqrt}(\text{pi})*\cos(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) - \text{sqrt}(\text{pi})*\sin(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + \text{sqrt}(\text{pi})*\sin(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))))*d*\text{sqrt}(\text{abs}(b)/\text{abs}(d))*\cos(-3*(b*c - a*d)/d) - (\text{sqrt}(\text{pi})*\cos(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + \text{sqrt}(\text{pi})*\cos(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) - I*\text{sqrt}(\text{pi})*\sin(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + I*\text{sqrt}(\text{pi})*\sin(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))))*d*\text{sqrt}(\text{abs}(b)/\text{abs}(d))*\sin(-3*(b*c - a*d)/d))*\text{erf}(\text{sqrt}(d*x + c)*\text{sqrt}(3*I*b/d)) + (\text{sqrt}(3)*3*I*\text{sqrt}(\text{pi})*\cos(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + 3*I*\text{sqrt}(\text{pi})*\cos(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + 3*\text{sqrt}(\text{pi})*\sin(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) - 3*\text{sqrt}(\text{pi})*\sin(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))))*d*\text{sqrt}(\text{abs}(b)/\text{abs}(d))*\cos(-(b*c - a*d)/d) + \text{sqrt}(3)*(3*\text{sqrt}(\text{pi})*\cos(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + 3*\text{sqrt}(\text{pi})*\cos(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) - 3*I*\text{sqrt}(\text{pi})*\sin(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + 3*I*\text{sqrt}(\text{pi})*\sin(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))))*d*\text{sqrt}(\text{abs}(b)/\text{abs}(d))*\sin(-(b*c - a*d)/d))*\text{erf}(\text{sqrt}(d*x + c)*\text{sqrt}(I*b/d)) + (\text{sqrt}(3)*(-3*I*\text{sqrt}(\text{pi})*\cos(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) - 3*I*\text{sqrt}(\text{pi})*\cos(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + 3*\text{sqrt}(\text{pi})*\sin(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) - 3*\text{sqrt}(\text{pi})*\sin(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))))*d*\text{sqrt}(\text{abs}(b)/\text{abs}(d))*\sin(-(b*c - a*d)/d))$

```

^2))) - 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))))*d*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) + sqrt(3)*(3*sqrt(pi)*cos
(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*
sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)
)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(ab
s(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((I*sq
rt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sq
rt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sq
rt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(p
i)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(a
bs(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d
)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d))*abs(d)/(b*d*abs(b))

```

Fricas [A] time = 0.583368, size = 639, normalized size = 2.1

$$\frac{\sqrt{6}\pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{3(bc-ad)}{d}\right) S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 9\sqrt{2}\pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{bc-ad}{d}\right) S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 9\sqrt{2}\pi d \sqrt{\frac{b}{\pi d}} C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")

[Out] 1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*cos(b*x + a)^2 - b)*sqrt(d*x + c)*sin(b*x + a)/b^2

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c+dx} \sin^2(a+bx) \cos(a+bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**2,x)

[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x), x)

Giac [C] time = 1.24356, size = 662, normalized size = 2.18

$$\frac{i\sqrt{6}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right) e^{\left(\frac{3ibc-3iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b} - \frac{9i\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right) e^{\left(\frac{ibc-iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b} + \frac{9i\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right) e^{\left(\frac{-ibc-ia}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/144*(I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c))*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) - 9*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt
(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(
b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sq
rt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sq
rt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3
*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*I*sqrt(d*x + c)*d*e
^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*I*sqrt(d*x + c)*d*e^((I*(
d*x + c)*b - I*b*c + I*a*d)/d)/b - 18*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b
+ I*b*c - I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c
- 3*I*a*d)/d)/b)/d
```

3.61 $\int \sqrt{c + dx} \cos(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=304

$$\frac{\sqrt{\frac{\pi}{6}} \sqrt{d} \sin\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \sin\left(a - \frac{bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(a - \frac{bc}{d}\right) S\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}$$

[Out] $-(\text{Sqrt}[d] * \text{Sqrt}[\text{Pi}/2] * \text{Cos}[a - (b*c)/d] * \text{FresnelS}[(\text{Sqrt}[b] * \text{Sqrt}[2/\text{Pi}] * \text{Sqrt}[c + d*x]) / \text{Sqrt}[d]]) / (4*b^(3/2)) + (\text{Sqrt}[d] * \text{Sqrt}[\text{Pi}/6] * \text{Cos}[3*a - (3*b*c)/d] * \text{FresnelS}[(\text{Sqrt}[b] * \text{Sqrt}[6/\text{Pi}] * \text{Sqrt}[c + d*x]) / \text{Sqrt}[d]]) / (12*b^(3/2)) + (\text{Sqrt}[d] * \text{Sqrt}[\text{Pi}/6] * \text{FresnelC}[(\text{Sqrt}[b] * \text{Sqrt}[6/\text{Pi}] * \text{Sqrt}[c + d*x]) / \text{Sqrt}[d]] * \text{Sin}[3*a - (3*b*c)/d]) / (12*b^(3/2)) - (\text{Sqrt}[d] * \text{Sqrt}[\text{Pi}/2] * \text{FresnelC}[(\text{Sqrt}[b] * \text{Sqrt}[2/\text{Pi}] * \text{Sqrt}[c + d*x]) / \text{Sqrt}[d]] * \text{Sin}[a - (b*c)/d]) / (4*b^(3/2)) + (\text{Sqrt}[c + d*x] * \text{Sin}[a + b*x]) / (4*b) - (\text{Sqrt}[c + d*x] * \text{Sin}[3*a + 3*b*x]) / (12*b)$

Rubi [A] time = 0.46789, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{6}} \sqrt{d} \sin\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \sin\left(a - \frac{bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(a - \frac{bc}{d}\right) S\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[c + d*x] * \text{Cos}[a + b*x] * \text{Sin}[a + b*x]^2, x]$

[Out] $-(\text{Sqrt}[d] * \text{Sqrt}[\text{Pi}/2] * \text{Cos}[a - (b*c)/d] * \text{FresnelS}[(\text{Sqrt}[b] * \text{Sqrt}[2/\text{Pi}] * \text{Sqrt}[c + d*x]) / \text{Sqrt}[d]]) / (4*b^(3/2)) + (\text{Sqrt}[d] * \text{Sqrt}[\text{Pi}/6] * \text{Cos}[3*a - (3*b*c)/d] * \text{FresnelS}[(\text{Sqrt}[b] * \text{Sqrt}[6/\text{Pi}] * \text{Sqrt}[c + d*x]) / \text{Sqrt}[d]]) / (12*b^(3/2)) + (\text{Sqrt}[d] * \text{Sqrt}[\text{Pi}/6] * \text{FresnelC}[(\text{Sqrt}[b] * \text{Sqrt}[6/\text{Pi}] * \text{Sqrt}[c + d*x]) / \text{Sqrt}[d]] * \text{Sin}[3*a - (3*b*c)/d]) / (12*b^(3/2)) - (\text{Sqrt}[d] * \text{Sqrt}[\text{Pi}/2] * \text{FresnelC}[(\text{Sqrt}[b] * \text{Sqrt}[2/\text{Pi}] * \text{Sqrt}[c + d*x]) / \text{Sqrt}[d]] * \text{Sin}[a - (b*c)/d]) / (4*b^(3/2)) + (\text{Sqrt}[c + d*x] * \text{Sin}[a + b*x]) / (4*b) - (\text{Sqrt}[c + d*x] * \text{Sin}[3*a + 3*b*x]) / (12*b)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^(p_.)*((c_.) + (d_.)*(x_.))^(m_.)*\text{Sin}[(a_.) + (b_.)*(x_.)]^(n_.), x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^(n)*\text{Cos}[a + b*x]^p, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^(m_.)*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m * \text{Cos}[e + f*x] / f, x] + \text{Dist}[(d*m) / f, \text{Int}[(c + d*x)^(m-1) * \text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \&\& \text{GtQ}[m, 0]$

Rule 3306

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)] / \text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f) / d], \text{Int}[\text{Sin}[(c*f) / d + f*x] / \text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d*e - c*f) / d], \text{Int}[\text{Cos}[(c*f) / d + f*x] / \text{Sqrt}[c + d*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \&\& \text{ComplexFreeQ}[f] \&\& \text{NeQ}[d*e - c*f, 0]$

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d,
Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f},
x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[
Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d,
Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[
Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{c+dx} \cos(a+bx) \sin^2(a+bx) dx &= \int \left(\frac{1}{4} \sqrt{c+dx} \cos(a+bx) - \frac{1}{4} \sqrt{c+dx} \cos(3a+3bx) \right) dx \\
&= \frac{1}{4} \int \sqrt{c+dx} \cos(a+bx) dx - \frac{1}{4} \int \sqrt{c+dx} \cos(3a+3bx) dx \\
&= \frac{\sqrt{c+dx} \sin(a+bx)}{4b} - \frac{\sqrt{c+dx} \sin(3a+3bx)}{12b} + \frac{d \int \frac{\sin(3a+3bx)}{\sqrt{c+dx}} dx}{24b} - \frac{d \int \frac{\sin(a+bx)}{\sqrt{c+dx}} dx}{8b} \\
&= \frac{\sqrt{c+dx} \sin(a+bx)}{4b} - \frac{\sqrt{c+dx} \sin(3a+3bx)}{12b} + \frac{\left(d \cos\left(3a - \frac{3bc}{d}\right) \right) \int \frac{\sin\left(\frac{3bc}{d} - \frac{3bx}{d}\right)}{\sqrt{c+dx}} dx}{24b} \\
&= \frac{\sqrt{c+dx} \sin(a+bx)}{4b} - \frac{\sqrt{c+dx} \sin(3a+3bx)}{12b} + \frac{\cos\left(3a - \frac{3bc}{d}\right) \text{Subst}\left(\int \frac{\sin\left(\frac{3bc}{d} - \frac{3bx}{d}\right)}{\sqrt{c+dx}} dx\right)}{12b} \\
&= -\frac{\sqrt{d} \sqrt{\frac{\pi}{2}} \cos\left(a - \frac{bc}{d}\right) S\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{d} \sqrt{\frac{\pi}{6}} \cos\left(3a - \frac{3bc}{d}\right) S\left(\frac{\sqrt{b} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}
\end{aligned}$$

Mathematica [C] time = 6.48926, size = 280, normalized size = 0.92

$$\frac{-\sqrt{2\pi} \sin\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\sqrt{\frac{6}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c+dx}\right) - \sqrt{2\pi} \cos\left(3a - \frac{3bc}{d}\right) S\left(\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right) + 2\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left(3a - \frac{3bc}{d}\right)}{24\sqrt{3}b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-I/8)*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[
((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/S
qrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) - ((Sqrt[2*Pi]*Cos[3*a
- (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*Fre
```

snelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]/(24*Sqrt[3]*b*Sqrt[b/d])

Maple [A] time = 0.035, size = 294, normalized size = 1.

$$2 \frac{1}{d} \left(\frac{1}{8} \frac{d\sqrt{dx+c}}{b} \sin\left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d}\right) - \frac{1}{16} \frac{d\sqrt{2}\sqrt{\pi}}{b} \left(\cos\left(\frac{ad-bc}{d}\right) \text{FresnelS}\left(\frac{\sqrt{2}\sqrt{dx+cb}}{\sqrt{\pi d}} \frac{1}{\sqrt{\frac{b}{d}}}\right) + \sin\left(\frac{ad-bc}{d}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x)

[Out] 2/d*(1/8/b*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/16/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))-1/24/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))

Maxima [C] time = 2.43116, size = 1648, normalized size = 5.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")

[Out] -1/288*sqrt(3)*(8*sqrt(3)*sqrt(d*x + c)*d*abs(b)*sin(3*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 24*sqrt(3)*sqrt(d*x + c)*d*abs(b)*sin(((d*x + c)*b - b*c + a*d)/d)/abs(d) + ((-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (sqrt(3)*(3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) + sqrt(3)*(3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d*sqrt(abs(b)/abs(d))*sin(-(b*c - a*d)/d)*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (sqrt(3)*(-3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d)/d)*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (sqrt(3)*(-3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d*sqrt(abs(b)/abs(d))*sin(-(b*c - a*d)/d)*erf(sqrt(d*x + c)*sqrt(I*b/d))

$$\begin{aligned} & \left(\sqrt{d^2} \right) - 3\sqrt{\pi} \sin\left(-\frac{1}{4}\pi + \frac{1}{2}\arctan\left(\frac{b}{d}\right) + \frac{1}{2}\arctan\left(\frac{d}{\sqrt{d^2}}\right)\right) \\ & + \sqrt{3} \left(3\sqrt{\pi} \cos\left(\frac{1}{4}\pi + \frac{1}{2}\arctan\left(\frac{b}{d}\right) + \frac{1}{2}\arctan\left(\frac{d}{\sqrt{d^2}}\right)\right) + 3\sqrt{\pi} \cos\left(-\frac{1}{4}\pi + \frac{1}{2}\arctan\left(\frac{b}{d}\right) + \frac{1}{2}\arctan\left(\frac{d}{\sqrt{d^2}}\right)\right) \right. \\ & + 3I\sqrt{\pi} \sin\left(\frac{1}{4}\pi + \frac{1}{2}\arctan\left(\frac{b}{d}\right) + \frac{1}{2}\arctan\left(\frac{d}{\sqrt{d^2}}\right)\right) - 3I\sqrt{\pi} \sin\left(-\frac{1}{4}\pi + \frac{1}{2}\arctan\left(\frac{b}{d}\right) + \frac{1}{2}\arctan\left(\frac{d}{\sqrt{d^2}}\right)\right) \\ & \left. + d\sqrt{\frac{b}{d}} \cos\left(-\frac{b^*c - a*d}{d}\right) + \sqrt{3} \left(3\sqrt{\pi} \cos\left(\frac{1}{4}\pi + \frac{1}{2}\arctan\left(\frac{b}{d}\right) + \frac{1}{2}\arctan\left(\frac{d}{\sqrt{d^2}}\right)\right) + 3\sqrt{\pi} \cos\left(-\frac{1}{4}\pi + \frac{1}{2}\arctan\left(\frac{b}{d}\right) + \frac{1}{2}\arctan\left(\frac{d}{\sqrt{d^2}}\right)\right) \right. \right. \\ & + 3I\sqrt{\pi} \sin\left(\frac{1}{4}\pi + \frac{1}{2}\arctan\left(\frac{b}{d}\right) + \frac{1}{2}\arctan\left(\frac{d}{\sqrt{d^2}}\right)\right) - 3I\sqrt{\pi} \sin\left(-\frac{1}{4}\pi + \frac{1}{2}\arctan\left(\frac{b}{d}\right) + \frac{1}{2}\arctan\left(\frac{d}{\sqrt{d^2}}\right)\right) \\ & \left. + d\sqrt{\frac{b}{d}} \cos\left(-\frac{b^*c - a*d}{d}\right) \right) \operatorname{erf}\left(\sqrt{d^*x + c}\sqrt{-I*b/d}\right) + \left(I\sqrt{\pi} \cos\left(\frac{1}{4}\pi + \frac{1}{2}\arctan\left(\frac{b}{d}\right) + \frac{1}{2}\arctan\left(\frac{d}{\sqrt{d^2}}\right)\right) + I\sqrt{\pi} \cos\left(-\frac{1}{4}\pi + \frac{1}{2}\arctan\left(\frac{b}{d}\right) + \frac{1}{2}\arctan\left(\frac{d}{\sqrt{d^2}}\right)\right) \right. \\ & - \sqrt{\pi} \sin\left(\frac{1}{4}\pi + \frac{1}{2}\arctan\left(\frac{b}{d}\right) + \frac{1}{2}\arctan\left(\frac{d}{\sqrt{d^2}}\right)\right) + \sqrt{\pi} \sin\left(-\frac{1}{4}\pi + \frac{1}{2}\arctan\left(\frac{b}{d}\right) + \frac{1}{2}\arctan\left(\frac{d}{\sqrt{d^2}}\right)\right) \\ & \left. + d\sqrt{\frac{b}{d}} \cos\left(-\frac{b^*c - a*d}{d}\right) - \left(\sqrt{\pi} \cos\left(\frac{1}{4}\pi + \frac{1}{2}\arctan\left(\frac{b}{d}\right) + \frac{1}{2}\arctan\left(\frac{d}{\sqrt{d^2}}\right)\right) + \sqrt{\pi} \cos\left(-\frac{1}{4}\pi + \frac{1}{2}\arctan\left(\frac{b}{d}\right) + \frac{1}{2}\arctan\left(\frac{d}{\sqrt{d^2}}\right)\right) \right. \right. \\ & + I\sqrt{\pi} \sin\left(\frac{1}{4}\pi + \frac{1}{2}\arctan\left(\frac{b}{d}\right) + \frac{1}{2}\arctan\left(\frac{d}{\sqrt{d^2}}\right)\right) - I\sqrt{\pi} \sin\left(-\frac{1}{4}\pi + \frac{1}{2}\arctan\left(\frac{b}{d}\right) + \frac{1}{2}\arctan\left(\frac{d}{\sqrt{d^2}}\right)\right) \\ & \left. + d\sqrt{\frac{b}{d}} \cos\left(-\frac{b^*c - a*d}{d}\right) \right) \operatorname{erf}\left(\sqrt{d^*x + c}\sqrt{-3*I*b/d}\right) \left| \frac{d}{b^*d^*b} \right| \end{aligned}$$

Fricas [A] time = 0.586352, size = 639, normalized size = 2.1

$$\frac{\sqrt{6}\pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{3(bc-ad)}{d}\right) S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 9\sqrt{2}\pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{bc-ad}{d}\right) S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 9\sqrt{2}\pi d \sqrt{\frac{b}{\pi d}} C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")

[Out] $\frac{1}{72} \left(\sqrt{6} \pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{3(bc-ad)}{d}\right) \operatorname{fresnel_sin}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 9\sqrt{2} \pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{bc-ad}{d}\right) \operatorname{fresnel_sin}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 9\sqrt{2} \pi d \sqrt{\frac{b}{\pi d}} \operatorname{fresnel_cos}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{bc-ad}{d}\right) + \sqrt{6} \pi d \sqrt{\frac{b}{\pi d}} \operatorname{fresnel_cos}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{3(bc-ad)}{d}\right) - 24(b \cos(bx+a)^2 - b) \sqrt{dx+c} \sin(bx+a) \right) / b^2$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c+dx} \sin^2(a+bx) \cos(a+bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**2,x)

[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x), x)

Giac [C] time = 1.27978, size = 662, normalized size = 2.18

$$\frac{i\sqrt{6}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{2d}\right) e^{\left(\frac{3ibc-3iad}{d}\right)} - 9i\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{2d}\right) e^{\left(\frac{ibc-iad}{d}\right)} + 9i\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{2d}\right) e^{\left(-\frac{ibc-iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} - \frac{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} + \frac{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)b}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/144*(I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) - 9*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt
(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(
b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sq
rt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sq
rt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3
*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*I*sqrt(d*x + c)*d*e
^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*I*sqrt(d*x + c)*d*e^((I*(
d*x + c)*b - I*b*c + I*a*d)/d)/b - 18*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b
+ I*b*c - I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c
- 3*I*a*d)/d)/b)/d
```

3.62 $\int (c + dx)^{3/2} \cos(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=353

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} + \frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}} - \frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a - \frac{3bc}{d}\right)}{24b^{5/2}}$$

```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(8*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(24*b^2) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])*Sin[a - (b*c)/d])/(8*b^(5/2)) + ((c + d*x)^(3/2)*Sin[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(12*b)
```

Rubi [A] time = 0.57135, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} + \frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}} - \frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a - \frac{3bc}{d}\right)}{24b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2, x]
```

```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(8*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(24*b^2) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])*Sin[a - (b*c)/d])/(8*b^(5/2)) + ((c + d*x)^(3/2)*Sin[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(12*b)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_.)]^(p_.)*((c_.) + (d_.)*(x_.))^(m_.)*Sin[(a_.) + (b_.)*(x_.)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n * Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] := -Simp[(c + d*x)^m * Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1) * Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_.)]/Sqrt[(c_.) + (d_.)*(x_.)], x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
```

$*e - c*f)/d$, Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]

Rule 3305

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)]]/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3304

Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)]]/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}
 \int (c + dx)^{3/2} \cos(a + bx) \sin^2(a + bx) dx &= \int \left(\frac{1}{4}(c + dx)^{3/2} \cos(a + bx) - \frac{1}{4}(c + dx)^{3/2} \cos(3a + 3bx) \right) dx \\
 &= \frac{1}{4} \int (c + dx)^{3/2} \cos(a + bx) dx - \frac{1}{4} \int (c + dx)^{3/2} \cos(3a + 3bx) dx \\
 &= \frac{(c + dx)^{3/2} \sin(a + bx)}{4b} - \frac{(c + dx)^{3/2} \sin(3a + 3bx)}{12b} + \frac{d \int \sqrt{c + dx} \sin(3a + 3bx) dx}{8b} \\
 &= \frac{3d\sqrt{c + dx} \cos(a + bx)}{8b^2} - \frac{d\sqrt{c + dx} \cos(3a + 3bx)}{24b^2} + \frac{(c + dx)^{3/2} \sin(a + bx)}{4b} \\
 &= \frac{3d\sqrt{c + dx} \cos(a + bx)}{8b^2} - \frac{d\sqrt{c + dx} \cos(3a + 3bx)}{24b^2} + \frac{(c + dx)^{3/2} \sin(a + bx)}{4b} \\
 &= \frac{3d\sqrt{c + dx} \cos(a + bx)}{8b^2} - \frac{d\sqrt{c + dx} \cos(3a + 3bx)}{24b^2} + \frac{(c + dx)^{3/2} \sin(a + bx)}{4b} \\
 &= \frac{3d\sqrt{c + dx} \cos(a + bx)}{8b^2} - \frac{d\sqrt{c + dx} \cos(3a + 3bx)}{24b^2} - \frac{3d^{3/2} \sqrt{\frac{\pi}{2}} \cos\left(a - \frac{bc}{d}\right) C\left(\sqrt{\frac{2}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c + dx}\right)}{8b^{5/2}}
 \end{aligned}$$

Mathematica [C] time = 9.30628, size = 677, normalized size = 1.92

$$\frac{ic\sqrt{c + dx} e^{-\frac{i(ad+bc)}{d}} \left(\frac{e^{2ia} \Gamma\left(\frac{3}{2}, -\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}} \Gamma\left(\frac{3}{2}, \frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}} \right)}{8b} + \frac{d \left(\sqrt{2\pi} \sqrt{\frac{b}{d}} \text{FresnelC} \left(\sqrt{\frac{2}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c + dx} \right) \right) (2bc \sin(a - \frac{bc}{d}))}{8b^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]

[Out]
$$\begin{aligned} &((-I/8)*c*\text{Sqrt}[c + d*x]*((E^{((2*I)*a)}*\text{Gamma}[3/2, ((-I)*b*(c + d*x))/d])/ \text{Sqrt} \\ & \text{t}[((-I)*b*(c + d*x))/d] - (E^{((2*I)*b*c)/d}*\text{Gamma}[3/2, (I*b*(c + d*x))/d]) \\ & / \text{Sqrt}[(I*b*(c + d*x))/d])/(b*E^{(I*(b*c + a*d))/d}) + (d*(\text{Sqrt}[b/d]*\text{Sqrt}[2 \\ & *Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[2/Pi]*\text{Sqrt}[c + d*x]]*(-3*d*\text{Cos}[a - (b*c)/d] + \\ & 2*b*c*\text{Sin}[a - (b*c)/d]) + \text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[2/Pi \\ &]*\text{Sqrt}[c + d*x]]*(2*b*c*\text{Cos}[a - (b*c)/d] + 3*d*\text{Sin}[a - (b*c)/d]) + 2*b*\text{Sqrt} \\ & [c + d*x]*(3*\text{Cos}[a + b*x] + 2*b*x*\text{Sin}[a + b*x]))/(16*b^3) - (c*(-\text{Sqrt}[2*P \\ & i]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]) - \text{Sqr} \\ & \text{t}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*\text{Sin}[3*a - (3*b*c)/d] + \\ & 2*\text{Sqrt}[3]*\text{Sqrt}[b/d]*\text{Sqrt}[c + d*x]*\text{Sin}[3*(a + b*x)]))/(24*\text{Sqrt}[3]*b*\text{Sqrt}[b/ \\ & d]) - (d*(\text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]] \\ & *(-d*\text{Cos}[3*a - (3*b*c)/d]) + 2*b*c*\text{Sin}[3*a - (3*b*c)/d]) + \text{Sqrt}[b/d]*\text{Sqrt}[\\ & 2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*(2*b*c*\text{Cos}[3*a - (3*b*c) \\ & /d] + d*\text{Sin}[3*a - (3*b*c)/d]) + 2*\text{Sqrt}[3]*b*\text{Sqrt}[c + d*x]*(\text{Cos}[3*(a + b*x)] \\ & + 2*b*x*\text{Sin}[3*(a + b*x)])))/(48*\text{Sqrt}[3]*b^3) \end{aligned}$$

Maple [A] time = 0.04, size = 386, normalized size = 1.1

$$2 \frac{1}{d} \left(\frac{1}{8} \frac{d(dx+c)^{3/2}}{b} \sin\left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d}\right) - \frac{3}{8} \frac{d}{b} \left(-\frac{1}{2} \frac{d\sqrt{dx+c}}{b} \cos\left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d}\right) + \frac{1}{4} \frac{d\sqrt{2}\sqrt{\pi}}{b} \left(\cos\left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x)

[Out]
$$\begin{aligned} &2/d*(1/8/b*d*(d*x+c)^{(3/2)}*\text{sin}(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/8/b*d*(-1/2/b*d \\ & *(d*x+c)^{(1/2)}*\text{cos}(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^{(1/2)}*Pi^{(1/2)}/(b/d \\ &)^{(1/2)}*(\text{cos}((a*d-b*c)/d)*\text{FresnelC}(2^{(1/2)}/Pi^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/ \\ & 2)}*b/d)-\text{sin}((a*d-b*c)/d)*\text{FresnelS}(2^{(1/2)}/Pi^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2 \\ &)}*b/d))-1/24/b*d*(d*x+c)^{(3/2)}*\text{sin}(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/8/b*d*(- \\ & 1/6/b*d*(d*x+c)^{(1/2)}*\text{cos}(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^{(1/2)}*Pi^{ \\ & (1/2)}*3^{(1/2)}/(b/d)^{(1/2)}*(\text{cos}(3*(a*d-b*c)/d)*\text{FresnelC}(2^{(1/2)}/Pi^{(1/2)}*3^{(\\ & 1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d)-\text{sin}(3*(a*d-b*c)/d)*\text{FresnelS}(2^{(1/2)}/Pi^{ \\ & (1/2)}*3^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d))) \end{aligned}$$

Maxima [C] time = 2.37528, size = 1787, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")

[Out]
$$\begin{aligned} &-1/576*\text{sqrt}(3)*(16*\text{sqrt}(3)*(d*x + c)^{(3/2)}*b*d*\text{abs}(b)*\text{sin}(3*((d*x + c)*b - \\ & b*c + a*d)/d)/\text{abs}(d) - 48*\text{sqrt}(3)*(d*x + c)^{(3/2)}*b*d*\text{abs}(b)*\text{sin}(((d*x + c) \\ & *b - b*c + a*d)/d)/\text{abs}(d) + 8*\text{sqrt}(3)*\text{sqrt}(d*x + c)*d^2*\text{abs}(b)*\text{cos}(3*((d*x \\ & + c)*b - b*c + a*d)/d)/\text{abs}(d) - 72*\text{sqrt}(3)*\text{sqrt}(d*x + c)*d^2*\text{abs}(b)*\text{cos}(((d \\ & *x + c)*b - b*c + a*d)/d)/\text{abs}(d) - ((\text{sqrt}(pi)*\text{cos}(1/4*pi + 1/2*\text{arctan2}(0, b \\ &) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + \text{sqrt}(pi)*\text{cos}(-1/4*pi + 1/2*\text{arctan2}(0, b \\ &) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) - I*\text{sqrt}(pi)*\text{sin}(1/4*pi + 1/2*\text{arctan2}(0, b \\ &) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + I*\text{sqrt}(pi)*\text{sin}(-1/4*pi + 1/2*\text{arctan2}(0, b \end{aligned}$$

```

) + 1/2*arctan2(0, d/sqrt(d^2))) * d^2 * sqrt(abs(b)/abs(d)) * cos(-3*(b*c - a*d)/d) - (I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * sin(-3*(b*c - a*d)/d) * erf(sqrt(d*x + c) * sqrt(3*I*b/d)) + (sqrt(3)*(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * cos(-(b*c - a*d)/d) + sqrt(3)*(-9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * sin(-(b*c - a*d)/d) * erf(sqrt(d*x + c) * sqrt(I*b/d)) + (sqrt(3)*(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * cos(-(b*c - a*d)/d) + sqrt(3)*(9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * sin(-(b*c - a*d)/d) * erf(sqrt(d*x + c) * sqrt(-I*b/d)) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * cos(-3*(b*c - a*d)/d) - (-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * sin(-3*(b*c - a*d)/d) * erf(sqrt(d*x + c) * sqrt(-3*I*b/d))) * abs(d)/(b^2*d*abs(b))

```

Fricas [A] time = 0.607138, size = 756, normalized size = 2.14

$$\sqrt{6}\pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{3(bc-ad)}{d}\right) C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 27\sqrt{2}\pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{bc-ad}{d}\right) C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) + 27\sqrt{2}\pi d^2 \sqrt{\frac{b}{\pi d}} S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*d*cos(b*x + a)^3 - 3*b*d*cos(b*x + a) - 2*(b^2*d*x + b^2*c - (b^2*d*x + b^2*c)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**2,x)

[Out] Timed out

Giac [C] time = 1.37434, size = 1517, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/288*(2*(I*\sqrt{6}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d})*\sqrt{d*x+c}) * \\ & (I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((3*I*b*c-3*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2}+1)*b)} - 9*I*\sqrt{2}*\sqrt{\pi}*d^2*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d})*\sqrt{d*x+c} * \\ & (I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((I*b*c-I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2}+1)*b)} + 9*I*\sqrt{2}*\sqrt{\pi}*d^2*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d})*\sqrt{d*x+c} * \\ & (-I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((-I*b*c+I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2}+1)*b)} - I*\sqrt{6}*\sqrt{\pi}*d^2*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d})*\sqrt{d*x+c} * \\ & (-I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((-3*I*b*c+3*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2}+1)*b)} - 6*I*\sqrt{d*x+c} * \\ & d*e^{((3*I*(d*x+c)*b-3*I*b*c+3*I*a*d)/d)/b} + 18*I*\sqrt{d*x+c}*d*e^{((I*(d*x+c)*b-I*b*c+I*a*d)/d)/b} - 18*I*\sqrt{d*x+c}*d*e^{((-I*(d*x+c)*b+I*b*c-I*a*d)/d)/b} + 6*I*\sqrt{d*x+c} * \\ & d*e^{((-3*I*(d*x+c)*b+3*I*b*c-3*I*a*d)/d)/b} *c - \sqrt{6}*\sqrt{\pi}*(2*I*b*c*d-d^2)*d*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d})*\sqrt{d*x+c} * \\ & (I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((3*I*b*c-3*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2}+1)*b^2)} - 9*\sqrt{2}*\sqrt{\pi}*(-2*I*b*c*d+3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d})*\sqrt{d*x+c} * \\ & (I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((I*b*c-I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2}+1)*b^2)} - 9*\sqrt{2}*\sqrt{\pi}*(2*I*b*c*d+3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d})*\sqrt{d*x+c} * \\ & (-I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((-I*b*c+I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2}+1)*b^2)} - \sqrt{6}*\sqrt{\pi}*(-2*I*b*c*d-d^2)*d*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d})*\sqrt{d*x+c} * \\ & (-I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((-3*I*b*c+3*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2}+1)*b^2)} + 6*(-2*I*(d*x+c)^(3/2)*b*d+2*I*\sqrt{d*x+c})*b*c*d + \sqrt{d*x+c} * \\ & d^2)*e^{((3*I*(d*x+c)*b-3*I*b*c+3*I*a*d)/d)/b^2} + 18*(2*I*(d*x+c)^(3/2)*b*d-2*I*\sqrt{d*x+c})*b*c*d - 3*\sqrt{d*x+c} * \\ & d^2)*e^{((-I*(d*x+c)*b+I*b*c-I*a*d)/d)/b^2} + 6*(2*I*(d*x+c)^(3/2)*b*d-2*I*\sqrt{d*x+c})*b*c*d + \sqrt{d*x+c} * \\ & d^2)*e^{((-3*I*(d*x+c)*b+3*I*b*c-3*I*a*d)/d)/b^2}/d \end{aligned}$$

3.63 $\int (c + dx)^{5/2} \cos(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=406

$$\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}} + \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} + \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a - \frac{bc}{d}\right)}{16b^{7/2}}$$

[Out] $(5*d*(c + d*x)^{(3/2)}*\text{Cos}[a + b*x])/(8*b^2) - (5*d*(c + d*x)^{(3/2)}*\text{Cos}[3*a + 3*b*x])/(72*b^2) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a - (b*c)/d]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(16*b^{(7/2)}) - (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(144*b^{(7/2)}) - (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[3*a - (3*b*c)/d])/(144*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[a - (b*c)/d])/(16*b^{(7/2)}) - (15*d^2*\text{Sqrt}[c + d*x]*\text{Sin}[a + b*x])/(16*b^3) + ((c + d*x)^{(5/2)}*\text{Sin}[a + b*x])/(4*b) + (5*d^2*\text{Sqrt}[c + d*x]*\text{Sin}[3*a + 3*b*x])/(144*b^3) - ((c + d*x)^{(5/2)}*\text{Sin}[3*a + 3*b*x])/(12*b)$

Rubi [A] time = 0.667682, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}} + \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} + \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a - \frac{bc}{d}\right)}{16b^{7/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^{(5/2)}*\text{Cos}[a + b*x]*\text{Sin}[a + b*x]^2, x]$

[Out] $(5*d*(c + d*x)^{(3/2)}*\text{Cos}[a + b*x])/(8*b^2) - (5*d*(c + d*x)^{(3/2)}*\text{Cos}[3*a + 3*b*x])/(72*b^2) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a - (b*c)/d]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(16*b^{(7/2)}) - (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(144*b^{(7/2)}) - (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[3*a - (3*b*c)/d])/(144*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[a - (b*c)/d])/(16*b^{(7/2)}) - (15*d^2*\text{Sqrt}[c + d*x]*\text{Sin}[a + b*x])/(16*b^3) + ((c + d*x)^{(5/2)}*\text{Sin}[a + b*x])/(4*b) + (5*d^2*\text{Sqrt}[c + d*x]*\text{Sin}[3*a + 3*b*x])/(144*b^3) - ((c + d*x)^{(5/2)}*\text{Sin}[3*a + 3*b*x])/(12*b)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.))^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3306


```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^{5/2} \cos(a + bx) \sin^2(a + bx) dx &= \int \left(\frac{1}{4}(c + dx)^{5/2} \cos(a + bx) - \frac{1}{4}(c + dx)^{5/2} \cos(3a + 3bx) \right) dx \\
&= \frac{1}{4} \int (c + dx)^{5/2} \cos(a + bx) dx - \frac{1}{4} \int (c + dx)^{5/2} \cos(3a + 3bx) dx \\
&= \frac{(c + dx)^{5/2} \sin(a + bx)}{4b} - \frac{(c + dx)^{5/2} \sin(3a + 3bx)}{12b} + \frac{(5d) \int (c + dx)^{3/2} \sin(a + bx) dx}{24b} \\
&= \frac{5d(c + dx)^{3/2} \cos(a + bx)}{8b^2} - \frac{5d(c + dx)^{3/2} \cos(3a + 3bx)}{72b^2} + \frac{(c + dx)^{5/2} \sin(a + bx)}{4b} \\
&= \frac{5d(c + dx)^{3/2} \cos(a + bx)}{8b^2} - \frac{5d(c + dx)^{3/2} \cos(3a + 3bx)}{72b^2} - \frac{15d^2 \sqrt{c + dx} \sin(a + bx)}{16b^3} \\
&= \frac{5d(c + dx)^{3/2} \cos(a + bx)}{8b^2} - \frac{5d(c + dx)^{3/2} \cos(3a + 3bx)}{72b^2} - \frac{15d^2 \sqrt{c + dx} \sin(a + bx)}{16b^3} \\
&= \frac{5d(c + dx)^{3/2} \cos(a + bx)}{8b^2} - \frac{5d(c + dx)^{3/2} \cos(3a + 3bx)}{72b^2} - \frac{15d^2 \sqrt{c + dx} \sin(a + bx)}{16b^3} \\
&= \frac{5d(c + dx)^{3/2} \cos(a + bx)}{8b^2} - \frac{5d(c + dx)^{3/2} \cos(3a + 3bx)}{72b^2} + \frac{15d^{5/2} \sqrt{\frac{\pi}{2}} \cos(a + bx)}{16b^3}
\end{aligned}$$

Mathematica [C] time = 14.6632, size = 1171, normalized size = 2.88

$$\frac{ie^{-\frac{i(bc+ad)}{d}} \sqrt{c + dx} \left(\frac{e^{2ia} \Gamma\left(\frac{3}{2}, -\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}} \Gamma\left(\frac{3}{2}, \frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}} \right) c^2}{8b} - \frac{\left(-\sqrt{2\pi} \cos\left(3a - \frac{3bc}{d}\right) S\left(\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c + dx}\right) - \sqrt{2\pi} \right)}{8b}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]

[Out]
$$\begin{aligned} &((-I/8)*c^2*\text{Sqrt}[c + d*x]*((E^{((2*I)*a)}*\text{Gamma}[3/2, ((-I)*b*(c + d*x))/d])/S \\ &\text{qrt}[((-I)*b*(c + d*x))/d] - (E^{((2*I)*b*c)/d}*\text{Gamma}[3/2, (I*b*(c + d*x))/d \\ &])/ \text{Sqrt}[(I*b*(c + d*x))/d]))/(b*E^{(I*(b*c + a*d))/d}) + (c*d*(\text{Sqrt}[b/d]*\text{Sq} \\ &\text{rt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[2/Pi]*\text{Sqrt}[c + d*x]]*(-3*d*\text{Cos}[a - (b*c)/d \\ &] + 2*b*c*\text{Sin}[a - (b*c)/d]) + \text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[\\ &2/Pi]*\text{Sqrt}[c + d*x]]*(2*b*c*\text{Cos}[a - (b*c)/d] + 3*d*\text{Sin}[a - (b*c)/d]) + 2*b* \\ &\text{Sqrt}[c + d*x]*(3*\text{Cos}[a + b*x] + 2*b*x*\text{Sin}[a + b*x])))/(8*b^3) + ((b/d)^(3/2) \\ &)*d^2*(-(\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[2/Pi]*\text{Sqrt}[c + d*x]]*((4*b^2*c^2 \\ &- 15*d^2)*\text{Cos}[a - (b*c)/d] + 12*b*c*d*\text{Sin}[a - (b*c)/d])) - \text{Sqrt}[2*Pi]*\text{Fre} \\ &\text{snelC}[\text{Sqrt}[b/d]*\text{Sqrt}[2/Pi]*\text{Sqrt}[c + d*x]]*(-12*b*c*d*\text{Cos}[a - (b*c)/d] + (4* \\ &b^2*c^2 - 15*d^2)*\text{Sin}[a - (b*c)/d]) + 2*\text{Sqrt}[b/d]*d*\text{Sqrt}[c + d*x]*(-2*b*(c \\ &- 5*d*x)*\text{Cos}[a + b*x] + d*(-15 + 4*b^2*x^2)*\text{Sin}[a + b*x]))/(32*b^5) - (c^2 \\ &*(-(\text{Sqrt}[2*Pi]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + \\ &d*x]]) - \text{Sqrt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*\text{Sin}[3*a - \\ &(3*b*c)/d] + 2*\text{Sqrt}[3]*\text{Sqrt}[b/d]*\text{Sqrt}[c + d*x]*\text{Sin}[3*(a + b*x)])))/(24*\text{Sqrt}[\\ &3]*b*\text{Sqrt}[b/d]) - (c*d*(\text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]* \\ &\text{Sqrt}[c + d*x]]*(-(d*\text{Cos}[3*a - (3*b*c)/d]) + 2*b*c*\text{Sin}[3*a - (3*b*c)/d]) + \text{S} \\ &\text{qrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*(2*b*c*\text{Cos} \\ &[3*a - (3*b*c)/d] + d*\text{Sin}[3*a - (3*b*c)/d]) + 2*\text{Sqrt}[3]*b*\text{Sqrt}[c + d*x]*(\text{Co} \\ &\text{s}[3*(a + b*x)] + 2*b*x*\text{Sin}[3*(a + b*x)])))/(24*\text{Sqrt}[3]*b^3) - ((b/d)^(3/2)* \\ &d^2*(-(\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*((12*b^2*c^2 \\ &- 5*d^2)*\text{Cos}[3*a - (3*b*c)/d] + 12*b*c*d*\text{Sin}[3*a - (3*b*c)/d])) - \text{Sqrt}[2*P \\ &i]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*(-12*b*c*d*\text{Cos}[3*a - (3*b*c \\ &)/d] + (12*b^2*c^2 - 5*d^2)*\text{Sin}[3*a - (3*b*c)/d]) + 2*\text{Sqrt}[3]*\text{Sqrt}[b/d]*d*\text{S} \\ &\text{qrt}[c + d*x]*(-2*b*(c - 5*d*x)*\text{Cos}[3*(a + b*x)] + d*(-5 + 12*b^2*x^2)*\text{Sin}[3 \\ &*(a + b*x)])))/(288*\text{Sqrt}[3]*b^5) \end{aligned}$$

Maple [A] time = 0.036, size = 474, normalized size = 1.2

$$2 \frac{1}{d} \left(\frac{1}{8} \frac{d(dx+c)^{5/2}}{b} \sin \left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d} \right) - \frac{5}{8} \frac{d}{b} \left(-\frac{1}{2} \frac{d(dx+c)^{3/2}}{b} \cos \left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d} \right) + \frac{3}{2} \frac{d}{b} \left(\frac{1}{2} \frac{d\sqrt{dx}}{b} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x)

[Out]
$$\begin{aligned} &2/d*(1/8/b*d*(d*x+c)^(5/2)*\text{sin}(1/d*(d*x+c)*b+(a*d-b*c)/d)-5/8/b*d*(-1/2/b*d \\ &*(d*x+c)^(3/2)*\text{cos}(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2) \\ &)*\text{sin}(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*\text{Pi}^(1/2)/(b/d)^(1/2)*(\text{cos}((\\ &a*d-b*c)/d)*\text{FresnelS}(2^(1/2)/\text{Pi}^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+\text{sin}((\\ &a*d-b*c)/d)*\text{FresnelC}(2^(1/2)/\text{Pi}^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))) -1/2 \\ &4/b*d*(d*x+c)^(5/2)*\text{sin}(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/24/b*d*(-1/6/b*d*(d* \\ &x+c)^(3/2)*\text{cos}(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2)* \\ &\text{sin}(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*\text{Pi}^(1/2)*3^(1/2)/(b/d)^(1 \\ &/2)*(\text{cos}(3*(a*d-b*c)/d)*\text{FresnelS}(2^(1/2)/\text{Pi}^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+ \\ &c)^(1/2)*b/d)+\text{sin}(3*(a*d-b*c)/d)*\text{FresnelC}(2^(1/2)/\text{Pi}^(1/2)*3^(1/2)/(b/d)^(1 \\ &/2)*(d*x+c)^(1/2)*b/d)))) \end{aligned}$$

Maxima [C] time = 2.45579, size = 1866, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/3456\sqrt{3}*(80\sqrt{3}*(d*x + c)^{(3/2)}*b*d^2*\text{abs}(b)*\cos(3*((d*x + c)*b \\ & - b*c + a*d)/d)/\text{abs}(d) - 720\sqrt{3}*(d*x + c)^{(3/2)}*b*d^2*\text{abs}(b)*\cos(((d*x \\ & + c)*b - b*c + a*d)/d)/\text{abs}(d) + ((5*I*\sqrt{\pi})*\cos(1/4*\pi + 1/2*\arctan2(0 \\ & , b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 5*I*\sqrt{\pi})*\cos(-1/4*\pi + 1/2*\arctan \\ & 2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 5*\sqrt{\pi})*\sin(1/4*\pi + 1/2*\arctan \\ & 2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 5*\sqrt{\pi})*\sin(-1/4*\pi + 1/2*\arcta \\ & n2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^3*\sqrt{\text{abs}(b)/\text{abs}(d)}*\cos(-3*(b* \\ & c - a*d)/d) + (5*\sqrt{\pi})*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d \\ & / \sqrt{d^2})) + 5*\sqrt{\pi})*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, \\ & d/\sqrt{d^2})) - 5*I*\sqrt{\pi})*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0 \\ & , d/\sqrt{d^2})) + 5*I*\sqrt{\pi})*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan \\ & 2(0, d/\sqrt{d^2}))) * d^3*\sqrt{\text{abs}(b)/\text{abs}(d)}*\sin(-3*(b*c - a*d)/d))*\text{erf}(\sqrt{ \\ & (d*x + c)*\sqrt{3*I*b/d}}) + (\sqrt{3})*(-135*I*\sqrt{\pi})*\cos(1/4*\pi + 1/2*\arcta \\ & n2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 135*I*\sqrt{\pi})*\cos(-1/4*\pi + 1/2* \\ & \arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 135*\sqrt{\pi})*\sin(1/4*\pi + 1/ \\ & 2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 135*\sqrt{\pi})*\sin(-1/4*\pi + \\ & 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^3*\sqrt{\text{abs}(b)/\text{abs}(d)}* \\ & \cos(-(b*c - a*d)/d) - \sqrt{3}*(135*\sqrt{\pi})*\cos(1/4*\pi + 1/2*\arctan2(0, b) \\ & + 1/2*\arctan2(0, d/\sqrt{d^2})) + 135*\sqrt{\pi})*\cos(-1/4*\pi + 1/2*\arctan2(0, \\ & b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 135*I*\sqrt{\pi})*\sin(1/4*\pi + 1/2*\arctan2 \\ & (0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 135*I*\sqrt{\pi})*\sin(-1/4*\pi + 1/2*\ar \\ & ctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^3*\sqrt{\text{abs}(b)/\text{abs}(d)}*\sin(-(b \\ & *c - a*d)/d))*\text{erf}(\sqrt{(d*x + c)*\sqrt{I*b/d}}) + (\sqrt{3})*(135*I*\sqrt{\pi})*\cos \\ & (1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 135*I*\sqrt{\pi} \\ & *\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 135*\sqrt{ \\ & \pi})*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 135*\sqrt{ \\ & \pi})*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^3*s \\ & \sqrt{\text{abs}(b)/\text{abs}(d)}*\cos(-(b*c - a*d)/d) - \sqrt{3}*(135*\sqrt{\pi})*\cos(1/4*\pi + \\ & 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 135*\sqrt{\pi})*\cos(-1/4*\pi \\ & + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 135*I*\sqrt{\pi})*\sin(1 \\ & /4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 135*I*\sqrt{\pi})*s \\ & \sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^3*\sqrt{\text{abs} \\ & (b)/\text{abs}(d)}*\sin(-(b*c - a*d)/d))*\text{erf}(\sqrt{(d*x + c)*\sqrt{-I*b/d}}) + ((-5*I*s \\ & \sqrt{\pi})*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 5*I \\ & *\sqrt{\pi})*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + \\ & 5*\sqrt{\pi})*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - \\ & 5*\sqrt{\pi})*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * \\ & d^3*\sqrt{\text{abs}(b)/\text{abs}(d)}*\cos(-3*(b*c - a*d)/d) + (5*\sqrt{\pi})*\cos(1/4*\pi + 1/ \\ & 2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 5*\sqrt{\pi})*\cos(-1/4*\pi + 1 \\ & /2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 5*I*\sqrt{\pi})*\sin(1/4*\pi + \\ & 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 5*I*\sqrt{\pi})*\sin(-1/4*\pi \\ & + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^3*\sqrt{\text{abs}(b)/\text{abs}(d)} \\ &))*\sin(-3*(b*c - a*d)/d))*\text{erf}(\sqrt{(d*x + c)*\sqrt{-3*I*b/d}}) + 8*(12*\sqrt{3} \\ & *(d*x + c)^{(5/2)}*b^2*d*\text{abs}(b)/\text{abs}(d) - 5*\sqrt{3})*\sqrt{(d*x + c)}*d^3*\text{abs}(b)/a \\ & \text{bs}(d))*\sin(3*((d*x + c)*b - b*c + a*d)/d) - 72*(4*\sqrt{3})*(d*x + c)^{(5/2)}*b \\ & ^2*d*\text{abs}(b)/\text{abs}(d) - 15*\sqrt{3})*\sqrt{(d*x + c)}*d^3*\text{abs}(b)/\text{abs}(d))*\sin(((d*x \\ & + c)*b - b*c + a*d)/d))*\text{abs}(d)/(b^3*d*\text{abs}(b)) \end{aligned}$$

Fricas [A] time = 0.638412, size = 918, normalized size = 2.26

$$\frac{5\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/864*(5*\sqrt{6}*\pi*d^3*\sqrt{b/(pi*d)}*\cos(-3*(b*c - a*d)/d)*\text{fresnel_sin}(\sqrt{6}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) \\ & - 405*\sqrt{2}*\pi*d^3*\sqrt{b/(pi*d)}*\cos(-3*(b*c - a*d)/d)*\text{fresnel_sin}(\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) \\ & - 405*\sqrt{2}*\pi*d^3*\sqrt{b/(pi*d)}*\text{fresnel_cos}(\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) \\ & * \sin(-3*(b*c - a*d)/d) + 5*\sqrt{6}*\pi*d^3*\sqrt{b/(pi*d)}*\text{fresnel_cos}(\sqrt{6}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) \\ & * \sin(-3*(b*c - a*d)/d) + 24*(10*(b^2*d^2*x + b^2*c*d)*\cos(b*x + a)^3 - 30*(b^2*d^2*x + b^2*c*d)*\cos(b*x + a) \\ & - (12*b^3*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 35*b*d^2 - (12*b^3*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*\cos(b*x + a)^2)*\sin(b*x + a))*\sqrt{d*x + c})/b^4 \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**2,x)

[Out] Timed out

Giac [C] time = 1.5621, size = 2722, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/1728*(12*(I*\sqrt{6}*\sqrt{\pi})*d^2*\text{erf}(-1/2*\sqrt{6}*\sqrt{b*d})*\sqrt{d*x + c} \\ & *(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((3*I*b*c - 3*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b)} \\ & - 9*I*\sqrt{2}*\sqrt{\pi}*d^2*\text{erf}(-1/2*\sqrt{2}*\sqrt{b*d})*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d \\ & *e^{((I*b*c - I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b)} + 9*I*\sqrt{2}*\sqrt{\pi}*d^2*\text{erf}(-1/2*\sqrt{2}*\sqrt{b*d})*\sqrt{d*x + c} \\ & *(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-I*b*c + I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b)} \\ & - I*\sqrt{6}*\sqrt{\pi}*d^2*\text{erf}(-1/2*\sqrt{6}*\sqrt{b*d})*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d \\ & *e^{((-3*I*b*c + 3*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b)} - 6*I*\sqrt{d*x + c} \\ & *d*e^{((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b} + 18*I*\sqrt{d*x + c}*d*e^{((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b} \\ & - 18*I*\sqrt{d*x + c}*d*e^{((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b} + 6*I*\sqrt{d*x + c}*d*e^{((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b} \\ & *c^2 - d^2*((\sqrt{6}*\sqrt{\pi})*(-12*I*b^2*c^2*d + 12*b*c*d^2 + 5*I*d^3)*d*\text{erf}(-1/2*\sqrt{6}*\sqrt{b*d})*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d) \\ & *e^{((3*I*b*c - 3*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1))} \end{aligned}$$

$$\begin{aligned}
&) * b^3) - 6 * (12 * I * (d * x + c)^{(5/2)} * b^2 * d - 24 * I * (d * x + c)^{(3/2)} * b^2 * c * d + 12 * \\
& I * \sqrt{d * x + c} * b^2 * c^2 * d + 10 * (d * x + c)^{(3/2)} * b * d^2 - 12 * \sqrt{d * x + c} * b * c \\
& * d^2 - 5 * I * \sqrt{d * x + c} * d^3) * e^{((-3 * I * (d * x + c) * b + 3 * I * b * c - 3 * I * a * d) / d) /} \\
& b^3) / d^2 + 27 * (\sqrt{2} * \sqrt{\pi}) * (4 * I * b^2 * c^2 * d - 12 * b * c * d^2 - 15 * I * d^3) * d * e \\
& \operatorname{rf}(-1/2 * \sqrt{2} * \sqrt{b * d} * \sqrt{d * x + c} * (I * b * d / \sqrt{b^2 * d^2} + 1) / d) * e^{((I * \\
& b * c - I * a * d) / d) / (\sqrt{b * d} * (I * b * d / \sqrt{b^2 * d^2} + 1) * b^3) - 2 * (-4 * I * (d * x + \\
& c)^{(5/2)} * b^2 * d + 8 * I * (d * x + c)^{(3/2)} * b^2 * c * d - 4 * I * \sqrt{d * x + c} * b^2 * c^2 * d \\
& - 10 * (d * x + c)^{(3/2)} * b * d^2 + 12 * \sqrt{d * x + c} * b * c * d^2 + 15 * I * \sqrt{d * x + c} * \\
& d^3) * e^{((-I * (d * x + c) * b + I * b * c - I * a * d) / d) / b^3) / d^2 + 27 * (\sqrt{2} * \sqrt{\pi}) \\
& * (-4 * I * b^2 * c^2 * d - 12 * b * c * d^2 + 15 * I * d^3) * d * \operatorname{erf}(-1/2 * \sqrt{2} * \sqrt{b * d} * \sqrt{d * x + c} * \\
& (-I * b * d / \sqrt{b^2 * d^2} + 1) / d) * e^{((-I * b * c + I * a * d) / d) / (\sqrt{b * d} * (- \\
& I * b * d / \sqrt{b^2 * d^2} + 1) * b^3) - 2 * (4 * I * (d * x + c)^{(5/2)} * b^2 * d - 8 * I * (d * x + \\
& c)^{(3/2)} * b^2 * c * d + 4 * I * \sqrt{d * x + c} * b^2 * c^2 * d - 10 * (d * x + c)^{(3/2)} * b * d^2 + \\
& 12 * \sqrt{d * x + c} * b * c * d^2 - 15 * I * \sqrt{d * x + c} * d^3) * e^{((I * (d * x + c) * b - I * b \\
& * c + I * a * d) / d) / b^3) / d^2 + (\sqrt{6} * \sqrt{\pi}) * (12 * I * b^2 * c^2 * d + 12 * b * c * d^2 - \\
& 5 * I * d^3) * d * \operatorname{erf}(-1/2 * \sqrt{6} * \sqrt{b * d} * \sqrt{d * x + c} * (-I * b * d / \sqrt{b^2 * d^2} + \\
& 1) / d) * e^{((-3 * I * b * c + 3 * I * a * d) / d) / (\sqrt{b * d} * (-I * b * d / \sqrt{b^2 * d^2} + 1) * b^3 \\
&) - 6 * (-12 * I * (d * x + c)^{(5/2)} * b^2 * d + 24 * I * (d * x + c)^{(3/2)} * b^2 * c * d - 12 * I * \sqrt{d * x + c} * b^2 * c^2 * d + 10 * (d * x + c)^{(3/2)} * b * d^2 - 12 * \sqrt{d * x + c} * b * c * d^2 + 5 * I * \sqrt{d * x + c} * d^3) * e^{((3 * I * (d * x + c) * b - 3 * I * b * c + 3 * I * a * d) / d) / b^3) / d^2} - 12 * (\sqrt{6} * \sqrt{\pi}) * (2 * I * b * c * d - d^2) * d * \operatorname{erf}(-1/2 * \sqrt{6} * \sqrt{b * d} * \sqrt{d * x + c} * (I * b * d / \sqrt{b^2 * d^2} + 1) / d) * e^{((3 * I * b * c - 3 * I * a * d) / d) / (\sqrt{b * d} * (I * b * d / \sqrt{b^2 * d^2} + 1) * b^2) + 9 * \sqrt{2} * \sqrt{\pi}) * (-2 * I * b * c * d + 3 * d^2) * d * \operatorname{erf}(-1/2 * \sqrt{2} * \sqrt{b * d} * \sqrt{d * x + c} * (I * b * d / \sqrt{b^2 * d^2} + 1) / d) * e^{((I * b * c - I * a * d) / d) / (\sqrt{b * d} * (I * b * d / \sqrt{b^2 * d^2} + 1) * b^2) + 9 * \sqrt{2} * \sqrt{\pi}) * (2 * I * b * c * d + 3 * d^2) * d * \operatorname{erf}(-1/2 * \sqrt{2} * \sqrt{b * d} * \sqrt{d * x + c} * (-I * b * d / \sqrt{b^2 * d^2} + 1) / d) * e^{((-I * b * c + I * a * d) / d) / (\sqrt{b * d} * (-I * b * d / \sqrt{b^2 * d^2} + 1) * b^2) + \sqrt{6} * \sqrt{\pi}) * (-2 * I * b * c * d - d^2) * d * \operatorname{erf}(-1/2 * \sqrt{6} * \sqrt{b * d} * \sqrt{d * x + c} * (-I * b * d / \sqrt{b^2 * d^2} + 1) / d) * e^{((-3 * I * b * c + 3 * I * a * d) / d) / (\sqrt{b * d} * (-I * b * d / \sqrt{b^2 * d^2} + 1) * b^2) - 6 * (-2 * I * (d * x + c)^{(3/2)} * b * d + 2 * I * \sqrt{d * x + c} * b * c * d + \sqrt{d * x + c} * d^2) * e^{((3 * I * (d * x + c) * b - 3 * I * b * c + 3 * I * a * d) / d) / b^2} - 18 * (2 * I * (d * x + c)^{(3/2)} * b * d - 2 * I * \sqrt{d * x + c} * b * c * d - 3 * \sqrt{d * x + c} * d^2) * e^{((I * (d * x + c) * b - I * b * c + I * a * d) / d) / b^2} - 18 * (-2 * I * (d * x + c)^{(3/2)} * b * d + 2 * I * \sqrt{d * x + c} * b * c * d - 3 * \sqrt{d * x + c} * d^2) * e^{((-I * (d * x + c) * b + I * b * c - I * a * d) / d) / b^2} - 6 * (2 * I * (d * x + c)^{(3/2)} * b * d - 2 * I * \sqrt{d * x + c} * b * c * d + \sqrt{d * x + c} * d^2) * e^{((-3 * I * (d * x + c) * b + 3 * I * b * c - 3 * I * a * d) / d) / b^2) * c) / d
\end{aligned}$$

3.64 $\int (c + dx)^{5/2} \cos(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=407

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} - \frac{15\sqrt{\pi}d^{5/2}\cos\left(2a - \frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}} - \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a - \frac{4bc}{d}\right)}{4096b^{7/2}}$$

[Out] $(15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[2*a + 2*b*x])/(8*b) - (15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[4*a + 4*b*x])/(2048*b^3) + ((c + d*x)^{(5/2)}*\text{Cos}[4*a + 4*b*x])/(32*b) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[4*a - (4*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(4096*b^{(7/2)}) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]])/(256*b^{(7/2)}) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d])* \text{Sin}[4*a - (4*b*c)/d])/(4096*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}])* \text{Sin}[2*a - (2*b*c)/d])/(256*b^{(7/2)}) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[2*a + 2*b*x])/(32*b^2) - (5*d*(c + d*x)^{(3/2)}*\text{Sin}[4*a + 4*b*x])/(256*b^2)$

Rubi [A] time = 1.05142, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} - \frac{15\sqrt{\pi}d^{5/2}\cos\left(2a - \frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}} - \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a - \frac{4bc}{d}\right)}{4096b^{7/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^{(5/2)}*\text{Cos}[a + b*x]*\text{Sin}[a + b*x]^3, x]$

[Out] $(15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[2*a + 2*b*x])/(8*b) - (15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[4*a + 4*b*x])/(2048*b^3) + ((c + d*x)^{(5/2)}*\text{Cos}[4*a + 4*b*x])/(32*b) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[4*a - (4*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(4096*b^{(7/2)}) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]])/(256*b^{(7/2)}) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d])* \text{Sin}[4*a - (4*b*c)/d])/(4096*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}])* \text{Sin}[2*a - (2*b*c)/d])/(256*b^{(7/2)}) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[2*a + 2*b*x])/(32*b^2) - (5*d*(c + d*x)^{(3/2)}*\text{Sin}[4*a + 4*b*x])/(256*b^2)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{GtQ}[m, 0]$

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
1S[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
1C[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^{5/2} \cos(a + bx) \sin^3(a + bx) dx &= \int \left(\frac{1}{4}(c + dx)^{5/2} \sin(2a + 2bx) - \frac{1}{8}(c + dx)^{5/2} \sin(4a + 4bx) \right) dx \\
&= -\left(\frac{1}{8} \int (c + dx)^{5/2} \sin(4a + 4bx) dx \right) + \frac{1}{4} \int (c + dx)^{5/2} \sin(2a + 2bx) dx \\
&= -\frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} + \frac{(c + dx)^{5/2} \cos(4a + 4bx)}{32b} - \frac{(5d) \int (c + dx)^{3/2} \sin(2a + 2bx) dx}{6} \\
&= -\frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} + \frac{(c + dx)^{5/2} \cos(4a + 4bx)}{32b} + \frac{5d(c + dx)^{3/2} \sin(2a + 2bx)}{32b^2} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{128b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} - \frac{15d^2 \sqrt{c + dx} \cos(4a + 4bx)}{2048b^3} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{128b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} - \frac{15d^2 \sqrt{c + dx} \cos(4a + 4bx)}{2048b^3} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{128b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} - \frac{15d^2 \sqrt{c + dx} \cos(4a + 4bx)}{2048b^3} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{128b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} - \frac{15d^2 \sqrt{c + dx} \cos(4a + 4bx)}{2048b^3}
\end{aligned}$$

Mathematica [A] time = 15.219, size = 550, normalized size = 1.35

$$-1024b^3 c^2 \sqrt{c + dx} \cos(2(a + bx)) + 256b^3 c^2 \sqrt{c + dx} \cos(4(a + bx)) - 1024b^3 d^2 x^2 \sqrt{c + dx} \cos(2(a + bx)) + 256b^3 d^2 x^2 \sqrt{c + dx} \cos(4(a + bx))$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]

[Out] (-1024*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 960*b*d^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 2048*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 1024*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 256*b^3*c^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 60*b*d^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 512*b^3*c*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 256*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 480*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 480*Sqrt[b/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] + 1280*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 1280*b^2*d^2*x*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 160*b^2*c*d*Sqrt[c + d*x]*Sin[4*(a + b*x)] - 160*b^2*d^2*x*Sqrt[c + d*x]*Sin[4*(a + b*x)]/(8192*b^4)

Maple [A] time = 0.036, size = 470, normalized size = 1.2

$$2 \frac{1}{d} \left(-1/16 \frac{d(dx+c)^{5/2}}{b} \cos \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) + \frac{5d}{16b} \left(\frac{1}{4} \frac{d(dx+c)^{3/2}}{b} \sin \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) - 3/4 \frac{d}{b} \left(-1 \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x)

[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/16/b*d*(1/4/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))) + 1/64/b*d*(d*x+c)^(5/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-5/64/b*d*(1/8/b*d*(d*x+c)^(3/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))

Maxima [C] time = 2.36066, size = 1874, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")

[Out] -1/65536*sqrt(2)*(640*sqrt(2)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(4*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 5120*sqrt(2)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 16*(64*sqrt(2)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 15*sqrt(2)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(4*((d*x + c)*b - b*c + a*d)/d) + 256*(16*sqrt(2)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 15*sqrt(2)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(2*((d*x + c)*b - b*c + a*d)/d) + ((480*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq


```

rt(d^2))) + 480*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d
/sqrt(d^2))) - 480*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + 480*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) *d^3*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + (-48
0*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
- 480*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) - 480*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) + 480*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) *d^3*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d)*erf(sqrt(d*x + c)
*sqrt(2*I*b/d)) - (sqrt(2)*(15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^3*sqrt(abs(b)/abs(d))*cos(-4*(b*c -
a*d)/d) - sqrt(2)*(15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) *d^3*sqrt(abs(b)/abs(d))*sin(-4*(b*c - a*d)/d))*
erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - (sqrt(2)*(15*sqrt(pi)*cos(1/4*pi + 1/2*a
rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(-1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^3*sqrt(abs(b)/abs(
d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(-15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^3*sqrt(abs(b)/abs(d))*sin(-
4*(b*c - a*d)/d)*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + ((480*sqrt(pi)*cos(1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*sqrt(pi)*cos(
-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*I*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 480*I*sqrt
(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^3*sq
rt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + (480*I*sqrt(pi)*cos(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*I*sqrt(pi)*cos(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 480*sqrt(pi)*sin(1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*sqrt(pi)*sin(-1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^3*sqrt(abs(b)/abs
(d))*sin(-2*(b*c - a*d)/d)*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*abs(d)/(b^3*
d*abs(b))

```

Fricas [A] time = 0.693492, size = 1011, normalized size = 2.48

$$15 \sqrt{2} \pi d^3 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{4(bc-ad)}{d}\right) C\left(2 \sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) - 15 \sqrt{2} \pi d^3 \sqrt{\frac{b}{\pi d}} S\left(2 \sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{4(bc-ad)}{d}\right) - 480 \pi d^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")

[Out] 1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d))) + 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 4*(320*b^3*d^2*x^2 + 640*b^3*c*d*x +

$$320*b^3*c^2 + 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*x + 64*b^3*c^2 - 15*b*d^2)*\cos(b*x + a)^4 - 255*b*d^2 - 8*(128*b^3*d^2*x^2 + 256*b^3*c*d*x + 128*b^3*c^2 - 75*b*d^2)*\cos(b*x + a)^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*\cos(b*x + a)^3 - 5*(b^2*d^2*x + b^2*c*d)*\cos(b*x + a))*\sin(b*x + a))*\sqrt{d*x + c})/b^4$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**3,x)

[Out] Timed out

Giac [C] time = 1.86076, size = 2681, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")

[Out]
$$\frac{1}{16384} * (64 * (\sqrt{2} * \sqrt{\pi} * d^2 * \operatorname{erf}(-\sqrt{2} * \sqrt{b*d}) * \sqrt{d*x + c}) * (I*b*d/\sqrt{b^2*d^2} + 1)/d) * e^{((4*I*b*c - 4*I*a*d)/d)} / (\sqrt{b*d}) * (I*b*d/\sqrt{b^2*d^2} + 1)*b + \sqrt{2} * \sqrt{\pi} * d^2 * \operatorname{erf}(-\sqrt{2} * \sqrt{b*d}) * \sqrt{d*x + c}) * (-I*b*d/\sqrt{b^2*d^2} + 1)/d) * e^{((-4*I*b*c + 4*I*a*d)/d)} / (\sqrt{b*d}) * (-I*b*d/\sqrt{b^2*d^2} + 1)*b - 8 * \sqrt{\pi} * d^2 * \operatorname{erf}(-\sqrt{b*d}) * \sqrt{d*x + c}) * (I*b*d/\sqrt{b^2*d^2} + 1)/d) * e^{((2*I*b*c - 2*I*a*d)/d)} / (\sqrt{b*d}) * (I*b*d/\sqrt{b^2*d^2} + 1)*b - 8 * \sqrt{\pi} * d^2 * \operatorname{erf}(-\sqrt{b*d}) * \sqrt{d*x + c}) * (-I*b*d/\sqrt{b^2*d^2} + 1)/d) * e^{((-2*I*b*c + 2*I*a*d)/d)} / (\sqrt{b*d}) * (-I*b*d/\sqrt{b^2*d^2} + 1)*b + 4 * \sqrt{d*x + c}) * d * e^{((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)} / b - 16 * \sqrt{d*x + c}) * d * e^{((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)} / b - 16 * \sqrt{d*x + c}) * d * e^{((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)} / b + 4 * \sqrt{d*x + c}) * d * e^{((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)} / b) * c^2 - d^2 * ((I * \sqrt{2}) * \sqrt{\pi}) * (64 * I * b^2 * c^2 * d - 48 * b * c * d^2 - 15 * I * d^3) * d * \operatorname{erf}(-\sqrt{2} * \sqrt{b*d}) * \sqrt{d*x + c}) * (I*b*d/\sqrt{b^2*d^2} + 1)/d) * e^{((4*I*b*c - 4*I*a*d)/d)} / (\sqrt{b*d}) * (I*b*d/\sqrt{b^2*d^2} + 1)*b^3 - 4 * I * (-64 * I * (d*x + c)^{(5/2)} * b^2 * d + 128 * I * (d*x + c)^{(3/2)} * b^2 * c * d - 64 * I * \sqrt{d*x + c}) * b^2 * c^2 * d - 40 * (d*x + c)^{(3/2)} * b * d^2 + 48 * \sqrt{d*x + c}) * b * c * d^2 + 15 * I * \sqrt{d*x + c}) * d^3) * e^{((-4 * I * (d*x + c) * b + 4 * I * b * c - 4 * I * a * d) / d) / b^3} / d^2 + (I * \sqrt{2}) * \sqrt{\pi}) * (64 * I * b^2 * c^2 * d + 48 * b * c * d^2 - 15 * I * d^3) * d * \operatorname{erf}(-\sqrt{2} * \sqrt{b*d}) * \sqrt{d*x + c}) * (-I*b*d/\sqrt{b^2*d^2} + 1)/d) * e^{((-4 * I * b * c + 4 * I * a * d) / d) / (\sqrt{b*d}) * (-I*b*d/\sqrt{b^2*d^2} + 1)*b^3 - 4 * I * (-64 * I * (d*x + c)^{(5/2)} * b^2 * d + 128 * I * (d*x + c)^{(3/2)} * b^2 * c * d - 64 * I * \sqrt{d*x + c}) * b^2 * c^2 * d + 40 * (d*x + c)^{(3/2)} * b * d^2 - 48 * \sqrt{d*x + c}) * b * c * d^2 + 15 * I * \sqrt{d*x + c}) * d^3) * e^{((4 * I * (d*x + c) * b - 4 * I * b * c + 4 * I * a * d) / d) / b^3} / d^2 + 32 * (I * \sqrt{\pi}) * (-16 * I * b^2 * c^2 * d + 24 * b * c * d^2 + 15 * I * d^3) * d * \operatorname{erf}(-\sqrt{b*d}) * \sqrt{d*x + c}) * (I*b*d/\sqrt{b^2*d^2} + 1)/d) * e^{((2 * I * b * c - 2 * I * a * d) / d) / (\sqrt{b*d}) * (I*b*d/\sqrt{b^2*d^2} + 1)*b^3 - 2 * I * (16 * I * (d*x + c)^{(5/2)} * b^2 * d - 32 * I * (d*x + c)^{(3/2)} * b^2 * c * d + 16 * I * \sqrt{d*x + c}) * b^2 * c^2 * d + 20 * (d*x + c)^{(3/2)} * b * d^2 - 24 * \sqrt{d*x + c}) * b * c * d^2 - 15 * I * \sqrt{d*x + c}) * d^3) * e^{((-2 * I * (d*x + c) * b + 2 * I * b * c - 2 * I * a * d) / d) / b^3} / d^2 + 32 * (I * \sqrt{\pi}) * (-16 * I * b^2 * c^2 * d - 24 * b * c * d^2 + 15 * I * d^3) * d * \operatorname{erf}(-\sqrt{b*d}) * \sqrt{d*x + c}) * (-I*b*d/\sqrt{b^2*d^2} + 1)/d) * e^{((-2 * I * b * c + 2 * I * a * d) / d) / (\sqrt{b*d})$$

$$\begin{aligned}
& *(-I*b*d/\sqrt{b^2*d^2} + 1)*b^3) - 2*I*(16*I*(d*x + c)^{(5/2)}*b^2*d - 32*I*(\\
& d*x + c)^{(3/2)}*b^2*c*d + 16*I*\sqrt{d*x + c}*b^2*c^2*d - 20*(d*x + c)^{(3/2)}* \\
& b*d^2 + 24*\sqrt{d*x + c}*b*c*d^2 - 15*I*\sqrt{d*x + c}*d^3)*e^{((2*I*(d*x + c) \\
&)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^2) - 16*(I*\sqrt{2}*\sqrt{\pi})*(-8*I*b*c*d \\
& + 3*d^2)*d*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d \\
&)*e^{((4*I*b*c - 4*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2) + I*s \\
& \sqrt{2}*\sqrt{\pi})*(-8*I*b*c*d - 3*d^2)*d*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c} \\
& *(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-4*I*b*c + 4*I*a*d)/d)/(\sqrt{b*d}*(-I*b* \\
& d/\sqrt{b^2*d^2} + 1)*b^2) + 16*I*\sqrt{\pi}*(4*I*b*c*d - 3*d^2)*d*\operatorname{erf}(-\sqrt{b \\
& *d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((2*I*b*c - 2*I*a*d)/d)/(s \\
& \sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2) + 16*I*\sqrt{\pi}*(4*I*b*c*d + 3*d^2) \\
& *d*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-2*I*b*c \\
& + 2*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2) - 4*I*(-8*I*(d*x + \\
& c)^{(3/2)}*b*d + 8*I*\sqrt{d*x + c}*b*c*d + 3*\sqrt{d*x + c}*d^2)*e^{((4*I*(d*x \\
& + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2 - 32*I*(4*I*(d*x + c)^{(3/2)}*b*d - 4*I*s \\
& \sqrt{d*x + c}*b*c*d - 3*\sqrt{d*x + c}*d^2)*e^{((2*I*(d*x + c)*b - 2*I*b*c + 2 \\
& *I*a*d)/d)/b^2 - 32*I*(4*I*(d*x + c)^{(3/2)}*b*d - 4*I*\sqrt{d*x + c}*b*c*d + \\
& 3*\sqrt{d*x + c}*d^2)*e^{((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2 - 4*I \\
& *(-8*I*(d*x + c)^{(3/2)}*b*d + 8*I*\sqrt{d*x + c}*b*c*d - 3*\sqrt{d*x + c}*d^2) \\
& *e^{((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)*c)/d
\end{aligned}$$

3.65 $\int (c + dx)^{3/2} \cos(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=351

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3\sqrt{\pi}d^{3/2}\sin\left(2a - \frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}} + \frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a - \frac{4bc}{d}\right)}{512b^5}$$

[Out] $-\left((c + d*x)^{(3/2)}*\text{Cos}[2*a + 2*b*x]\right)/(8*b) + \left((c + d*x)^{(3/2)}*\text{Cos}[4*a + 4*b*x]\right)/(32*b) + (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[4*a - (4*b*c)/d]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(512*b^{(5/2)}) - (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]))/(64*b^{(5/2)}) + (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[4*a - (4*b*c)/d])/(512*b^{(5/2)}) - (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]))*\text{Sin}[2*a - (2*b*c)/d])/(64*b^{(5/2)}) + (3*d*\text{Sqrt}[c + d*x]*\text{Sin}[2*a + 2*b*x])/(32*b^2) - (3*d*\text{Sqrt}[c + d*x]*\text{Sin}[4*a + 4*b*x])/(256*b^2)$

Rubi [A] time = 0.673582, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3\sqrt{\pi}d^{3/2}\sin\left(2a - \frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}} + \frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a - \frac{4bc}{d}\right)}{512b^5}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^{(3/2)}*\text{Cos}[a + b*x]*\text{Sin}[a + b*x]^3, x]$

[Out] $-\left((c + d*x)^{(3/2)}*\text{Cos}[2*a + 2*b*x]\right)/(8*b) + \left((c + d*x)^{(3/2)}*\text{Cos}[4*a + 4*b*x]\right)/(32*b) + (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[4*a - (4*b*c)/d]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(512*b^{(5/2)}) - (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]))/(64*b^{(5/2)}) + (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[4*a - (4*b*c)/d])/(512*b^{(5/2)}) - (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]))*\text{Sin}[2*a - (2*b*c)/d])/(64*b^{(5/2)}) + (3*d*\text{Sqrt}[c + d*x]*\text{Sin}[2*a + 2*b*x])/(32*b^2) - (3*d*\text{Sqrt}[c + d*x]*\text{Sin}[4*a + 4*b*x])/(256*b^2)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3306

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[c*f/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d$

*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]

Rule 3305

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3304

Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}
 \int (c + dx)^{3/2} \cos(a + bx) \sin^3(a + bx) dx &= \int \left(\frac{1}{4}(c + dx)^{3/2} \sin(2a + 2bx) - \frac{1}{8}(c + dx)^{3/2} \sin(4a + 4bx) \right) dx \\
 &= -\left(\frac{1}{8} \int (c + dx)^{3/2} \sin(4a + 4bx) dx \right) + \frac{1}{4} \int (c + dx)^{3/2} \sin(2a + 2bx) dx \\
 &= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{8b} + \frac{(c + dx)^{3/2} \cos(4a + 4bx)}{32b} - \frac{(3d) \int \sqrt{c + dx} \cos(4a + 4bx) dx}{64b^2} \\
 &= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{8b} + \frac{(c + dx)^{3/2} \cos(4a + 4bx)}{32b} + \frac{3d\sqrt{c + dx} \sin(2a + 2bx)}{32b^2} \\
 &= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{8b} + \frac{(c + dx)^{3/2} \cos(4a + 4bx)}{32b} + \frac{3d\sqrt{c + dx} \sin(2a + 2bx)}{32b^2} \\
 &= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{8b} + \frac{(c + dx)^{3/2} \cos(4a + 4bx)}{32b} + \frac{3d\sqrt{c + dx} \sin(2a + 2bx)}{32b^2} \\
 &= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{8b} + \frac{(c + dx)^{3/2} \cos(4a + 4bx)}{32b} + \frac{3d^{3/2} \sqrt{\frac{\pi}{2}} \cos\left(4a + \frac{2bx}{\sqrt{\pi}}\right)}{32b}
 \end{aligned}$$

Mathematica [A] time = 3.34361, size = 393, normalized size = 1.12

$$3\sqrt{2\pi}d \sin\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c + dx}\right) - 48\sqrt{\pi}d \sin\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c + dx}}{\sqrt{\pi}}\right) + 3\sqrt{2\pi}d \cos\left(4a - \frac{4bc}{d}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3, x]

```
[Out] (-128*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 128*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 32*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 32*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 3*d*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 48*d*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] - 48*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] + 96*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 12*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[4*(a + b*x)]/(1024*b^2*Sqrt[b/d])
```

Maple [A] time = 0.039, size = 376, normalized size = 1.1

$$2 \frac{1}{d} \left(-1/16 \frac{d(dx+c)^{3/2}}{b} \cos \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) + 3/16 \frac{d}{b} \left(1/4 \frac{d\sqrt{dx+c}}{b} \sin \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) - 1/8 \frac{d\sqrt{\pi}}{b} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/16/b*d*(1/4/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))+1/64/b*d*(d*x+c)^(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/64/b*d*(1/8/b*d*(d*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))
```

Maxima [C] time = 2.31838, size = 1804, normalized size = 5.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8192*sqrt(2)*(128*sqrt(2)*(d*x + c)^(3/2)*b*d*abs(b)*cos(4*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 512*sqrt(2)*(d*x + c)^(3/2)*b*d*abs(b)*cos(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 48*sqrt(2)*sqrt(d*x + c)*d^2*abs(b)*sin(4*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 384*sqrt(2)*sqrt(d*x + c)*d^2*abs(b)*sin(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - ((48*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + (48*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - (sqrt(2)*(-3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos(-
```

$$\begin{aligned}
& 1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}) - 3*\sqrt{\pi}*\sin(\\
& 1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*\sqrt{\pi}*\sin(\\
& -1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) * d^2 * \sqrt{\text{abs}(b) / \text{abs}(d)} * \cos(-4*(b*c - a*d)/d) - \sqrt{2} * (3*\sqrt{\pi} * \cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*\sqrt{\pi} * \cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 3*I*\sqrt{\pi} * \sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*I*\sqrt{\pi} * \sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) * d^2 * \sqrt{\text{abs}(b) / \text{abs}(d)} * \sin(-4*(b*c - a*d)/d) * \text{erf}(2*\sqrt{d*x + c} * \sqrt{I*b/d}) - (\sqrt{2} * (3*I*\sqrt{\pi} * \cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*I*\sqrt{\pi} * \cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 3*\sqrt{\pi} * \sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*\sqrt{\pi} * \sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) * d^2 * \sqrt{\text{abs}(b) / \text{abs}(d)} * \cos(-4*(b*c - a*d)/d) - \sqrt{2} * (3*\sqrt{\pi} * \cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*\sqrt{\pi} * \cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*I*\sqrt{\pi} * \sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 3*I*\sqrt{\pi} * \sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) * d^2 * \sqrt{\text{abs}(b) / \text{abs}(d)} * \sin(-4*(b*c - a*d)/d) * \text{erf}(2*\sqrt{d*x + c} * \sqrt{-I*b/d}) - ((-48*I*\sqrt{\pi} * \cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 48*I*\sqrt{\pi} * \cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 48*\sqrt{\pi} * \sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 48*\sqrt{\pi} * \sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) * d^2 * \sqrt{\text{abs}(b) / \text{abs}(d)} * \cos(-2*(b*c - a*d)/d) + (48*\sqrt{\pi} * \cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 48*\sqrt{\pi} * \cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 48*I*\sqrt{\pi} * \sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 48*I*\sqrt{\pi} * \sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) * d^2 * \sqrt{\text{abs}(b) / \text{abs}(d)} * \sin(-2*(b*c - a*d)/d) * \text{erf}(\sqrt{d*x + c} * \sqrt{-2*I*b/d})) * \text{abs}(d) / (b^2 * d * \text{abs}(b))
\end{aligned}$$

Fricas [A] time = 0.646811, size = 802, normalized size = 2.28

$$3\sqrt{2}\pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{4(bc-ad)}{d}\right) S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) + 3\sqrt{2}\pi d^2 \sqrt{\frac{b}{\pi d}} C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{4(bc-ad)}{d}\right) - 48\pi d^2 \sqrt{\frac{b}{\pi d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")

[Out] 1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 48*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x + a)^4 + 10*b^2*d*x + 10*b^2*c - 32*(b^2*d*x + b^2*c)*cos(b*x + a)^2 - 3*(2*b*d*cos(b*x + a)^3 - 5*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^3

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.55835, size = 1485, normalized size = 4.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2048*(8*(sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)*b) - 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)*b) - 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)*b) + 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b -
16*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b - 16*sqrt(
d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)
*d*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c - I*sqrt(2)*sqrt(pi)*(
-8*I*b*c*d + 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b^2) - I*sqrt(2)*sqrt(pi)*(-8*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sq
rt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(
b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 16*I*sqrt(pi)*(4*I*b*c*d - 3*d^2)*d*
erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 16*I*sqrt(pi)*(4*I*b*c
*d + 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 4*I*(
-8*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e
^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2 + 32*I*(4*I*(d*x + c)^(3/2)*
b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b -
2*I*b*c + 2*I*a*d)/d)/b^2 + 32*I*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x +
c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d
)/b^2 + 4*I*(-8*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*
x + c)*d^2)*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d
```


3.66 $\int \sqrt{c + dx} \cos(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=299

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} + \frac{\sqrt{\pi} \sqrt{d} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b} \sqrt{c+dx}}{\sqrt{\pi} \sqrt{d}}\right)}{16b^{3/2}} + \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \sin\left(4a - \frac{4bc}{d}\right)}{64b^{3/2}}$$

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(8*b) + (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(
(32*b) - (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) + (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rubi [A] time = 0.499154, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} + \frac{\sqrt{\pi} \sqrt{d} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b} \sqrt{c+dx}}{\sqrt{\pi} \sqrt{d}}\right)}{16b^{3/2}} + \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \sin\left(4a - \frac{4bc}{d}\right)}{64b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(8*b) + (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(
(32*b) - (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) + (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d,
Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f},
x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^-2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d,
Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^-2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned} \int \sqrt{c+dx} \cos(a+bx) \sin^3(a+bx) dx &= \int \left(\frac{1}{4} \sqrt{c+dx} \sin(2a+2bx) - \frac{1}{8} \sqrt{c+dx} \sin(4a+4bx) \right) dx \\ &= -\left(\frac{1}{8} \int \sqrt{c+dx} \sin(4a+4bx) dx \right) + \frac{1}{4} \int \sqrt{c+dx} \sin(2a+2bx) dx \\ &= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx} \cos(4a+4bx)}{32b} - \frac{d \int \frac{\cos(4a+4bx)}{\sqrt{c+dx}} dx}{64b} + \frac{d \int \frac{\cos(2a+2bx)}{\sqrt{c+dx}} dx}{32b} \\ &= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx} \cos(4a+4bx)}{32b} - \frac{\left(d \cos\left(4a - \frac{4bc}{d}\right) \right) \int \frac{\cos\left(\frac{4b}{d}\sqrt{c+dx}\right)}{\sqrt{c+dx}} dx}{64b} \\ &= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx} \cos(4a+4bx)}{32b} - \frac{\cos\left(4a - \frac{4bc}{d}\right) \text{Subst}\left(\int \frac{\cos\left(\frac{4b}{d}\sqrt{c+dx}\right)}{\sqrt{c+dx}} dx\right)}{32b} \\ &= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx} \cos(4a+4bx)}{32b} - \frac{\sqrt{d} \sqrt{\frac{\pi}{2}} \cos\left(4a - \frac{4bc}{d}\right) C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)}{64b^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.873951, size = 264, normalized size = 0.88

$$\frac{-\sqrt{2\pi} \cos\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right) + 8\sqrt{\pi} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + \sqrt{2\pi} \sin\left(4a - \frac{4bc}{d}\right) S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)}{128b\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-16*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 4*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*x)] - Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] + 8*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]])/Sqrt[Pi]
```



```

^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) *d*sqrt(abs(b)/abs(d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(-I*sqrt(pi)*
cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*
cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*s
in(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d*sqrt(abs(b)/
abs(d))*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + ((8*sqrt
(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*sqrt
(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*I*s
qrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 8*I
*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d
*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + (8*I*sqrt(pi)*cos(1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*I*sqrt(pi)*cos(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 8*sqrt(pi)*sin(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*sqrt(pi)*sin(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d*sqrt(abs(b)/abs(d))*si
n(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*abs(d)/(b*d*abs(b))

```

Fricas [A] time = 0.607041, size = 630, normalized size = 2.11

$$\frac{\sqrt{2}\pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{4(bc-ad)}{d}\right) C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - \sqrt{2}\pi d \sqrt{\frac{b}{\pi d}} S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{4(bc-ad)}{d}\right) - 8\pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{4(bc-ad)}{d}\right) - 8\pi d \sqrt{\frac{b}{\pi d}} \sin\left(-\frac{4(bc-ad)}{d}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")

[Out] -1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 8*pi*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d))) + 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 16*b*cos(b*x + a)^2 + 5*b)*sqrt(d*x + c)/b^2

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**3,x)

[Out] Timed out

Giac [C] time = 1.337, size = 643, normalized size = 2.15

$$\frac{\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right) e^{\left(\frac{4ibc-4iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} + \frac{\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right) e^{\left(\frac{-4ibc+4iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} - \frac{8\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right) e^{\left(\frac{2ibc-2iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/256*(sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b - 16*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b - 16*sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)/d
```

3.67 $\int \sqrt{c + dx} \cos(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=299

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}} \sqrt{b\sqrt{c+dx}}}{\sqrt{d}}\right)}{64b^{3/2}} + \frac{\sqrt{\pi} \sqrt{d} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{16b^{3/2}} + \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \sin\left(4a - \frac{4bc}{d}\right)}{64b^{3/2}}$$

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(8*b) + (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(
(32*b) - (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) + (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rubi [A] time = 0.459801, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}} \sqrt{b\sqrt{c+dx}}}{\sqrt{d}}\right)}{64b^{3/2}} + \frac{\sqrt{\pi} \sqrt{d} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{16b^{3/2}} + \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \sin\left(4a - \frac{4bc}{d}\right)}{64b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3, x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(8*b) + (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(
(32*b) - (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) + (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3304

Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}
 \int \sqrt{c+dx} \cos(a+bx) \sin^3(a+bx) dx &= \int \left(\frac{1}{4} \sqrt{c+dx} \sin(2a+2bx) - \frac{1}{8} \sqrt{c+dx} \sin(4a+4bx) \right) dx \\
 &= -\left(\frac{1}{8} \int \sqrt{c+dx} \sin(4a+4bx) dx \right) + \frac{1}{4} \int \sqrt{c+dx} \sin(2a+2bx) dx \\
 &= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx} \cos(4a+4bx)}{32b} - \frac{d \int \frac{\cos(4a+4bx)}{\sqrt{c+dx}} dx}{64b} + \frac{d \int \frac{\cos(2a+2bx)}{\sqrt{c+dx}} dx}{64b} \\
 &= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx} \cos(4a+4bx)}{32b} - \frac{\left(d \cos\left(4a - \frac{4bc}{d}\right) \right) \int \frac{\cos\left(4a - \frac{4bc}{d}\right)}{\sqrt{c+dx}} dx}{64b} \\
 &= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx} \cos(4a+4bx)}{32b} - \frac{\cos\left(4a - \frac{4bc}{d}\right) \text{Subst}\left(\int \frac{\cos\left(4a - \frac{4bc}{d}\right)}{\sqrt{c+dx}} dx\right)}{64b} \\
 &= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx} \cos(4a+4bx)}{32b} - \frac{\sqrt{d} \sqrt{\frac{\pi}{2}} \cos\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}} \sqrt{c+dx}}{\sqrt{\pi}}\right)}{64b^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.325591, size = 264, normalized size = 0.88

$$-\sqrt{2\pi} \cos\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c+dx}\right) + 8\sqrt{\pi} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}} \sqrt{c+dx}}{\sqrt{\pi}}\right) + \sqrt{2\pi} \sin\left(4a - \frac{4bc}{d}\right) \text{FresnelS}\left(\frac{2\sqrt{\frac{b}{d}} \sqrt{c+dx}}{\sqrt{\pi}}\right)$$

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Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]

[Out] (-16*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 4*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*x)] - Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] + 8*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]])/d

$\text{rt}[c + d*x]]*\text{Sin}[4*a - (4*b*c)/d] - 8*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[b/d]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[\text{Pi}]]*\text{Sin}[2*a - (2*b*c)/d]/(128*b*\text{Sqrt}[b/d])$

Maple [A] time = 0.035, size = 286, normalized size = 1.

$$2 \frac{1}{d} \left(-1/16 \frac{d\sqrt{dx+c}}{b} \cos \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) + 1/32 \frac{d\sqrt{\pi}}{b} \left(\cos \left(2 \frac{ad-bc}{d} \right) \text{FresnelC} \left(2 \frac{\sqrt{dx+cb}}{d\sqrt{\pi}} \frac{1}{\sqrt{\frac{b}{d}}} \right) - \sin \left(2 \frac{ad-bc}{d} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d*x+c)^{(1/2)}*\text{cos}(b*x+a)*\text{sin}(b*x+a)^3,x)$

[Out] $2/d*(-1/16/b*d*(d*x+c)^{(1/2)}*\text{cos}(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/32/b*d*\text{Pi}^{(1/2)}/(b/d)^{(1/2)}*(\text{cos}(2*(a*d-b*c)/d)*\text{FresnelC}(2/\text{Pi}^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d)-\text{sin}(2*(a*d-b*c)/d)*\text{FresnelS}(2/\text{Pi}^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d))+1/64/b*d*(d*x+c)^{(1/2)}*\text{cos}(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/256/b*d*2^{(1/2)}*\text{Pi}^{(1/2)}/(b/d)^{(1/2)}*(\text{cos}(4*(a*d-b*c)/d)*\text{FresnelC}(2*2^{(1/2)}/\text{Pi}^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d)-\text{sin}(4*(a*d-b*c)/d)*\text{FresnelS}(2*2^{(1/2)}/\text{Pi}^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d))$

Maxima [C] time = 2.44712, size = 1655, normalized size = 5.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((d*x+c)^{(1/2)}*\text{cos}(b*x+a)*\text{sin}(b*x+a)^3,x, \text{algorithm}=\text{"maxima"})$

[Out] $1/1024*\text{sqrt}(2)*(16*\text{sqrt}(2)*\text{sqrt}(d*x + c)*d*\text{abs}(b)*\text{cos}(4*((d*x + c)*b - b*c + a*d)/d)/\text{abs}(d) - 64*\text{sqrt}(2)*\text{sqrt}(d*x + c)*d*\text{abs}(b)*\text{cos}(2*((d*x + c)*b - b*c + a*d)/d)/\text{abs}(d) + ((8*\text{sqrt}(\text{pi})*\text{cos}(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + 8*\text{sqrt}(\text{pi})*\text{cos}(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) - 8*I*\text{sqrt}(\text{pi})*\text{sin}(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + 8*I*\text{sqrt}(\text{pi})*\text{sin}(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))))*d*\text{sqrt}(\text{abs}(b)/\text{abs}(d))*\text{cos}(-2*(b*c - a*d)/d) + (-8*I*\text{sqrt}(\text{pi})*\text{cos}(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) - 8*I*\text{sqrt}(\text{pi})*\text{cos}(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) - 8*\text{sqrt}(\text{pi})*\text{sin}(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + 8*\text{sqrt}(\text{pi})*\text{sin}(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))))*d*\text{sqrt}(\text{abs}(b)/\text{abs}(d))*\text{sin}(-2*(b*c - a*d)/d))*\text{erf}(\text{sqrt}(d*x + c)*\text{sqrt}(2*I*b/d)) - (\text{sqrt}(2)*(\text{sqrt}(\text{pi})*\text{cos}(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + \text{sqrt}(\text{pi})*\text{cos}(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) - I*\text{sqrt}(\text{pi})*\text{sin}(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + I*\text{sqrt}(\text{pi})*\text{sin}(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))))*d*\text{sqrt}(\text{abs}(b)/\text{abs}(d))*\text{cos}(-4*(b*c - a*d)/d) - \text{sqrt}(2)*(I*\text{sqrt}(\text{pi})*\text{cos}(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + I*\text{sqrt}(\text{pi})*\text{cos}(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + \text{sqrt}(\text{pi})*\text{sin}(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) - \text{sqrt}(\text{pi})*\text{sin}(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))))*d*\text{sqrt}(\text{abs}(b)/\text{abs}(d))*\text{sin}(-4*(b*c - a*d)/d))*\text{erf}(2*\text{sqrt}(d*x + c)*\text{sqrt}(I*b/d)) - (\text{sqrt}(2)*(\text{sqrt}(\text{pi})*\text{cos}(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + \text{sqrt}(\text{pi})*\text{cos}(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + I*\text{sqrt}(\text{pi})*\text{sin}(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + I*\text{sqrt}(\text{pi})*\text{sin}(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2)))$

$$\begin{aligned} &^2)) - I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d*\sqrt{\pi}*\cos(-4*(b*c - a*d)/d) - \sqrt{2}*(-I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + \sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - \sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d*\sqrt{\pi}*\sin(-4*(b*c - a*d)/d))*\operatorname{erf}(2*\sqrt{d*x + c}*\sqrt{-I*b/d}) + ((8*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 8*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 8*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 8*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d*\sqrt{\pi}*\cos(-2*(b*c - a*d)/d) + (8*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 8*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 8*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 8*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d*\sqrt{\pi}*\sin(-2*(b*c - a*d)/d))*\operatorname{erf}(\sqrt{d*x + c}*\sqrt{-2*I*b/d}))*\operatorname{abs}(d)/(b*d*\operatorname{abs}(b)) \end{aligned}$$

Fricas [A] time = 0.608352, size = 630, normalized size = 2.11

$$\frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)-8\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)}{\sqrt{2}\sqrt{\pi d^2}\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\frac{4i(bc-4iad)}{d}}+\sqrt{2}\sqrt{\pi d^2}\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\frac{-4i(bc+4iad)}{d}}-8\sqrt{\pi d^2}\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\frac{2i(bc-2iad)}{d}}-\sqrt{2}\sqrt{\pi d^2}\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\frac{-2i(bc+2iad)}{d}}\right)}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} + \frac{\sqrt{2}\sqrt{\pi d^2}\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\frac{-4i(bc+4iad)}{d}}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} - \frac{8\sqrt{\pi d^2}\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\frac{2i(bc-2iad)}{d}}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} - \frac{\sqrt{2}\sqrt{\pi d^2}\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\frac{-2i(bc+2iad)}{d}}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")

[Out]
$$-1/128*(\sqrt{2}*\pi*d*\sqrt{b/(\pi*d)}*\cos(-4*(b*c - a*d)/d)*\operatorname{fresnel_cos}(2*\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/(\pi*d)}) - \sqrt{2}*\pi*d*\sqrt{b/(\pi*d)}*\operatorname{fresnel_sin}(2*\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/(\pi*d)})*\sin(-4*(b*c - a*d)/d) - 8*\pi*d*\sqrt{b/(\pi*d)}*\cos(-2*(b*c - a*d)/d)*\operatorname{fresnel_cos}(2*\sqrt{d*x + c}*\sqrt{b/(\pi*d)}) + 8*\pi*d*\sqrt{b/(\pi*d)}*\operatorname{fresnel_sin}(2*\sqrt{d*x + c}*\sqrt{b/(\pi*d)})*\sin(-2*(b*c - a*d)/d) - 4*(8*b*\cos(b*x + a)^4 - 16*b*\cos(b*x + a)^2 + 5*b)*\sqrt{d*x + c})/b^2$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**3,x)

[Out] Timed out

Giac [C] time = 1.36449, size = 643, normalized size = 2.15

$$\frac{\sqrt{2}\sqrt{\pi d^2}\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\frac{4i(bc-4iad)}{d}}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} + \frac{\sqrt{2}\sqrt{\pi d^2}\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\frac{-4i(bc+4iad)}{d}}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} - \frac{8\sqrt{\pi d^2}\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\frac{2i(bc-2iad)}{d}}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} - \frac{\sqrt{2}\sqrt{\pi d^2}\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\frac{-2i(bc+2iad)}{d}}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")

[Out] $\frac{1}{256}(\sqrt{2}\sqrt{\pi}d^2\operatorname{erf}(-\sqrt{2}\sqrt{bd})\sqrt{dx+c}(Ibd/\sqrt{b^2d^2+1})/d)e^{((4Ib^2c-4Ia^2d)/d)/(\sqrt{bd}(Ibd/\sqrt{b^2d^2+1})+1)b} + \sqrt{2}\sqrt{\pi}d^2\operatorname{erf}(-\sqrt{2}\sqrt{bd})\sqrt{dx+c}(-Ibd/\sqrt{b^2d^2+1})/d)e^{((-4Ib^2c+4Ia^2d)/d)/(\sqrt{bd}(-Ibd/\sqrt{b^2d^2+1})+1)b} - 8\sqrt{\pi}d^2\operatorname{erf}(-\sqrt{bd})\sqrt{dx+c}(Ibd/\sqrt{b^2d^2+1})/d)e^{((2Ib^2c-2Ia^2d)/d)/(\sqrt{bd}(Ibd/\sqrt{b^2d^2+1})+1)b} - 8\sqrt{\pi}d^2\operatorname{erf}(-\sqrt{bd})\sqrt{dx+c}(-Ibd/\sqrt{b^2d^2+1})/d)e^{((-2Ib^2c+2Ia^2d)/d)/(\sqrt{bd}(-Ibd/\sqrt{b^2d^2+1})+1)b} + 4\sqrt{dx+c}d e^{((4I(d*x+c)b-4Ib^2c+4Ia^2d)/d)/b} - 16\sqrt{dx+c}d e^{((2I(d*x+c)b-2Ib^2c+2Ia^2d)/d)/b} - 16\sqrt{dx+c}d e^{((-2I(d*x+c)b+2Ib^2c-2Ia^2d)/d)/b} + 4\sqrt{dx+c}d e^{((-4I(d*x+c)b+4Ib^2c-4Ia^2d)/d)/b}/d$

3.68 $\int (c + dx)^{3/2} \cos(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=351

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3\sqrt{\pi}d^{3/2}\sin\left(2a - \frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}} + \frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a - \frac{4bc}{d}\right)}{512b^{5/2}}$$

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(8*b) + ((c + d*x)^(3/2)*Cos[4*a + 4*b*x])/(32*b) + (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]*Sqrt[Pi]])/(64*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]*Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) - (3*d*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rubi [A] time = 0.566206, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3\sqrt{\pi}d^{3/2}\sin\left(2a - \frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}} + \frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a - \frac{4bc}{d}\right)}{512b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3, x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(8*b) + ((c + d*x)^(3/2)*Cos[4*a + 4*b*x])/(32*b) + (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]*Sqrt[Pi]])/(64*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]*Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) - (3*d*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n * Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m * Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1) * Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
```

$*e - c*f)/d]$, $\text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /;$ $\text{FreeQ}[\{c, d, e, f\}, x]$ && $\text{ComplexFreeQ}[f]$ && $\text{NeQ}[d*e - c*f, 0]$

Rule 3305

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] := \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Sin}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /;$ $\text{FreeQ}[\{c, d, e, f\}, x]$ && $\text{ComplexFreeQ}[f]$ && $\text{EqQ}[d*e - c*f, 0]$

Rule 3351

$\text{Int}[\text{Sin}[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] := \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /;$ $\text{FreeQ}[\{d, e, f\}, x]$

Rule 3304

$\text{Int}[\sin[\text{Pi}/2 + (e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] := \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Cos}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /;$ $\text{FreeQ}[\{c, d, e, f\}, x]$ && $\text{ComplexFreeQ}[f]$ && $\text{EqQ}[d*e - c*f, 0]$

Rule 3352

$\text{Int}[\text{Cos}[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] := \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /;$ $\text{FreeQ}[\{d, e, f\}, x]$

Rubi steps

$$\begin{aligned} \int (c + dx)^{3/2} \cos(a + bx) \sin^3(a + bx) dx &= \int \left(\frac{1}{4}(c + dx)^{3/2} \sin(2a + 2bx) - \frac{1}{8}(c + dx)^{3/2} \sin(4a + 4bx) \right) dx \\ &= -\left(\frac{1}{8} \int (c + dx)^{3/2} \sin(4a + 4bx) dx \right) + \frac{1}{4} \int (c + dx)^{3/2} \sin(2a + 2bx) dx \\ &= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{8b} + \frac{(c + dx)^{3/2} \cos(4a + 4bx)}{32b} - \frac{(3d) \int \sqrt{c + dx} \cos(2a + 2bx) dx}{64b} \\ &= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{8b} + \frac{(c + dx)^{3/2} \cos(4a + 4bx)}{32b} + \frac{3d\sqrt{c + dx} \sin(2a + 2bx)}{32b^2} \\ &= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{8b} + \frac{(c + dx)^{3/2} \cos(4a + 4bx)}{32b} + \frac{3d\sqrt{c + dx} \sin(2a + 2bx)}{32b^2} \\ &= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{8b} + \frac{(c + dx)^{3/2} \cos(4a + 4bx)}{32b} + \frac{3d\sqrt{c + dx} \sin(2a + 2bx)}{32b^2} \\ &= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{8b} + \frac{(c + dx)^{3/2} \cos(4a + 4bx)}{32b} + \frac{3d^{3/2} \sqrt{\frac{\pi}{2}} \cos\left(4a - \frac{4bc}{d}\right)}{512b} \end{aligned}$$

Mathematica [A] time = 3.12884, size = 393, normalized size = 1.12

$$3\sqrt{2\pi}d \sin\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c + dx}\right) - 48\sqrt{\pi}d \sin\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c + dx}}{\sqrt{\pi}}\right) + 3\sqrt{2\pi}d \cos\left(4a - \frac{4bc}{d}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]

```
[Out] (-128*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 128*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 32*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 32*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 3*d*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 48*d*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] - 48*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] + 96*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 12*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[4*(a + b*x)]/(1024*b^2*Sqrt[b/d])
```

Maple [A] time = 0.033, size = 376, normalized size = 1.1

$$2 \frac{1}{d} \left(-1/16 \frac{d(dx+c)^{3/2}}{b} \cos \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) + 3/16 \frac{d}{b} \left(1/4 \frac{d\sqrt{dx+c}}{b} \sin \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) - 1/8 \frac{d\sqrt{dx+c}}{b} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/16/b*d*(1/4/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))+1/64/b*d*(d*x+c)^(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/64/b*d*(1/8/b*d*(d*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))
```

Maxima [C] time = 2.47109, size = 1804, normalized size = 5.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8192*sqrt(2)*(128*sqrt(2)*(d*x + c)^(3/2)*b*d*abs(b)*cos(4*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 512*sqrt(2)*(d*x + c)^(3/2)*b*d*abs(b)*cos(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 48*sqrt(2)*sqrt(d*x + c)*d^2*abs(b)*sin(4*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 384*sqrt(2)*sqrt(d*x + c)*d^2*abs(b)*sin(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - ((48*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + (48*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d)*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - (sqrt(2)*(-3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos(-
```

$$\begin{aligned}
& 1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}) - 3*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^2*\sqrt{\text{abs}(b)/\text{abs}(d)}*\cos(-4*(b*c - a*d)/d) - \sqrt{2}*(3*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 3*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^2*\sqrt{\text{abs}(b)/\text{abs}(d)}*\sin(-4*(b*c - a*d)/d))*\text{erf}(2*\sqrt{d*x + c}*\sqrt{I*b/d}) - (\sqrt{2}*(3*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 3*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^2*\sqrt{\text{abs}(b)/\text{abs}(d)}*\cos(-4*(b*c - a*d)/d) - \sqrt{2}*(3*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 3*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^2*\sqrt{\text{abs}(b)/\text{abs}(d)}*\sin(-4*(b*c - a*d)/d))*\text{erf}(2*\sqrt{d*x + c}*\sqrt{-I*b/d}) - ((-48*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 48*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 48*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 48*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^2*\sqrt{\text{abs}(b)/\text{abs}(d)}*\cos(-2*(b*c - a*d)/d) + (48*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 48*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 48*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 48*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^2*\sqrt{\text{abs}(b)/\text{abs}(d)}*\sin(-2*(b*c - a*d)/d))*\text{erf}(\sqrt{d*x + c}*\sqrt{-2*I*b/d}))*\text{abs}(d)/(b^2*d*\text{abs}(b))
\end{aligned}$$

Fricas [A] time = 0.645748, size = 802, normalized size = 2.28

$$3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)-48\pi d^2\sqrt{\frac{b}{\pi d}}C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\cos\left(-\frac{4(bc-ad)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")

[Out] 1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 48*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x + a)^4 + 10*b^2*d*x + 10*b^2*c - 32*(b^2*d*x + b^2*c)*cos(b*x + a)^2 - 3*(2*b*d*cos(b*x + a)^3 - 5*b*d*cos(b*x + a))*sin(b*x + a)*sqrt(d*x + c))/b^3

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.53918, size = 1485, normalized size = 4.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2048*(8*(sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)*b) - 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)*b) - 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)*b) + 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b -
16*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b - 16*sqrt(
d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)
*d*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b*c - I*sqrt(2)*sqrt(pi)*(
-8*I*b*c*d + 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b^2) - I*sqrt(2)*sqrt(pi)*(-8*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sq
rt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(
b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 16*I*sqrt(pi)*(4*I*b*c*d - 3*d^2)*d*
erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 16*I*sqrt(pi)*(4*I*b*c
*d + 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 4*I*(
-8*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e
^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2 + 32*I*(4*I*(d*x + c)^(3/2)*
b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b -
2*I*b*c + 2*I*a*d)/d)/b^2 + 32*I*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x +
c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d
)/b^2 + 4*I*(-8*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*
x + c)*d^2)*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d
```

3.69 $\int (c + dx)^{5/2} \cos(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=407

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} - \frac{15\sqrt{\pi}d^{5/2}\cos\left(2a - \frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}} - \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a - \frac{4bc}{d}\right)}{4096b^{7/2}}$$

[Out] $(15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[2*a + 2*b*x])/(8*b) - (15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[4*a + 4*b*x])/(2048*b^3) + ((c + d*x)^{(5/2)}*\text{Cos}[4*a + 4*b*x])/(32*b) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[4*a - (4*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(4096*b^{(7/2)}) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]])/(256*b^{(7/2)}) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d])* \text{Sin}[4*a - (4*b*c)/d])/(4096*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}])* \text{Sin}[2*a - (2*b*c)/d])/(256*b^{(7/2)}) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[2*a + 2*b*x])/(32*b^2) - (5*d*(c + d*x)^{(3/2)}*\text{Sin}[4*a + 4*b*x])/(256*b^2)$

Rubi [A] time = 0.696998, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} - \frac{15\sqrt{\pi}d^{5/2}\cos\left(2a - \frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}} - \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a - \frac{4bc}{d}\right)}{4096b^{7/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^{(5/2)}*\text{Cos}[a + b*x]*\text{Sin}[a + b*x]^3, x]$

[Out] $(15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[2*a + 2*b*x])/(8*b) - (15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[4*a + 4*b*x])/(2048*b^3) + ((c + d*x)^{(5/2)}*\text{Cos}[4*a + 4*b*x])/(32*b) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[4*a - (4*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(4096*b^{(7/2)}) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]])/(256*b^{(7/2)}) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d])* \text{Sin}[4*a - (4*b*c)/d])/(4096*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}])* \text{Sin}[2*a - (2*b*c)/d])/(256*b^{(7/2)}) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[2*a + 2*b*x])/(32*b^2) - (5*d*(c + d*x)^{(3/2)}*\text{Sin}[4*a + 4*b*x])/(256*b^2)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^{5/2} \cos(a + bx) \sin^3(a + bx) dx &= \int \left(\frac{1}{4}(c + dx)^{5/2} \sin(2a + 2bx) - \frac{1}{8}(c + dx)^{5/2} \sin(4a + 4bx) \right) dx \\
&= -\left(\frac{1}{8} \int (c + dx)^{5/2} \sin(4a + 4bx) dx \right) + \frac{1}{4} \int (c + dx)^{5/2} \sin(2a + 2bx) dx \\
&= -\frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} + \frac{(c + dx)^{5/2} \cos(4a + 4bx)}{32b} - \frac{(5d) \int (c + dx)^{3/2} \sin(2a + 2bx) dx}{6} \\
&= -\frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} + \frac{(c + dx)^{5/2} \cos(4a + 4bx)}{32b} + \frac{5d(c + dx)^{3/2} \sin(2a + 2bx)}{32b^2} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{128b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} - \frac{15d^2 \sqrt{c + dx} \cos(4a + 4bx)}{2048b^3} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{128b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} - \frac{15d^2 \sqrt{c + dx} \cos(4a + 4bx)}{2048b^3} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{128b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} - \frac{15d^2 \sqrt{c + dx} \cos(4a + 4bx)}{2048b^3} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{128b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} - \frac{15d^2 \sqrt{c + dx} \cos(4a + 4bx)}{2048b^3}
\end{aligned}$$

Mathematica [A] time = 11.7879, size = 550, normalized size = 1.35

$$-1024b^3 c^2 \sqrt{c + dx} \cos(2(a + bx)) + 256b^3 c^2 \sqrt{c + dx} \cos(4(a + bx)) - 1024b^3 d^2 x^2 \sqrt{c + dx} \cos(2(a + bx)) + 256b^3 d^2 x^2 \sqrt{c + dx} \cos(4(a + bx))$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]

[Out] (-1024*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 960*b*d^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 2048*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 1024*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 256*b^3*c^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 60*b*d^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 512*b^3*c*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 256*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 480*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 480*Sqrt[b/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] + 1280*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 1280*b^2*d^2*x*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 160*b^2*c*d*Sqrt[c + d*x]*Sin[4*(a + b*x)] - 160*b^2*d^2*x*Sqrt[c + d*x]*Sin[4*(a + b*x)]/(8192*b^4)

Maple [A] time = 0.033, size = 470, normalized size = 1.2

$$2 \frac{1}{d} \left(-1/16 \frac{d(dx+c)^{5/2}}{b} \cos \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) + \frac{5d}{16b} \left(\frac{1}{4} \frac{d(dx+c)^{3/2}}{b} \sin \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) - 3/4 \frac{d}{b} \left(-1 \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x)

[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/16/b*d*(1/4/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))) + 1/64/b*d*(d*x+c)^(5/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-5/64/b*d*(1/8/b*d*(d*x+c)^(3/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))

Maxima [C] time = 2.41457, size = 1874, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")

[Out] -1/65536*sqrt(2)*(640*sqrt(2)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(4*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 5120*sqrt(2)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 16*(64*sqrt(2)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 15*sqrt(2)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(4*((d*x + c)*b - b*c + a*d)/d) + 256*(16*sqrt(2)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 15*sqrt(2)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(2*((d*x + c)*b - b*c + a*d)/d) + ((480*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq

```

rt(d^2))) + 480*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d
/sqrt(d^2))) - 480*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + 480*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) *d^3*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + (-48
0*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
- 480*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) - 480*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) + 480*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) *d^3*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d)*erf(sqrt(d*x + c)
*sqrt(2*I*b/d)) - (sqrt(2)*(15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^3*sqrt(abs(b)/abs(d))*cos(-4*(b*c -
a*d)/d) - sqrt(2)*(15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) *d^3*sqrt(abs(b)/abs(d))*sin(-4*(b*c - a*d)/d))*
erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - (sqrt(2)*(15*sqrt(pi)*cos(1/4*pi + 1/2*a
rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(-1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^3*sqrt(abs(b)/abs(
d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(-15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^3*sqrt(abs(b)/abs(d))*sin(-
4*(b*c - a*d)/d)*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + ((480*sqrt(pi)*cos(1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*sqrt(pi)*cos(
-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*I*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 480*I*sqrt
(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^3*sq
rt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + (480*I*sqrt(pi)*cos(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*I*sqrt(pi)*cos(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 480*sqrt(pi)*sin(1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*sqrt(pi)*sin(-1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^3*sqrt(abs(b)/abs
(d))*sin(-2*(b*c - a*d)/d)*erf(sqrt(d*x + c)*sqrt(-2*I*b/d))*abs(d)/(b^3*
d*abs(b))

```

Fricas [A] time = 0.68742, size = 1011, normalized size = 2.48

$$15 \sqrt{2} \pi d^3 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{4(bc-ad)}{d}\right) C\left(2 \sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) - 15 \sqrt{2} \pi d^3 \sqrt{\frac{b}{\pi d}} S\left(2 \sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{4(bc-ad)}{d}\right) - 480 \pi d^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")

[Out] 1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d))) + 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 4*(320*b^3*d^2*x^2 + 640*b^3*c*d*x +

$$320*b^3*c^2 + 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*x + 64*b^3*c^2 - 15*b*d^2)*\cos(b*x + a)^4 - 255*b*d^2 - 8*(128*b^3*d^2*x^2 + 256*b^3*c*d*x + 128*b^3*c^2 - 75*b*d^2)*\cos(b*x + a)^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*\cos(b*x + a)^3 - 5*(b^2*d^2*x + b^2*c*d)*\cos(b*x + a))*\sin(b*x + a))*\sqrt{d*x + c})/b^4$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**3,x)

[Out] Timed out

Giac [C] time = 1.79902, size = 2681, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")

[Out]
$$\frac{1}{16384} * (64 * (\sqrt{2} * \sqrt{\pi} * d^2 * \operatorname{erf}(-\sqrt{2} * \sqrt{b*d}) * \sqrt{d*x + c}) * (I*b*d/\sqrt{b^2*d^2} + 1)/d) * e^{((4*I*b*c - 4*I*a*d)/d)} / (\sqrt{b*d}) * (I*b*d/\sqrt{b^2*d^2} + 1)*b + \sqrt{2} * \sqrt{\pi} * d^2 * \operatorname{erf}(-\sqrt{2} * \sqrt{b*d}) * \sqrt{d*x + c}) * (-I*b*d/\sqrt{b^2*d^2} + 1)/d) * e^{((-4*I*b*c + 4*I*a*d)/d)} / (\sqrt{b*d}) * (-I*b*d/\sqrt{b^2*d^2} + 1)*b - 8 * \sqrt{\pi} * d^2 * \operatorname{erf}(-\sqrt{b*d}) * \sqrt{d*x + c}) * (I*b*d/\sqrt{b^2*d^2} + 1)/d) * e^{((2*I*b*c - 2*I*a*d)/d)} / (\sqrt{b*d}) * (I*b*d/\sqrt{b^2*d^2} + 1)*b - 8 * \sqrt{\pi} * d^2 * \operatorname{erf}(-\sqrt{b*d}) * \sqrt{d*x + c}) * (-I*b*d/\sqrt{b^2*d^2} + 1)/d) * e^{((-2*I*b*c + 2*I*a*d)/d)} / (\sqrt{b*d}) * (-I*b*d/\sqrt{b^2*d^2} + 1)*b + 4 * \sqrt{d*x + c}) * d * e^{((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)} / b - 16 * \sqrt{d*x + c}) * d * e^{((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)} / b - 16 * \sqrt{d*x + c}) * d * e^{((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)} / b + 4 * \sqrt{d*x + c}) * d * e^{((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)} / b) * c^2 - d^2 * ((I * \sqrt{2} * \sqrt{\pi}) * (64 * I * b^2 * c^2 * d - 48 * b * c * d^2 - 15 * I * d^3) * d * \operatorname{erf}(-\sqrt{2} * \sqrt{b*d}) * \sqrt{d*x + c}) * (I*b*d/\sqrt{b^2*d^2} + 1)/d) * e^{((4*I*b*c - 4*I*a*d)/d)} / (\sqrt{b*d}) * (I*b*d/\sqrt{b^2*d^2} + 1)*b^3 - 4 * I * (-64 * I * (d*x + c)^{(5/2)} * b^2 * d + 128 * I * (d*x + c)^{(3/2)} * b^2 * c * d - 64 * I * \sqrt{d*x + c}) * b^2 * c^2 * d - 40 * (d*x + c)^{(3/2)} * b * d^2 + 48 * \sqrt{d*x + c}) * b * c * d^2 + 15 * I * \sqrt{d*x + c}) * d^3) * e^{((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)} / b^3) / d^2 + (I * \sqrt{2} * \sqrt{\pi}) * (64 * I * b^2 * c^2 * d + 48 * b * c * d^2 - 15 * I * d^3) * d * \operatorname{erf}(-\sqrt{2} * \sqrt{b*d}) * \sqrt{d*x + c}) * (-I*b*d/\sqrt{b^2*d^2} + 1)/d) * e^{((-4*I*b*c + 4*I*a*d)/d)} / (\sqrt{b*d}) * (-I*b*d/\sqrt{b^2*d^2} + 1)*b^3 - 4 * I * (-64 * I * (d*x + c)^{(5/2)} * b^2 * d + 128 * I * (d*x + c)^{(3/2)} * b^2 * c * d - 64 * I * \sqrt{d*x + c}) * b^2 * c^2 * d + 40 * (d*x + c)^{(3/2)} * b * d^2 - 48 * \sqrt{d*x + c}) * b * c * d^2 + 15 * I * \sqrt{d*x + c}) * d^3) * e^{((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)} / b^3) / d^2 + 32 * (I * \sqrt{\pi}) * (-16 * I * b^2 * c^2 * d + 24 * b * c * d^2 + 15 * I * d^3) * d * \operatorname{erf}(-\sqrt{b*d}) * \sqrt{d*x + c}) * (I*b*d/\sqrt{b^2*d^2} + 1)/d) * e^{((2*I*b*c - 2*I*a*d)/d)} / (\sqrt{b*d}) * (I*b*d/\sqrt{b^2*d^2} + 1)*b^3 - 2 * I * (16 * I * (d*x + c)^{(5/2)} * b^2 * d - 32 * I * (d*x + c)^{(3/2)} * b^2 * c * d + 16 * I * \sqrt{d*x + c}) * b^2 * c^2 * d + 20 * (d*x + c)^{(3/2)} * b * d^2 - 24 * \sqrt{d*x + c}) * b * c * d^2 - 15 * I * \sqrt{d*x + c}) * d^3) * e^{((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)} / b^3) / d^2 + 32 * (I * \sqrt{\pi}) * (-16 * I * b^2 * c^2 * d - 24 * b * c * d^2 + 15 * I * d^3) * d * \operatorname{erf}(-\sqrt{b*d}) * \sqrt{d*x + c}) * (-I*b*d/\sqrt{b^2*d^2} + 1)/d) * e^{((-2*I*b*c + 2*I*a*d)/d)} / (\sqrt{b*d})$$

$$\begin{aligned}
& *(-I*b*d/\sqrt{b^2*d^2} + 1)*b^3) - 2*I*(16*I*(d*x + c)^{(5/2)}*b^2*d - 32*I*(\\
& d*x + c)^{(3/2)}*b^2*c*d + 16*I*\sqrt{d*x + c}*b^2*c^2*d - 20*(d*x + c)^{(3/2)}* \\
& b*d^2 + 24*\sqrt{d*x + c}*b*c*d^2 - 15*I*\sqrt{d*x + c}*d^3)*e^{((2*I*(d*x + c) \\
&)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^2) - 16*(I*\sqrt{2}*\sqrt{\pi})*(-8*I*b*c*d \\
& + 3*d^2)*d*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d \\
&)*e^{((4*I*b*c - 4*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2) + I*s \\
& \sqrt{2}*\sqrt{\pi})*(-8*I*b*c*d - 3*d^2)*d*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c} \\
& *(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-4*I*b*c + 4*I*a*d)/d)/(\sqrt{b*d}*(-I*b* \\
& d/\sqrt{b^2*d^2} + 1)*b^2) + 16*I*\sqrt{\pi}*(4*I*b*c*d - 3*d^2)*d*\operatorname{erf}(-\sqrt{b \\
& *d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((2*I*b*c - 2*I*a*d)/d)/(s \\
& \sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2) + 16*I*\sqrt{\pi}*(4*I*b*c*d + 3*d^2) \\
& *d*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-2*I*b*c \\
& + 2*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2) - 4*I*(-8*I*(d*x + \\
& c)^{(3/2)}*b*d + 8*I*\sqrt{d*x + c}*b*c*d + 3*\sqrt{d*x + c}*d^2)*e^{((4*I*(d*x \\
& + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2 - 32*I*(4*I*(d*x + c)^{(3/2)}*b*d - 4*I*s \\
& \sqrt{d*x + c}*b*c*d - 3*\sqrt{d*x + c}*d^2)*e^{((2*I*(d*x + c)*b - 2*I*b*c + 2 \\
& *I*a*d)/d)/b^2 - 32*I*(4*I*(d*x + c)^{(3/2)}*b*d - 4*I*\sqrt{d*x + c}*b*c*d + \\
& 3*\sqrt{d*x + c}*d^2)*e^{((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2 - 4*I \\
& *(-8*I*(d*x + c)^{(3/2)}*b*d + 8*I*\sqrt{d*x + c}*b*c*d - 3*\sqrt{d*x + c}*d^2) \\
& *e^{((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)*c)/d
\end{aligned}$$

3.70 $\int (c + dx)^m \cos^2(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=267

$$\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b} - \frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{8b}$$

[Out] $-(E^{(I*(a - (b*c)/d))}*(c + d*x)^m*\Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(8*b*((-I)*b*(c + d*x))/d)^m - ((c + d*x)^m*\Gamma[1 + m, (I*b*(c + d*x))/d])/(8*b*E^{(I*(a - (b*c)/d))}*((I*b*(c + d*x))/d)^m - (3^{(-1 - m)}*E^{((3*I)*(a - (b*c)/d))}*(c + d*x)^m*\Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(8*b*((-I)*b*(c + d*x))/d)^m - (3^{(-1 - m)}*(c + d*x)^m*\Gamma[1 + m, ((3*I)*b*(c + d*x))/d])/(8*b*E^{((3*I)*(a - (b*c)/d))}*((I*b*(c + d*x))/d)^m)$

Rubi [A] time = 0.282696, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4406, 3308, 2181}

$$\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b} - \frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{8b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x], x]

[Out] $-(E^{(I*(a - (b*c)/d))}*(c + d*x)^m*\Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(8*b*((-I)*b*(c + d*x))/d)^m - ((c + d*x)^m*\Gamma[1 + m, (I*b*(c + d*x))/d])/(8*b*E^{(I*(a - (b*c)/d))}*((I*b*(c + d*x))/d)^m - (3^{(-1 - m)}*E^{((3*I)*(a - (b*c)/d))}*(c + d*x)^m*\Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(8*b*((-I)*b*(c + d*x))/d)^m - (3^{(-1 - m)}*(c + d*x)^m*\Gamma[1 + m, ((3*I)*b*(c + d*x))/d])/(8*b*E^{((3*I)*(a - (b*c)/d))}*((I*b*(c + d*x))/d)^m)$

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3308

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 2181

Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int (c + dx)^m \cos^2(a + bx) \sin(a + bx) dx &= \int \left(\frac{1}{4}(c + dx)^m \sin(a + bx) + \frac{1}{4}(c + dx)^m \sin(3a + 3bx) \right) dx \\
&= \frac{1}{4} \int (c + dx)^m \sin(a + bx) dx + \frac{1}{4} \int (c + dx)^m \sin(3a + 3bx) dx \\
&= \frac{1}{8} i \int e^{-i(a+bx)} (c + dx)^m dx - \frac{1}{8} i \int e^{i(a+bx)} (c + dx)^m dx + \frac{1}{8} i \int e^{-i(3a+3bx)} (c + dx)^m dx \\
&\quad + \frac{1}{8} i \int e^{i(3a+3bx)} (c + dx)^m dx \\
&= -\frac{e^{i\left(a-\frac{bc}{d}\right)} (c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, -\frac{ib(c+dx)}{d}\right)}{8b} - \frac{e^{-i\left(a-\frac{bc}{d}\right)} (c + dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, \frac{ib(c+dx)}{d}\right)}{8b}
\end{aligned}$$

Mathematica [A] time = 0.476431, size = 250, normalized size = 0.94

$$\frac{e^{-\frac{3i(ad+bc)}{d}} (c + dx)^m \left(3e^{\frac{2i(ad+bc)}{d}} \left(-e^{2ia} \left(-\frac{ib(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m + 1, -\frac{ib(c+dx)}{d}\right) - e^{\frac{2ibc}{d}} \left(\frac{ib(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m + 1, \frac{ib(c+dx)}{d}\right)\right)}{24}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x], x]

[Out] ((c + d*x)^m*(3*E^(((2*I)*(b*c + a*d))/d))*(-((E^((2*I)*a))*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/((-I)*b*(c + d*x))/d)^m - (E^(((2*I)*b*c)/d))*Gamma[1 + m, (I*b*(c + d*x))/d])/((I*b*(c + d*x))/d)^m - (E^(((6*I)*a))*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d] + E^(((6*I)*b*c)/d))*((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((3*I)*b*(c + d*x))/d])/((3^m*((b^2*(c + d*x)^2)/d^2)^m))/((24*b*E^(((3*I)*(b*c + a*d))/d))

Maple [F] time = 0.239, size = 0, normalized size = 0.

$$\int (dx + c)^m (\cos(bx + a))^2 \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a), x)

[Out] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a)^2 \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a), x, algorithm="maxima")

[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a), x)

Fricas [A] time = 0.547447, size = 460, normalized size = 1.72

$$\frac{e^{\left(-\frac{dm \log\left(\frac{3ib}{d}\right) - 3ibc + 3iad}{d}\right)} \Gamma\left(m + 1, \frac{3ibdx + 3ibc}{d}\right) + 3e^{\left(-\frac{dm \log\left(\frac{ib}{d}\right) - ibc + iad}{d}\right)} \Gamma\left(m + 1, \frac{ibdx + ibc}{d}\right) + 3e^{\left(-\frac{dm \log\left(-\frac{ib}{d}\right) + ibc - iad}{d}\right)} \Gamma\left(m + 1, \frac{-ibdx}{d}\right)}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")

[Out] $-1/24*(e^{-(d*m*\log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d}*\gamma(m + 1, (3*I*b*d*x + 3*I*b*c)/d) + 3*e^{-(d*m*\log(I*b/d) - I*b*c + I*a*d)/d}*\gamma(m + 1, (I*b*d*x + I*b*c)/d) + 3*e^{-(d*m*\log(-I*b/d) + I*b*c - I*a*d)/d}*\gamma(m + 1, (-I*b*d*x - I*b*c)/d) + e^{-(d*m*\log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d}*\gamma(m + 1, (-3*I*b*d*x - 3*I*b*c)/d))/b$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**m*cos(b*x+a)**2*sin(b*x+a),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a)^2 \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a), x)

3.71 $\int (c + dx)^4 \cos^2(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=205

$$\frac{160d^3(c + dx) \sin(a + bx)}{27b^4} + \frac{4d^2(c + dx)^2 \cos^3(a + bx)}{9b^3} + \frac{8d^2(c + dx)^2 \cos(a + bx)}{3b^3} - \frac{8d^3(c + dx) \sin(a + bx) \cos^2(a + bx)}{27b^4}$$

```
[Out] (-160*d^4*Cos[a + b*x])/(27*b^5) + (8*d^2*(c + d*x)^2*Cos[a + b*x])/(3*b^3)
- (8*d^4*Cos[a + b*x]^3)/(81*b^5) + (4*d^2*(c + d*x)^2*Cos[a + b*x]^3)/(9*
b^3) - ((c + d*x)^4*Cos[a + b*x]^3)/(3*b) - (160*d^3*(c + d*x)*Sin[a + b*x]
)/(27*b^4) + (8*d*(c + d*x)^3*SIN[a + b*x])/(9*b^2) - (8*d^3*(c + d*x)*Cos[
a + b*x]^2*SIN[a + b*x])/(27*b^4) + (4*d*(c + d*x)^3*Cos[a + b*x]^2*SIN[a +
b*x])/(9*b^2)
```

Rubi [A] time = 0.202537, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4405, 3311, 3296, 2638, 3310}

$$\frac{160d^3(c + dx) \sin(a + bx)}{27b^4} + \frac{4d^2(c + dx)^2 \cos^3(a + bx)}{9b^3} + \frac{8d^2(c + dx)^2 \cos(a + bx)}{3b^3} - \frac{8d^3(c + dx) \sin(a + bx) \cos^2(a + bx)}{27b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^2*SIN[a + b*x], x]
```

```
[Out] (-160*d^4*Cos[a + b*x])/(27*b^5) + (8*d^2*(c + d*x)^2*Cos[a + b*x])/(3*b^3)
- (8*d^4*Cos[a + b*x]^3)/(81*b^5) + (4*d^2*(c + d*x)^2*Cos[a + b*x]^3)/(9*
b^3) - ((c + d*x)^4*Cos[a + b*x]^3)/(3*b) - (160*d^3*(c + d*x)*Sin[a + b*x]
)/(27*b^4) + (8*d*(c + d*x)^3*SIN[a + b*x])/(9*b^2) - (8*d^3*(c + d*x)*Cos[
a + b*x]^2*SIN[a + b*x])/(27*b^4) + (4*d*(c + d*x)^3*Cos[a + b*x]^2*SIN[a +
b*x])/(9*b^2)
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:= Simp[(d*m*(c + d*x)^(m - 1)*(b*SIN[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*SIN[e + f*x])^(n - 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*SIN[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*SIN[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_.))*(b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_), x_Symbol] :=
Simp[(d*(b*Sine + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sine + f*x)]^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sine + f*x)]^(n - 1)/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^4 \cos^2(a + bx) \sin(a + bx) dx &= -\frac{(c + dx)^4 \cos^3(a + bx)}{3b} + \frac{(4d) \int (c + dx)^3 \cos^3(a + bx) dx}{3b} \\ &= \frac{4d^2(c + dx)^2 \cos^3(a + bx)}{9b^3} - \frac{(c + dx)^4 \cos^3(a + bx)}{3b} + \frac{4d(c + dx)^3 \cos^2(a + bx)}{9b^2} \\ &= -\frac{8d^4 \cos^3(a + bx)}{81b^5} + \frac{4d^2(c + dx)^2 \cos^3(a + bx)}{9b^3} - \frac{(c + dx)^4 \cos^3(a + bx)}{3b} + \frac{8d(c + dx)^3 \cos^2(a + bx)}{9b^2} \\ &= \frac{8d^2(c + dx)^2 \cos(a + bx)}{3b^3} - \frac{8d^4 \cos^3(a + bx)}{81b^5} + \frac{4d^2(c + dx)^2 \cos^3(a + bx)}{9b^3} - \frac{(c + dx)^4 \cos^3(a + bx)}{3b} \\ &= -\frac{16d^4 \cos(a + bx)}{27b^5} + \frac{8d^2(c + dx)^2 \cos(a + bx)}{3b^3} - \frac{8d^4 \cos^3(a + bx)}{81b^5} + \frac{4d^2(c + dx)^2 \cos^3(a + bx)}{9b^3} \\ &= -\frac{160d^4 \cos(a + bx)}{27b^5} + \frac{8d^2(c + dx)^2 \cos(a + bx)}{3b^3} - \frac{8d^4 \cos^3(a + bx)}{81b^5} + \frac{4d^2(c + dx)^2 \cos^3(a + bx)}{9b^3} \end{aligned}$$

Mathematica [A] time = 1.62255, size = 150, normalized size = 0.73

$$\frac{81 \cos(a + bx) (-12b^2 d^2 (c + dx)^2 + b^4 (c + dx)^4 + 24d^4) + \cos(3(a + bx)) (-36b^2 d^2 (c + dx)^2 + 27b^4 (c + dx)^4 + 8d^4) - 24d^4 \cos^3(a + bx)}{324b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x], x]
```

```
[Out] -(81*(24*d^4 - 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cos[a + b*x] + (8*d^4 - 36*b^2*d^2*(c + d*x)^2 + 27*b^4*(c + d*x)^4)*Cos[3*(a + b*x)] - 24*b*d*(c + d*x)*(-82*d^2 + 15*b^2*(c + d*x)^2 + (-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/(324*b^5)
```

Maple [B] time = 0.043, size = 835, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a), x)
```

```
[Out] 1/b*(1/b^4*d^4*(-1/3*(b*x+a)^4*cos(b*x+a)^3+4/9*(b*x+a)^3*(2+cos(b*x+a)^2)*sin(b*x+a)+8/3*(b*x+a)^2*cos(b*x+a)-160/27*cos(b*x+a)-16/3*(b*x+a)*sin(b*x+a)+4/9*(b*x+a)^2*cos(b*x+a)^3-8/27*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)-8/81*cos(b*x+a)^3)-4/b^4*a*d^4*(-1/3*(b*x+a)^3*cos(b*x+a)^3+1/3*(b*x+a)^2*(2+co
```

$$\begin{aligned} & s(b*x+a)^2*\sin(b*x+a)-4/3*\sin(b*x+a)+4/3*(b*x+a)*\cos(b*x+a)+2/9*(b*x+a)*\cos \\ & s(b*x+a)^3-2/27*(2+\cos(b*x+a)^2)*\sin(b*x+a))+4/b^3*c*d^3*(-1/3*(b*x+a)^3*\cos \\ & s(b*x+a)^3+1/3*(b*x+a)^2*(2+\cos(b*x+a)^2)*\sin(b*x+a)-4/3*\sin(b*x+a)+4/3*(b* \\ & x+a)*\cos(b*x+a)+2/9*(b*x+a)*\cos(b*x+a)^3-2/27*(2+\cos(b*x+a)^2)*\sin(b*x+a))+ \\ & 6/b^4*a^2*d^4*(-1/3*(b*x+a)^2*\cos(b*x+a)^3+2/9*(b*x+a)*(2+\cos(b*x+a)^2)*\sin \\ & (b*x+a)+2/27*\cos(b*x+a)^3+4/9*\cos(b*x+a))-12/b^3*a*c*d^3*(-1/3*(b*x+a)^2*\cos \\ & s(b*x+a)^3+2/9*(b*x+a)*(2+\cos(b*x+a)^2)*\sin(b*x+a)+2/27*\cos(b*x+a)^3+4/9*\cos \\ & s(b*x+a))+6/b^2*c^2*d^2*(-1/3*(b*x+a)^2*\cos(b*x+a)^3+2/9*(b*x+a)*(2+\cos(b*x \\ & +a)^2)*\sin(b*x+a)+2/27*\cos(b*x+a)^3+4/9*\cos(b*x+a))-4/b^4*a^3*d^4*(-1/3*(b* \\ & x+a)*\cos(b*x+a)^3+1/9*(2+\cos(b*x+a)^2)*\sin(b*x+a))+12/b^3*a^2*c*d^3*(-1/3*(\\ & b*x+a)*\cos(b*x+a)^3+1/9*(2+\cos(b*x+a)^2)*\sin(b*x+a))-12/b^2*a*c^2*d^2*(-1/3 \\ & *(b*x+a)*\cos(b*x+a)^3+1/9*(2+\cos(b*x+a)^2)*\sin(b*x+a))+4/b*c^3*d*(-1/3*(b*x \\ & +a)*\cos(b*x+a)^3+1/9*(2+\cos(b*x+a)^2)*\sin(b*x+a))-1/3/b^4*a^4*d^4*\cos(b*x+a) \\ &)^3+4/3/b^3*a^3*c*d^3*\cos(b*x+a)^3-2/b^2*a^2*c^2*d^2*\cos(b*x+a)^3+4/3/b*a*c \\ & ^3*d*\cos(b*x+a)^3-1/3*c^4*\cos(b*x+a)^3) \end{aligned}$$

Maxima [B] time = 1.39513, size = 1200, normalized size = 5.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/324*(108*c^4*\cos(b*x + a)^3 - 432*a*c^3*d*\cos(b*x + a)^3/b + 648*a^2*c^2 \\ & *d^2*\cos(b*x + a)^3/b^2 - 432*a^3*c*d^3*\cos(b*x + a)^3/b^3 + 108*a^4*d^4*\cos \\ & s(b*x + a)^3/b^4 + 36*(3*(b*x + a)*\cos(3*b*x + 3*a) + 9*(b*x + a)*\cos(b*x + \\ & a) - \sin(3*b*x + 3*a) - 9*\sin(b*x + a))*c^3*d/b - 108*(3*(b*x + a)*\cos(3*b \\ & *x + 3*a) + 9*(b*x + a)*\cos(b*x + a) - \sin(3*b*x + 3*a) - 9*\sin(b*x + a))*a \\ & *c^2*d^2/b^2 + 108*(3*(b*x + a)*\cos(3*b*x + 3*a) + 9*(b*x + a)*\cos(b*x + a) \\ & - \sin(3*b*x + 3*a) - 9*\sin(b*x + a))*a^2*c*d^3/b^3 - 36*(3*(b*x + a)*\cos(3 \\ & *b*x + 3*a) + 9*(b*x + a)*\cos(b*x + a) - \sin(3*b*x + 3*a) - 9*\sin(b*x + a)) \\ & *a^3*d^4/b^4 + 18*((9*(b*x + a)^2 - 2)*\cos(3*b*x + 3*a) + 27*((b*x + a)^2 - \\ & 2)*\cos(b*x + a) - 6*(b*x + a)*\sin(3*b*x + 3*a) - 54*(b*x + a)*\sin(b*x + a) \\ &)*c^2*d^2/b^2 - 36*((9*(b*x + a)^2 - 2)*\cos(3*b*x + 3*a) + 27*((b*x + a)^2 \\ & - 2)*\cos(b*x + a) - 6*(b*x + a)*\sin(3*b*x + 3*a) - 54*(b*x + a)*\sin(b*x + a) \\ &))*a*c*d^3/b^3 + 18*((9*(b*x + a)^2 - 2)*\cos(3*b*x + 3*a) + 27*((b*x + a)^2 \\ & - 2)*\cos(b*x + a) - 6*(b*x + a)*\sin(3*b*x + 3*a) - 54*(b*x + a)*\sin(b*x + \\ & a))*a^2*d^4/b^4 + 12*(3*(3*(b*x + a)^3 - 2*b*x - 2*a)*\cos(3*b*x + 3*a) + 27 \\ & *((b*x + a)^3 - 6*b*x - 6*a)*\cos(b*x + a) - (9*(b*x + a)^2 - 2)*\sin(3*b*x + \\ & 3*a) - 81*((b*x + a)^2 - 2)*\sin(b*x + a))*c*d^3/b^3 - 12*(3*(3*(b*x + a)^3 \\ & - 2*b*x - 2*a)*\cos(3*b*x + 3*a) + 27*((b*x + a)^3 - 6*b*x - 6*a)*\cos(b*x + \\ & a) - (9*(b*x + a)^2 - 2)*\sin(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*\sin(b*x + \\ & a))*a*d^4/b^4 + ((27*(b*x + a)^4 - 36*(b*x + a)^2 + 8)*\cos(3*b*x + 3*a) + \\ & 81*((b*x + a)^4 - 12*(b*x + a)^2 + 24)*\cos(b*x + a) - 12*(3*(b*x + a)^3 - 2 \\ & *b*x - 2*a)*\sin(3*b*x + 3*a) - 324*((b*x + a)^3 - 6*b*x - 6*a)*\sin(b*x + a) \\ &)*d^4/b^4)/b \end{aligned}$$

Fricas [A] time = 0.525951, size = 636, normalized size = 3.1

$$(27 b^4 d^4 x^4 + 108 b^4 c d^3 x^3 + 27 b^4 c^4 - 36 b^2 c^2 d^2 + 8 d^4 + 18 (9 b^4 c^2 d^2 - 2 b^2 d^4) x^2 + 36 (3 b^4 c^3 d - 2 b^2 c d^3) x) \cos(bx + a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/81*((27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 - 36*b^2*c^2*d^2 +
8*d^4 + 18*(9*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 36*(3*b^4*c^3*d - 2*b^2*c*d^3)
*x)*cos(b*x + a)^3 - 24*(9*b^2*d^4*x^2 + 18*b^2*c*d^3*x + 9*b^2*c^2*d^2 - 2
0*d^4)*cos(b*x + a) - 12*(6*b^3*d^4*x^3 + 18*b^3*c*d^3*x^2 + 6*b^3*c^3*d -
40*b*c*d^3 + (3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d - 2*b*c*d^3 + (
9*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 + 2*(9*b^3*c^2*d^2 - 20*b*d^4)*x
)*sin(b*x + a))/b^5
```

Sympy [A] time = 10.4695, size = 646, normalized size = 3.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**4*cos(a + b*x)**3/(3*b) - 4*c**3*d*x*cos(a + b*x)**3/(3*b) -
2*c**2*d**2*x**2*cos(a + b*x)**3/b - 4*c*d**3*x**3*cos(a + b*x)**3/(3*b) -
d**4*x**4*cos(a + b*x)**3/(3*b) + 8*c**3*d*sin(a + b*x)**3/(9*b**2) + 4*c*
*3*d*sin(a + b*x)*cos(a + b*x)**2/(3*b**2) + 8*c**2*d**2*x*sin(a + b*x)**3/
(3*b**2) + 4*c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**2/b**2 + 8*c*d**3*x**2*
sin(a + b*x)**3/(3*b**2) + 4*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**2/b**2
+ 8*d**4*x**3*sin(a + b*x)**3/(9*b**2) + 4*d**4*x**3*sin(a + b*x)*cos(a + b
*x)**2/(3*b**2) + 8*c**2*d**2*sin(a + b*x)**2*cos(a + b*x)/(3*b**3) + 28*c*
*2*d**2*cos(a + b*x)**3/(9*b**3) + 16*c*d**3*x*sin(a + b*x)**2*cos(a + b*x)
/(3*b**3) + 56*c*d**3*x*cos(a + b*x)**3/(9*b**3) + 8*d**4*x**2*sin(a + b*x)
**2*cos(a + b*x)/(3*b**3) + 28*d**4*x**2*cos(a + b*x)**3/(9*b**3) - 160*c*d
**3*sin(a + b*x)**3/(27*b**4) - 56*c*d**3*sin(a + b*x)*cos(a + b*x)**2/(9*b
**4) - 160*d**4*x*sin(a + b*x)**3/(27*b**4) - 56*d**4*x*sin(a + b*x)*cos(a
+ b*x)**2/(9*b**4) - 160*d**4*sin(a + b*x)**2*cos(a + b*x)/(27*b**5) - 488*
d**4*cos(a + b*x)**3/(81*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**
2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*cos(a)**2, True))
```

Giac [A] time = 1.20536, size = 473, normalized size = 2.31

$$\frac{(27b^4d^4x^4 + 108b^4cd^3x^3 + 162b^4c^2d^2x^2 + 108b^4c^3dx + 27b^4c^4 - 36b^2d^4x^2 - 72b^2cd^3x - 36b^2c^2d^2 + 8d^4)\cos(3bx + 3a)}{324b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/324*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*
c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8
*d^4)*cos(3*b*x + 3*a)/b^5 - 1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2
*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b
^2*c^2*d^2 + 24*d^4)*cos(b*x + a)/b^5 + 1/27*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x
^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*sin(3*b*x + 3*a
)/b^5 + (b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b*
d^4*x - 6*b*c*d^3)*sin(b*x + a)/b^5
```

3.72 $\int (c + dx)^3 \cos^2(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=151

$$\frac{2d^2(c + dx) \cos^3(a + bx)}{9b^3} + \frac{4d^2(c + dx) \cos(a + bx)}{3b^3} + \frac{2d(c + dx)^2 \sin(a + bx)}{3b^2} + \frac{d(c + dx)^2 \sin(a + bx) \cos^2(a + bx)}{3b^2}$$

[Out] $(4*d^2*(c + d*x)*Cos[a + b*x])/(3*b^3) + (2*d^2*(c + d*x)*Cos[a + b*x]^3)/(9*b^3) - ((c + d*x)^3*Cos[a + b*x]^3)/(3*b) - (14*d^3*Sin[a + b*x])/(9*b^4) + (2*d*(c + d*x)^2*Sin[a + b*x])/(3*b^2) + (d*(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x])/(3*b^2) + (2*d^3*Sin[a + b*x]^3)/(27*b^4)$

Rubi [A] time = 0.13308, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4405, 3311, 3296, 2637, 2633}

$$\frac{2d^2(c + dx) \cos^3(a + bx)}{9b^3} + \frac{4d^2(c + dx) \cos(a + bx)}{3b^3} + \frac{2d(c + dx)^2 \sin(a + bx)}{3b^2} + \frac{d(c + dx)^2 \sin(a + bx) \cos^2(a + bx)}{3b^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x], x]

[Out] $(4*d^2*(c + d*x)*Cos[a + b*x])/(3*b^3) + (2*d^2*(c + d*x)*Cos[a + b*x]^3)/(9*b^3) - ((c + d*x)^3*Cos[a + b*x]^3)/(3*b) - (14*d^3*Sin[a + b*x])/(9*b^4) + (2*d*(c + d*x)^2*Sin[a + b*x])/(3*b^2) + (d*(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x])/(3*b^2) + (2*d^3*Sin[a + b*x]^3)/(27*b^4)$

Rule 4405

Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 3311

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^3 \cos^2(a + bx) \sin(a + bx) dx &= -\frac{(c + dx)^3 \cos^3(a + bx)}{3b} + \frac{d \int (c + dx)^2 \cos^3(a + bx) dx}{b} \\ &= \frac{2d^2(c + dx) \cos^3(a + bx)}{9b^3} - \frac{(c + dx)^3 \cos^3(a + bx)}{3b} + \frac{d(c + dx)^2 \cos^2(a + bx) \sin(a + bx)}{3b^2} \\ &= \frac{2d^2(c + dx) \cos^3(a + bx)}{9b^3} - \frac{(c + dx)^3 \cos^3(a + bx)}{3b} + \frac{2d(c + dx)^2 \sin(a + bx)}{3b^2} + \frac{d^2 \sin^2(a + bx)}{3b} \\ &= \frac{4d^2(c + dx) \cos(a + bx)}{3b^3} + \frac{2d^2(c + dx) \cos^3(a + bx)}{9b^3} - \frac{(c + dx)^3 \cos^3(a + bx)}{3b} \\ &= \frac{4d^2(c + dx) \cos(a + bx)}{3b^3} + \frac{2d^2(c + dx) \cos^3(a + bx)}{9b^3} - \frac{(c + dx)^3 \cos^3(a + bx)}{3b} \end{aligned}$$

Mathematica [A] time = 0.960798, size = 127, normalized size = 0.84

$$\frac{-27b(c + dx) \cos(a + bx) (b^2(c + dx)^2 - 6d^2) - 3b(c + dx) \cos(3(a + bx)) (3b^2(c + dx)^2 - 2d^2) + 2d \sin(a + bx) (\cos(2(a + bx)) - 1)}{108b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x], x]
```

```
[Out] (-27*b*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 3*b*(c + d*x)*(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 2*d*(-82*d^2 + 45*b^2*(c + d*x)^2 + (-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x]/(108*b^4)
```

Maple [B] time = 0.018, size = 447, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a), x)
```

```
[Out] 1/b*(1/b^3*d^3*(-1/3*(b*x+a)^3*cos(b*x+a)^3+1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/3*sin(b*x+a)+4/3*(b*x+a)*cos(b*x+a)+2/9*(b*x+a)*cos(b*x+a)^3-2/27*(2+cos(b*x+a)^2)*sin(b*x+a))-3/b^3*a*d^3*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2/9*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+2/27*cos(b*x+a)^3+4/9*cos(b*x+a))+3/b^2*c*d^2*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2/9*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+2/27*cos(b*x+a)^3+4/9*cos(b*x+a))+3/b^3*a^2*d^3*(-1/3*(b*x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))-6/b^2*a*c*d^2*(-1/3*(b*x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))+3/b*c^2*d*(-1/3*(b*x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))+1/3/b^3*a^3*d^3*cos(b*x+a)^3-1/b^2*a^2*c*d^2*cos(b*x+a)^3+1/b*a*c^2*d*cos(b*x+a)^3-1/3*c^3*cos(b*x+a)^3)
```

Maxima [B] time = 1.17515, size = 682, normalized size = 4.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")

[Out]
$$-1/108*(36*c^3*\cos(b*x + a)^3 - 108*a*c^2*d*\cos(b*x + a)^3/b + 108*a^2*c*d^2*\cos(b*x + a)^3/b^2 - 36*a^3*d^3*\cos(b*x + a)^3/b^3 + 9*(3*(b*x + a)*\cos(3*b*x + 3*a) + 9*(b*x + a)*\cos(b*x + a) - \sin(3*b*x + 3*a) - 9*\sin(b*x + a))*c^2*d/b - 18*(3*(b*x + a)*\cos(3*b*x + 3*a) + 9*(b*x + a)*\cos(b*x + a) - \sin(3*b*x + 3*a) - 9*\sin(b*x + a))*a*c*d^2/b^2 + 9*(3*(b*x + a)*\cos(3*b*x + 3*a) + 9*(b*x + a)*\cos(b*x + a) - \sin(3*b*x + 3*a) - 9*\sin(b*x + a))*a^2*d^3/b^3 + 3*((9*(b*x + a)^2 - 2)*\cos(3*b*x + 3*a) + 27*((b*x + a)^2 - 2)*\cos(b*x + a) - 6*(b*x + a)*\sin(3*b*x + 3*a) - 54*(b*x + a)*\sin(b*x + a))*c*d^2/b^2 - 3*((9*(b*x + a)^2 - 2)*\cos(3*b*x + 3*a) + 27*((b*x + a)^2 - 2)*\cos(b*x + a) - 6*(b*x + a)*\sin(3*b*x + 3*a) - 54*(b*x + a)*\sin(b*x + a))*a*d^3/b^3 + (3*(3*(b*x + a)^3 - 2*b*x - 2*a)*\cos(3*b*x + 3*a) + 27*((b*x + a)^3 - 6*b*x - 6*a)*\cos(b*x + a) - (9*(b*x + a)^2 - 2)*\sin(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*\sin(b*x + a))*d^3/b^3)/b$$

Fricas [A] time = 0.496975, size = 404, normalized size = 2.68

$$\frac{3(3b^3d^3x^3 + 9b^3cd^2x^2 + 3b^3c^3 - 2bcd^2 + (9b^3c^2d - 2bd^3)x)\cos(bx + a)^3 - 36(bd^3x + bcd^2)\cos(bx + a) - (18b^2d^3x^2 + 36b^2cd^2x + 18b^2c^2d - 40d^3 + (9b^2d^3x^2 + 18b^2cd^2x + 9b^2c^2d - 2d^3)\cos(bx + a)^2)\sin(bx + a)}{27b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")

[Out]
$$-1/27*(3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 2*b*c*d^2 + (9*b^3*c^2*d - 2*b*d^3)*x)*\cos(b*x + a)^3 - 36*(b*d^3*x + b*c*d^2)*\cos(b*x + a) - (18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - 40*d^3 + (9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*\cos(b*x + a)^2)*\sin(b*x + a))/b^4$$

Sympy [A] time = 5.38453, size = 391, normalized size = 2.59

$$\left\{ \begin{array}{l} \frac{c^3 \cos^3(a+bx)}{3b} - \frac{c^2 dx \cos^3(a+bx)}{b} - \frac{cd^2 x^2 \cos^3(a+bx)}{b} - \frac{d^3 x^3 \cos^3(a+bx)}{3b} + \frac{2c^2 d \sin^3(a+bx)}{3b^2} + \frac{c^2 d \sin(a+bx) \cos^2(a+bx)}{b^2} + \frac{4cd^2 x \sin^3(a+bx)}{3b^2} \\ \left(c^3 x + \frac{3c^2 dx^2}{2} + cd^2 x^3 + \frac{d^3 x^4}{4} \right) \sin(a) \cos^2(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*cos(b*x+a)**2*sin(b*x+a),x)

[Out]
$$\text{Piecewise}((-c**3*\cos(a + b*x)**3/(3*b) - c**2*d*x*\cos(a + b*x)**3/b - c*d**2*x**2*\cos(a + b*x)**3/b - d**3*x**3*\cos(a + b*x)**3/(3*b) + 2*c**2*d*\sin(a + b*x)**3/(3*b**2) + c**2*d*\sin(a + b*x)*\cos(a + b*x)**2/b**2 + 4*c*d**2*x*\sin(a + b*x)**3/(3*b**2) + 2*c*d**2*x*\sin(a + b*x)*\cos(a + b*x)**2/b**2 + 2*d**3*x**2*\sin(a + b*x)**3/(3*b**2) + d**3*x**2*\sin(a + b*x)*\cos(a + b*x)**2/b**2 + 4*c*d**2*\sin(a + b*x)**2*\cos(a + b*x)/(3*b**3) + 14*c*d**2*\cos(a + b*x)**3/(9*b**3) + 4*d**3*x*\sin(a + b*x)**2*\cos(a + b*x)/(3*b**3) + 14*d**3*x*\cos(a + b*x)**3/(9*b**3) - 40*d**3*\sin(a + b*x)**3/(27*b**4) - 14*d**3$$

```
*sin(a + b*x)*cos(a + b*x)**2/(9*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2
/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)*cos(a)**2, True))
```

Giac [A] time = 1.20255, size = 312, normalized size = 2.07

$$\frac{(3b^3d^3x^3 + 9b^3cd^2x^2 + 9b^3c^2dx + 3b^3c^3 - 2bd^3x - 2bcd^2)\cos(3bx + 3a)}{36b^4} - \frac{(b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + b^3c^3 - 2bd^3x - 2bcd^2)\cos(bx + a)}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/36*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^
3*x - 2*b*c*d^2)*cos(3*b*x + 3*a)/b^4 - 1/4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
+ 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*cos(b*x + a)/b^4 + 1/108
*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*sin(3*b*x + 3*a)/b^
4 + 3/4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a)/b^4
```


3.73 $\int (c + dx)^2 \cos^2(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=103

$$\frac{4d(c + dx) \sin(a + bx)}{9b^2} + \frac{2d(c + dx) \sin(a + bx) \cos^2(a + bx)}{9b^2} + \frac{2d^2 \cos^3(a + bx)}{27b^3} + \frac{4d^2 \cos(a + bx)}{9b^3} - \frac{(c + dx)^2 \cos^3(a + bx)}{3b}$$

```
[Out] (4*d^2*Cos[a + b*x])/(9*b^3) + (2*d^2*Cos[a + b*x]^3)/(27*b^3) - ((c + d*x)^2*Cos[a + b*x]^3)/(3*b) + (4*d*(c + d*x)*Sin[a + b*x])/(9*b^2) + (2*d*(c + d*x)*Cos[a + b*x]^2*Sine[a + b*x])/(9*b^2)
```

Rubi [A] time = 0.0790108, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4405, 3310, 3296, 2638}

$$\frac{4d(c + dx) \sin(a + bx)}{9b^2} + \frac{2d(c + dx) \sin(a + bx) \cos^2(a + bx)}{9b^2} + \frac{2d^2 \cos^3(a + bx)}{27b^3} + \frac{4d^2 \cos(a + bx)}{9b^3} - \frac{(c + dx)^2 \cos^3(a + bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (4*d^2*Cos[a + b*x])/(9*b^3) + (2*d^2*Cos[a + b*x]^3)/(27*b^3) - ((c + d*x)^2*Cos[a + b*x]^3)/(3*b) + (4*d*(c + d*x)*Sin[a + b*x])/(9*b^2) + (2*d*(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(9*b^2)
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(d*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)*(b*Sine[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b*Sine[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2 \cos^2(a+bx) \sin(a+bx) dx &= -\frac{(c+dx)^2 \cos^3(a+bx)}{3b} + \frac{(2d) \int (c+dx) \cos^3(a+bx) dx}{3b} \\
&= \frac{2d^2 \cos^3(a+bx)}{27b^3} - \frac{(c+dx)^2 \cos^3(a+bx)}{3b} + \frac{2d(c+dx) \cos^2(a+bx) \sin(a+bx)}{9b^2} \\
&= \frac{2d^2 \cos^3(a+bx)}{27b^3} - \frac{(c+dx)^2 \cos^3(a+bx)}{3b} + \frac{4d(c+dx) \sin(a+bx)}{9b^2} + \frac{2d(c+dx) \cos^2(a+bx) \sin(a+bx)}{9b^2} \\
&= \frac{4d^2 \cos(a+bx)}{9b^3} + \frac{2d^2 \cos^3(a+bx)}{27b^3} - \frac{(c+dx)^2 \cos^3(a+bx)}{3b} + \frac{4d(c+dx) \sin(a+bx)}{9b^2}
\end{aligned}$$

Mathematica [A] time = 0.509733, size = 86, normalized size = 0.83

$$\frac{27 \cos(a+bx) (b^2(c+dx)^2 - 2d^2) + \cos(3(a+bx)) (9b^2(c+dx)^2 - 2d^2) - 6bd(c+dx)(9 \sin(a+bx) + \sin(3(a+bx)))}{108b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x], x]

[Out] -(27*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + (-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] - 6*b*d*(c + d*x)*(9*Sin[a + b*x] + Sin[3*(a + b*x)]))/(108*b^3)

Maple [B] time = 0.018, size = 204, normalized size = 2.

$$\frac{1}{b} \left(\frac{d^2}{b^2} \left(-\frac{(bx+a)^2 (\cos(bx+a))^3}{3} + \frac{(2bx+2a) (2 + (\cos(bx+a))^2) \sin(bx+a)}{9} + \frac{2 (\cos(bx+a))^3}{27} + \frac{4 \cos(bx+a)}{9} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a), x)

[Out] 1/b*(1/b^2*d^2*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2/9*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+2/27*cos(b*x+a)^3+4/9*cos(b*x+a))-2/b^2*a*d^2*(-1/3*(b*x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))+2/b*c*d*(-1/3*(b*x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))-1/3/b^2*a^2*d^2*cos(b*x+a)^3+2/3/b*a*c*d*cos(b*x+a)^3-1/3*c^2*cos(b*x+a)^3)

Maxima [B] time = 1.20785, size = 328, normalized size = 3.18

$$\frac{36c^2 \cos(bx+a)^3 - \frac{72acd \cos(bx+a)^3}{b} + \frac{36a^2d^2 \cos(bx+a)^3}{b^2} + \frac{6(3(bx+a) \cos(3bx+3a)+9(bx+a) \cos(bx+a)-\sin(3bx+3a)-9 \sin(bx+a))cd}{b} - \frac{6(3(bx+a) \cos(3bx+3a)+9(bx+a) \cos(bx+a)-\sin(3bx+3a)-9 \sin(bx+a))}{b}}{108b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a), x, algorithm="maxima")

[Out] -1/108*(36*c^2*cos(b*x + a)^3 - 72*a*c*d*cos(b*x + a)^3/b + 36*a^2*d^2*cos(b*x + a)^3/b^2 + 6*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*c*d/b - 6*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))

$$a) + 9*(b*x + a)*\cos(b*x + a) - \sin(3*b*x + 3*a) - 9*\sin(b*x + a))*a*d^2/b^2 + ((9*(b*x + a)^2 - 2)*\cos(3*b*x + 3*a) + 27*((b*x + a)^2 - 2)*\cos(b*x + a) - 6*(b*x + a)*\sin(3*b*x + 3*a) - 54*(b*x + a)*\sin(b*x + a))*d^2/b^2)/b$$

Fricas [A] time = 0.482611, size = 236, normalized size = 2.29

$$\frac{(9b^2d^2x^2 + 18b^2cdx + 9b^2c^2 - 2d^2)\cos(bx + a)^3 - 12d^2\cos(bx + a) - 6(2bd^2x + 2bcd + (bd^2x + bcd)\cos(bx + a))}{27b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")

[Out] -1/27*((9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(b*x + a)^3 - 12*d^2*cos(b*x + a) - 6*(2*b*d^2*x + 2*b*c*d + (b*d^2*x + b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/b^3

Sympy [A] time = 2.62664, size = 216, normalized size = 2.1

$$\left\{ \begin{array}{l} -\frac{c^2\cos^3(a+bx)}{3b} - \frac{2cdx\cos^3(a+bx)}{3b} - \frac{d^2x^2\cos^3(a+bx)}{3b} + \frac{4cd\sin^3(a+bx)}{9b^2} + \frac{2cd\sin(a+bx)\cos^2(a+bx)}{3b^2} + \frac{4d^2x\sin^3(a+bx)}{9b^2} + \frac{2d^2x\sin(a+bx)\cos^2(a+bx)}{3b^2} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin(a)\cos^2(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*cos(b*x+a)**2*sin(b*x+a),x)

[Out] Piecewise((-c**2*cos(a + b*x)**3/(3*b) - 2*c*d*x*cos(a + b*x)**3/(3*b) - d**2*x**2*cos(a + b*x)**3/(3*b) + 4*c*d*sin(a + b*x)**3/(9*b**2) + 2*c*d*sin(a + b*x)*cos(a + b*x)**2/(3*b**2) + 4*d**2*x*sin(a + b*x)**3/(9*b**2) + 2*d**2*x*sin(a + b*x)*cos(a + b*x)**2/(3*b**2) + 4*d**2*sin(a + b*x)**2*cos(a + b*x)/(9*b**3) + 14*d**2*cos(a + b*x)**3/(27*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)*cos(a)**2, True))

Giac [A] time = 1.15306, size = 185, normalized size = 1.8

$$\frac{(9b^2d^2x^2 + 18b^2cdx + 9b^2c^2 - 2d^2)\cos(3bx + 3a)}{108b^3} - \frac{(b^2d^2x^2 + 2b^2cdx + b^2c^2 - 2d^2)\cos(bx + a)}{4b^3} + \frac{(bd^2x + bcd)\cos(bx + a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")

[Out] -1/108*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(3*b*x + 3*a)/b^3 - 1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)/b^3 + 1/18*(b*d^2*x + b*c*d)*sin(3*b*x + 3*a)/b^3 + 1/2*(b*d^2*x + b*c*d)*sin(b*x + a)/b^3

3.74 $\int (c + dx) \cos^2(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=51

$$-\frac{d \sin^3(a + bx)}{9b^2} + \frac{d \sin(a + bx)}{3b^2} - \frac{(c + dx) \cos^3(a + bx)}{3b}$$

[Out] $-\frac{(c + d*x)*\text{Cos}[a + b*x]^3}{(3*b)} + \frac{(d*\text{Sin}[a + b*x])}{(3*b^2)} - \frac{(d*\text{Sin}[a + b*x]^3)}{(9*b^2)}$

Rubi [A] time = 0.0342699, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {4405, 2633}

$$-\frac{d \sin^3(a + bx)}{9b^2} + \frac{d \sin(a + bx)}{3b^2} - \frac{(c + dx) \cos^3(a + bx)}{3b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x], x]

[Out] $-\frac{(c + d*x)*\text{Cos}[a + b*x]^3}{(3*b)} + \frac{(d*\text{Sin}[a + b*x])}{(3*b^2)} - \frac{(d*\text{Sin}[a + b*x]^3)}{(9*b^2)}$

Rule 4405

Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*cos[a + b*x]^(n + 1))/(b*(n + 1)), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rubi steps

$$\begin{aligned} \int (c + dx) \cos^2(a + bx) \sin(a + bx) dx &= -\frac{(c + dx) \cos^3(a + bx)}{3b} + \frac{d \int \cos^3(a + bx) dx}{3b} \\ &= -\frac{(c + dx) \cos^3(a + bx)}{3b} - \frac{d \text{Subst}\left(\int (1 - x^2) dx, x, -\sin(a + bx)\right)}{3b^2} \\ &= -\frac{(c + dx) \cos^3(a + bx)}{3b} + \frac{d \sin(a + bx)}{3b^2} - \frac{d \sin^3(a + bx)}{9b^2} \end{aligned}$$

Mathematica [A] time = 0.153405, size = 71, normalized size = 1.39

$$\frac{d(\sin(a + bx) - bx \cos(a + bx))}{4b^2} + \frac{d(\sin(3(a + bx)) - 3bx \cos(3(a + bx)))}{36b^2} - \frac{c \cos^3(a + bx)}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x], x]

[Out] $-(c \cos[a + b*x]^3)/(3*b) + (d*(-(b*x*\cos[a + b*x]) + \sin[a + b*x]))/(4*b^2) + (d*(-3*b*x*\cos[3*(a + b*x)] + \sin[3*(a + b*x)]))/(36*b^2)$

Maple [A] time = 0.018, size = 71, normalized size = 1.4

$$\frac{1}{b} \left(\frac{d}{b} \left(-\frac{(bx+a)(\cos(bx+a))^3}{3} + \frac{(2+(\cos(bx+a))^2)\sin(bx+a)}{9} \right) + \frac{ad(\cos(bx+a))^3}{3b} - \frac{(\cos(bx+a))^3 c}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x)`

[Out] $1/b*(d/b*(-1/3*(b*x+a)*\cos(b*x+a)^3+1/9*(2+\cos(b*x+a)^2)*\sin(b*x+a))+1/3/b*d*a*\cos(b*x+a)^3-1/3*\cos(b*x+a)^3*c)$

Maxima [A] time = 1.11705, size = 116, normalized size = 2.27

$$\frac{12c \cos(bx+a)^3 - \frac{12ad \cos(bx+a)^3}{b} + \frac{(3(bx+a)\cos(3bx+3a)+9(bx+a)\cos(bx+a)-\sin(3bx+3a)-9\sin(bx+a))d}{b}}{36b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")`

[Out] $-1/36*(12*c*\cos(b*x + a)^3 - 12*a*d*\cos(b*x + a)^3/b + (3*(b*x + a)*\cos(3*b*x + 3*a) + 9*(b*x + a)*\cos(b*x + a) - \sin(3*b*x + 3*a) - 9*\sin(b*x + a))*d/b)/b$

Fricas [A] time = 0.474018, size = 112, normalized size = 2.2

$$\frac{3(bdx+bc)\cos(bx+a)^3 - (d\cos(bx+a)^2 + 2d)\sin(bx+a)}{9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")`

[Out] $-1/9*(3*(b*d*x + b*c)*\cos(b*x + a)^3 - (d*\cos(b*x + a)^2 + 2*d)*\sin(b*x + a))/b^2$

Sympy [A] time = 1.13508, size = 85, normalized size = 1.67

$$\begin{cases} -\frac{c \cos^3(a+bx)}{3b} - \frac{dx \cos^3(a+bx)}{3b} + \frac{2d \sin^3(a+bx)}{9b^2} + \frac{d \sin(a+bx) \cos^2(a+bx)}{3b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2} \right) \sin(a) \cos^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*cos(b*x+a)**2*sin(b*x+a),x)`

```
[Out] Piecewise((-c*cos(a + b*x)**3/(3*b) - d*x*cos(a + b*x)**3/(3*b) + 2*d*sin(a
+ b*x)**3/(9*b**2) + d*sin(a + b*x)*cos(a + b*x)**2/(3*b**2), Ne(b, 0)), (
(c*x + d*x**2/2)*sin(a)*cos(a)**2, True))
```

Giac [A] time = 1.20394, size = 93, normalized size = 1.82

$$-\frac{(bdx + bc) \cos(3bx + 3a)}{12b^2} - \frac{(bdx + bc) \cos(bx + a)}{4b^2} + \frac{d \sin(3bx + 3a)}{36b^2} + \frac{d \sin(bx + a)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/12*(b*d*x + b*c)*cos(3*b*x + 3*a)/b^2 - 1/4*(b*d*x + b*c)*cos(b*x + a)/b
^2 + 1/36*d*sin(3*b*x + 3*a)/b^2 + 1/4*d*sin(b*x + a)/b^2
```

$$3.75 \quad \int \frac{\cos^2(a+bx) \sin(a+bx)}{c+dx} dx$$

Optimal. Leaf size=121

$$\frac{\sin\left(3a - \frac{3bc}{d}\right) \operatorname{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{4d} + \frac{\sin\left(a - \frac{bc}{d}\right) \operatorname{CosIntegral}\left(\frac{bc}{d} + bx\right)}{4d} + \frac{\cos\left(a - \frac{bc}{d}\right) \operatorname{Si}\left(\frac{bc}{d} + bx\right)}{4d} + \frac{\cos\left(3a - \frac{3bc}{d}\right)}{4d}$$

[Out] (CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(4*d) + (CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(4*d) + (Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(4*d) + (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(4*d)

Rubi [A] time = 0.22444, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4406, 3303, 3299, 3302}

$$\frac{\sin\left(3a - \frac{3bc}{d}\right) \operatorname{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{4d} + \frac{\sin\left(a - \frac{bc}{d}\right) \operatorname{CosIntegral}\left(\frac{bc}{d} + bx\right)}{4d} + \frac{\cos\left(a - \frac{bc}{d}\right) \operatorname{Si}\left(\frac{bc}{d} + bx\right)}{4d} + \frac{\cos\left(3a - \frac{3bc}{d}\right)}{4d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x),x]

[Out] (CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(4*d) + (CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(4*d) + (Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(4*d) + (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(4*d)

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx} dx &= \int \left(\frac{\sin(a+bx)}{4(c+dx)} + \frac{\sin(3a+3bx)}{4(c+dx)} \right) dx \\
&= \frac{1}{4} \int \frac{\sin(a+bx)}{c+dx} dx + \frac{1}{4} \int \frac{\sin(3a+3bx)}{c+dx} dx \\
&= \frac{1}{4} \cos\left(3a - \frac{3bc}{d}\right) \int \frac{\sin\left(\frac{3bc}{d} + 3bx\right)}{c+dx} dx + \frac{1}{4} \cos\left(a - \frac{bc}{d}\right) \int \frac{\sin\left(\frac{bc}{d} + bx\right)}{c+dx} dx + \frac{1}{4} \sin\left(3a - \frac{3bc}{d}\right) \\
&= \frac{\text{Ci}\left(\frac{3bc}{d} + 3bx\right) \sin\left(3a - \frac{3bc}{d}\right)}{4d} + \frac{\text{Ci}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{4d} + \frac{\cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{4d} + \dots
\end{aligned}$$

Mathematica [A] time = 0.304082, size = 100, normalized size = 0.83

$$\frac{\sin\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right) + \sin\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(b\left(\frac{c}{d} + x\right)\right) + \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(b\left(\frac{c}{d} + x\right)\right) + \cos\left(3a - \frac{3bc}{d}\right)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x), x]

[Out] (CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + CosIntegral[b*(c/d + x)]*Sin[a - (b*c)/d] + Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d)

Maple [A] time = 0.02, size = 167, normalized size = 1.4

$$\frac{1}{b} \left(\frac{b}{12} \left(3 \frac{1}{d} \text{Si} \left(3bx + 3a + 3 \frac{-ad+bc}{d} \right) \cos \left(3 \frac{-ad+bc}{d} \right) - 3 \frac{1}{d} \text{Ci} \left(3bx + 3a + 3 \frac{-ad+bc}{d} \right) \sin \left(3 \frac{-ad+bc}{d} \right) \right) + \frac{b}{4} \left(\frac{1}{d} \text{Si} \left(b \left(\frac{c}{d} + x \right) \right) \cos \left(a - \frac{bc}{d} \right) + \frac{1}{d} \text{Ci} \left(b \left(\frac{c}{d} + x \right) \right) \sin \left(a - \frac{bc}{d} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c), x)

[Out] 1/b*(1/12*b*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)+1/4*b*(Si(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)

Maxima [C] time = 1.38316, size = 369, normalized size = 3.05

$$\frac{b \left(i E_1 \left(\frac{i bc + i (bx+a)d - i ad}{d} \right) - i E_1 \left(-\frac{i bc + i (bx+a)d - i ad}{d} \right) \right) \cos \left(-\frac{bc-ad}{d} \right) + b \left(i E_1 \left(\frac{3i bc + 3i (bx+a)d - 3i ad}{d} \right) - i E_1 \left(-\frac{3i bc + 3i (bx+a)d - 3i ad}{d} \right) \right) \cos \left(-\frac{3(bc-ad)}{d} \right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c), x, algorithm="maxima")

[Out] -1/8*(b*(I*exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b*(I*exp_integral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_integral_e(1, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d) + b*

$(\exp_{\text{integral_e}}(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + \exp_{\text{integral_e}}(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*\sin(-(b*c - a*d)/d) + b*(\exp_{\text{integral_e}}(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + \exp_{\text{integral_e}}(1, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*\sin(-3*(b*c - a*d)/d))/(b*d)$

Fricas [A] time = 0.481807, size = 404, normalized size = 3.34

$$\frac{\left(\text{Ci}\left(\frac{bdx+bc}{d}\right) + \text{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right) + \left(\text{Ci}\left(\frac{3(bdx+bc)}{d}\right) + \text{Ci}\left(-\frac{3(bdx+bc)}{d}\right)\right)\sin\left(-\frac{3(bc-ad)}{d}\right) + 2\cos\left(-\frac{3(bc-ad)}{d}\right)\text{Si}\left(\frac{3(bdx+bc)}{d}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x, algorithm="fricas")

[Out] 1/8*((cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/d) + (cos_integral(3*(b*d*x + b*c)/d) + cos_integral(-3*(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d) + 2*cos(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) + 2*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a + bx) \cos^2(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c),x)

[Out] Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x), x)

Giac [C] time = 1.77063, size = 8477, normalized size = 70.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x, algorithm="giac")

[Out] 1/8*(imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(b*x + b*c/d))*tan(

$$\begin{aligned}
& 3/2*a)^2*\tan(1/2*a)*\tan(3/2*b*c/d)^2*\tan(1/2*b*c/d)^2 - 2*\text{real_part}(\text{cos_int} \\
& \text{egral}(-b*x - b*c/d))*\tan(3/2*a)^2*\tan(1/2*a)*\tan(3/2*b*c/d)^2*\tan(1/2*b*c/d \\
&)^2 - 2*\text{real_part}(\text{cos_integral}(3*b*x + 3*b*c/d))*\tan(3/2*a)*\tan(1/2*a)^2*\tan \\
& (3/2*b*c/d)^2*\tan(1/2*b*c/d)^2 - 2*\text{real_part}(\text{cos_integral}(-3*b*x - 3*b*c/d \\
&))*\tan(3/2*a)*\tan(1/2*a)^2*\tan(3/2*b*c/d)^2*\tan(1/2*b*c/d)^2 + \text{imag_part}(\text{co} \\
& \text{s_integral}(3*b*x + 3*b*c/d))*\tan(3/2*a)^2*\tan(1/2*a)^2*\tan(3/2*b*c/d)^2 - i \\
& \text{mag_part}(\text{cos_integral}(b*x + b*c/d))*\tan(3/2*a)^2*\tan(1/2*a)^2*\tan(3/2*b*c/d \\
&)^2 + \text{imag_part}(\text{cos_integral}(-b*x - b*c/d))*\tan(3/2*a)^2*\tan(1/2*a)^2*\tan(3 \\
& /2*b*c/d)^2 - \text{imag_part}(\text{cos_integral}(-3*b*x - 3*b*c/d))*\tan(3/2*a)^2*\tan(1/ \\
& 2*a)^2*\tan(3/2*b*c/d)^2 + 2*\text{sin_integral}(3*(b*d*x + b*c)/d)*\tan(3/2*a)^2*\tan \\
& (1/2*a)^2*\tan(3/2*b*c/d)^2 - 2*\text{sin_integral}((b*d*x + b*c)/d)*\tan(3/2*a)^2* \\
& \tan(1/2*a)^2*\tan(3/2*b*c/d)^2 + 4*\text{imag_part}(\text{cos_integral}(b*x + b*c/d))*\tan(\\
& 3/2*a)^2*\tan(1/2*a)*\tan(3/2*b*c/d)^2*\tan(1/2*b*c/d) - 4*\text{imag_part}(\text{cos_integ} \\
& \text{ral}(-b*x - b*c/d))*\tan(3/2*a)^2*\tan(1/2*a)*\tan(3/2*b*c/d)^2*\tan(1/2*b*c/d) \\
& + 8*\text{sin_integral}((b*d*x + b*c)/d)*\tan(3/2*a)^2*\tan(1/2*a)*\tan(3/2*b*c/d)^2* \\
& \tan(1/2*b*c/d) - \text{imag_part}(\text{cos_integral}(3*b*x + 3*b*c/d))*\tan(3/2*a)^2*\tan(\\
& 1/2*a)^2*\tan(1/2*b*c/d)^2 + \text{imag_part}(\text{cos_integral}(b*x + b*c/d))*\tan(3/2*a) \\
& ^2*\tan(1/2*a)^2*\tan(1/2*b*c/d)^2 - \text{imag_part}(\text{cos_integral}(-b*x - b*c/d))*\tan \\
& (3/2*a)^2*\tan(1/2*a)^2*\tan(1/2*b*c/d)^2 + \text{imag_part}(\text{cos_integral}(-3*b*x - \\
& 3*b*c/d))*\tan(3/2*a)^2*\tan(1/2*a)^2*\tan(1/2*b*c/d)^2 - 2*\text{sin_integral}(3*(b* \\
& d*x + b*c)/d)*\tan(3/2*a)^2*\tan(1/2*a)^2*\tan(1/2*b*c/d)^2 + 2*\text{sin_integral}((\\
& b*d*x + b*c)/d)*\tan(3/2*a)^2*\tan(1/2*a)^2*\tan(1/2*b*c/d)^2 + 4*\text{imag_part}(\text{co} \\
& \text{s_integral}(3*b*x + 3*b*c/d))*\tan(3/2*a)*\tan(1/2*a)^2*\tan(3/2*b*c/d)*\tan(1/2 \\
& *b*c/d)^2 - 4*\text{imag_part}(\text{cos_integral}(-3*b*x - 3*b*c/d))*\tan(3/2*a)*\tan(1/2* \\
& a)^2*\tan(3/2*b*c/d)*\tan(1/2*b*c/d)^2 + 8*\text{sin_integral}(3*(b*d*x + b*c)/d)*\tan \\
& (3/2*a)*\tan(1/2*a)^2*\tan(3/2*b*c/d)*\tan(1/2*b*c/d)^2 + \text{imag_part}(\text{cos_integ} \\
& \text{ral}(3*b*x + 3*b*c/d))*\tan(3/2*a)^2*\tan(3/2*b*c/d)^2*\tan(1/2*b*c/d)^2 - \text{imag} \\
& _ \text{part}(\text{cos_integral}(b*x + b*c/d))*\tan(3/2*a)^2*\tan(3/2*b*c/d)^2*\tan(1/2*b*c/d \\
&)^2 + \text{imag_part}(\text{cos_integral}(-b*x - b*c/d))*\tan(3/2*a)^2*\tan(3/2*b*c/d)^2* \\
& \tan(1/2*b*c/d)^2 - \text{imag_part}(\text{cos_integral}(-3*b*x - 3*b*c/d))*\tan(3/2*a)^2*\tan \\
& (3/2*b*c/d)^2*\tan(1/2*b*c/d)^2 + 2*\text{sin_integral}(3*(b*d*x + b*c)/d)*\tan(3/ \\
& 2*a)^2*\tan(3/2*b*c/d)^2*\tan(1/2*b*c/d)^2 - 2*\text{sin_integral}((b*d*x + b*c)/d)* \\
& \tan(3/2*a)^2*\tan(3/2*b*c/d)^2*\tan(1/2*b*c/d)^2 - \text{imag_part}(\text{cos_integral}(3*b \\
& *x + 3*b*c/d))*\tan(1/2*a)^2*\tan(3/2*b*c/d)^2*\tan(1/2*b*c/d)^2 + \text{imag_part}(\text{c} \\
& \text{os_integral}(b*x + b*c/d))*\tan(1/2*a)^2*\tan(3/2*b*c/d)^2*\tan(1/2*b*c/d)^2 - \\
& \text{imag_part}(\text{cos_integral}(-b*x - b*c/d))*\tan(1/2*a)^2*\tan(3/2*b*c/d)^2*\tan(1/2 \\
& *b*c/d)^2 + \text{imag_part}(\text{cos_integral}(-3*b*x - 3*b*c/d))*\tan(1/2*a)^2*\tan(3/2* \\
& b*c/d)^2*\tan(1/2*b*c/d)^2 - 2*\text{sin_integral}(3*(b*d*x + b*c)/d)*\tan(1/2*a)^2* \\
& \tan(3/2*b*c/d)^2*\tan(1/2*b*c/d)^2 + 2*\text{sin_integral}((b*d*x + b*c)/d)*\tan(1/2 \\
& *a)^2*\tan(3/2*b*c/d)^2*\tan(1/2*b*c/d)^2 + 2*\text{real_part}(\text{cos_integral}(3*b*x + \\
& 3*b*c/d))*\tan(3/2*a)^2*\tan(1/2*a)^2*\tan(3/2*b*c/d) + 2*\text{real_part}(\text{cos_integr} \\
& \text{al}(-3*b*x - 3*b*c/d))*\tan(3/2*a)^2*\tan(1/2*a)^2*\tan(3/2*b*c/d) + 2*\text{real_par} \\
& \text{t}(\text{cos_integral}(b*x + b*c/d))*\tan(3/2*a)^2*\tan(1/2*a)*\tan(3/2*b*c/d)^2 + 2*r \\
& \text{eal_part}(\text{cos_integral}(-b*x - b*c/d))*\tan(3/2*a)^2*\tan(1/2*a)*\tan(3/2*b*c/d) \\
& ^2 - 2*\text{real_part}(\text{cos_integral}(3*b*x + 3*b*c/d))*\tan(3/2*a)*\tan(1/2*a)^2*\tan \\
& (3/2*b*c/d)^2 - 2*\text{real_part}(\text{cos_integral}(-3*b*x - 3*b*c/d))*\tan(3/2*a)*\tan(\\
& 1/2*a)^2*\tan(3/2*b*c/d)^2 + 2*\text{real_part}(\text{cos_integral}(b*x + b*c/d))*\tan(3/2* \\
& a)^2*\tan(1/2*a)^2*\tan(1/2*b*c/d) + 2*\text{real_part}(\text{cos_integral}(-b*x - b*c/d))* \\
& \tan(3/2*a)^2*\tan(1/2*a)^2*\tan(1/2*b*c/d) - 2*\text{real_part}(\text{cos_integral}(b*x + b \\
& *c/d))*\tan(3/2*a)^2*\tan(3/2*b*c/d)^2*\tan(1/2*b*c/d) - 2*\text{real_part}(\text{cos_integ} \\
& \text{ral}(-b*x - b*c/d))*\tan(3/2*a)^2*\tan(3/2*b*c/d)^2*\tan(1/2*b*c/d) + 2*\text{real_pa} \\
& \text{rt}(\text{cos_integral}(b*x + b*c/d))*\tan(1/2*a)^2*\tan(3/2*b*c/d)^2*\tan(1/2*b*c/d) \\
& + 2*\text{real_part}(\text{cos_integral}(-b*x - b*c/d))*\tan(1/2*a)^2*\tan(3/2*b*c/d)^2*\tan \\
& (1/2*b*c/d) - 2*\text{real_part}(\text{cos_integral}(b*x + b*c/d))*\tan(3/2*a)^2*\tan(1/2*a) \\
&)*\tan(1/2*b*c/d)^2 - 2*\text{real_part}(\text{cos_integral}(-b*x - b*c/d))*\tan(3/2*a)^2*\tan \\
& (1/2*a)*\tan(1/2*b*c/d)^2 + 2*\text{real_part}(\text{cos_integral}(3*b*x + 3*b*c/d))*\tan \\
& (3/2*a)*\tan(1/2*a)^2*\tan(1/2*b*c/d)^2 + 2*\text{real_part}(\text{cos_integral}(-3*b*x - 3 \\
& *b*c/d))*\tan(3/2*a)*\tan(1/2*a)^2*\tan(1/2*b*c/d)^2 + 2*\text{real_part}(\text{cos_integra} \\
& \text{l}(3*b*x + 3*b*c/d))*\tan(3/2*a)^2*\tan(3/2*b*c/d)*\tan(1/2*b*c/d)^2 + 2*\text{real_p}
\end{aligned}$$

$$\begin{aligned}
& /2*b*c/d) + 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/ \\
& 2*b*c/d) - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2* \\
& b*c/d) - 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b \\
& *c/d) - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d \\
&)^2 - 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) \\
& ^2 + 2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2 + 2 \\
& *real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2 - 2*real \\
& _part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d) - 2*real_part(\\
& cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d) + 2*real_part(cos_i \\
& ntegral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*real_part(cos_integra \\
& l(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*real_part(cos_integral(b*x \\
& + b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*real_part(cos_integral(-b*x \\
& - b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*real_part(cos_integral(3*b*x \\
& + 3*b*c/d))*tan(3/2*a)*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(-3*b*x - \\
& 3*b*c/d))*tan(3/2*a)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(b*x + b*c \\
& /d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-b*x - b*c/d))* \\
& tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*ta \\
& n(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-3*b*x - 3*b*c/d)) \\
& *tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - imag_part(cos_integral(3*b*x + 3*b*c/d)) \\
& *tan(3/2*a)^2 + imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2 - imag_pa \\
& rt(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2 + imag_part(cos_integral(-3*b*x \\
& - 3*b*c/d))*tan(3/2*a)^2 - 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2 \\
& + 2*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2 + imag_part(cos_integral(3*b \\
& *x + 3*b*c/d))*tan(1/2*a)^2 - imag_part(cos_integral(b*x + b*c/d))*tan(1/2* \\
& a)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 - imag_part(cos_i \\
& ntegral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2 + 2*sin_integral(3*(b*d*x + b*c)/d) \\
& *tan(1/2*a)^2 - 2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2 + 4*imag_part(\\
& cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) - 4*imag_part(cos_ \\
& integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) + 8*sin_integral(3*(b \\
& *d*x + b*c)/d)*tan(3/2*a)*tan(3/2*b*c/d) - imag_part(cos_integral(3*b*x + 3 \\
& *b*c/d))*tan(3/2*b*c/d)^2 + imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b* \\
& c/d)^2 - imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*c/d)^2 + imag_part \\
& (cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2 - 2*sin_integral(3*(b*d*x \\
& + b*c)/d)*tan(3/2*b*c/d)^2 + 2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*c/d \\
&)^2 + 4*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) - 4* \\
& imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + 8*sin_int \\
& egral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d) + imag_part(cos_integral(3 \\
& *b*x + 3*b*c/d))*tan(1/2*b*c/d)^2 - imag_part(cos_integral(b*x + b*c/d))*ta \\
& n(1/2*b*c/d)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 - i \\
& mag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*c/d)^2 + 2*sin_integral(\\
& 3*(b*d*x + b*c)/d)*tan(1/2*b*c/d)^2 - 2*sin_integral((b*d*x + b*c)/d)*tan(1 \\
& /2*b*c/d)^2 + 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a) + 2*rea \\
& l_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a) + 2*real_part(cos_integra \\
& l(b*x + b*c/d))*tan(1/2*a) + 2*real_part(cos_integral(-b*x - b*c/d))*tan(1/ \\
& 2*a) - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d) - 2*real_p \\
& art(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d) - 2*real_part(cos_integr \\
& al(b*x + b*c/d))*tan(1/2*b*c/d) - 2*real_part(cos_integral(-b*x - b*c/d))*t \\
& an(1/2*b*c/d) + imag_part(cos_integral(3*b*x + 3*b*c/d)) + imag_part(cos_in \\
& tegral(b*x + b*c/d)) - imag_part(cos_integral(-b*x - b*c/d)) - imag_part(co \\
& s_integral(-3*b*x - 3*b*c/d)) + 2*sin_integral(3*(b*d*x + b*c)/d) + 2*sin_i \\
& ntegral((b*d*x + b*c)/d))/(d*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan \\
& (1/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + d*tan(3/2*a) \\
& ^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2* \\
& b*c/d)^2 + d*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a) \\
& ^2*tan(1/2*a)^2 + d*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + d*tan(1/2*a)^2*tan(3/2*b \\
& *c/d)^2 + d*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(1/2*a)^2*tan(1/2*b*c/d)^2 \\
& + d*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^2 + d*tan(1/2*a)^2 + \\
& d*tan(3/2*b*c/d)^2 + d*tan(1/2*b*c/d)^2 + d)
\end{aligned}$$

$$3.76 \quad \int \frac{\cos^2(a+bx) \sin(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=168

$$\frac{b \cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{4d^2} + \frac{3b \cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{4d^2} - \frac{b \sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{4d^2} - \frac{3b \sin\left(3a - \frac{3bc}{d}\right) \text{Si}\left(\frac{3bc}{d} + 3bx\right)}{4d^2}$$

[Out] (b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d^2) + (3*b*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d^2) - Sin[a + b*x]/(4*d*(c + d*x)) - Sin[3*a + 3*b*x]/(4*d*(c + d*x)) - (b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(4*d^2) - (3*b*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(4*d^2)

Rubi [A] time = 0.266386, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b \cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{4d^2} + \frac{3b \cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{4d^2} - \frac{b \sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{4d^2} - \frac{3b \sin\left(3a - \frac{3bc}{d}\right) \text{Si}\left(\frac{3bc}{d} + 3bx\right)}{4d^2}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^2, x]

[Out] (b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d^2) + (3*b*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d^2) - Sin[a + b*x]/(4*d*(c + d*x)) - Sin[3*a + 3*b*x]/(4*d*(c + d*x)) - (b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(4*d^2) - (3*b*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(4*d^2)

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n * Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

`Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]`

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(a + bx) \sin(a + bx)}{(c + dx)^2} dx &= \int \left(\frac{\sin(a + bx)}{4(c + dx)^2} + \frac{\sin(3a + 3bx)}{4(c + dx)^2} \right) dx \\ &= \frac{1}{4} \int \frac{\sin(a + bx)}{(c + dx)^2} dx + \frac{1}{4} \int \frac{\sin(3a + 3bx)}{(c + dx)^2} dx \\ &= -\frac{\sin(a + bx)}{4d(c + dx)} - \frac{\sin(3a + 3bx)}{4d(c + dx)} + \frac{b \int \frac{\cos(a+bx)}{c+dx} dx}{4d} + \frac{(3b) \int \frac{\cos(3a+3bx)}{c+dx} dx}{4d} \\ &= -\frac{\sin(a + bx)}{4d(c + dx)} - \frac{\sin(3a + 3bx)}{4d(c + dx)} + \frac{\left(3b \cos\left(3a - \frac{3bc}{d}\right)\right) \int \frac{\cos\left(\frac{3bc}{d} + 3bx\right)}{c+dx} dx}{4d} + \frac{\left(b \cos\left(a - \frac{bc}{d}\right)\right) \int \frac{\cos\left(\frac{bc}{d} + bx\right)}{c+dx} dx}{4d} \\ &= \frac{b \cos\left(a - \frac{bc}{d}\right) \text{Ci}\left(\frac{bc}{d} + bx\right)}{4d^2} + \frac{3b \cos\left(3a - \frac{3bc}{d}\right) \text{Ci}\left(\frac{3bc}{d} + 3bx\right)}{4d^2} - \frac{\sin(a + bx)}{4d(c + dx)} - \frac{\sin(3a + 3bx)}{4d(c + dx)} \end{aligned}$$

Mathematica [A] time = 1.13457, size = 139, normalized size = 0.83

$$\frac{-b \cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(b\left(\frac{c}{d} + x\right)\right) - 3b \cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right) + b \sin\left(a - \frac{bc}{d}\right) \text{Si}\left(b\left(\frac{c}{d} + x\right)\right) + 3b \sin\left(3a - \frac{3bc}{d}\right) \text{Si}\left(\frac{3b(c+dx)}{d}\right)}{4d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^2,x]

[Out] -(-(b*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)]) - 3*b*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*(c + d*x))/d] + (d*Sin[a + b*x])/(c + d*x) + (d*Sin[3*(a + b*x)])/(c + d*x) + b*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 3*b*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d^2)

Maple [A] time = 0.022, size = 240, normalized size = 1.4

$$\frac{1}{b} \left(\frac{b^2}{12} \left(-3 \frac{\sin(3bx + 3a)}{((bx + a)d - ad + bc)d} + 3 \frac{1}{d} \left(3 \frac{1}{d} \text{Si}\left(3bx + 3a + 3 \frac{-ad + bc}{d}\right) \sin\left(3 \frac{-ad + bc}{d}\right) + 3 \frac{1}{d} \text{Ci}\left(3bx + 3a + 3 \frac{-ad + bc}{d}\right) \cos\left(3 \frac{-ad + bc}{d}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x)

[Out] 1/b*(1/12*b^2*(-3*sin(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)/d+3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d)+1/4*b^2*(-sin(b*x+a)/((b*x+a)*d-a*d+b*c)/d+(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d/d))

Maxima [C] time = 1.72511, size = 405, normalized size = 2.41

$$b^2 \left(i E_2 \left(\frac{ibc+i(bx+a)d-iad}{d} \right) - i E_2 \left(-\frac{ibc+i(bx+a)d-iad}{d} \right) \right) \cos \left(-\frac{bc-ad}{d} \right) + b^2 \left(i E_2 \left(\frac{3ibc+3i(bx+a)d-3iad}{d} \right) - i E_2 \left(-\frac{3ibc+3i(bx+a)d-3iad}{d} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")

[Out] $-1/8*(b^2*(I*\exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*\exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*\cos(-(b*c - a*d)/d) + b^2*(I*\exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*\exp_integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*\cos(-3*(b*c - a*d)/d) + b^2*(\exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + \exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*\sin(-(b*c - a*d)/d) + b^2*(\exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + \exp_integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*\sin(-3*(b*c - a*d)/d)/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)$

Fricas [A] time = 0.538292, size = 586, normalized size = 3.49

$$8d \cos(bx+a)^2 \sin(bx+a) + 6(bdx+bc) \sin\left(-\frac{3(bc-ad)}{d}\right) \text{Si}\left(\frac{3(bdx+bc)}{d}\right) + 2(bdx+bc) \sin\left(-\frac{bc-ad}{d}\right) \text{Si}\left(\frac{bdx+bc}{d}\right) - \left(b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")

[Out] $-1/8*(8*d*\cos(b*x + a)^2*\sin(b*x + a) + 6*(b*d*x + b*c)*\sin(-3*(b*c - a*d)/d)*\sin_integral(3*(b*d*x + b*c)/d) + 2*(b*d*x + b*c)*\sin(-(b*c - a*d)/d)*\sin_integral((b*d*x + b*c)/d) - ((b*d*x + b*c)*\cos_integral((b*d*x + b*c)/d) + (b*d*x + b*c)*\cos_integral(-(b*d*x + b*c)/d))*\cos(-(b*c - a*d)/d) - 3*((b*d*x + b*c)*\cos_integral(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*\cos_integral(-3*(b*d*x + b*c)/d))*\cos(-3*(b*c - a*d)/d)/((d^3*x + c*d^2)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx) \cos^2(a+bx)}{(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c)**2,x)

[Out] Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^2 \sin(bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2*sin(b*x + a)/(d*x + c)^2, x)
```


$$3.77 \quad \int \frac{\cos^2(a+bx) \sin(a+bx)}{(c+dx)^3} dx$$

Optimal. Leaf size=221

$$\frac{9b^2 \sin\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{8d^3} - \frac{b^2 \sin\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{8d^3} - \frac{b^2 \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{8d^3}$$

[Out] $-(b \cos[a + b x]) / (8 d^2 (c + d x)) - (3 b \cos[3 a + 3 b x]) / (8 d^2 (c + d x)) - (9 b^2 \cos \text{Integral}[(3 b c) / d + 3 b x] \sin[3 a - (3 b c) / d]) / (8 d^3) - (b^2 \cos \text{Integral}[(b c) / d + b x] \sin[a - (b c) / d]) / (8 d^3) - \sin[a + b x] / (8 d (c + d x)^2) - \sin[3 a + 3 b x] / (8 d (c + d x)^2) - (b^2 \cos[a - (b c) / d] \text{SinIntegral}[(b c) / d + b x]) / (8 d^3) - (9 b^2 \cos[3 a - (3 b c) / d] \text{SinIntegral}[(3 b c) / d + 3 b x]) / (8 d^3)$

Rubi [A] time = 0.324418, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{9b^2 \sin\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{8d^3} - \frac{b^2 \sin\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{8d^3} - \frac{b^2 \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{8d^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\cos[a + b x])^2 \sin[a + b x] / (c + d x)^3, x]$

[Out] $-(b \cos[a + b x]) / (8 d^2 (c + d x)) - (3 b \cos[3 a + 3 b x]) / (8 d^2 (c + d x)) - (9 b^2 \cos \text{Integral}[(3 b c) / d + 3 b x] \sin[3 a - (3 b c) / d]) / (8 d^3) - (b^2 \cos \text{Integral}[(b c) / d + b x] \sin[a - (b c) / d]) / (8 d^3) - \sin[a + b x] / (8 d (c + d x)^2) - \sin[3 a + 3 b x] / (8 d (c + d x)^2) - (b^2 \cos[a - (b c) / d] \text{SinIntegral}[(b c) / d + b x]) / (8 d^3) - (9 b^2 \cos[3 a - (3 b c) / d] \text{SinIntegral}[(3 b c) / d + 3 b x]) / (8 d^3)$

Rule 4406

$\text{Int}[\cos[(a_.) + (b_.)(x_.)]^{(p_.)} ((c_.) + (d_.)(x_.))^{(m_.)} \sin[(a_.) + (b_.)(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d x)^m, \sin[a + b x]^n \cos[a + b x]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3297

$\text{Int}[(c_.) + (d_.)(x_.)]^{(m_.)} \sin[(e_.) + (f_.)(x_.)], x_Symbol] \rightarrow \text{Simp}[(c + d x)^{(m+1)} \sin[e + f x] / (d(m+1)), x] - \text{Dist}[f / (d(m+1)), \text{Int}[(c + d x)^{(m+1)} \cos[e + f x], x], x] /;$ FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

$\text{Int}[\sin[(e_.) + (f_.)(x_.)] / ((c_.) + (d_.)(x_.)), x_Symbol] \rightarrow \text{Dist}[\cos[(d e - c f) / d], \text{Int}[\sin[(c f) / d + f x] / (c + d x), x], x] + \text{Dist}[\sin[(d e - c f) / d], \text{Int}[\cos[(c f) / d + f x] / (c + d x), x], x] /;$ FreeQ[{c, d, e, f}, x] && NeQ[d e - c f, 0]

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3} dx &= \int \left(\frac{\sin(a+bx)}{4(c+dx)^3} + \frac{\sin(3a+3bx)}{4(c+dx)^3} \right) dx \\ &= \frac{1}{4} \int \frac{\sin(a+bx)}{(c+dx)^3} dx + \frac{1}{4} \int \frac{\sin(3a+3bx)}{(c+dx)^3} dx \\ &= -\frac{\sin(a+bx)}{8d(c+dx)^2} - \frac{\sin(3a+3bx)}{8d(c+dx)^2} + \frac{b \int \frac{\cos(a+bx)}{(c+dx)^2} dx}{8d} + \frac{(3b) \int \frac{\cos(3a+3bx)}{(c+dx)^2} dx}{8d} \\ &= -\frac{b \cos(a+bx)}{8d^2(c+dx)} - \frac{3b \cos(3a+3bx)}{8d^2(c+dx)} - \frac{\sin(a+bx)}{8d(c+dx)^2} - \frac{\sin(3a+3bx)}{8d(c+dx)^2} - \frac{b^2 \int \frac{\sin(a+bx)}{c+dx} dx}{8d^2} \\ &= -\frac{b \cos(a+bx)}{8d^2(c+dx)} - \frac{3b \cos(3a+3bx)}{8d^2(c+dx)} - \frac{\sin(a+bx)}{8d(c+dx)^2} - \frac{\sin(3a+3bx)}{8d(c+dx)^2} - \frac{\left(9b^2 \cos\left(3a - \frac{3bc}{d}\right)\right)}{8d^2} \\ &= -\frac{b \cos(a+bx)}{8d^2(c+dx)} - \frac{3b \cos(3a+3bx)}{8d^2(c+dx)} - \frac{9b^2 \text{Ci}\left(\frac{3bc}{d} + 3bx\right) \sin\left(3a - \frac{3bc}{d}\right)}{8d^3} - \frac{b^2 \text{Ci}\left(\frac{bc}{d} + bx\right)}{8d^2} \end{aligned}$$

Mathematica [A] time = 2.6147, size = 181, normalized size = 0.82

$$\frac{9b^2 \sin\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right) + b^2 \sin\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(b\left(\frac{c}{d} + x\right)\right) + b^2 \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(b\left(\frac{c}{d} + x\right)\right) + 9b^2 \text{Ci}\left(\frac{bc}{d} + bx\right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^3,x]
```

```
[Out] -(9*b^2*CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + b^2*CosIntegr
al[b*(c/d + x)]*Sin[a - (b*c)/d] + (d*(b*(c + d*x)*Cos[a + b*x] + d*Sin[a +
b*x]))/(c + d*x)^2 + (d*(3*b*(c + d*x)*Cos[3*(a + b*x)] + d*Sin[3*(a + b*x
)]))/(c + d*x)^2 + b^2*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 9*b^2*Co
s[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(8*d^3)
```

Maple [A] time = 0.02, size = 313, normalized size = 1.4

$$\frac{1}{b} \left(\frac{b^3}{12} \left(-\frac{3 \sin(3bx+3a)}{2((bx+a)d-ad+bc)^2 d} + \frac{3}{2d} \left(-3 \frac{\cos(3bx+3a)}{((bx+a)d-ad+bc)d} - 3 \frac{1}{d} \text{Si}\left(3bx+3a+3\frac{-ad+bc}{d}\right) \cos\left(3\frac{-ad+bc}{d}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x)
```

```
[Out] 1/b*(1/12*b^3*(-3/2*sin(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)^2/d+3/2*(-3*cos(3*b*
x+3*a)/((b*x+a)*d-a*d+b*c)/d-3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+
```

$$b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d)+1/4*b^3*(-1/2*sin(b*x+a)/((b*x+a)*d-a*d+b*c)^2/d+1/2*(-cos(b*x+a)/((b*x+a)*d-a*d+b*c)/d-(Si(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)$$

Maxima [C] time = 2.17134, size = 452, normalized size = 2.05

$$\frac{b^3 \left(i E_3 \left(\frac{ibc+i(bx+a)d-iad}{d} \right) - i E_3 \left(-\frac{ibc+i(bx+a)d-iad}{d} \right) \right) \cos \left(-\frac{bc-ad}{d} \right) + b^3 \left(i E_3 \left(\frac{3ibc+3i(bx+a)d-3iad}{d} \right) - i E_3 \left(-\frac{3ibc+3i(bx+a)d-3iad}{d} \right) \right)}{8(b^2c^2d - 2a^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")

[Out] $-1/8*(b^3*(I*\exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*\exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*\cos(-(b*c - a*d)/d) + b^3*(I*\exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*\exp_integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*\cos(-3*(b*c - a*d)/d) + b^3*(\exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + \exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*\sin(-(b*c - a*d)/d) + b^3*(\exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + \exp_integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*\sin(-3*(b*c - a*d)/d))/((b^2*c^2*d^2 - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)$

Fricas [A] time = 0.589684, size = 907, normalized size = 4.1

$$\frac{8d^2 \cos(bx+a)^2 \sin(bx+a) + 24(bd^2x + bcd) \cos(bx+a)^3 + 18(b^2d^2x^2 + 2b^2cdx + b^2c^2) \cos\left(-\frac{3(bc-ad)}{d}\right) \operatorname{Si}\left(\frac{3(bd^2x + bcd)}{d}\right)}{d^5x^2 + 2cd^4x + c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")

[Out] $-1/16*(8*d^2*\cos(b*x + a)^2*\sin(b*x + a) + 24*(b*d^2*x + b*c*d)*\cos(b*x + a)^3 + 18*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(-3*(b*c - a*d)/d)*\sin_integral(3*(b*d*x + b*c)/d) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(-(b*c - a*d)/d)*\sin_integral((b*d*x + b*c)/d) - 16*(b*d^2*x + b*c*d)*\cos(b*x + a) + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos_integral((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos_integral(-(b*d*x + b*c)/d))*\sin(-(b*c - a*d)/d) + 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos_integral(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos_integral(-3*(b*d*x + b*c)/d))*\sin(-3*(b*c - a*d)/d))/((d^5*x^2 + 2*c*d^4*x + c^2*d^3)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a + bx) \cos^2(a + bx)}{(c + dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.78 \quad \int \frac{\cos^2(a+bx) \sin(a+bx)}{(c+dx)^4} dx$$

Optimal. Leaf size=270

$$\frac{b^3 \cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{24d^4} - \frac{9b^3 \cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{8d^4} + \frac{b^3 \sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{24d^4}$$

[Out] $-(b \cdot \text{Cos}[a + b \cdot x]) / (24 \cdot d^2 \cdot (c + d \cdot x)^2) - (b \cdot \text{Cos}[3a + 3 \cdot b \cdot x]) / (8 \cdot d^2 \cdot (c + d \cdot x)^2) - (b^3 \cdot \text{Cos}[a - (b \cdot c) / d] \cdot \text{CosIntegral}[(b \cdot c) / d + b \cdot x]) / (24 \cdot d^4) - (9 \cdot b^3 \cdot \text{Cos}[3a - (3 \cdot b \cdot c) / d] \cdot \text{CosIntegral}[(3 \cdot b \cdot c) / d + 3 \cdot b \cdot x]) / (8 \cdot d^4) - \text{Sin}[a + b \cdot x] / (12 \cdot d \cdot (c + d \cdot x)^3) + (b^2 \cdot \text{Sin}[a + b \cdot x]) / (24 \cdot d^3 \cdot (c + d \cdot x)) - \text{Sin}[3a + 3 \cdot b \cdot x] / (12 \cdot d \cdot (c + d \cdot x)^3) + (3 \cdot b^2 \cdot \text{Sin}[3a + 3 \cdot b \cdot x]) / (8 \cdot d^3 \cdot (c + d \cdot x)) + (b^3 \cdot \text{Sin}[a - (b \cdot c) / d] \cdot \text{SinIntegral}[(b \cdot c) / d + b \cdot x]) / (24 \cdot d^4) + (9 \cdot b^3 \cdot \text{Sin}[3a - (3 \cdot b \cdot c) / d] \cdot \text{SinIntegral}[(3 \cdot b \cdot c) / d + 3 \cdot b \cdot x]) / (8 \cdot d^4)$

Rubi [A] time = 0.377452, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b^3 \cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{24d^4} - \frac{9b^3 \cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{8d^4} + \frac{b^3 \sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{24d^4}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^4,x]

[Out] $-(b \cdot \text{Cos}[a + b \cdot x]) / (24 \cdot d^2 \cdot (c + d \cdot x)^2) - (b \cdot \text{Cos}[3a + 3 \cdot b \cdot x]) / (8 \cdot d^2 \cdot (c + d \cdot x)^2) - (b^3 \cdot \text{Cos}[a - (b \cdot c) / d] \cdot \text{CosIntegral}[(b \cdot c) / d + b \cdot x]) / (24 \cdot d^4) - (9 \cdot b^3 \cdot \text{Cos}[3a - (3 \cdot b \cdot c) / d] \cdot \text{CosIntegral}[(3 \cdot b \cdot c) / d + 3 \cdot b \cdot x]) / (8 \cdot d^4) - \text{Sin}[a + b \cdot x] / (12 \cdot d \cdot (c + d \cdot x)^3) + (b^2 \cdot \text{Sin}[a + b \cdot x]) / (24 \cdot d^3 \cdot (c + d \cdot x)) - \text{Sin}[3a + 3 \cdot b \cdot x] / (12 \cdot d \cdot (c + d \cdot x)^3) + (3 \cdot b^2 \cdot \text{Sin}[3a + 3 \cdot b \cdot x]) / (8 \cdot d^3 \cdot (c + d \cdot x)) + (b^3 \cdot \text{Sin}[a - (b \cdot c) / d] \cdot \text{SinIntegral}[(b \cdot c) / d + b \cdot x]) / (24 \cdot d^4) + (9 \cdot b^3 \cdot \text{Sin}[3a - (3 \cdot b \cdot c) / d] \cdot \text{SinIntegral}[(3 \cdot b \cdot c) / d + 3 \cdot b \cdot x]) / (8 \cdot d^4)$

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n * Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1) * Sin[e + f*x]) / (d * (m + 1)), x] - Dist[f / (d * (m + 1)), Int[(c + d*x)^(m + 1) * Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)] / ((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f) / d], Int[Sin[(c*f) / d + f*x] / (c + d*x), x], x] + Dist[Sin[(d*e - c*f) / d], Int[Cos[(c*f) / d + f*x] / (c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4} dx &= \int \left(\frac{\sin(a+bx)}{4(c+dx)^4} + \frac{\sin(3a+3bx)}{4(c+dx)^4} \right) dx \\
 &= \frac{1}{4} \int \frac{\sin(a+bx)}{(c+dx)^4} dx + \frac{1}{4} \int \frac{\sin(3a+3bx)}{(c+dx)^4} dx \\
 &= -\frac{\sin(a+bx)}{12d(c+dx)^3} - \frac{\sin(3a+3bx)}{12d(c+dx)^3} + \frac{b \int \frac{\cos(a+bx)}{(c+dx)^3} dx}{12d} + \frac{b \int \frac{\cos(3a+3bx)}{(c+dx)^3} dx}{4d} \\
 &= -\frac{b \cos(a+bx)}{24d^2(c+dx)^2} - \frac{b \cos(3a+3bx)}{8d^2(c+dx)^2} - \frac{\sin(a+bx)}{12d(c+dx)^3} - \frac{\sin(3a+3bx)}{12d(c+dx)^3} - \frac{b^2 \int \frac{\sin(a+bx)}{(c+dx)^2} dx}{24d^2} \\
 &= -\frac{b \cos(a+bx)}{24d^2(c+dx)^2} - \frac{b \cos(3a+3bx)}{8d^2(c+dx)^2} - \frac{\sin(a+bx)}{12d(c+dx)^3} + \frac{b^2 \sin(a+bx)}{24d^3(c+dx)} - \frac{\sin(3a+3bx)}{12d(c+dx)^3} + \\
 &= -\frac{b \cos(a+bx)}{24d^2(c+dx)^2} - \frac{b \cos(3a+3bx)}{8d^2(c+dx)^2} - \frac{\sin(a+bx)}{12d(c+dx)^3} + \frac{b^2 \sin(a+bx)}{24d^3(c+dx)} - \frac{\sin(3a+3bx)}{12d(c+dx)^3} + \\
 &= -\frac{b \cos(a+bx)}{24d^2(c+dx)^2} - \frac{b \cos(3a+3bx)}{8d^2(c+dx)^2} - \frac{b^3 \cos\left(a - \frac{bc}{d}\right) \text{Ci}\left(\frac{bc}{d} + bx\right)}{24d^4} - \frac{9b^3 \cos\left(3a - \frac{3bc}{d}\right)}{8d^4}
 \end{aligned}$$

Mathematica [A] time = 1.93298, size = 300, normalized size = 1.11

$$b^3(c+dx)^3 \left(\cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(b\left(\frac{c}{d} + x\right)\right) - \sin\left(a - \frac{bc}{d}\right) \text{Si}\left(b\left(\frac{c}{d} + x\right)\right) \right) + 27b^3(c+dx)^3 \left(\cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(3b\left(\frac{c}{d} + x\right)\right) - \sin\left(3a - \frac{3bc}{d}\right) \text{Si}\left(3b\left(\frac{c}{d} + x\right)\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^4, x]

[Out] -(d*Cos[b*x]*(b*d*(c + d*x)*Cos[a] - (-2*d^2 + b^2*(c + d*x)^2)*Sin[a]) + d*Cos[3*b*x]*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)*Sin[3*a]) - d*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*d*(c + d*x)*Sin[a])*Sin[b*x] - d*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*x)*Sin[3*a])*Sin[3*b*x] + b^3*(c + d*x)^3*(Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] - Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)]) + 27*b^3*(c + d*x)^3*(Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*(c + d*x))/d] - Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d]))/(24*d^4*(c + d*x)^3)

Maple [A] time = 0.023, size = 381, normalized size = 1.4

$$\frac{1}{b} \left(\frac{b^4}{12} \left(-\frac{\sin(3bx+3a)}{((bx+a)d - ad + bc)^3 d} + \frac{1}{d} \left(-\frac{3 \cos(3bx+3a)}{2((bx+a)d - ad + bc)^2 d} - \frac{3}{2d} \left(-3 \frac{\sin(3bx+3a)}{((bx+a)d - ad + bc)d} + 3 \frac{1}{d} \left(3 \frac{1}{d} \text{Si}\left(3bx + \frac{3a}{d}\right) \right) \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x)

[Out] $\frac{1}{b} \left(\frac{1}{12} b^4 \frac{-\sin(3bx+3a)}{((bx+a)d-ad+bc)^3/d} + \frac{-3/2 \cos(3bx+3a)}{((bx+a)d-ad+bc)^2/d} - \frac{3/2 \sin(3bx+3a)}{((bx+a)d-ad+bc)/d} + 3 \left(\frac{3 \operatorname{Si}(3bx+3a+3(-ad+bc)/d) \sin(3(-ad+bc)/d)}{d} + \frac{3 \operatorname{Ci}(3bx+3a+3(-ad+bc)/d) \cos(3(-ad+bc)/d)}{d} \right) + \frac{1}{4} b^4 \frac{-1/3 \sin(bx+a)}{((bx+a)d-ad+bc)^3/d} + \frac{1}{3} \frac{-1/2 \cos(bx+a)}{((bx+a)d-ad+bc)^2/d} - \frac{1}{2} \frac{-\sin(bx+a)}{((bx+a)d-ad+bc)/d} + \left(\frac{\operatorname{Si}(bx+a+(-ad+bc)/d) \sin((-ad+bc)/d)}{d} + \frac{\operatorname{Ci}(bx+a+(-ad+bc)/d) \cos((-ad+bc)/d)}{d} \right) \right)$

Maxima [C] time = 2.78691, size = 520, normalized size = 1.93

$$\frac{b^4 \left(i E_4 \left(\frac{ibc+i(bx+a)d-id}{d} \right) - i E_4 \left(-\frac{ibc+i(bx+a)d-id}{d} \right) \right) \cos \left(-\frac{bc-ad}{d} \right) + b^4 \left(i E_4 \left(\frac{3ibc+3i(bx+a)d-3iad}{d} \right) - i E_4 \left(-\frac{3ibc+3i(bx+a)d-3iad}{d} \right) \right)}{8(b^3c^3d - 3ab^2c^2d^2 + 3a^2bcd^3 + (bx + a)^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x, algorithm="maxima")

[Out] $\frac{-1}{8} b^4 \left(\operatorname{I} \exp_{\text{integral_e}}(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - \operatorname{I} \exp_{\text{integral_e}}(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d) \right) \cos(-\frac{b*c - a*d}{d}) + b^4 \left(\operatorname{I} \exp_{\text{integral_e}}(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - \operatorname{I} \exp_{\text{integral_e}}(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) \right) \cos(-\frac{3*(b*c - a*d)}{d}) + b^4 \left(\exp_{\text{integral_e}}(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + \exp_{\text{integral_e}}(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d) \right) \sin(-\frac{b*c - a*d}{d}) + b^4 \left(\exp_{\text{integral_e}}(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + \exp_{\text{integral_e}}(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) \right) \sin(-\frac{3*(b*c - a*d)}{d}) \left((b^3c^3d - 3a*b^2c^2d^2 + 3a^2b*c*d^3 + (b*x + a)^3d^4 - a^3d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2c^2d^2 - 2a*b*c*d^3 + a^2d^4)*(b*x + a)) * b \right)$

Fricas [B] time = 0.625508, size = 1231, normalized size = 4.56

$$\frac{24(bd^3x + bcd^2) \cos(bx + a)^3 - 54(b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + b^3c^3) \sin\left(-\frac{3(bc-ad)}{d}\right) \operatorname{Si}\left(\frac{3(bdx+bc)}{d}\right) - 2(b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + b^3c^3) \cos(bx + a) + ((b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + b^3c^3) \cos_{\text{integral}}((b*d*x + b*c)/d) + (b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + b^3c^3) \cos_{\text{integral}}(-(b*d*x + b*c)/d)) \cos(-\frac{b*c - a*d}{d}) + 27*((b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + b^3c^3) \cos_{\text{integral}}(3*(b*d*x + b*c)/d) + (b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + b^3c^3) \cos_{\text{integral}}(-3*(b*d*x + b*c)/d)) \cos(-\frac{3*(b*c - a*d)}{d}) + 8*(2*b^2*d^3*x^2 + 4*b^2*c*d^3*x + b^2*c^2*d^3) \sin(-\frac{b*c - a*d}{d})}{(b^3c^3d - 3ab^2c^2d^2 + 3a^2bcd^3 + (bx + a)^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x, algorithm="fricas")

[Out] $\frac{-1}{48} (24*(b*d^3*x + b*c*d^2) \cos(b*x + a)^3 - 54*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3) \sin(-\frac{3*(b*c - a*d)}{d}) \sin_{\text{integral}}(3*(b*d*x + b*c)/d) - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3) \sin(-\frac{b*c - a*d}{d}) \sin_{\text{integral}}((b*d*x + b*c)/d) - 16*(b^3*d^3*x + b*c*d^2) \cos(b*x + a) + ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3) \cos_{\text{integral}}((b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3) \cos_{\text{integral}}(-(b*d*x + b*c)/d)) \cos(-\frac{b*c - a*d}{d}) + 27*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3) \cos_{\text{integral}}(3*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3) \cos_{\text{integral}}(-3*(b*d*x + b*c)/d)) \cos(-\frac{3*(b*c - a*d)}{d}) + 8*(2*b^2*d^3*x^2 + 4*b^2*c*d^3*x + b^2*c^2*d^3) \sin(-\frac{b*c - a*d}{d})}{(b^3c^3d - 3ab^2c^2d^2 + 3a^2bcd^3 + (bx + a)^2d^3)}$

$$b^2cd^2x + 2b^2c^2d - (9b^2d^3x^2 + 18b^2cd^2x + 9b^2c^2d - 2d^3)\cos(bx + a)^2\sin(bx + a)/(d^7x^3 + 3cd^6x^2 + 3c^2d^5x + c^3d^4)$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a + bx) \cos^2(a + bx)}{(c + dx)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c)**4,x)

[Out] Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x)**4, x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")

[Out] Exception raised: NotImplementedError

3.79 $\int (c + dx)^m \cos^2(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=162

$$\frac{i^{2-2(m+3)} e^{4i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{4ib(c+dx)}{d}\right)}{b} - \frac{i^{2-2(m+3)} e^{-4i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, \frac{4ib(c+dx)}{d}\right)}{b}$$

```
[Out] (c + d*x)^(1 + m)/(8*d*(1 + m)) + (I*E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*(((I)*b*(c + d*x))/d)^m) - (I*(c + d*x)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*E^((4*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.2048, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3307, 2181}

$$\frac{i^{2-2(m+3)} e^{4i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{4ib(c+dx)}{d}\right)}{b} - \frac{i^{2-2(m+3)} e^{-4i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, \frac{4ib(c+dx)}{d}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m * Cos[a + b*x]^2 * Sin[a + b*x]^2, x]
```

```
[Out] (c + d*x)^(1 + m)/(8*d*(1 + m)) + (I*E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*(((I)*b*(c + d*x))/d)^m) - (I*(c + d*x)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*E^((4*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_.)]^(p_.)*((c_.) + (d_.)*(x_.))^(m_.)*Sin[(a_.) + (b_.)*(x_.)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n * Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_.)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_.)))*((c_.) + (d_.)*(x_.))^(m_), x_Symbol] := -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^m \cos^2(a + bx) \sin^2(a + bx) dx &= \int \left(\frac{1}{8}(c + dx)^m - \frac{1}{8}(c + dx)^m \cos(4a + 4bx) \right) dx \\
&= \frac{(c + dx)^{1+m}}{8d(1+m)} - \frac{1}{8} \int (c + dx)^m \cos(4a + 4bx) dx \\
&= \frac{(c + dx)^{1+m}}{8d(1+m)} - \frac{1}{16} \int e^{-i(4a+4bx)}(c + dx)^m dx - \frac{1}{16} \int e^{i(4a+4bx)}(c + dx)^m dx \\
&= \frac{(c + dx)^{1+m}}{8d(1+m)} + \frac{i4^{-3-m}e^{4i\left(a-\frac{bc}{d}\right)}(c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{4ib(c+dx)}{d}\right)}{b} - \dots
\end{aligned}$$

Mathematica [A] time = 1.10385, size = 213, normalized size = 1.31

$$\frac{4^{-m-3}(c + dx)^m \left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m} \left(-id(m+1)\left(-\frac{ib(c+dx)}{d}\right)^m \left(\cos\left(4a - \frac{4bc}{d}\right) - i \sin\left(4a - \frac{4bc}{d}\right)\right) \Gamma\left(m+1, \frac{4ib(c+dx)}{d}\right) + id(m+1)}{bd(m+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^m * Cos[a + b*x]^2 * Sin[a + b*x]^2, x]

[Out] $(4^{-3-m})(c + d*x)^m * (2^{(3+2*m)*b*(c + d*x)} * ((b^2*(c + d*x)^2)/d^2)^m - I*d*(1+m)*((-I)*b*(c + d*x))/d)^m * \Gamma[1+m, ((4*I)*b*(c + d*x))/d] * (\text{Cos}[4*a - (4*b*c)/d] - I*\text{Sin}[4*a - (4*b*c)/d]) + I*d*(1+m)*((I*b*(c + d*x))/d)^m * \Gamma[1+m, ((-4*I)*b*(c + d*x))/d] * (\text{Cos}[4*a - (4*b*c)/d] + I*\text{Sin}[4*a - (4*b*c)/d]) / (b*d*(1+m)*((b^2*(c + d*x)^2)/d^2)^m$

Maple [F] time = 0.209, size = 0, normalized size = 0.

$$\int (dx + c)^m (\cos(bx + a))^2 (\sin(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x)

[Out] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(dm + d) \int (dx + c)^m \cos(4bx + 4a) dx - e^{(m \log(dx+c) + \log(dx+c))}}{8(dm + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")

[Out] $-1/8*((d*m + d)*integrate((d*x + c)^m*cos(4*b*x + 4*a), x) - e^{(m*\log(d*x + c) + \log(d*x + c))})/(d*m + d)$

Fricas [A] time = 0.526546, size = 342, normalized size = 2.11

$$\frac{(-idm - id)e^{\left(-\frac{dm \log\left(\frac{4ib}{d}\right) - 4ibc + 4iad}{d}\right)} \Gamma\left(m + 1, \frac{4ibdx + 4ibc}{d}\right) + (idm + id)e^{\left(-\frac{dm \log\left(-\frac{4ib}{d}\right) + 4ibc - 4iad}{d}\right)} \Gamma\left(m + 1, \frac{-4ibdx - 4ibc}{d}\right) + 8(bdx + c)^m}{64(bdm + bd)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")

[Out] 1/64*((-I*d*m - I*d)*e^(-(d*m*log(4*I*b/d) - 4*I*b*c + 4*I*a*d)/d)*gamma(m + 1, (4*I*b*d*x + 4*I*b*c)/d) + (I*d*m + I*d)*e^(-(d*m*log(-4*I*b/d) + 4*I*b*c - 4*I*a*d)/d)*gamma(m + 1, (-4*I*b*d*x - 4*I*b*c)/d) + 8*(b*d*x + b*c)*(d*x + c)^m)/(b*d*m + b*d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^m \sin^2(a + bx) \cos^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**m*cos(b*x+a)**2*sin(b*x+a)**2,x)

[Out] Integral((c + d*x)**m*sin(a + b*x)**2*cos(a + b*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a)^2 \sin(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a)^2, x)

3.80 $\int (c + dx)^4 \cos^2(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=131

$$\frac{3d^2(c + dx)^2 \sin(4a + 4bx)}{128b^3} + \frac{3d^3(c + dx) \cos(4a + 4bx)}{256b^4} - \frac{d(c + dx)^3 \cos(4a + 4bx)}{32b^2} - \frac{3d^4 \sin(4a + 4bx)}{1024b^5} - \frac{(c + dx)^4 \sin(4a + 4bx)}{32b^2}$$

[Out] (c + d*x)^5/(40*d) + (3*d^3*(c + d*x)*Cos[4*a + 4*b*x])/(256*b^4) - (d*(c + d*x)^3*Cos[4*a + 4*b*x])/(32*b^2) - (3*d^4*Sin[4*a + 4*b*x])/(1024*b^5) + (3*d^2*(c + d*x)^2*Sin[4*a + 4*b*x])/(128*b^3) - ((c + d*x)^4*Sin[4*a + 4*b*x])/(32*b)

Rubi [A] time = 0.1644, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3296, 2637}

$$\frac{3d^2(c + dx)^2 \sin(4a + 4bx)}{128b^3} + \frac{3d^3(c + dx) \cos(4a + 4bx)}{256b^4} - \frac{d(c + dx)^3 \cos(4a + 4bx)}{32b^2} - \frac{3d^4 \sin(4a + 4bx)}{1024b^5} - \frac{(c + dx)^4 \sin(4a + 4bx)}{32b^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^2,x]

[Out] (c + d*x)^5/(40*d) + (3*d^3*(c + d*x)*Cos[4*a + 4*b*x])/(256*b^4) - (d*(c + d*x)^3*Cos[4*a + 4*b*x])/(32*b^2) - (3*d^4*Sin[4*a + 4*b*x])/(1024*b^5) + (3*d^2*(c + d*x)^2*Sin[4*a + 4*b*x])/(128*b^3) - ((c + d*x)^4*Sin[4*a + 4*b*x])/(32*b)

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (c + dx)^4 \cos^2(a + bx) \sin^2(a + bx) dx &= \int \left(\frac{1}{8}(c + dx)^4 - \frac{1}{8}(c + dx)^4 \cos(4a + 4bx) \right) dx \\
&= \frac{(c + dx)^5}{40d} - \frac{1}{8} \int (c + dx)^4 \cos(4a + 4bx) dx \\
&= \frac{(c + dx)^5}{40d} - \frac{(c + dx)^4 \sin(4a + 4bx)}{32b} + \frac{d \int (c + dx)^3 \sin(4a + 4bx) dx}{8b} \\
&= \frac{(c + dx)^5}{40d} - \frac{d(c + dx)^3 \cos(4a + 4bx)}{32b^2} - \frac{(c + dx)^4 \sin(4a + 4bx)}{32b} + \frac{(3d^2) \int (c + dx)^2 \sin(4a + 4bx) dx}{128b^3} \\
&= \frac{(c + dx)^5}{40d} - \frac{d(c + dx)^3 \cos(4a + 4bx)}{32b^2} + \frac{3d^2(c + dx)^2 \sin(4a + 4bx)}{128b^3} - \frac{(c + dx)^4 \sin(4a + 4bx)}{32b} \\
&= \frac{(c + dx)^5}{40d} + \frac{3d^3(c + dx) \cos(4a + 4bx)}{256b^4} - \frac{d(c + dx)^3 \cos(4a + 4bx)}{32b^2} + \frac{3d^2(c + dx)^2 \sin(4a + 4bx)}{128b^3} \\
&= \frac{(c + dx)^5}{40d} + \frac{3d^3(c + dx) \cos(4a + 4bx)}{256b^4} - \frac{d(c + dx)^3 \cos(4a + 4bx)}{32b^2} - \frac{3d^4(c + dx)^2 \sin(4a + 4bx)}{128b^3}
\end{aligned}$$

Mathematica [A] time = 1.36655, size = 132, normalized size = 1.01

$$\frac{-5 \sin(4(a + bx)) (-24b^2d^2(c + dx)^2 + 32b^4(c + dx)^4 + 3d^4) + 20bd(c + dx) \cos(4(a + bx)) (3d^2 - 8b^2(c + dx)^2) + 128b^5}{5120b^5}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^2,x]

[Out] (128*b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4) + 20*b*d*(c + d*x)*(3*d^2 - 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] - 5*(3*d^4 - 24*b^2*d^2*(c + d*x)^2 + 32*b^4*(c + d*x)^4)*Sin[4*(a + b*x)])/(5120*b^5)

Maple [B] time = 0.076, size = 1915, normalized size = 14.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x)

[Out] 1/b*(1/b^4*d^4*((b*x+a)^4*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^3*cos(b*x+a)^2+3/4*(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/32*(b*x+a)*cos(b*x+a)^2-3/64*cos(b*x+a)*sin(b*x+a)-21/256*b*x-21/256*a-7/16*(b*x+a)^3-1/10*(b*x+a)^5-(b*x+a)^4*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/4*(b*x+a)^3*sin(b*x+a)^4+3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)*sin(b*x+a)^4+3/128*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-4/b^4*a*d^4*((b*x+a)^3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/16*(b*x+a)^2*cos(b*x+a)^2+3/8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-21/128*(b*x+a)^2-3/128*sin(b*x+a)^2-3/32*(b*x+a)^4-(b*x+a)^3*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/16*(b*x+a)^2*sin(b*x+a)^4+3/8*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/128*sin(b*x+a)^4)+4/b^3*c*d^3*((b*x+a)^3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/16*(b*x+a)^2*cos(b*x+a)^2+3/8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-21/128*(b*x+a)^2-3/128*sin(b*x+a)^2-3/32*(b*x+a)^4-(b*x+a)^3*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/16*(b*x+a)^2*sin(b*x+a)^4+3/8*

```
(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/128
*sin(b*x+a)^4)+6/b^4*a^2*d^4*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x
+1/2*a)-1/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a
-1/12*(b*x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/
8*b*x+3/8*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*co
s(b*x+a))-12/b^3*a*c*d^3*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2
*a)-1/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a-1/1
2*(b*x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*
x+3/8*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*
x+a))+6/b^2*c^2*d^2*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1
/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a-1/12*(b*
x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8
*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a))
-4/b^4*a^3*d^4*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*
x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*
x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)+12/b^3*a^2*c*d^3*((b*x+a)*(-1/2*cos(
b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*
(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a
)^4)-12/b^2*a*c^2*d^2*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1
/16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))
*cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)+4/b*c^3*d*((b*x+a)*(-1/2*cos(
b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*
(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a
)^4)+1/b^4*a^4*d^4*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+
1/8*b*x+1/8*a)-4/b^3*a^3*c*d^3*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)
*sin(b*x+a)+1/8*b*x+1/8*a)+6/b^2*a^2*c^2*d^2*(-1/4*sin(b*x+a)*cos(b*x+a)^3+
1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)-4/b*a*c^3*d*(-1/4*sin(b*x+a)*cos(b
*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)+c^4*(-1/4*sin(b*x+a)*cos(b
*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a))
```

Maxima [B] time = 1.37141, size = 992, normalized size = 7.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")

```
[Out] 1/5120*(160*(4*b*x + 4*a - sin(4*b*x + 4*a))*c^4 - 640*(4*b*x + 4*a - sin(4
*b*x + 4*a))*a*c^3*d/b + 960*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^2*c^2*d^2/b
^2 - 640*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^3*c*d^3/b^3 + 160*(4*b*x + 4*a
- sin(4*b*x + 4*a))*a^4*d^4/b^4 + 160*(8*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*
x + 4*a) - cos(4*b*x + 4*a))*c^3*d/b - 480*(8*(b*x + a)^2 - 4*(b*x + a)*sin
(4*b*x + 4*a) - cos(4*b*x + 4*a))*a*c^2*d^2/b^2 + 480*(8*(b*x + a)^2 - 4*(b
*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a^2*c*d^3/b^3 - 160*(8*(b*x +
a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a^3*d^4/b^4 + 40*(3
2*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*sin(4
*b*x + 4*a))*c^2*d^2/b^2 - 80*(32*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*
a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a))*a*c*d^3/b^3 + 40*(32*(b*x + a)
^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a)
)*a^2*d^4/b^4 + 20*(32*(b*x + a)^4 - 3*(8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a)
- 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*b*x + 4*a))*c*d^3/b^3 - 20*(32*(b*
x + a)^4 - 3*(8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 4*(8*(b*x + a)^3 - 3*b*
x - 3*a)*sin(4*b*x + 4*a))*a*d^4/b^4 + (128*(b*x + a)^5 - 20*(8*(b*x + a)^3
- 3*b*x - 3*a)*cos(4*b*x + 4*a) - 5*(32*(b*x + a)^4 - 24*(b*x + a)^2 + 3)*
sin(4*b*x + 4*a))*d^4/b^4)/b
```

Fricas [B] time = 0.531492, size = 986, normalized size = 7.53

$$32b^5d^4x^5 + 160b^5cd^3x^4 - 40(8b^3d^4x^3 + 24b^3cd^3x^2 + 8b^3c^3d - 3bcd^3 + 3(8b^3c^2d^2 - bd^4)x) \cos(bx + a)^4 + 40(8b^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")

[Out] 1/1280*(32*b^5*d^4*x^5 + 160*b^5*c*d^3*x^4 - 40*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(8*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^4 + 40*(8*b^5*c^2*d^2 - b^3*d^4)*x^3 + 40*(8*b^5*c^3*d - 3*b^3*c*d^3)*x^2 + 40*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(8*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^2 + 5*(32*b^5*c^4 - 24*b^3*c^2*d^2 + 3*b*d^4)*x - 5*(2*(32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 32*b^4*c^4 - 24*b^2*c^2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*x^2 + 16*(8*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^3 - (32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 32*b^4*c^4 - 24*b^2*c^2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*x^2 + 16*(8*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a))*sin(b*x + a))/b^5

Sympy [A] time = 17.6012, size = 1209, normalized size = 9.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**4*cos(b*x+a)**2*sin(b*x+a)**2,x)

[Out] Piecewise((c**4*x*sin(a + b*x)**4/8 + c**4*x*sin(a + b*x)**2*cos(a + b*x)**2/4 + c**4*x*cos(a + b*x)**4/8 + c**3*d*x**2*sin(a + b*x)**4/4 + c**3*d*x**2*sin(a + b*x)**2*cos(a + b*x)**2/2 + c**3*d*x**2*cos(a + b*x)**4/4 + c**2*d**2*x**3*sin(a + b*x)**4/4 + c**2*d**2*x**3*sin(a + b*x)**2*cos(a + b*x)**2/2 + c**2*d**2*x**3*cos(a + b*x)**4/4 + c*d**3*x**4*sin(a + b*x)**4/8 + c*d**3*x**4*sin(a + b*x)**2*cos(a + b*x)**2/4 + c*d**3*x**4*cos(a + b*x)**4/8 + d**4*x**5*sin(a + b*x)**4/40 + d**4*x**5*sin(a + b*x)**2*cos(a + b*x)**2/20 + d**4*x**5*cos(a + b*x)**4/40 + c**4*sin(a + b*x)**3*cos(a + b*x)/(8*b) - c**4*sin(a + b*x)*cos(a + b*x)**3/(8*b) + c**3*d*x*sin(a + b*x)**3*cos(a + b*x)/(2*b) - c**3*d*x*sin(a + b*x)*cos(a + b*x)**3/(2*b) + 3*c**2*d**2*x**2*sin(a + b*x)**3*cos(a + b*x)/(4*b) - 3*c**2*d**2*x**2*sin(a + b*x)*cos(a + b*x)**3/(4*b) + c*d**3*x**3*sin(a + b*x)**3*cos(a + b*x)/(2*b) - c*d**3*x**3*sin(a + b*x)*cos(a + b*x)**3/(2*b) + d**4*x**4*sin(a + b*x)**3*cos(a + b*x)/(8*b) - d**4*x**4*sin(a + b*x)*cos(a + b*x)**3/(8*b) + c**3*d*sin(a + b*x)**2*cos(a + b*x)**2/(4*b**2) - 3*c**2*d**2*x*sin(a + b*x)**4/(32*b**2) + 9*c**2*d**2*x*sin(a + b*x)**2*cos(a + b*x)**2/(16*b**2) - 3*c**2*d**2*x*cos(a + b*x)**4/(32*b**2) - 3*c*d**3*x**2*sin(a + b*x)**4/(32*b**2) + 9*c*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(16*b**2) - 3*c*d**3*x**2*cos(a + b*x)**4/(32*b**2) - d**4*x**3*sin(a + b*x)**4/(32*b**2) + 3*d**4*x**3*sin(a + b*x)**2*cos(a + b*x)**2/(16*b**2) - d**4*x**3*cos(a + b*x)**4/(32*b**2) - 3*c**2*d**2*sin(a + b*x)**3*cos(a + b*x)/(32*b**3) + 3*c**2*d**2*sin(a + b*x)*cos(a + b*x)**3/(32*b**3) - 3*c*d**3*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**3) + 3*c*d**3*x*sin(a + b*x)*cos(a + b*x)**3/(16*b**3) - 3*d**4*x**2*sin(a + b*x)*cos(a + b*x)**3/(32*b**3) - 3*c*d**3*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**4) + 3*d**4*x*sin(a + b*x)**4/(256*b**4) - 9*d**4*x*sin(a + b*x)**2*cos(a + b*x)**2/(128*b**4) + 3*d**4*x*cos(a + b*x)**4/(256*b**4) + 3*d**4*sin(a + b*x)**3

```
*cos(a + b*x)/(256*b**5) - 3*d**4*sin(a + b*x)*cos(a + b*x)**3/(256*b**5),
Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4
*x**5/5)*sin(a)**2*cos(a)**2, True))
```

Giac [A] time = 1.16027, size = 302, normalized size = 2.31

$$\frac{1}{40}d^4x^5 + \frac{1}{8}cd^3x^4 + \frac{1}{4}c^2d^2x^3 + \frac{1}{4}c^3dx^2 + \frac{1}{8}c^4x - \frac{(8b^3d^4x^3 + 24b^3cd^3x^2 + 24b^3c^2d^2x + 8b^3c^3d - 3bd^4x - 3bcd^3)\cos(a)}{256b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/40*d^4*x^5 + 1/8*c*d^3*x^4 + 1/4*c^2*d^2*x^3 + 1/4*c^3*d*x^2 + 1/8*c^4*x
- 1/256*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 24*b^3*c^2*d^2*x + 8*b^3*c^3*d
- 3*b*d^4*x - 3*b*c*d^3)*cos(4*b*x + 4*a)/b^5 - 1/1024*(32*b^4*d^4*x^4 + 12
8*b^4*c*d^3*x^3 + 192*b^4*c^2*d^2*x^2 + 128*b^4*c^3*d*x + 32*b^4*c^4 - 24*b
^2*d^4*x^2 - 48*b^2*c*d^3*x - 24*b^2*c^2*d^2 + 3*d^4)*sin(4*b*x + 4*a)/b^5
```


3.81 $\int (c + dx)^3 \cos^2(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=105

$$\frac{3d^2(c + dx) \sin(4a + 4bx)}{256b^3} - \frac{3d(c + dx)^2 \cos(4a + 4bx)}{128b^2} + \frac{3d^3 \cos(4a + 4bx)}{1024b^4} - \frac{(c + dx)^3 \sin(4a + 4bx)}{32b} + \frac{(c + dx)^4}{32d}$$

[Out] $(c + d*x)^4/(32*d) + (3*d^3*\text{Cos}[4*a + 4*b*x])/(1024*b^4) - (3*d*(c + d*x)^2*\text{Cos}[4*a + 4*b*x])/(128*b^2) + (3*d^2*(c + d*x)*\text{Sin}[4*a + 4*b*x])/(256*b^3) - ((c + d*x)^3*\text{Sin}[4*a + 4*b*x])/(32*b)$

Rubi [A] time = 0.130323, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3296, 2638}

$$\frac{3d^2(c + dx) \sin(4a + 4bx)}{256b^3} - \frac{3d(c + dx)^2 \cos(4a + 4bx)}{128b^2} + \frac{3d^3 \cos(4a + 4bx)}{1024b^4} - \frac{(c + dx)^3 \sin(4a + 4bx)}{32b} + \frac{(c + dx)^4}{32d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^3*\text{Cos}[a + b*x]^2*\text{Sin}[a + b*x]^2, x]$

[Out] $(c + d*x)^4/(32*d) + (3*d^3*\text{Cos}[4*a + 4*b*x])/(1024*b^4) - (3*d*(c + d*x)^2*\text{Cos}[4*a + 4*b*x])/(128*b^2) + (3*d^2*(c + d*x)*\text{Sin}[4*a + 4*b*x])/(256*b^3) - ((c + d*x)^3*\text{Sin}[4*a + 4*b*x])/(32*b)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] := \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] := -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

$\text{Int}[\text{sin}[(c_.) + (d_.)*(x_.)], x_Symbol] := -\text{Simp}[\text{Cos}[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (c+dx)^3 \cos^2(a+bx) \sin^2(a+bx) dx &= \int \left(\frac{1}{8}(c+dx)^3 - \frac{1}{8}(c+dx)^3 \cos(4a+4bx) \right) dx \\
&= \frac{(c+dx)^4}{32d} - \frac{1}{8} \int (c+dx)^3 \cos(4a+4bx) dx \\
&= \frac{(c+dx)^4}{32d} - \frac{(c+dx)^3 \sin(4a+4bx)}{32b} + \frac{(3d) \int (c+dx)^2 \sin(4a+4bx) dx}{32b} \\
&= \frac{(c+dx)^4}{32d} - \frac{3d(c+dx)^2 \cos(4a+4bx)}{128b^2} - \frac{(c+dx)^3 \sin(4a+4bx)}{32b} + \frac{(3d^2) \int (c+dx) \sin(4a+4bx) dx}{256b^3} \\
&= \frac{(c+dx)^4}{32d} - \frac{3d(c+dx)^2 \cos(4a+4bx)}{128b^2} + \frac{3d^2(c+dx) \sin(4a+4bx)}{256b^3} - \frac{(c+dx)^3 \cos(4a+4bx)}{32b} \\
&= \frac{(c+dx)^4}{32d} + \frac{3d^3 \cos(4a+4bx)}{1024b^4} - \frac{3d(c+dx)^2 \cos(4a+4bx)}{128b^2} + \frac{3d^2(c+dx) \sin(4a+4bx)}{256b^3}
\end{aligned}$$

Mathematica [A] time = 0.685076, size = 106, normalized size = 1.01

$$\frac{-4b(c+dx)\sin(4(a+bx))(8b^2(c+dx)^2-3d^2)-3d\cos(4(a+bx))(8b^2(c+dx)^2-d^2)+32b^4x(6c^2dx+4c^3+4cd^2x^2+1024b^4)}{1024b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^2,x]

[Out] (32*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 3*d*(-d^2 + 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] - 4*b*(c + d*x)*(-3*d^2 + 8*b^2*(c + d*x)^2)*Sin[4*(a + b*x)])/(1024*b^4)

Maple [B] time = 0.022, size = 1074, normalized size = 10.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x)

[Out] 1/b*(1/b^3*d^3*((b*x+a)^3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/16*(b*x+a)^2*cos(b*x+a)^2+3/8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-21/128*(b*x+a)^2-3/128*sin(b*x+a)^2-3/32*(b*x+a)^4-(b*x+a)^3*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/16*(b*x+a)^2*sin(b*x+a)^4+3/8*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/128*sin(b*x+a)^4)-3/b^3*a*d^3*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a-1/12*(b*x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a))+3/b^2*c*d^2*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a-1/12*(b*x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a))+3/b^3*a^2*d^3*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)-6/b^2*a*c*d^2*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)

$$\begin{aligned} & x+a)^4)+3/b*c^2*d*((b*x+a)*(-1/2*\cos(b*x+a)*\sin(b*x+a)+1/2*b*x+1/2*a)-1/16* \\ & (b*x+a)^2+1/16*\sin(b*x+a)^2-(b*x+a)*(-1/4*(\sin(b*x+a)^3+3/2*\sin(b*x+a))*\cos \\ & (b*x+a)+3/8*b*x+3/8*a)-1/16*\sin(b*x+a)^4)-1/b^3*a^3*d^3*(-1/4*\sin(b*x+a)*\cos \\ & (b*x+a)^3+1/8*\cos(b*x+a)*\sin(b*x+a)+1/8*b*x+1/8*a)+3/b^2*a^2*c*d^2*(-1/4*\sin \\ & (b*x+a)*\cos(b*x+a)^3+1/8*\cos(b*x+a)*\sin(b*x+a)+1/8*b*x+1/8*a)-3/b*a*c^2*d \\ & *(-1/4*\sin(b*x+a)*\cos(b*x+a)^3+1/8*\cos(b*x+a)*\sin(b*x+a)+1/8*b*x+1/8*a)+c^3 \\ & *(-1/4*\sin(b*x+a)*\cos(b*x+a)^3+1/8*\cos(b*x+a)*\sin(b*x+a)+1/8*b*x+1/8*a)) \end{aligned}$$

Maxima [B] time = 1.30413, size = 597, normalized size = 5.69

$$\frac{32(4bx+4a-\sin(4bx+4a))c^3 - \frac{96(4bx+4a-\sin(4bx+4a))ac^2d}{b} + \frac{96(4bx+4a-\sin(4bx+4a))a^2cd^2}{b^2} - \frac{32(4bx+4a-\sin(4bx+4a))a^3d^3}{b^3}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")

[Out] $\frac{1}{1024}*(32*(4*b*x + 4*a - \sin(4*b*x + 4*a))*c^3 - 96*(4*b*x + 4*a - \sin(4*b*x + 4*a))*a*c^2*d/b + 96*(4*b*x + 4*a - \sin(4*b*x + 4*a))*a^2*c*d^2/b^2 - 32*(4*b*x + 4*a - \sin(4*b*x + 4*a))*a^3*d^3/b^3 + 24*(8*(b*x + a)^2 - 4*(b*x + a)*\sin(4*b*x + 4*a) - \cos(4*b*x + 4*a))*c^2*d/b - 48*(8*(b*x + a)^2 - 4*(b*x + a)*\sin(4*b*x + 4*a) - \cos(4*b*x + 4*a))*a*c*d^2/b^2 + 24*(8*(b*x + a)^2 - 4*(b*x + a)*\sin(4*b*x + 4*a) - \cos(4*b*x + 4*a))*a^2*d^3/b^3 + 4*(32*(b*x + a)^3 - 12*(b*x + a)*\cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*\sin(4*b*x + 4*a))*c*d^2/b^2 - 4*(32*(b*x + a)^3 - 12*(b*x + a)*\cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*\sin(4*b*x + 4*a))*a*d^3/b^3 + (32*(b*x + a)^4 - 3*(8*(b*x + a)^2 - 1)*\cos(4*b*x + 4*a) - 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*\sin(4*b*x + 4*a))*d^3/b^3)/b$

Fricas [B] time = 0.513568, size = 647, normalized size = 6.16

$$\frac{4b^4d^3x^4 + 16b^4cd^2x^3 - 3(8b^2d^3x^2 + 16b^2cd^2x + 8b^2c^2d - d^3)\cos(bx+a)^4 + 3(8b^4c^2d - b^2d^3)x^2 + 3(8b^2d^3x^2 + 16b^2cd^2x + 8b^2c^2d - d^3)\sin(bx+a)^4}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")

[Out] $\frac{1}{128}*(4*b^4*d^3*x^4 + 16*b^4*c*d^2*x^3 - 3*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*\cos(b*x + a)^4 + 3*(8*b^4*c^2*d - b^2*d^3)*x^2 + 3*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*\cos(b*x + a)^2 + 2*(8*b^4*c^3 - 3*b^2*c*d^2)*x - 2*(2*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 8*b^3*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*\cos(b*x + a)^3 - (8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 8*b^3*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*\cos(b*x + a))*\sin(b*x + a))/b^4$

Sympy [A] time = 9.97599, size = 813, normalized size = 7.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*cos(b*x+a)**2*sin(b*x+a)**2,x)

[Out] Piecewise((c**3*x*sin(a + b*x)**4/8 + c**3*x*sin(a + b*x)**2*cos(a + b*x)**2/4 + c**3*x*cos(a + b*x)**4/8 + 3*c**2*d*x**2*sin(a + b*x)**4/16 + 3*c**2*d*x**2*sin(a + b*x)**2*cos(a + b*x)**2/8 + 3*c**2*d*x**2*cos(a + b*x)**4/16 + c*d**2*x**3*sin(a + b*x)**4/8 + c*d**2*x**3*sin(a + b*x)**2*cos(a + b*x)**2/4 + c*d**2*x**3*cos(a + b*x)**4/8 + d**3*x**4*sin(a + b*x)**4/32 + d**3*x**4*sin(a + b*x)**2*cos(a + b*x)**2/16 + d**3*x**4*cos(a + b*x)**4/32 + c**3*sin(a + b*x)**3*cos(a + b*x)/(8*b) - c**3*sin(a + b*x)*cos(a + b*x)**3/(8*b) + 3*c**2*d*x*sin(a + b*x)**3*cos(a + b*x)/(8*b) - 3*c**2*d*x*sin(a + b*x)*cos(a + b*x)**3/(8*b) + 3*c*d**2*x**2*sin(a + b*x)**3*cos(a + b*x)/(8*b) - 3*c*d**2*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b) + d**3*x**3*sin(a + b*x)**3*cos(a + b*x)/(8*b) - d**3*x**3*sin(a + b*x)*cos(a + b*x)**3/(8*b) + 3*c**2*d*sin(a + b*x)**2*cos(a + b*x)**2/(16*b**2) - 3*c*d**2*x*sin(a + b*x)**4/(64*b**2) + 9*c*d**2*x*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**2) - 3*c*d**2*x*cos(a + b*x)**4/(64*b**2) - 3*d**3*x**2*sin(a + b*x)**4/(128*b**2) + 9*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(64*b**2) - 3*d**3*x**2*cos(a + b*x)**4/(128*b**2) - 3*c*d**2*sin(a + b*x)**3*cos(a + b*x)/(64*b**3) + 3*c*d**2*sin(a + b*x)*cos(a + b*x)**3/(64*b**3) - 3*d**3*x*sin(a + b*x)**3*cos(a + b*x)/(64*b**3) + 3*d**3*x*sin(a + b*x)*cos(a + b*x)**3/(64*b**3) - 3*d**3*sin(a + b*x)**2*cos(a + b*x)**2/(128*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**2*cos(a)**2, True))

Giac [A] time = 1.14039, size = 207, normalized size = 1.97

$$\frac{1}{32}d^3x^4 + \frac{1}{8}cd^2x^3 + \frac{3}{16}c^2dx^2 + \frac{1}{8}c^3x - \frac{3(8b^2d^3x^2 + 16b^2cd^2x + 8b^2c^2d - d^3)\cos(4bx + 4a)}{1024b^4} - \frac{(8b^3d^3x^3 + 24b^3cd^2x^2 + 24b^3c^2dx + 8b^3c^3 - 3bd^3x - 3b^2cd^2)\sin(4bx + 4a)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")

[Out] 1/32*d^3*x^4 + 1/8*c*d^2*x^3 + 3/16*c^2*d*x^2 + 1/8*c^3*x - 3/1024*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(4*b*x + 4*a)/b^4 - 1/256*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 3*b*d^3*x - 3*b^2*c*d^2)*sin(4*b*x + 4*a)/b^4

3.82 $\int (c + dx)^2 \cos^2(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=79

$$-\frac{d(c + dx) \cos(4a + 4bx)}{64b^2} + \frac{d^2 \sin(4a + 4bx)}{256b^3} - \frac{(c + dx)^2 \sin(4a + 4bx)}{32b} + \frac{(c + dx)^3}{24d}$$

[Out] $(c + d*x)^3/(24*d) - (d*(c + d*x)*\text{Cos}[4*a + 4*b*x])/(64*b^2) + (d^2*\text{Sin}[4*a + 4*b*x])/(256*b^3) - ((c + d*x)^2*\text{Sin}[4*a + 4*b*x])/(32*b)$

Rubi [A] time = 0.122533, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3296, 2637}

$$-\frac{d(c + dx) \cos(4a + 4bx)}{64b^2} + \frac{d^2 \sin(4a + 4bx)}{256b^3} - \frac{(c + dx)^2 \sin(4a + 4bx)}{32b} + \frac{(c + dx)^3}{24d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^2*\text{Cos}[a + b*x]^2*\text{Sin}[a + b*x]^2,x]$

[Out] $(c + d*x)^3/(24*d) - (d*(c + d*x)*\text{Cos}[4*a + 4*b*x])/(64*b^2) + (d^2*\text{Sin}[4*a + 4*b*x])/(256*b^3) - ((c + d*x)^2*\text{Sin}[4*a + 4*b*x])/(32*b)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] := \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] := -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

$\text{Int}[\text{sin}[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := \text{Simp}[\text{Sin}[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \cos^2(a + bx) \sin^2(a + bx) dx &= \int \left(\frac{1}{8}(c + dx)^2 - \frac{1}{8}(c + dx)^2 \cos(4a + 4bx) \right) dx \\ &= \frac{(c + dx)^3}{24d} - \frac{1}{8} \int (c + dx)^2 \cos(4a + 4bx) dx \\ &= \frac{(c + dx)^3}{24d} - \frac{(c + dx)^2 \sin(4a + 4bx)}{32b} + \frac{d \int (c + dx) \sin(4a + 4bx) dx}{16b} \\ &= \frac{(c + dx)^3}{24d} - \frac{d(c + dx) \cos(4a + 4bx)}{64b^2} - \frac{(c + dx)^2 \sin(4a + 4bx)}{32b} + \frac{d^2 \int \cos(4a + 4bx) dx}{16b} \\ &= \frac{(c + dx)^3}{24d} - \frac{d(c + dx) \cos(4a + 4bx)}{64b^2} + \frac{d^2 \sin(4a + 4bx)}{256b^3} - \frac{(c + dx)^2 \sin(4a + 4bx)}{32b} \end{aligned}$$

Mathematica [A] time = 0.43819, size = 77, normalized size = 0.97

$$\frac{-3 \sin(4(a + bx)) (8b^2(c + dx)^2 - d^2) - 12bd(c + dx) \cos(4(a + bx)) + 32b^3x(3c^2 + 3cdx + d^2x^2)}{768b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^2,x]

[Out] (32*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) - 12*b*d*(c + d*x)*Cos[4*(a + b*x)] - 3*(-d^2 + 8*b^2*(c + d*x)^2)*Sin[4*(a + b*x)])/(768*b^3)

Maple [B] time = 0.02, size = 519, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x)

[Out] 1/b*(1/b^2*d^2*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a-1/12*(b*x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a))-2/b^2*a*d^2*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)+2/b*c*d*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)+1/b^2*a^2*d^2*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)-2/b*a*c*d*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)+c^2*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a))

Maxima [B] time = 1.22758, size = 313, normalized size = 3.96

$$\frac{24(4bx + 4a - \sin(4bx + 4a))c^2 - \frac{48(4bx + 4a - \sin(4bx + 4a))acd}{b} + \frac{24(4bx + 4a - \sin(4bx + 4a))a^2d^2}{b^2} + \frac{12(8(bx+a)^2 - 4(bx+a)\sin(4bx + 4a) - \cos(4bx + 4a))d^2}{b}}{768b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")

[Out] 1/768*(24*(4*b*x + 4*a - sin(4*b*x + 4*a))*c^2 - 48*(4*b*x + 4*a - sin(4*b*x + 4*a))*a*c*d/b + 24*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^2*d^2/b^2 + 12*(8*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*c*d/b - 12*(8*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a*d^2/b^2 + (32*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a))*d^2/b^2)/b

Fricas [B] time = 0.500507, size = 398, normalized size = 5.04

$$\frac{8b^3d^2x^3 + 24b^3cdx^2 - 24(bd^2x + bcd)\cos(bx + a)^4 + 24(bd^2x + bcd)\cos(bx + a)^2 + 3(8b^3c^2 - bd^2)x - 3(2(8b^2d^2x^2 + 16b^2cdx + 8b^2c^2 - d^2)\cos(bx + a)^3 - (8b^2d^2x^2 + 16b^2cdx + 8b^2c^2 - d^2)\cos(bx + a))\sin(bx + a)}{192b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")

[Out] 1/192*(8*b^3*d^2*x^3 + 24*b^3*c*d*x^2 - 24*(b*d^2*x + b*c*d)*cos(b*x + a)^4 + 24*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 3*(8*b^3*c^2 - b*d^2)*x - 3*(2*(8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(b*x + a)^3 - (8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(b*x + a))*sin(b*x + a))/b^3

Sympy [A] time = 5.12042, size = 484, normalized size = 6.13

$$\left\{ \begin{array}{l} \frac{c^2x \sin^4(a+bx)}{8} + \frac{c^2x \sin^2(a+bx) \cos^2(a+bx)}{4} + \frac{c^2x \cos^4(a+bx)}{8} + \frac{cdx^2 \sin^4(a+bx)}{8} + \frac{cdx^2 \sin^2(a+bx) \cos^2(a+bx)}{4} + \frac{cdx^2 \cos^4(a+bx)}{8} + \frac{d^2x^3 \sin^4(a+bx)}{24} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3} \right) \sin^2(a) \cos^2(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*cos(b*x+a)**2*sin(b*x+a)**2,x)

[Out] Piecewise((c**2*x*sin(a + b*x)**4/8 + c**2*x*sin(a + b*x)**2*cos(a + b*x)**2/4 + c**2*x*cos(a + b*x)**4/8 + c*d*x**2*sin(a + b*x)**4/8 + c*d*x**2*sin(a + b*x)**2*cos(a + b*x)**2/4 + c*d*x**2*cos(a + b*x)**4/8 + d**2*x**3*sin(a + b*x)**4/24 + d**2*x**3*sin(a + b*x)**2*cos(a + b*x)**2/12 + d**2*x**3*cos(a + b*x)**4/24 + c**2*sin(a + b*x)**3*cos(a + b*x)/(8*b) - c**2*sin(a + b*x)*cos(a + b*x)**3/(8*b) + c*d*x*sin(a + b*x)**3*cos(a + b*x)/(4*b) - c*d*x*sin(a + b*x)*cos(a + b*x)**3/(4*b) + d**2*x**2*sin(a + b*x)**3*cos(a + b*x)/(8*b) - d**2*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b) + c*d*sin(a + b*x)**2*cos(a + b*x)**2/(8*b**2) - d**2*x*sin(a + b*x)**4/(64*b**2) + 3*d**2*x*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**2) - d**2*x*cos(a + b*x)**4/(64*b**2) - d**2*sin(a + b*x)**3*cos(a + b*x)/(64*b**3) + d**2*sin(a + b*x)*cos(a + b*x)**3/(64*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**2*cos(a)**2, True))

Giac [A] time = 1.08375, size = 127, normalized size = 1.61

$$\frac{1}{24}d^2x^3 + \frac{1}{8}cdx^2 + \frac{1}{8}c^2x - \frac{(bd^2x + bcd)\cos(4bx + 4a)}{64b^3} - \frac{(8b^2d^2x^2 + 16b^2cdx + 8b^2c^2 - d^2)\sin(4bx + 4a)}{256b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")

[Out] 1/24*d^2*x^3 + 1/8*c*d*x^2 + 1/8*c^2*x - 1/64*(b*d^2*x + b*c*d)*cos(4*b*x + 4*a)/b^3 - 1/256*(8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*sin(4*b*x + 4*a)/b^3

3.83 $\int (c + dx) \cos^2(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=53

$$-\frac{d \cos(4a + 4bx)}{128b^2} - \frac{(c + dx) \sin(4a + 4bx)}{32b} + \frac{(c + dx)^2}{16d}$$

[Out] $(c + d*x)^2/(16*d) - (d*\text{Cos}[4*a + 4*b*x])/(128*b^2) - ((c + d*x)*\text{Sin}[4*a + 4*b*x])/(32*b)$

Rubi [A] time = 0.0537823, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4406, 3296, 2638}

$$-\frac{d \cos(4a + 4bx)}{128b^2} - \frac{(c + dx) \sin(4a + 4bx)}{32b} + \frac{(c + dx)^2}{16d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)*\text{Cos}[a + b*x]^2*\text{Sin}[a + b*x]^2, x]$

[Out] $(c + d*x)^2/(16*d) - (d*\text{Cos}[4*a + 4*b*x])/(128*b^2) - ((c + d*x)*\text{Sin}[4*a + 4*b*x])/(32*b)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /; \text{FreeQ}\{a, b, c, d, m, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[p, 0]$

Rule 3296

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f, x\} \ \&\& \ \text{GtQ}[m, 0]$

Rule 2638

$\text{Int}[\text{sin}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}\{c, d, x\}$

Rubi steps

$$\begin{aligned} \int (c + dx) \cos^2(a + bx) \sin^2(a + bx) dx &= \int \left(\frac{1}{8}(c + dx) - \frac{1}{8}(c + dx) \cos(4a + 4bx) \right) dx \\ &= \frac{(c + dx)^2}{16d} - \frac{1}{8} \int (c + dx) \cos(4a + 4bx) dx \\ &= \frac{(c + dx)^2}{16d} - \frac{(c + dx) \sin(4a + 4bx)}{32b} + \frac{d \int \sin(4a + 4bx) dx}{32b} \\ &= \frac{(c + dx)^2}{16d} - \frac{d \cos(4a + 4bx)}{128b^2} - \frac{(c + dx) \sin(4a + 4bx)}{32b} \end{aligned}$$

Mathematica [A] time = 0.297987, size = 54, normalized size = 1.02

$$\frac{8(a+bx)(ad-2bc-bdx)+4b(c+dx)\sin(4(a+bx))+d\cos(4(a+bx))}{128b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]

[Out] $-(8*(a + b*x)*(-2*b*c + a*d - b*d*x) + d*\cos[4*(a + b*x)] + 4*b*(c + d*x)*\sin[4*(a + b*x)])/(128*b^2)$

Maple [B] time = 0.02, size = 194, normalized size = 3.7

$$\frac{1}{b} \left(\frac{d}{b} \left((bx+a) \left(-\frac{\cos(bx+a)\sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right) - \frac{(bx+a)^2}{16} + \frac{(\sin(bx+a))^2}{16} - (bx+a) \left(-\frac{\cos(bx+a)}{4} \left(\sin(bx+a) \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x)

[Out] $1/b*(d/b*((b*x+a)*(-1/2*\cos(b*x+a)*\sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/16*\sin(b*x+a)^2-(b*x+a)*(-1/4*(\sin(b*x+a)^3+3/2*\sin(b*x+a))*\cos(b*x+a)+3/8*b*x+3/8*a)-1/16*\sin(b*x+a)^4)-1/b*d*a*(-1/4*\sin(b*x+a)*\cos(b*x+a)^3+1/8*\cos(b*x+a)*\sin(b*x+a)+1/8*b*x+1/8*a)+c*(-1/4*\sin(b*x+a)*\cos(b*x+a)^3+1/8*\cos(b*x+a)*\sin(b*x+a)+1/8*b*x+1/8*a))$

Maxima [B] time = 1.25609, size = 130, normalized size = 2.45

$$\frac{4(4bx+4a-\sin(4bx+4a))c - \frac{4(4bx+4a-\sin(4bx+4a))ad}{b} + \frac{(8(bx+a)^2-4(bx+a)\sin(4bx+4a)-\cos(4bx+4a))d}{b}}{128b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")

[Out] $1/128*(4*(4*b*x + 4*a - \sin(4*b*x + 4*a))*c - 4*(4*b*x + 4*a - \sin(4*b*x + 4*a))*a*d/b + (8*(b*x + a)^2 - 4*(b*x + a)*\sin(4*b*x + 4*a) - \cos(4*b*x + 4*a))*d/b)/b$

Fricas [A] time = 0.481737, size = 204, normalized size = 3.85

$$\frac{b^2dx^2 - d\cos(bx+a)^4 + 2b^2cx + d\cos(bx+a)^2 - 2(2(bdx+bc)\cos(bx+a)^3 - (bdx+bc)\cos(bx+a))\sin(bx+a)}{16b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")

[Out] $1/16*(b^2*d*x^2 - d*\cos(b*x + a)^4 + 2*b^2*c*x + d*\cos(b*x + a)^2 - 2*(2*(b*d*x + b*c)*\cos(b*x + a)^3 - (b*d*x + b*c)*\cos(b*x + a))*\sin(b*x + a))/b^2$

Sympy [A] time = 2.355, size = 231, normalized size = 4.36

$$\left\{ \begin{array}{l} \frac{cx \sin^4(a+bx)}{8} + \frac{cx \sin^2(a+bx) \cos^2(a+bx)}{4} + \frac{cx \cos^4(a+bx)}{8} + \frac{dx^2 \sin^4(a+bx)}{16} + \frac{dx^2 \sin^2(a+bx) \cos^2(a+bx)}{8} + \frac{dx^2 \cos^4(a+bx)}{16} + \frac{c \sin^3(a+bx) \cos(a+bx)}{8b} \\ \left(cx + \frac{dx^2}{2} \right) \sin^2(a) \cos^2(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)**2*sin(b*x+a)**2,x)

[Out] Piecewise((c*x*sin(a + b*x)**4/8 + c*x*sin(a + b*x)**2*cos(a + b*x)**2/4 + c*x*cos(a + b*x)**4/8 + d*x**2*sin(a + b*x)**4/16 + d*x**2*sin(a + b*x)**2*cos(a + b*x)**2/8 + d*x**2*cos(a + b*x)**4/16 + c*sin(a + b*x)**3*cos(a + b*x)/(8*b) - c*sin(a + b*x)*cos(a + b*x)**3/(8*b) + d*x*sin(a + b*x)**3*cos(a + b*x)/(8*b) - d*x*sin(a + b*x)*cos(a + b*x)**3/(8*b) + d*sin(a + b*x)**2*cos(a + b*x)**2/(16*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*sin(a)**2*cos(a)**2, True))

Giac [A] time = 1.08702, size = 65, normalized size = 1.23

$$\frac{1}{16} dx^2 + \frac{1}{8} cx - \frac{d \cos(4bx + 4a)}{128b^2} - \frac{(bdx + bc) \sin(4bx + 4a)}{32b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")

[Out] 1/16*d*x^2 + 1/8*c*x - 1/128*d*cos(4*b*x + 4*a)/b^2 - 1/32*(b*d*x + b*c)*sin(4*b*x + 4*a)/b^2

$$3.84 \quad \int \frac{\cos^2(a+bx) \sin^2(a+bx)}{c+dx} dx$$

Optimal. Leaf size=78

$$-\frac{\cos\left(4a - \frac{4bc}{d}\right) \operatorname{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{8d} + \frac{\sin\left(4a - \frac{4bc}{d}\right) \operatorname{Si}\left(\frac{4bc}{d} + 4bx\right)}{8d} + \frac{\log(c+dx)}{8d}$$

[Out] $-(\operatorname{Cos}[4*a - (4*b*c)/d]*\operatorname{CosIntegral}[(4*b*c)/d + 4*b*x])/(8*d) + \operatorname{Log}[c + d*x]/(8*d) + (\operatorname{Sin}[4*a - (4*b*c)/d]*\operatorname{SinIntegral}[(4*b*c)/d + 4*b*x])/(8*d)$

Rubi [A] time = 0.139741, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4406, 3303, 3299, 3302}

$$-\frac{\cos\left(4a - \frac{4bc}{d}\right) \operatorname{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{8d} + \frac{\sin\left(4a - \frac{4bc}{d}\right) \operatorname{Si}\left(\frac{4bc}{d} + 4bx\right)}{8d} + \frac{\log(c+dx)}{8d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Cos}[a + b*x]^2*\operatorname{Sin}[a + b*x]^2)/(c + d*x), x]$

[Out] $-(\operatorname{Cos}[4*a - (4*b*c)/d]*\operatorname{CosIntegral}[(4*b*c)/d + 4*b*x])/(8*d) + \operatorname{Log}[c + d*x]/(8*d) + (\operatorname{Sin}[4*a - (4*b*c)/d]*\operatorname{SinIntegral}[(4*b*c)/d + 4*b*x])/(8*d)$

Rule 4406

$\operatorname{Int}[\operatorname{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\operatorname{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandTrigReduce}[(c + d*x)^m, \operatorname{Sin}[a + b*x]^n*\operatorname{Cos}[a + b*x]^p, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, m\}, x\} \ \&\& \ \operatorname{IGtQ}[n, 0] \ \&\& \ \operatorname{IGtQ}[p, 0]$

Rule 3303

$\operatorname{Int}[\operatorname{sin}[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] \rightarrow \operatorname{Dist}[\operatorname{Cos}[(d*e - c*f)/d], \operatorname{Int}[\operatorname{Sin}[(c*f)/d + f*x]/(c + d*x), x], x] + \operatorname{Dist}[\operatorname{Sin}[(d*e - c*f)/d], \operatorname{Int}[\operatorname{Cos}[(c*f)/d + f*x]/(c + d*x), x], x] /;$ $\operatorname{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \operatorname{NeQ}[d*e - c*f, 0]$

Rule 3299

$\operatorname{Int}[\operatorname{sin}[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{SinIntegral}[e + f*x]/d, x] /;$ $\operatorname{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \operatorname{EqQ}[d*e - c*f, 0]$

Rule 3302

$\operatorname{Int}[\operatorname{sin}[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{CosIntegral}[e - \operatorname{Pi}/2 + f*x]/d, x] /;$ $\operatorname{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \operatorname{EqQ}[d*(e - \operatorname{Pi}/2) - c*f, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx} dx &= \int \left(\frac{1}{8(c+dx)} - \frac{\cos(4a+4bx)}{8(c+dx)} \right) dx \\
&= \frac{\log(c+dx)}{8d} - \frac{1}{8} \int \frac{\cos(4a+4bx)}{c+dx} dx \\
&= \frac{\log(c+dx)}{8d} - \frac{1}{8} \cos\left(4a - \frac{4bc}{d}\right) \int \frac{\cos\left(\frac{4bc}{d} + 4bx\right)}{c+dx} dx + \frac{1}{8} \sin\left(4a - \frac{4bc}{d}\right) \int \frac{\sin\left(\frac{4bc}{d} + 4bx\right)}{c+dx} dx \\
&= -\frac{\cos\left(4a - \frac{4bc}{d}\right) \operatorname{Ci}\left(\frac{4bc}{d} + 4bx\right)}{8d} + \frac{\log(c+dx)}{8d} + \frac{\sin\left(4a - \frac{4bc}{d}\right) \operatorname{Si}\left(\frac{4bc}{d} + 4bx\right)}{8d}
\end{aligned}$$

Mathematica [A] time = 0.164507, size = 65, normalized size = 0.83

$$\frac{-\cos\left(4a - \frac{4bc}{d}\right) \operatorname{CosIntegral}\left(\frac{4b(c+dx)}{d}\right) + \sin\left(4a - \frac{4bc}{d}\right) \operatorname{Si}\left(\frac{4b(c+dx)}{d}\right) + \log(c+dx)}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x), x]

[Out] $(-\cos[4a - (4b*c)/d] \operatorname{CosIntegral}[(4b*(c + d*x))/d]) + \log[c + d*x] + \sin[4a - (4b*c)/d] \operatorname{SinIntegral}[(4b*(c + d*x))/d]) / (8*d)$

Maple [A] time = 0.026, size = 105, normalized size = 1.4

$$-\frac{1}{8d} \operatorname{Si}\left(4bx + 4a + 4\frac{-ad+bc}{d}\right) \sin\left(4\frac{-ad+bc}{d}\right) - \frac{1}{8d} \operatorname{Ci}\left(4bx + 4a + 4\frac{-ad+bc}{d}\right) \cos\left(4\frac{-ad+bc}{d}\right) + \frac{\ln((bx+a)d - a*d+bc)/d}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c), x)

[Out] $-1/8*\operatorname{Si}(4*b*x+4*a+4*(-a*d+b*c)/d)*\sin(4*(-a*d+b*c)/d)/d-1/8*\operatorname{Ci}(4*b*x+4*a+4*(-a*d+b*c)/d)*\cos(4*(-a*d+b*c)/d)/d+1/8*\ln((b*x+a)*d-a*d+b*c)/d$

Maxima [C] time = 1.45638, size = 216, normalized size = 2.77

$$\frac{b\left(E_1\left(\frac{4i bc+4i (bx+a)d-4i ad}{d}\right) + E_1\left(-\frac{4i bc+4i (bx+a)d-4i ad}{d}\right)\right) \cos\left(-\frac{4(bc-ad)}{d}\right) + b\left(-i E_1\left(\frac{4i bc+4i (bx+a)d-4i ad}{d}\right) + i E_1\left(-\frac{4i bc+4i (bx+a)d-4i ad}{d}\right)\right) \sin\left(-\frac{4(bc-ad)}{d}\right)}{16bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c), x, algorithm="maxima")

[Out] $1/16*(b*(\exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) + \exp_integral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*\cos(-4*(b*c - a*d)/d) + b*(-I*\exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) + I*\exp_integral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*\sin(-4*(b*c - a*d)/d) + 2*b*\log(b*c + (b*x + a)*d - a*d)/(b*d)$

Fricas [A] time = 0.487259, size = 239, normalized size = 3.06

$$\frac{\left(\operatorname{Ci}\left(\frac{4(bdx+bc)}{d}\right) + \operatorname{Ci}\left(-\frac{4(bdx+bc)}{d}\right)\right) \cos\left(-\frac{4(bc-ad)}{d}\right) - 2 \sin\left(-\frac{4(bc-ad)}{d}\right) \operatorname{Si}\left(\frac{4(bdx+bc)}{d}\right) - 2 \log(dx+c)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c), x, algorithm="fricas")

[Out] -1/16*((cos_integral(4*(b*d*x + b*c)/d) + cos_integral(-4*(b*d*x + b*c)/d)) * cos(-4*(b*c - a*d)/d) - 2*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) - 2*log(d*x + c))/d

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a + bx) \cos^2(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c), x)

[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x), x)

Giac [C] time = 1.19668, size = 903, normalized size = 11.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c), x, algorithm="giac")

[Out] 1/16*(2*log(abs(d*x + c))*tan(2*a)^2*tan(2*b*c/d)^2 - real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 - real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) - 2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(2*b*c/d) - 2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d)^2 + 2*log(abs(d*x + c))*tan(2*a)^2 + real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2 + real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2 - 4*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d) - 4*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d) + 2*log(abs(d*x + c))*tan(2*b*c/d)^2 + real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2 + real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a) - 2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a) + 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a) - 2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d) + 2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d) - 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d) + 2*log(abs(d*x + c)) - real_part(cos_integral(4*b*x + 4*b*c/d)) - real_part(cos_integral(-4*b*x - 4*b*c/d)))/(d*tan(2*a)^2*tan(2*b*c/d)^2 + d*tan(2*a)^2 + d*tan(2*b*c/d)^2 + d)

$$3.85 \quad \int \frac{\cos^2(a+bx) \sin^2(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=104

$$\frac{b \sin\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{2d^2} + \frac{b \cos\left(4a - \frac{4bc}{d}\right) \text{Si}\left(\frac{4bc}{d} + 4bx\right)}{2d^2} + \frac{\cos(4a + 4bx)}{8d(c + dx)} - \frac{1}{8d(c + dx)}$$

[Out] -1/(8*d*(c + d*x)) + Cos[4*a + 4*b*x]/(8*d*(c + d*x)) + (b*CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(2*d^2) + (b*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(2*d^2)

Rubi [A] time = 0.168982, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b \sin\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{2d^2} + \frac{b \cos\left(4a - \frac{4bc}{d}\right) \text{Si}\left(\frac{4bc}{d} + 4bx\right)}{2d^2} + \frac{\cos(4a + 4bx)}{8d(c + dx)} - \frac{1}{8d(c + dx)}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^2,x]

[Out] -1/(8*d*(c + d*x)) + Cos[4*a + 4*b*x]/(8*d*(c + d*x)) + (b*CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(2*d^2) + (b*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(2*d^2)

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_.)]^(p_.)*((c_.) + (d_.)*(x_.))^(m_.)*Sin[(a_.) + (b_.)*(x_.)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3297

Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] :> Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2} dx &= \int \left(\frac{1}{8(c+dx)^2} - \frac{\cos(4a+4bx)}{8(c+dx)^2} \right) dx \\ &= -\frac{1}{8d(c+dx)} - \frac{1}{8} \int \frac{\cos(4a+4bx)}{(c+dx)^2} dx \\ &= -\frac{1}{8d(c+dx)} + \frac{\cos(4a+4bx)}{8d(c+dx)} + \frac{b \int \frac{\sin(4a+4bx)}{c+dx} dx}{2d} \\ &= -\frac{1}{8d(c+dx)} + \frac{\cos(4a+4bx)}{8d(c+dx)} + \frac{\left(b \cos\left(4a - \frac{4bc}{d}\right)\right) \int \frac{\sin\left(\frac{4bc}{d} + 4bx\right)}{c+dx} dx}{2d} + \frac{\left(b \sin\left(4a - \frac{4bc}{d}\right)\right) \int \frac{\cos\left(\frac{4bc}{d} + 4bx\right)}{c+dx} dx}{2d} \\ &= -\frac{1}{8d(c+dx)} + \frac{\cos(4a+4bx)}{8d(c+dx)} + \frac{b \operatorname{Ci}\left(\frac{4bc}{d} + 4bx\right) \sin\left(4a - \frac{4bc}{d}\right)}{2d^2} + \frac{b \cos\left(4a - \frac{4bc}{d}\right) \operatorname{Si}\left(\frac{4bc}{d} + 4bx\right)}{2d^2} \end{aligned}$$

Mathematica [A] time = 0.462863, size = 81, normalized size = 0.78

$$\frac{4b \sin\left(4a - \frac{4bc}{d}\right) \operatorname{CosIntegral}\left(\frac{4b(c+dx)}{d}\right) + 4b \cos\left(4a - \frac{4bc}{d}\right) \operatorname{Si}\left(\frac{4b(c+dx)}{d}\right) + \frac{d(\cos(4(a+bx))-1)}{c+dx}}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] ((d*(-1 + Cos[4*(a + b*x)]))/(c + d*x) + 4*b*CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + 4*b*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(8*d^2)
```

Maple [A] time = 0.025, size = 156, normalized size = 1.5

$$\frac{1}{b} \left(-\frac{b^2}{32} \left(-4 \frac{\cos(4bx+4a)}{((bx+a)d-ad+bc)d} - 4 \frac{1}{d} \left(4 \frac{1}{d} \operatorname{Si}\left(4bx+4a+4\frac{-ad+bc}{d}\right) \cos\left(4\frac{-ad+bc}{d}\right) - 4 \frac{1}{d} \operatorname{Ci}\left(4bx+4a+4\frac{-ad+bc}{d}\right) \sin\left(4\frac{-ad+bc}{d}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] 1/b*(-1/32*b^2*(-4*cos(4*b*x+4*a)/((b*x+a)*d-a*d+b*c)/d-4*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)/d)-1/8*b^2/((b*x+a)*d-a*d+b*c)/d
```

Maxima [C] time = 1.46927, size = 231, normalized size = 2.22

$$\frac{64b^2 \left(E_2 \left(\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) + E_2 \left(-\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) \right) \cos \left(-\frac{4(bc-ad)}{d} \right) - b^2 \left(64i E_2 \left(\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) - 64i E_2 \left(-\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) \right) \sin \left(-\frac{4(bc-ad)}{d} \right)}{1024 (bcd + (bx+a)d^2 - ad^2)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")

[Out] 1/1024*(64*b^2*(exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) + exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*(b*c - a*d)/d) - b^2*(64*I*exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) - 64*I*exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*sin(-4*(b*c - a*d)/d) - 128*b^2)/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)

Fricas [A] time = 0.54973, size = 346, normalized size = 3.33

$$\frac{4 d \cos (b x+a)^4-4 d \cos (b x+a)^2+2(b d x+b c) \cos \left(-\frac{4(b c-a d)}{d}\right) \operatorname{Si}\left(\frac{4(b d x+b c)}{d}\right)+\left((b d x+b c) \operatorname{Ci}\left(\frac{4(b d x+b c)}{d}\right)+(b d x+b c)\right)}{4\left(d^3 x+c d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")

[Out] 1/4*(4*d*cos(b*x + a)^4 - 4*d*cos(b*x + a)^2 + 2*(b*d*x + b*c)*cos(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + ((b*d*x + b*c)*cos_integral(4*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-4*(b*d*x + b*c)/d))*sin(-4*(b*c - a*d)/d))/(d^3*x + c*d^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a + b x) \cos^2(a + b x)}{(c + d x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c)**2,x)

[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (b x+a)^2 \sin (b x+a)^2}{(d x+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(cos(b*x + a)^2*sin(b*x + a)^2/(d*x + c)^2, x)

$$3.86 \quad \int \frac{\cos^2(a+bx) \sin^2(a+bx)}{(c+dx)^3} dx$$

Optimal. Leaf size=127

$$\frac{b^2 \cos\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{d^3} - \frac{b^2 \sin\left(4a - \frac{4bc}{d}\right) \text{Si}\left(\frac{4bc}{d} + 4bx\right)}{d^3} - \frac{b \sin(4a + 4bx)}{4d^2(c + dx)} + \frac{\cos(4a + 4bx)}{16d(c + dx)^2} - \frac{1}{16d^2(c + dx)^3}$$

[Out] $-1/(16*d*(c + d*x)^2) + \text{Cos}[4*a + 4*b*x]/(16*d*(c + d*x)^2) + (b^2*\text{Cos}[4*a - (4*b*c)/d]*\text{CosIntegral}[(4*b*c)/d + 4*b*x])/d^3 - (b*\text{Sin}[4*a + 4*b*x])/(4*d^2*(c + d*x)) - (b^2*\text{Sin}[4*a - (4*b*c)/d]*\text{SinIntegral}[(4*b*c)/d + 4*b*x])/d^3$

Rubi [A] time = 0.198285, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b^2 \cos\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{d^3} - \frac{b^2 \sin\left(4a - \frac{4bc}{d}\right) \text{Si}\left(\frac{4bc}{d} + 4bx\right)}{d^3} - \frac{b \sin(4a + 4bx)}{4d^2(c + dx)} + \frac{\cos(4a + 4bx)}{16d(c + dx)^2} - \frac{1}{16d^2(c + dx)^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[a + b*x]^2*\text{Sin}[a + b*x]^2)/(c + d*x)^3, x]$

[Out] $-1/(16*d*(c + d*x)^2) + \text{Cos}[4*a + 4*b*x]/(16*d*(c + d*x)^2) + (b^2*\text{Cos}[4*a - (4*b*c)/d]*\text{CosIntegral}[(4*b*c)/d + 4*b*x])/d^3 - (b*\text{Sin}[4*a + 4*b*x])/(4*d^2*(c + d*x)) - (b^2*\text{Sin}[4*a - (4*b*c)/d]*\text{SinIntegral}[(4*b*c)/d + 4*b*x])/d^3$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)*((c_.) + (d_.)*(x_.))^{(m_.)*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] := \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3297

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] := \text{Simp}[(c + d*x)^{(m + 1)*\text{Sin}[e + f*x]} / (d*(m + 1)), x] - \text{Dist}[f / (d*(m + 1)), \text{Int}[(c + d*x)^{(m + 1)*\text{Cos}[e + f*x]}, x], x] /;$ FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)] / ((c_.) + (d_.)*(x_.)), x_Symbol] := \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x] / (c + d*x), x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x] / (c + d*x), x], x] /;$ FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)] / ((c_.) + (d_.)*(x_.)), x_Symbol] := \text{Simp}[\text{SinIntegral}[e + f*x]/d, x] /;$ FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\cos^2(a + bx) \sin^2(a + bx)}{(c + dx)^3} dx &= \int \left(\frac{1}{8(c + dx)^3} - \frac{\cos(4a + 4bx)}{8(c + dx)^3} \right) dx \\
 &= -\frac{1}{16d(c + dx)^2} - \frac{1}{8} \int \frac{\cos(4a + 4bx)}{(c + dx)^3} dx \\
 &= -\frac{1}{16d(c + dx)^2} + \frac{\cos(4a + 4bx)}{16d(c + dx)^2} + \frac{b \int \frac{\sin(4a + 4bx)}{(c + dx)^2} dx}{4d} \\
 &= -\frac{1}{16d(c + dx)^2} + \frac{\cos(4a + 4bx)}{16d(c + dx)^2} - \frac{b \sin(4a + 4bx)}{4d^2(c + dx)} + \frac{b^2 \int \frac{\cos(4a + 4bx)}{c + dx} dx}{d^2} \\
 &= -\frac{1}{16d(c + dx)^2} + \frac{\cos(4a + 4bx)}{16d(c + dx)^2} - \frac{b \sin(4a + 4bx)}{4d^2(c + dx)} + \frac{\left(b^2 \cos\left(4a - \frac{4bc}{d}\right) \right) \int \frac{\cos\left(\frac{4bc}{d} + 4bx\right)}{c + dx}}{d^2} \\
 &= -\frac{1}{16d(c + dx)^2} + \frac{\cos(4a + 4bx)}{16d(c + dx)^2} + \frac{b^2 \cos\left(4a - \frac{4bc}{d}\right) \text{Ci}\left(\frac{4bc}{d} + 4bx\right)}{d^3} - \frac{b \sin(4a + 4bx)}{4d^2(c + dx)}
 \end{aligned}$$

Mathematica [A] time = 0.88326, size = 105, normalized size = 0.83

$$\frac{16b^2 \cos\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right) - 16b^2 \sin\left(4a - \frac{4bc}{d}\right) \text{Si}\left(\frac{4b(c+dx)}{d}\right) + \frac{d(-4b(c+dx) \sin(4(a+bx)) + d \cos(4(a+bx)) - d)}{(c+dx)^2}}{16d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^3,x]

[Out] (16*b^2*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] + (d*(-d + d*Cos[4*(a + b*x)] - 4*b*(c + d*x)*Sin[4*(a + b*x)]))/(c + d*x)^2 - 16*b^2*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(16*d^3)

Maple [A] time = 0.024, size = 193, normalized size = 1.5

$$\frac{1}{b} \left(-\frac{b^3}{32} \left(-2 \frac{\cos(4bx + 4a)}{((bx + a)d - ad + bc)^2 d} - 2 \frac{1}{d} \left(-4 \frac{\sin(4bx + 4a)}{((bx + a)d - ad + bc)d} + 4 \frac{1}{d} \left(4 \frac{1}{d} \text{Si}\left(4bx + 4a + 4 \frac{-ad + bc}{d}\right) \sin\left(4 \frac{-ad + bc}{d}\right) \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x)

[Out] 1/b*(-1/32*b^3*(-2*cos(4*b*x+4*a)/((b*x+a)*d-a*d+b*c)^2/d-2*(-4*sin(4*b*x+4*a)/((b*x+a)*d-a*d+b*c)/d+4*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d)/d)-1/16*b^3/((b*x+a)*d-a*d+b*c)^2/d)

Maxima [C] time = 1.83377, size = 278, normalized size = 2.19

$$\frac{64b^3 \left(E_3 \left(\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) + E_3 \left(-\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) \right) \cos \left(-\frac{4(bc-ad)}{d} \right) - b^3 \left(64i E_3 \left(\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) - 64i E_3 \left(-\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) \right)}{1024 \left(b^2 c^2 d - 2abcd^2 + (bx+a)^2 d^3 + a^2 d^3 + 2(bcd^2 - ad^3)(bx+a) \right) b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")

[Out] 1/1024*(64*b^3*(exp_integral_e(3, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) + exp_integral_e(3, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*(b*c - a*d)/d) - b^3*(64*I*exp_integral_e(3, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) - 64*I*exp_integral_e(3, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*sin(-4*(b*c - a*d)/d) - 64*b^3/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)

Fricas [B] time = 0.564002, size = 582, normalized size = 4.58

$$d^2 \cos(bx+a)^4 - d^2 \cos(bx+a)^2 - 2(b^2 d^2 x^2 + 2b^2 cdx + b^2 c^2) \sin\left(-\frac{4(bc-ad)}{d}\right) \text{Si}\left(\frac{4(bdx+bc)}{d}\right) + \left((b^2 d^2 x^2 + 2b^2 cdx + b^2 c^2) \cos\left(-\frac{4(bc-ad)}{d}\right) - 2(b^2 d^2 x^2 + 2b^2 cdx + b^2 c^2) \sin\left(-\frac{4(bc-ad)}{d}\right)\right) / (d^5 x^2 + 2c d^4 x + c^2 d^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")

[Out] 1/2*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(4*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-4*(b*d*x + b*c)/d))*cos(-4*(b*c - a*d)/d) - 2*(2*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - (b*d^2*x + b*c*d)*cos(b*x + a))*sin(b*x + a)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a+bx) \cos^2(a+bx)}{(c+dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c)**3,x)

[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x)**3, x)

Giac [C] time = 1.64119, size = 7560, normalized size = 59.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")

$$\begin{aligned}
& \text{eal_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)^2*\tan(2*b*c/d)^2 + 4*b^2*d^2*x^2*\text{real_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2 + 4*b^2*d^2*x^2 \\
& \text{real_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2 - 16*b^2*c*d*x*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a) + 16*b^2*c*d*x \\
& *\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a) - 32*b^2*c*d*x*\sin_integral(4*(b*d*x + b*c)/d)*\tan(2*b*x)^2*\tan(2*a) - 4*b^2*d^2*x^2 \\
& \text{real_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a)^2 - 4*b^2*d^2*x^2*\text{real_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)^2 - 4*b^2*c^2*\text{real_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)^2 - 4*b^2*c^2*\text{real_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)^2 + 16*b^2*c*d*x*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2*\tan(2*b*c/d) - 16*b^2*c*d*x*\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2*\tan(2*b*c/d) + 32*b^2*c*d*x*\sin_integral(4*(b*d*x + b*c)/d)*\tan(2*b*x)^2*\tan(2*b*c/d) + 16*b^2*d^2*x^2*\text{real_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a)*\tan(2*b*c/d) + 16*b^2*d^2*x^2*\text{real_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)*\tan(2*b*c/d) + 16*b^2*c^2*\text{real_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)*\tan(2*b*c/d) + 16*b^2*c^2*\text{real_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)*\tan(2*b*c/d) - 16*b^2*c*d*x*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a)^2*\tan(2*b*c/d) + 16*b^2*c*d*x*\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)^2*\tan(2*b*c/d) - 32*b^2*c*d*x*\sin_integral(4*(b*d*x + b*c)/d)*\tan(2*a)^2*\tan(2*b*c/d) - 4*b^2*d^2*x^2*\text{real_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*c/d)^2 - 4*b^2*d^2*x^2*\text{real_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*c/d)^2 - 4*b^2*c^2*\text{real_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2*\tan(2*b*c/d)^2 - 4*b^2*c^2*\text{real_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2*\tan(2*b*c/d)^2 + 16*b^2*c*d*x*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a)*\tan(2*b*c/d)^2 - 16*b^2*c*d*x*\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)*\tan(2*b*c/d)^2 + 32*b^2*c*d*x*\sin_integral(4*(b*d*x + b*c)/d)*\tan(2*a)*\tan(2*b*c/d)^2 + 4*b*d^2*x*\tan(2*b*x)^2*\tan(2*a)*\tan(2*b*c/d)^2 + 4*b^2*c^2*\text{real_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a)^2*\tan(2*b*c/d)^2 + 4*b*d^2*x*\tan(2*b*x)*\tan(2*a)^2*\tan(2*b*c/d)^2 + 8*b^2*c*d*x*\text{real_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2 + 8*b^2*c*d*x*\text{real_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2 - 8*b^2*d^2*x^2*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a) + 8*b^2*d^2*x^2*\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a) - 16*b^2*d^2*x^2*\sin_integral(4*(b*d*x + b*c)/d)*\tan(2*a) - 8*b^2*c^2*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a) + 8*b^2*c^2*\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a) - 16*b^2*c^2*\sin_integral(4*(b*d*x + b*c)/d)*\tan(2*b*x)^2*\tan(2*a) - 8*b^2*c*d*x*\text{real_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a)^2 - 8*b^2*c*d*x*\text{real_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)^2 + 8*b^2*d^2*x^2*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*c/d) - 8*b^2*d^2*x^2*\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*c/d) + 16*b^2*d^2*x^2*\sin_integral(4*(b*d*x + b*c)/d)*\tan(2*b*c/d) + 8*b^2*c^2*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2*\tan(2*b*c/d) - 8*b^2*c^2*\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2*\tan(2*b*c/d) + 16*b^2*c^2*\sin_integral(4*(b*d*x + b*c)/d)*\tan(2*b*x)^2*\tan(2*b*c/d) + 32*b^2*c*d*x*\text{real_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a)*\tan(2*b*c/d) + 32*b^2*c*d*x*\text{real_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)*\tan(2*b*c/d) - 8*b^2*c^2*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a)^2*\tan(2*b*c/d) + 8*b^2*c^2*\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)^2*\tan(2*b*c/d) - 16*b^2*c^2*\sin_integral(4*(b*d*x + b*c)/d)*\tan(2*a)^2*\tan(2*b*c/d) - 8*b^2*c*d*x*\text{real_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*c/d)^2 - 8*b^2*c*d*x*\text{real_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*c/d)^2 + 8*b^2*c^2*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a)*\tan(2*b*c/d)^2 - 8*b^2*c^2*\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)*\tan(2*b*c/d)^2 + 16*b^2*c^2*\sin_integral(4*(b*d*x + b*c)/d)*\tan(2*a)*\tan(2*b*c/d)^2 + 4*b*c*d*\tan(2*b*x)*\tan(2*a)^2*\tan(2*b*c/d)^2 + 4*b^2*d^2*x^2*\text{real_part}(\cos_integral(4*b*x + 4*b*c/d)) + 4*b^2*d^2*x^2*\text{real_part}(\cos_integral(-4*b*x - 4*b*c/d)) + 4*b^2*c^2*\text{real_part}(\cos
\end{aligned}$$

$$\begin{aligned}
& _integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2 + 4*b^2*c^2*real_part(cos_integral \\
& (-4*b*x - 4*b*c/d))*tan(2*b*x)^2 - 16*b^2*c*d*x*imag_part(cos_integral(4*b* \\
& x + 4*b*c/d))*tan(2*a) + 16*b^2*c*d*x*imag_part(cos_integral(-4*b*x - 4*b*c \\
& /d))*tan(2*a) - 32*b^2*c*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a) + 4*b \\
& *d^2*x*tan(2*b*x)^2*tan(2*a) - 4*b^2*c^2*real_part(cos_integral(4*b*x + 4*b \\
& *c/d))*tan(2*a)^2 - 4*b^2*c^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan \\
& (2*a)^2 + 4*b*d^2*x*tan(2*b*x)*tan(2*a)^2 + 16*b^2*c*d*x*imag_part(cos_inte \\
& gral(4*b*x + 4*b*c/d))*tan(2*b*c/d) - 16*b^2*c*d*x*imag_part(cos_integral(- \\
& 4*b*x - 4*b*c/d))*tan(2*b*c/d) + 32*b^2*c*d*x*sin_integral(4*(b*d*x + b*c)/ \\
& d)*tan(2*b*c/d) + 16*b^2*c^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2 \\
& *a)*tan(2*b*c/d) + 16*b^2*c^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan \\
& (2*a)*tan(2*b*c/d) - 4*b^2*c^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan \\
& (2*b*c/d)^2 - 4*b^2*c^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c \\
& /d)^2 - 4*b*d^2*x*tan(2*b*x)*tan(2*b*c/d)^2 - 4*b*d^2*x*tan(2*a)*tan(2*b*c/ \\
& d)^2 + 8*b^2*c*d*x*real_part(cos_integral(4*b*x + 4*b*c/d)) + 8*b^2*c*d*x*r \\
& eal_part(cos_integral(-4*b*x - 4*b*c/d)) - 8*b^2*c^2*imag_part(cos_integral \\
& (4*b*x + 4*b*c/d))*tan(2*a) + 8*b^2*c^2*imag_part(cos_integral(-4*b*x - 4*b \\
& *c/d))*tan(2*a) - 16*b^2*c^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a) + 4*b \\
& *c*d*tan(2*b*x)^2*tan(2*a) + 4*b*c*d*tan(2*b*x)*tan(2*a)^2 + 8*b^2*c^2*imag \\
& _part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d) - 8*b^2*c^2*imag_part(cos \\
& _integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d) + 16*b^2*c^2*sin_integral(4*(b*d* \\
& x + b*c)/d)*tan(2*b*c/d) - 4*b*c*d*tan(2*b*x)*tan(2*b*c/d)^2 - d^2*tan(2*b* \\
& x)^2*tan(2*b*c/d)^2 - 4*b*c*d*tan(2*a)*tan(2*b*c/d)^2 - 2*d^2*tan(2*b*x)*ta \\
& n(2*a)*tan(2*b*c/d)^2 - d^2*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*c^2*real_part \\
& (cos_integral(4*b*x + 4*b*c/d)) + 4*b^2*c^2*real_part(cos_integral(-4*b*x - \\
& 4*b*c/d)) - 4*b*d^2*x*tan(2*b*x) - 4*b*d^2*x*tan(2*a) - 4*b*c*d*tan(2*b*x) \\
& - d^2*tan(2*b*x)^2 - 4*b*c*d*tan(2*a) - 2*d^2*tan(2*b*x)*tan(2*a) - d^2*ta \\
& n(2*a)^2)/(d^5*x^2*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*c*d^4*x*tan(2 \\
& *b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + d^5*x^2*tan(2*b*x)^2*tan(2*a)^2 + d^5*x \\
& ^2*tan(2*b*x)^2*tan(2*b*c/d)^2 + d^5*x^2*tan(2*a)^2*tan(2*b*c/d)^2 + c^2*d^ \\
& 3*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*c*d^4*x*tan(2*b*x)^2*tan(2*a)^ \\
& 2 + 2*c*d^4*x*tan(2*b*x)^2*tan(2*b*c/d)^2 + 2*c*d^4*x*tan(2*a)^2*tan(2*b*c/ \\
& d)^2 + d^5*x^2*tan(2*b*x)^2 + d^5*x^2*tan(2*a)^2 + c^2*d^3*tan(2*b*x)^2*tan \\
& (2*a)^2 + d^5*x^2*tan(2*b*c/d)^2 + c^2*d^3*tan(2*b*x)^2*tan(2*b*c/d)^2 + c^ \\
& 2*d^3*tan(2*a)^2*tan(2*b*c/d)^2 + 2*c*d^4*x*tan(2*b*x)^2 + 2*c*d^4*x*tan(2* \\
& a)^2 + 2*c*d^4*x*tan(2*b*c/d)^2 + d^5*x^2 + c^2*d^3*tan(2*b*x)^2 + c^2*d^3* \\
& tan(2*a)^2 + c^2*d^3*tan(2*b*c/d)^2 + 2*c*d^4*x + c^2*d^3)
\end{aligned}$$

$$3.87 \quad \int \frac{\cos^2(a+bx) \sin^2(a+bx)}{(c+dx)^4} dx$$

Optimal. Leaf size=158

$$\frac{4b^3 \sin\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{3d^4} - \frac{4b^3 \cos\left(4a - \frac{4bc}{d}\right) \text{Si}\left(\frac{4bc}{d} + 4bx\right)}{3d^4} - \frac{b^2 \cos(4a + 4bx)}{3d^3(c + dx)} - \frac{b \sin(4a + 4bx)}{12d^2(c + dx)}$$

[Out] $-1/(24*d*(c + d*x)^3) + \text{Cos}[4*a + 4*b*x]/(24*d*(c + d*x)^3) - (b^2*\text{Cos}[4*a + 4*b*x])/(3*d^3*(c + d*x)) - (4*b^3*\text{CosIntegral}[(4*b*c)/d + 4*b*x]*\text{Sin}[4*a - (4*b*c)/d])/(3*d^4) - (b*\text{Sin}[4*a + 4*b*x])/(12*d^2*(c + d*x)^2) - (4*b^3*\text{Cos}[4*a - (4*b*c)/d]*\text{SinIntegral}[(4*b*c)/d + 4*b*x])/(3*d^4)$

Rubi [A] time = 0.227787, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{4b^3 \sin\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{3d^4} - \frac{4b^3 \cos\left(4a - \frac{4bc}{d}\right) \text{Si}\left(\frac{4bc}{d} + 4bx\right)}{3d^4} - \frac{b^2 \cos(4a + 4bx)}{3d^3(c + dx)} - \frac{b \sin(4a + 4bx)}{12d^2(c + dx)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[a + b*x]^2*\text{Sin}[a + b*x]^2)/(c + d*x)^4, x]$

[Out] $-1/(24*d*(c + d*x)^3) + \text{Cos}[4*a + 4*b*x]/(24*d*(c + d*x)^3) - (b^2*\text{Cos}[4*a + 4*b*x])/(3*d^3*(c + d*x)) - (4*b^3*\text{CosIntegral}[(4*b*c)/d + 4*b*x]*\text{Sin}[4*a - (4*b*c)/d])/(3*d^4) - (b*\text{Sin}[4*a + 4*b*x])/(12*d^2*(c + d*x)^2) - (4*b^3*\text{Cos}[4*a - (4*b*c)/d]*\text{SinIntegral}[(4*b*c)/d + 4*b*x])/(3*d^4)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)*((c_.) + (d_.)*(x_.))^{(m_.)*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] := \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3297

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] := \text{Simp}[(c + d*x)^{(m + 1)*\text{Sin}[e + f*x]} / (d*(m + 1)), x] - \text{Dist}[f / (d*(m + 1)), \text{Int}[(c + d*x)^{(m + 1)*\text{Cos}[e + f*x]}, x], x] /;$ FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)] / ((c_.) + (d_.)*(x_.)), x_Symbol] := \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x] / (c + d*x), x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x] / (c + d*x), x], x] /;$ FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)] / ((c_.) + (d_.)*(x_.)), x_Symbol] := \text{Simp}[\text{SinIntegral}[e + f*x]/d, x] /;$ FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(a + bx) \sin^2(a + bx)}{(c + dx)^4} dx &= \int \left(\frac{1}{8(c + dx)^4} - \frac{\cos(4a + 4bx)}{8(c + dx)^4} \right) dx \\ &= -\frac{1}{24d(c + dx)^3} - \frac{1}{8} \int \frac{\cos(4a + 4bx)}{(c + dx)^4} dx \\ &= -\frac{1}{24d(c + dx)^3} + \frac{\cos(4a + 4bx)}{24d(c + dx)^3} + \frac{b \int \frac{\sin(4a + 4bx)}{(c + dx)^3} dx}{6d} \\ &= -\frac{1}{24d(c + dx)^3} + \frac{\cos(4a + 4bx)}{24d(c + dx)^3} - \frac{b \sin(4a + 4bx)}{12d^2(c + dx)^2} + \frac{b^2 \int \frac{\cos(4a + 4bx)}{(c + dx)^2} dx}{3d^2} \\ &= -\frac{1}{24d(c + dx)^3} + \frac{\cos(4a + 4bx)}{24d(c + dx)^3} - \frac{b^2 \cos(4a + 4bx)}{3d^3(c + dx)} - \frac{b \sin(4a + 4bx)}{12d^2(c + dx)^2} - \frac{(4b^3) \int \frac{\sin(4a + 4bx)}{c + dx} dx}{3d^3} \\ &= -\frac{1}{24d(c + dx)^3} + \frac{\cos(4a + 4bx)}{24d(c + dx)^3} - \frac{b^2 \cos(4a + 4bx)}{3d^3(c + dx)} - \frac{b \sin(4a + 4bx)}{12d^2(c + dx)^2} - \frac{(4b^3) \cos\left(4a - \frac{4bc}{d} + 4bx\right)}{3d^3} \\ &= -\frac{1}{24d(c + dx)^3} + \frac{\cos(4a + 4bx)}{24d(c + dx)^3} - \frac{b^2 \cos(4a + 4bx)}{3d^3(c + dx)} - \frac{4b^3 \operatorname{Ci}\left(\frac{4bc}{d} + 4bx\right) \sin\left(4a - \frac{4bc}{d} + 4bx\right)}{3d^4} \end{aligned}$$

Mathematica [A] time = 1.7543, size = 123, normalized size = 0.78

$$\frac{32b^3 \sin\left(4a - \frac{4bc}{d}\right) \operatorname{CosIntegral}\left(\frac{4b(c+dx)}{d}\right) + \frac{d(\cos(4(a+bx))(8b^2(c+dx)^2 - d^2) + d(2b(c+dx) \sin(4(a+bx)) + d))}{(c+dx)^3} + 32b^3 \cos\left(4a - \frac{4bc}{d}\right) \operatorname{Si}\left(\frac{4b(c+dx)}{d}\right)}{24d^4}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^4, x]

[Out] -(32*b^3*CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + (d*((-d^2 + 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] + d*(d + 2*b*(c + d*x)*Sin[4*(a + b*x)])))/(c + d*x)^3 + 32*b^3*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(24*d^4)

Maple [A] time = 0.026, size = 230, normalized size = 1.5

$$\frac{1}{b} \left(-\frac{b^4}{32} \left(-\frac{4 \cos(4bx + 4a)}{3((bx + a)d - ad + bc)^3 d} - \frac{4}{3d} \left(-2 \frac{\sin(4bx + 4a)}{((bx + a)d - ad + bc)^2 d} + 2 \frac{1}{d} \left(-4 \frac{\cos(4bx + 4a)}{((bx + a)d - ad + bc)d} - 4 \frac{1}{d} \operatorname{Si}\left(\frac{4b(c+dx)}{d}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4, x)

[Out] 1/b*(-1/32*b^4*(-4/3*cos(4*b*x+4*a))/((b*x+a)*d-a*d+b*c)^3/d-4/3*(-2*sin(4*b*x+4*a))/((b*x+a)*d-a*d+b*c)^2/d+2*(-4*cos(4*b*x+4*a))/((b*x+a)*d-a*d+b*c)/d-4*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d))/24*d^4

$a*d+b*c)/d)*\sin(4*(-a*d+b*c)/d)/d)/d)/d)-1/24*b^4/((b*x+a)*d-a*d+b*c)^3/d)$

Maxima [C] time = 2.338, size = 346, normalized size = 2.19

$$\frac{3b^4 \left(E_4 \left(\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) + E_4 \left(-\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) \right) \cos \left(-\frac{4(bc-ad)}{d} \right) - b^4 \left(3i E_4 \left(\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) - 3i E_4 \left(-\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) \right)}{48 \left(b^3 c^3 d - 3 ab^2 c^2 d^2 + 3 a^2 b c d^3 + (bx+a)^3 d^4 - a^3 d^4 + 3 (bcd^3 - ad^4)(bx+a)^2 + 3 (b^2 c^2 d^2 - 2 abcd^3) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")

[Out] $1/48*(3*b^4*(\exp_integral_e(4, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) + \exp_integral_e(4, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*\cos(-4*(b*c - a*d)/d) - b^4*(3*I*\exp_integral_e(4, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) - 3*I*\exp_integral_e(4, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*\sin(-4*(b*c - a*d)/d) - 2*b^4/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)$

Fricas [B] time = 0.600936, size = 880, normalized size = 5.57

$$\frac{b^2 d^3 x^2 + 2 b^2 c d^2 x + b^2 c^2 d + (8 b^2 d^3 x^2 + 16 b^2 c d^2 x + 8 b^2 c^2 d - d^3) \cos(bx + a)^4 - (8 b^2 d^3 x^2 + 16 b^2 c d^2 x + 8 b^2 c^2 d - d^3) \sin(bx + a)^4}{(d^7 x^3 + 3 c d^6 x^2 + 3 c^2 d^5 x + c^3 d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")

[Out] $-1/3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d + (8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3))*\cos(b*x + a)^4 - (8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3))*\sin(b*x + a)^4 + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3))*\cos(-4*(b*c - a*d)/d)*\sin_integral(4*(b*d*x + b*c)/d) + (2*(b*d^3*x + b*c*d^2))*\cos(b*x + a)^3 - (b*d^3*x + b*c*d^2))*\cos(b*x + a)^3)*\sin(b*x + a) + 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3))*\cos_integral(4*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3))*\cos_integral(-4*(b*d*x + b*c)/d))*\sin(-4*(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a + bx) \cos^2(a + bx)}{(c + dx)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c)**4,x)

[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x)**4, x)

Giac [C] time = 1.87002, size = 11486, normalized size = 72.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/12*(8*b^3*d^3*x^3*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2* \\ & \tan(2*a)^2*\tan(2*b*c/d)^2 - 8*b^3*d^3*x^3*\text{imag_part}(\cos_integral(-4*b*x - 4 \\ & *b*c/d))*\tan(2*b*x)^2*\tan(2*a)^2*\tan(2*b*c/d)^2 + 16*b^3*d^3*x^3*\text{sin_integr} \\ & \text{al}(4*(b*d*x + b*c)/d)*\tan(2*b*x)^2*\tan(2*a)^2*\tan(2*b*c/d)^2 + 16*b^3*d^3*x \\ & ^3*\text{real_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)^2*\tan(2*b \\ & *c/d) + 16*b^3*d^3*x^3*\text{real_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*x) \\ & ^2*\tan(2*a)^2*\tan(2*b*c/d) - 16*b^3*d^3*x^3*\text{real_part}(\cos_integral(4*b*x + \\ & 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)*\tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*\text{real_part}(c \\ & \text{os_integral}(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)*\tan(2*b*c/d)^2 + 24*b^ \\ & 3*c*d^2*x^2*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)^ \\ & 2*\tan(2*b*c/d)^2 - 24*b^3*c*d^2*x^2*\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d \\ &))*\tan(2*b*x)^2*\tan(2*a)^2*\tan(2*b*c/d)^2 + 48*b^3*c*d^2*x^2*\text{sin_integral}(4 \\ & *(b*d*x + b*c)/d)*\tan(2*b*x)^2*\tan(2*a)^2*\tan(2*b*c/d)^2 - 8*b^3*d^3*x^3*\text{im} \\ & \text{ag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)^2 + 8*b^3*d^3*x \\ & ^3*\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)^2 - 16* \\ & b^3*d^3*x^3*\text{sin_integral}(4*(b*d*x + b*c)/d)*\tan(2*b*x)^2*\tan(2*a)^2 + 32*b^ \\ & 3*d^3*x^3*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)*\text{ta} \\ & \text{n}(2*b*c/d) - 32*b^3*d^3*x^3*\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2 \\ & *b*x)^2*\tan(2*a)*\tan(2*b*c/d) + 64*b^3*d^3*x^3*\text{sin_integral}(4*(b*d*x + b*c) \\ & /d)*\tan(2*b*x)^2*\tan(2*a)*\tan(2*b*c/d) + 48*b^3*c*d^2*x^2*\text{real_part}(\cos_int \\ & \text{egral}(4*b*x + 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)^2*\tan(2*b*c/d) + 48*b^3*c*d^2 \\ & *x^2*\text{real_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)^2*\tan(\\ & 2*b*c/d) - 8*b^3*d^3*x^3*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x) \\ & ^2*\tan(2*b*c/d)^2 + 8*b^3*d^3*x^3*\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d) \\ &)*\tan(2*b*x)^2*\tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*\text{sin_integral}(4*(b*d*x + b*c) \\ & /d)*\tan(2*b*x)^2*\tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*\text{real_part}(\cos_integral(4 \\ & *b*x + 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)*\tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*\text{re} \\ & \text{al_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)*\tan(2*b*c/d)^ \\ & 2 + 8*b^3*d^3*x^3*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a)^2*\tan(2 \\ & *b*c/d)^2 - 8*b^3*d^3*x^3*\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a \\ &)^2*\tan(2*b*c/d)^2 + 16*b^3*d^3*x^3*\text{sin_integral}(4*(b*d*x + b*c)/d)*\tan(2*a \\ &)^2*\tan(2*b*c/d)^2 + 24*b^3*c^2*d*x*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d) \\ &)*\tan(2*b*x)^2*\tan(2*a)^2*\tan(2*b*c/d)^2 - 24*b^3*c^2*d*x*\text{imag_part}(\cos_int \\ & \text{egral}(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)^2*\tan(2*b*c/d)^2 + 48*b^3*c^ \\ & 2*d*x*\text{sin_integral}(4*(b*d*x + b*c)/d)*\tan(2*b*x)^2*\tan(2*a)^2*\tan(2*b*c/d) \\ & ^2 + 16*b^3*d^3*x^3*\text{real_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2*\text{ta} \\ & \text{n}(2*a) + 16*b^3*d^3*x^3*\text{real_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*x) \\ & ^2*\tan(2*a) - 24*b^3*c*d^2*x^2*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\text{ta} \\ & \text{n}(2*b*x)^2*\tan(2*a)^2 + 24*b^3*c*d^2*x^2*\text{imag_part}(\cos_integral(-4*b*x - 4* \\ & b*c/d))*\tan(2*b*x)^2*\tan(2*a)^2 - 48*b^3*c*d^2*x^2*\text{sin_integral}(4*(b*d*x + \\ & b*c)/d)*\tan(2*b*x)^2*\tan(2*a)^2 - 16*b^3*d^3*x^3*\text{real_part}(\cos_integral(4*b \\ & *x + 4*b*c/d))*\tan(2*b*x)^2*\tan(2*b*c/d) - 16*b^3*d^3*x^3*\text{real_part}(\cos_int \\ & \text{egral}(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2*\tan(2*b*c/d) + 96*b^3*c*d^2*x^2*\text{imag_} \\ & \text{part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)*\tan(2*b*c/d) - 96 \\ & *b^3*c*d^2*x^2*\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2*\tan(2 \\ & *a)*\tan(2*b*c/d) + 192*b^3*c*d^2*x^2*\text{sin_integral}(4*(b*d*x + b*c)/d)*\tan(2* \\ & b*x)^2*\tan(2*a)*\tan(2*b*c/d) + 16*b^3*d^3*x^3*\text{real_part}(\cos_integral(4*b*x \\ & + 4*b*c/d))*\tan(2*a)^2*\tan(2*b*c/d) + 16*b^3*d^3*x^3*\text{real_part}(\cos_integral \\ & (-4*b*x - 4*b*c/d))*\tan(2*a)^2*\tan(2*b*c/d) + 48*b^3*c^2*d*x*\text{real_part}(\cos_ \\ & \text{integral}(4*b*x + 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)^2*\tan(2*b*c/d) + 48*b^3*c^ \\ & \end{aligned}$$

$$\begin{aligned}
& 2*d*x*real_part(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)^2*\tan \\
& (2*b*c/d) - 24*b^3*c*d^2*x^2*imag_part(\cos_integral(4*b*x + 4*b*c/d))*\tan(2 \\
& *b*x)^2*\tan(2*b*c/d)^2 + 24*b^3*c*d^2*x^2*imag_part(\cos_integral(-4*b*x - 4 \\
& *b*c/d))*\tan(2*b*x)^2*\tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*\sin_integral(4*(b*d \\
& *x + b*c)/d)*\tan(2*b*x)^2*\tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*real_part(\cos_int \\
& egral(4*b*x + 4*b*c/d))*\tan(2*a)*\tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*real_part(\\
& \cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)*\tan(2*b*c/d)^2 - 48*b^3*c^2*d*x*re \\
& al_part(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)*\tan(2*b*c/d)^2 \\
& - 48*b^3*c^2*d*x*real_part(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2*ta \\
& n(2*a)*\tan(2*b*c/d)^2 + 24*b^3*c*d^2*x^2*imag_part(\cos_integral(4*b*x + 4*b \\
& *c/d))*\tan(2*a)^2*\tan(2*b*c/d)^2 - 24*b^3*c*d^2*x^2*imag_part(\cos_integral(\\
& -4*b*x - 4*b*c/d))*\tan(2*a)^2*\tan(2*b*c/d)^2 + 48*b^3*c*d^2*x^2*\sin_integra \\
& l(4*(b*d*x + b*c)/d)*\tan(2*a)^2*\tan(2*b*c/d)^2 + 4*b^2*d^3*x^2*\tan(2*b*x)^2 \\
& *\tan(2*a)^2*\tan(2*b*c/d)^2 + 8*b^3*c^3*imag_part(\cos_integral(4*b*x + 4*b*c \\
& /d))*\tan(2*b*x)^2*\tan(2*a)^2*\tan(2*b*c/d)^2 - 8*b^3*c^3*imag_part(\cos_integ \\
& ral(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)^2*\tan(2*b*c/d)^2 + 16*b^3*c^3* \\
& \sin_integral(4*(b*d*x + b*c)/d)*\tan(2*b*x)^2*\tan(2*a)^2*\tan(2*b*c/d)^2 + 8* \\
& b^3*d^3*x^3*imag_part(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2 - 8*b^3*d \\
& ^3*x^3*imag_part(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2 + 16*b^3*d^3*x \\
& ^3*\sin_integral(4*(b*d*x + b*c)/d)*\tan(2*b*x)^2 + 48*b^3*c*d^2*x^2*real_pa \\
& rt(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a) + 48*b^3*c*d^2*x^2* \\
& real_part(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a) - 8*b^3*d^3 \\
& *x^3*imag_part(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a)^2 + 8*b^3*d^3*x^3*im \\
& ag_part(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)^2 - 16*b^3*d^3*x^3*\sin_int \\
& egral(4*(b*d*x + b*c)/d)*\tan(2*a)^2 - 24*b^3*c^2*d*x*imag_part(\cos_integral \\
& (4*b*x + 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)^2 + 24*b^3*c^2*d*x*imag_part(\cos_i \\
& ntegral(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)^2 - 48*b^3*c^2*d*x*\sin_int \\
& egral(4*(b*d*x + b*c)/d)*\tan(2*b*x)^2*\tan(2*a)^2 - 48*b^3*c*d^2*x^2*real_pa \\
& rt(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2*\tan(2*b*c/d) - 48*b^3*c*d^2*x \\
& ^2*real_part(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2*\tan(2*b*c/d) + 3 \\
& 2*b^3*d^3*x^3*imag_part(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a)*\tan(2*b*c/d) \\
&) - 32*b^3*d^3*x^3*imag_part(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)*\tan(2 \\
& *b*c/d) + 64*b^3*d^3*x^3*\sin_integral(4*(b*d*x + b*c)/d)*\tan(2*a)*\tan(2*b*c \\
& /d) + 96*b^3*c^2*d*x*imag_part(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2* \\
& \tan(2*a)*\tan(2*b*c/d) - 96*b^3*c^2*d*x*imag_part(\cos_integral(-4*b*x - 4*b* \\
& c/d))*\tan(2*b*x)^2*\tan(2*a)*\tan(2*b*c/d) + 192*b^3*c^2*d*x*\sin_integral(4*(\\
& b*d*x + b*c)/d)*\tan(2*b*x)^2*\tan(2*a)*\tan(2*b*c/d) + 48*b^3*c*d^2*x^2*real_ \\
& part(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a)^2*\tan(2*b*c/d) + 48*b^3*c*d^2*x \\
& ^2*real_part(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)^2*\tan(2*b*c/d) + 16* \\
& b^3*c^3*real_part(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)^2*ta \\
& n(2*b*c/d) + 16*b^3*c^3*real_part(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*x \\
&)^2*\tan(2*a)^2*\tan(2*b*c/d) - 8*b^3*d^3*x^3*imag_part(\cos_integral(4*b*x + \\
& 4*b*c/d))*\tan(2*b*c/d)^2 + 8*b^3*d^3*x^3*imag_part(\cos_integral(-4*b*x - 4* \\
& b*c/d))*\tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*\sin_integral(4*(b*d*x + b*c)/d)*\tan \\
& (2*b*c/d)^2 - 24*b^3*c^2*d*x*imag_part(\cos_integral(4*b*x + 4*b*c/d))*\tan(2 \\
& *b*x)^2*\tan(2*b*c/d)^2 + 24*b^3*c^2*d*x*imag_part(\cos_integral(-4*b*x - 4*b \\
& *c/d))*\tan(2*b*x)^2*\tan(2*b*c/d)^2 - 48*b^3*c^2*d*x*\sin_integral(4*(b*d*x + \\
& b*c)/d)*\tan(2*b*x)^2*\tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*real_part(\cos_integ \\
& ral(4*b*x + 4*b*c/d))*\tan(2*a)*\tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*real_part(\\
& \cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)*\tan(2*b*c/d)^2 - 16*b^3*c^3*real_p \\
& art(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)*\tan(2*b*c/d)^2 - 1 \\
& 6*b^3*c^3*real_part(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*x)^2*\tan(2*a)*t \\
& an(2*b*c/d)^2 + 24*b^3*c^2*d*x*imag_part(\cos_integral(4*b*x + 4*b*c/d))*\tan \\
& (2*a)^2*\tan(2*b*c/d)^2 - 24*b^3*c^2*d*x*imag_part(\cos_integral(-4*b*x - 4*b \\
& *c/d))*\tan(2*a)^2*\tan(2*b*c/d)^2 + 48*b^3*c^2*d*x*\sin_integral(4*(b*d*x + b \\
& *c)/d)*\tan(2*a)^2*\tan(2*b*c/d)^2 + 8*b^2*c*d^2*x*\tan(2*b*x)^2*\tan(2*a)^2*ta \\
& n(2*b*c/d)^2 + 24*b^3*c*d^2*x^2*imag_part(\cos_integral(4*b*x + 4*b*c/d))*ta \\
& n(2*b*x)^2 - 24*b^3*c*d^2*x^2*imag_part(\cos_integral(-4*b*x - 4*b*c/d))*\tan \\
& (2*b*x)^2 + 48*b^3*c*d^2*x^2*\sin_integral(4*(b*d*x + b*c)/d)*\tan(2*b*x)^2 +
\end{aligned}$$

$$\begin{aligned}
& 16b^3d^3x^3\text{real_part}(\cos_integral(4bx + 4bc/d))\tan(2a) + 16b^3d^3x^3\text{real_part}(\cos_integral(-4bx - 4bc/d))\tan(2a) + 48b^3c^2dx \\
& * \text{real_part}(\cos_integral(4bx + 4bc/d))\tan(2bx)^2\tan(2a) + 48b^3c^2dx * \text{real_part}(\cos_integral(-4bx - 4bc/d))\tan(2bx)^2\tan(2a) - 24b^3c^2dx^2 \\
& * \text{imag_part}(\cos_integral(4bx + 4bc/d))\tan(2a)^2 + 24b^3c^2dx^2 * \text{imag_part}(\cos_integral(-4bx - 4bc/d))\tan(2a)^2 - 48b^3cd^2x^2 \\
& * \sin_integral(4(bdx + bc)/d)\tan(2a)^2 + 4b^2d^3x^2\tan(2bx)^2\tan(2a)^2 - 8b^3c^3\text{imag_part}(\cos_integral(4bx + 4bc/d))\tan(2bx)^2 \\
& * \tan(2a)^2 + 8b^3c^3\text{imag_part}(\cos_integral(-4bx - 4bc/d))\tan(2bx)^2\tan(2a)^2 - 16b^3c^3\sin_integral(4(bdx + bc)/d)\tan(2bx)^2 \\
& * \tan(2a)^2 - 16b^3d^3x^3\text{real_part}(\cos_integral(4bx + 4bc/d))\tan(2bc/d) - 16b^3d^3x^3\text{real_part}(\cos_integral(-4bx - 4bc/d))\tan(2bc/d) \\
& - 48b^3c^2dx * \text{real_part}(\cos_integral(4bx + 4bc/d))\tan(2bx)^2\tan(2bc/d) - 48b^3c^2dx * \text{real_part}(\cos_integral(-4bx - 4bc/d))\tan(2bx)^2 \\
& * \tan(2bc/d) + 96b^3cd^2x^2 * \text{imag_part}(\cos_integral(4bx + 4bc/d))\tan(2a)\tan(2bc/d) - 96b^3cd^2x^2 * \text{imag_part}(\cos_integral(-4bx - 4bc/d))\tan(2a)\tan(2bc/d) \\
& + 192b^3cd^2x^2 * \sin_integral(4(bdx + bc)/d)\tan(2a)\tan(2bc/d) + 32b^3c^3\text{imag_part}(\cos_integral(4bx + 4bc/d))\tan(2bx)^2 \\
& * \tan(2a)\tan(2bc/d) - 32b^3c^3\text{imag_part}(\cos_integral(-4bx - 4bc/d))\tan(2bx)^2 * \tan(2a)\tan(2bc/d) + 64b^3c^3\sin_integral(4(bdx + bc)/d) \\
& * \tan(2bx)^2 * \tan(2a)\tan(2bc/d) + 48b^3c^2dx * \text{real_part}(\cos_integral(4bx + 4bc/d))\tan(2a)^2 * \tan(2bc/d) + 48b^3c^2dx * \text{real_part}(\cos_integral(-4bx - 4bc/d))\tan(2a)^2 \\
& * \tan(2bc/d) - 24b^3cd^2x^2 * \text{imag_part}(\cos_integral(4bx + 4bc/d))\tan(2bc/d)^2 + 24b^3cd^2x^2 * \text{imag_part}(\cos_integral(-4bx - 4bc/d))\tan(2bc/d)^2 \\
& - 48b^3cd^2x^2 * \sin_integral(4(bdx + bc)/d)\tan(2bc/d)^2 - 4b^2d^3x^2 * \tan(2bx)^2 * \tan(2bc/d)^2 - 8b^3c^3\text{imag_part}(\cos_integral(4bx + 4bc/d))\tan(2bx)^2 \\
& * \tan(2bc/d)^2 + 8b^3c^3\text{imag_part}(\cos_integral(-4bx - 4bc/d))\tan(2bx)^2 * \tan(2bc/d)^2 - 16b^3c^3\sin_integral(4(bdx + bc)/d)\tan(2bx)^2 \\
& * \tan(2bc/d)^2 - 48b^3cd^2dx * \text{real_part}(\cos_integral(4bx + 4bc/d))\tan(2a)\tan(2bc/d)^2 - 48b^3cd^2dx * \text{real_part}(\cos_integral(-4bx - 4bc/d))\tan(2a)\tan(2bc/d)^2 \\
& - 16b^2d^3x^2 * \tan(2bx)\tan(2a)\tan(2bc/d)^2 - 4b^2d^3x^2 * \tan(2a)^2 * \tan(2bc/d)^2 + 8b^3c^3\text{imag_part}(\cos_integral(4bx + 4bc/d))\tan(2a)^2 \\
& * \tan(2bc/d)^2 - 8b^3c^3\text{imag_part}(\cos_integral(-4bx - 4bc/d))\tan(2a)^2 * \tan(2bc/d)^2 + 16b^3c^3\sin_integral(4(bdx + bc)/d)\tan(2a)^2 \\
& * \tan(2bc/d)^2 + 4b^2c^2d * \tan(2bx)^2 * \tan(2a)^2 * \tan(2bc/d)^2 + 8b^3d^3x^3 * \text{imag_part}(\cos_integral(4bx + 4bc/d)) - 8b^3d^3x^3 * \text{imag_part}(\cos_integral(-4bx - 4bc/d)) \\
& + 16b^3d^3x^3 * \sin_integral(4(bdx + bc)/d) + 24b^3c^2dx * \text{imag_part}(\cos_integral(4bx + 4bc/d))\tan(2bx)^2 - 24b^3c^2dx * \text{imag_part}(\cos_integral(-4bx - 4bc/d))\tan(2bx)^2 \\
& + 48b^3c^2dx * \sin_integral(4(bdx + bc)/d)\tan(2bx)^2 + 48b^3cd^2x^2 * \text{real_part}(\cos_integral(4bx + 4bc/d))\tan(2a) + 48b^3cd^2x^2 * \text{real_part}(\cos_integral(-4bx - 4bc/d))\tan(2a) \\
& + 16b^3c^3 * \text{real_part}(\cos_integral(4bx + 4bc/d))\tan(2bx)^2 * \tan(2a) + 16b^3c^3 * \text{real_part}(\cos_integral(-4bx - 4bc/d))\tan(2bx)^2 * \tan(2a) - 24b^3c^2dx * \text{imag_part}(\cos_integral(4bx + 4bc/d))\tan(2a)^2 \\
& + 24b^3c^2dx * \text{imag_part}(\cos_integral(-4bx - 4bc/d))\tan(2a)^2 - 48b^3cd^2dx * \sin_integral(4(bdx + bc)/d)\tan(2a)^2 + 8b^2cd^2x * \tan(2bx)^2 * \tan(2a)^2 \\
& - 48b^3cd^2x^2 * \text{real_part}(\cos_integral(4bx + 4bc/d))\tan(2bc/d) - 48b^3cd^2x^2 * \text{real_part}(\cos_integral(-4bx - 4bc/d))\tan(2bc/d) - 16b^3c^3 * \text{real_part}(\cos_integral(4bx + 4bc/d))\tan(2bx)^2 \\
& * \tan(2bc/d) - 16b^3c^3 * \text{real_part}(\cos_integral(-4bx - 4bc/d))\tan(2bx)^2 * \tan(2bc/d) + 96b^3c^2dx * \text{imag_part}(\cos_integral(4bx + 4bc/d))\tan(2a)\tan(2bc/d) - 96b^3c^2dx * \text{imag_part}(\cos_integral(-4bx - 4bc/d))\tan(2a)\tan(2bc/d) \\
& + 192b^3cd^2dx * \sin_integral(4(bdx + bc)/d)\tan(2a)\tan(2bc/d) + 16b^3c^3 * \text{real_part}(\cos_integral(4bx + 4bc/d))\tan(2a)^2 * \tan(2bc/d) + 16b^3c^3 * \text{real_part}(\cos_integral(-4bx - 4bc/d))\tan(2a)^2 \\
& * \tan(2bc/d) - 24b^3cd^2dx * \text{imag_part}(\cos_integral(4bx + 4bc/d))\tan(2a) + 24b^3cd^2dx * \text{imag_part}(\cos_integral(-4bx - 4bc/d))\tan(2a)
\end{aligned}$$

$$\begin{aligned}
& *b*c/d))\tan(2*b*c/d)^2 + 24*b^3*c^2*d*x*\text{imag_part}(\cos_integral(-4*b*x - 4* \\
& b*c/d))\tan(2*b*c/d)^2 - 48*b^3*c^2*d*x*\sin_integral(4*(b*d*x + b*c)/d)\tan \\
& (2*b*c/d)^2 - 8*b^2*c*d^2*x*\tan(2*b*x)^2*\tan(2*b*c/d)^2 - 16*b^3*c^3*\text{real_p} \\
& \text{art}(\cos_integral(4*b*x + 4*b*c/d))\tan(2*a)*\tan(2*b*c/d)^2 - 16*b^3*c^3*\text{rea} \\
& \text{l_part}(\cos_integral(-4*b*x - 4*b*c/d))\tan(2*a)*\tan(2*b*c/d)^2 - 32*b^2*c*d \\
& ^2*x*\tan(2*b*x)*\tan(2*a)*\tan(2*b*c/d)^2 - 2*b*d^3*x*\tan(2*b*x)^2*\tan(2*a)*\t \\
& \text{an}(2*b*c/d)^2 - 8*b^2*c*d^2*x*\tan(2*a)^2*\tan(2*b*c/d)^2 - 2*b*d^3*x*\tan(2*b \\
& *x)*\tan(2*a)^2*\tan(2*b*c/d)^2 + 24*b^3*c*d^2*x^2*\text{imag_part}(\cos_integral(4*b \\
& *x + 4*b*c/d)) - 24*b^3*c*d^2*x^2*\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d)) \\
& + 48*b^3*c*d^2*x^2*\sin_integral(4*(b*d*x + b*c)/d) - 4*b^2*d^3*x^2*\tan(2*b \\
& *x)^2 + 8*b^3*c^3*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))\tan(2*b*x)^2 - 8 \\
& *b^3*c^3*\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))\tan(2*b*x)^2 + 16*b^3*c^ \\
& 3*\sin_integral(4*(b*d*x + b*c)/d)\tan(2*b*x)^2 + 48*b^3*c^2*d*x*\text{real_part}(c \\
& \text{os_integral}(4*b*x + 4*b*c/d))\tan(2*a) + 48*b^3*c^2*d*x*\text{real_part}(\cos_integ \\
& \text{ral}(-4*b*x - 4*b*c/d))\tan(2*a) - 16*b^2*d^3*x^2*\tan(2*b*x)*\tan(2*a) - 4*b^ \\
& 2*d^3*x^2*\tan(2*a)^2 - 8*b^3*c^3*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))\t \\
& \text{an}(2*a)^2 + 8*b^3*c^3*\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))\tan(2*a)^2 \\
& - 16*b^3*c^3*\sin_integral(4*(b*d*x + b*c)/d)\tan(2*a)^2 + 4*b^2*c^2*d*\tan(2 \\
& *b*x)^2*\tan(2*a)^2 - 48*b^3*c^2*d*x*\text{real_part}(\cos_integral(4*b*x + 4*b*c/d) \\
&)\tan(2*b*c/d) - 48*b^3*c^2*d*x*\text{real_part}(\cos_integral(-4*b*x - 4*b*c/d))\t \\
& \text{an}(2*b*c/d) + 32*b^3*c^3*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))\tan(2*a)* \\
& \tan(2*b*c/d) - 32*b^3*c^3*\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))\tan(2*a) \\
& *\tan(2*b*c/d) + 64*b^3*c^3*\sin_integral(4*(b*d*x + b*c)/d)\tan(2*a)*\tan(2* \\
& b*c/d) + 4*b^2*d^3*x^2*\tan(2*b*c/d)^2 - 8*b^3*c^3*\text{imag_part}(\cos_integral(4* \\
& b*x + 4*b*c/d))\tan(2*b*c/d)^2 + 8*b^3*c^3*\text{imag_part}(\cos_integral(-4*b*x - \\
& 4*b*c/d))\tan(2*b*c/d)^2 - 16*b^3*c^3*\sin_integral(4*(b*d*x + b*c)/d)\tan(2 \\
& *b*c/d)^2 - 4*b^2*c^2*d*\tan(2*b*x)^2*\tan(2*b*c/d)^2 - 16*b^2*c^2*d*\tan(2*b* \\
& x)*\tan(2*a)*\tan(2*b*c/d)^2 - 2*b*c*d^2*\tan(2*b*x)^2*\tan(2*a)*\tan(2*b*c/d)^2 \\
& - 4*b^2*c^2*d*\tan(2*a)^2*\tan(2*b*c/d)^2 - 2*b*c*d^2*\tan(2*b*x)*\tan(2*a)^2* \\
& \tan(2*b*c/d)^2 + 24*b^3*c^2*d*x*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d)) - \\
& 24*b^3*c^2*d*x*\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d)) + 48*b^3*c^2*d*x*\text{sin} \\
& \text{in_integral}(4*(b*d*x + b*c)/d) - 8*b^2*c*d^2*x*\tan(2*b*x)^2 + 16*b^3*c^3*\text{rea} \\
& \text{al_part}(\cos_integral(4*b*x + 4*b*c/d))\tan(2*a) + 16*b^3*c^3*\text{real_part}(\cos_ \\
& \text{integral}(-4*b*x - 4*b*c/d))\tan(2*a) - 32*b^2*c*d^2*x*\tan(2*b*x)*\tan(2*a) - \\
& 2*b*d^3*x*\tan(2*b*x)^2*\tan(2*a) - 8*b^2*c*d^2*x*\tan(2*a)^2 - 2*b*d^3*x*\tan \\
& (2*b*x)*\tan(2*a)^2 - 16*b^3*c^3*\text{real_part}(\cos_integral(4*b*x + 4*b*c/d))\t \\
& \text{an}(2*b*c/d) - 16*b^3*c^3*\text{real_part}(\cos_integral(-4*b*x - 4*b*c/d))\tan(2*b*c \\
& /d) + 8*b^2*c*d^2*x*\tan(2*b*c/d)^2 + 2*b*d^3*x*\tan(2*b*x)*\tan(2*b*c/d)^2 + \\
& 2*b*d^3*x*\tan(2*a)*\tan(2*b*c/d)^2 + 4*b^2*d^3*x^2 + 8*b^3*c^3*\text{imag_part}(\cos \\
& _integral(4*b*x + 4*b*c/d)) - 8*b^3*c^3*\text{imag_part}(\cos_integral(-4*b*x - 4*b \\
& *c/d)) + 16*b^3*c^3*\sin_integral(4*(b*d*x + b*c)/d) - 4*b^2*c^2*d*\tan(2*b*x \\
&)^2 - 16*b^2*c^2*d*\tan(2*b*x)*\tan(2*a) - 2*b*c*d^2*\tan(2*b*x)^2*\tan(2*a) - \\
& 4*b^2*c^2*d*\tan(2*a)^2 - 2*b*c*d^2*\tan(2*b*x)*\tan(2*a)^2 + 4*b^2*c^2*d*\tan(\\
& 2*b*c/d)^2 + 2*b*c*d^2*\tan(2*b*x)*\tan(2*b*c/d)^2 + d^3*\tan(2*b*x)^2*\tan(2*b \\
& *c/d)^2 + 2*b*c*d^2*\tan(2*a)*\tan(2*b*c/d)^2 + 2*d^3*\tan(2*b*x)*\tan(2*a)*\tan \\
& (2*b*c/d)^2 + d^3*\tan(2*a)^2*\tan(2*b*c/d)^2 + 8*b^2*c*d^2*x + 2*b*d^3*x*\tan \\
& (2*b*x) + 2*b*d^3*x*\tan(2*a) + 4*b^2*c^2*d + 2*b*c*d^2*\tan(2*b*x) + d^3*\tan \\
& (2*b*x)^2 + 2*b*c*d^2*\tan(2*a) + 2*d^3*\tan(2*b*x)*\tan(2*a) + d^3*\tan(2*a)^2 \\
&)/(d^7*x^3*\tan(2*b*x)^2*\tan(2*a)^2*\tan(2*b*c/d)^2 + 3*c*d^6*x^2*\tan(2*b*x) \\
& ^2*\tan(2*a)^2*\tan(2*b*c/d)^2 + d^7*x^3*\tan(2*b*x)^2*\tan(2*a)^2 + d^7*x^3*\tan \\
& (2*b*x)^2*\tan(2*b*c/d)^2 + d^7*x^3*\tan(2*a)^2*\tan(2*b*c/d)^2 + 3*c^2*d^5*x* \\
& \tan(2*b*x)^2*\tan(2*a)^2*\tan(2*b*c/d)^2 + 3*c*d^6*x^2*\tan(2*b*x)^2*\tan(2*a) \\
& ^2 + 3*c*d^6*x^2*\tan(2*b*x)^2*\tan(2*b*c/d)^2 + 3*c*d^6*x^2*\tan(2*a)^2*\tan(2* \\
& b*c/d)^2 + c^3*d^4*\tan(2*b*x)^2*\tan(2*a)^2*\tan(2*b*c/d)^2 + d^7*x^3*\tan(2*b \\
& *x)^2 + d^7*x^3*\tan(2*a)^2 + 3*c^2*d^5*x*\tan(2*b*x)^2*\tan(2*a)^2 + d^7*x^3* \\
& \tan(2*b*c/d)^2 + 3*c^2*d^5*x*\tan(2*b*x)^2*\tan(2*b*c/d)^2 + 3*c^2*d^5*x*\tan(\\
& 2*a)^2*\tan(2*b*c/d)^2 + 3*c*d^6*x^2*\tan(2*b*x)^2 + 3*c*d^6*x^2*\tan(2*a)^2 + \\
& c^3*d^4*\tan(2*b*x)^2*\tan(2*a)^2 + 3*c*d^6*x^2*\tan(2*b*c/d)^2 + c^3*d^4*\tan \\
& (2*b*x)^2*\tan(2*b*c/d)^2 + c^3*d^4*\tan(2*a)^2*\tan(2*b*c/d)^2 + d^7*x^3 + 3*
\end{aligned}$$

$$c^2d^5x\tan(2bx)^2 + 3c^2d^5x\tan(2a)^2 + 3c^2d^5x\tan(2b*c/d)^2 + 3c*d^6x^2 + c^3d^4\tan(2bx)^2 + c^3d^4\tan(2a)^2 + c^3d^4\tan(2*b*c/d)^2 + 3c^2d^5x + c^3d^4$$

3.88 $\int (c + dx)^m \cos^2(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=407

$$\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{16b} - \frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{32b}$$

```
[Out] -(E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(16*b
*(((I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(
(16*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - (3^(-1 - m)*E^((3*I)*(a
- (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(32*b*(((I)*
b*(c + d*x))/d)^m) - (3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((3*I)*b*(c + d*x
))/d])/(32*b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + (5^(-1 - m)*E
^((5*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-5*I)*b*(c + d*x))/d])/(3
2*b*(((I)*b*(c + d*x))/d)^m) + (5^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((5*I)
*b*(c + d*x))/d])/(32*b*E^((5*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.402562, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3308, 2181}

$$\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{16b} - \frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m * Cos[a + b*x]^2 * Sin[a + b*x]^3, x]
```

```
[Out] -(E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(16*b
*(((I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(
(16*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - (3^(-1 - m)*E^((3*I)*(a
- (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(32*b*(((I)*
b*(c + d*x))/d)^m) - (3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((3*I)*b*(c + d*x
))/d])/(32*b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + (5^(-1 - m)*E
^((5*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-5*I)*b*(c + d*x))/d])/(3
2*b*(((I)*b*(c + d*x))/d)^m) + (5^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((5*I)
*b*(c + d*x))/d])/(32*b*E^((5*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n * Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:= -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, -(f*g*Lo
```

$g[F])/d))*(c + d*x)]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F]]*(c + d*x))/d))^FracPart[m]), x) /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]$

Rubi steps

$$\begin{aligned} \int (c + dx)^m \cos^2(a + bx) \sin^3(a + bx) dx &= \int \left(\frac{1}{8} (c + dx)^m \sin(a + bx) + \frac{1}{16} (c + dx)^m \sin(3a + 3bx) - \frac{1}{16} (c + dx)^m \sin(5a + 5bx) \right) dx \\ &= \frac{1}{16} \int (c + dx)^m \sin(3a + 3bx) dx - \frac{1}{16} \int (c + dx)^m \sin(5a + 5bx) dx + \frac{1}{8} \int (c + dx)^m \sin(a + bx) dx \\ &= \frac{1}{32} i \int e^{-i(3a+3bx)} (c + dx)^m dx - \frac{1}{32} i \int e^{i(3a+3bx)} (c + dx)^m dx - \frac{1}{32} i \int e^{-i(5a+5bx)} (c + dx)^m dx \\ &\quad + \frac{1}{32} i \int e^{i(5a+5bx)} (c + dx)^m dx + \frac{1}{8} \int e^{-i(a+bx)} (c + dx)^m dx - \frac{1}{8} \int e^{i(a+bx)} (c + dx)^m dx \\ &= -\frac{e^{i\left(a-\frac{bc}{d}\right)} (c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, -\frac{ib(c+dx)}{d}\right)}{16b} - \frac{e^{-i\left(a-\frac{bc}{d}\right)} (c + dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, \frac{ib(c+dx)}{d}\right)}{16b} \end{aligned}$$

Mathematica [A] time = 0.66986, size = 376, normalized size = 0.92

$$e^{-\frac{5i(ad+bc)}{d}} (c + dx)^m \left(-5 \cdot 3^{-m} e^{\frac{2i(ad+bc)}{d}} \left(\frac{b^2(c+dx)^2}{d^2} \right)^{-m} \left(e^{6ia} \left(\frac{ib(c+dx)}{d} \right)^m \Gamma\left(m + 1, -\frac{3ib(c+dx)}{d}\right) + e^{\frac{6ibc}{d}} \left(-\frac{ib(c+dx)}{d} \right)^m \Gamma\left(m + 1, \frac{3ib(c+dx)}{d}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^m * Cos[a + b*x]^2 * Sin[a + b*x]^3, x]

[Out] ((c + d*x)^m * (30 * E^(((4*I)*(b*c + a*d))/d)) * (-((E^(((2*I)*a)) * Gamma[1 + m, ((-I)*b*(c + d*x))/d]) / (((-I)*b*(c + d*x))/d)^m) - (E^(((2*I)*b*c)/d) * Gamma[1 + m, (I*b*(c + d*x))/d]) / ((I*b*(c + d*x))/d)^m) - (5 * E^(((2*I)*(b*c + a*d))/d) * (E^(((6*I)*a)) * ((I*b*(c + d*x))/d)^m * Gamma[1 + m, ((-3*I)*b*(c + d*x))/d] + E^(((6*I)*b*c)/d) * (((-I)*b*(c + d*x))/d)^m * Gamma[1 + m, ((3*I)*b*(c + d*x))/d])) / (3^m * ((b^2*(c + d*x)^2)/d^2)^m) + (3 * (E^(((10*I)*a)) * ((I*b*(c + d*x))/d)^m * Gamma[1 + m, ((-5*I)*b*(c + d*x))/d] + E^(((10*I)*b*c)/d) * (((-I)*b*(c + d*x))/d)^m * Gamma[1 + m, ((5*I)*b*(c + d*x))/d])) / (5^m * ((b^2*(c + d*x)^2)/d^2)^m)) / (480 * b * E^(((5*I)*(b*c + a*d))/d))

Maple [F] time = 0.334, size = 0, normalized size = 0.

$$\int (dx + c)^m (\cos(bx + a))^2 (\sin(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x)

[Out] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a)^2 \sin(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")

[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a)^3, x)

Fricas [A] time = 0.569725, size = 709, normalized size = 1.74

$$3e^{\left(-\frac{dm \log\left(\frac{5ib}{d}\right) - 5ibc + 5iad}{d}\right)} \Gamma\left(m+1, \frac{5ibdx + 5ibc}{d}\right) - 5e^{\left(-\frac{dm \log\left(\frac{3ib}{d}\right) - 3ibc + 3iad}{d}\right)} \Gamma\left(m+1, \frac{3ibdx + 3ibc}{d}\right) - 30e^{\left(-\frac{dm \log\left(\frac{ib}{d}\right) - ibc + iad}{d}\right)} \Gamma\left(m+1, \frac{ibdx + ibc}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")

[Out] $\frac{1}{480} \left(3e^{-(d*m*\log(5*I*b/d) - 5*I*b*c + 5*I*a*d)/d} * \text{gamma}(m + 1, (5*I*b*d*x + 5*I*b*c)/d) - 5e^{-(d*m*\log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d} * \text{gamma}(m + 1, (3*I*b*d*x + 3*I*b*c)/d) - 30e^{-(d*m*\log(I*b/d) - I*b*c + I*a*d)/d} * \text{gamma}(m + 1, (I*b*d*x + I*b*c)/d) - 30e^{-(d*m*\log(-I*b/d) + I*b*c - I*a*d)/d} * \text{gamma}(m + 1, (-I*b*d*x - I*b*c)/d) - 5e^{-(d*m*\log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d} * \text{gamma}(m + 1, (-3*I*b*d*x - 3*I*b*c)/d) + 3e^{-(d*m*\log(-5*I*b/d) + 5*I*b*c - 5*I*a*d)/d} * \text{gamma}(m + 1, (-5*I*b*d*x - 5*I*b*c)/d) \right) / b$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**m*cos(b*x+a)**2*sin(b*x+a)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a)^2 \sin(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")

[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a)^3, x)

3.89 $\int (c + dx)^4 \cos^2(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=330

$$-\frac{3d^3(c + dx) \sin(a + bx)}{b^4} - \frac{d^3(c + dx) \sin(3a + 3bx)}{54b^4} + \frac{3d^3(c + dx) \sin(5a + 5bx)}{1250b^4} + \frac{3d^2(c + dx)^2 \cos(a + bx)}{2b^3} + \frac{d^2(c + dx)}{b^2}$$

[Out] $(-3*d^4*\text{Cos}[a + b*x])/b^5 + (3*d^2*(c + d*x)^2*\text{Cos}[a + b*x])/(2*b^3) - ((c + d*x)^4*\text{Cos}[a + b*x])/(8*b) - (d^4*\text{Cos}[3*a + 3*b*x])/(162*b^5) + (d^2*(c + d*x)^2*\text{Cos}[3*a + 3*b*x])/(36*b^3) - ((c + d*x)^4*\text{Cos}[3*a + 3*b*x])/(48*b) + (3*d^4*\text{Cos}[5*a + 5*b*x])/(6250*b^5) - (3*d^2*(c + d*x)^2*\text{Cos}[5*a + 5*b*x])/(500*b^3) + ((c + d*x)^4*\text{Cos}[5*a + 5*b*x])/(80*b) - (3*d^3*(c + d*x)*\text{Sin}[a + b*x])/b^4 + (d*(c + d*x)^3*\text{Sin}[a + b*x])/(2*b^2) - (d^3*(c + d*x)*\text{Sin}[3*a + 3*b*x])/(54*b^4) + (d*(c + d*x)^3*\text{Sin}[3*a + 3*b*x])/(36*b^2) + (3*d^3*(c + d*x)*\text{Sin}[5*a + 5*b*x])/(1250*b^4) - (d*(c + d*x)^3*\text{Sin}[5*a + 5*b*x])/(100*b^2)$

Rubi [A] time = 0.391063, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3296, 2638}

$$-\frac{3d^3(c + dx) \sin(a + bx)}{b^4} - \frac{d^3(c + dx) \sin(3a + 3bx)}{54b^4} + \frac{3d^3(c + dx) \sin(5a + 5bx)}{1250b^4} + \frac{3d^2(c + dx)^2 \cos(a + bx)}{2b^3} + \frac{d^2(c + dx)}{b^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^4*\text{Cos}[a + b*x]^2*\text{Sin}[a + b*x]^3, x]$

[Out] $(-3*d^4*\text{Cos}[a + b*x])/b^5 + (3*d^2*(c + d*x)^2*\text{Cos}[a + b*x])/(2*b^3) - ((c + d*x)^4*\text{Cos}[a + b*x])/(8*b) - (d^4*\text{Cos}[3*a + 3*b*x])/(162*b^5) + (d^2*(c + d*x)^2*\text{Cos}[3*a + 3*b*x])/(36*b^3) - ((c + d*x)^4*\text{Cos}[3*a + 3*b*x])/(48*b) + (3*d^4*\text{Cos}[5*a + 5*b*x])/(6250*b^5) - (3*d^2*(c + d*x)^2*\text{Cos}[5*a + 5*b*x])/(500*b^3) + ((c + d*x)^4*\text{Cos}[5*a + 5*b*x])/(80*b) - (3*d^3*(c + d*x)*\text{Sin}[a + b*x])/b^4 + (d*(c + d*x)^3*\text{Sin}[a + b*x])/(2*b^2) - (d^3*(c + d*x)*\text{Sin}[3*a + 3*b*x])/(54*b^4) + (d*(c + d*x)^3*\text{Sin}[3*a + 3*b*x])/(36*b^2) + (3*d^3*(c + d*x)*\text{Sin}[5*a + 5*b*x])/(1250*b^4) - (d*(c + d*x)^3*\text{Sin}[5*a + 5*b*x])/(100*b^2)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

$\text{Int}[\text{sin}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (c+dx)^4 \cos^2(a+bx) \sin^3(a+bx) dx &= \int \left(\frac{1}{8}(c+dx)^4 \sin(a+bx) + \frac{1}{16}(c+dx)^4 \sin(3a+3bx) - \frac{1}{16}(c+dx)^4 \sin(5a+5bx) \right) dx \\
&= \frac{1}{16} \int (c+dx)^4 \sin(3a+3bx) dx - \frac{1}{16} \int (c+dx)^4 \sin(5a+5bx) dx + \frac{1}{8} \int (c+dx)^4 \sin(a+bx) dx \\
&= -\frac{(c+dx)^4 \cos(a+bx)}{8b} - \frac{(c+dx)^4 \cos(3a+3bx)}{48b} + \frac{(c+dx)^4 \cos(5a+5bx)}{80b} \\
&= -\frac{(c+dx)^4 \cos(a+bx)}{8b} - \frac{(c+dx)^4 \cos(3a+3bx)}{48b} + \frac{(c+dx)^4 \cos(5a+5bx)}{80b} \\
&= \frac{3d^2(c+dx)^2 \cos(a+bx)}{2b^3} - \frac{(c+dx)^4 \cos(a+bx)}{8b} + \frac{d^2(c+dx)^2 \cos(3a+3bx)}{36b^3} \\
&= \frac{3d^2(c+dx)^2 \cos(a+bx)}{2b^3} - \frac{(c+dx)^4 \cos(a+bx)}{8b} + \frac{d^2(c+dx)^2 \cos(3a+3bx)}{36b^3} \\
&= -\frac{3d^4 \cos(a+bx)}{b^5} + \frac{3d^2(c+dx)^2 \cos(a+bx)}{2b^3} - \frac{(c+dx)^4 \cos(a+bx)}{8b}
\end{aligned}$$

Mathematica [A] time = 3.4162, size = 238, normalized size = 0.72

$$120bd(c+dx)\sin(a+bx)\left(16\cos(2(a+bx))\left(75b^2(c+dx)^2-68d^2\right)-27\cos(4(a+bx))\left(25b^2(c+dx)^2-6d^2\right)+1747d^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^3,x]

[Out] (-506250*(24*d^4 - 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cos[a + b*x] - 3125*(8*d^4 - 36*b^2*d^2*(c + d*x)^2 + 27*b^4*(c + d*x)^4)*Cos[3*(a + b*x)] + 81*(24*d^4 - 300*b^2*d^2*(c + d*x)^2 + 625*b^4*(c + d*x)^4)*Cos[5*(a + b*x)] + 120*b*d*(c + d*x)*(17475*b^2*c^2 - 101794*d^2 + 34950*b^2*c*d*x + 17475*b^2*d^2*x^2 + 16*(-68*d^2 + 75*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] - 27*(-6*d^2 + 25*b^2*(c + d*x)^2)*Cos[4*(a + b*x)])*Sin[a + b*x]/(4050000*b^5)

Maple [B] time = 0.087, size = 1812, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x)

[Out] 1/b*(1/b^4*d^4*(-1/3*(b*x+a)^4*(2+sin(b*x+a)^2)*cos(b*x+a)+8/15*(b*x+a)^3*sin(b*x+a)+8/5*(b*x+a)^2*cos(b*x+a)-3424/1125*cos(b*x+a)-3424/1125*(b*x+a)*sin(b*x+a)+4/45*(b*x+a)^3*sin(b*x+a)^3+4/45*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+88/3375*(b*x+a)*sin(b*x+a)^3+88/10125*(2+sin(b*x+a)^2)*cos(b*x+a)+1/5*(b*x+a)^4*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-4/25*(b*x+a)^3*sin(b*x+a)^5-12/125*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)+24/625*(b*x+a)*sin(b*x+a)^5+24/3125*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-4/b^4*a*d^4*(-1/3*(b*x+a)^3*(2+sin(b*x+a)^2)*cos(b*x+a)+2/5*(b*x+a)^2*sin(b*x+a)-856/1125*sin(b*x+a)+4/5*(b*x+a)*cos(b*x+a)+1/15*(b*x+a)^2*sin(b*x+a)^3+2/45*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+22/3375*sin(b*x+a)^3+1/5*(b*x+a)^3*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-3/25*(b*x+a)^2*sin(b*x+a)^5-6/125*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)+6/625*sin(b*x+a)^5)+4/b^3*c*d^3*(-1/3*(b*x+a)^3*(2+sin(b*x+a)^2)*cos(b*x+a)+2

$$\begin{aligned} & /5*(b*x+a)^2*\sin(b*x+a)-856/1125*\sin(b*x+a)+4/5*(b*x+a)*\cos(b*x+a)+1/15*(b*x+a)^2*\sin(b*x+a)^3+2/45*(b*x+a)*(2+\sin(b*x+a)^2)*\cos(b*x+a)+22/3375*\sin(b*x+a)^3+1/5*(b*x+a)^3*(8/3+\sin(b*x+a)^4+4/3*\sin(b*x+a)^2)*\cos(b*x+a)-3/25*(b*x+a)^2*\sin(b*x+a)^5-6/125*(b*x+a)*(8/3+\sin(b*x+a)^4+4/3*\sin(b*x+a)^2)*\cos(b*x+a)+6/625*\sin(b*x+a)^5)+6/b^4*a^2*d^4*(-1/3*(b*x+a)^2*(2+\sin(b*x+a)^2)*\cos(b*x+a)+4/15*\cos(b*x+a)+4/15*(b*x+a)*\sin(b*x+a)+2/45*(b*x+a)*\sin(b*x+a)^3+2/135*(2+\sin(b*x+a)^2)*\cos(b*x+a)+1/5*(b*x+a)^2*(8/3+\sin(b*x+a)^4+4/3*\sin(b*x+a)^2)*\cos(b*x+a)-2/25*(b*x+a)*\sin(b*x+a)^5-2/125*(8/3+\sin(b*x+a)^4+4/3*\sin(b*x+a)^2)*\cos(b*x+a))-12/b^3*a*c*d^3*(-1/3*(b*x+a)^2*(2+\sin(b*x+a)^2)*\cos(b*x+a)+4/15*\cos(b*x+a)+4/15*(b*x+a)*\sin(b*x+a)+2/45*(b*x+a)*\sin(b*x+a)^3+2/135*(2+\sin(b*x+a)^2)*\cos(b*x+a)+1/5*(b*x+a)^2*(8/3+\sin(b*x+a)^4+4/3*\sin(b*x+a)^2)*\cos(b*x+a)-2/25*(b*x+a)*\sin(b*x+a)^5-2/125*(8/3+\sin(b*x+a)^4+4/3*\sin(b*x+a)^2)*\cos(b*x+a))+6/b^2*c^2*d^2*(-1/3*(b*x+a)^2*(2+\sin(b*x+a)^2)*\cos(b*x+a)+4/15*\cos(b*x+a)+4/15*(b*x+a)*\sin(b*x+a)+2/45*(b*x+a)*\sin(b*x+a)^3+2/135*(2+\sin(b*x+a)^2)*\cos(b*x+a)+1/5*(b*x+a)^2*(8/3+\sin(b*x+a)^4+4/3*\sin(b*x+a)^2)*\cos(b*x+a)-2/25*(b*x+a)*\sin(b*x+a)^5-2/125*(8/3+\sin(b*x+a)^4+4/3*\sin(b*x+a)^2)*\cos(b*x+a))-4/b^4*a^3*d^4*(-1/3*(b*x+a)*(2+\sin(b*x+a)^2)*\cos(b*x+a)+1/45*\sin(b*x+a)^3+2/15*\sin(b*x+a)+1/5*(b*x+a)*(8/3+\sin(b*x+a)^4+4/3*\sin(b*x+a)^2)*\cos(b*x+a)-1/25*\sin(b*x+a)^5)+12/b^3*a^2*c*d^3*(-1/3*(b*x+a)*(2+\sin(b*x+a)^2)*\cos(b*x+a)+1/45*\sin(b*x+a)^3+2/15*\sin(b*x+a)+1/5*(b*x+a)*(8/3+\sin(b*x+a)^4+4/3*\sin(b*x+a)^2)*\cos(b*x+a)-1/25*\sin(b*x+a)^5)-12/b^2*a*c^2*d^2*(-1/3*(b*x+a)*(2+\sin(b*x+a)^2)*\cos(b*x+a)+1/45*\sin(b*x+a)^3+2/15*\sin(b*x+a)+1/5*(b*x+a)*(8/3+\sin(b*x+a)^4+4/3*\sin(b*x+a)^2)*\cos(b*x+a)-1/25*\sin(b*x+a)^5)+4/b*c^3*d*(-1/3*(b*x+a)*(2+\sin(b*x+a)^2)*\cos(b*x+a)+1/45*\sin(b*x+a)^3+2/15*\sin(b*x+a)+1/5*(b*x+a)*(8/3+\sin(b*x+a)^4+4/3*\sin(b*x+a)^2)*\cos(b*x+a)-1/25*\sin(b*x+a)^5)+1/b^4*a^4*d^4*(-1/5*\sin(b*x+a)^2*\cos(b*x+a)^3-2/15*\cos(b*x+a)^3)-4/b^3*a^3*c*d^3*(-1/5*\sin(b*x+a)^2*\cos(b*x+a)^3-2/15*\cos(b*x+a)^3)+6/b^2*a^2*c^2*d^2*(-1/5*\sin(b*x+a)^2*\cos(b*x+a)^3-2/15*\cos(b*x+a)^3)-4/b*a*c^3*d*(-1/5*\sin(b*x+a)^2*\cos(b*x+a)^3-2/15*\cos(b*x+a)^3)+c^4*(-1/5*\sin(b*x+a)^2*\cos(b*x+a)^3-2/15*\cos(b*x+a)^3)) \end{aligned}$$

Maxima [B] time = 1.42847, size = 1808, normalized size = 5.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")

[Out] $1/4050000*(270000*(3*\cos(b*x + a)^5 - 5*\cos(b*x + a)^3)*c^4 - 1080000*(3*\cos(b*x + a)^5 - 5*\cos(b*x + a)^3)*a*c^3*d/b + 1620000*(3*\cos(b*x + a)^5 - 5*\cos(b*x + a)^3)*a^2*c^2*d^2/b^2 - 1080000*(3*\cos(b*x + a)^5 - 5*\cos(b*x + a)^3)*a^3*c*d^3/b^3 + 270000*(3*\cos(b*x + a)^5 - 5*\cos(b*x + a)^3)*a^4*d^4/b^4 + 4500*(45*(b*x + a)*\cos(5*b*x + 5*a) - 75*(b*x + a)*\cos(3*b*x + 3*a) - 450*(b*x + a)*\cos(b*x + a) - 9*\sin(5*b*x + 5*a) + 25*\sin(3*b*x + 3*a) + 450*\sin(b*x + a))*c^3*d/b - 13500*(45*(b*x + a)*\cos(5*b*x + 5*a) - 75*(b*x + a)*\cos(3*b*x + 3*a) - 450*(b*x + a)*\cos(b*x + a) - 9*\sin(5*b*x + 5*a) + 25*\sin(3*b*x + 3*a) + 450*\sin(b*x + a))*a*c^2*d^2/b^2 + 13500*(45*(b*x + a)*\cos(5*b*x + 5*a) - 75*(b*x + a)*\cos(3*b*x + 3*a) - 450*(b*x + a)*\cos(b*x + a) - 9*\sin(5*b*x + 5*a) + 25*\sin(3*b*x + 3*a) + 450*\sin(b*x + a))*a^2*c*d^3/b^3 - 4500*(45*(b*x + a)*\cos(5*b*x + 5*a) - 75*(b*x + a)*\cos(3*b*x + 3*a) - 450*(b*x + a)*\cos(b*x + a) - 9*\sin(5*b*x + 5*a) + 25*\sin(3*b*x + 3*a) + 450*\sin(b*x + a))*a^3*d^4/b^4 + 450*(27*(25*(b*x + a)^2 - 2)*\cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*\cos(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*\cos(b*x + a) - 270*(b*x + a)*\sin(5*b*x + 5*a) + 750*(b*x + a)*\sin(3*b*x + 3*a) + 13500*(b*x + a)*\sin(b*x + a))*c^2*d^2/b^2 - 900*(27*(25*(b*x + a)^2 - 2)*\cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*\cos(3*b*x + 3*a) - 6750*((b*x + a)^2$

```

- 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*x + a)*sin(3*b
*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*a*c*d^3/b^3 + 450*(27*(25*(b*x +
a)^2 - 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 675
0*((b*x + a)^2 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*
x + a)*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*a^2*d^4/b^4 + 60*(1
35*(25*(b*x + a)^3 - 6*b*x - 6*a)*cos(5*b*x + 5*a) - 1875*(3*(b*x + a)^3 -
2*b*x - 2*a)*cos(3*b*x + 3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x +
a) - 81*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 625*(9*(b*x + a)^2 - 2)*si
n(3*b*x + 3*a) + 101250*((b*x + a)^2 - 2)*sin(b*x + a))*c*d^3/b^3 - 60*(135
*(25*(b*x + a)^3 - 6*b*x - 6*a)*cos(5*b*x + 5*a) - 1875*(3*(b*x + a)^3 - 2*
b*x - 2*a)*cos(3*b*x + 3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a
) - 81*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 625*(9*(b*x + a)^2 - 2)*sin(
3*b*x + 3*a) + 101250*((b*x + a)^2 - 2)*sin(b*x + a))*a*d^4/b^4 + (81*(625*
(b*x + a)^4 - 300*(b*x + a)^2 + 24)*cos(5*b*x + 5*a) - 3125*(27*(b*x + a)^4
- 36*(b*x + a)^2 + 8)*cos(3*b*x + 3*a) - 506250*((b*x + a)^4 - 12*(b*x + a
)^2 + 24)*cos(b*x + a) - 1620*(25*(b*x + a)^3 - 6*b*x - 6*a)*sin(5*b*x + 5*
a) + 37500*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) + 2025000*((b*x +
a)^3 - 6*b*x - 6*a)*sin(b*x + a))*d^4/b^4)/b

```

Fricas [A] time = 0.610837, size = 1121, normalized size = 3.4

$$81 \left(625 b^4 d^4 x^4 + 2500 b^4 c d^3 x^3 + 625 b^4 c^4 - 300 b^2 c^2 d^2 + 24 d^4 + 150 \left(25 b^4 c^2 d^2 - 2 b^2 d^4 \right) x^2 + 100 \left(25 b^4 c^3 d - 6 b^2 c d^3 \right) x + 100 \left(25 b^4 c^3 d - 6 b^2 c d^3 \right) \right) / b^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")

[Out] 1/253125*(81*(625*b^4*d^4*x^4 + 2500*b^4*c*d^3*x^3 + 625*b^4*c^4 - 300*b^2*c^2*d^2 + 24*d^4 + 150*(25*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 100*(25*b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a)^5 - 5*(16875*b^4*d^4*x^4 + 67500*b^4*c*d^3*x^3 + 16875*b^4*c^4 - 11700*b^2*c^2*d^2 + 1736*d^4 + 450*(225*b^4*c^2*d^2 - 26*b^2*d^4)*x^2 + 900*(75*b^4*c^3*d - 26*b^2*c*d^3)*x)*cos(b*x + a)^3 + 120*(2925*b^2*d^4*x^2 + 5850*b^2*c*d^3*x + 2925*b^2*c^2*d^2 - 6284*d^4)*cos(b*x + a) + 60*(1950*b^3*d^4*x^3 + 5850*b^3*c*d^3*x^2 + 1950*b^3*c^3*d - 12568*b*c*d^3 - 27*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 25*b^3*c^3*d - 6*b*c*d^3 + 3*(25*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^4 + (975*b^3*d^4*x^3 + 2925*b^3*c*d^3*x^2 + 975*b^3*c^3*d - 434*b*c*d^3 + (2925*b^3*c^2*d^2 - 434*b*d^4)*x)*cos(b*x + a)^2 + 2*(2925*b^3*c^2*d^2 - 6284*b*d^4)*x)*sin(b*x + a))/b^5

Sympy [A] time = 79.2152, size = 1098, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**4*cos(b*x+a)**2*sin(b*x+a)**3,x)

[Out] Piecewise((-c**4*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c**4*cos(a + b*x)**5/(15*b) - 4*c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 8*c**3*d*x*cos(a + b*x)**5/(15*b) - 2*c**2*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**3/b - 4*c**2*d**2*x**2*cos(a + b*x)**5/(5*b) - 4*c*d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 8*c*d**3*x**3*cos(a + b*x)**5/(15*b) - d**4*x**4*si

```

n(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*d**4*x**4*cos(a + b*x)**5/(15*b) +
104*c**3*d*sin(a + b*x)**5/(225*b**2) + 52*c**3*d*sin(a + b*x)**3*cos(a + b
*x)**2/(45*b**2) + 8*c**3*d*sin(a + b*x)*cos(a + b*x)**4/(15*b**2) + 104*c*
*2*d**2*x*sin(a + b*x)**5/(75*b**2) + 52*c**2*d**2*x*sin(a + b*x)**3*cos(a
+ b*x)**2/(15*b**2) + 8*c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**4/(5*b**2) +
104*c*d**3*x**2*sin(a + b*x)**5/(75*b**2) + 52*c*d**3*x**2*sin(a + b*x)**3
*cos(a + b*x)**2/(15*b**2) + 8*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**4/(5*
b**2) + 104*d**4*x**3*sin(a + b*x)**5/(225*b**2) + 52*d**4*x**3*sin(a + b*x
)**3*cos(a + b*x)**2/(45*b**2) + 8*d**4*x**3*sin(a + b*x)*cos(a + b*x)**4/(
15*b**2) + 104*c**2*d**2*sin(a + b*x)**4*cos(a + b*x)/(75*b**3) + 676*c**2*
d**2*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 1712*c**2*d**2*cos(a + b*
x)**5/(1125*b**3) + 208*c*d**3*x*sin(a + b*x)**4*cos(a + b*x)/(75*b**3) + 1
352*c*d**3*x*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 3424*c*d**3*x*cos
(a + b*x)**5/(1125*b**3) + 104*d**4*x**2*sin(a + b*x)**4*cos(a + b*x)/(75*b
**3) + 676*d**4*x**2*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 1712*d**4
*x**2*cos(a + b*x)**5/(1125*b**3) - 50272*c*d**3*sin(a + b*x)**5/(16875*b**
4) - 20456*c*d**3*sin(a + b*x)**3*cos(a + b*x)**2/(3375*b**4) - 3424*c*d**3
*sin(a + b*x)*cos(a + b*x)**4/(1125*b**4) - 50272*d**4*x*sin(a + b*x)**5/(1
6875*b**4) - 20456*d**4*x*sin(a + b*x)**3*cos(a + b*x)**2/(3375*b**4) - 342
4*d**4*x*sin(a + b*x)*cos(a + b*x)**4/(1125*b**4) - 50272*d**4*sin(a + b*x)
**4*cos(a + b*x)/(16875*b**5) - 303368*d**4*sin(a + b*x)**2*cos(a + b*x)**3
/(50625*b**5) - 760816*d**4*cos(a + b*x)**5/(253125*b**5), Ne(b, 0)), ((c**
4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*
*3*cos(a)**2, True))

```

Giac [A] time = 1.13131, size = 717, normalized size = 2.17

$$\frac{(625b^4d^4x^4 + 2500b^4cd^3x^3 + 3750b^4c^2d^2x^2 + 2500b^4c^3dx + 625b^4c^4 - 300b^2d^4x^2 - 600b^2cd^3x - 300b^2c^2d^2 + 24d^4)c^3\cos(a+b^2x)}{50000b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/50000*(625*b^4*d^4*x^4 + 2500*b^4*c*d^3*x^3 + 3750*b^4*c^2*d^2*x^2 + 2500
*b^4*c^3*d*x + 625*b^4*c^4 - 300*b^2*d^4*x^2 - 600*b^2*c*d^3*x - 300*b^2*c^
2*d^2 + 24*d^4)*cos(5*b*x + 5*a)/b^5 - 1/1296*(27*b^4*d^4*x^4 + 108*b^4*c*d
^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^
2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8*d^4)*cos(3*b*x + 3*a)/b^5 - 1/8*(b^
4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 -
12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^2*c^2*d^2 + 24*d^4)*cos(b*x + a)/b^
5 - 1/2500*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 75*b^3*c^2*d^2*x + 25*b^3*c
^3*d - 6*b*d^4*x - 6*b*c*d^3)*sin(5*b*x + 5*a)/b^5 + 1/108*(3*b^3*d^4*x^3 +
9*b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*s
in(3*b*x + 3*a)/b^5 + 1/2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x
+ b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*sin(b*x + a)/b^5
```

3.90 $\int (c + dx)^3 \cos^2(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=259

$$\frac{3d^2(c + dx) \cos(a + bx)}{4b^3} + \frac{d^2(c + dx) \cos(3a + 3bx)}{72b^3} - \frac{3d^2(c + dx) \cos(5a + 5bx)}{1000b^3} + \frac{3d(c + dx)^2 \sin(a + bx)}{8b^2} + \frac{d(c + dx)}{b}$$

```
[Out] (3*d^2*(c + d*x)*Cos[a + b*x])/(4*b^3) - ((c + d*x)^3*Cos[a + b*x])/(8*b) +
(d^2*(c + d*x)*Cos[3*a + 3*b*x])/(72*b^3) - ((c + d*x)^3*Cos[3*a + 3*b*x])
/(48*b) - (3*d^2*(c + d*x)*Cos[5*a + 5*b*x])/(1000*b^3) + ((c + d*x)^3*Cos[
5*a + 5*b*x])/(80*b) - (3*d^3*Sin[a + b*x])/(4*b^4) + (3*d*(c + d*x)^2*Sin[
a + b*x])/(8*b^2) - (d^3*Sin[3*a + 3*b*x])/(216*b^4) + (d*(c + d*x)^2*Sin[3
*a + 3*b*x])/(48*b^2) + (3*d^3*Sin[5*a + 5*b*x])/(5000*b^4) - (3*d*(c + d*x
)^2*Sin[5*a + 5*b*x])/(400*b^2)
```

Rubi [A] time = 0.279157, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3296, 2637}

$$\frac{3d^2(c + dx) \cos(a + bx)}{4b^3} + \frac{d^2(c + dx) \cos(3a + 3bx)}{72b^3} - \frac{3d^2(c + dx) \cos(5a + 5bx)}{1000b^3} + \frac{3d(c + dx)^2 \sin(a + bx)}{8b^2} + \frac{d(c + dx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (3*d^2*(c + d*x)*Cos[a + b*x])/(4*b^3) - ((c + d*x)^3*Cos[a + b*x])/(8*b) +
(d^2*(c + d*x)*Cos[3*a + 3*b*x])/(72*b^3) - ((c + d*x)^3*Cos[3*a + 3*b*x])
/(48*b) - (3*d^2*(c + d*x)*Cos[5*a + 5*b*x])/(1000*b^3) + ((c + d*x)^3*Cos[
5*a + 5*b*x])/(80*b) - (3*d^3*Sin[a + b*x])/(4*b^4) + (3*d*(c + d*x)^2*Sin[
a + b*x])/(8*b^2) - (d^3*Sin[3*a + 3*b*x])/(216*b^4) + (d*(c + d*x)^2*Sin[3
*a + 3*b*x])/(48*b^2) + (3*d^3*Sin[5*a + 5*b*x])/(5000*b^4) - (3*d*(c + d*x
)^2*Sin[5*a + 5*b*x])/(400*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3 \cos^2(a+bx) \sin^3(a+bx) dx &= \int \left(\frac{1}{8}(c+dx)^3 \sin(a+bx) + \frac{1}{16}(c+dx)^3 \sin(3a+3bx) - \frac{1}{16}(c+dx)^3 \sin(5a+5bx) \right) dx \\
&= \frac{1}{16} \int (c+dx)^3 \sin(3a+3bx) dx - \frac{1}{16} \int (c+dx)^3 \sin(5a+5bx) dx + \frac{1}{8} \int (c+dx)^3 \sin(a+bx) dx \\
&= -\frac{(c+dx)^3 \cos(a+bx)}{8b} - \frac{(c+dx)^3 \cos(3a+3bx)}{48b} + \frac{(c+dx)^3 \cos(5a+5bx)}{80b} \\
&= -\frac{(c+dx)^3 \cos(a+bx)}{8b} - \frac{(c+dx)^3 \cos(3a+3bx)}{48b} + \frac{(c+dx)^3 \cos(5a+5bx)}{80b} \\
&= \frac{3d^2(c+dx) \cos(a+bx)}{4b^3} - \frac{(c+dx)^3 \cos(a+bx)}{8b} + \frac{d^2(c+dx) \cos(3a+3bx)}{72b^3} \\
&= \frac{3d^2(c+dx) \cos(a+bx)}{4b^3} - \frac{(c+dx)^3 \cos(a+bx)}{8b} + \frac{d^2(c+dx) \cos(3a+3bx)}{72b^3}
\end{aligned}$$

Mathematica [A] time = 1.59901, size = 369, normalized size = 1.42

$$101250b^2c^2d \sin(a+bx) + 5625b^2c^2d \sin(3(a+bx)) - 2025b^2c^2d \sin(5(a+bx)) + 10125b^3c^2dx \cos(5(a+bx)) + 3375b^3c^2dx \cos(3(a+bx)) - 3375b^3c^2dx \cos(a+bx)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3 * Cos[a + b*x]^2 * Sin[a + b*x]^3, x]

[Out] (-33750*b*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 1875*b*(c + d*x)*(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 3375*b^3*c^3*Cos[5*(a + b*x)] - 810*b*c*d^2*Cos[5*(a + b*x)] + 10125*b^3*c^2*d*x*Cos[5*(a + b*x)] - 810*b*d^3*x*Cos[5*(a + b*x)] + 10125*b^3*c*d^2*x^2*Cos[5*(a + b*x)] + 3375*b^3*d^3*x^3*Cos[5*(a + b*x)] + 101250*b^2*c^2*d*Sin[a + b*x] - 202500*d^3*Sin[a + b*x] + 202500*b^2*c*d^2*x*Sin[a + b*x] + 101250*b^2*d^3*x^2*Sin[a + b*x] + 5625*b^2*c^2*d*Sin[3*(a + b*x)] - 1250*d^3*Sin[3*(a + b*x)] + 11250*b^2*c*d^2*x*Sin[3*(a + b*x)] + 5625*b^2*d^3*x^2*Sin[3*(a + b*x)] - 2025*b^2*c^2*d*Sin[5*(a + b*x)] + 162*d^3*Sin[5*(a + b*x)] - 4050*b^2*c*d^2*x*Sin[5*(a + b*x)] - 2025*b^2*d^3*x^2*Sin[5*(a + b*x)])/(270000*b^4)

Maple [B] time = 0.024, size = 992, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x)

[Out] 1/b*(1/b^3*d^3*(-1/3*(b*x+a)^3*(2+sin(b*x+a)^2)*cos(b*x+a)+2/5*(b*x+a)^2*sin(b*x+a)-856/1125*sin(b*x+a)+4/5*(b*x+a)*cos(b*x+a)+1/15*(b*x+a)^2*sin(b*x+a)^3+2/45*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+22/3375*sin(b*x+a)^3+1/5*(b*x+a)^3*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-3/25*(b*x+a)^2*sin(b*x+a)^5-6/125*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)+6/625*sin(b*x+a)^5)-3/b^3*a*d^3*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/15*cos(b*x+a)+4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3+2/135*(2+sin(b*x+a)^2)*cos(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-2/25*(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a))+3/b^2*c*d^2*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/15*cos(b*x+a)+4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3+2/135*(2+sin(b*x+a)^2)*cos(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)

$$\begin{aligned}
& -2/25*(b*x+a)*\sin(b*x+a)^5-2/125*(8/3+\sin(b*x+a)^4+4/3*\sin(b*x+a)^2)*\cos(b*x+a) \\
& +3/b^3*a^2*d^3*(-1/3*(b*x+a)*(2+\sin(b*x+a)^2)*\cos(b*x+a)+1/45*\sin(b*x+a)^3+2/15*\sin(b*x+a)+1/5*(b*x+a)*(8/3+\sin(b*x+a)^4+4/3*\sin(b*x+a)^2)*\cos(b*x+a)-1/25*\sin(b*x+a)^5)-6/b^2*a*c*d^2*(-1/3*(b*x+a)*(2+\sin(b*x+a)^2)*\cos(b*x+a)+1/45*\sin(b*x+a)^3+2/15*\sin(b*x+a)+1/5*(b*x+a)*(8/3+\sin(b*x+a)^4+4/3*\sin(b*x+a)^2)*\cos(b*x+a)-1/25*\sin(b*x+a)^5)+3/b*c^2*d*(-1/3*(b*x+a)*(2+\sin(b*x+a)^2)*\cos(b*x+a)+1/45*\sin(b*x+a)^3+2/15*\sin(b*x+a)+1/5*(b*x+a)*(8/3+\sin(b*x+a)^4+4/3*\sin(b*x+a)^2)*\cos(b*x+a)-1/25*\sin(b*x+a)^5)-1/b^3*a^3*d^3*(-1/5*\sin(b*x+a)^2*\cos(b*x+a)^3-2/15*\cos(b*x+a)^3)+3/b^2*a^2*c*d^2*(-1/5*\sin(b*x+a)^2*\cos(b*x+a)^3-2/15*\cos(b*x+a)^3)-3/b*a*c^2*d*(-1/5*\sin(b*x+a)^2*\cos(b*x+a)^3-2/15*\cos(b*x+a)^3)+c^3*(-1/5*\sin(b*x+a)^2*\cos(b*x+a)^3-2/15*\cos(b*x+a)^3)
\end{aligned}$$

Maxima [B] time = 1.38457, size = 1034, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned}
& 1/270000*(18000*(3*\cos(b*x + a)^5 - 5*\cos(b*x + a)^3)*c^3 - 54000*(3*\cos(b*x + a)^5 - 5*\cos(b*x + a)^3)*a^2*c*d/b + 54000*(3*\cos(b*x + a)^5 - 5*\cos(b*x + a)^3)*a^2*c*d^2/b^2 - 18000*(3*\cos(b*x + a)^5 - 5*\cos(b*x + a)^3)*a^3*d^3/b^3 + 225*(45*(b*x + a)*\cos(5*b*x + 5*a) - 75*(b*x + a)*\cos(3*b*x + 3*a) - 450*(b*x + a)*\cos(b*x + a) - 9*\sin(5*b*x + 5*a) + 25*\sin(3*b*x + 3*a) + 450*\sin(b*x + a))*c^2*d/b - 450*(45*(b*x + a)*\cos(5*b*x + 5*a) - 75*(b*x + a)*\cos(3*b*x + 3*a) - 450*(b*x + a)*\cos(b*x + a) - 9*\sin(5*b*x + 5*a) + 25*\sin(3*b*x + 3*a) + 450*\sin(b*x + a))*a*c*d^2/b^2 + 225*(45*(b*x + a)*\cos(5*b*x + 5*a) - 75*(b*x + a)*\cos(3*b*x + 3*a) - 450*(b*x + a)*\cos(b*x + a) - 9*\sin(5*b*x + 5*a) + 25*\sin(3*b*x + 3*a) + 450*\sin(b*x + a))*a^2*d^3/b^3 + 15*(27*(25*(b*x + a)^2 - 2)*\cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*\cos(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*\cos(b*x + a) - 270*(b*x + a)*\sin(5*b*x + 5*a) + 750*(b*x + a)*\sin(3*b*x + 3*a) + 13500*(b*x + a)*\sin(b*x + a))*c*d^2/b^2 - 15*(27*(25*(b*x + a)^2 - 2)*\cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*\cos(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*\cos(b*x + a) - 270*(b*x + a)*\sin(5*b*x + 5*a) + 750*(b*x + a)*\sin(3*b*x + 3*a) + 13500*(b*x + a)*\sin(b*x + a))*a*d^3/b^3 + (135*(25*(b*x + a)^3 - 6*b*x - 6*a)*\cos(5*b*x + 5*a) - 1875*(3*(b*x + a)^3 - 2*b*x - 2*a)*\cos(3*b*x + 3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*\cos(b*x + a) - 81*(25*(b*x + a)^2 - 2)*\sin(5*b*x + 5*a) + 625*(9*(b*x + a)^2 - 2)*\sin(3*b*x + 3*a) + 101250*((b*x + a)^2 - 2)*\sin(b*x + a))*d^3/b^3)/b
\end{aligned}$$

Fricas [A] time = 0.538011, size = 706, normalized size = 2.73

$$135(25b^3d^3x^3 + 75b^3cd^2x^2 + 25b^3c^3 - 6bcd^2 + 3(25b^3c^2d - 2bd^3)x)\cos(bx + a)^5 - 75(75b^3d^3x^3 + 225b^3cd^2x^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned}
& 1/16875*(135*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 25*b^3*c^3 - 6*b*c*d^2 + 3*(25*b^3*c^2*d - 2*b*d^3)*x)*\cos(b*x + a)^5 - 75*(75*b^3*d^3*x^3 + 225*b^3*c*d^2*x^2 + 75*b^3*c^3 - 26*b*c*d^2 + (225*b^3*c^2*d - 26*b*d^3)*x)*\cos(b
\end{aligned}$$

$$x + a)^3 + 11700*(b*d^3*x + b*c*d^2)*\cos(b*x + a) + (5850*b^2*d^3*x^2 + 11700*b^2*c*d^2*x + 5850*b^2*c^2*d - 81*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*\cos(b*x + a)^4 - 12568*d^3 + (2925*b^2*d^3*x^2 + 5850*b^2*c*d^2*x + 2925*b^2*c^2*d - 434*d^3)*\cos(b*x + a)^2)*\sin(b*x + a))/b^4$$

Sympy [A] time = 23.6664, size = 690, normalized size = 2.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*cos(b*x+a)**2*sin(b*x+a)**3,x)

[Out] Piecewise((-c**3*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c**3*cos(a + b*x)**5/(15*b) - c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**3/b - 2*c**2*d*x*cos(a + b*x)**5/(5*b) - c*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**3/b - 2*c*d**2*x**2*cos(a + b*x)**5/(5*b) - d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*d**3*x**3*cos(a + b*x)**5/(15*b) + 26*c**2*d*sin(a + b*x)**5/(75*b**2) + 13*c**2*d*sin(a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 2*c**2*d*sin(a + b*x)*cos(a + b*x)**4/(5*b**2) + 52*c*d**2*x*sin(a + b*x)**5/(75*b**2) + 26*c*d**2*x*sin(a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 4*c*d**2*x*sin(a + b*x)*cos(a + b*x)**4/(5*b**2) + 26*d**3*x**2*sin(a + b*x)**5/(75*b**2) + 13*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 2*d**3*x**2*sin(a + b*x)*cos(a + b*x)**4/(5*b**2) + 52*c*d**2*sin(a + b*x)**4*cos(a + b*x)/(75*b**3) + 338*c*d**2*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 856*c*d**2*cos(a + b*x)**5/(1125*b**3) + 52*d**3*x*sin(a + b*x)**4*cos(a + b*x)/(75*b**3) + 338*d**3*x*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 856*d**3*x*cos(a + b*x)**5/(1125*b**3) - 12568*d**3*sin(a + b*x)**5/(16875*b**4) - 5114*d**3*sin(a + b*x)**3*cos(a + b*x)**2/(3375*b**4) - 856*d**3*sin(a + b*x)*cos(a + b*x)**4/(1125*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**3*cos(a)**2, True))

Giac [A] time = 1.11185, size = 474, normalized size = 1.83

$$\frac{(25b^3d^3x^3 + 75b^3cd^2x^2 + 75b^3c^2dx + 25b^3c^3 - 6bd^3x - 6bcd^2)\cos(5bx + 5a)}{2000b^4} - \frac{(3b^3d^3x^3 + 9b^3cd^2x^2 + 9b^3c^2dx + 3b^3c^3 - 6bd^3x - 6bcd^2)\cos(3bx + 3a)}{144b^4} - \frac{(b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + b^3c^3 - 6bd^3x - 6bcd^2)\cos(bx + a)}{8b^4} - \frac{3(25b^2d^3x^2 + 50b^2cd^2x + 25b^2c^2d - 2d^3)\sin(5bx + 5a)}{10000b^4} + \frac{1(9b^2d^3x^2 + 18b^2cd^2x + 9b^2c^2d - 2d^3)\sin(3bx + 3a)}{432b^4} + \frac{3(b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3)\sin(bx + a)}{8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")

[Out] 1/2000*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 75*b^3*c^2*d*x + 25*b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*cos(5*b*x + 5*a)/b^4 - 1/144*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^3*x - 2*b*c*d^2)*cos(3*b*x + 3*a)/b^4 - 1/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*cos(b*x + a)/b^4 - 3/10000*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*sin(5*b*x + 5*a)/b^4 + 1/432*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*sin(3*b*x + 3*a)/b^4 + 3/8*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a)/b^4

3.91 $\int (c + dx)^2 \cos^2(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=184

$$\frac{d(c + dx) \sin(a + bx)}{4b^2} + \frac{d(c + dx) \sin(3a + 3bx)}{72b^2} - \frac{d(c + dx) \sin(5a + 5bx)}{200b^2} + \frac{d^2 \cos(a + bx)}{4b^3} + \frac{d^2 \cos(3a + 3bx)}{216b^3} - \frac{d^2}{216b^3}$$

[Out] (d^2*cos[a + b*x])/(4*b^3) - ((c + d*x)^2*cos[a + b*x])/(8*b) + (d^2*cos[3*a + 3*b*x])/(216*b^3) - ((c + d*x)^2*cos[3*a + 3*b*x])/(48*b) - (d^2*cos[5*a + 5*b*x])/(1000*b^3) + ((c + d*x)^2*cos[5*a + 5*b*x])/(80*b) + (d*(c + d*x)*Sin[a + b*x])/(4*b^2) + (d*(c + d*x)*Sin[3*a + 3*b*x])/(72*b^2) - (d*(c + d*x)*Sin[5*a + 5*b*x])/(200*b^2)

Rubi [A] time = 0.197179, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3296, 2638}

$$\frac{d(c + dx) \sin(a + bx)}{4b^2} + \frac{d(c + dx) \sin(3a + 3bx)}{72b^2} - \frac{d(c + dx) \sin(5a + 5bx)}{200b^2} + \frac{d^2 \cos(a + bx)}{4b^3} + \frac{d^2 \cos(3a + 3bx)}{216b^3} - \frac{d^2}{216b^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2*cos[a + b*x]^2*sin[a + b*x]^3,x]

[Out] (d^2*cos[a + b*x])/(4*b^3) - ((c + d*x)^2*cos[a + b*x])/(8*b) + (d^2*cos[3*a + 3*b*x])/(216*b^3) - ((c + d*x)^2*cos[3*a + 3*b*x])/(48*b) - (d^2*cos[5*a + 5*b*x])/(1000*b^3) + ((c + d*x)^2*cos[5*a + 5*b*x])/(80*b) + (d*(c + d*x)*Sin[a + b*x])/(4*b^2) + (d*(c + d*x)*Sin[3*a + 3*b*x])/(72*b^2) - (d*(c + d*x)*Sin[5*a + 5*b*x])/(200*b^2)

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (c+dx)^2 \cos^2(a+bx) \sin^3(a+bx) dx &= \int \left(\frac{1}{8}(c+dx)^2 \sin(a+bx) + \frac{1}{16}(c+dx)^2 \sin(3a+3bx) - \frac{1}{16}(c+dx)^2 \sin(5a+5bx) \right) dx \\
&= \frac{1}{16} \int (c+dx)^2 \sin(3a+3bx) dx - \frac{1}{16} \int (c+dx)^2 \sin(5a+5bx) dx + \frac{1}{8} \int (c+dx)^2 \sin(a+bx) dx \\
&= -\frac{(c+dx)^2 \cos(a+bx)}{8b} - \frac{(c+dx)^2 \cos(3a+3bx)}{48b} + \frac{(c+dx)^2 \cos(5a+5bx)}{80b} \\
&= -\frac{(c+dx)^2 \cos(a+bx)}{8b} - \frac{(c+dx)^2 \cos(3a+3bx)}{48b} + \frac{(c+dx)^2 \cos(5a+5bx)}{80b} \\
&= \frac{d^2 \cos(a+bx)}{4b^3} - \frac{(c+dx)^2 \cos(a+bx)}{8b} + \frac{d^2 \cos(3a+3bx)}{216b^3} - \frac{(c+dx)^2 \cos(3a+3bx)}{48b}
\end{aligned}$$

Mathematica [A] time = 0.923184, size = 127, normalized size = 0.69

$$\frac{-6750 \cos(a+bx) (b^2(c+dx)^2 - 2d^2) - 125 \cos(3(a+bx)) (9b^2(c+dx)^2 - 2d^2) + 27 \cos(5(a+bx)) (25b^2(c+dx)^2 - 2d^2)}{54000b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^3,x]

[Out] (-6750*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 125*(-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 27*(-2*d^2 + 25*b^2*(c + d*x)^2)*Cos[5*(a + b*x)] + 30*b*d*(c + d*x)*(450*Sin[a + b*x] + 25*Sin[3*(a + b*x)] - 9*Sin[5*(a + b*x)]))/(54000*b^3)

Maple [B] time = 0.026, size = 466, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x)

[Out] 1/b*(1/b^2*d^2*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/15*cos(b*x+a)+4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3+2/135*(2+sin(b*x+a)^2)*cos(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-2/25*(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a))-2/b^2*a*d^2*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+a)^5)+2/b*c*d*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+a)^5)+1/b^2*a^2*d^2*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)-2/b*a*c*d*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)+c^2*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3))

Maxima [B] time = 1.25098, size = 506, normalized size = 2.75

$$\frac{3600(3 \cos(bx+a)^5 - 5 \cos(bx+a)^3)c^2 - \frac{7200(3 \cos(bx+a)^5 - 5 \cos(bx+a)^3)acd}{b} + \frac{3600(3 \cos(bx+a)^5 - 5 \cos(bx+a)^3)a^2d^2}{b^2} + \frac{30(45(bx+a)^5 - 75(bx+a)^3)c^2}{b^3}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")

[Out] 1/54000*(3600*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c^2 - 7200*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*a*c*d/b + 3600*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*a^2*d^2/b^2 + 30*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*c*d/b - 30*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a*d^2/b^2 + (27*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*x + a)*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*d^2/b^2)/b

Fricas [A] time = 0.505668, size = 406, normalized size = 2.21

$$\frac{27(25b^2d^2x^2 + 50b^2cdx + 25b^2c^2 - 2d^2)\cos(bx + a)^5 - 5(225b^2d^2x^2 + 450b^2cdx + 225b^2c^2 - 26d^2)\cos(bx + a)^3}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")

[Out] 1/3375*(27*(25*b^2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*cos(b*x + a)^5 - 5*(225*b^2*d^2*x^2 + 450*b^2*c*d*x + 225*b^2*c^2 - 26*d^2)*cos(b*x + a)^3 + 780*d^2*cos(b*x + a) - 30*(9*(b*d^2*x + b*c*d)*cos(b*x + a)^4 - 26*b*d^2*x - 26*b*c*d - 13*(b*d^2*x + b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/b^3

Sympy [A] time = 20.5763, size = 382, normalized size = 2.08

$$\left\{ \frac{c^2 \sin^2(a+bx) \cos^3(a+bx)}{3b} - \frac{2c^2 \cos^5(a+bx)}{15b} - \frac{2cdx \sin^2(a+bx) \cos^3(a+bx)}{3b} - \frac{4cdx \cos^5(a+bx)}{15b} - \frac{d^2x^2 \sin^2(a+bx) \cos^3(a+bx)}{3b} - \frac{2d^2x^2 \cos^5(a+bx)}{15b} \right\} \left(c^2x + cdx^2 + \frac{d^2x^3}{3} \right) \sin^3(a) \cos^2(a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*cos(b*x+a)**2*sin(b*x+a)**3,x)

[Out] Piecewise((-c**2*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c**2*cos(a + b*x)**5/(15*b) - 2*c*d*x*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 4*c*d*x*cos(a + b*x)**5/(15*b) - d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*d**2*x**2*cos(a + b*x)**5/(15*b) + 52*c*d*sin(a + b*x)**5/(225*b**2) + 26*c*d*sin(a + b*x)**3*cos(a + b*x)**2/(45*b**2) + 4*c*d*sin(a + b*x)*cos(a + b*x)**4/(15*b**2) + 52*d**2*x*sin(a + b*x)**5/(225*b**2) + 26*d**2*x*sin(a + b*x)**3*cos(a + b*x)**2/(45*b**2) + 4*d**2*x*sin(a + b*x)*cos(a + b*x)**4/(15*b**2) + 52*d**2*sin(a + b*x)**4*cos(a + b*x)/(225*b**3) + 338*d**2*sin(a + b*x)**2*cos(a + b*x)**3/(675*b**3) + 856*d**2*cos(a + b*x)**5/(3375*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**3*cos(a)**2, True))

Giac [A] time = 1.12945, size = 282, normalized size = 1.53

$$\frac{(25b^2d^2x^2 + 50b^2cdx + 25b^2c^2 - 2d^2)\cos(5bx + 5a)}{2000b^3} - \frac{(9b^2d^2x^2 + 18b^2cdx + 9b^2c^2 - 2d^2)\cos(3bx + 3a)}{432b^3} - \frac{(b^2d^2x^2 + 2b^2cdx + b^2c^2 - 2d^2)\cos(bx + a)}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2000*(25*b^2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*cos(5*b*x + 5*a)
)/b^3 - 1/432*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(3*b*x
+ 3*a)/b^3 - 1/8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)
/b^3 - 1/200*(b*d^2*x + b*c*d)*sin(5*b*x + 5*a)/b^3 + 1/72*(b*d^2*x + b*c*d
)*sin(3*b*x + 3*a)/b^3 + 1/4*(b*d^2*x + b*c*d)*sin(b*x + a)/b^3
```

3.92 $\int (c + dx) \cos^2(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=109

$$\frac{d \sin(a + bx)}{8b^2} + \frac{d \sin(3a + 3bx)}{144b^2} - \frac{d \sin(5a + 5bx)}{400b^2} - \frac{(c + dx) \cos(a + bx)}{8b} - \frac{(c + dx) \cos(3a + 3bx)}{48b} + \frac{(c + dx) \cos(5a + 5bx)}{80b}$$

[Out] $-\frac{(c + dx) \cos(a + bx)}{8b} - \frac{(c + dx) \cos(3a + 3bx)}{48b} + \frac{(c + dx) \cos(5a + 5bx)}{80b} + \frac{d \sin(a + bx)}{8b^2} + \frac{d \sin(3a + 3bx)}{144b^2} - \frac{d \sin(5a + 5bx)}{400b^2}$

Rubi [A] time = 0.0969841, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4406, 3296, 2637}

$$\frac{d \sin(a + bx)}{8b^2} + \frac{d \sin(3a + 3bx)}{144b^2} - \frac{d \sin(5a + 5bx)}{400b^2} - \frac{(c + dx) \cos(a + bx)}{8b} - \frac{(c + dx) \cos(3a + 3bx)}{48b} + \frac{(c + dx) \cos(5a + 5bx)}{80b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]

[Out] $-\frac{(c + dx) \cos(a + bx)}{8b} - \frac{(c + dx) \cos(3a + 3bx)}{48b} + \frac{(c + dx) \cos(5a + 5bx)}{80b} + \frac{d \sin(a + bx)}{8b^2} + \frac{d \sin(3a + 3bx)}{144b^2} - \frac{d \sin(5a + 5bx)}{400b^2}$

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n * Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m * Cos[e + f*x] / f, x] + Dist[(d*m) / f, Int[(c + d*x)^(m - 1) * Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x] / d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (c + dx) \cos^2(a + bx) \sin^3(a + bx) dx &= \int \left(\frac{1}{8}(c + dx) \sin(a + bx) + \frac{1}{16}(c + dx) \sin(3a + 3bx) - \frac{1}{16}(c + dx) \sin(5a + 5bx) \right) dx \\ &= \frac{1}{16} \int (c + dx) \sin(3a + 3bx) dx - \frac{1}{16} \int (c + dx) \sin(5a + 5bx) dx + \frac{1}{8} \int (c + dx) \sin(a + bx) dx \\ &= -\frac{(c + dx) \cos(a + bx)}{8b} - \frac{(c + dx) \cos(3a + 3bx)}{48b} + \frac{(c + dx) \cos(5a + 5bx)}{80b} \\ &= -\frac{(c + dx) \cos(a + bx)}{8b} - \frac{(c + dx) \cos(3a + 3bx)}{48b} + \frac{(c + dx) \cos(5a + 5bx)}{80b} + \end{aligned}$$

Mathematica [A] time = 0.324346, size = 94, normalized size = 0.86

$$\frac{-450b(c + dx) \cos(a + bx) - 75b(c + dx) \cos(3(a + bx)) + 45bc \cos(5(a + bx)) + 450d \sin(a + bx) + 25d \sin(3(a + bx)) - 3600b^2}{3600b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]

[Out] (-450*b*(c + d*x)*Cos[a + b*x] - 75*b*(c + d*x)*Cos[3*(a + b*x)] + 45*b*c*cos[5*(a + b*x)] + 45*b*d*x*cos[5*(a + b*x)] + 450*d*Sin[a + b*x] + 25*d*Sin[3*(a + b*x)] - 9*d*Sin[5*(a + b*x)])/(3600*b^2)

Maple [A] time = 0.023, size = 163, normalized size = 1.5

$$\frac{1}{b} \left(\frac{d}{b} \left(-\frac{(bx+a)(2+(\sin(bx+a))^2)\cos(bx+a)}{3} + \frac{(\sin(bx+a))^3}{45} + \frac{2\sin(bx+a)}{15} + \frac{(bx+a)\cos(bx+a)}{5} \right) \left(\frac{8}{3} + \sin(bx+a) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x)

[Out] 1/b*(d/b*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+a)^5)-1/b*d*a*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)+c*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3))

Maxima [A] time = 1.23504, size = 188, normalized size = 1.72

$$\frac{240(3 \cos(bx+a)^5 - 5 \cos(bx+a)^3)c - \frac{240(3 \cos(bx+a)^5 - 5 \cos(bx+a)^3)ad}{b} + \frac{(45(bx+a)\cos(5bx+5a) - 75(bx+a)\cos(3bx+3a) - 450(bx+a)\cos(bx+a) - 9\sin(5bx+5a) + 25\sin(3bx+3a) + 450\sin(bx+a))d}{3600b}}{3600b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")

[Out] 1/3600*(240*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c - 240*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*a*d/b + (45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*d/b)/b

Fricas [A] time = 0.490192, size = 194, normalized size = 1.78

$$\frac{45(bdx + bc) \cos(bx + a)^5 - 75(bdx + bc) \cos(bx + a)^3 - (9d \cos(bx + a)^4 - 13d \cos(bx + a)^2 - 26d) \sin(bx + a)}{225b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")

[Out] $\frac{1}{225}*(45*(b*d*x + b*c)*\cos(b*x + a)^5 - 75*(b*d*x + b*c)*\cos(b*x + a)^3 - (9*d*\cos(b*x + a)^4 - 13*d*\cos(b*x + a)^2 - 26*d)*\sin(b*x + a))/b^2$

Sympy [A] time = 9.73825, size = 163, normalized size = 1.5

$$\left\{ \begin{array}{l} -\frac{c \sin^2(a+bx) \cos^3(a+bx)}{3b} - \frac{2c \cos^5(a+bx)}{15b} - \frac{dx \sin^2(a+bx) \cos^3(a+bx)}{3b} - \frac{2dx \cos^5(a+bx)}{15b} + \frac{26d \sin^5(a+bx)}{225b^2} + \frac{13d \sin^3(a+bx) \cos^2(a+bx)}{45b^2} + 2 \\ \left(cx + \frac{dx^2}{2} \right) \sin^3(a) \cos^2(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)**2*sin(b*x+a)**3,x)

[Out] Piecewise((-c*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c*cos(a + b*x)**5/(15*b) - d*x*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*d*x*cos(a + b*x)**5/(15*b) + 26*d*sin(a + b*x)**5/(225*b**2) + 13*d*sin(a + b*x)**3*cos(a + b*x)**2/(45*b**2) + 2*d*sin(a + b*x)*cos(a + b*x)**4/(15*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*sin(a)**3*cos(a)**2, True))

Giac [A] time = 1.20442, size = 143, normalized size = 1.31

$$\frac{(bdx + bc) \cos(5bx + 5a)}{80b^2} - \frac{(bdx + bc) \cos(3bx + 3a)}{48b^2} - \frac{(bdx + bc) \cos(bx + a)}{8b^2} - \frac{d \sin(5bx + 5a)}{400b^2} + \frac{d \sin(3bx + 3a)}{144b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")

[Out] $\frac{1}{80}*(b*d*x + b*c)*\cos(5*b*x + 5*a)/b^2 - \frac{1}{48}*(b*d*x + b*c)*\cos(3*b*x + 3*a)/b^2 - \frac{1}{8}*(b*d*x + b*c)*\cos(b*x + a)/b^2 - \frac{1}{400}*d*\sin(5*b*x + 5*a)/b^2 + \frac{1}{144}*d*\sin(3*b*x + 3*a)/b^2 + \frac{1}{8}*d*\sin(b*x + a)/b^2$

3.93 $\int \frac{\cos^2(a+bx) \sin^3(a+bx)}{c+dx} dx$

Optimal. Leaf size=185

$$\frac{\sin\left(5a - \frac{5bc}{d}\right) \operatorname{CosIntegral}\left(\frac{5bc}{d} + 5bx\right)}{16d} + \frac{\sin\left(3a - \frac{3bc}{d}\right) \operatorname{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{16d} + \frac{\sin\left(a - \frac{bc}{d}\right) \operatorname{CosIntegral}\left(\frac{bc}{d} + bx\right)}{8d}$$

[Out] -(CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a - (5*b*c)/d])/(16*d) + (CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(16*d) + (CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(8*d) + (Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d) + (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d) - (Cos[5*a - (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d)

Rubi [A] time = 0.338659, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4406, 3303, 3299, 3302}

$$\frac{\sin\left(5a - \frac{5bc}{d}\right) \operatorname{CosIntegral}\left(\frac{5bc}{d} + 5bx\right)}{16d} + \frac{\sin\left(3a - \frac{3bc}{d}\right) \operatorname{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{16d} + \frac{\sin\left(a - \frac{bc}{d}\right) \operatorname{CosIntegral}\left(\frac{bc}{d} + bx\right)}{8d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x), x]

[Out] -(CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a - (5*b*c)/d])/(16*d) + (CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(16*d) + (CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(8*d) + (Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d) + (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d) - (Cos[5*a - (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d)

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx} dx &= \int \left(\frac{\sin(a+bx)}{8(c+dx)} + \frac{\sin(3a+3bx)}{16(c+dx)} - \frac{\sin(5a+5bx)}{16(c+dx)} \right) dx \\
&= \frac{1}{16} \int \frac{\sin(3a+3bx)}{c+dx} dx - \frac{1}{16} \int \frac{\sin(5a+5bx)}{c+dx} dx + \frac{1}{8} \int \frac{\sin(a+bx)}{c+dx} dx \\
&= - \left(\frac{1}{16} \cos \left(5a - \frac{5bc}{d} \right) \int \frac{\sin \left(\frac{5bc}{d} + 5bx \right)}{c+dx} dx \right) + \frac{1}{16} \cos \left(3a - \frac{3bc}{d} \right) \int \frac{\sin \left(\frac{3bc}{d} + 3bx \right)}{c+dx} dx \\
&\quad + \frac{1}{8} \cos \left(a - \frac{bc}{d} \right) \int \frac{\sin \left(\frac{bc}{d} + bx \right)}{c+dx} dx \\
&= - \frac{\text{Ci} \left(\frac{5bc}{d} + 5bx \right) \sin \left(5a - \frac{5bc}{d} \right)}{16d} + \frac{\text{Ci} \left(\frac{3bc}{d} + 3bx \right) \sin \left(3a - \frac{3bc}{d} \right)}{16d} + \frac{\text{Ci} \left(\frac{bc}{d} + bx \right) \sin \left(a - \frac{bc}{d} \right)}{8d}
\end{aligned}$$

Mathematica [A] time = 0.523339, size = 154, normalized size = 0.83

$$\frac{\sin \left(5a - \frac{5bc}{d} \right) \left(-\text{CosIntegral} \left(\frac{5b(c+dx)}{d} \right) \right) + \sin \left(3a - \frac{3bc}{d} \right) \text{CosIntegral} \left(\frac{3b(c+dx)}{d} \right) + 2 \sin \left(a - \frac{bc}{d} \right) \text{CosIntegral} \left(b \left(\frac{c}{d} + x \right) \right)}{16d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x), x]

[Out] $(-\text{CosIntegral}[(5*b*(c + d*x))/d]*\text{Sin}[5*a - (5*b*c)/d]) + \text{CosIntegral}[(3*b*(c + d*x))/d]*\text{Sin}[3*a - (3*b*c)/d] + 2*\text{CosIntegral}[b*(c/d + x)]*\text{Sin}[a - (b*c)/d] + 2*\text{Cos}[a - (b*c)/d]*\text{SinIntegral}[b*(c/d + x)] + \text{Cos}[3*a - (3*b*c)/d]*\text{SinIntegral}[(3*b*(c + d*x))/d] - \text{Cos}[5*a - (5*b*c)/d]*\text{SinIntegral}[(5*b*(c + d*x))/d])/(16*d)$

Maple [A] time = 0.024, size = 253, normalized size = 1.4

$$\frac{1}{b} \left(-\frac{b}{80} \left(5 \frac{1}{d} \text{Si} \left(5bx + 5a + 5 \frac{-ad+bc}{d} \right) \cos \left(5 \frac{-ad+bc}{d} \right) - 5 \frac{1}{d} \text{Ci} \left(5bx + 5a + 5 \frac{-ad+bc}{d} \right) \sin \left(5 \frac{-ad+bc}{d} \right) \right) + \frac{b}{8} \left(\dots \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c), x)

[Out] $1/b*(-1/80*b*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d-5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+1/8*b*(Si(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+1/48*b*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)$

Maxima [C] time = 1.72738, size = 549, normalized size = 2.97

$$\frac{b \left(-2i E_1 \left(\frac{ibc+i(bx+a)d-id}{d} \right) + 2i E_1 \left(-\frac{ibc+i(bx+a)d-id}{d} \right) \right) \cos \left(-\frac{bc-ad}{d} \right) + b \left(-i E_1 \left(\frac{3ibc+3i(bx+a)d-3iad}{d} \right) + i E_1 \left(-\frac{3ibc+3i(bx+a)d-3iad}{d} \right) \right)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")

[Out] $\frac{1}{32} * (b * (-2 * I * \exp_integral_e(1, (I * b * c + I * (b * x + a) * d - I * a * d) / d) + 2 * I * \exp_integral_e(1, -(I * b * c + I * (b * x + a) * d - I * a * d) / d)) * \cos(-(b * c - a * d) / d) + b * (-I * \exp_integral_e(1, (3 * I * b * c + 3 * I * (b * x + a) * d - 3 * I * a * d) / d) + I * \exp_integral_e(1, -(3 * I * b * c + 3 * I * (b * x + a) * d - 3 * I * a * d) / d)) * \cos(-3 * (b * c - a * d) / d) + b * (I * \exp_integral_e(1, (5 * I * b * c + 5 * I * (b * x + a) * d - 5 * I * a * d) / d) - I * \exp_integral_e(1, -(5 * I * b * c + 5 * I * (b * x + a) * d - 5 * I * a * d) / d)) * \cos(-5 * (b * c - a * d) / d) - 2 * b * (\exp_integral_e(1, (I * b * c + I * (b * x + a) * d - I * a * d) / d) + \exp_integral_e(1, -(I * b * c + I * (b * x + a) * d - I * a * d) / d)) * \sin(-(b * c - a * d) / d) - b * (\exp_integral_e(1, (3 * I * b * c + 3 * I * (b * x + a) * d - 3 * I * a * d) / d) + \exp_integral_e(1, -(3 * I * b * c + 3 * I * (b * x + a) * d - 3 * I * a * d) / d)) * \sin(-3 * (b * c - a * d) / d) + b * (\exp_integral_e(1, (5 * I * b * c + 5 * I * (b * x + a) * d - 5 * I * a * d) / d) + \exp_integral_e(1, -(5 * I * b * c + 5 * I * (b * x + a) * d - 5 * I * a * d) / d)) * \sin(-5 * (b * c - a * d) / d)) / (b * d)$

Fricas [A] time = 0.50882, size = 612, normalized size = 3.31

$$2 \left(\operatorname{Ci} \left(\frac{bdx+bc}{d} \right) + \operatorname{Ci} \left(-\frac{bdx+bc}{d} \right) \right) \sin \left(-\frac{bc-ad}{d} \right) + \left(\operatorname{Ci} \left(\frac{3(bdx+bc)}{d} \right) + \operatorname{Ci} \left(-\frac{3(bdx+bc)}{d} \right) \right) \sin \left(-\frac{3(bc-ad)}{d} \right) - \left(\operatorname{Ci} \left(\frac{5(bdx+bc)}{d} \right) + \operatorname{Ci} \left(-\frac{5(bdx+bc)}{d} \right) \right) \sin \left(-\frac{5(bc-ad)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")

[Out] $\frac{1}{32} * (2 * (\cos_integral((b * d * x + b * c) / d) + \cos_integral(-(b * d * x + b * c) / d)) * \sin(-(b * c - a * d) / d) + (\cos_integral(3 * (b * d * x + b * c) / d) + \cos_integral(-3 * (b * d * x + b * c) / d)) * \sin(-3 * (b * c - a * d) / d) - (\cos_integral(5 * (b * d * x + b * c) / d) + \cos_integral(-5 * (b * d * x + b * c) / d)) * \sin(-5 * (b * c - a * d) / d) - 2 * \cos(-5 * (b * c - a * d) / d) * \sin_integral(5 * (b * d * x + b * c) / d) + 2 * \cos(-3 * (b * c - a * d) / d) * \sin_integral(3 * (b * d * x + b * c) / d) + 4 * \cos(-(b * c - a * d) / d) * \sin_integral((b * d * x + b * c) / d)) / d$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3(a + bx) \cos^2(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c),x)

[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x), x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.94 \quad \int \frac{\cos^2(a+bx) \sin^3(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=257

$$\frac{b \cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{8d^2} + \frac{3b \cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{16d^2} - \frac{5b \cos\left(5a - \frac{5bc}{d}\right) \text{CosIntegral}\left(\frac{5bc}{d} + 5bx\right)}{16d^2}$$

```
[Out] (b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d^2) + (3*b*Cos[3*a - (3
*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) - (5*b*Cos[5*a - (5*b*c)/
d]*CosIntegral[(5*b*c)/d + 5*b*x])/(16*d^2) - Sin[a + b*x]/(8*d*(c + d*x))
- Sin[3*a + 3*b*x]/(16*d*(c + d*x)) + Sin[5*a + 5*b*x]/(16*d*(c + d*x)) - (
b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^2) - (3*b*Sin[3*a - (3*
b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) + (5*b*Sin[5*a - (5*b*c)/d
]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d^2)
```

Rubi [A] time = 0.416478, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b \cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{8d^2} + \frac{3b \cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{16d^2} - \frac{5b \cos\left(5a - \frac{5bc}{d}\right) \text{CosIntegral}\left(\frac{5bc}{d} + 5bx\right)}{16d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] (b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d^2) + (3*b*Cos[3*a - (3
*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) - (5*b*Cos[5*a - (5*b*c)/
d]*CosIntegral[(5*b*c)/d + 5*b*x])/(16*d^2) - Sin[a + b*x]/(8*d*(c + d*x))
- Sin[3*a + 3*b*x]/(16*d*(c + d*x)) + Sin[5*a + 5*b*x]/(16*d*(c + d*x)) - (
b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^2) - (3*b*Sin[3*a - (3*
b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) + (5*b*Sin[5*a - (5*b*c)/d
]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(a + bx) \sin^3(a + bx)}{(c + dx)^2} dx &= \int \left(\frac{\sin(a + bx)}{8(c + dx)^2} + \frac{\sin(3a + 3bx)}{16(c + dx)^2} - \frac{\sin(5a + 5bx)}{16(c + dx)^2} \right) dx \\ &= \frac{1}{16} \int \frac{\sin(3a + 3bx)}{(c + dx)^2} dx - \frac{1}{16} \int \frac{\sin(5a + 5bx)}{(c + dx)^2} dx + \frac{1}{8} \int \frac{\sin(a + bx)}{(c + dx)^2} dx \\ &= -\frac{\sin(a + bx)}{8d(c + dx)} - \frac{\sin(3a + 3bx)}{16d(c + dx)} + \frac{\sin(5a + 5bx)}{16d(c + dx)} + \frac{b \int \frac{\cos(a+bx)}{c+dx} dx}{8d} + \frac{(3b) \int \frac{\cos(3a+3bx)}{c+dx} dx}{16d} \\ &= -\frac{\sin(a + bx)}{8d(c + dx)} - \frac{\sin(3a + 3bx)}{16d(c + dx)} + \frac{\sin(5a + 5bx)}{16d(c + dx)} - \frac{\left(5b \cos\left(5a - \frac{5bc}{d}\right)\right) \int \frac{\cos\left(\frac{5bc}{d} + 5bx\right)}{c+dx} dx}{16d} \\ &= \frac{b \cos\left(a - \frac{bc}{d}\right) \text{Ci}\left(\frac{bc}{d} + bx\right)}{8d^2} + \frac{3b \cos\left(3a - \frac{3bc}{d}\right) \text{Ci}\left(\frac{3bc}{d} + 3bx\right)}{16d^2} - \frac{5b \cos\left(5a - \frac{5bc}{d}\right) \text{Ci}\left(\frac{5bc}{d} + 5bx\right)}{16d^2} \end{aligned}$$

Mathematica [A] time = 1.56163, size = 213, normalized size = 0.83

$$2b \cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(b\left(\frac{c}{d} + x\right)\right) + 3b \cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right) - 5b \cos\left(5a - \frac{5bc}{d}\right) \text{CosIntegral}\left(\frac{5b(c+dx)}{d}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^2,x]

[Out] (2*b*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] + 3*b*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*(c + d*x))/d] - 5*b*Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b*(c + d*x))/d] - (2*d*Sin[a + b*x])/(c + d*x) - (d*Sin[3*(a + b*x)])/(c + d*x) + (d*Sin[5*(a + b*x)])/(c + d*x) - 2*b*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 3*b*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] + 5*b*Sin[5*a - (5*b*c)/d]*SinIntegral[(5*b*(c + d*x))/d])/(16*d^2)

Maple [A] time = 0.027, size = 365, normalized size = 1.4

$$\frac{1}{b} \left(-\frac{b^2}{80} \left(-5 \frac{\sin(5bx + 5a)}{((bx + a)d - ad + bc)d} + 5 \frac{1}{d} \left(5 \frac{1}{d} \text{Si}\left(5bx + 5a + 5 \frac{-ad + bc}{d}\right) \sin\left(5 \frac{-ad + bc}{d}\right) + 5 \frac{1}{d} \text{Ci}\left(5bx + 5a + 5 \frac{-ad + bc}{d}\right) \cos\left(5 \frac{-ad + bc}{d}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x)

[Out] 1/b*(-1/80*b^2*(-5*sin(5*b*x+5*a)/((b*x+a)*d-a*d+b*c)/d+5*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d))

$-a*d+b*c)/d)/d)/d)+1/8*b^2*(-\sin(b*x+a)/((b*x+a)*d-a*d+b*c)/d+(\text{Si}(b*x+a+(-a*d+b*c)/d)*\sin((-a*d+b*c)/d)/d+\text{Ci}(b*x+a+(-a*d+b*c)/d)*\cos((-a*d+b*c)/d)/d)/d)+1/48*b^2*(-3*\sin(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)/d+3*(3*\text{Si}(3*b*x+3*a+3*(-a*d+b*c)/d)*\sin(3*(-a*d+b*c)/d)/d+3*\text{Ci}(3*b*x+3*a+3*(-a*d+b*c)/d)*\cos(3*(-a*d+b*c)/d)/d)/d))$

Maxima [C] time = 2.12228, size = 591, normalized size = 2.3

$$b^2 \left(-2i E_2 \left(\frac{ibc+i(bx+a)d-iad}{d} \right) + 2i E_2 \left(-\frac{ibc+i(bx+a)d-iad}{d} \right) \right) \cos \left(-\frac{bc-ad}{d} \right) + b^2 \left(-i E_2 \left(\frac{3ibc+3i(bx+a)d-3iad}{d} \right) + i E_2 \left(-\frac{3ibc+3i(bx+a)d-3iad}{d} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")

[Out] $1/32*(b^2*(-2*I*\exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 2*I*\exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*\cos(-(b*c - a*d)/d) + b^2*(-I*\exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + I*\exp_integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*\cos(-3*(b*c - a*d)/d) + b^2*(I*\exp_integral_e(2, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) - I*\exp_integral_e(2, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*\cos(-5*(b*c - a*d)/d) - 2*b^2*(\exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + \exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*\sin(-(b*c - a*d)/d) - b^2*(\exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + \exp_integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*\sin(-3*(b*c - a*d)/d) + b^2*(\exp_integral_e(2, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) + \exp_integral_e(2, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*\sin(-5*(b*c - a*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)$

Fricas [A] time = 0.64037, size = 883, normalized size = 3.44

$$10(bdx + bc) \sin \left(-\frac{5(bc-ad)}{d} \right) \text{Si} \left(\frac{5(bdx+bc)}{d} \right) - 6(bdx + bc) \sin \left(-\frac{3(bc-ad)}{d} \right) \text{Si} \left(\frac{3(bdx+bc)}{d} \right) - 4(bdx + bc) \sin \left(-\frac{bc-ad}{d} \right) \text{Si} \left(\frac{bdx+bc}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")

[Out] $1/32*(10*(b*d*x + b*c)*\sin(-5*(b*c - a*d)/d)*\sin_integral(5*(b*d*x + b*c)/d) - 6*(b*d*x + b*c)*\sin(-3*(b*c - a*d)/d)*\sin_integral(3*(b*d*x + b*c)/d) - 4*(b*d*x + b*c)*\sin(-(b*c - a*d)/d)*\sin_integral((b*d*x + b*c)/d) + 2*((b*d*x + b*c)*\cos_integral((b*d*x + b*c)/d) + (b*d*x + b*c)*\cos_integral(-(b*d*x + b*c)/d))*\cos(-(b*c - a*d)/d) + 3*((b*d*x + b*c)*\cos_integral(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*\cos_integral(-3*(b*d*x + b*c)/d))*\cos(-3*(b*c - a*d)/d) - 5*((b*d*x + b*c)*\cos_integral(5*(b*d*x + b*c)/d) + (b*d*x + b*c)*\cos_integral(-5*(b*d*x + b*c)/d))*\cos(-5*(b*c - a*d)/d) + 32*(d*\cos(b*x + a)^4 - d*\cos(b*x + a)^2)*\sin(b*x + a))/(d^3*x + c*d^2)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3(a + bx) \cos^2(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c)**2,x)`

[Out] `Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x)**2, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(bx + a) \sin^3(bx + a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")`

[Out] `integrate(cos(b*x + a)^2*sin(b*x + a)^3/(d*x + c)^2, x)`

$$3.95 \quad \int \frac{\cos^2(a+bx) \sin^3(a+bx)}{(c+dx)^3} dx$$

Optimal. Leaf size=338

$$\frac{25b^2 \sin\left(5a - \frac{5bc}{d}\right) \text{CosIntegral}\left(\frac{5bc}{d} + 5bx\right)}{32d^3} - \frac{9b^2 \sin\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{32d^3} - \frac{b^2 \sin\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{16d^3}$$

```
[Out] -(b*cos[a + b*x])/(16*d^2*(c + d*x)) - (3*b*cos[3*a + 3*b*x])/(32*d^2*(c +
d*x)) + (5*b*cos[5*a + 5*b*x])/(32*d^2*(c + d*x)) + (25*b^2*cosIntegral[(5*
b*c)/d + 5*b*x]*Sin[5*a - (5*b*c)/d])/(32*d^3) - (9*b^2*cosIntegral[(3*b*c)
/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(32*d^3) - (b^2*cosIntegral[(b*c)/d + b*x
]*Sin[a - (b*c)/d])/(16*d^3) - Sin[a + b*x]/(16*d*(c + d*x)^2) - Sin[3*a +
3*b*x]/(32*d*(c + d*x)^2) + Sin[5*a + 5*b*x]/(32*d*(c + d*x)^2) - (b^2*cos[
a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(16*d^3) - (9*b^2*cos[3*a - (3*b*c
)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(32*d^3) + (25*b^2*cos[5*a - (5*b*c)/d
]*SinIntegral[(5*b*c)/d + 5*b*x])/(32*d^3)
```

Rubi [A] time = 0.50475, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{25b^2 \sin\left(5a - \frac{5bc}{d}\right) \text{CosIntegral}\left(\frac{5bc}{d} + 5bx\right)}{32d^3} - \frac{9b^2 \sin\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{32d^3} - \frac{b^2 \sin\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] -(b*cos[a + b*x])/(16*d^2*(c + d*x)) - (3*b*cos[3*a + 3*b*x])/(32*d^2*(c +
d*x)) + (5*b*cos[5*a + 5*b*x])/(32*d^2*(c + d*x)) + (25*b^2*cosIntegral[(5*
b*c)/d + 5*b*x]*Sin[5*a - (5*b*c)/d])/(32*d^3) - (9*b^2*cosIntegral[(3*b*c)
/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(32*d^3) - (b^2*cosIntegral[(b*c)/d + b*x
]*Sin[a - (b*c)/d])/(16*d^3) - Sin[a + b*x]/(16*d*(c + d*x)^2) - Sin[3*a +
3*b*x]/(32*d*(c + d*x)^2) + Sin[5*a + 5*b*x]/(32*d*(c + d*x)^2) - (b^2*cos[
a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(16*d^3) - (9*b^2*cos[3*a - (3*b*c
)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(32*d^3) + (25*b^2*cos[5*a - (5*b*c)/d
]*SinIntegral[(5*b*c)/d + 5*b*x])/(32*d^3)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3} dx &= \int \left(\frac{\sin(a+bx)}{8(c+dx)^3} + \frac{\sin(3a+3bx)}{16(c+dx)^3} - \frac{\sin(5a+5bx)}{16(c+dx)^3} \right) dx \\
&= \frac{1}{16} \int \frac{\sin(3a+3bx)}{(c+dx)^3} dx - \frac{1}{16} \int \frac{\sin(5a+5bx)}{(c+dx)^3} dx + \frac{1}{8} \int \frac{\sin(a+bx)}{(c+dx)^3} dx \\
&= -\frac{\sin(a+bx)}{16d(c+dx)^2} - \frac{\sin(3a+3bx)}{32d(c+dx)^2} + \frac{\sin(5a+5bx)}{32d(c+dx)^2} + \frac{b \int \frac{\cos(a+bx)}{(c+dx)^2} dx}{16d} + \frac{(3b) \int \frac{\cos(3a+3bx)}{(c+dx)^2} dx}{32d} \\
&= -\frac{b \cos(a+bx)}{16d^2(c+dx)} - \frac{3b \cos(3a+3bx)}{32d^2(c+dx)} + \frac{5b \cos(5a+5bx)}{32d^2(c+dx)} - \frac{\sin(a+bx)}{16d(c+dx)^2} - \frac{\sin(3a+3bx)}{32d(c+dx)^2} \\
&= -\frac{b \cos(a+bx)}{16d^2(c+dx)} - \frac{3b \cos(3a+3bx)}{32d^2(c+dx)} + \frac{5b \cos(5a+5bx)}{32d^2(c+dx)} - \frac{\sin(a+bx)}{16d(c+dx)^2} - \frac{\sin(3a+3bx)}{32d(c+dx)^2} \\
&= -\frac{b \cos(a+bx)}{16d^2(c+dx)} - \frac{3b \cos(3a+3bx)}{32d^2(c+dx)} + \frac{5b \cos(5a+5bx)}{32d^2(c+dx)} + \frac{25b^2 \operatorname{Ci}\left(\frac{5bc}{d} + 5bx\right) \sin\left(5a - \frac{5bc}{d} + 5bx\right)}{32d^3}
\end{aligned}$$

Mathematica [A] time = 4.21656, size = 279, normalized size = 0.83

$$-2 \left(b^2 \sin\left(a - \frac{bc}{d}\right) \operatorname{CosIntegral}\left(b\left(\frac{c}{d} + x\right)\right) + b^2 \cos\left(a - \frac{bc}{d}\right) \operatorname{Si}\left(b\left(\frac{c}{d} + x\right)\right) + \frac{d(b(c+dx)\cos(a+bx)+d\sin(a+bx))}{(c+dx)^2} \right) + 25b^2 \sin\left(5a - \frac{5bc}{d} + 5bx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] (25*b^2*CosIntegral[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d] - 9*b^2*CosInte
gral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] - (d*(3*b*(c + d*x)*Cos[3*(a +
b*x)] + d*Sin[3*(a + b*x)]))/(c + d*x)^2 + (d*(5*b*(c + d*x)*Cos[5*(a + b*
x)] + d*Sin[5*(a + b*x)]))/(c + d*x)^2 - 2*(b^2*CosIntegral[b*(c/d + x)]*Si
n[a - (b*c)/d] + (d*(b*(c + d*x)*Cos[a + b*x] + d*Sin[a + b*x]))/(c + d*x)^
2 + b^2*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)]) - 9*b^2*Cos[3*a - (3*b*c
)/d]*SinIntegral[(3*b*(c + d*x))/d] + 25*b^2*Cos[5*a - (5*b*c)/d]*SinIntegr
al[(5*b*(c + d*x))/d]/(32*d^3)
```

Maple [A] time = 0.027, size = 475, normalized size = 1.4

$$\frac{1}{b} \left(-\frac{b^3}{80} \left(-\frac{5 \sin(5bx + 5a)}{2((bx + a)d - ad + bc)^2 d} + \frac{5}{2d} \left(-5 \frac{\cos(5bx + 5a)}{((bx + a)d - ad + bc)d} - 5 \frac{1}{d} \left(5 \frac{1}{d} \operatorname{Si} \left(5bx + 5a + 5 \frac{-ad + bc}{d} \right) \cos \left(5 \right. \right. \right. \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x)

[Out] 1/b*(-1/80*b^3*(-5/2*sin(5*b*x+5*a)/((b*x+a)*d-a*d+b*c)^2/d+5/2*(-5*cos(5*b*x+5*a)/((b*x+a)*d-a*d+b*c)/d-5*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d-5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d)/d+1/8*b^3*(-1/2*sin(b*x+a)/((b*x+a)*d-a*d+b*c)^2/d+1/2*(-cos(b*x+a)/((b*x+a)*d-a*d+b*c)/d-(Si(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)+1/48*b^3*(-3/2*sin(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)^2/d+3/2*(-3*cos(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)/d-3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d)

Maxima [C] time = 3.00987, size = 639, normalized size = 1.89

$$b^3 \left(-2i E_3 \left(\frac{ibc+i(bx+a)d-iad}{d} \right) + 2i E_3 \left(-\frac{ibc+i(bx+a)d-iad}{d} \right) \right) \cos \left(-\frac{bc-ad}{d} \right) + b^3 \left(-i E_3 \left(\frac{3ibc+3i(bx+a)d-3iad}{d} \right) + i E_3 \left(-\frac{3ibc+3i(bx+a)d-3iad}{d} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")

[Out] 1/32*(b^3*(-2*I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 2*I*exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^3*(-I*exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + I*exp_integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d) + b^3*(I*exp_integral_e(3, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) - I*exp_integral_e(3, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*cos(-5*(b*c - a*d)/d) - 2*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^3*(exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d) + b^3*(exp_integral_e(3, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) + exp_integral_e(3, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*sin(-5*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)

Fricas [A] time = 0.759665, size = 1358, normalized size = 4.02

$$160(bd^2x + bcd) \cos(bx + a)^5 - 224(bd^2x + bcd) \cos(bx + a)^3 + 50(b^2d^2x^2 + 2b^2cdx + b^2c^2) \cos\left(-\frac{5(bc-ad)}{d}\right) \operatorname{Si}\left(\frac{5(bc-ad)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")

```
[Out] 1/64*(160*(b*d^2*x + b*c*d)*cos(b*x + a)^5 - 224*(b*d^2*x + b*c*d)*cos(b*x + a)^3 + 50*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) - 18*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 64*(b*d^2*x + b*c*d)*cos(b*x + a) + 32*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)*sin(b*x + a) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/d) - 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-3*(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d) + 25*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(5*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-5*(b*d*x + b*c)/d))*sin(-5*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.96 \quad \int \frac{\cos^2(a+bx) \sin^3(a+bx)}{(c+dx)^4} dx$$

Optimal. Leaf size=413

$$\frac{b^3 \cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{48d^4} - \frac{9b^3 \cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{32d^4} + \frac{125b^3 \cos\left(5a - \frac{5bc}{d}\right) \text{CosIntegral}\left(\frac{5bc}{d} + 5bx\right)}{96d^4}$$

```
[Out] -(b*cos[a + b*x])/(48*d^2*(c + d*x)^2) - (b*cos[3*a + 3*b*x])/(32*d^2*(c +
d*x)^2) + (5*b*cos[5*a + 5*b*x])/(96*d^2*(c + d*x)^2) - (b^3*cos[a - (b*c)/
d]*CosIntegral[(b*c)/d + b*x])/(48*d^4) - (9*b^3*cos[3*a - (3*b*c)/d]*CosIn
tegral[(3*b*c)/d + 3*b*x])/(32*d^4) + (125*b^3*cos[5*a - (5*b*c)/d]*CosInte
gral[(5*b*c)/d + 5*b*x])/(96*d^4) - Sin[a + b*x]/(24*d*(c + d*x)^3) + (b^2*
Sin[a + b*x])/(48*d^3*(c + d*x)) - Sin[3*a + 3*b*x]/(48*d*(c + d*x)^3) + (3
*b^2*sin[3*a + 3*b*x])/(32*d^3*(c + d*x)) + Sin[5*a + 5*b*x]/(48*d*(c + d*x
)^3) - (25*b^2*sin[5*a + 5*b*x])/(96*d^3*(c + d*x)) + (b^3*sin[a - (b*c)/d]
*SinIntegral[(b*c)/d + b*x])/(48*d^4) + (9*b^3*sin[3*a - (3*b*c)/d]*SinInte
gral[(3*b*c)/d + 3*b*x])/(32*d^4) - (125*b^3*sin[5*a - (5*b*c)/d]*SinIntegr
al[(5*b*c)/d + 5*b*x])/(96*d^4)
```

Rubi [A] time = 0.5904, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b^3 \cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{48d^4} - \frac{9b^3 \cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{32d^4} + \frac{125b^3 \cos\left(5a - \frac{5bc}{d}\right) \text{CosIntegral}\left(\frac{5bc}{d} + 5bx\right)}{96d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] -(b*cos[a + b*x])/(48*d^2*(c + d*x)^2) - (b*cos[3*a + 3*b*x])/(32*d^2*(c +
d*x)^2) + (5*b*cos[5*a + 5*b*x])/(96*d^2*(c + d*x)^2) - (b^3*cos[a - (b*c)/
d]*CosIntegral[(b*c)/d + b*x])/(48*d^4) - (9*b^3*cos[3*a - (3*b*c)/d]*CosIn
tegral[(3*b*c)/d + 3*b*x])/(32*d^4) + (125*b^3*cos[5*a - (5*b*c)/d]*CosInte
gral[(5*b*c)/d + 5*b*x])/(96*d^4) - Sin[a + b*x]/(24*d*(c + d*x)^3) + (b^2*
Sin[a + b*x])/(48*d^3*(c + d*x)) - Sin[3*a + 3*b*x]/(48*d*(c + d*x)^3) + (3
*b^2*sin[3*a + 3*b*x])/(32*d^3*(c + d*x)) + Sin[5*a + 5*b*x]/(48*d*(c + d*x
)^3) - (25*b^2*sin[5*a + 5*b*x])/(96*d^3*(c + d*x)) + (b^3*sin[a - (b*c)/d]
*SinIntegral[(b*c)/d + b*x])/(48*d^4) + (9*b^3*sin[3*a - (3*b*c)/d]*SinInte
gral[(3*b*c)/d + 3*b*x])/(32*d^4) - (125*b^3*sin[5*a - (5*b*c)/d]*SinIntegr
al[(5*b*c)/d + 5*b*x])/(96*d^4)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4} dx &= \int \left(\frac{\sin(a+bx)}{8(c+dx)^4} + \frac{\sin(3a+3bx)}{16(c+dx)^4} - \frac{\sin(5a+5bx)}{16(c+dx)^4} \right) dx \\
&= \frac{1}{16} \int \frac{\sin(3a+3bx)}{(c+dx)^4} dx - \frac{1}{16} \int \frac{\sin(5a+5bx)}{(c+dx)^4} dx + \frac{1}{8} \int \frac{\sin(a+bx)}{(c+dx)^4} dx \\
&= -\frac{\sin(a+bx)}{24d(c+dx)^3} - \frac{\sin(3a+3bx)}{48d(c+dx)^3} + \frac{\sin(5a+5bx)}{48d(c+dx)^3} + \frac{b \int \frac{\cos(a+bx)}{(c+dx)^3} dx}{24d} + \frac{b \int \frac{\cos(3a+3bx)}{(c+dx)^3} dx}{16d} \\
&= -\frac{b \cos(a+bx)}{48d^2(c+dx)^2} - \frac{b \cos(3a+3bx)}{32d^2(c+dx)^2} + \frac{5b \cos(5a+5bx)}{96d^2(c+dx)^2} - \frac{\sin(a+bx)}{24d(c+dx)^3} - \frac{\sin(3a+3bx)}{48d(c+dx)^3} \\
&= -\frac{b \cos(a+bx)}{48d^2(c+dx)^2} - \frac{b \cos(3a+3bx)}{32d^2(c+dx)^2} + \frac{5b \cos(5a+5bx)}{96d^2(c+dx)^2} - \frac{\sin(a+bx)}{24d(c+dx)^3} + \frac{b^2 \sin(a+bx)}{48d^3(c+dx)^3} \\
&= -\frac{b \cos(a+bx)}{48d^2(c+dx)^2} - \frac{b \cos(3a+3bx)}{32d^2(c+dx)^2} + \frac{5b \cos(5a+5bx)}{96d^2(c+dx)^2} - \frac{\sin(a+bx)}{24d(c+dx)^3} + \frac{b^2 \sin(a+bx)}{48d^3(c+dx)^3} \\
&= -\frac{b \cos(a+bx)}{48d^2(c+dx)^2} - \frac{b \cos(3a+3bx)}{32d^2(c+dx)^2} + \frac{5b \cos(5a+5bx)}{96d^2(c+dx)^2} - \frac{b^3 \cos\left(a - \frac{bc}{d}\right) \text{Ci}\left(\frac{bc}{d} + bx\right)}{48d^4}
\end{aligned}$$

Mathematica [A] time = 3.17416, size = 457, normalized size = 1.11

$$-2 \left(b^3 (c+dx)^3 \left(\cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(b\left(\frac{c}{d} + x\right)\right) - \sin\left(a - \frac{bc}{d}\right) \text{Si}\left(b\left(\frac{c}{d} + x\right)\right) \right) + d \cos(bx) \left(bd \cos(a)(c+dx) - \sin\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] -(d*Cos[3*b*x]*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)*Si
n[3*a])) + d*Cos[5*b*x]*(5*b*d*(c + d*x)*Cos[5*a] - (-2*d^2 + 25*b^2*(c + d
*x)^2)*Sin[5*a]) + d*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*
x)*Sin[3*a])*Sin[3*b*x] - d*((-2*d^2 + 25*b^2*(c + d*x)^2)*Cos[5*a] + 5*b*d
*(c + d*x)*Sin[5*a])*Sin[5*b*x] - 2*(d*Cos[b*x]*(b*d*(c + d*x)*Cos[a] - (-2
```

$$*d^2 + b^2*(c + d*x)^2)*\sin[a]) - d*((-2*d^2 + b^2*(c + d*x)^2)*\cos[a] + b*d*(c + d*x)*\sin[a])* \sin[b*x] + b^3*(c + d*x)^3*(\cos[a - (b*c)/d]*\cos\text{Integral}[b*(c/d + x)] - \sin[a - (b*c)/d]*\sin\text{Integral}[b*(c/d + x)]) - 27*b^3*(c + d*x)^3*(\cos[3*a - (3*b*c)/d]*\cos\text{Integral}[(3*b*(c + d*x))/d] - \sin[3*a - (3*b*c)/d]*\sin\text{Integral}[(3*b*(c + d*x))/d]) + 125*b^3*(c + d*x)^3*(\cos[5*a - (5*b*c)/d]*\cos\text{Integral}[(5*b*(c + d*x))/d] - \sin[5*a - (5*b*c)/d]*\sin\text{Integral}[(5*b*(c + d*x))/d]))/(96*d^4*(c + d*x)^3)$$

Maple [A] time = 0.028, size = 580, normalized size = 1.4

$$\frac{1}{b} \left(-\frac{b^4}{80} \left(\frac{5 \sin(5bx + 5a)}{3((bx + a)d - ad + bc)^3 d} + \frac{5}{3d} \left(-\frac{5 \cos(5bx + 5a)}{2((bx + a)d - ad + bc)^2 d} - \frac{5}{2d} \left(-5 \frac{\sin(5bx + 5a)}{((bx + a)d - ad + bc)d} + 5 \frac{1}{d} \left(5 \frac{\sin(5bx + 5a)}{((bx + a)d - ad + bc)d} + \dots \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x)

[Out] 1/b*(-1/80*b^4*(-5/3*sin(5*b*x+5*a)/((b*x+a)*d-a*d+b*c)^3/d+5/3*(-5/2*cos(5*b*x+5*a)/((b*x+a)*d-a*d+b*c)^2/d-5/2*(-5*sin(5*b*x+5*a)/((b*x+a)*d-a*d+b*c)/d+5*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d)/d)+1/8*b^4*(-1/3*sin(b*x+a)/((b*x+a)*d-a*d+b*c)^3/d+1/3*(-1/2*cos(b*x+a)/((b*x+a)*d-a*d+b*c)^2/d-1/2*(-sin(b*x+a)/((b*x+a)*d-a*d+b*c)/d+(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)+1/48*b^4*(-sin(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)^3/d+(-3/2*cos(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)^2/d-3/2*(-3*sin(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)/d+3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d)/d)/d)

Maxima [C] time = 4.00839, size = 706, normalized size = 1.71

$$b^4 \left(-2i E_4 \left(\frac{ibc+i(bx+a)d-iad}{d} \right) + 2i E_4 \left(-\frac{ibc+i(bx+a)d-iad}{d} \right) \right) \cos \left(-\frac{bc-ad}{d} \right) + b^4 \left(-i E_4 \left(\frac{3ibc+3i(bx+a)d-3iad}{d} \right) + i E_4 \left(-\frac{3ibc+3i(bx+a)d-3iad}{d} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="maxima")

[Out] 1/32*(b^4*(-2*I*exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 2*I*exp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^4*(-I*exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + I*exp_integral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d) + b^4*(I*exp_integral_e(4, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) - I*exp_integral_e(4, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*cos(-5*(b*c - a*d)/d) - 2*b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^4*(exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d) + b^4*(exp_integral_e(4, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) + exp_integral_e(4, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*sin(-5*(b*c - a*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)

Fricas [B] time = 0.81492, size = 1841, normalized size = 4.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="fricas")

[Out]
$$\frac{1}{192} \cdot (160 \cdot (b \cdot d^3 \cdot x + b \cdot c \cdot d^2) \cdot \cos(b \cdot x + a)^5 - 224 \cdot (b \cdot d^3 \cdot x + b \cdot c \cdot d^2) \cdot \cos(b \cdot x + a)^3 - 250 \cdot (b^3 \cdot d^3 \cdot x^3 + 3 \cdot b^3 \cdot c \cdot d^2 \cdot x^2 + 3 \cdot b^3 \cdot c^2 \cdot d \cdot x + b^3 \cdot c^3) \cdot \sin(-5 \cdot (b \cdot c - a \cdot d) / d) \cdot \sin_integral(5 \cdot (b \cdot d \cdot x + b \cdot c) / d) + 54 \cdot (b^3 \cdot d^3 \cdot x^3 + 3 \cdot b^3 \cdot c \cdot d^2 \cdot x^2 + 3 \cdot b^3 \cdot c^2 \cdot d \cdot x + b^3 \cdot c^3) \cdot \sin(-3 \cdot (b \cdot c - a \cdot d) / d) \cdot \sin_integral(3 \cdot (b \cdot d \cdot x + b \cdot c) / d) + 4 \cdot (b^3 \cdot d^3 \cdot x^3 + 3 \cdot b^3 \cdot c \cdot d^2 \cdot x^2 + 3 \cdot b^3 \cdot c^2 \cdot d \cdot x + b^3 \cdot c^3) \cdot \sin(-(b \cdot c - a \cdot d) / d) \cdot \sin_integral((b \cdot d \cdot x + b \cdot c) / d) + 64 \cdot (b \cdot d^3 \cdot x + b \cdot c \cdot d^2) \cdot \cos(b \cdot x + a) - 2 \cdot ((b^3 \cdot d^3 \cdot x^3 + 3 \cdot b^3 \cdot c \cdot d^2 \cdot x^2 + 3 \cdot b^3 \cdot c^2 \cdot d \cdot x + b^3 \cdot c^3) \cdot \cos_integral((b \cdot d \cdot x + b \cdot c) / d) + (b^3 \cdot d^3 \cdot x^3 + 3 \cdot b^3 \cdot c \cdot d^2 \cdot x^2 + 3 \cdot b^3 \cdot c^2 \cdot d \cdot x + b^3 \cdot c^3) \cdot \cos_integral(-(b \cdot d \cdot x + b \cdot c) / d)) \cdot \cos(-(b \cdot c - a \cdot d) / d) - 27 \cdot ((b^3 \cdot d^3 \cdot x^3 + 3 \cdot b^3 \cdot c \cdot d^2 \cdot x^2 + 3 \cdot b^3 \cdot c^2 \cdot d \cdot x + b^3 \cdot c^3) \cdot \cos_integral(3 \cdot (b \cdot d \cdot x + b \cdot c) / d) + (b^3 \cdot d^3 \cdot x^3 + 3 \cdot b^3 \cdot c \cdot d^2 \cdot x^2 + 3 \cdot b^3 \cdot c^2 \cdot d \cdot x + b^3 \cdot c^3) \cdot \cos_integral(-3 \cdot (b \cdot d \cdot x + b \cdot c) / d)) \cdot \cos(-3 \cdot (b \cdot c - a \cdot d) / d) + 125 \cdot ((b^3 \cdot d^3 \cdot x^3 + 3 \cdot b^3 \cdot c \cdot d^2 \cdot x^2 + 3 \cdot b^3 \cdot c^2 \cdot d \cdot x + b^3 \cdot c^3) \cdot \cos_integral(5 \cdot (b \cdot d \cdot x + b \cdot c) / d) + (b^3 \cdot d^3 \cdot x^3 + 3 \cdot b^3 \cdot c \cdot d^2 \cdot x^2 + 3 \cdot b^3 \cdot c^2 \cdot d \cdot x + b^3 \cdot c^3) \cdot \cos_integral(-5 \cdot (b \cdot d \cdot x + b \cdot c) / d)) \cdot \cos(-5 \cdot (b \cdot c - a \cdot d) / d) - 32 \cdot (2 \cdot b^2 \cdot d^3 \cdot x^2 + 4 \cdot b^2 \cdot c \cdot d^2 \cdot x + 2 \cdot b^2 \cdot c^2 \cdot d + (25 \cdot b^2 \cdot d^3 \cdot x^2 + 50 \cdot b^2 \cdot c \cdot d^2 \cdot x + 25 \cdot b^2 \cdot c^2 \cdot d - 2 \cdot d^3) \cdot \cos(b \cdot x + a)^4 - (21 \cdot b^2 \cdot d^3 \cdot x^2 + 42 \cdot b^2 \cdot c \cdot d^2 \cdot x + 21 \cdot b^2 \cdot c^2 \cdot d - 2 \cdot d^3) \cdot \cos(b \cdot x + a)^2) \cdot \sin(b \cdot x + a)) / (d^7 \cdot x^3 + 3 \cdot c \cdot d^6 \cdot x^2 + 3 \cdot c^2 \cdot d^5 \cdot x + c^3 \cdot d^4)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c)**4,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")

[Out] Exception raised: NotImplementedError

3.97 $\int (c + dx)^m \cos(a + bx) \cot(a + bx) dx$

Optimal. Leaf size=143

$$\text{Unintegrable}(\csc(a + bx)(c + dx)^m, x) + \frac{e^{i\left(a - \frac{bc}{d}\right)}(c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, -\frac{ib(c+dx)}{d}\right)}{2b} + \frac{e^{-i\left(a - \frac{bc}{d}\right)}(c + dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, \frac{ib(c+dx)}{d}\right)}{2b}$$

[Out] (E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(2*b*((-I)*b*(c + d*x))/d)^m + ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(2*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + Unintegrable[(c + d*x)^m*Csc[a + b*x], x]

Rubi [A] time = 0.126564, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \cos(a + bx) \cot(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x], x]

[Out] (E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(2*b*((-I)*b*(c + d*x))/d)^m + ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(2*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + Defer[Int][(c + d*x)^m*Csc[a + b*x], x]

Rubi steps

$$\begin{aligned} \int (c + dx)^m \cos(a + bx) \cot(a + bx) dx &= \int (c + dx)^m \csc(a + bx) dx - \int (c + dx)^m \sin(a + bx) dx \\ &= -\left(\frac{1}{2}i \int e^{-i(a+bx)}(c + dx)^m dx\right) + \frac{1}{2}i \int e^{i(a+bx)}(c + dx)^m dx + \int (c + dx)^m \csc(a + bx) dx \\ &= \frac{e^{i\left(a - \frac{bc}{d}\right)}(c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, -\frac{ib(c+dx)}{d}\right)}{2b} + \frac{e^{-i\left(a - \frac{bc}{d}\right)}(c + dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, \frac{ib(c+dx)}{d}\right)}{2b} \end{aligned}$$

Mathematica [A] time = 6.43278, size = 0, normalized size = 0.

$$\int (c + dx)^m \cos(a + bx) \cot(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x], x]

[Out] Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x], x]

Maple [A] time = 0.297, size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) \cot(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x)`

[Out] `int((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x)`

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) \cot(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")`

[Out] `integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a), x)`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx + c)^m \cos(bx + a) \cot(bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")`

[Out] `integral((d*x + c)^m*cos(b*x + a)*cot(b*x + a), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)**m*cos(b*x+a)*cot(b*x+a),x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) \cot(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")`

[Out] `integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a), x)`

3.98 $\int (c + dx)^4 \cos(a + bx) \cot(a + bx) dx$

Optimal. Leaf size=333

$$\frac{24id^3(c + dx)\text{PolyLog}\left(4, -e^{i(a+bx)}\right)}{b^4} + \frac{24id^3(c + dx)\text{PolyLog}\left(4, e^{i(a+bx)}\right)}{b^4} - \frac{12d^2(c + dx)^2\text{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^3} + \frac{12d^2(c + dx)^2\text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3} + \dots$$

```
[Out] (-2*(c + d*x)^4*ArcTanh[E^(I*(a + b*x))])/b + (24*d^4*Cos[a + b*x])/b^5 - (12*d^2*(c + d*x)^2*Cos[a + b*x])/b^3 + ((c + d*x)^4*Cos[a + b*x])/b + ((4*I)*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((4*I)*d*(c + d*x)^3*PolyLog[2, E^(I*(a + b*x))])/b^2 - (12*d^2*(c + d*x)^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (12*d^2*(c + d*x)^2*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((24*I)*d^3*(c + d*x)*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((24*I)*d^3*(c + d*x)*PolyLog[4, E^(I*(a + b*x))])/b^4 + (24*d^4*PolyLog[5, -E^(I*(a + b*x))])/b^5 - (24*d^4*PolyLog[5, E^(I*(a + b*x))])/b^5 + (24*d^3*(c + d*x)*Sin[a + b*x])/b^4 - (4*d*(c + d*x)^3*Sin[a + b*x])/b^2
```

Rubi [A] time = 0.284133, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {4408, 3296, 2638, 4183, 2531, 6609, 2282, 6589}

$$\frac{24id^3(c + dx)\text{PolyLog}\left(4, -e^{i(a+bx)}\right)}{b^4} + \frac{24id^3(c + dx)\text{PolyLog}\left(4, e^{i(a+bx)}\right)}{b^4} - \frac{12d^2(c + dx)^2\text{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^3} + \frac{12d^2(c + dx)^2\text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3} + \dots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x], x]
```

```
[Out] (-2*(c + d*x)^4*ArcTanh[E^(I*(a + b*x))])/b + (24*d^4*Cos[a + b*x])/b^5 - (12*d^2*(c + d*x)^2*Cos[a + b*x])/b^3 + ((c + d*x)^4*Cos[a + b*x])/b + ((4*I)*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((4*I)*d*(c + d*x)^3*PolyLog[2, E^(I*(a + b*x))])/b^2 - (12*d^2*(c + d*x)^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (12*d^2*(c + d*x)^2*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((24*I)*d^3*(c + d*x)*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((24*I)*d^3*(c + d*x)*PolyLog[4, E^(I*(a + b*x))])/b^4 + (24*d^4*PolyLog[5, -E^(I*(a + b*x))])/b^5 - (24*d^4*PolyLog[5, E^(I*(a + b*x))])/b^5 + (24*d^3*(c + d*x)*Sin[a + b*x])/b^4 - (4*d*(c + d*x)^3*Sin[a + b*x])/b^2
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/ (b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/ (b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_.)*((a_.)*(v_)^(n_.))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^4 \cos(a + bx) \cot(a + bx) dx &= \int (c + dx)^4 \csc(a + bx) dx - \int (c + dx)^4 \sin(a + bx) dx \\
&= -\frac{2(c + dx)^4 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{(c + dx)^4 \cos(a + bx)}{b} - \frac{(4d) \int (c + dx)^3 \cos(a + bx) dx}{b} \\
&= -\frac{2(c + dx)^4 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{(c + dx)^4 \cos(a + bx)}{b} + \frac{4id(c + dx)^3 \text{Li}_2(-e^{i(a+bx)})}{b^2} \\
&= -\frac{2(c + dx)^4 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{12d^2(c + dx)^2 \cos(a + bx)}{b^3} + \frac{(c + dx)^4 \cos(a + bx)}{b} \\
&= -\frac{2(c + dx)^4 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{12d^2(c + dx)^2 \cos(a + bx)}{b^3} + \frac{(c + dx)^4 \cos(a + bx)}{b} \\
&= -\frac{2(c + dx)^4 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{24d^4 \cos(a + bx)}{b^5} - \frac{12d^2(c + dx)^2 \cos(a + bx)}{b^3} + \frac{(c + dx)^4 \cos(a + bx)}{b} \\
&= -\frac{2(c + dx)^4 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{24d^4 \cos(a + bx)}{b^5} - \frac{12d^2(c + dx)^2 \cos(a + bx)}{b^3} + \frac{(c + dx)^4 \cos(a + bx)}{b}
\end{aligned}$$

Mathematica [B] time = 1.37105, size = 837, normalized size = 2.51

$$c^4 \cos(ax) b^4 + d^4 x^4 \cos(ax) b^4 + 4cd^3 x^3 \cos(ax) b^4 + 6c^2 d^2 x^2 \cos(ax) b^4 + 4c^3 dx \cos(ax) b^4 + c^4 \log$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x], x]

[Out] (b^4*c^4*Cos[a + b*x] - 12*b^2*c^2*d^2*Cos[a + b*x] + 24*d^4*Cos[a + b*x] + 4*b^4*c^3*d*x*Cos[a + b*x] - 24*b^2*c*d^3*x*Cos[a + b*x] + 6*b^4*c^2*d^2*x^2*Cos[a + b*x] - 12*b^2*d^4*x^2*Cos[a + b*x] + 4*b^4*c*d^3*x^3*Cos[a + b*x] + b^4*d^4*x^4*Cos[a + b*x] + b^4*c^4*Log[1 - E^(I*(a + b*x))] + 4*b^4*c^3*d*x*Log[1 - E^(I*(a + b*x))] + 6*b^4*c^2*d^2*x^2*Log[1 - E^(I*(a + b*x))] + 4*b^4*c*d^3*x^3*Log[1 - E^(I*(a + b*x))] + b^4*d^4*x^4*Log[1 - E^(I*(a + b*x))] - b^4*c^4*Log[1 + E^(I*(a + b*x))] - 4*b^4*c^3*d*x*Log[1 + E^(I*(a + b*x))] - 6*b^4*c^2*d^2*x^2*Log[1 + E^(I*(a + b*x))] - 4*b^4*c*d^3*x^3*Log[1 + E^(I*(a + b*x))] - b^4*d^4*x^4*Log[1 + E^(I*(a + b*x))] + (4*I)*b^3*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))] - (4*I)*b^3*d*(c + d*x)^3*PolyLog[2, E^(I*(a + b*x))] - 12*b^2*c^2*d^2*PolyLog[3, -E^(I*(a + b*x))] - 24*b^2*c*d^3*x*PolyLog[3, -E^(I*(a + b*x))] - 12*b^2*d^4*x^2*PolyLog[3, -E^(I*(a + b*x))] + 12*b^2*c^2*d^2*PolyLog[3, E^(I*(a + b*x))] + 24*b^2*c*d^3*x*PolyLog[3, E^(I*(a + b*x))] + 12*b^2*d^4*x^2*PolyLog[3, E^(I*(a + b*x))] - (24*I)*b*c*d^3*PolyLog[4, -E^(I*(a + b*x))] - (24*I)*b*d^4*x*PolyLog[4, -E^(I*(a + b*x))] + (24*I)*b*c*d^3*PolyLog[4, E^(I*(a + b*x))] + (24*I)*b*d^4*x*PolyLog[4, E^(I*(a + b*x))] + 24*d^4*PolyLog[5, -E^(I*(a + b*x))] - 24*d^4*PolyLog[5, E^(I*(a + b*x))] - 4*b^3*c^3*d*Sin[a + b*x] + 24*b*c*d^3*Sin[a + b*x] - 12*b^3*c^2*d^2*x*Sin[a + b*x] + 24*b*d^4*x*Sin[a + b*x] - 12*b^3*c*d^3*x^2*Sin[a + b*x] - 4*b^3*d^4*x^3*Sin[a + b*x])/b^5

Maple [B] time = 0.354, size = 1295, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^4*cos(b*x+a)*cot(b*x+a), x)

[Out] 1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4-1/b^5*d^4*ln(1-exp(I*(b*x+a)))*a^4+8/b^4*c*d^3*a^3*arctanh(exp(I*(b*x+a)))-12/b^3*c^2*d^2*a^2*arctanh(exp(I*(b*x+a)))+8/b^2*c^3*d*a*arctanh(exp(I*(b*x+a)))+4/b*c*d^3*ln(1-exp(I*(b*x+a)))*x^3-4/b^4*c*d^3*ln(exp(I*(b*x+a))+1)*a^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3+12*I/b^2*c^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-12*I/b^2*c^2*d^2*polylog(2,exp(I*(b*x+a)))*x-12*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2-4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3+12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x^2+1/b^5*d^4*a^4*ln(exp(I*(b*x+a))+1)+12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))-12/b^3*c^2*d^2*polylog(3,-exp(I*(b*x+a)))-2/b^5*d^4*a^4*arctanh(exp(I*(b*x+a)))-12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2+6/b*c^2*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b*d^4*ln(exp(I*(b*x+a))+1)*x^4-6/b^3*c^2*d^2*a^2*ln(1-exp(I*(b*x+a)))+4/b*c^3*d*ln(1-exp(I*(b*x+a)))*x+4/b^2*c^3*d*ln(1-exp(I*(b*x+a)))*a-4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x-24/b^3*c*d^3*polylog(3,-exp(I*(b*x+a)))*x+6/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a))+1)-6/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2+24/b^3*c*d^3*polylog(3,exp(I*(b*x+a)))*x-4/b^2*c^3*d*ln(exp(I*(b*x+a))+1)*a-24*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)))*x-4*I/b^2*d^4*polylog(2,exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-24*I/b^4*c*d^3*polylog(4,-exp(I*(b*x+a)))+4*I/b^2*c^3*d*polylog(2,-exp(I*(b*x+a)))+24*I/b^4*c*d^3*polylog(4,exp(I

$$\begin{aligned} &*(b*x+a))) - 4*I/b^2*c^3*d*polylog(2, \exp(I*(b*x+a))) + 4*I/b^2*d^4*polylog(2, -\exp(I*(b*x+a))) * x^3 + 1/2*(d^4*x^4*b^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x - 4*I*b^3*d^4*x^3 + b^4*c^4 - 12*b^2*d^4*x^2 - 12*I*b^3*c*d^3*x^2 - 24*b^2*c*d^3*x - 12*I*b^3*c^2*d^2*x - 12*c^2*d^2*b^2 - 4*I*b^3*c^3*d + 24*I*b*d^4*x + 24*d^4 + 24*I*b*c*d^3)/b^5*\exp(-I*(b*x+a)) + 1/2*(d^4*x^4*b^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*I*b^3*d^4*x^3 + b^4*c^4 - 12*b^2*d^4*x^2 + 12*I*b^3*c*d^3*x^2 - 24*b^2*c*d^3*x + 12*I*b^3*c^2*d^2*x - 12*c^2*d^2*b^2 + 4*I*b^3*c^3*d - 24*I*b*d^4*x + 24*d^4 - 24*I*b*c*d^3)/b^5*\exp(I*(b*x+a)) + 24*d^4*polylog(5, -\exp(I*(b*x+a)))/b^5 - 24*d^4*polylog(5, \exp(I*(b*x+a)))/b^5 - 2/b*c^4*\operatorname{arctanh}(\exp(I*(b*x+a))) + 12*I/b^2*c*d^3*polylog(2, -\exp(I*(b*x+a))) * x^2 \end{aligned}$$

Maxima [B] time = 2.32893, size = 2076, normalized size = 6.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a), x, algorithm="maxima")

[Out] $\frac{1}{2}(c^4(2\cos(bx+a) - \log(\cos(bx+a) + 1) + \log(\cos(bx+a) - 1)) - 4ac^3d(2\cos(bx+a) - \log(\cos(bx+a) + 1) + \log(\cos(bx+a) - 1)) / b + 6a^2c^2d^2(2\cos(bx+a) - \log(\cos(bx+a) + 1) + \log(\cos(bx+a) - 1)) / b^2 - 4a^3c^3d^3(2\cos(bx+a) - \log(\cos(bx+a) + 1) + \log(\cos(bx+a) - 1)) / b^3 + a^4d^4(2\cos(bx+a) - \log(\cos(bx+a) + 1) + \log(\cos(bx+a) - 1)) / b^4 + (48d^4\operatorname{polylog}(5, -e^{Ibx+Ia}) - 48d^4\operatorname{polylog}(5, e^{Ibx+Ia}) - (2I(bx+a)^4d^4 + (8Ib^3cd^3 - 8Ia^2d^4))(bx+a)^3 + (12Ib^2c^2d^2 - 24Ia^2b^2cd^3 + 12Ia^2d^4)(bx+a)^2 + (8Ib^3c^3d - 24Ia^2b^2c^2d^2 + 24Ia^2b^2cd^3 - 8Ia^3d^4)(bx+a)\operatorname{arctan2}(\sin(bx+a), \cos(bx+a) + 1) - (2I(bx+a)^4d^4 + (8Ib^3cd^3 - 8Ia^2d^4)(bx+a)^3 + (12Ib^2c^2d^2 - 24Ia^2b^2cd^3 + 12Ia^2d^4)(bx+a)^2 + (8Ib^3c^3d - 24Ia^2b^2c^2d^2 + 24Ia^2b^2cd^3 - 8Ia^3d^4)(bx+a)\operatorname{arctan2}(\sin(bx+a), -\cos(bx+a) + 1) + 2((bx+a)^4d^4 - 12b^2c^2d^2 + 24a^2b^2cd^3 - 12(a^2 - 2)d^4 + 4(b^3cd^3 - ad^4)(bx+a)^3 + 6(b^2c^2d^2 - 2a^2b^2cd^3 + (a^2 - 2)d^4)(bx+a)^2 + 4(b^3c^3d - 3a^2b^2c^2d^2 + 3(a^2 - 2)b^2cd^3 - (a^3 - 6a)d^4)(bx+a))\cos(bx+a) - (-8Ib^3c^3d + 24Ia^2b^2c^2d^2 - 24Ia^2b^2cd^3 - 8I(bx+a)^3d^4 + 8Ia^3d^4 + (-24Ib^3cd^3 + 24Ia^2d^4)(bx+a)^2 + (-24Ib^2c^2d^2 + 48Ia^2b^2cd^3 - 24Ia^2d^4)(bx+a))\operatorname{dilog}(-e^{Ibx+Ia}) - (8Ib^3c^3d - 24Ia^2b^2c^2d^2 + 24Ia^2b^2cd^3 + 8I(bx+a)^3d^4 - 8Ia^3d^4 + (24Ib^3cd^3 - 24Ia^2d^4)(bx+a)^2 + (24Ib^2c^2d^2 - 48Ia^2b^2cd^3 + 24Ia^2d^4)(bx+a))\operatorname{dilog}(e^{Ibx+Ia}) - ((bx+a)^4d^4 + 4(b^3cd^3 - ad^4)(bx+a)^3 + 6(b^2c^2d^2 - 2a^2b^2cd^3 + a^2d^4)(bx+a)^2 + 4(b^3c^3d - 3a^2b^2c^2d^2 + 3a^2b^2cd^3 - a^3d^4)(bx+a))\log(\cos(bx+a)^2 + \sin(bx+a)^2 + 2\cos(bx+a) + 1) + ((bx+a)^4d^4 + 4(b^3cd^3 - ad^4)(bx+a)^3 + 6(b^2c^2d^2 - 2a^2b^2cd^3 + a^2d^4)(bx+a)^2 + 4(b^3c^3d - 3a^2b^2c^2d^2 + 3a^2b^2cd^3 - a^3d^4)(bx+a))\log(\cos(bx+a)^2 + \sin(bx+a)^2 - 2\cos(bx+a) + 1) - (48Ib^3cd^3 + 48I(bx+a)d^4 - 48Ia^2d^4)\operatorname{polylog}(4, -e^{Ibx+Ia}) - (-48Ib^3cd^3 - 48I(bx+a)d^4 + 48Ia^2d^4)\operatorname{polylog}(4, e^{Ibx+Ia}) - 24(b^2c^2d^2 - 2a^2b^2cd^3 + (bx+a)^2d^4 + a^2d^4 + 2(b^3cd^3 - ad^4)(bx+a))\operatorname{polylog}(3, -e^{Ibx+Ia}) + 24(b^2c^2d^2 - 2a^2b^2cd^3 + (bx+a)^2d^4 + a^2d^4 + 2(b^3cd^3 - ad^4)(bx+a))\operatorname{polylog}(3, e^{Ibx+Ia}) - 8(b^3c^3d - 3a^2b^2c^2d^2 + (bx+a)^3d^4 + 3(a^2 - 2)b^2cd^3 - (a^3 - 6a)d^4 + 3(b^3cd^3 - ad^4)(bx+a)^2 + 3(b^2c^2d^2 - 2a^2b^2cd^3 + (a^2 - 2)d^4)(bx+a))\sin(bx+a) / b^4) / b$

Fricas [C] time = 0.849061, size = 3281, normalized size = 9.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")

[Out]
$$-1/2*(24*d^4*\text{polylog}(5, \cos(b*x + a) + I*\sin(b*x + a)) + 24*d^4*\text{polylog}(5, \cos(b*x + a) - I*\sin(b*x + a)) - 24*d^4*\text{polylog}(5, -\cos(b*x + a) + I*\sin(b*x + a)) - 24*d^4*\text{polylog}(5, -\cos(b*x + a) - I*\sin(b*x + a)) - 2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 24*d^4 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*\cos(b*x + a) - (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*\text{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*\text{dilog}(\cos(b*x + a) - I*\sin(b*x + a)) - (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*\text{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*\text{dilog}(-\cos(b*x + a) - I*\sin(b*x + a)) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*\log(-1/2*\cos(b*x + a) + 1/2*I*\sin(b*x + a) + 1/2) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*\log(-1/2*\cos(b*x + a) - 1/2*I*\sin(b*x + a) + 1/2) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*\log(-\cos(b*x + a) - I*\sin(b*x + a) + 1) - (24*I*b*d^4*x + 24*I*b*c*d^3)*\text{polylog}(4, \cos(b*x + a) + I*\sin(b*x + a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*\text{polylog}(4, \cos(b*x + a) - I*\sin(b*x + a)) - (24*I*b*d^4*x + 24*I*b*c*d^3)*\text{polylog}(4, -\cos(b*x + a) + I*\sin(b*x + a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*\text{polylog}(4, -\cos(b*x + a) - I*\sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*\text{polylog}(3, \cos(b*x + a) + I*\sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*\text{polylog}(3, \cos(b*x + a) - I*\sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*\text{polylog}(3, -\cos(b*x + a) + I*\sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*\text{polylog}(3, -\cos(b*x + a) - I*\sin(b*x + a)) + 8*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d - 6*b*c*d^3 + 3*(b^3*c^2*d^2 - 2*b*d^4)*x)*\sin(b*x + a))/b^5$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^4 \cos(a + bx) \cot(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**4*cos(b*x+a)*cot(b*x+a),x)

[Out] Integral((c + d*x)**4*cos(a + b*x)*cot(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^4 \cos(bx + a) \cot(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)*cot(b*x + a), x)
```


3.99 $\int (c + dx)^3 \cos(a + bx) \cot(a + bx) dx$

Optimal. Leaf size=254

$$\frac{6d^2(c + dx)\text{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^3} + \frac{6d^2(c + dx)\text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3} + \frac{3id(c + dx)^2\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^2} - \frac{3id(c + dx)^2\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^2}$$

```
[Out] (-2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b - (6*d^2*(c + d*x)*Cos[a + b*x])/b^3 + ((c + d*x)^3*Cos[a + b*x])/b + ((3*I)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^2 - (6*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((6*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4 + (6*d^3*Sin[a + b*x])/b^4 - (3*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rubi [A] time = 0.19835, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {4408, 3296, 2637, 4183, 2531, 6609, 2282, 6589}

$$\frac{6d^2(c + dx)\text{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^3} + \frac{6d^2(c + dx)\text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3} + \frac{3id(c + dx)^2\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^2} - \frac{3id(c + dx)^2\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x], x]
```

```
[Out] (-2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b - (6*d^2*(c + d*x)*Cos[a + b*x])/b^3 + ((c + d*x)^3*Cos[a + b*x])/b + ((3*I)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^2 - (6*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((6*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4 + (6*d^3*Sin[a + b*x])/b^4 - (3*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]
```

```
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_.))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 \cos(a + bx) \cot(a + bx) dx &= \int (c + dx)^3 \csc(a + bx) dx - \int (c + dx)^3 \sin(a + bx) dx \\
&= -\frac{2(c + dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c + dx)^3 \cos(a + bx)}{b} - \frac{(3d) \int (c + dx)^2 \cos(a + bx) dx}{b} \\
&= -\frac{2(c + dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c + dx)^3 \cos(a + bx)}{b} + \frac{3id(c + dx)^2 \operatorname{Li}_2\left(-e^{i(a+bx)}\right)}{b^2} \\
&= -\frac{2(c + dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d^2(c + dx) \cos(a + bx)}{b^3} + \frac{(c + dx)^3 \cos(a + bx)}{b} \\
&= -\frac{2(c + dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d^2(c + dx) \cos(a + bx)}{b^3} + \frac{(c + dx)^3 \cos(a + bx)}{b} \\
&= -\frac{2(c + dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d^2(c + dx) \cos(a + bx)}{b^3} + \frac{(c + dx)^3 \cos(a + bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.982757, size = 330, normalized size = 1.3

$$3id\left(b^2(c + dx)^2 \operatorname{PolyLog}(2, -\cos(a + bx) - i \sin(a + bx)) + 2ibd(c + dx) \operatorname{PolyLog}(3, -\cos(a + bx) - i \sin(a + bx)) - 2d^2\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (-2*b^3*(c + d*x)^3*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]] + (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] - 2*d^2*PolyLog[4, -Cos[a + b*x] - I*Sin[a + b*x]]) - (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, Cos[a + b*x] + I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, Cos[a + b*x] + I*Sin[a + b*x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Sin[a + b*x]]) + Cos[b*x]*(b*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a] - 3*d*(-2*d^2 + b^2*(c + d*x)^2)*Sin[a]) - (3*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x])/b^4
```

Maple [B] time = 0.218, size = 847, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] -3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b^3*c*d^2*ln(exp(I*(b*x+a))+1)*a^2-6/b^3*c*d^2*a^2*arctanh(exp(I*(b*x+a)))+2/b^4*d^3*a^3*arctanh(exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))-6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))-6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x+1/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))-1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a+1/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3-6*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4+6*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+a)))*x-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4-3*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2+3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-3*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))+6/b^2*c^2*d*a*arctanh(exp(I*(b*x+a)))+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3-1/b^4*d^3*ln(exp(I*(b*x+a))+1)*a^3-2/b*c^3*arctanh(exp(I*(b*x+a)))+3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))-3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x-3/b^2*c^2*d*ln(exp(I*(b*x+a))+1)*a
```

Maxima [B] time = 1.99071, size = 1241, normalized size = 4.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c^3*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) - 3*a*c^2*d*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)))/b + 3*a^2*c*d^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b^2 - a^3*d^3*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b^3 - (12*I*d^3*polylog(4, -e^(I*b*x + I*a)) - 12*I*d^3*polylog(4, e^(I*b*x + I*a)) + (2*I*(b*x + a)^3*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*a^2*d^3)*(b*x + a))*arctan2
```

```
(sin(b*x + a), cos(b*x + a) + 1) + (2*I*(b*x + a)^3*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos(b*x + a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*dilog(-e^(I*b*x + I*a)) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*dilog(e^(I*b*x + I*a)) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, -e^(I*b*x + I*a)) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, e^(I*b*x + I*a)) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*x + a))/b^3)/b
```

Fricas [C] time = 0.742391, size = 2280, normalized size = 8.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a))/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \cos(a + bx) \cot(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*cos(b*x+a)*cot(b*x+a),x)

[Out] Integral((c + d*x)**3*cos(a + b*x)*cot(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \cos(bx + a) \cot(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^3*cos(b*x + a)*cot(b*x + a), x)

3.100 $\int (c + dx)^2 \cos(a + bx) \cot(a + bx) dx$

Optimal. Leaf size=171

$$\frac{2id(c + dx)\text{PolyLog}(2, -e^{i(a+bx)})}{b^2} - \frac{2id(c + dx)\text{PolyLog}(2, e^{i(a+bx)})}{b^2} - \frac{2d^2\text{PolyLog}(3, -e^{i(a+bx)})}{b^3} + \frac{2d^2\text{PolyLog}(3, e^{i(a+bx)})}{b^3}$$

[Out] $(-2*(c + d*x)^2*\text{ArcTanh}[E^{(I*(a + b*x))}])/b - (2*d^2*\text{Cos}[a + b*x])/b^3 + ((c + d*x)^2*\text{Cos}[a + b*x])/b + ((2*I)*d*(c + d*x)*\text{PolyLog}[2, -E^{(I*(a + b*x))}])/b^2 - ((2*I)*d*(c + d*x)*\text{PolyLog}[2, E^{(I*(a + b*x))}])/b^2 - (2*d^2*\text{PolyLog}[3, -E^{(I*(a + b*x))}])/b^3 + (2*d^2*\text{PolyLog}[3, E^{(I*(a + b*x))}])/b^3 - (2*d*(c + d*x)*\text{Sin}[a + b*x])/b^2$

Rubi [A] time = 0.138333, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$, Rules used = {4408, 3296, 2638, 4183, 2531, 2282, 6589}

$$\frac{2id(c + dx)\text{PolyLog}(2, -e^{i(a+bx)})}{b^2} - \frac{2id(c + dx)\text{PolyLog}(2, e^{i(a+bx)})}{b^2} - \frac{2d^2\text{PolyLog}(3, -e^{i(a+bx)})}{b^3} + \frac{2d^2\text{PolyLog}(3, e^{i(a+bx)})}{b^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2*cos[a + b*x]*Cot[a + b*x], x]

[Out] $(-2*(c + d*x)^2*\text{ArcTanh}[E^{(I*(a + b*x))}])/b - (2*d^2*\text{Cos}[a + b*x])/b^3 + ((c + d*x)^2*\text{Cos}[a + b*x])/b + ((2*I)*d*(c + d*x)*\text{PolyLog}[2, -E^{(I*(a + b*x))}])/b^2 - ((2*I)*d*(c + d*x)*\text{PolyLog}[2, E^{(I*(a + b*x))}])/b^2 - (2*d^2*\text{PolyLog}[3, -E^{(I*(a + b*x))}])/b^3 + (2*d^2*\text{PolyLog}[3, E^{(I*(a + b*x))}])/b^3 - (2*d*(c + d*x)*\text{Sin}[a + b*x])/b^2$

Rule 4408

Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Int[(c + d*x)^m*cos[a + b*x]^n*Cot[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4183

Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_ /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \cos(a + bx) \cot(a + bx) dx &= \int (c + dx)^2 \csc(a + bx) dx - \int (c + dx)^2 \sin(a + bx) dx \\ &= -\frac{2(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{(c + dx)^2 \cos(a + bx)}{b} - \frac{(2d) \int (c + dx) \cos(a + bx) dx}{b} \\ &= -\frac{2(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{(c + dx)^2 \cos(a + bx)}{b} + \frac{2id(c + dx) \text{Li}_2(-e^{i(a+bx)})}{b^2} \\ &= -\frac{2(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{2d^2 \cos(a + bx)}{b^3} + \frac{(c + dx)^2 \cos(a + bx)}{b} + \frac{2id(c + dx) \text{Li}_2(-e^{i(a+bx)})}{b^2} \\ &= -\frac{2(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{2d^2 \cos(a + bx)}{b^3} + \frac{(c + dx)^2 \cos(a + bx)}{b} + \frac{2id(c + dx) \text{Li}_2(-e^{i(a+bx)})}{b^2} \end{aligned}$$

Mathematica [A] time = 0.874117, size = 221, normalized size = 1.29

$$\frac{2ibd(c + dx)\text{PolyLog}\left(2, -e^{i(a+bx)}\right) - 2ibd(c + dx)\text{PolyLog}\left(2, e^{i(a+bx)}\right) - 2d^2\text{PolyLog}\left(3, -e^{i(a+bx)}\right) + 2d^2\text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x], x]
```

```
[Out] (b^2*(c + d*x)^2*Log[1 - E^(I*(a + b*x))] - b^2*(c + d*x)^2*Log[1 + E^(I*(a
+ b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] - (2*I)*b*d*(c
+ d*x)*PolyLog[2, E^(I*(a + b*x))] - 2*d^2*PolyLog[3, -E^(I*(a + b*x))] +
2*d^2*PolyLog[3, E^(I*(a + b*x))] + Cos[b*x]*((-2*d^2 + b^2*(c + d*x)^2)*Co
s[a] - 2*b*d*(c + d*x)*Sin[a]) - (2*b*d*(c + d*x)*Cos[a] + (-2*d^2 + b^2*(c
+ d*x)^2)*Sin[a])*Sin[b*x])/b^3
```

Maple [B] time = 0.184, size = 479, normalized size = 2.8

$$\frac{(d^2x^2b^2 + 2b^2cdx + b^2c^2 + 2ibd^2x - 2d^2 + 2ibcd)e^{i(bx+a)}}{2b^3} + \frac{(d^2x^2b^2 + 2b^2cdx + b^2c^2 - 2ibd^2x - 2d^2 - 2ibcd)e^{-i(bx+a)}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x)`

[Out] $\frac{1}{2}(d^2x^2b^2+2b^2cdx+b^2c^2+2Ibd^2x-2d^2+2Ib^2cd)/b^3\exp(I(bx+a))+\frac{1}{2}(d^2x^2b^2+2b^2cdx+b^2c^2-2Ibd^2x-2d^2-2Ib^2cd)/b^3\exp(-I(bx+a))+2d^2\text{polylog}(3,\exp(I(bx+a)))/b^3-2/b^3d^2a^2\text{arctanh}(\exp(I(bx+a)))-2d^2\text{polylog}(3,-\exp(I(bx+a)))/b^3-2/b^3cd\ln(\exp(I(bx+a))+1)*x-2/b^2cd\ln(\exp(I(bx+a))+1)*a+2/b^2cd\ln(1-\exp(I(bx+a)))*x+2/b^2cd\ln(1-\exp(I(bx+a)))*a-2/b^2cd\text{arctanh}(\exp(I(bx+a)))+1/b^2d^2\ln(1-\exp(I(bx+a)))*x^2-1/b^3d^2\ln(1-\exp(I(bx+a)))*a^2-2I/b^2cd\text{polylog}(2,\exp(I(bx+a)))-1/b^2d^2\ln(\exp(I(bx+a))+1)*x^2+1/b^3d^2\ln(\exp(I(bx+a))+1)*a^2-2I/b^2d^2\text{polylog}(2,\exp(I(bx+a)))*x+2I/b^2d^2\text{polylog}(2,-\exp(I(bx+a)))*x+4/b^2cd*a*\text{arctanh}(\exp(I(bx+a)))+2I/b^2cd\text{polylog}(2,-\exp(I(bx+a)))$

Maxima [B] time = 1.47035, size = 684, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")`

[Out] $\frac{1}{2}(c^2(2\cos(bx+a) - \log(\cos(bx+a) + 1) + \log(\cos(bx+a) - 1)) - 2a^2cd(2\cos(bx+a) - \log(\cos(bx+a) + 1) + \log(\cos(bx+a) - 1)))/b^2 - (4d^2\text{polylog}(3, -e^{Ibx+Ia}) - 4d^2\text{polylog}(3, e^{Ibx+Ia})) + (2I(bx+a)^2d^2 + (4Ib^2cd - 4Ia^2d^2)(bx+a))\text{arctan2}(\sin(bx+a), \cos(bx+a) + 1) + (2I(bx+a)^2d^2 + (4Ib^2cd - 4Ia^2d^2)(bx+a))\text{arctan2}(\sin(bx+a), -\cos(bx+a) + 1) - 2((bx+a)^2d^2 + 2(b^2cd - a^2d^2)(bx+a) - 2d^2)\cos(bx+a) + (-4Ib^2cd - 4I(bx+a)d^2 + 4Ia^2d^2)\text{dilog}(-e^{Ibx+Ia}) + (4Ib^2cd + 4I(bx+a)d^2 - 4Ia^2d^2)\text{dilog}(e^{Ibx+Ia}) + ((bx+a)^2d^2 + 2(b^2cd - a^2d^2)(bx+a))\log(\cos(bx+a)^2 + \sin(bx+a)^2 + 2\cos(bx+a) + 1) - ((bx+a)^2d^2 + 2(b^2cd - a^2d^2)(bx+a))\log(\cos(bx+a)^2 + \sin(bx+a)^2 - 2\cos(bx+a) + 1) + 4(b^2cd + (bx+a)d^2 - a^2d^2)\sin(bx+a))/b^2/b$

Fricas [C] time = 0.638458, size = 1462, normalized size = 8.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")`

[Out] $\frac{1}{2}(2d^2\text{polylog}(3, \cos(bx+a) + I\sin(bx+a)) + 2d^2\text{polylog}(3, \cos(bx+a) - I\sin(bx+a)) - 2d^2\text{polylog}(3, -\cos(bx+a) + I\sin(bx+a)) - 2d^2\text{polylog}(3, -\cos(bx+a) - I\sin(bx+a)) + 2(b^2d^2x^2 + 2b^2cdx + b^2c^2 - 2d^2)\cos(bx+a) + (-2Ibd^2x - 2Ib^2cd)\text{dilog}(\cos(bx+a) + I\sin(bx+a)) + (2Ibd^2x + 2Ib^2cd)\text{dilog}(\cos(bx+a) - I\sin(bx+a)) + (-2Ibd^2x - 2Ib^2cd)\text{dilog}(-\cos(bx+a) + I\sin(bx+a)) + (2Ibd^2x + 2Ib^2cd)\text{dilog}(-\cos(bx+a) - I\sin(bx+a))$

$$\begin{aligned}
& + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\log(\cos(b*x + a) + I*\sin(b*x \\
& + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\log(\cos(b*x + a) - I*\sin \\
& (b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\log(-1/2*\cos(b*x + a) + 1/ \\
& 2*I*\sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\log(-1/2*\cos(b*x \\
& + a) - 1/2*I*\sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - \\
& a^2*d^2)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c* \\
& d*x + 2*a*b*c*d - a^2*d^2)*\log(-\cos(b*x + a) - I*\sin(b*x + a) + 1) - 4*(b*d \\
& ^2*x + b*c*d)*\sin(b*x + a))/b^3
\end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \cos(a + bx) \cot(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*cos(b*x+a)*cot(b*x+a),x)

[Out] Integral((c + d*x)**2*cos(a + b*x)*cot(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \cos(bx + a) \cot(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^2*cos(b*x + a)*cot(b*x + a), x)

3.101 $\int (c + dx) \cos(a + bx) \cot(a + bx) dx$

Optimal. Leaf size=94

$$\frac{idPolyLog(2, -e^{i(a+bx)})}{b^2} - \frac{idPolyLog(2, e^{i(a+bx)})}{b^2} - \frac{d \sin(a + bx)}{b^2} + \frac{(c + dx) \cos(a + bx)}{b} - \frac{2(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b}$$

[Out] $(-2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + ((c + d*x)*Cos[a + b*x])/b + (I*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^(I*(a + b*x))])/b^2 - (d*Sin[a + b*x])/b^2$

Rubi [A] time = 0.06174, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4408, 3296, 2637, 4183, 2279, 2391}

$$\frac{idPolyLog(2, -e^{i(a+bx)})}{b^2} - \frac{idPolyLog(2, e^{i(a+bx)})}{b^2} - \frac{d \sin(a + bx)}{b^2} + \frac{(c + dx) \cos(a + bx)}{b} - \frac{2(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Cos[a + b*x]*Cot[a + b*x], x]

[Out] $(-2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + ((c + d*x)*Cos[a + b*x])/b + (I*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^(I*(a + b*x))])/b^2 - (d*Sin[a + b*x])/b^2$

Rule 4408

Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4183

Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_.) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))]^(n_.), x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))]

)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned} \int (c + dx) \cos(a + bx) \cot(a + bx) dx &= \int (c + dx) \csc(a + bx) dx - \int (c + dx) \sin(a + bx) dx \\ &= -\frac{2(c + dx) \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c + dx) \cos(a + bx)}{b} - \frac{d \int \cos(a + bx) dx}{b} - \frac{d}{b} \\ &= -\frac{2(c + dx) \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c + dx) \cos(a + bx)}{b} - \frac{d \sin(a + bx)}{b^2} + \frac{(id) \operatorname{Su}}{b^2} \\ &= -\frac{2(c + dx) \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c + dx) \cos(a + bx)}{b} + \frac{id \operatorname{Li}_2\left(-e^{i(a+bx)}\right)}{b^2} - \frac{id \operatorname{Li}_2\left(-e^{i(a+bx)}\right)}{b^2} \end{aligned}$$

Mathematica [A] time = 0.17497, size = 176, normalized size = 1.87

$$\frac{d \left(i \left(\operatorname{PolyLog} \left(2, -e^{i(a+bx)} \right) - \operatorname{PolyLog} \left(2, e^{i(a+bx)} \right) \right) + (a + bx) \left(\log \left(1 - e^{i(a+bx)} \right) - \log \left(1 + e^{i(a+bx)} \right) \right) - a \log \left(\tan \left(\frac{1}{2}(a + bx) \right) \right) \right)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Cos[a + b*x]*Cot[a + b*x], x]

[Out] (c*cos[a + b*x])/b - (c*log[Cos[(a + b*x)/2]])/b + (c*log[Sin[(a + b*x)/2]])/b + (d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) - a*Log[Tan[(a + b*x)/2]] + I*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a + b*x))])))/b^2 + (d*cos[b*x]*(b*x*cos[a] - Sin[a]))/b^2 - (d*(cos[a] + b*x*sin[a])*sin[b*x])/b^2

Maple [B] time = 0.164, size = 203, normalized size = 2.2

$$\frac{(dxb + bc + id) e^{i(bx+a)}}{2b^2} + \frac{(dxb + bc - id) e^{-i(bx+a)}}{2b^2} - 2 \frac{c \operatorname{Artanh}\left(e^{i(bx+a)}\right)}{b} + \frac{d \ln\left(1 - e^{i(bx+a)}\right) x}{b} + \frac{d \ln\left(1 - e^{i(bx+a)}\right) a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*cos(b*x+a)*cot(b*x+a), x)

[Out] 1/2*(d*x*b+b*c+I*d)/b^2*exp(I*(b*x+a))+1/2*(d*x*b+b*c-I*d)/b^2*exp(-I*(b*x+a))-2/b*c*arctanh(exp(I*(b*x+a)))+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(b*x+a)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2-1/b*d*ln(exp(I*(b*x+a)))+1)*x-1/b^2*d*ln(exp(I*(b*x+a)))+1)*a+I*d*polylog(2,-exp(I*(b*x+a)))/b^2+2/b^2*d*a*arctanh(exp(I*(b*x+a)))

Maxima [B] time = 1.6541, size = 269, normalized size = 2.86

$$\frac{2i b d x \arctan(\sin(bx + a), -\cos(bx + a) + 1) - 2i b c \arctan(\sin(bx + a), \cos(bx + a) - 1) + (2i b d x + 2i b c) \arctan(\sin(bx + a), \cos(bx + a) - 1)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")

[Out] $-1/2*(2*I*b*d*x*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) - 2*I*b*c*\arctan2(\sin(b*x + a), \cos(b*x + a) - 1) + (2*I*b*d*x + 2*I*b*c)*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) - 2*(b*d*x + b*c)*\cos(b*x + a) - 2*I*d*\operatorname{dilog}(-e^{(I*b*x + I*a)}) + 2*I*d*\operatorname{dilog}(e^{(I*b*x + I*a)}) + (b*d*x + b*c)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - (b*d*x + b*c)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) + 2*d*\sin(b*x + a))/b^2$

Fricas [B] time = 0.607487, size = 787, normalized size = 8.37

$2(bdx + bc)\cos(bx + a) - i d\operatorname{Li}_2(\cos(bx + a) + i \sin(bx + a)) + i d\operatorname{Li}_2(\cos(bx + a) - i \sin(bx + a)) - i d\operatorname{Li}_2(-\cos(bx + a) + i \sin(bx + a)) - i d\operatorname{Li}_2(-\cos(bx + a) - i \sin(bx + a))$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")

[Out] $1/2*(2*(b*d*x + b*c)*\cos(b*x + a) - I*d*\operatorname{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) + I*d*\operatorname{dilog}(\cos(b*x + a) - I*\sin(b*x + a)) - I*d*\operatorname{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) + I*d*\operatorname{dilog}(-\cos(b*x + a) - I*\sin(b*x + a)) - (b*d*x + b*c)*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) - (b*d*x + b*c)*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1) + (b*c - a*d)*\log(-1/2*\cos(b*x + a) + 1/2*I*\sin(b*x + a) + 1/2) + (b*c - a*d)*\log(-1/2*\cos(b*x + a) - 1/2*I*\sin(b*x + a) + 1/2) + (b*d*x + a*d)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1) + (b*d*x + a*d)*\log(-\cos(b*x + a) - I*\sin(b*x + a) + 1) - 2*d*\sin(b*x + a))/b^2$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \cos(a + bx) \cot(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x)

[Out] Integral((c + d*x)*cos(a + b*x)*cot(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \cos(bx + a) \cot(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)*cos(b*x + a)*cot(b*x + a), x)

$$3.102 \quad \int \frac{\cos(a+bx) \cot(a+bx)}{c+dx} dx$$

Optimal. Leaf size=69

$$\text{Unintegrable}\left(\frac{\csc(a+bx)}{c+dx}, x\right) - \frac{\sin\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{d} - \frac{\cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d}$$

[Out] -((CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d) - (Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d + Unintegrable[Csc[a + b*x]/(c + d*x), x]

Rubi [A] time = 0.106625, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cos(a+bx) \cot(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x), x]

[Out] -((CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d) - (Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d + Defer[Int][Csc[a + b*x]/(c + d*x), x]

Rubi steps

$$\begin{aligned} \int \frac{\cos(a+bx) \cot(a+bx)}{c+dx} dx &= \int \frac{\csc(a+bx)}{c+dx} dx - \int \frac{\sin(a+bx)}{c+dx} dx \\ &= -\left(\cos\left(a - \frac{bc}{d}\right) \int \frac{\sin\left(\frac{bc}{d} + bx\right)}{c+dx} dx\right) - \sin\left(a - \frac{bc}{d}\right) \int \frac{\cos\left(\frac{bc}{d} + bx\right)}{c+dx} dx + \int \frac{\csc(a+bx)}{c+dx} dx \\ &= -\frac{\text{Ci}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{d} - \frac{\cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d} + \int \frac{\csc(a+bx)}{c+dx} dx \end{aligned}$$

Mathematica [A] time = 8.49259, size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx) \cot(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x), x]

[Out] Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x), x]

Maple [A] time = 0.302, size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a) \cot(bx+a)}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(b*x+a)*cot(b*x+a)/(d*x+c),x)`

[Out] `int(cos(b*x+a)*cot(b*x+a)/(d*x+c),x)`

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(i E_1\left(\frac{i b d x+i b c}{d}\right)-i E_1\left(-\frac{i b d x+i b c}{d}\right)\right) \cos\left(-\frac{b c-a d}{d}\right)+2 d \int \frac{\sin(b x+a)}{(d x+c)\left(\cos(b x+a)^2+\sin(b x+a)^2+2 \cos(b x+a)+1\right)} d x+2 d \int \frac{\sin(b x+a)}{(d x+c)\left(\cos(b x+a)^2+\sin(b x+a)^2+2 \cos(b x+a)+1\right)} d x}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x, algorithm="maxima")`

[Out] `1/2*((I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + 2*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)*cos(b*x + a) + c), x) + 2*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) + (exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d)/d`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(bx+a)\cot(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x, algorithm="fricas")`

[Out] `integral(cos(b*x + a)*cot(b*x + a)/(d*x + c), x)`

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x)`

[Out] `Integral(cos(a + b*x)*cot(a + b*x)/(c + d*x), x)`

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)\cot(bx+a)}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*cot(b*x + a)/(d*x + c), x)
```

3.103 $\int \frac{\cos(a+bx) \cot(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=87

$$\text{Unintegrable}\left(\frac{\csc(a+bx)}{(c+dx)^2}, x\right) - \frac{b \cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{d^2} + \frac{b \sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d^2} + \frac{\sin(a+bx)}{d(c+dx)}$$

[Out] -((b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2) + Sin[a + b*x]/(d*(c + d*x)) + (b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Unintegrable[Csc[a + b*x]/(c + d*x)^2, x]

Rubi [A] time = 0.130614, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0$., Rules used = {}

$$\int \frac{\cos(a+bx) \cot(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x)^2, x]

[Out] -((b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2) + Sin[a + b*x]/(d*(c + d*x)) + (b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Defer[Int][Csc[a + b*x]/(c + d*x)^2, x]

Rubi steps

$$\begin{aligned} \int \frac{\cos(a+bx) \cot(a+bx)}{(c+dx)^2} dx &= \int \frac{\csc(a+bx)}{(c+dx)^2} dx - \int \frac{\sin(a+bx)}{(c+dx)^2} dx \\ &= \frac{\sin(a+bx)}{d(c+dx)} - \frac{b \int \frac{\cos(a+bx)}{c+dx} dx}{d} + \int \frac{\csc(a+bx)}{(c+dx)^2} dx \\ &= \frac{\sin(a+bx)}{d(c+dx)} - \frac{\left(b \cos\left(a - \frac{bc}{d}\right)\right) \int \frac{\cos\left(\frac{bc}{d} + bx\right)}{c+dx} dx}{d} + \frac{\left(b \sin\left(a - \frac{bc}{d}\right)\right) \int \frac{\sin\left(\frac{bc}{d} + bx\right)}{c+dx} dx}{d} + \int \frac{\csc(a+bx)}{(c+dx)^2} dx \\ &= -\frac{b \cos\left(a - \frac{bc}{d}\right) \text{Ci}\left(\frac{bc}{d} + bx\right)}{d^2} + \frac{\sin(a+bx)}{d(c+dx)} + \frac{b \sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d^2} + \int \frac{\csc(a+bx)}{(c+dx)^2} dx \end{aligned}$$

Mathematica [A] time = 4.12491, size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx) \cot(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x)^2, x]

[Out] Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x)^2, x]

Maple [A] time = 0.63, size = 0, normalized size = 0.

$$\int \frac{\cos (bx+a) \cot (bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x)

[Out] int(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos (bx+a) \cot (bx+a)}{d^2 x^2+2 c d x+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(cos(b*x + a)*cot(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (a+bx) \cot (a+bx)}{(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)**2,x)

[Out] Integral(cos(a + b*x)*cot(a + b*x)/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (bx+a) \cot (bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*cot(b*x + a)/(d*x + c)^2, x)
```

3.104 $\int (c + dx)^m \cot^2(a + bx) dx$

Optimal. Leaf size=18

Unintegrable($\cot^2(a + bx)(c + dx)^m, x$)

[Out] Unintegrable[(c + d*x)^m*Cot[a + b*x]^2, x]

Rubi [A] time = 0.0347689, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \cot^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Cot[a + b*x]^2, x]

[Out] Defer[Int] [(c + d*x)^m*Cot[a + b*x]^2, x]

Rubi steps

$$\int (c + dx)^m \cot^2(a + bx) dx = \int (c + dx)^m \cot^2(a + bx) dx$$

Mathematica [A] time = 1.17278, size = 0, normalized size = 0.

$$\int (c + dx)^m \cot^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Cot[a + b*x]^2, x]

[Out] Integrate[(c + d*x)^m*Cot[a + b*x]^2, x]

Maple [A] time = 0.16, size = 0, normalized size = 0.

$$\int (dx + c)^m (\cot (bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*cot(b*x+a)^2, x)

[Out] int((d*x+c)^m*cot(b*x+a)^2, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cot (bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cot(b*x+a)^2,x, algorithm="maxima")

[Out] integrate((d*x + c)^m*cot(b*x + a)^2, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left((dx + c)^m \cot (bx + a)^2, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cot(b*x+a)^2,x, algorithm="fricas")

[Out] integral((d*x + c)^m*cot(b*x + a)^2, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^m \cot^2 (a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**m*cot(b*x+a)**2,x)

[Out] Integral((c + d*x)**m*cot(a + b*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cot (bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cot(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^m*cot(b*x + a)^2, x)

3.105 $\int (c + dx)^4 \cot^2(a + bx) dx$

Optimal. Leaf size=155

$$\frac{6id^2(c + dx)^2 \text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^3} + \frac{6d^3(c + dx) \text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{b^4} + \frac{3id^4 \text{PolyLog}\left(4, e^{2i(a+bx)}\right)}{b^5} + \frac{4d(c + dx)^3}{b^2}$$

[Out] $((-1)*(c + d*x)^4)/b - (c + d*x)^5/(5*d) - ((c + d*x)^4*\text{Cot}[a + b*x])/b + (4*d*(c + d*x)^3*\text{Log}[1 - E^{((2*I)*(a + b*x))}])/b^2 - ((6*I)*d^2*(c + d*x)^2*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^3 + (6*d^3*(c + d*x)*\text{PolyLog}[3, E^{((2*I)*(a + b*x))}])/b^4 + ((3*I)*d^4*\text{PolyLog}[4, E^{((2*I)*(a + b*x))}])/b^5$

Rubi [A] time = 0.22885, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3720, 3717, 2190, 2531, 6609, 2282, 6589, 32}

$$\frac{6id^2(c + dx)^2 \text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^3} + \frac{6d^3(c + dx) \text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{b^4} + \frac{3id^4 \text{PolyLog}\left(4, e^{2i(a+bx)}\right)}{b^5} + \frac{4d(c + dx)^3}{b^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^4*Cot[a + b*x]^2,x]

[Out] $((-1)*(c + d*x)^4)/b - (c + d*x)^5/(5*d) - ((c + d*x)^4*\text{Cot}[a + b*x])/b + (4*d*(c + d*x)^3*\text{Log}[1 - E^{((2*I)*(a + b*x))}])/b^2 - ((6*I)*d^2*(c + d*x)^2*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^3 + (6*d^3*(c + d*x)*\text{PolyLog}[3, E^{((2*I)*(a + b*x))}])/b^4 + ((3*I)*d^4*\text{PolyLog}[4, E^{((2*I)*(a + b*x))}])/b^5$

Rule 3720

Int[((c_.) + (d_.)*(x_.))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^(m*(b*Tan[e + f*x])^(n - 2), x], x)] /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 3717

Int[((c_.) + (d_.)*(x_.))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_.)], x_Symbol] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[(c + d*x)^m*E^(2*I*k*Pi)*E^(2*I*(e + f*x))]/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_.))))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_.))))^(n_.)), x_Symbol] :> Simp[(((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n]/a)]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n]/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_.))))^(n_.)]*((f_.) + (g_.)*(x_.))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/ (b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f}

, g, n}, x] && GtQ[m, 0]

Rule 6609

```
Int[((e_.) + (f_.)*(x_.))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_.)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_.))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_.))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^4 \cot^2(a + bx) dx &= -\frac{(c + dx)^4 \cot(a + bx)}{b} + \frac{(4d) \int (c + dx)^3 \cot(a + bx) dx}{b} - \int (c + dx)^4 dx \\
&= -\frac{i(c + dx)^4}{b} - \frac{(c + dx)^5}{5d} - \frac{(c + dx)^4 \cot(a + bx)}{b} - \frac{(8id) \int \frac{e^{2i(a+bx)}(c+dx)^3}{1-e^{2i(a+bx)}} dx}{b} \\
&= -\frac{i(c + dx)^4}{b} - \frac{(c + dx)^5}{5d} - \frac{(c + dx)^4 \cot(a + bx)}{b} + \frac{4d(c + dx)^3 \log(1 - e^{2i(a+bx)})}{b^2} - \frac{(12d^2)}{b^2} \\
&= -\frac{i(c + dx)^4}{b} - \frac{(c + dx)^5}{5d} - \frac{(c + dx)^4 \cot(a + bx)}{b} + \frac{4d(c + dx)^3 \log(1 - e^{2i(a+bx)})}{b^2} - \frac{6id^2(c + dx)}{b^2} \\
&= -\frac{i(c + dx)^4}{b} - \frac{(c + dx)^5}{5d} - \frac{(c + dx)^4 \cot(a + bx)}{b} + \frac{4d(c + dx)^3 \log(1 - e^{2i(a+bx)})}{b^2} - \frac{6id^2(c + dx)}{b^2} \\
&= -\frac{i(c + dx)^4}{b} - \frac{(c + dx)^5}{5d} - \frac{(c + dx)^4 \cot(a + bx)}{b} + \frac{4d(c + dx)^3 \log(1 - e^{2i(a+bx)})}{b^2} - \frac{6id^2(c + dx)}{b^2} \\
&= -\frac{i(c + dx)^4}{b} - \frac{(c + dx)^5}{5d} - \frac{(c + dx)^4 \cot(a + bx)}{b} + \frac{4d(c + dx)^3 \log(1 - e^{2i(a+bx)})}{b^2} - \frac{6id^2(c + dx)}{b^2}
\end{aligned}$$

Mathematica [B] time = 6.72704, size = 795, normalized size = 5.13

$$e^{ia} \csc(a) (b^4 e^{-2ia} x^4 + 2ib^3 (1 - e^{-2ia}) \log(1 - e^{-i(a+bx)}) x^3 + 2ib^3 (1 - e^{-2ia}) \log(1 + e^{-i(a+bx)}) x^3 - 6e^{-2ia} (-1 + e^{2ia}) (b^2 F$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^4*Cot[a + b*x]^2,x]

[Out] $-(x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4))/5 - (2*c*d^3*E^{I*a}*Csc[a]*((2*b^3*x^3)/E^{(2*I)*a} + (3*I)*b^2*(1 - E^{(-2*I)*a}))*x^2*Log[1 - E^{(-I)*(a + b*x)}] + (3*I)*b^2*(1 - E^{(-2*I)*a})*x^2*Log[1 + E^{(-I)*(a + b*x)}] - (6*(-1 + E^{(2*I)*a})*(b*x*PolyLog[2, -E^{(-I)*(a + b*x)}]) - I*PolyLog[3, -E^{(-I)*(a + b*x)}]))/E^{(2*I)*a} - (6*(-1 + E^{(2*I)*a})*(b*x*PolyLog[2, E^{(-I)*(a + b*x)}]) - I*PolyLog[3, E^{(-I)*(a + b*x)}]))/E^{(2*I)*a}))/b^4 - (d^4*E^{I*a}*Csc[a]*((b^4*x^4)/E^{(2*I)*a} + (2*I)*b^3*(1 - E^{(-2*I)*a}))*x^3*Log[1 - E^{(-I)*(a + b*x)}] + (2*I)*b^3*(1 - E^{(-2*I)*a}))*x^3*Log[1 + E^{(-I)*(a + b*x)}] - (6*(-1 + E^{(2*I)*a})*(b^2*x^2*PolyLog[2, -E^{(-I)*(a + b*x)}] - (2*I)*b*x*PolyLog[3, -E^{(-I)*(a + b*x)}]) - 2*PolyLog[4, -E^{(-I)*(a + b*x)}]))/E^{(2*I)*a} - (6*(-1 + E^{(2*I)*a})*(b^2*x^2*PolyLog[2, E^{(-I)*(a + b*x)}] - (2*I)*b*x*PolyLog[3, E^{(-I)*(a + b*x)}]) - 2*PolyLog[4, E^{(-I)*(a + b*x)}]))/E^{(2*I)*a}))/b^5 + (4*c^3*d*Csc[a]*(-b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^2*(Cos[a]^2 + Sin[a]^2)) + (Csc[a]*Csc[a + b*x]*(c^4*Sin[b*x] + 4*c^3*d*x*Sin[b*x] + 6*c^2*d^2*x^2*Sin[b*x] + 4*c*d^3*x^3*Sin[b*x] + d^4*x^4*Sin[b*x]))/b - (6*c^2*d^2*Csc[a]*Sec[a]*(b^2*E^{I*ArcTan[Tan[a]]})*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^{(-2*I)*b*x}] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^{(2*I)*(b*x + ArcTan[Tan[a]])}]) + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]]) + I*PolyLog[2, E^{(2*I)*(b*x + ArcTan[Tan[a]])}])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^3*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])$

Maple [B] time = 0.213, size = 913, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^4*cot(b*x+a)^2,x)

[Out] $-24*I*d^3/b^3*c*polylog(2, \exp(I*(b*x+a)))*x - 2*I*(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)/b / (\exp(2*I*(b*x+a)) - 1) + 4*d^4/b^2*\ln(1 - \exp(I*(b*x+a)))*x^3 - 2*I*d^4/b*x^4 - 8*d/b^2*c^3*\ln(\exp(I*(b*x+a))) + 4*d/b^2*c^3*\ln(\exp(I*(b*x+a)) - 1) + 24*I*d^4/b^5*polylog(4, -\exp(I*(b*x+a))) - 6*I*d^4/b^5*a^4 + 4*d/b^2*c^3*\ln(\exp(I*(b*x+a)) + 1) + 24*d^4/b^4*polylog(3, \exp(I*(b*x+a)))*x + 24*d^4/b^4*polylog(3, -\exp(I*(b*x+a)))*x - 4*d^4/b^5*a^3*\ln(\exp(I*(b*x+a)) - 1) + 24*d^3/b^4*c*polylog(3, -\exp(I*(b*x+a))) + 24*d^3/b^4*c*polylog(3, \exp(I*(b*x+a))) + 8*d^4/b^5*a^3*\ln(\exp(I*(b*x+a))) + 24*I*d^4*polylog(4, \exp(I*(b*x+a)))/b^5 - 1/5*d^4*x^5 - c^4*x - 12*d^2/b^3*c^2*a*\ln(\exp(I*(b*x+a)) - 1) + 12*d^3/b^4*c*a^2*\ln(\exp(I*(b*x+a)) - 1) - 24*I*d^2/b^2*c^2*a*x - 24*I*d^3/b^3*c*polylog(2, -\exp(I*(b*x+a)))*x + 12*d^3/b^2*c*\ln(\exp(I*(b*x+a)) + 1)*x^2 + 12*d^3/b^2*c*\ln(1 - \exp(I*(b*x+a)))*x^2 - 12*d^3/b^4*c*\ln(1 - \exp(I*(b*x+a)))*a^2 - 12*I*d^2/b^3*c^2*polylog(2, \exp(I*(b*x+a))) - 12*I*d^4/b^3*polylog(2, \exp(I*(b*x+a)))*x^2 - 2*c^2*d^2*x^3 - 2*c^3*d*x^2 - 24*d^3/b^4*c*a^2*\ln(\exp(I*(b*x+a))) + 24*d^2/b^3*c^2*a*\ln(\exp(I*(b*x+a))) - c*d^3*x^4 + 24*I*d^3/b^3*c*a^2*x - 12*I*d^2/b^3*c^2*polylog(2, -\exp(I*(b*x+a))) - 12*I*d^4/b^3*polylog(2, -\exp(I*(b*x+a)))*x^2 - 12*I*d^2/b*c^2*x^2 - 8*I*d^3/b*c*x^3 - 8*I*d^4/b^4*a^3*x + 16*I*d^3/b^4*c*a^3 - 12*I*d^2/b^3*c^2*a^2 + 4*d^4/b^5*\ln(1 - \exp(I*(b*x+a)))*a^3 + 4*d^4/b^2*\ln(\exp(I*(b*x+a)) + 1)*x^3 + 12*d^2/b^2*c^2*\ln(1 - \exp(I*(b*x+a)))*x + 12*d^2/b^3*c^2*\ln(1 - \exp(I*(b*x+a)))*a + 12*d^2/b^2*c^2*\ln(\exp(I*(b*x+a)) + 1)*x$

Maxima [B] time = 3.24618, size = 4359, normalized size = 28.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cot(b*x+a)^2,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -((b*x + a + 1/\tan(b*x + a))*c^4 - 4*(b*x + a + 1/\tan(b*x + a))*a*c^3*d/b + \\ & 6*(b*x + a + 1/\tan(b*x + a))*a^2*c^2*d^2/b^2 - 4*(b*x + a + 1/\tan(b*x + a)) \\ &)*a^3*c*d^3/b^3 + (b*x + a + 1/\tan(b*x + a))*a^4*d^4/b^4 + 2*((b*x + a)^2*c \\ & \cos(2*b*x + 2*a)^2 + (b*x + a)^2*\sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*\cos(2*b* \\ & x + 2*a) + (b*x + a)^2 - (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2 \\ & *b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) \\ & - (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\log(\cos \\ & (b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) + 4*(b*x + a)*\sin(2*b*x \\ & + 2*a))*c^3*d/((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2* \\ & a) + 1)*b) - 6*((b*x + a)^2*\cos(2*b*x + 2*a)^2 + (b*x + a)^2*\sin(2*b*x + 2* \\ & a)^2 - 2*(b*x + a)^2*\cos(2*b*x + 2*a) + (b*x + a)^2 - (\cos(2*b*x + 2*a)^2 + \\ & \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x \\ & + a)^2 + 2*\cos(b*x + a) + 1) - (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2 \\ & *\cos(2*b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) \\ & + 1) + 4*(b*x + a)*\sin(2*b*x + 2*a))*a*c^2*d^2/((\cos(2*b*x + 2*a)^2 + \sin(\\ & 2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*b^2) + 6*((b*x + a)^2*\cos(2*b*x + \\ & 2*a)^2 + (b*x + a)^2*\sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*\cos(2*b*x + 2*a) + \\ & (b*x + a)^2 - (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) \\ & + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - (\cos(2*b* \\ & x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(b*x + a)^ \\ & 2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) + 4*(b*x + a)*\sin(2*b*x + 2*a))*a^ \\ & 2*c*d^3/((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1) \\ & *b^3) - 2*((b*x + a)^2*\cos(2*b*x + 2*a)^2 + (b*x + a)^2*\sin(2*b*x + 2*a)^2 \\ & - 2*(b*x + a)^2*\cos(2*b*x + 2*a) + (b*x + a)^2 - (\cos(2*b*x + 2*a)^2 + \sin(\\ & 2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^ \\ & 2 + 2*\cos(b*x + a) + 1) - (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(\\ & 2*b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) \\ & + 4*(b*x + a)*\sin(2*b*x + 2*a))*a^3*d^4/((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + \\ & 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*b^4) - (-I*(b*x + a)^5*d^4 + (-5*I*b*c*d^ \\ & 3 + 5*I*a*d^4)*(b*x + a)^4 + (-10*I*b^2*c^2*d^2 + 20*I*a*b*c*d^3 - 10*I*a^2 \\ & *d^4)*(b*x + a)^3 - (20*(b*x + a)^3*d^4 + 60*(b*c*d^3 - a*d^4)*(b*x + a)^2 \\ & + 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) - 20*((b*x + a)^3*d^4 \\ & + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4) \\ & *(b*x + a))*\cos(2*b*x + 2*a) - (20*I*(b*x + a)^3*d^4 + (60*I*b*c*d^3 - 60*I \\ & *a*d^4)*(b*x + a)^2 + (60*I*b^2*c^2*d^2 - 120*I*a*b*c*d^3 + 60*I*a^2*d^4)*(\\ & b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) + (20*(\\ & b*x + a)^3*d^4 + 60*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 60*(b^2*c^2*d^2 - 2*a*b \\ & *c*d^3 + a^2*d^4)*(b*x + a) - 20*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b* \\ & x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*\cos(2*b*x + 2 \\ & *a) + (-20*I*(b*x + a)^3*d^4 + (-60*I*b*c*d^3 + 60*I*a*d^4)*(b*x + a)^2 + (\\ & -60*I*b^2*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*a^2*d^4)*(b*x + a))*\sin(2*b*x + \\ & 2*a))*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) + (I*(b*x + a)^5*d^4 + (5*I \\ & b*c*d^3 - 5*(I*a + 2)*d^4)*(b*x + a)^4 + (10*I*b^2*c^2*d^2 - 20*(I*a + 2)*b \\ & *c*d^3 + (10*I*a^2 + 40*a)*d^4)*(b*x + a)^3 - 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 \\ & + a^2*d^4)*(b*x + a)^2*\cos(2*b*x + 2*a) + (60*b^2*c^2*d^2 - 120*a*b*c*d^3 \\ & + 60*(b*x + a)^2*d^4 + 60*a^2*d^4 + 120*(b*c*d^3 - a*d^4)*(b*x + a) - 60*(\\ & b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4) \\ & *(b*x + a))*\cos(2*b*x + 2*a) + (-60*I*b^2*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I* \\ & (b*x + a)^2*d^4 - 60*I*a^2*d^4 + (-120*I*b*c*d^3 + 120*I*a*d^4)*(b*x + a))* \\ & \sin(2*b*x + 2*a))*\operatorname{dilog}(-e^{I*b*x + I*a}) + (60*b^2*c^2*d^2 - 120*a*b*c*d^3 \end{aligned}$$

$$\begin{aligned}
& + 60*(b*x + a)^2*d^4 + 60*a^2*d^4 + 120*(b*c*d^3 - a*d^4)*(b*x + a) - 60*(\\
& b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4) \\
& *(b*x + a))*\cos(2*b*x + 2*a) + (-60*I*b^2*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I* \\
& (b*x + a)^2*d^4 - 60*I*a^2*d^4 + (-120*I*b*c*d^3 + 120*I*a*d^4)*(b*x + a))* \\
& \sin(2*b*x + 2*a))*\operatorname{dilog}(e^{(I*b*x + I*a)}) + (10*I*(b*x + a)^3*d^4 + (30*I*b* \\
& c*d^3 - 30*I*a*d^4)*(b*x + a)^2 + (30*I*b^2*c^2*d^2 - 60*I*a*b*c*d^3 + 30*I \\
& *a^2*d^4)*(b*x + a) + (-10*I*(b*x + a)^3*d^4 + (-30*I*b*c*d^3 + 30*I*a*d^4) \\
& *(b*x + a)^2 + (-30*I*b^2*c^2*d^2 + 60*I*a*b*c*d^3 - 30*I*a^2*d^4)*(b*x + a \\
&))*\cos(2*b*x + 2*a) + 10*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 \\
& + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\log \\
& (\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) + (10*I*(b*x + a)^3*d^4 \\
& + (30*I*b*c*d^3 - 30*I*a*d^4)*(b*x + a)^2 + (30*I*b^2*c^2*d^2 - 60*I*a* \\
& b*c*d^3 + 30*I*a^2*d^4)*(b*x + a) + (-10*I*(b*x + a)^3*d^4 + (-30*I*b*c*d^3 \\
& + 30*I*a*d^4)*(b*x + a)^2 + (-30*I*b^2*c^2*d^2 + 60*I*a*b*c*d^3 - 30*I*a^2 \\
& *d^4)*(b*x + a))*\cos(2*b*x + 2*a) + 10*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^ \\
& 4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*\sin(2*b \\
& *x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) + 120* \\
& (d^4*\cos(2*b*x + 2*a) + I*d^4*\sin(2*b*x + 2*a) - d^4)*\operatorname{polylog}(4, -e^{(I*b*x \\
& + I*a)}) + 120*(d^4*\cos(2*b*x + 2*a) + I*d^4*\sin(2*b*x + 2*a) - d^4)*\operatorname{polylog} \\
& (4, e^{(I*b*x + I*a)}) + (120*I*b*c*d^3 + 120*I*(b*x + a)*d^4 - 120*I*a*d^4 + \\
& (-120*I*b*c*d^3 - 120*I*(b*x + a)*d^4 + 120*I*a*d^4)*\cos(2*b*x + 2*a) + 12 \\
& 0*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*\sin(2*b*x + 2*a))*\operatorname{polylog}(3, -e^{(I*b*x \\
& + I*a)}) + (120*I*b*c*d^3 + 120*I*(b*x + a)*d^4 - 120*I*a*d^4 + (-120*I*b*c* \\
& d^3 - 120*I*(b*x + a)*d^4 + 120*I*a*d^4)*\cos(2*b*x + 2*a) + 120*(b*c*d^3 + \\
& (b*x + a)*d^4 - a*d^4)*\sin(2*b*x + 2*a))*\operatorname{polylog}(3, e^{(I*b*x + I*a)}) - ((b* \\
& x + a)^5*d^4 + (5*b*c*d^3 - (5*a - 10*I)*d^4)*(b*x + a)^4 + (10*b^2*c^2*d^2 \\
& - (20*a - 40*I)*b*c*d^3 + 10*(a^2 - 4*I*a)*d^4)*(b*x + a)^3 - (-60*I*b^2*c \\
& ^2*d^2 + 120*I*a*b*c*d^3 - 60*I*a^2*d^4)*(b*x + a)^2)*\sin(2*b*x + 2*a))/(-5 \\
& *I*b^4*\cos(2*b*x + 2*a) + 5*b^4*\sin(2*b*x + 2*a) + 5*I*b^4))/b
\end{aligned}$$

Fricas [C] time = 0.618904, size = 2034, normalized size = 13.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cot(b*x+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned}
& -1/10*(10*b^4*d^4*x^4 + 40*b^4*c*d^3*x^3 + 60*b^4*c^2*d^2*x^2 + 40*b^4*c^3* \\
& d*x + 10*b^4*c^4 - 15*I*d^4*\operatorname{polylog}(4, \cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a) \\
&))*\sin(2*b*x + 2*a) + 15*I*d^4*\operatorname{polylog}(4, \cos(2*b*x + 2*a) - I*\sin(2*b*x + \\
& 2*a))*\sin(2*b*x + 2*a) - (-30*I*b^2*d^4*x^2 - 60*I*b^2*c*d^3*x - 30*I*b^2*c \\
& ^2*d^2)*\operatorname{dilog}(\cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a))*\sin(2*b*x + 2*a) - (30 \\
& *I*b^2*d^4*x^2 + 60*I*b^2*c*d^3*x + 30*I*b^2*c^2*d^2)*\operatorname{dilog}(\cos(2*b*x + 2*a) \\
&) - I*\sin(2*b*x + 2*a))*\sin(2*b*x + 2*a) - 20*(b^3*c^3*d - 3*a*b^2*c^2*d^2 \\
& + 3*a^2*b*c*d^3 - a^3*d^4)*\log(-1/2*\cos(2*b*x + 2*a) + 1/2*I*\sin(2*b*x + 2* \\
& a) + 1/2)*\sin(2*b*x + 2*a) - 20*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 \\
& - a^3*d^4)*\log(-1/2*\cos(2*b*x + 2*a) - 1/2*I*\sin(2*b*x + 2*a) + 1/2)*\sin(\\
& 2*b*x + 2*a) - 20*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^ \\
& 2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*\log(-\cos(2*b*x + 2*a) + I*\sin(2*b*x + \\
& 2*a) + 1)*\sin(2*b*x + 2*a) - 20*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2* \\
& d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*\log(-\cos(2*b*x + 2*a) - \\
& I*\sin(2*b*x + 2*a) + 1)*\sin(2*b*x + 2*a) - 30*(b*d^4*x + b*c*d^3)*\operatorname{polylog}(3 \\
& , \cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a))*\sin(2*b*x + 2*a) - 30*(b*d^4*x + b \\
& *c*d^3)*\operatorname{polylog}(3, \cos(2*b*x + 2*a) - I*\sin(2*b*x + 2*a))*\sin(2*b*x + 2*a) \\
& + 10*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b \\
& ^4*c^4)*\cos(2*b*x + 2*a) + 2*(b^5*d^4*x^5 + 5*b^5*c*d^3*x^4 + 10*b^5*c^2*d^
\end{aligned}$$

$2*x^3 + 10*b^5*c^3*d*x^2 + 5*b^5*c^4*x)*\sin(2*b*x + 2*a))/(b^5*\sin(2*b*x + 2*a))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^4 \cot^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**4*cot(b*x+a)**2,x)

[Out] Integral((c + d*x)**4*cot(a + b*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^4 \cot(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cot(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^4*cot(b*x + a)^2, x)

3.106 $\int (c + dx)^3 \cot^2(a + bx) dx$

Optimal. Leaf size=127

$$\frac{3id^2(c + dx)\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^3} + \frac{3d^3\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^4} + \frac{3d(c + dx)^2 \log\left(1 - e^{2i(a+bx)}\right)}{b^2} - \frac{(c + dx)^3 \cot(a + bx)}{b}$$

[Out] $((-I)*(c + d*x)^3)/b - (c + d*x)^4/(4*d) - ((c + d*x)^3*\text{Cot}[a + b*x])/b + (3*d*(c + d*x)^2*\text{Log}[1 - E^{((2*I)*(a + b*x))}])/b^2 - ((3*I)*d^2*(c + d*x)*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^3 + (3*d^3*\text{PolyLog}[3, E^{((2*I)*(a + b*x))}])/(2*b^4)$

Rubi [A] time = 0.197258, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3720, 3717, 2190, 2531, 2282, 6589, 32}

$$\frac{3id^2(c + dx)\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^3} + \frac{3d^3\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^4} + \frac{3d(c + dx)^2 \log\left(1 - e^{2i(a+bx)}\right)}{b^2} - \frac{(c + dx)^3 \cot(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^3*\text{Cot}[a + b*x]^2, x]$

[Out] $((-I)*(c + d*x)^3)/b - (c + d*x)^4/(4*d) - ((c + d*x)^3*\text{Cot}[a + b*x])/b + (3*d*(c + d*x)^2*\text{Log}[1 - E^{((2*I)*(a + b*x))}])/b^2 - ((3*I)*d^2*(c + d*x)*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^3 + (3*d^3*\text{PolyLog}[3, E^{((2*I)*(a + b*x))}])/(2*b^4)$

Rule 3720

$\text{Int}[(c + d*x)^m*(b*\text{Tan}[e + f*x])^{n-1}, x] + (-\text{Dist}[(b*d*m)/(f*(n-1)], \text{Int}[(c + d*x)^{m-1}*(b*\text{Tan}[e + f*x])^{n-1}, x], x] - \text{Dist}[b^2, \text{Int}[(c + d*x)^m*(b*\text{Tan}[e + f*x])^{n-2}, x], x]) /;$ FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 3717

$\text{Int}[(c + d*x)^m*\text{Tan}[e + \text{Pi}*k + (f*x)], x] + (-\text{Dist}[2*I, \text{Int}[(c + d*x)^m*\text{E}^{(2*I*k*\text{Pi})}*\text{E}^{(2*I*(e + f*x))}/(1 + \text{E}^{(2*I*k*\text{Pi})}*\text{E}^{(2*I*(e + f*x))}), x], x] /;$ FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 2190

$\text{Int}[(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x))})^n)/a], x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{m-1}*\text{Log}[1 + (b*(F^{(g*(e + f*x))})^n)/a], x], x] /;$ FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

$\text{Int}[\text{Log}[1 + (e*(F^{(c*(a + b*x))})^n)], x] + \text{Dist}[(g*m)/(b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^{m-1}*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x))})^n)], x], x] /;$ FreeQ[{F, a, b, c, e, f, g, n}, x]

, g, n}, x] && GtQ[m, 0]

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^3 \cot^2(a + bx) dx &= -\frac{(c + dx)^3 \cot(a + bx)}{b} + \frac{(3d) \int (c + dx)^2 \cot(a + bx) dx}{b} - \int (c + dx)^3 dx \\ &= -\frac{i(c + dx)^3}{b} - \frac{(c + dx)^4}{4d} - \frac{(c + dx)^3 \cot(a + bx)}{b} - \frac{(6id) \int \frac{e^{2i(a+bx)}(c+dx)^2}{1-e^{2i(a+bx)}} dx}{b} \\ &= -\frac{i(c + dx)^3}{b} - \frac{(c + dx)^4}{4d} - \frac{(c + dx)^3 \cot(a + bx)}{b} + \frac{3d(c + dx)^2 \log(1 - e^{2i(a+bx)})}{b^2} - \frac{(6d^2) \int}{b^2} \\ &= -\frac{i(c + dx)^3}{b} - \frac{(c + dx)^4}{4d} - \frac{(c + dx)^3 \cot(a + bx)}{b} + \frac{3d(c + dx)^2 \log(1 - e^{2i(a+bx)})}{b^2} - \frac{3id^2(c + dx)}{b^2} \\ &= -\frac{i(c + dx)^3}{b} - \frac{(c + dx)^4}{4d} - \frac{(c + dx)^3 \cot(a + bx)}{b} + \frac{3d(c + dx)^2 \log(1 - e^{2i(a+bx)})}{b^2} - \frac{3id^2(c + dx)}{b^2} \\ &= -\frac{i(c + dx)^3}{b} - \frac{(c + dx)^4}{4d} - \frac{(c + dx)^3 \cot(a + bx)}{b} + \frac{3d(c + dx)^2 \log(1 - e^{2i(a+bx)})}{b^2} - \frac{3id^2(c + dx)}{b^2} \end{aligned}$$

Mathematica [B] time = 6.44499, size = 374, normalized size = 2.94

$$3cd^2 \left(-i \operatorname{PolyLog} \left(2, e^{2i(\tan^{-1}(\tan(a))+bx)} \right) - b^2 x^2 e^{i \tan^{-1}(\tan(a))} \cot(a) \sqrt{\sec^2(a)} + ibx \left(\pi - 2 \tan^{-1}(\tan(a)) \right) + 2 \left(\tan^{-1}(\tan(a)) \right) \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x]^2,x]
```

```
[Out] -(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 - (3*c^2*d*(b*x*Cot[a] -
Log[Sin[a + b*x]]))/b^2 + (3*c*d^2*(I*b*x*(Pi - 2*ArcTan[Tan[a]]) + Pi*Log
[1 + E^((-2*I)*b*x)] + 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + Arc
Tan[Tan[a]])]) - Pi*Log[Cos[b*x]] - 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[T
an[a]]]]) - I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]])]) - b^2*E^(I*ArcTan
[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^2])/b^3 + (d^3*(I + Cot[a])*(I*b^3*x^3 -
b^3*x^3*Cot[a] + 3*b^2*x^2*Log[1 - E^((-I)*(a + b*x))] + 3*b^2*x^2*Log[1 +
E^((-I)*(a + b*x))] + (6*I)*b*x*PolyLog[2, -E^((-I)*(a + b*x))] + (6*I)*b*x
*PolyLog[2, E^((-I)*(a + b*x))] + 6*PolyLog[3, -E^((-I)*(a + b*x))] + 6*Pol
```

$y \log[3, E^{(-I)(a + b*x)}] * \sin[a] / (b^4 * E^{I*a}) + ((c + d*x)^3 * \text{Csc}[a] * \text{Cs}c[a + b*x] * \sin[b*x]) / b$

Maple [B] time = 0.194, size = 573, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int ((d*x+c)^3 * \cot(b*x+a)^2, x)$

[Out] $6*d^3 * \text{polylog}(3, -\exp(I*(b*x+a))) / b^4 + 6*d^3 * \text{polylog}(3, \exp(I*(b*x+a))) / b^4 - 3*d^3 / b^4 * \ln(1 - \exp(I*(b*x+a))) * a^2 + 3*d^3 / b^2 * \ln(\exp(I*(b*x+a)) + 1) * x^2 + 3*d^3 / b^2 * \ln(1 - \exp(I*(b*x+a))) * x^2 + 3*d^3 / b^4 * a^2 * \ln(\exp(I*(b*x+a)) - 1) - c*d^2 * x^3 - 1/4*d^3 * x^4 - c^3 * x - 2*I*(d^3 * x^3 + 3*c*d^2 * x^2 + 3*c^2 * d * x + c^3) / b / (\exp(2*I*(b*x+a)) - 1) - 6*d^3 / b^4 * a^2 * \ln(\exp(I*(b*x+a))) - 6*d / b^2 * c^2 * \ln(\exp(I*(b*x+a))) + 3*d / b^2 * c^2 * \ln(\exp(I*(b*x+a)) + 1) + 3*d / b^2 * c^2 * \ln(\exp(I*(b*x+a)) - 1) - 2*I*d^3 / b * x^3 + 4*I*d^3 / b^4 * a^3 - 12*I*d^2 / b^2 * c * a * x + 6*d^2 / b^2 * c * \ln(\exp(I*(b*x+a)) + 1) * x + 6*d^2 / b^2 * c * \ln(1 - \exp(I*(b*x+a))) * x + 6*d^2 / b^3 * c * \ln(1 - \exp(I*(b*x+a))) * a - 6*I*d^3 / b^3 * \text{polylog}(2, \exp(I*(b*x+a))) * x - 6*d^2 / b^3 * c * a * \ln(\exp(I*(b*x+a)) - 1) - 6*I*d^3 / b^3 * \text{polylog}(2, -\exp(I*(b*x+a))) * x - 3/2 * c^2 * d * x^2 + 12*d^2 / b^3 * c * a * \ln(\exp(I*(b*x+a))) - 6*I*d^2 / b^3 * c * \text{polylog}(2, \exp(I*(b*x+a))) + 6*I*d^3 / b^3 * a^2 * x - 6*I*d^2 / b^3 * c * a^2 - 6*I*d^2 / b * c * x^2 - 6*I*d^2 / b^3 * c * \text{polylog}(2, -\exp(I*(b*x+a)))$

Maxima [B] time = 2.36897, size = 2626, normalized size = 20.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((d*x+c)^3 * \cot(b*x+a)^2, x, \text{algorithm}="maxima")$

[Out] $-1/2 * (2*(b*x + a + 1/\tan(b*x + a)) * c^3 - 6*(b*x + a + 1/\tan(b*x + a)) * a * c^2 * d / b + 6*(b*x + a + 1/\tan(b*x + a)) * a^2 * c * d^2 / b^2 - 2*(b*x + a + 1/\tan(b*x + a)) * a^3 * d^3 / b^3 + 3*((b*x + a)^2 * \cos(2*b*x + 2*a)^2 + (b*x + a)^2 * \sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2 * \cos(2*b*x + 2*a) + (b*x + a)^2 - (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1) * \log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1) * \log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) + 4*(b*x + a) * \sin(2*b*x + 2*a)) * c^2 * d / ((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1) * b) - 6*((b*x + a)^2 * \cos(2*b*x + 2*a)^2 + (b*x + a)^2 * \sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2 * \cos(2*b*x + 2*a) + (b*x + a)^2 - (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1) * \log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1) * \log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) + 4*(b*x + a) * \sin(2*b*x + 2*a)) * a * c * d^2 / ((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1) * b^2) + 3*((b*x + a)^2 * \cos(2*b*x + 2*a)^2 + (b*x + a)^2 * \sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2 * \cos(2*b*x + 2*a) + (b*x + a)^2 - (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1) * \log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1) * \log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) + 4*(b*x + a) * \sin(2*b*x + 2*a)) * a^2 * d^3 / ((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1) * b^3) - 2*(-I*(b*x + a)^4 * d^3 + (-4*I*b*c*d$

$$\begin{aligned} &^2 + 4*I*a*d^3)*(b*x + a)^3 - (12*(b*x + a)^2*d^3 + 24*(b*c*d^2 - a*d^3)*(b \\ &*x + a) - 12*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + \\ &2*a) - (12*I*(b*x + a)^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*\sin(2 \\ &*b*x + 2*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) + (12*(b*x + a)^2*d^3 \\ &+ 24*(b*c*d^2 - a*d^3)*(b*x + a) - 12*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3 \\ &)*(b*x + a))*\cos(2*b*x + 2*a) + (-12*I*(b*x + a)^2*d^3 + (-24*I*b*c*d^2 + 2 \\ &4*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), -\cos(b*x + a) \\ &+ 1) + (I*(b*x + a)^4*d^3 + (4*I*b*c*d^2 - 4*(I*a + 2)*d^3)*(b*x + a)^3 - \\ &24*(b*c*d^2 - a*d^3)*(b*x + a)^2*\cos(2*b*x + 2*a) + (24*b*c*d^2 + 24*(b*x \\ &+ a)*d^3 - 24*a*d^3 - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3))*\cos(2*b*x + 2*a) \\ &+ (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3))*\sin(2*b*x + 2*a))*\text{dilo} \\ &\text{g}(-e^{(I*b*x + I*a)}) + (24*b*c*d^2 + 24*(b*x + a)*d^3 - 24*a*d^3 - 24*(b*c*d \\ &^2 + (b*x + a)*d^3 - a*d^3))*\cos(2*b*x + 2*a) + (-24*I*b*c*d^2 - 24*I*(b*x + \\ &a)*d^3 + 24*I*a*d^3))*\sin(2*b*x + 2*a))*\text{dilog}(e^{(I*b*x + I*a)}) + (6*I*(b*x \\ &+ a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a) + (-6*I*(b*x + a)^2*d^3 \\ &+ (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + 6*((b*x + a)^2 \\ &*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 \\ &+ \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) + (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d \\ &^2 - 12*I*a*d^3)*(b*x + a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I \\ &a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^ \\ &3)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos \\ &(b*x + a) + 1) + (-24*I*d^3*\cos(2*b*x + 2*a) + 24*d^3*\sin(2*b*x + 2*a) + 24 \\ &*I*d^3)*\text{polylog}(3, -e^{(I*b*x + I*a)}) + (-24*I*d^3*\cos(2*b*x + 2*a) + 24*d^3 \\ &*\sin(2*b*x + 2*a) + 24*I*d^3)*\text{polylog}(3, e^{(I*b*x + I*a)}) - ((b*x + a)^4*d^ \\ &3 + (4*b*c*d^2 - (4*a - 8*I)*d^3)*(b*x + a)^3 - (-24*I*b*c*d^2 + 24*I*a*d^3 \\ &)*(b*x + a)^2*\sin(2*b*x + 2*a))/(-4*I*b^3*\cos(2*b*x + 2*a) + 4*b^3*\sin(2*b \\ &*x + 2*a) + 4*I*b^3))/b \end{aligned}$$

Fricas [C] time = 0.565249, size = 1439, normalized size = 11.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cot(b*x+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} &-1/4*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x + 4*b^3*c^3 - 3*d^3 \\ &*\text{polylog}(3, \cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a))*\sin(2*b*x + 2*a) - 3*d^3 \\ &*\text{polylog}(3, \cos(2*b*x + 2*a) - I*\sin(2*b*x + 2*a))*\sin(2*b*x + 2*a) - (-6*I \\ &*b*d^3*x - 6*I*b*c*d^2)*\text{dilog}(\cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a))*\sin(2* \\ &b*x + 2*a) - (6*I*b*d^3*x + 6*I*b*c*d^2)*\text{dilog}(\cos(2*b*x + 2*a) - I*\sin(2*b \\ &*x + 2*a))*\sin(2*b*x + 2*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*\log(-1/ \\ &2*\cos(2*b*x + 2*a) + 1/2*I*\sin(2*b*x + 2*a) + 1/2)*\sin(2*b*x + 2*a) - 6*(b^ \\ &2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*\log(-1/2*\cos(2*b*x + 2*a) - 1/2*I*\sin(2*b* \\ &x + 2*a) + 1/2)*\sin(2*b*x + 2*a) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c \\ &*d^2 - a^2*d^3)*\log(-\cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a) + 1)*\sin(2*b*x + \\ &2*a) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*\log(-\cos(2* \\ &b*x + 2*a) - I*\sin(2*b*x + 2*a) + 1)*\sin(2*b*x + 2*a) + 4*(b^3*d^3*x^3 + 3* \\ &b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\cos(2*b*x + 2*a) + (b^4*d^3*x^4 + \\ &4*b^4*c*d^2*x^3 + 6*b^4*c^2*d*x^2 + 4*b^4*c^3*x)*\sin(2*b*x + 2*a))/ (b^4*\sin \\ &(2*b*x + 2*a)) \end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \cot^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*cot(b*x+a)**2,x)

[Out] Integral((c + d*x)**3*cot(a + b*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \cot (bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cot(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^3*cot(b*x + a)^2, x)

3.107 $\int (c + dx)^2 \cot^2(a + bx) dx$

Optimal. Leaf size=97

$$-\frac{id^2 \text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^3} + \frac{2d(c+dx) \log\left(1 - e^{2i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2 \cot(a+bx)}{b} - \frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d}$$

[Out] $((-I)*(c + d*x)^2)/b - (c + d*x)^3/(3*d) - ((c + d*x)^2*\text{Cot}[a + b*x])/b + (2*d*(c + d*x)*\text{Log}[1 - E^{((2*I)*(a + b*x))}])/b^2 - (I*d^2*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^3$

Rubi [A] time = 0.129615, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3720, 3717, 2190, 2279, 2391, 32}

$$-\frac{id^2 \text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^3} + \frac{2d(c+dx) \log\left(1 - e^{2i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2 \cot(a+bx)}{b} - \frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2*Cot[a + b*x]^2,x]

[Out] $((-I)*(c + d*x)^2)/b - (c + d*x)^3/(3*d) - ((c + d*x)^2*\text{Cot}[a + b*x])/b + (2*d*(c + d*x)*\text{Log}[1 - E^{((2*I)*(a + b*x))}])/b^2 - (I*d^2*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^3$

Rule 3720

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 3717

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[(c + d*x)^m*E^(2*I*k*Pi)*E^(2*I*(e + f*x))]/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] := Simp[(c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 32

Int[((a_.) + (b_.)*(x_)^(m_)), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \cot^2(a + bx) dx &= -\frac{(c + dx)^2 \cot(a + bx)}{b} + \frac{(2d) \int (c + dx) \cot(a + bx) dx}{b} - \int (c + dx)^2 dx \\ &= -\frac{i(c + dx)^2}{b} - \frac{(c + dx)^3}{3d} - \frac{(c + dx)^2 \cot(a + bx)}{b} - \frac{(4id) \int \frac{e^{2i(a+bx)(c+dx)}}{1-e^{2i(a+bx)}} dx}{b} \\ &= -\frac{i(c + dx)^2}{b} - \frac{(c + dx)^3}{3d} - \frac{(c + dx)^2 \cot(a + bx)}{b} + \frac{2d(c + dx) \log(1 - e^{2i(a+bx)})}{b^2} - \frac{(2d^2)}{b^2} \\ &= -\frac{i(c + dx)^2}{b} - \frac{(c + dx)^3}{3d} - \frac{(c + dx)^2 \cot(a + bx)}{b} + \frac{2d(c + dx) \log(1 - e^{2i(a+bx)})}{b^2} + \frac{(id^2)}{b^2} \\ &= -\frac{i(c + dx)^2}{b} - \frac{(c + dx)^3}{3d} - \frac{(c + dx)^2 \cot(a + bx)}{b} + \frac{2d(c + dx) \log(1 - e^{2i(a+bx)})}{b^2} - \frac{id^2 \operatorname{Li}_2(e^{2i(a+bx)})}{b^2} \end{aligned}$$

Mathematica [B] time = 6.31777, size = 198, normalized size = 2.04

$$d^2 \left(-i \operatorname{PolyLog} \left(2, e^{2i(\tan^{-1}(\tan(a))+bx)} \right) - b^2 x^2 e^{i \tan^{-1}(\tan(a))} \cot(a) \sqrt{\sec^2(a)} + ibx (\pi - 2 \tan^{-1}(\tan(a))) + 2 (\tan^{-1}(\tan(a))) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^2*Cot[a + b*x]^2,x]

[Out] $-(x*(3*c^2 + 3*c*d*x + d^2*x^2))/3 - (2*c*d*(b*x*\cot[a] - \log[\sin[a + b*x]]))/b^2 + (d^2*(I*b*x*(\pi - 2*\operatorname{ArcTan}[\tan[a]]) + \pi*\log[1 + E^{((-2*I)*b*x)]} + 2*(b*x + \operatorname{ArcTan}[\tan[a]])*\log[1 - E^{((2*I)*(b*x + \operatorname{ArcTan}[\tan[a]])})}] - \pi*\log[\cos[b*x]] - 2*\operatorname{ArcTan}[\tan[a]]*\log[\sin[b*x + \operatorname{ArcTan}[\tan[a]]]] - I*\operatorname{PolyLog}[2, E^{((2*I)*(b*x + \operatorname{ArcTan}[\tan[a]])})}] - b^2*E^{(I*\operatorname{ArcTan}[\tan[a]])}*x^2*\cot[a]*\operatorname{Sqrt}[\sec[a]^2]))/b^3 + ((c + d*x)^2*\csc[a]*\csc[a + b*x]*\sin[b*x])/b$

Maple [B] time = 0.152, size = 297, normalized size = 3.1

$$-\frac{d^2 x^3}{3} - cdx^2 - c^2 x - \frac{2id^2 x^2}{b} + 2 \frac{cd \ln(e^{i(bx+a)} - 1)}{b^2} - 4 \frac{cd \ln(e^{i(bx+a)})}{b^2} + 2 \frac{cd \ln(e^{i(bx+a)} + 1)}{b^2} - \frac{2id^2 a^2}{b^3} - \frac{2i(d^2 x^2 + \dots)}{b(e^{2i(bx+a)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*cot(b*x+a)^2,x)

[Out] $-1/3*d^2*x^3 - c*d*x^2 - c^2*x - 2*I*d^2/b*x^2 + 2*d/b^2*c*\ln(\exp(I*(b*x+a)) - 1) - 4*d/b^2*c*\ln(\exp(I*(b*x+a))) + 2*d/b^2*c*\ln(\exp(I*(b*x+a)) + 1) - 2*I*d^2/b^3*a^2 - 2*I*(d^2*x^2 + 2*c*d*x + c^2)/b / (\exp(2*I*(b*x+a)) - 1) - 2*I*d^2*\operatorname{polylog}(2, \exp(I*(b*x+a))) / b^3 + 2*d^2/b^2*\ln(1 - \exp(I*(b*x+a)))*x + 2*d^2/b^3*\ln(1 - \exp(I*(b*x+a)))*a$

```
-2*I*d^2/b^3*polylog(2,-exp(I*(b*x+a)))+2*d^2/b^2*ln(exp(I*(b*x+a))+1)*x-4*
I*d^2/b^2*a*x-2*d^2/b^3*a*ln(exp(I*(b*x+a))-1)+4*d^2/b^3*a*ln(exp(I*(b*x+a)
))
```

Maxima [B] time = 2.2818, size = 872, normalized size = 8.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (-I*b^3*d^2*x^3 - 3*I*b^3*c*d*x^2 - 3*I*b^3*c^2*x - 6*b^2*c^2 - (6*b*d^2*x
+ 6*b*c*d - 6*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a) - (6*I*b*d^2*x + 6*I*b*c*d
)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (6*b*c*d*cos(
2*b*x + 2*a) + 6*I*b*c*d*sin(2*b*x + 2*a) - 6*b*c*d)*arctan2(sin(b*x + a),
cos(b*x + a) - 1) - (6*b*d^2*x*cos(2*b*x + 2*a) + 6*I*b*d^2*x*sin(2*b*x + 2
*a) - 6*b*d^2*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (I*b^3*d^2*x^3
+ (3*I*b^3*c*d - 6*b^2*d^2)*x^2 - 3*(-I*b^3*c^2 + 4*b^2*c*d)*x)*cos(2*b*x +
2*a) - 6*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(-e^(I
*b*x + I*a)) - 6*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilo
g(e^(I*b*x + I*a)) + (3*I*b*d^2*x + 3*I*b*c*d + (-3*I*b*d^2*x - 3*I*b*c*d)*
cos(2*b*x + 2*a) + 3*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2
+ sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (3*I*b*d^2*x + 3*I*b*c*d + (-3*I*
b*d^2*x - 3*I*b*c*d)*cos(2*b*x + 2*a) + 3*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a
))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (b^3*d^2*x^3
+ 3*(b^3*c*d + 2*I*b^2*d^2)*x^2 + (3*b^3*c^2 + 12*I*b^2*c*d)*x)*sin(2*b*x
+ 2*a))/(-3*I*b^3*cos(2*b*x + 2*a) + 3*b^3*sin(2*b*x + 2*a) + 3*I*b^3)
```

Fricas [B] time = 0.543463, size = 950, normalized size = 9.79

$$6b^2d^2x^2 + 12b^2cdx + 6b^2c^2 + 3id^2\text{Li}_2(\cos(2bx + 2a) + i\sin(2bx + 2a))\sin(2bx + 2a) - 3id^2\text{Li}_2(\cos(2bx + 2a) - i\sin(2bx + 2a))\sin(2bx + 2a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*b^2*d^2*x^2 + 12*b^2*c*d*x + 6*b^2*c^2 + 3*I*d^2*dilog(cos(2*b*x +
2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 3*I*d^2*dilog(cos(2*b*x + 2*a
) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 6*(b*c*d - a*d^2)*log(-1/2*cos(2
*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) - 6*(b*c*d - a
*d^2)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x +
2*a) - 6*(b*d^2*x + a*d^2)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1)
*sin(2*b*x + 2*a) - 6*(b*d^2*x + a*d^2)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x
+ 2*a) + 1)*sin(2*b*x + 2*a) + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos
(2*b*x + 2*a) + 2*(b^3*d^2*x^3 + 3*b^3*c*d*x^2 + 3*b^3*c^2*x)*sin(2*b*x + 2
*a))/(b^3*sin(2*b*x + 2*a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \cot^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*cot(b*x+a)**2,x)

[Out] Integral((c + d*x)**2*cot(a + b*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \cot (bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cot(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^2*cot(b*x + a)^2, x)

3.108 $\int (c + dx) \cot^2(a + bx) dx$

Optimal. Leaf size=41

$$\frac{d \log(\sin(a + bx))}{b^2} - \frac{(c + dx) \cot(a + bx)}{b} - cx - \frac{dx^2}{2}$$

[Out] $-(c*x) - (d*x^2)/2 - ((c + d*x)*\text{Cot}[a + b*x])/b + (d*\text{Log}[\text{Sin}[a + b*x]])/b^2$

Rubi [A] time = 0.0261395, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3720, 3475}

$$\frac{d \log(\sin(a + bx))}{b^2} - \frac{(c + dx) \cot(a + bx)}{b} - cx - \frac{dx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Cot[a + b*x]^2,x]

[Out] $-(c*x) - (d*x^2)/2 - ((c + d*x)*\text{Cot}[a + b*x])/b + (d*\text{Log}[\text{Sin}[a + b*x]])/b^2$

Rule 3720

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (c + dx) \cot^2(a + bx) dx &= -\frac{(c + dx) \cot(a + bx)}{b} + \frac{d \int \cot(a + bx) dx}{b} - \int (c + dx) dx \\ &= -cx - \frac{dx^2}{2} - \frac{(c + dx) \cot(a + bx)}{b} + \frac{d \log(\sin(a + bx))}{b^2} \end{aligned}$$

Mathematica [C] time = 0.450996, size = 82, normalized size = 2.

$$-\frac{c \cot(a + bx) \text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, -\tan^2(a + bx)\right)}{b} + \frac{d \log(\sin(a + bx))}{b^2} + \frac{dx \csc(a) \sin(bx) \csc(a + bx)}{b} - \frac{d}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Cot[a + b*x]^2,x]

[Out] $-\frac{((c*\text{Cot}[a + b*x]*\text{Hypergeometric2F1}[-1/2, 1, 1/2, -\text{Tan}[a + b*x]^2])}{b} + (d*\text{Log}[\text{Sin}[a + b*x]])/b^2 - (d*x*\text{Csc}[a]*(2*\text{Cos}[a] + b*x*\text{Sin}[a]))/(2*b) + (d*x$

*Csc[a]*Csc[a + b*x]*Sin[b*x])/b

Maple [A] time = 0.041, size = 49, normalized size = 1.2

$$-\frac{dx^2}{2} - cx - \frac{d \cot(bx+a)x}{b} + \frac{d \ln(\sin(bx+a))}{b^2} - \frac{c \cot(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*cot(b*x+a)^2,x)

[Out] -1/2*d*x^2-c*x-1/b*d*cot(b*x+a)*x+d*ln(sin(b*x+a))/b^2-1/b*c*cot(b*x+a)

Maxima [B] time = 1.68273, size = 394, normalized size = 9.61

$$2 \left(bx + a + \frac{1}{\tan(bx+a)} \right) c - \frac{2 \left(bx+a + \frac{1}{\tan(bx+a)} \right) ad}{b} + \frac{\left((bx+a)^2 \cos(2bx+2a)^2 + (bx+a)^2 \sin(2bx+2a)^2 - 2(bx+a)^2 \cos(2bx+2a) + (bx+a)^2 - (\cos(2bx+2a))^2 \right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cot(b*x+a)^2,x, algorithm="maxima")

[Out] -1/2*(2*(b*x + a + 1/tan(b*x + a))*c - 2*(b*x + a + 1/tan(b*x + a))*a*d/b + ((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b))/b

Fricas [B] time = 0.490468, size = 242, normalized size = 5.9

$$\frac{2 b d x - d \log \left(-\frac{1}{2} \cos (2 b x + 2 a) + \frac{1}{2} \right) \sin (2 b x + 2 a) + 2 b c + 2 (b d x + b c) \cos (2 b x + 2 a) + \left(b^2 d x^2 + 2 b^2 c x \right) \sin (2 b x + 2 a)}{2 b^2 \sin (2 b x + 2 a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cot(b*x+a)^2,x, algorithm="fricas")

[Out] -1/2*(2*b*d*x - d*log(-1/2*cos(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) + 2*b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d*x^2 + 2*b^2*c*x)*sin(2*b*x + 2*a))/(b^2*sin(2*b*x + 2*a))

Sympy [A] time = 1.29077, size = 100, normalized size = 2.44

$$\begin{cases} \infty \left(cx + \frac{dx^2}{2} \right) & \text{for } (a = 0 \vee a = -bx) \wedge (a = -bx \vee b = 0) \\ \left(cx + \frac{dx^2}{2} \right) \cot^2(a) & \text{for } b = 0 \\ -cx - \frac{dx^2}{2} - \frac{c}{b \tan(a+bx)} - \frac{dx}{b \tan(a+bx)} - \frac{d \log(\tan^2(a+bx)+1)}{2b^2} + \frac{d \log(\tan(a+bx))}{b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*(c*x + d*x**2/2), (Eq(a, 0) | Eq(a, -b*x)) & (Eq(b, 0) | Eq(a, -b*x))), ((c*x + d*x**2/2)*cot(a)**2, Eq(b, 0)), (-c*x - d*x**2/2 - c/(b*tan(a + b*x)) - d*x/(b*tan(a + b*x)) - d*log(tan(a + b*x)**2 + 1)/(2*b**2) + d*log(tan(a + b*x))/b**2, True))
```

Giac [B] time = 1.60698, size = 1856, normalized size = 45.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*d*x^2*tan(1/2*b*x)^2*tan(1/2*a) + b^2*d*x^2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*b^2*c*x*tan(1/2*b*x)^2*tan(1/2*a) + 2*b^2*c*x*tan(1/2*b*x)*tan(1/2*a)^2 - b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - b^2*d*x^2*tan(1/2*b*x) - b^2*d*x^2*tan(1/2*a) - b*c*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^2*c*x*tan(1/2*b*x) + b*d*x*tan(1/2*b*x)^2 - 2*b^2*c*x*tan(1/2*a) + 4*b*d*x*tan(1/2*b*x)*tan(1/2*a) - d*log(16*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2))*tan(1/2*b*x)^2*tan(1/2*a) + b*d*x*tan(1/2*a)^2 - d*log(16*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2))*tan(1/2*b*x)*tan(1/2*a)^2 + b*c*tan(1/2*b*x)^2 + 4*b*c*tan(1/2*b*x)*tan(1/2*a) + b*c*tan(1/2*a)^2 - b*d*x + d*log(16*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2))*tan(1/2*a) - b*c)/(b^2*tan(1/2*b*x)^2*tan(1/2*a) + b^2*tan(1/2*b*x)*
```

$$\tan(1/2*a)^2 - b^2*\tan(1/2*b*x) - b^2*\tan(1/2*a))$$

$$3.109 \quad \int \frac{\cot^2(a+bx)}{c+dx} dx$$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\cot^2(a+bx)}{c+dx}, x\right)$$

[Out] Unintegrable[Cot[a + b*x]^2/(c + d*x), x]

Rubi [A] time = 0.0355474, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cot^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[Cot[a + b*x]^2/(c + d*x), x]

[Out] Defer[Int][Cot[a + b*x]^2/(c + d*x), x]

Rubi steps

$$\int \frac{\cot^2(a+bx)}{c+dx} dx = \int \frac{\cot^2(a+bx)}{c+dx} dx$$

Mathematica [A] time = 4.64655, size = 0, normalized size = 0.

$$\int \frac{\cot^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[Cot[a + b*x]^2/(c + d*x), x]

[Out] Integrate[Cot[a + b*x]^2/(c + d*x), x]

Maple [A] time = 0.332, size = 0, normalized size = 0.

$$\int \frac{(\cot(bx+a))^2}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(b*x+a)^2/(d*x+c), x)

[Out] int(cot(b*x+a)^2/(d*x+c), x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)^2/(d*x+c),x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cot^2(bx+a)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)^2/(d*x+c),x, algorithm="fricas")

[Out] integral(cot(b*x + a)^2/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(a+bx)}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)**2/(d*x+c),x)

[Out] Integral(cot(a + b*x)**2/(c + d*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(bx+a)}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)^2/(d*x+c),x, algorithm="giac")

[Out] integrate(cot(b*x + a)^2/(d*x + c), x)

$$3.110 \quad \int \frac{\cot^2(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\cot^2(a+bx)}{(c+dx)^2}, x\right)$$

[Out] Unintegrable[Cot[a + b*x]^2/(c + d*x)^2, x]

Rubi [A] time = 0.0344833, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[Cot[a + b*x]^2/(c + d*x)^2,x]

[Out] Defer[Int][Cot[a + b*x]^2/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx = \int \frac{\cot^2(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 2.41796, size = 0, normalized size = 0.

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Cot[a + b*x]^2/(c + d*x)^2,x]

[Out] Integrate[Cot[a + b*x]^2/(c + d*x)^2, x]

Maple [A] time = 0.447, size = 0, normalized size = 0.

$$\int \frac{(\cot(bx+a))^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(b*x+a)^2/(d*x+c)^2,x)

[Out] int(cot(b*x+a)^2/(d*x+c)^2,x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cot (bx+a)^2}{d^2x^2+2cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(cot(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)**2/(d*x+c)**2,x)

[Out] Integral(cot(a + b*x)**2/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot (bx+a)^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(cot(b*x + a)^2/(d*x + c)^2, x)

3.111 $\int (c + dx)^m \cot^2(a + bx) \csc(a + bx) dx$

Optimal. Leaf size=37

$$\text{Unintegrable}(\csc^3(a + bx)(c + dx)^m, x) - \text{Unintegrable}(\csc(a + bx)(c + dx)^m, x)$$

[Out] -Unintegrable[(c + d*x)^m*Csc[a + b*x], x] + Unintegrable[(c + d*x)^m*Csc[a + b*x]^3, x]

Rubi [A] time = 0.0752131, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \cot^2(a + bx) \csc(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Cot[a + b*x]^2*Csc[a + b*x], x]

[Out] -Defer[Int][(c + d*x)^m*Csc[a + b*x], x] + Defer[Int][(c + d*x)^m*Csc[a + b*x]^3, x]

Rubi steps

$$\int (c + dx)^m \cot^2(a + bx) \csc(a + bx) dx = - \int (c + dx)^m \csc(a + bx) dx + \int (c + dx)^m \csc^3(a + bx) dx$$

Mathematica [A] time = 10.2968, size = 0, normalized size = 0.

$$\int (c + dx)^m \cot^2(a + bx) \csc(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Cot[a + b*x]^2*Csc[a + b*x], x]

[Out] Integrate[(c + d*x)^m*Cot[a + b*x]^2*Csc[a + b*x], x]

Maple [A] time = 0.18, size = 0, normalized size = 0.

$$\int (dx + c)^m (\cot(bx + a))^2 \csc(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a), x)

[Out] int((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cot (bx + a)^2 \csc (bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")

[Out] integrate((d*x + c)^m*cot(b*x + a)^2*csc(b*x + a), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left((dx + c)^m \cot (bx + a)^2 \csc (bx + a), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")

[Out] integral((d*x + c)^m*cot(b*x + a)^2*csc(b*x + a), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^m \cot^2 (a + bx) \csc (a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**m*cot(b*x+a)**2*csc(b*x+a),x)

[Out] Integral((c + d*x)**m*cot(a + b*x)**2*csc(a + b*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cot (bx + a)^2 \csc (bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^m*cot(b*x + a)^2*csc(b*x + a), x)

3.112 $\int (c + dx)^4 \cot^2(a + bx) \csc(a + bx) dx$

Optimal. Leaf size=416

$$\frac{12id^3(c + dx)\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^4} - \frac{12id^3(c + dx)\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^4} + \frac{12id^3(c + dx)\text{PolyLog}\left(4, -e^{i(a+bx)}\right)}{b^4} - \frac{12id^3(c + dx)\text{PolyLog}\left(4, e^{i(a+bx)}\right)}{b^4}$$

[Out] $(-12*d^2*(c + d*x)^2*\text{ArcTanh}[E^{(I*(a + b*x))}])/b^3 + ((c + d*x)^4*\text{ArcTanh}[E^{(I*(a + b*x))}])/b - (2*d*(c + d*x)^3*\text{Csc}[a + b*x])/b^2 - ((c + d*x)^4*\text{Cot}[a + b*x]*\text{Csc}[a + b*x])/(2*b) + ((12*I)*d^3*(c + d*x)*\text{PolyLog}[2, -E^{(I*(a + b*x))}])/b^4 - ((2*I)*d*(c + d*x)^3*\text{PolyLog}[2, -E^{(I*(a + b*x))}])/b^2 - ((12*I)*d^3*(c + d*x)*\text{PolyLog}[2, E^{(I*(a + b*x))}])/b^4 + ((2*I)*d*(c + d*x)^3*\text{PolyLog}[2, E^{(I*(a + b*x))}])/b^2 - (12*d^4*\text{PolyLog}[3, -E^{(I*(a + b*x))}])/b^5 + (6*d^2*(c + d*x)^2*\text{PolyLog}[3, -E^{(I*(a + b*x))}])/b^3 + (12*d^4*\text{PolyLog}[3, E^{(I*(a + b*x))}])/b^5 - (6*d^2*(c + d*x)^2*\text{PolyLog}[3, E^{(I*(a + b*x))}])/b^3 + ((12*I)*d^3*(c + d*x)*\text{PolyLog}[4, -E^{(I*(a + b*x))}])/b^4 - ((12*I)*d^3*(c + d*x)*\text{PolyLog}[4, E^{(I*(a + b*x))}])/b^4 - (12*d^4*\text{PolyLog}[5, -E^{(I*(a + b*x))}])/b^5 + (12*d^4*\text{PolyLog}[5, E^{(I*(a + b*x))}])/b^5$

Rubi [A] time = 0.501959, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {4415, 4183, 2531, 6609, 2282, 6589, 4186}

$$\frac{12id^3(c + dx)\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^4} - \frac{12id^3(c + dx)\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^4} + \frac{12id^3(c + dx)\text{PolyLog}\left(4, -e^{i(a+bx)}\right)}{b^4} - \frac{12id^3(c + dx)\text{PolyLog}\left(4, e^{i(a+bx)}\right)}{b^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^4*\text{Cot}[a + b*x]^2*\text{Csc}[a + b*x], x]$

[Out] $(-12*d^2*(c + d*x)^2*\text{ArcTanh}[E^{(I*(a + b*x))}])/b^3 + ((c + d*x)^4*\text{ArcTanh}[E^{(I*(a + b*x))}])/b - (2*d*(c + d*x)^3*\text{Csc}[a + b*x])/b^2 - ((c + d*x)^4*\text{Cot}[a + b*x]*\text{Csc}[a + b*x])/(2*b) + ((12*I)*d^3*(c + d*x)*\text{PolyLog}[2, -E^{(I*(a + b*x))}])/b^4 - ((2*I)*d*(c + d*x)^3*\text{PolyLog}[2, -E^{(I*(a + b*x))}])/b^2 - ((12*I)*d^3*(c + d*x)*\text{PolyLog}[2, E^{(I*(a + b*x))}])/b^4 + ((2*I)*d*(c + d*x)^3*\text{PolyLog}[2, E^{(I*(a + b*x))}])/b^2 - (12*d^4*\text{PolyLog}[3, -E^{(I*(a + b*x))}])/b^5 + (6*d^2*(c + d*x)^2*\text{PolyLog}[3, -E^{(I*(a + b*x))}])/b^3 + (12*d^4*\text{PolyLog}[3, E^{(I*(a + b*x))}])/b^5 - (6*d^2*(c + d*x)^2*\text{PolyLog}[3, E^{(I*(a + b*x))}])/b^3 + ((12*I)*d^3*(c + d*x)*\text{PolyLog}[4, -E^{(I*(a + b*x))}])/b^4 - ((12*I)*d^3*(c + d*x)*\text{PolyLog}[4, E^{(I*(a + b*x))}])/b^4 - (12*d^4*\text{PolyLog}[5, -E^{(I*(a + b*x))}])/b^5 + (12*d^4*\text{PolyLog}[5, E^{(I*(a + b*x))}])/b^5$

Rule 4415

$\text{Int}[\text{Cot}[(a_.) + (b_.)*(x_.)]^{(p_.)}*\text{Csc}[(a_.) + (b_.)*(x_.)]^{(m_.)}, x_Symbol] \rightarrow -\text{Int}[(c + d*x)^m*\text{Csc}[a + b*x]*\text{Cot}[a + b*x]^{(p - 2)}, x] + \text{Int}[(c + d*x)^m*\text{Csc}[a + b*x]^3*\text{Cot}[a + b*x]^{(p - 2)}, x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[p/2, 0]$

Rule 4183

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{ArcTanh}[E^{(I*(e + f*x))}]/f, x] + (-\text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 - E^{(I*(e + f*x))}], x], x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + E^{(I*(e + f*x))}], x], x]) /; \text{FreeQ}\{c, d, e, f\}, x] \ \&\& \ \text{IGtQ}[m, 0]$

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_)^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)^(m_)), x_Symbo
l] := -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^4 \cot^2(a + bx) \csc(a + bx) dx &= - \int (c + dx)^4 \csc(a + bx) dx + \int (c + dx)^4 \csc^3(a + bx) dx \\
&= \frac{2(c + dx)^4 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{2d(c + dx)^3 \csc(a + bx)}{b^2} - \frac{(c + dx)^4 \cot(a + bx)}{2b} \\
&= -\frac{12d^2(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^3} + \frac{(c + dx)^4 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{2d(c + dx)^3 \csc(a + bx)}{b^2} \\
&= -\frac{12d^2(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^3} + \frac{(c + dx)^4 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{2d(c + dx)^3 \csc(a + bx)}{b^2} \\
&= -\frac{12d^2(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^3} + \frac{(c + dx)^4 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{2d(c + dx)^3 \csc(a + bx)}{b^2} \\
&= -\frac{12d^2(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^3} + \frac{(c + dx)^4 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{2d(c + dx)^3 \csc(a + bx)}{b^2} \\
&= -\frac{12d^2(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^3} + \frac{(c + dx)^4 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{2d(c + dx)^3 \csc(a + bx)}{b^2} \\
&= -\frac{12d^2(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^3} + \frac{(c + dx)^4 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{2d(c + dx)^3 \csc(a + bx)}{b^2}
\end{aligned}$$

Mathematica [B] time = 8.43926, size = 966, normalized size = 2.32

$$-\frac{c^4 \log(1 - e^{i(a+bx)}) b^4 - d^4 x^4 \log(1 - e^{i(a+bx)}) b^4 - 4cd^3 x^3 \log(1 - e^{i(a+bx)}) b^4 - 6c^2 d^2 x^2 \log(1 - e^{i(a+bx)}) b^4 - 4c^3 dx \log(1 - e^{i(a+bx)}) b^4}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^4*Cot[a + b*x]^2*Csc[a + b*x], x]

[Out] $(-(b^4 c^4 \text{Log}[1 - E^{i(a+bx)}]) + 12 b^2 c^2 d^2 \text{Log}[1 - E^{i(a+bx)}]) - 4 b^4 c^3 d x \text{Log}[1 - E^{i(a+bx)}] + 24 b^2 c d^3 x^2 \text{Log}[1 - E^{i(a+bx)}] - 6 b^4 c^2 d^2 x^2 \text{Log}[1 - E^{i(a+bx)}] + 12 b^2 d^4 x^2 \text{Log}[1 - E^{i(a+bx)}] - 4 b^4 c d^3 x^3 \text{Log}[1 - E^{i(a+bx)}] - b^4 d^4 x^4 \text{Log}[1 - E^{i(a+bx)}] + b^4 c^4 \text{Log}[1 + E^{i(a+bx)}] - 12 b^2 c^2 d^2 \text{Log}[1 + E^{i(a+bx)}] + 4 b^4 c^3 d x \text{Log}[1 + E^{i(a+bx)}] - 24 b^2 c d^3 x^2 \text{Log}[1 + E^{i(a+bx)}] + 6 b^4 c^2 d^2 x^2 \text{Log}[1 + E^{i(a+bx)}] - 12 b^2 d^4 x^2 \text{Log}[1 + E^{i(a+bx)}] + 4 b^4 c d^3 x^3 \text{Log}[1 + E^{i(a+bx)}] + b^4 d^4 x^4 \text{Log}[1 + E^{i(a+bx)}] - (4 I) b d (c + d x) (-6 d^2 + b^2 (c + d x)^2) \text{PolyLog}[2, -E^{i(a+bx)}] + (4 I) b d (c + d x) (-6 d^2 + b^2 (c + d x)^2) \text{PolyLog}[2, E^{i(a+bx)}] + 12 b^2 c^2 d^2 \text{PolyLog}[3, -E^{i(a+bx)}] - 24 d^4 \text{PolyLog}[3, -E^{i(a+bx)}] + 24 b^2 c d^3 x \text{PolyLog}[3, -E^{i(a+bx)}] + 12 b^2 d^4 x^2 \text{PolyLog}[3, -E^{i(a+bx)}] - 12 b^2 c^2 d^2 \text{PolyLog}[3, E^{i(a+bx)}] + 24 d^4 \text{PolyLog}[3, E^{i(a+bx)}] - 24 b^2 c d^3 x \text{PolyLog}[3, E^{i(a+bx)}] - 12 b^2 d^4 x^2 \text{PolyLog}[3, E^{i(a+bx)}] + (24 I) b c d^3 \text{PolyLog}[4, -E^{i(a+bx)}] + (24 I) b d^4 x \text{PolyLog}[4, -E^{i(a+bx)}] - (24 I) b c d^3 \text{PolyLog}[4, E^{i(a+bx)}] - (24 I) b d^4 x \text{PolyLog}[4, E^{i(a+bx)}] - 24 d^4 \text{PolyLog}[5, -E^{i(a+bx)}] + 24 d^4 \text{PolyLog}[5, E^{i(a+bx)}]) / (2 b^5) - (\text{Csc}[a + b x]^2 (b c^4 \text{Cos}[a + b x] + 4 b c^3 d x \text{Cos}[a + b x] + 6 b c^2 d^2 x^2 \text{Cos}[a + b x] + 4 b c d^3 x^3 \text{Cos}[a + b x] + b d^4 x^4 \text{Cos}[a + b x] + 4 c^3 d \text{Sin}[a + b x] + 12 c^2 d^2 x \text{Sin}[a + b x] + 12 c d^3 x^2 \text{Sin}[a + b x] + 4 d^4 x^3 \text{Sin}[a + b x])) / (2 b^2)$

Maple [B] time = 0.381, size = 1673, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d*x+c)^4*\cot(b*x+a)^2*\csc(b*x+a), x)$

[Out]
$$\begin{aligned} & -1/2/b*d^4*\ln(1-\exp(I*(b*x+a)))*x^4+1/2/b^5*d^4*\ln(1-\exp(I*(b*x+a)))*a^4-4/ \\ & b^4*c*d^3*a^3*\operatorname{arctanh}(\exp(I*(b*x+a)))+6/b^3*c^2*d^2*a^2*\operatorname{arctanh}(\exp(I*(b*x+ \\ & a)))-4/b^2*c^3*d*a*\operatorname{arctanh}(\exp(I*(b*x+a)))+24/b^4*c*d^3*a*\operatorname{arctanh}(\exp(I*(b* \\ & x+a)))+2*I/b^2*c^3*d*\operatorname{polylog}(2, \exp(I*(b*x+a)))+12*I/b^4*c*d^3*\operatorname{polylog}(2, -\exp \\ & (I*(b*x+a)))-2*I/b^2*d^4*\operatorname{polylog}(2, -\exp(I*(b*x+a)))*x^3+12*I/b^4*d^4*\operatorname{polyl} \\ & \operatorname{og}(4, -\exp(I*(b*x+a)))*x-2/b*c*d^3*\ln(1-\exp(I*(b*x+a)))*x^3+6*d^4/b^3*\ln(1-\exp(I*(b*x+a)))*x^2-6*d^4/b^5*\ln(1-\exp(I*(b*x+a)))*a^2-6*d^4/b^3*\ln(\exp(I*(b*x+a))+1)*x^2+2/b^4*c*d^3*\ln(\exp(I*(b*x+a))+1)*a^3-2/b^4*c*d^3*\ln(1-\exp(I*(b*x+a)))*a^3-12*d^4*\operatorname{polylog}(3, -\exp(I*(b*x+a)))/b^5+12*d^4*\operatorname{polylog}(3, \exp(I*(b*x+a)))/b^5+2/b*c*d^3*\ln(\exp(I*(b*x+a))+1)*x^3-6/b^3*d^4*\operatorname{polylog}(3, \exp(I*(b*x+a)))*x^2-1/2/b^5*d^4*a^4*\ln(\exp(I*(b*x+a))+1)-6/b^3*c^2*d^2*\operatorname{polylog}(3, \exp(I*(b*x+a)))+6/b^3*c^2*d^2*\operatorname{polylog}(3, -\exp(I*(b*x+a)))+1/b^5*d^4*a^4*\operatorname{arctanh}(\exp(I*(b*x+a)))+6/b^3*d^4*\operatorname{polylog}(3, -\exp(I*(b*x+a)))*x^2-3/b*c^2*d^2*\ln(1-\exp(I*(b*x+a)))*x^2+1/2/b*d^4*\ln(\exp(I*(b*x+a))+1)*x^4+3/b^3*c^2*d^2*a^2*\ln(1-\exp(I*(b*x+a)))-2/b*c^3*d*\ln(1-\exp(I*(b*x+a)))*x-2/b^2*c^3*d*\ln(1-\exp(I*(b*x+a)))*a+2/b*c^3*d*\ln(\exp(I*(b*x+a))+1)*x+12/b^3*c*d^3*\operatorname{polylog}(3, -\exp(I*(b*x+a)))*x-3/b^3*c^2*d^2*a^2*\ln(\exp(I*(b*x+a))+1)+3/b*c^2*d^2*\ln(\exp(I*(b*x+a))+1)*x^2-12/b^3*c*d^3*\operatorname{polylog}(3, \exp(I*(b*x+a)))*x+2/b^2*c^3*d*\ln(\exp(I*(b*x+a))+1)*a+12*d^3/b^3*c*\ln(1-\exp(I*(b*x+a)))*x+12*d^3/b^4*c*\ln(1-\exp(I*(b*x+a)))*a-12*d^3/b^3*c*\ln(\exp(I*(b*x+a))+1)*x-12*I*d^4/b^4*\operatorname{polylog}(2, \exp(I*(b*x+a)))*x-12*I*d^3/b^4*c*\operatorname{polylog}(2, \exp(I*(b*x+a)))+1/b^2/(\exp(2*I*(b*x+a))-1)^2*(d^4*x^4*b*\exp(3*I*(b*x+a))+4*c*d^3*x^3*b*\exp(3*I*(b*x+a))+6*c^2*d^2*x^2*b*\exp(3*I*(b*x+a))+d^4*x^4*b*\exp(I*(b*x+a))+4*c^3*d*x*b*\exp(3*I*(b*x+a))+4*c*d^3*x^3*b*\exp(I*(b*x+a))+12*I*c*d^3*x^2*\exp(I*(b*x+a))+c^4*b*\exp(3*I*(b*x+a))+6*c^2*d^2*x^2*b*\exp(I*(b*x+a))+4*I*c^3*d*\exp(I*(b*x+a))+4*c^3*d*x*b*\exp(I*(b*x+a))-12*I*c*d^3*x^2*\exp(3*I*(b*x+a))-4*I*c^3*d*\exp(3*I*(b*x+a))+c^4*b*\exp(I*(b*x+a))+4*I*d^4*x^3*\exp(I*(b*x+a))-4*I*d^4*x^3*\exp(3*I*(b*x+a))-12*I*c^2*d^2*x*\exp(3*I*(b*x+a))+12*I*c^2*d^2*x*\exp(I*(b*x+a))+6/b^5*d^4*\ln(\exp(I*(b*x+a))+1)*a^2-12/b^5*d^4*a^2*\operatorname{arctanh}(\exp(I*(b*x+a)))-12/b^3*c^2*d^2*\operatorname{arctanh}(\exp(I*(b*x+a)))-12*d^4*\operatorname{polylog}(5, -\exp(I*(b*x+a)))/b^5+12*d^4*\operatorname{polylog}(5, \exp(I*(b*x+a)))/b^5+1/b*c^4*\operatorname{arctanh}(\exp(I*(b*x+a)))+6*I/b^2*\operatorname{polylog}(2, \exp(I*(b*x+a)))*c^2*d^2*x-6*I/b^2*\operatorname{polylog}(2, -\exp(I*(b*x+a)))*c^2*d^2*x+6*I/b^2*c*d^3*\operatorname{polylog}(2, \exp(I*(b*x+a)))*x^2-12/b^4*c*d^3*\ln(\exp(I*(b*x+a))+1)*a-12*I/b^4*d^4*\operatorname{polylog}(4, \exp(I*(b*x+a)))*x+2*I/b^2*d^4*\operatorname{polylog}(2, \exp(I*(b*x+a)))*x^3+12*I/b^4*d^4*\operatorname{polylog}(2, -\exp(I*(b*x+a)))*x+12*I/b^4*c*d^3*\operatorname{polylog}(4, -\exp(I*(b*x+a)))-2*I/b^2*c^3*d*\operatorname{polylog}(2, -\exp(I*(b*x+a)))-12*I/b^4*c*d^3*\operatorname{polylog}(4, \exp(I*(b*x+a)))-6*I/b^2*c*d^3*\operatorname{polylog}(2, -\exp(I*(b*x+a)))*x^2 \end{aligned}$$

Maxima [B] time = 18.8847, size = 9385, normalized size = 22.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((d*x+c)^4*\cot(b*x+a)^2*\csc(b*x+a), x, \text{algorithm}=\text{"maxima"})$

[Out]
$$\begin{aligned} & 1/4*(c^4*(2*\cos(b*x + a)/(\cos(b*x + a)^2 - 1) + \log(\cos(b*x + a) + 1) - \log \\ & (\cos(b*x + a) - 1)) - 4*a*c^3*d*(2*\cos(b*x + a)/(\cos(b*x + a)^2 - 1) + \log(\cos(b*x + a) + 1) - \log(\cos(b*x + a) - 1))/b + 6*a^2*c^2*d^2*(2*\cos(b*x + a) \\ &)/(\cos(b*x + a)^2 - 1) + \log(\cos(b*x + a) + 1) - \log(\cos(b*x + a) - 1))/b^2 \\ & - 4*a^3*c*d^3*(2*\cos(b*x + a)/(\cos(b*x + a)^2 - 1) + \log(\cos(b*x + a) + 1) \\ & - \log(\cos(b*x + a) - 1))/b^3 + a^4*d^4*(2*\cos(b*x + a)/(\cos(b*x + a)^2 - 1 \end{aligned}$$

$$\begin{aligned}
&) + \log(\cos(b*x + a) + 1) - \log(\cos(b*x + a) - 1))/b^4 + 4*((2*(b*x + a)^4*d^4 - 24*b^2*c^2*d^2 + 48*a*b*c*d^3 - 24*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a) + 2*((b*x + a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*\cos(4*b*x + 4*a) - 4*((b*x + a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*\cos(2*b*x + 2*a) + (2*I*(b*x + a)^4*d^4 - 24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*a^2*d^4 + (8*I*b*c*d^3 - 8*I*a*d^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + (12*I*a^2 - 24*I)*d^4)*(b*x + a)^2 + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2 + (24*I*a^2 - 48*I)*b*c*d^3 + (-8*I*a^3 + 48*I*a)*d^4)*(b*x + a))*\sin(4*b*x + 4*a) + (-4*I*(b*x + a)^4*d^4 + 48*I*b^2*c^2*d^2 - 96*I*a*b*c*d^3 + 48*I*a^2*d^4 + (-16*I*b*c*d^3 + 16*I*a*d^4)*(b*x + a)^3 + (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 + (-24*I*a^2 + 48*I)*d^4)*(b*x + a)^2 + (-16*I*b^3*c^3*d + 48*I*a*b^2*c^2*d^2 + (-48*I*a^2 + 96*I)*b*c*d^3 + (16*I*a^3 - 96*I*a)*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) + (24*b^2*c^2*d^2 - 48*a*b*c*d^3 + 24*a^2*d^4 + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*\cos(4*b*x + 4*a) - 48*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*\cos(2*b*x + 2*a) + (24*I*b^2*c^2*d^2 - 48*I*a*b*c*d^3 + 24*I*a^2*d^4)*\sin(4*b*x + 4*a) + (-48*I*b^2*c^2*d^2 + 96*I*a*b*c*d^3 - 48*I*a^2*d^4)*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) - 1) + (2*(b*x + a)^4*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a) + 2*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*\cos(4*b*x + 4*a) - 4*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*\cos(2*b*x + 2*a) + (2*I*(b*x + a)^4*d^4 + (8*I*b*c*d^3 - 8*I*a*d^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + (12*I*a^2 - 24*I)*d^4)*(b*x + a)^2 + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2 + (24*I*a^2 - 48*I)*b*c*d^3 + (-8*I*a^3 + 48*I*a)*d^4)*(b*x + a))*\sin(4*b*x + 4*a) + (-4*I*(b*x + a)^4*d^4 + (-16*I*b*c*d^3 + 16*I*a*d^4)*(b*x + a)^3 + (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 + (-24*I*a^2 + 48*I)*d^4)*(b*x + a)^2 + (-16*I*b^3*c^3*d + 48*I*a*b^2*c^2*d^2 + (-48*I*a^2 + 96*I)*b*c*d^3 + (16*I*a^3 - 96*I*a)*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) + (-4*I*(b*x + a)^4*d^4 - 16*b^3*c^3*d + 48*a*b^2*c^2*d^2 - 48*a^2*b*c*d^3 + 16*a^3*d^4 + (-16*I*b*c*d^3 - 16*(-I*a + 1)*d^4)*(b*x + a)^3 + (-24*I*b^2*c^2*d^2 - 48*(-I*a + 1)*b*c*d^3 + (-24*I*a^2 + 48*a)*d^4)*(b*x + a)^2 + (-16*I*b^3*c^3*d - 48*(-I*a + 1)*b^2*c^2*d^2 + (-48*I*a^2 + 96*a)*b*c*d^3 + (16*I*a^3 - 48*a^2)*d^4)*(b*x + a))*\cos(3*b*x + 3*a) + (-4*I*(b*x + a)^4*d^4 + 16*b^3*c^3*d - 48*a*b^2*c^2*d^2 + 48*a^2*b*c*d^3 - 16*a^3*d^4 + (-16*I*b*c*d^3 - 16*(-I*a - 1)*d^4)*(b*x + a)^3 + (-24*I*b^2*c^2*d^2 - 48*(-I*a - 1)*b*c*d^3 + (-24*I*a^2 - 48*a)*d^4)*(b*x + a)^2 + (-16*I*b^3*c^3*d - 48*(-I*a - 1)*b^2*c^2*d^2 + (-48*I*a^2 - 96*a)*b*c*d^3 + (16*I*a^3 + 48*a^2)*d^4)*(b*x + a))*\cos(b*x + a) - (8*b^3*c^3*d - 24*a*b^2*c^2*d^2 + 8*(b*x + a)^3*d^4 + 24*(a^2 - 2)*b*c*d^3 - 8*(a^3 - 6*a)*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a) + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a))*\cos(4*b*x + 4*a) - 16*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a))*\cos(2*b*x + 2*a) - (-8*I*b^3*c^3*d + 24*I*a*b^2*c^2*d^2 - 8*I*(b*x + a)^3*d^4 + (-24*I*a^2 + 48*I)*b*c*d
\end{aligned}$$

$$\begin{aligned}
&^3 + (8Ia^3 - 48Ia)d^4 + (-24Ib^3c^3d + 24Ia^2d^4)(bx + a)^2 + (- \\
&24Ib^2c^2d^2 + 48Ia^2b^3c^3d + (-24Ia^2 + 48I)d^4)(bx + a))\sin(\\
&4bx + 4a) - (16Ib^3c^3d - 48Ia^2b^2c^2d^2 + 16I(bx + a)^3d^4 \\
&+ (48Ia^2 - 96I)b^3c^3d + (-16Ia^3 + 96Ia)d^4 + (48Ib^3c^3d - 48 \\
&Ia^2d^4)(bx + a)^2 + (48Ib^2c^2d^2 - 96Ia^2b^3c^3d + (48Ia^2 - 96 \\
&I)d^4)(bx + a))\sin(2bx + 2a))\operatorname{dilog}(-e^{(Ibx + Ia)}) + (8b^3c^3 \\
&d - 24a^2b^2c^2d^2 + 8(bx + a)^3d^4 + 24(a^2 - 2)b^3c^3d - 8(a^3 - \\
&6a)d^4 + 24(b^3c^3d - a^2d^4)(bx + a)^2 + 24(b^2c^2d^2 - 2a^2b^3c^3 \\
&+ (a^2 - 2)d^4)(bx + a) + 8(b^3c^3d - 3a^2b^2c^2d^2 + (bx + a)^3d^4 \\
&+ 3(a^2 - 2)b^3c^3d - (a^3 - 6a)d^4 + 3(b^3c^3d - a^2d^4)(bx + a) \\
&^2 + 3(b^2c^2d^2 - 2a^2b^3c^3d + (a^2 - 2)d^4)(bx + a))\cos(4bx + 4 \\
&a) - 16(b^3c^3d - 3a^2b^2c^2d^2 + (bx + a)^3d^4 + 3(a^2 - 2)b^3c^3 \\
&d - (a^3 - 6a)d^4 + 3(b^3c^3d - a^2d^4)(bx + a)^2 + 3(b^2c^2d^2 - 2 \\
&a^2b^3c^3d + (a^2 - 2)d^4)(bx + a))\cos(2bx + 2a) + (8Ib^3c^3d - \\
&24Ia^2b^2c^2d^2 + 8I(bx + a)^3d^4 + (24Ia^2 - 48I)b^3c^3d + (-8I \\
&a^3 + 48Ia)d^4 + (24Ib^3c^3d - 24Ia^2d^4)(bx + a)^2 + (24Ib^2c^2 \\
&d^2 - 48Ia^2b^3c^3d + (24Ia^2 - 48I)d^4)(bx + a))\sin(4bx + 4a) \\
&+ (-16Ib^3c^3d + 48Ia^2b^2c^2d^2 - 16I(bx + a)^3d^4 + (-48Ia^2 \\
&+ 96I)b^3c^3d + (16Ia^3 - 96Ia)d^4 + (-48Ib^3c^3d + 48Ia^2d^4) \\
&)(bx + a)^2 + (-48Ib^2c^2d^2 + 96Ia^2b^3c^3d + (-48Ia^2 + 96I)d^4 \\
&)(bx + a))\sin(2bx + 2a))\operatorname{dilog}(e^{(Ibx + Ia)}) + (-I(bx + a)^4d^4 \\
&+ 12Ib^2c^2d^2 - 24Ia^2b^3c^3d + 12Ia^2d^4 + (-4Ib^3c^3d + 4Ia^2 \\
&d^4)(bx + a)^3 + (-6Ib^2c^2d^2 + 12Ia^2b^3c^3d + (-6Ia^2 + 12I) \\
&d^4)(bx + a)^2 + (-4Ib^3c^3d + 12Ia^2b^2c^2d^2 + (-12Ia^2 + 24I) \\
&)(bx + a)^3 + (4Ia^3 - 24Ia)d^4)(bx + a) + (-I(bx + a)^4d^4 + 12I \\
&b^2c^2d^2 - 24Ia^2b^3c^3d + 12Ia^2d^4 + (-4Ib^3c^3d + 4Ia^2d^4)(b \\
&>x + a)^3 + (-6Ib^2c^2d^2 + 12Ia^2b^3c^3d + (-6Ia^2 + 12I)d^4)(b \\
&>x + a)^2 + (-4Ib^3c^3d + 12Ia^2b^2c^2d^2 + (-12Ia^2 + 24I)b^3c^3 \\
&d + (4Ia^3 - 24Ia)d^4)(bx + a))\cos(4bx + 4a) + (2I(bx + a)^4d^4 \\
&- 24Ib^2c^2d^2 + 48Ia^2b^3c^3d - 24Ia^2d^4 + (8Ib^3c^3d - 8I \\
&a^2d^4)(bx + a)^3 + (12Ib^2c^2d^2 - 24Ia^2b^3c^3d + (12Ia^2 - 24I) \\
&)(bx + a)^2 + (8Ib^3c^3d - 24Ia^2b^2c^2d^2 + (24Ia^2 - 48I) \\
&)(bx + a)^3 + (-8Ia^3 + 48Ia)d^4)(bx + a))\cos(2bx + 2a) + ((bx + \\
&a)^4d^4 - 12b^2c^2d^2 + 24a^2b^3c^3d - 12a^2d^4 + 4(b^3c^3d - a^2d^4) \\
&)(bx + a)^3 + 6(b^2c^2d^2 - 2a^2b^3c^3d + (a^2 - 2)d^4)(bx + a)^2 + \\
&4(b^3c^3d - 3a^2b^2c^2d^2 + 3(a^2 - 2)b^3c^3d - (a^3 - 6a)d^4)(bx \\
&+ a))\sin(4bx + 4a) - 2((bx + a)^4d^4 - 12b^2c^2d^2 + 24a^2b^3c^3 \\
&d - 12a^2d^4 + 4(b^3c^3d - a^2d^4)(bx + a)^3 + 6(b^2c^2d^2 - 2a^2b^3 \\
&c^3d + (a^2 - 2)d^4)(bx + a)^2 + 4(b^3c^3d - 3a^2b^2c^2d^2 + 3(a^2 \\
&- 2)b^3c^3d - (a^3 - 6a)d^4)(bx + a))\sin(2bx + 2a))\log(\cos(bx \\
&+ a)^2 + \sin(bx + a)^2 + 2\cos(bx + a) + 1) + (I(bx + a)^4d^4 - 12Ib^2 \\
&c^2d^2 + 24Ia^2b^3c^3d - 12Ia^2d^4 + (4Ib^3c^3d - 4Ia^2d^4)(bx \\
&+ a)^3 + (6Ib^2c^2d^2 - 12Ia^2b^3c^3d + (6Ia^2 - 12I)d^4)(bx + \\
&a)^2 + (4Ib^3c^3d - 12Ia^2b^2c^2d^2 + (12Ia^2 - 24I)b^3c^3d + (- \\
&4Ia^3 + 24Ia)d^4)(bx + a) + (I(bx + a)^4d^4 - 12Ib^2c^2d^2 + \\
&24Ia^2b^3c^3d - 12Ia^2d^4 + (4Ib^3c^3d - 4Ia^2d^4)(bx + a)^3 + (6I \\
&b^2c^2d^2 - 12Ia^2b^3c^3d + (6Ia^2 - 12I)d^4)(bx + a)^2 + (4Ib^3 \\
&c^3d - 12Ia^2b^2c^2d^2 + (12Ia^2 - 24I)b^3c^3d + (-4Ia^3 + 24I \\
&Ia)d^4)(bx + a))\cos(4bx + 4a) + (-2I(bx + a)^4d^4 + 24Ib^2c^2 \\
&d^2 - 48Ia^2b^3c^3d + 24Ia^2d^4 + (-8Ib^3c^3d + 8Ia^2d^4)(bx + a) \\
&)^3 + (-12Ib^2c^2d^2 + 24Ia^2b^3c^3d + (-12Ia^2 + 24I)d^4)(bx + \\
&a)^2 + (-8Ib^3c^3d + 24Ia^2b^2c^2d^2 + (-24Ia^2 + 48I)b^3c^3d + \\
&(8Ia^3 - 48Ia)d^4)(bx + a))\cos(2bx + 2a) - ((bx + a)^4d^4 - 12 \\
&b^2c^2d^2 + 24a^2b^3c^3d - 12a^2d^4 + 4(b^3c^3d - a^2d^4)(bx + a)^3 \\
&+ 6(b^2c^2d^2 - 2a^2b^3c^3d + (a^2 - 2)d^4)(bx + a)^2 + 4(b^3c^3d \\
&- 3a^2b^2c^2d^2 + 3(a^2 - 2)b^3c^3d - (a^3 - 6a)d^4)(bx + a))\sin(4 \\
&bx + 4a) + 2((bx + a)^4d^4 - 12b^2c^2d^2 + 24a^2b^3c^3d - 12a^2d^4 \\
&+ 4(b^3c^3d - a^2d^4)(bx + a)^3 + 6(b^2c^2d^2 - 2a^2b^3c^3d + (a^2 \\
&- 2)d^4)(bx + a)^2 + 4(b^3c^3d - 3a^2b^2c^2d^2 + 3(a^2 - 2)b^3c^3d
\end{aligned}$$

$$\begin{aligned}
& 3 - (a^3 - 6a)d^4(bx + a)\sin(2bx + 2a)\log(\cos(bx + a)^2 + \sin(bx + a)^2 - 2\cos(bx + a) + 1) + (48Id^4\cos(4bx + 4a) - 96Id^4\cos(2bx + 2a) - 48d^4\sin(4bx + 4a) + 96d^4\sin(2bx + 2a) + 48Id^4) \\
& \text{polylog}(5, -e^{(Ibx + Ia)}) + (-48Id^4\cos(4bx + 4a) + 96Id^4\cos(2bx + 2a) + 48d^4\sin(4bx + 4a) - 96d^4\sin(2bx + 2a) - 48Id^4) \\
& \text{polylog}(5, e^{(Ibx + Ia)}) + (48b^3cd^3 + 48(bx + a)d^4 - 48ad^4 + 48(b^3cd^3 + (bx + a)d^4 - ad^4)\cos(4bx + 4a) - 96(b^3cd^3 + (bx + a)d^4 - ad^4)\cos(2bx + 2a) + (48Ib^3cd^3 + 48I(bx + a)d^4 - 48Iad^4)\sin(4bx + 4a) + (-96Ib^3cd^3 - 96I(bx + a)d^4 + 96Iad^4)\sin(2bx + 2a)) \\
& \text{polylog}(4, -e^{(Ibx + Ia)}) - (48b^3cd^3 + 48(bx + a)d^4 - 48ad^4 + 48(b^3cd^3 + (bx + a)d^4 - ad^4)\cos(4bx + 4a) - 96(b^3cd^3 + (bx + a)d^4 - ad^4)\cos(2bx + 2a) - (-48Ib^3cd^3 - 48I(bx + a)d^4 + 48Iad^4)\sin(4bx + 4a) - (96Ib^3cd^3 + 96I(bx + a)d^4 - 96Iad^4)\sin(2bx + 2a)) \\
& \text{polylog}(4, e^{(Ibx + Ia)}) + (-24Ib^2c^2d^2 + 48Iab^3cd^3 - 24I(bx + a)^2d^4 + (-24Ia^2 + 48I)d^4 + (-48Ib^3cd^3 + 48Iad^4)(bx + a) + (-24Ib^2c^2d^2 + 48Iab^3cd^3 - 24I(bx + a)^2d^4 + (-24Ia^2 + 48I)d^4 + (-48Ib^3cd^3 + 48Iad^4)(bx + a))\cos(4bx + 4a) + (48Ib^2c^2d^2 - 96Iab^3cd^3 + 48I(bx + a)^2d^4 + (48Ia^2 - 96I)d^4 + (96Ib^3cd^3 - 96Iad^4)(bx + a))\cos(2bx + 2a) + 24(b^2c^2d^2 - 2ab^3cd^3 + (bx + a)^2d^4 + (a^2 - 2)d^4 + 2(b^3cd^3 - ad^4)(bx + a))\sin(4bx + 4a) - 48(b^2c^2d^2 - 2ab^3cd^3 + (bx + a)^2d^4 + (a^2 - 2)d^4 + 2(b^3cd^3 - ad^4)(bx + a))\sin(2bx + 2a)) \\
& \text{polylog}(3, -e^{(Ibx + Ia)}) + (24Ib^2c^2d^2 - 48Iab^3cd^3 + 24I(bx + a)^2d^4 + (24Ia^2 - 48I)d^4 + (48Ib^3cd^3 - 48Iad^4)(bx + a) + (24Ib^2c^2d^2 - 48Iab^3cd^3 + 24I(bx + a)^2d^4 + (24Ia^2 - 48I)d^4 + (48Ib^3cd^3 - 48Iad^4)(bx + a))\cos(4bx + 4a) + (-48Ib^2c^2d^2 + 96Iab^3cd^3 - 48I(bx + a)^2d^4 + (-48Ia^2 + 96I)d^4 + (-96Ib^3cd^3 + 96Iad^4)(bx + a))\cos(2bx + 2a) - 24(b^2c^2d^2 - 2ab^3cd^3 + (bx + a)^2d^4 + (a^2 - 2)d^4 + 2(b^3cd^3 - ad^4)(bx + a))\sin(4bx + 4a) + 48(b^2c^2d^2 - 2ab^3cd^3 + (bx + a)^2d^4 + (a^2 - 2)d^4 + 2(b^3cd^3 - ad^4)(bx + a))\sin(2bx + 2a)) \\
& \text{polylog}(3, e^{(Ibx + Ia)}) + (4(bx + a)^4d^4 - 16Ib^3c^3d + 48Iab^2c^2d^2 - 48Ia^2b^3cd^3 + 16Ia^3d^4 + (16b^3cd^3 - (16a + 16I)d^4)(bx + a)^3 + (24b^2c^2d^2 - (48a + 48I)b^3cd^3 + 24(a^2 + 2Ia)d^4)(bx + a)^2 + (16b^3c^3d - (48a + 48I)b^2c^2d^2 + 48(a^2 + 2Ia)b^3cd^3 - 16(a^3 + 3Ia^2)d^4)(bx + a))\sin(3bx + 3a) + (4(bx + a)^4d^4 + 16Ib^3c^3d - 48Iab^2c^2d^2 + 48Ia^2b^3cd^3 - 16Ia^3d^4 + (16b^3cd^3 - (16a - 16I)d^4)(bx + a)^3 + (24b^2c^2d^2 - (48a - 48I)b^3cd^3 + 24(a^2 - 2Ia)d^4)(bx + a)^2 + (16b^3c^3d - (48a - 48I)b^2c^2d^2 + 48(a^2 - 2Ia)b^3cd^3 - 16(a^3 - 3Ia^2)d^4)(bx + a))\sin(bx + a))/(-4Ib^4\cos(4bx + 4a) + 8Ib^4\cos(2bx + 2a) + 4b^4\sin(4bx + 4a) - 8b^4\sin(2bx + 2a) - 4Ib^4))/b
\end{aligned}$$

Fricas [C] time = 1.04884, size = 6342, normalized size = 15.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")

[Out] $\frac{1}{4}*(2*(b^4d^4x^4 + 4b^4cd^3x^3 + 6b^4c^2d^2x^2 + 4b^4c^3dx + b^4c^4)\cos(bx + a) + (-4Ib^3d^4x^3 - 12Ib^3cd^3x^2 - 4Ib^3c^3d^3 + 24Ib^3cd^3 + (4Ib^3d^4x^3 + 12Ib^3cd^3x^2 + 4Ib^3c^3d^3 - 24Ib^3cd^3 + 12I(b^3c^2d^2 - 2bd^4)x)\cos(bx + a)^2 - 12I(b^3c^2d^2 - 2bd^4)x)\text{dilog}(\cos(bx + a) + I\sin(bx + a)) + (4Ib^3d^4$

$$\begin{aligned}
& *x^3 + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d - 24*I*b*c*d^3 + (-4*I*b^3*d^4*x^3 \\
& - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d + 24*I*b*c*d^3 - 12*I*(b^3*c^2*d^2 - \\
& 2*b*d^4)*x)*\cos(b*x + a)^2 + 12*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*\operatorname{dilog}(\cos(b*x \\
& + a) - I*\sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c^3* \\
& c^3*d + 24*I*b*c*d^3 + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d \\
& - 24*I*b*c*d^3 + 12*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*\cos(b*x + a)^2 - 12*I*(b \\
& ^3*c^2*d^2 - 2*b*d^4)*x)*\operatorname{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) + (4*I*b^3*d^4*x^3 \\
& + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d - 24*I*b*c*d^3 + (-4*I*b^3*d^4*x^3 \\
& - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d + 24*I*b*c*d^3 - 12*I*(b^3*c^2*d^2 \\
& - 2*b*d^4)*x)*\cos(b*x + a)^2 + 12*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*\operatorname{dilog}(-\cos(b*x \\
& + a) - I*\sin(b*x + a)) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12* \\
& b^2*c^2*d^2 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 \\
& + b^4*c^4 - 12*b^2*c^2*d^2 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d \\
& - 6*b^2*c*d^3)*x)*\cos(b*x + a)^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*\log(\cos(b*x + a) \\
& + I*\sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 \\
& + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 \\
& - 12*b^2*c^2*d^2 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*\cos(b*x + a)^2 \\
& + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d \\
& + 6*(a^2 - 2)*b^2*c^2*d^2 - 4*(a^3 - 6*a)*b*c*d^3 + (a^4 - 12*a^2)*d^4 - (b^4*c^4 \\
& - 4*a*b^3*c^3*d + 6*(a^2 - 2)*b^2*c^2*d^2 - 4*(a^3 - 6*a)*b*c*d^3 + (a^4 \\
& - 12*a^2)*d^4)*\cos(b*x + a)^2*\log(-1/2*\cos(b*x + a) + 1/2*I*\sin(b*x + a) + \\
& 1/2) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 2)*b^2*c^2*d^2 - 4*(a^3 - 6*a)* \\
& b*c*d^3 + (a^4 - 12*a^2)*d^4 - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 2)*b^2*c^2*d^2 \\
& - 4*(a^3 - 6*a)*b*c*d^3 + (a^4 - 12*a^2)*d^4)*\cos(b*x + a)^2*\log(-1 \\
& /2*\cos(b*x + a) - 1/2*I*\sin(b*x + a) + 1/2) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 \\
& + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 6*a)*b*c*d^3 - (a^4 - 12*a^2) \\
& *d^4 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 \\
& + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 6*a)*b*c*d^3 - (a^4 - 12*a^2) \\
& *d^4 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*\cos(b*x + a)^2 \\
& + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1) + (b^4*d^4*x^4 \\
& + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 6*a)*b*c*d^3 - (a^4 - 12*a^2) \\
& *d^4 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 \\
& + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 6*a)*b*c*d^3 - (a^4 - 12*a^2) \\
& *d^4 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*\cos(b*x + a)^2 \\
& + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*\log(-\cos(b*x + a) - I*\sin(b*x + a) + 1) + 24*(d^4*\cos(b*x + a) \\
& ^2 - d^4)*\operatorname{polylog}(5, \cos(b*x + a) + I*\sin(b*x + a)) + 24*(d^4*\cos(b*x + a)^2 \\
& - d^4)*\operatorname{polylog}(5, \cos(b*x + a) - I*\sin(b*x + a)) - 24*(d^4*\cos(b*x + a)^2 \\
& - d^4)*\operatorname{polylog}(5, -\cos(b*x + a) + I*\sin(b*x + a)) - 24*(d^4*\cos(b*x + a)^2 \\
& - d^4)*\operatorname{polylog}(5, -\cos(b*x + a) - I*\sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b \\
& *c*d^3 + (-24*I*b*d^4*x - 24*I*b*c*d^3)*\cos(b*x + a)^2)*\operatorname{polylog}(4, \cos(b*x \\
& + a) + I*\sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3 + (24*I*b*d^4*x + 24 \\
& *I*b*c*d^3)*\cos(b*x + a)^2)*\operatorname{polylog}(4, \cos(b*x + a) - I*\sin(b*x + a)) + (24 \\
& *I*b*d^4*x + 24*I*b*c*d^3 + (-24*I*b*d^4*x - 24*I*b*c*d^3)*\cos(b*x + a)^2)* \\
& \operatorname{polylog}(4, -\cos(b*x + a) + I*\sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3 \\
& + (24*I*b*d^4*x + 24*I*b*c*d^3)*\cos(b*x + a)^2)*\operatorname{polylog}(4, -\cos(b*x + a) - \\
& I*\sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4 - (\\
& b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4)*\cos(b*x + a)^2)*\operatorname{polylog}(\\
& 3, \cos(b*x + a) + I*\sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 \\
& - 2*d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4)*\cos(b*x + a)^2)*\operatorname{polylog}(\\
& 3, \cos(b*x + a) - I*\sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 \\
& - 2*d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4)*\cos(b*x + a)^2)*\operatorname{polylog}(\\
& 3, -\cos(b*x + a) + I*\sin(b*x + a)) - 12 \\
& *(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4 - (b^2*d^4*x^2 + 2*b^2*c \\
& *d^3*x + b^2*c^2*d^2 - 2*d^4)*\cos(b*x + a)^2)*\operatorname{polylog}(3, -\cos(b*x + a) - I \\
& *\sin(b*x + a)) + 8*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c \\
& ^3*d)*\sin(b*x + a))/(b^5*\cos(b*x + a)^2 - b^5)
\end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^4 \cot^2(a + bx) \csc(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**4*cot(b*x+a)**2*csc(b*x+a),x)

[Out] Integral((c + d*x)**4*cot(a + b*x)**2*csc(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^4 \cot(bx + a)^2 \csc(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^4*cot(b*x + a)^2*csc(b*x + a), x)

3.113 $\int (c + dx)^3 \cot^2(a + bx) \csc(a + bx) dx$

Optimal. Leaf size=308

$$\frac{3d^2(c + dx)\text{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^3} - \frac{3d^2(c + dx)\text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3} - \frac{3id(c + dx)^2\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{2b^2} + \frac{3id(c + dx)^2\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{2b^2}$$

```
[Out] (-6*d^2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^3 + ((c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b - (3*d*(c + d*x)^2*Csc[a + b*x])/(2*b^2) - ((c + d*x)^3*Cot[a + b*x]*Csc[a + b*x])/(2*b) + (((3*I)*d^3*PolyLog[2, -E^(I*(a + b*x))])/b^4 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((3*I)*d^3*PolyLog[2, E^(I*(a + b*x))])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 - (3*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 + ((3*I)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^4 - ((3*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4
```

Rubi [A] time = 0.344229, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {4415, 4183, 2531, 6609, 2282, 6589, 4186, 2279, 2391}

$$\frac{3d^2(c + dx)\text{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^3} - \frac{3d^2(c + dx)\text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3} - \frac{3id(c + dx)^2\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{2b^2} + \frac{3id(c + dx)^2\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x]^2*Csc[a + b*x], x]
```

```
[Out] (-6*d^2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^3 + ((c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b - (3*d*(c + d*x)^2*Csc[a + b*x])/(2*b^2) - ((c + d*x)^3*Cot[a + b*x]*Csc[a + b*x])/(2*b) + (((3*I)*d^3*PolyLog[2, -E^(I*(a + b*x))])/b^4 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((3*I)*d^3*PolyLog[2, E^(I*(a + b*x))])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 - (3*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 + ((3*I)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^4 - ((3*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4
```

Rule 4415

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))]^(n_.)*((f_.) + (g_.)*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/b*c*n*Log[F], x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f}
```

, g, n}, x] && GtQ[m, 0]

Rule 6609

Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rule 4186

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

Rule 2279

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 \cot^2(a + bx) \csc(a + bx) dx &= - \int (c + dx)^3 \csc(a + bx) dx + \int (c + dx)^3 \csc^3(a + bx) dx \\
&= \frac{2(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{3d(c + dx)^2 \csc(a + bx)}{2b^2} - \frac{(c + dx)^3 \cot(a + bx)}{2b} \\
&= -\frac{6d^2(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^3} + \frac{(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{3d(c + dx)^2 \csc(a + bx)}{2b^2} \\
&= -\frac{6d^2(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^3} + \frac{(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{3d(c + dx)^2 \csc(a + bx)}{2b^2} \\
&= -\frac{6d^2(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^3} + \frac{(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{3d(c + dx)^2 \csc(a + bx)}{2b^2} \\
&= -\frac{6d^2(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^3} + \frac{(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{3d(c + dx)^2 \csc(a + bx)}{2b^2} \\
&= -\frac{6d^2(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^3} + \frac{(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{3d(c + dx)^2 \csc(a + bx)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 5.05101, size = 528, normalized size = 1.71

$$3id \left(b^2(c + dx)^2 - 2d^2 \right) \text{PolyLog} \left(2, -e^{i(a+bx)} \right) - 3id \left(b^2(c + dx)^2 - 2d^2 \right) \text{PolyLog} \left(2, e^{i(a+bx)} \right) - 6bcd^2 \text{PolyLog} \left(3, -e^{i(a+bx)} \right) + 6bcd^2 \text{PolyLog} \left(3, e^{i(a+bx)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3*Cot[a + b*x]^2*Csc[a + b*x], x]

[Out] $-(b^2(c + dx)^2(3d + b(c + dx)) \cot(a + bx) \csc(a + bx) + b^3 c^3 \text{Log}[1 - E^{i(a+bx)}] - 6b^2 c d^2 \text{Log}[1 - E^{i(a+bx)}] + 3b^3 c^2 d x \text{Log}[1 - E^{i(a+bx)}] - 6b^2 d^3 x \text{Log}[1 - E^{i(a+bx)}] + 3b^3 c d^2 x^2 \text{Log}[1 - E^{i(a+bx)}] + b^3 d^3 x^3 \text{Log}[1 - E^{i(a+bx)}] - b^3 c^3 \text{Log}[1 + E^{i(a+bx)}] + 6b^2 c d^2 \text{Log}[1 + E^{i(a+bx)}] - 3b^3 c^2 d x \text{Log}[1 + E^{i(a+bx)}] + 6b^2 d^3 x \text{Log}[1 + E^{i(a+bx)}] - 3b^3 c d^2 x^2 \text{Log}[1 + E^{i(a+bx)}] - b^3 d^3 x^3 \text{Log}[1 + E^{i(a+bx)}]) + (3I) d (-2d^2 + b^2(c + dx)^2) \text{PolyLog}[2, -E^{i(a+bx)}] - (3I) d (-2d^2 + b^2(c + dx)^2) \text{PolyLog}[2, E^{i(a+bx)}] - 6b^2 c d^2 \text{PolyLog}[3, -E^{i(a+bx)}] - 6b^2 d^3 x \text{PolyLog}[3, -E^{i(a+bx)}] + 6b^2 c d^2 \text{PolyLog}[3, E^{i(a+bx)}] + 6b^2 d^3 x \text{PolyLog}[3, E^{i(a+bx)}] - (6I) d^3 \text{PolyLog}[4, -E^{i(a+bx)}] + (6I) d^3 \text{PolyLog}[4, E^{i(a+bx)}]) / (2b^4)$

Maple [B] time = 0.342, size = 1056, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a), x)

[Out] $3/2/b^2 c d^2 \ln(\exp(i(b*x+a))+1) x^2 - 3/2/b^3 c d^2 \ln(\exp(i(b*x+a))+1) a^2 + 3/b^3 c d^2 a^2 \operatorname{arctanh}(\exp(i(b*x+a))) - 3/2 I/b^2 d^3 \operatorname{polylog}(2, -\exp(i(b*x+a))) x^2 + 3/2 I/b^2 c^2 d \operatorname{polylog}(2, \exp(i(b*x+a))) - 3/2 I/b^2 c^2 d \operatorname{polylog}(2, -\exp(i(b*x+a))) + 3/2 I/b^2 d^3 \operatorname{polylog}(2, \exp(i(b*x+a))) x^2 + 3 I d^3 \operatorname{polylog}(2, -\exp(i(b*x+a))) / b^4 + 3 I d^3 \operatorname{polylog}(4, -\exp(i(b*x+a))) / b^4 - 1/b^4 d$

$$\begin{aligned} &^3*a^3*\operatorname{arctanh}(\exp(I*(b*x+a)))-3/b^3*c*d^2*\operatorname{polylog}(3,\exp(I*(b*x+a)))+3/b^3* \\ &c*d^2*\operatorname{polylog}(3,-\exp(I*(b*x+a)))+3/b^3*d^3*\operatorname{polylog}(3,-\exp(I*(b*x+a)))*x-3/b \\ &^3*d^3*\operatorname{polylog}(3,\exp(I*(b*x+a)))*x+1/2/b*d^3*\ln(\exp(I*(b*x+a))+1)*x^3-3/2/b \\ &*c*d^2*\ln(1-\exp(I*(b*x+a)))*x^2+3/2/b^3*c*d^2*\ln(1-\exp(I*(b*x+a)))*a^2-3/2/ \\ &b*c^2*d*\ln(1-\exp(I*(b*x+a)))*x-3/2/b^2*c^2*d*\ln(1-\exp(I*(b*x+a)))*a-3/b^4*d \\ &^3*\ln(\exp(I*(b*x+a))+1)*a+3/b^3*d^3*\ln(1-\exp(I*(b*x+a)))*x+3/b^4*d^3*\ln(1-\exp(I*(b*x+a)))*a \\ &+6/b^4*d^3*a*\operatorname{arctanh}(\exp(I*(b*x+a)))-6/b^3*d^2*c*\operatorname{arctanh}(\exp(I*(b*x+a)))-3/b^3*d^3*\ln(\exp(I*(b*x+a))+1)*x \\ &-3*I/b^2*\operatorname{polylog}(2,-\exp(I*(b*x+a)))*c*d^2*x+3*I/b^2*\operatorname{polylog}(2,\exp(I*(b*x+a)))*c*d^2*x-1/2/b^4*d^3*\ln(1-\exp(I*(b*x+a)))*a^3 \\ &+1/b^2/(\exp(2*I*(b*x+a))-1)^2*(d^3*x^3*b*\exp(3*I*(b*x+a))+3*c*d^2*x^2*b*\exp(3*I*(b*x+a))+3*c^2*d*x*b*\exp(3*I*(b*x+a))+d^3*x^3*b*\exp(I*(b*x+a))+c^3*b*\exp(3*I*(b*x+a))+3*c*d^2*x^2*b*\exp(I*(b*x+a))-3*I*d^3*x^2*\exp(3*I*(b*x+a))+3*c^2*d*x*b*\exp(I*(b*x+a))-6*I*c*d^2*x*\exp(3*I*(b*x+a))+c^3*b*\exp(I*(b*x+a))-3*I*c^2*d*\exp(3*I*(b*x+a))+3*I*d^3*x^2*\exp(I*(b*x+a))+6*I*c*d^2*x*\exp(I*(b*x+a))+3*I*c^2*d*\exp(I*(b*x+a)))-3/b^2*c^2*d*a*\operatorname{arctanh}(\exp(I*(b*x+a)))-1/2/b*d^3*\ln(1-\exp(I*(b*x+a)))*x^3+1/2/b^4*d^3*\ln(\exp(I*(b*x+a))+1)*a^3+1/b*c^3*\operatorname{arctanh}(\exp(I*(b*x+a)))+3/2/b*c^2*d*\ln(\exp(I*(b*x+a))+1)*x+3/2/b^2*c^2*d*\ln(\exp(I*(b*x+a))+1)*a-3*I*d^3*\operatorname{polylog}(2,\exp(I*(b*x+a)))/b^4-3*I*d^3*\operatorname{polylog}(4,\exp(I*(b*x+a)))/b^4 \end{aligned}$$

Maxima [B] time = 6.89686, size = 5227, normalized size = 16.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")

[Out] $\frac{1}{4}*(c^3*(2*\cos(b*x + a)/(\cos(b*x + a)^2 - 1) + \log(\cos(b*x + a) + 1) - \log(\cos(b*x + a) - 1)) - 3*a*c^2*d*(2*\cos(b*x + a)/(\cos(b*x + a)^2 - 1) + \log(\cos(b*x + a) + 1) - \log(\cos(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*\cos(b*x + a)/(\cos(b*x + a)^2 - 1) + \log(\cos(b*x + a) + 1) - \log(\cos(b*x + a) - 1))/b^2 - a^3*d^3*(2*\cos(b*x + a)/(\cos(b*x + a)^2 - 1) + \log(\cos(b*x + a) + 1) - \log(\cos(b*x + a) - 1))/b^3 + 4*((2*(b*x + a)^3*d^3 - 12*b*c*d^2 + 12*a*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*\cos(4*b*x + 4*a) - 4*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (2*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + (6*I*a^2 - 12*I)*d^3)*(b*x + a))*\sin(4*b*x + 4*a) + (-4*I*(b*x + a)^3*d^3 + 24*I*b*c*d^2 - 24*I*a*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 + (-12*I*a^2 + 24*I)*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{arctan2}(\sin(b*x + a), \cos(b*x + a) + 1) + (12*b*c*d^2 - 12*a*d^3 + 12*(b*c*d^2 - a*d^3)*\cos(4*b*x + 4*a) - 24*(b*c*d^2 - a*d^3)*\cos(2*b*x + 2*a) + (12*I*b*c*d^2 - 12*I*a*d^3)*\sin(4*b*x + 4*a) + (-24*I*b*c*d^2 + 24*I*a*d^3)*\sin(2*b*x + 2*a))*\operatorname{arctan2}(\sin(b*x + a), \cos(b*x + a) - 1) + (2*(b*x + a)^3*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*\cos(4*b*x + 4*a) - 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (2*I*(b*x + a)^3*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + (6*I*a^2 - 12*I)*d^3)*(b*x + a))*\sin(4*b*x + 4*a) + (-4*I*(b*x + a)^3*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 + (-12*I*a^2 + 24*I)*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{arctan2}(\sin(b*x$

$$\begin{aligned}
& + a), -\cos(b*x + a) + 1) + (-4*I*(b*x + a)^3*d^3 - 12*b^2*c^2*d + 24*a*b*c*d^2 - 12*a^2*d^3 + (-12*I*b*c*d^2 - 12*(-I*a + 1)*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d - 24*(-I*a + 1)*b*c*d^2 + (-12*I*a^2 + 24*a)*d^3)*(b*x + a))*\cos(3*b*x + 3*a) + (-4*I*(b*x + a)^3*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^2*d^3 + (-12*I*b*c*d^2 - 12*(-I*a - 1)*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d - 24*(-I*a - 1)*b*c*d^2 + (-12*I*a^2 - 24*a)*d^3)*(b*x + a))*\cos(b*x + a) - (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*(a^2 - 2)*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 + (-6*I*a^2 + 12*I)*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*\sin(4*b*x + 4*a) - (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*d^3 + (12*I*a^2 - 24*I)*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{dilog}(-e^(I*b*x + I*a)) + (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*(a^2 - 2)*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + (6*I*a^2 - 12*I)*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*\sin(4*b*x + 4*a) + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 + (-12*I*a^2 + 24*I)*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{dilog}(e^(I*b*x + I*a)) + (-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 + (-3*I*a^2 + 6*I)*d^3)*(b*x + a) + (-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 + (-3*I*a^2 + 6*I)*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + (2*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + (6*I*a^2 - 12*I)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + ((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*\sin(4*b*x + 4*a) - 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) + (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + (3*I*a^2 - 6*I)*d^3)*(b*x + a) + (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + (3*I*a^2 - 6*I)*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + (-2*I*(b*x + a)^3*d^3 + 12*I*b*c*d^2 - 12*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 12*I)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - ((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*\sin(4*b*x + 4*a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) + (12*d^3*\cos(4*b*x + 4*a) - 24*d^3*\cos(2*b*x + 2*a) + 12*I*d^3*\sin(4*b*x + 4*a) - 24*I*d^3*\sin(2*b*x + 2*a) + 12*d^3)*\operatorname{polylog}(4, -e^(I*b*x + I*a)) - (12*d^3*\cos(4*b*x + 4*a) - 24*d^3*\cos(2*b*x + 2*a) + 12*I*d^3*\sin(4*b*x + 4*a) - 24*I*d^3*\sin(2*b*x + 2*a) + 12*d^3)*\operatorname{polylog}(4, e^(I*b*x + I*a)) + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3 + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*\cos(4*b*x + 4*a) + (24*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 24*I*a*d^3)*\cos(2*b*x + 2*a) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\sin(4*b*x + 4*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\sin(2*b*x + 2*a))*\operatorname{polylog}(3, -e^(I*b*x + I*a)) + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3 + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*\cos(4*b*x + 4*a) + (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*\cos(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\sin(4*b*x + 4*a) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\sin(2*b*x + 2*a))*\operatorname{polylog}(3, e^(I*b*x + I*a)) + (4*(b*x + a)^3*d^3 - 12*
\end{aligned}$$

$$\begin{aligned} & I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*a^2*d^3 + (12*b*c*d^2 - (12*a + 12*I)*d \\ & ^3)*(b*x + a)^2 + (12*b^2*c^2*d - (24*a + 24*I)*b*c*d^2 + 12*(a^2 + 2*I*a)* \\ & d^3)*(b*x + a))*\sin(3*b*x + 3*a) + (4*(b*x + a)^3*d^3 + 12*I*b^2*c^2*d - 24 \\ & *I*a*b*c*d^2 + 12*I*a^2*d^3 + (12*b*c*d^2 - (12*a - 12*I)*d^3)*(b*x + a)^2 \\ & + (12*b^2*c^2*d - (24*a - 24*I)*b*c*d^2 + 12*(a^2 - 2*I*a)*d^3)*(b*x + a))* \\ & \sin(b*x + a))/(-4*I*b^3*\cos(4*b*x + 4*a) + 8*I*b^3*\cos(2*b*x + 2*a) + 4*b^3 \\ & *\sin(4*b*x + 4*a) - 8*b^3*\sin(2*b*x + 2*a) - 4*I*b^3))/b \end{aligned}$$

Fricas [C] time = 0.808976, size = 4091, normalized size = 13.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")

[Out] $\frac{1}{4}*(2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\cos(b*x + a) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + 6*I*d^3 + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d - 6*I*d^3)*\cos(b*x + a)^2)*\operatorname{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d - 6*I*d^3 + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + 6*I*d^3)*\cos(b*x + a)^2)*\operatorname{dilog}(\cos(b*x + a) - I*\sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + 6*I*d^3 + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d - 6*I*d^3)*\cos(b*x + a)^2)*\operatorname{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d - 6*I*d^3 + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + 6*I*d^3)*\cos(b*x + a)^2)*\operatorname{dilog}(-\cos(b*x + a) - I*\sin(b*x + a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*\cos(b*x + a)^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*\cos(b*x + a)^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*\cos(b*x + a)^2)*\log(-1/2*\cos(b*x + a) + 1/2*I*\sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*\cos(b*x + a)^2)*\log(-1/2*\cos(b*x + a) - 1/2*I*\sin(b*x + a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*\cos(b*x + a)^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*\cos(b*x + a)^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*\log(-\cos(b*x + a) - I*\sin(b*x + a) + 1) + (-6*I*d^3*\cos(b*x + a)^2 + 6*I*d^3)*\operatorname{polylog}(4, \cos(b*x + a) + I*\sin(b*x + a)) + (6*I*d^3*\cos(b*x + a)^2 - 6*I*d^3)*\operatorname{polylog}(4, -\cos(b*x + a) + I*\sin(b*x + a)) + (6*I*d^3*\cos(b*x + a)^2 - 6*I*d^3)*\operatorname{polylog}(4, -\cos(b*x + a) - I*\sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*\cos(b*x + a)^2)*\operatorname{polylog}(3, \cos(b*x + a) + I*\sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*\cos(b*x + a)^2)*\operatorname{polylog}(3, \cos(b*x + a) - I*\sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*\cos(b*x + a)^2)*\operatorname{polylog}(3, -\cos(b*x + a) + I*\sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*\cos(b*x + a)^2)*\operatorname{polylog}(3, -\cos(b*x + a) - I*\sin(b*x + a)) + 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*\sin(b*x + a))/(b^4*\cos(b*x + a)^2 - b^4)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \cot^2(a + bx) \csc(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*cot(b*x+a)**2*csc(b*x+a),x)

[Out] Integral((c + d*x)**3*cot(a + b*x)**2*csc(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \cot(bx + a)^2 \csc(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^3*cot(b*x + a)^2*csc(b*x + a), x)

3.114 $\int (c + dx)^2 \cot^2(a + bx) \csc(a + bx) dx$

Optimal. Leaf size=179

$$-\frac{id(c + dx)\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^2} + \frac{id(c + dx)\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^2} + \frac{d^2\text{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^3} - \frac{d^2\text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3}$$

```
[Out] ((c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (d^2*ArcTanh[Cos[a + b*x]])/b^3
- (d*(c + d*x)*Csc[a + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]*Csc[a + b*x])/
(2*b) - (I*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 + (I*d*(c + d*x)*P
olyLog[2, E^(I*(a + b*x))])/b^2 + (d^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 -
(d^2*PolyLog[3, E^(I*(a + b*x))])/b^3
```

Rubi [A] time = 0.223015, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {4415, 4183, 2531, 2282, 6589, 4186, 3770}

$$-\frac{id(c + dx)\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^2} + \frac{id(c + dx)\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^2} + \frac{d^2\text{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^3} - \frac{d^2\text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x]^2*Csc[a + b*x], x]
```

```
[Out] ((c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (d^2*ArcTanh[Cos[a + b*x]])/b^3
- (d*(c + d*x)*Csc[a + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]*Csc[a + b*x])/
(2*b) - (I*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 + (I*d*(c + d*x)*P
olyLog[2, E^(I*(a + b*x))])/b^2 + (d^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 -
(d^2*PolyLog[3, E^(I*(a + b*x))])/b^3
```

Rule 4415

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^m_], x_Symbol] := -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^m_], x_Symbol] := Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(
m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^m_], x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
```

```
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_ /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x]
+ (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*
(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^
m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*
(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x]
&& GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \cot^2(a + bx) \csc(a + bx) dx &= - \int (c + dx)^2 \csc(a + bx) dx + \int (c + dx)^2 \csc^3(a + bx) dx \\ &= \frac{2(c + dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d(c + dx) \csc(a + bx)}{b^2} - \frac{(c + dx)^2 \cot(a + bx)}{2b} \\ &= \frac{(c + dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2 \tanh^{-1}(\cos(a + bx))}{b^3} - \frac{d(c + dx) \csc(a + bx)}{b^2} \\ &= \frac{(c + dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2 \tanh^{-1}(\cos(a + bx))}{b^3} - \frac{d(c + dx) \csc(a + bx)}{b^2} \\ &= \frac{(c + dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2 \tanh^{-1}(\cos(a + bx))}{b^3} - \frac{d(c + dx) \csc(a + bx)}{b^2} \\ &= \frac{(c + dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2 \tanh^{-1}(\cos(a + bx))}{b^3} - \frac{d(c + dx) \csc(a + bx)}{b^2} \end{aligned}$$

Mathematica [B] time = 7.59481, size = 471, normalized size = 2.63

$$\frac{-2ibd(c + dx)\text{PolyLog}\left(2, -e^{i(a+bx)}\right) + 2ibd(c + dx)\text{PolyLog}\left(2, e^{i(a+bx)}\right) + 2d^2\text{PolyLog}\left(3, -e^{i(a+bx)}\right) - 2d^2\text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x]^2*Csc[a + b*x], x]
```

```
[Out] -((d*(c + d*x)*Csc[a])/b^2) + ((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a/2 + (b*x)/2]
^2)/(8*b) + (-(b^2*c^2*Log[1 - E^(I*(a + b*x))]) + 2*d^2*Log[1 - E^(I*(a +
b*x))] - 2*b^2*c*d*x*Log[1 - E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 - E^(I*(
a + b*x))] + b^2*c^2*Log[1 + E^(I*(a + b*x))] - 2*d^2*Log[1 + E^(I*(a + b*x
))] + 2*b^2*c*d*x*Log[1 + E^(I*(a + b*x))] + b^2*d^2*x^2*Log[1 + E^(I*(a +
```

$$b*x)) - (2*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] + 2*d^2*PolyLog[3, -E^(I*(a + b*x))] - 2*d^2*PolyLog[3, E^(I*(a + b*x))]/(2*b^3) + ((c^2 + 2*c*d*x + d^2*x^2)*Sec[a/2 + (b*x)/2]^2)/(8*b) + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c*d*Sin[(b*x)/2]) - d^2*x*Sin[(b*x)/2]))/(2*b^2) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*(c*d*Sin[(b*x)/2] + d^2*x*Sin[(b*x)/2]))/(2*b^2)$$

Maple [B] time = 0.281, size = 546, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x)

[Out] $\frac{1}{b^2} \left(\frac{\exp(2I(b*x+a)) - 1}{\exp(2I(b*x+a)) - 1} \right)^2 (d^2 x^2 b \exp(3I(b*x+a)) + 2c d x b \exp(3I(b*x+a)) + c^2 b \exp(3I(b*x+a)) + d^2 x^2 b \exp(I(b*x+a)) + 2c d x b \exp(I(b*x+a)) - 2I d^2 x \exp(3I(b*x+a)) + c^2 b \exp(I(b*x+a)) - 2I d c \exp(3I(b*x+a)) + 2I d^2 x \exp(I(b*x+a)) + 2I d c \exp(I(b*x+a))) + \frac{1}{b^3} d^2 a^2 \operatorname{arctanh}(\exp(I(b*x+a))) - \frac{1}{2} \frac{1}{b^3} d^2 \ln(\exp(I(b*x+a)) + 1) a^2 + \frac{1}{2} \frac{1}{b^3} d^2 \ln(1 - \exp(I(b*x+a))) a^2 + d^2 \operatorname{polylog}(3, -\exp(I(b*x+a))) / b^3 + \frac{1}{b^2} c d \ln(\exp(I(b*x+a)) + 1) a + \frac{1}{2} \frac{1}{b} d^2 \ln(\exp(I(b*x+a)) + 1) x^2 - \frac{1}{b^2} \operatorname{polylog}(2, -\exp(I(b*x+a))) d^2 x - \frac{1}{2} \frac{1}{b} d^2 \ln(1 - \exp(I(b*x+a))) x^2 + \frac{1}{b^2} c d \operatorname{polylog}(2, \exp(I(b*x+a))) - \frac{2}{b^2} c d a \operatorname{arctanh}(\exp(I(b*x+a))) + \frac{1}{b^2} \operatorname{polylog}(2, \exp(I(b*x+a))) d^2 x - \frac{1}{b^2} c d \operatorname{polylog}(2, -\exp(I(b*x+a))) - \frac{1}{b} c d \ln(1 - \exp(I(b*x+a))) x - \frac{1}{b^2} c d \ln(1 - \exp(I(b*x+a))) a + \frac{1}{b} c d \ln(\exp(I(b*x+a)) + 1) x - d^2 \operatorname{polylog}(3, \exp(I(b*x+a))) / b^3 - \frac{2}{b^3} d^2 \operatorname{arctanh}(\exp(I(b*x+a)))$

Maxima [B] time = 2.40362, size = 2607, normalized size = 14.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")

[Out] $\frac{1}{4} (c^2 (2 \cos(b*x + a) / (\cos(b*x + a)^2 - 1) + \log(\cos(b*x + a) + 1) - \log(\cos(b*x + a) - 1)) - 2a c d (2 \cos(b*x + a) / (\cos(b*x + a)^2 - 1) + \log(\cos(b*x + a) + 1) - \log(\cos(b*x + a) - 1))) / b + a^2 d^2 (2 \cos(b*x + a) / (\cos(b*x + a)^2 - 1) + \log(\cos(b*x + a) + 1) - \log(\cos(b*x + a) - 1)) / b^2 + 4 ((2 (b*x + a)^2 d^2 + 4 (b*c*d - a*d^2) (b*x + a) - 4 d^2 + 2 ((b*x + a)^2 d^2 + 2 (b*c*d - a*d^2) (b*x + a) - 2 d^2) \cos(4*b*x + 4*a) - 4 ((b*x + a)^2 d^2 + 2 (b*c*d - a*d^2) (b*x + a) - 2 d^2) \cos(2*b*x + 2*a) + (2*I*(b*x + a)^2 d^2 + (4*I*b*c*d - 4*I*a*d^2) (b*x + a) - 4*I*d^2) \sin(4*b*x + 4*a) + (-4*I*(b*x + a)^2 d^2 + (-8*I*b*c*d + 8*I*a*d^2) (b*x + a) + 8*I*d^2) \sin(2*b*x + 2*a)) \operatorname{arctan2}(\sin(b*x + a), \cos(b*x + a) + 1) + (4*d^2 \cos(4*b*x + 4*a) - 8*d^2 \cos(2*b*x + 2*a) + 4*I*d^2 \sin(4*b*x + 4*a) - 8*I*d^2 \sin(2*b*x + 2*a) + 4*d^2) \operatorname{arctan2}(\sin(b*x + a), \cos(b*x + a) - 1) + (2*(b*x + a)^2 d^2 + 4*(b*c*d - a*d^2) (b*x + a) + 2*((b*x + a)^2 d^2 + 2*(b*c*d - a*d^2) (b*x + a)) \cos(4*b*x + 4*a) - 4*((b*x + a)^2 d^2 + 2*(b*c*d - a*d^2) (b*x + a)) \cos(2*b*x + 2*a) + (2*I*(b*x + a)^2 d^2 + (4*I*b*c*d - 4*I*a*d^2) (b*x + a)) \sin(4*b*x + 4*a) + (-4*I*(b*x + a)^2 d^2 + (-8*I*b*c*d + 8*I*a*d^2) (b*x + a)) \sin(2*b*x + 2*a)) \operatorname{arctan2}(\sin(b*x + a), -\cos(b*x + a) + 1) + (-4*I*$

$$\begin{aligned}
& (bx + a)^2d^2 - 8b^2cd + 8a^2d^2 + (-8Ib^2cd - 8(-Ia + 1)d^2)(bx + a) \cos(3bx + 3a) + (-4I(bx + a)^2d^2 + 8b^2cd - 8a^2d^2 + (-8Ib^2cd - 8(-Ia - 1)d^2)(bx + a)) \cos(bx + a) - (4b^2cd + 4(bx + a)d^2 - 4a^2d^2 + 4(b^2cd + (bx + a)d^2 - a^2d^2)) \cos(4bx + 4a) - 8(b^2cd + (bx + a)d^2 - a^2d^2) \cos(2bx + 2a) - (-4Ib^2cd - 4I(bx + a)d^2 + 4Ia^2d^2) \sin(4bx + 4a) - (8Ib^2cd + 8I(bx + a)d^2 - 8Ia^2d^2) \sin(2bx + 2a) \operatorname{dilog}(-e^{Ibx + Ia}) + (4b^2cd + 4(bx + a)d^2 - 4a^2d^2 + 4(b^2cd + (bx + a)d^2 - a^2d^2)) \cos(4bx + 4a) - 8(b^2cd + (bx + a)d^2 - a^2d^2) \cos(2bx + 2a) + (4Ib^2cd + 4I(bx + a)d^2 - 4Ia^2d^2) \sin(4bx + 4a) + (-8Ib^2cd - 8I(bx + a)d^2 + 8Ia^2d^2) \sin(2bx + 2a) \operatorname{dilog}(e^{Ibx + Ia}) + (-I(bx + a)^2d^2 + (-2Ib^2cd + 2Ia^2d^2)(bx + a) + 2Id^2 + (-I(bx + a)^2d^2 + (-2Ib^2cd + 2Ia^2d^2)(bx + a) + 2Id^2)) \cos(4bx + 4a) + (2I(bx + a)^2d^2 + (4Ib^2cd - 4Ia^2d^2)(bx + a) - 4Id^2) \cos(2bx + 2a) + ((bx + a)^2d^2 + 2(b^2cd - a^2d^2)(bx + a) - 2d^2) \sin(4bx + 4a) - 2((bx + a)^2d^2 + 2(b^2cd - a^2d^2)(bx + a) - 2d^2) \sin(2bx + 2a) \log(\cos(bx + a)^2 + \sin(bx + a)^2 + 2\cos(bx + a) + 1) + (I(bx + a)^2d^2 + (2Ib^2cd - 2Ia^2d^2)(bx + a) - 2Id^2 + (I(bx + a)^2d^2 + (2Ib^2cd - 2Ia^2d^2)(bx + a) - 2Id^2)) \cos(4bx + 4a) + (-2I(bx + a)^2d^2 + (-4Ib^2cd + 4Ia^2d^2)(bx + a) + 4Id^2) \cos(2bx + 2a) - ((bx + a)^2d^2 + 2(b^2cd - a^2d^2)(bx + a) - 2d^2) \sin(4bx + 4a) + 2((bx + a)^2d^2 + 2(b^2cd - a^2d^2)(bx + a) - 2d^2) \sin(2bx + 2a) \log(\cos(bx + a)^2 + \sin(bx + a)^2 - 2\cos(bx + a) + 1) + (-4Id^2 \cos(4bx + 4a) + 8Id^2 \cos(2bx + 2a) + 4d^2 \sin(4bx + 4a) - 8d^2 \sin(2bx + 2a) - 4Id^2) \operatorname{polylog}(3, -e^{Ibx + Ia}) + (4Id^2 \cos(4bx + 4a) - 8Id^2 \cos(2bx + 2a) - 4d^2 \sin(4bx + 4a) + 8d^2 \sin(2bx + 2a) + 4Id^2) \operatorname{polylog}(3, e^{Ibx + Ia}) + (4(bx + a)^2d^2 - 8Ib^2cd + 8Ia^2d^2 + (8b^2cd - (8a + 8I)d^2)(bx + a)) \sin(3bx + 3a) + (4(bx + a)^2d^2 + 8Ib^2cd - 8Ia^2d^2 + (8b^2cd - (8a - 8I)d^2)(bx + a)) \sin(bx + a) / (-4Ib^2 \cos(4bx + 4a) + 8Ib^2 \cos(2bx + 2a) + 4b^2 \sin(4bx + 4a) - 8b^2 \sin(2bx + 2a) - 4Ib^2) / b
\end{aligned}$$

Fricas [C] time = 0.688199, size = 2379, normalized size = 13.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")

[Out] $1/4*(2*(b^2d^2x^2 + 2b^2cdx + b^2c^2)) \cos(bx + a) + (-2Ib^2d^2x - 2Ib^2cd + (2Ib^2d^2x + 2Ib^2cd)) \cos(bx + a)^2 \operatorname{dilog}(\cos(bx + a) + I \sin(bx + a)) + (2Ib^2d^2x + 2Ib^2cd + (-2Ib^2d^2x - 2Ib^2cd)) \cos(bx + a)^2 \operatorname{dilog}(\cos(bx + a) - I \sin(bx + a)) + (-2Ib^2d^2x - 2Ib^2cd + (2Ib^2d^2x + 2Ib^2cd)) \cos(bx + a)^2 \operatorname{dilog}(-\cos(bx + a) + I \sin(bx + a)) + (2Ib^2d^2x + 2Ib^2cd + (-2Ib^2d^2x - 2Ib^2cd)) \cos(bx + a)^2 \operatorname{dilog}(-\cos(bx + a) - I \sin(bx + a)) - (b^2d^2x^2 + 2b^2cdx + b^2c^2 - (b^2d^2x^2 + 2b^2cdx + b^2c^2 - 2d^2)) \cos(bx + a)^2 - 2d^2 \log(\cos(bx + a) + I \sin(bx + a) + 1) - (b^2d^2x^2 + 2b^2cdx + b^2c^2 - (b^2d^2x^2 + 2b^2cdx + b^2c^2 - 2d^2)) \cos(bx + a)^2 - 2d^2 \log(\cos(bx + a) - I \sin(bx + a) + 1) + (b^2c^2 - 2a^2bcd + (a^2 - 2)d^2 - (b^2c^2 - 2a^2bcd + (a^2 - 2)d^2)) \cos(bx + a)^2 \log(-1/2 \cos(bx + a) + 1/2 I \sin(bx + a) + 1/2) + (b^2c^2 - 2a^2bcd + (a^2 - 2)d^2 - (b^2c^2 - 2a^2bcd + (a^2 - 2)d^2)) \cos(bx + a)^2 \log(-1/2 \cos(bx + a) - 1/2 I \sin(bx + a) + 1/2) + (b^2d^2x^2 + 2b^2cdx + 2a^2bcd - a^2d^2 - (b^2d^2x^2 + 2b^2cdx + 2a^2bcd - a^2d^2)) \cos(bx + a)^2 \log(-\cos(bx + a) + I \sin(bx + a) + 1) + (b^2d^2x^2 + 2b^2cdx +$

$$\frac{2ab^2cd - a^2d^2 - (b^2d^2x^2 + 2b^2cdx + 2ab^2cd - a^2d^2)\cos(bx + a)^2 \log(-\cos(bx + a) - I\sin(bx + a) + 1) - 2(d^2\cos(bx + a)^2 - d^2)\text{polylog}(3, \cos(bx + a) + I\sin(bx + a)) - 2(d^2\cos(bx + a)^2 - d^2)\text{polylog}(3, \cos(bx + a) - I\sin(bx + a)) + 2(d^2\cos(bx + a)^2 - d^2)\text{polylog}(3, -\cos(bx + a) + I\sin(bx + a)) + 2(d^2\cos(bx + a)^2 - d^2)\text{polylog}(3, -\cos(bx + a) - I\sin(bx + a)) + 4(bd^2x + bcd)\sin(bx + a)}{(b^3\cos(bx + a)^2 - b^3)}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \cot^2(a + bx) \csc(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*cot(b*x+a)**2*csc(b*x+a),x)

[Out] Integral((c + d*x)**2*cot(a + b*x)**2*csc(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \cot(bx + a)^2 \csc(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^2*cot(b*x + a)^2*csc(b*x + a), x)

3.115 $\int (c + dx) \cot^2(a + bx) \csc(a + bx) dx$

Optimal. Leaf size=108

$$\frac{id\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{2b^2} + \frac{id\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{2b^2} - \frac{d \csc(a + bx)}{2b^2} + \frac{(c + dx) \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{(c + dx) \cot(a + bx)}{2b}$$

[Out] ((c + d*x)*ArcTanh[E^(I*(a + b*x))])/b - (d*Csc[a + b*x])/(2*b^2) - ((c + d*x)*Cot[a + b*x]*Csc[a + b*x])/(2*b) - ((I/2)*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 + ((I/2)*d*PolyLog[2, E^(I*(a + b*x))])/b^2

Rubi [A] time = 0.107197, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {4415, 4183, 2279, 2391, 4185}

$$\frac{id\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{2b^2} + \frac{id\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{2b^2} - \frac{d \csc(a + bx)}{2b^2} + \frac{(c + dx) \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{(c + dx) \cot(a + bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Cot[a + b*x]^2*Csc[a + b*x], x]

[Out] ((c + d*x)*ArcTanh[E^(I*(a + b*x))])/b - (d*Csc[a + b*x])/(2*b^2) - ((c + d*x)*Cot[a + b*x]*Csc[a + b*x])/(2*b) - ((I/2)*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 + ((I/2)*d*PolyLog[2, E^(I*(a + b*x))])/b^2

Rule 4415

Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p/2, 0]

Rule 4183

Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))]^(n_), x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 4185

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :> -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]

, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x] /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]

Rubi steps

$$\begin{aligned} \int (c + dx) \cot^2(a + bx) \csc(a + bx) dx &= - \int (c + dx) \csc(a + bx) dx + \int (c + dx) \csc^3(a + bx) dx \\ &= \frac{2(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{d \csc(a + bx)}{2b^2} - \frac{(c + dx) \cot(a + bx) \csc(a + bx)}{2b} + \\ &= \frac{(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{d \csc(a + bx)}{2b^2} - \frac{(c + dx) \cot(a + bx) \csc(a + bx)}{2b} - \\ &= \frac{(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{d \csc(a + bx)}{2b^2} - \frac{(c + dx) \cot(a + bx) \csc(a + bx)}{2b} - \\ &= \frac{(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{d \csc(a + bx)}{2b^2} - \frac{(c + dx) \cot(a + bx) \csc(a + bx)}{2b} - \end{aligned}$$

Mathematica [B] time = 1.83628, size = 260, normalized size = 2.41

$$\frac{d \left(i \left(\text{PolyLog} \left(2, -e^{i(a+bx)} \right) - \text{PolyLog} \left(2, e^{i(a+bx)} \right) \right) + (a + bx) \left(\log \left(1 - e^{i(a+bx)} \right) - \log \left(1 + e^{i(a+bx)} \right) \right) \right)}{2b^2} - \frac{d \tan \left(\frac{1}{2}(a + bx) \right)}{4b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Cot[a + b*x]^2*Csc[a + b*x], x]

[Out] -(d*Cot[(a + b*x)/2])/(4*b^2) - (c*Csc[(a + b*x)/2]^2)/(8*b) - (d*x*Csc[(a + b*x)/2]^2)/(8*b) + (c*Log[Cos[(a + b*x)/2]])/(2*b) - (c*Log[Sin[(a + b*x)/2]])/(2*b) + (a*d*Log[Tan[(a + b*x)/2]])/(2*b^2) - (d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a + b*x))]))/(2*b^2) + (c*Sec[(a + b*x)/2]^2)/(8*b) + (d*x*Sec[(a + b*x)/2]^2)/(8*b) - (d*Tan[(a + b*x)/2])/(4*b^2)

Maple [B] time = 0.139, size = 246, normalized size = 2.3

$$\frac{dxbe^{3i(bx+a)} + bce^{3i(bx+a)} + dxbe^{i(bx+a)} + bce^{i(bx+a)} - ide^{3i(bx+a)} + ide^{i(bx+a)}}{b^2(e^{2i(bx+a)} - 1)^2} + \frac{c \text{Artanh}(e^{i(bx+a)})}{b} - \frac{d \ln(1 - e^{i(bx+a)})}{2b} x - \frac{d \ln(1 - e^{i(bx+a)})}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*cot(b*x+a)^2*csc(b*x+a), x)

[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2*(d*x*b*exp(3*I*(b*x+a))+b*c*exp(3*I*(b*x+a))+d*x*b*exp(I*(b*x+a))+b*c*exp(I*(b*x+a))-I*d*exp(3*I*(b*x+a))+I*d*exp(I*(b*x+a)))+1/b*c*arctanh(exp(I*(b*x+a)))-1/2/b*d*ln(1-exp(I*(b*x+a)))*x-1/2/b^2*d*ln(1-exp(I*(b*x+a)))*a+1/2*I*d*polylog(2,exp(I*(b*x+a)))/b^2+1/2/b*d*ln(exp(I*(b*x+a))+1)*x+1/2/b^2*d*ln(exp(I*(b*x+a))+1)*a-1/2*I*d*polylog(2,-exp(I*(b*x+a)))/b^2-1/b^2*d*a*arctanh(exp(I*(b*x+a)))

Maxima [B] time = 1.90929, size = 1037, normalized size = 9.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")

[Out]
$$\begin{aligned} & ((2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*\cos(4*b*x + 4*a) - 4*(b*d*x + b*c)*\cos(\\ & 2*b*x + 2*a) + (2*I*b*d*x + 2*I*b*c)*\sin(4*b*x + 4*a) + (-4*I*b*d*x - 4*I*b \\ & *c)*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) - (2*b*c*\cos(\\ & 4*b*x + 4*a) - 4*b*c*\cos(2*b*x + 2*a) + 2*I*b*c*\sin(4*b*x + 4*a) - 4*I*b*c* \\ & \sin(2*b*x + 2*a) + 2*b*c)*\arctan2(\sin(b*x + a), \cos(b*x + a) - 1) + (2*b*d* \\ & x*\cos(4*b*x + 4*a) - 4*b*d*x*\cos(2*b*x + 2*a) + 2*I*b*d*x*\sin(4*b*x + 4*a) \\ & - 4*I*b*d*x*\sin(2*b*x + 2*a) + 2*b*d*x)*\arctan2(\sin(b*x + a), -\cos(b*x + a) \\ & + 1) + (-4*I*b*d*x - 4*I*b*c - 4*d)*\cos(3*b*x + 3*a) + (-4*I*b*d*x - 4*I*b \\ & *c + 4*d)*\cos(b*x + a) - (2*d*\cos(4*b*x + 4*a) - 4*d*\cos(2*b*x + 2*a) + 2*I \\ & *d*\sin(4*b*x + 4*a) - 4*I*d*\sin(2*b*x + 2*a) + 2*d)*\operatorname{dilog}(-e^{(I*b*x + I*a)}) \\ & + (2*d*\cos(4*b*x + 4*a) - 4*d*\cos(2*b*x + 2*a) + 2*I*d*\sin(4*b*x + 4*a) - \\ & 4*I*d*\sin(2*b*x + 2*a) + 2*d)*\operatorname{dilog}(e^{(I*b*x + I*a)}) + (-I*b*d*x - I*b*c + \\ & (-I*b*d*x - I*b*c)*\cos(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*\cos(2*b*x + 2*a) \\ &) + (b*d*x + b*c)*\sin(4*b*x + 4*a) - 2*(b*d*x + b*c)*\sin(2*b*x + 2*a))*\log(\\ & \cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) + (I*b*d*x + I*b*c + \\ & (I*b*d*x + I*b*c)*\cos(4*b*x + 4*a) + (-2*I*b*d*x - 2*I*b*c)*\cos(2*b*x + 2*a) \\ &) - (b*d*x + b*c)*\sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*\sin(2*b*x + 2*a))*\log(\\ & \cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) + 4*(b*d*x + b*c - I* \\ & d)*\sin(3*b*x + 3*a) + 4*(b*d*x + b*c + I*d)*\sin(b*x + a))/(-4*I*b^2*\cos(4*b \\ & *x + 4*a) + 8*I*b^2*\cos(2*b*x + 2*a) + 4*b^2*\sin(4*b*x + 4*a) - 8*b^2*\sin(2 \\ & *b*x + 2*a) - 4*I*b^2) \end{aligned}$$

Fricas [B] time = 0.576427, size = 1191, normalized size = 11.03

$$2(bdx + bc) \cos(bx + a) + (id \cos(bx + a)^2 - id) \operatorname{Li}_2(\cos(bx + a) + i \sin(bx + a)) + (-id \cos(bx + a)^2 + id) \operatorname{Li}_2(\cos(bx + a) - i \sin(bx + a))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & 1/4*(2*(b*d*x + b*c)*\cos(b*x + a) + (I*d*\cos(b*x + a)^2 - I*d)*\operatorname{dilog}(\cos(b*x \\ & + a) + I*\sin(b*x + a)) + (-I*d*\cos(b*x + a)^2 + I*d)*\operatorname{dilog}(\cos(b*x + a) - \\ & I*\sin(b*x + a)) + (I*d*\cos(b*x + a)^2 - I*d)*\operatorname{dilog}(-\cos(b*x + a) + I*\sin(b \\ & *x + a)) + (-I*d*\cos(b*x + a)^2 + I*d)*\operatorname{dilog}(-\cos(b*x + a) - I*\sin(b*x + a) \\ &) - (b*d*x - (b*d*x + b*c)*\cos(b*x + a)^2 + b*c)*\log(\cos(b*x + a) + I*\sin(b \\ & *x + a) + 1) - (b*d*x - (b*d*x + b*c)*\cos(b*x + a)^2 + b*c)*\log(\cos(b*x + a) \\ &) - I*\sin(b*x + a) + 1) - ((b*c - a*d)*\cos(b*x + a)^2 - b*c + a*d)*\log(-1/2 \\ & *\cos(b*x + a) + 1/2*I*\sin(b*x + a) + 1/2) - ((b*c - a*d)*\cos(b*x + a)^2 - b \\ & *c + a*d)*\log(-1/2*\cos(b*x + a) - 1/2*I*\sin(b*x + a) + 1/2) + (b*d*x - (b*d \\ & *x + a*d)*\cos(b*x + a)^2 + a*d)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1) + (\\ & b*d*x - (b*d*x + a*d)*\cos(b*x + a)^2 + a*d)*\log(-\cos(b*x + a) - I*\sin(b*x + \\ & a) + 1) + 2*d*\sin(b*x + a))/(b^2*\cos(b*x + a)^2 - b^2) \end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \cot^2(a + bx) \csc(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cot(b*x+a)**2*csc(b*x+a),x)

[Out] Integral((c + d*x)*cot(a + b*x)**2*csc(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \cot (bx + a)^2 \csc (bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)*cot(b*x + a)^2*csc(b*x + a), x)

$$3.116 \quad \int \frac{\cot^2(a+bx) \csc(a+bx)}{c+dx} dx$$

Optimal. Leaf size=37

$$\text{Unintegrable}\left(\frac{\csc^3(a+bx)}{c+dx}, x\right) - \text{Unintegrable}\left(\frac{\csc(a+bx)}{c+dx}, x\right)$$

[Out] -Unintegrable[Csc[a + b*x]/(c + d*x), x] + Unintegrable[Csc[a + b*x]^3/(c + d*x), x]

Rubi [A] time = 0.0810282, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cot^2(a+bx) \csc(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x), x]

[Out] -Defer[Int][Csc[a + b*x]/(c + d*x), x] + Defer[Int][Csc[a + b*x]^3/(c + d*x), x]

Rubi steps

$$\int \frac{\cot^2(a+bx) \csc(a+bx)}{c+dx} dx = - \int \frac{\csc(a+bx)}{c+dx} dx + \int \frac{\csc^3(a+bx)}{c+dx} dx$$

Mathematica [A] time = 36.7121, size = 0, normalized size = 0.

$$\int \frac{\cot^2(a+bx) \csc(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x), x]

[Out] Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x), x]

Maple [A] time = 2.23, size = 0, normalized size = 0.

$$\int \frac{(\cot(bx+a))^2 \csc(bx+a)}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c), x)

[Out] int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x, algorithm="maxima")

[Out] (((b*d*x + b*c)*cos(3*b*x + 3*a) + (b*d*x + b*c)*cos(b*x + a) - d*sin(3*b*x + 3*a) + d*sin(b*x + a))*cos(4*b*x + 4*a) + (b*d*x + b*c - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - 2*d*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) - 2*((b*d*x + b*c)*cos(b*x + a) + d*sin(b*x + a))*cos(2*b*x + 2*a) + (b*d*x + b*c)*cos(b*x + a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(b*x + a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)), x) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(b*x + a)^2 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)), x) + (d*cos(3*b*x + 3*a) - d*cos(b*x + a) + (b*d*x + b*c)*sin(3*b*x + 3*a) + (b*d*x + b*c)*sin(b*x + a))*sin(4*b*x + 4*a) + (2*d*cos(2*b*x + 2*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a) - d)*sin(3*b*x + 3*a) + 2*(d*cos(b*x + a) - (b*d*x + b*c)*sin(b*x + a))*sin(2*b*x + 2*a) + d*sin(b*x + a))/(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cot (bx+a)^2 \csc (bx+a)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x, algorithm="fricas")

[Out] `integral(cot(b*x + a)^2*csc(b*x + a)/(d*x + c), x)`

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(a + bx) \csc(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(b*x+a)**2*csc(b*x+a)/(d*x+c), x)`

[Out] `Integral(cot(a + b*x)**2*csc(a + b*x)/(c + d*x), x)`

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(bx + a)^2 \csc(bx + a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c), x, algorithm="giac")`

[Out] `integrate(cot(b*x + a)^2*csc(b*x + a)/(d*x + c), x)`

$$3.117 \quad \int \frac{\cot^2(a+bx) \csc(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=37

$$\text{Unintegrable}\left(\frac{\csc^3(a+bx)}{(c+dx)^2}, x\right) - \text{Unintegrable}\left(\frac{\csc(a+bx)}{(c+dx)^2}, x\right)$$

[Out] -Unintegrable[Csc[a + b*x]/(c + d*x)^2, x] + Unintegrable[Csc[a + b*x]^3/(c + d*x)^2, x]

Rubi [A] time = 0.0804008, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cot^2(a+bx) \csc(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x)^2,x]

[Out] -Defer[Int][Csc[a + b*x]/(c + d*x)^2, x] + Defer[Int][Csc[a + b*x]^3/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\cot^2(a+bx) \csc(a+bx)}{(c+dx)^2} dx = - \int \frac{\csc(a+bx)}{(c+dx)^2} dx + \int \frac{\csc^3(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 42.6071, size = 0, normalized size = 0.

$$\int \frac{\cot^2(a+bx) \csc(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x)^2,x]

[Out] Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x)^2, x]

Maple [A] time = 3.468, size = 0, normalized size = 0.

$$\int \frac{(\cot(bx+a))^2 \csc(bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x)

```
[Out] int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (((b*d*x + b*c)*cos(3*b*x + 3*a) + (b*d*x + b*c)*cos(b*x + a) - 2*d*sin(3*b*x + 3*a) + 2*d*sin(b*x + a))*cos(4*b*x + 4*a) + (b*d*x + b*c - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - 4*d*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) - 2*((b*d*x + b*c)*cos(b*x + a) + 2*d*sin(b*x + a))*cos(2*b*x + 2*a) + (b*d*x + b*c)*cos(b*x + a) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 6*d^2)*sin(b*x + a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(b*x + a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 6*d^2)*sin(b*x + a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(b*x + a)^2 - 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x) + (2*d*cos(3*b*x + 3*a) - 2*d*cos(b*x + a) + (b*d*x + b*c)*sin(3*b*x + 3*a) + (b*d*x + b*c)*sin(b*x + a))*sin(4*b*x + 4*a) + 2*(2*d*cos(2*b*x + 2*a) - (b*d*x + b*c)*sin(2*b*x + 2*a) - d)*sin(3*b*x + 3*a) + 2*(2*d*cos(b*x + a) - (b*d*x + b*c)*sin(b*x + a))*sin(2*b*x + 2*a) + 2*d*sin(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*c
```

os(4*b*x + 4*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cot (bx+a)^2 \csc (bx+a)}{d^2 x^2+2 c d x+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(cot(b*x + a)^2*csc(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(a+bx) \csc(a+bx)}{(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)**2*csc(b*x+a)/(d*x+c)**2,x)

[Out] Integral(cot(a + b*x)**2*csc(a + b*x)/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot (bx+a)^2 \csc (bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(cot(b*x + a)^2*csc(b*x + a)/(d*x + c)^2, x)

3.118 $\int (c + dx)^{5/2} \cos^2(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=406

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} - \frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}} + \frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a - \frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}$$

[Out] $(15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[a + b*x])/(16*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[a + b*x])/ (4*b) + (5*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[3*a + 3*b*x])/(144*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[3*a + 3*b*x])/(12*b) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a - (b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(16*b^{(7/2)}) - (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(144*b^{(7/2)}) + (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[3*a - (3*b*c)/d])/(144*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[a - (b*c)/d])/(16*b^{(7/2)}) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[a + b*x])/(8*b^2) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[3*a + 3*b*x])/(72*b^2)$

Rubi [A] time = 0.666837, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} - \frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}} + \frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a - \frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^{(5/2)}*\text{Cos}[a + b*x]^2*\text{Sin}[a + b*x], x]$

[Out] $(15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[a + b*x])/(16*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[a + b*x])/ (4*b) + (5*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[3*a + 3*b*x])/(144*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[3*a + 3*b*x])/(12*b) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a - (b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(16*b^{(7/2)}) - (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(144*b^{(7/2)}) + (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[3*a - (3*b*c)/d])/(144*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[a - (b*c)/d])/(16*b^{(7/2)}) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[a + b*x])/(8*b^2) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[3*a + 3*b*x])/(72*b^2)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[p, 0]$

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{GtQ}[m, 0]$

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^{5/2} \cos^2(a+bx) \sin(a+bx) dx &= \int \left(\frac{1}{4}(c+dx)^{5/2} \sin(a+bx) + \frac{1}{4}(c+dx)^{5/2} \sin(3a+3bx) \right) dx \\
&= \frac{1}{4} \int (c+dx)^{5/2} \sin(a+bx) dx + \frac{1}{4} \int (c+dx)^{5/2} \sin(3a+3bx) dx \\
&= -\frac{(c+dx)^{5/2} \cos(a+bx)}{4b} - \frac{(c+dx)^{5/2} \cos(3a+3bx)}{12b} + \frac{(5d) \int (c+dx)^{3/2} \cos(3a+3bx) dx}{24b} \\
&= -\frac{(c+dx)^{5/2} \cos(a+bx)}{4b} - \frac{(c+dx)^{5/2} \cos(3a+3bx)}{12b} + \frac{5d(c+dx)^{3/2} \sin(a+bx)}{8b^2} \\
&= \frac{15d^2 \sqrt{c+dx} \cos(a+bx)}{16b^3} - \frac{(c+dx)^{5/2} \cos(a+bx)}{4b} + \frac{5d^2 \sqrt{c+dx} \cos(3a+3bx)}{144b^3} \\
&= \frac{15d^2 \sqrt{c+dx} \cos(a+bx)}{16b^3} - \frac{(c+dx)^{5/2} \cos(a+bx)}{4b} + \frac{5d^2 \sqrt{c+dx} \cos(3a+3bx)}{144b^3} \\
&= \frac{15d^2 \sqrt{c+dx} \cos(a+bx)}{16b^3} - \frac{(c+dx)^{5/2} \cos(a+bx)}{4b} + \frac{5d^2 \sqrt{c+dx} \cos(3a+3bx)}{144b^3} \\
&= \frac{15d^2 \sqrt{c+dx} \cos(a+bx)}{16b^3} - \frac{(c+dx)^{5/2} \cos(a+bx)}{4b} + \frac{5d^2 \sqrt{c+dx} \cos(3a+3bx)}{144b^3}
\end{aligned}$$

Mathematica [C] time = 16.3898, size = 1168, normalized size = 2.88

$$\frac{e^{-\frac{i(bc+ad)}{d}} \sqrt{c+dx} \left(-\frac{e^{2ia} \Gamma\left(\frac{3}{2}, -\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}} \Gamma\left(\frac{3}{2}, \frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}} \right) c^2}{8b} - \left(2\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos(3(a+bx)) - \sqrt{2\pi} \cos\left(3a - \frac{3bc}{d}\right) \right)$$

Maxima [C] time = 2.533, size = 1874, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")

[Out] $\frac{1}{3456}\sqrt{3}(80\sqrt{3}(d^3x^2+c^3)^{3/2}bd^2\sqrt{b^2+d^2}\sin(3((d^3x^2+c^3)b-bc+ad)/d)/\sqrt{d}+720\sqrt{3}(d^3x^2+c^3)^{3/2}bd^2\sqrt{b^2+d^2}\sin(((d^3x^2+c^3)b-bc+ad)/d)/\sqrt{d}-8(12\sqrt{3}(d^3x^2+c^3)^{5/2}b^2d\sqrt{b^2+d^2}\sqrt{b^2+d^2}/\sqrt{d}-5\sqrt{3}\sqrt{d^3x^2+c^3}d^3\sqrt{b^2+d^2}/\sqrt{d})\cos(3((d^3x^2+c^3)b-bc+ad)/d)-72(4\sqrt{3}(d^3x^2+c^3)^{5/2}b^2d\sqrt{b^2+d^2}/\sqrt{d}-15\sqrt{3}\sqrt{d^3x^2+c^3}d^3\sqrt{b^2+d^2}/\sqrt{d})\cos(((d^3x^2+c^3)b-bc+ad)/d)-((5\sqrt{\pi}\cos(1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))+5\sqrt{\pi}\cos(-1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))-5I\sqrt{\pi}\sin(1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))+5I\sqrt{\pi}\sin(-1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2})))d^3\sqrt{b^2+d^2}/\sqrt{d}\cos(-3(bc-ad)/d)-(5I\sqrt{\pi}\cos(1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))+5I\sqrt{\pi}\cos(-1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))+5\sqrt{\pi}\sin(1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))-5\sqrt{\pi}\sin(-1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2})))d^3\sqrt{b^2+d^2}/\sqrt{d}\sin(-3(bc-ad)/d)*\operatorname{erf}(\sqrt{d^3x^2+c^3})\sqrt{3Ib/d})-(\sqrt{3}(135\sqrt{\pi}\cos(1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))+135\sqrt{\pi}\cos(-1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))-135I\sqrt{\pi}\sin(1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))+135I\sqrt{\pi}\sin(-1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2})))d^3\sqrt{b^2+d^2}/\sqrt{d}\cos(-(bc-ad)/d)-\sqrt{3}(135I\sqrt{\pi}\cos(1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))+135I\sqrt{\pi}\cos(-1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))+135\sqrt{\pi}\sin(1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))-135\sqrt{\pi}\sin(-1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2})))d^3\sqrt{b^2+d^2}/\sqrt{d}\sin(-(bc-ad)/d)*\operatorname{erf}(\sqrt{d^3x^2+c^3})\sqrt{Ib/d})-(\sqrt{3}(135\sqrt{\pi}\cos(1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))+135\sqrt{\pi}\cos(-1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))+135I\sqrt{\pi}\sin(1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))-135I\sqrt{\pi}\sin(-1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2})))d^3\sqrt{b^2+d^2}/\sqrt{d}\cos(-(bc-ad)/d)-\sqrt{3}(-135I\sqrt{\pi}\cos(1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))-135I\sqrt{\pi}\cos(-1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))+135\sqrt{\pi}\sin(1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))-135\sqrt{\pi}\sin(-1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2})))d^3\sqrt{b^2+d^2}/\sqrt{d}\sin(-(bc-ad)/d)*\operatorname{erf}(\sqrt{d^3x^2+c^3})\sqrt{-Ib/d})-(5\sqrt{\pi}\cos(1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))+5\sqrt{\pi}\cos(-1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))+5I\sqrt{\pi}\sin(1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))-5I\sqrt{\pi}\sin(-1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2})))d^3\sqrt{b^2+d^2}/\sqrt{d}\cos(-3(bc-ad)/d)-(-5I\sqrt{\pi}\cos(1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))-5I\sqrt{\pi}\cos(-1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))+5\sqrt{\pi}\sin(1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2}))-5\sqrt{\pi}\sin(-1/4\pi+1/2\arctan(0,b)+1/2\arctan(0,d/\sqrt{d^2})))d^3\sqrt{b^2+d^2}/\sqrt{d}\sin(-3(bc-ad)/d)*\operatorname{erf}(\sqrt{d^3x^2+c^3})\sqrt{-3Ib/d}))/\sqrt{b^2+d^2}$

Fricas [A] time = 0.653204, size = 856, normalized size = 2.11

$$\frac{5\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\sin\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\sin\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/864*(5*\sqrt{6}*\pi*d^3*\sqrt{b/(pi*d)}*\cos(-3*(b*c - a*d)/d)*\text{fresnel_cos}(\sqrt{6}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) \\ & + 405*\sqrt{2}*\pi*d^3*\sqrt{b/(pi*d)}*\cos(-3*(b*c - a*d)/d)*\text{fresnel_cos}(\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) \\ & - 405*\sqrt{2}*\pi*d^3*\sqrt{b/(pi*d)}*\text{fresnel_sin}(\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) \\ & * \sin(-3*(b*c - a*d)/d) - 5*\sqrt{6}*\pi*d^3*\sqrt{b/(pi*d)}*\text{fresnel_sin}(\sqrt{6}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) \\ & * \sin(-3*(b*c - a*d)/d) - 24*(30*b*d^2*\cos(b*x + a) - (12*b^3*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*\cos(b*x + a)^3 \\ & + 10*(2*b^2*d^2*x + 2*b^2*c*d + (b^2*d^2*x + b^2*c*d)*\cos(b*x + a)^2)*\sin(b*x + a)*\sqrt{d*x + c})/b^4 \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a),x)

[Out] Timed out

Giac [C] time = 1.52087, size = 2724, normalized size = 6.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/1728*(12*(\sqrt{6}*\sqrt{\pi})*d^2*\text{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((3*I*b*c - 3*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b)} \\ & + 9*\sqrt{2}*\sqrt{\pi}*d^2*\text{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((I*b*c - I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b)} \\ & + 9*\sqrt{2}*\sqrt{\pi}*d^2*\text{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-I*b*c + I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b)} \\ & + \sqrt{6}*\sqrt{\pi}*d^2*\text{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-3*I*b*c + 3*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b)} \\ & + 6*\sqrt{d*x + c}*d*e^{((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b} + 18*\sqrt{d*x + c}*d*e^{((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b} \\ & + 18*\sqrt{d*x + c}*d*e^{((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b} + 6*\sqrt{d*x + c}*d*e^{((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b} \\ & *c^2 + d^2*((I*\sqrt{6}*\sqrt{\pi})*(-12*I*b^2*c^2*d + 12*b*c*d^2 + 5*I*d^3)*d*\text{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((3*I*b*c - 3*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^3)} \\ & - 6*I*(12*I*(d*x + c)^{(5/2)}*b^2*d - 24*I*(d*x + c)^{(3/2)}*b^2*c*d + 12*I*\sqrt{d*x + c}) \end{aligned}$$

$$\begin{aligned}
& c) * b^2 * c^2 * d + 10 * (d * x + c)^{(3/2)} * b * d^2 - 12 * \text{sqrt}(d * x + c) * b * c * d^2 - 5 * I * \text{sqrt}(d * x + c) * d^3 * e^{((-3 * I * (d * x + c) * b + 3 * I * b * c - 3 * I * a * d) / d) / b^3} / d^2 + 2 \\
& 7 * (I * \text{sqrt}(2) * \text{sqrt}(\pi) * (-4 * I * b^2 * c^2 * d + 12 * b * c * d^2 + 15 * I * d^3) * d * \text{erf}(-1/2 * \text{sqrt}(2) * \text{sqrt}(b * d) * \text{sqrt}(d * x + c) * (I * b * d / \text{sqrt}(b^2 * d^2) + 1) / d) * e^{((I * b * c - I * a * d) / d) / (\text{sqrt}(b * d) * (I * b * d / \text{sqrt}(b^2 * d^2) + 1) * b^3)} - 2 * I * (4 * I * (d * x + c)^{(5/2)} * b^2 * d - 8 * I * (d * x + c)^{(3/2)} * b^2 * c * d + 4 * I * \text{sqrt}(d * x + c) * b^2 * c^2 * d + 10 * (d * x + c)^{(3/2)} * b * d^2 - 12 * \text{sqrt}(d * x + c) * b * c * d^2 - 15 * I * \text{sqrt}(d * x + c) * d^3) * e^{((-I * (d * x + c) * b + I * b * c - I * a * d) / d) / b^3} / d^2 + 27 * (I * \text{sqrt}(2) * \text{sqrt}(\pi) * (-4 * I * b^2 * c^2 * d - 12 * b * c * d^2 + 15 * I * d^3) * d * \text{erf}(-1/2 * \text{sqrt}(2) * \text{sqrt}(b * d) * \text{sqrt}(d * x + c) * (-I * b * d / \text{sqrt}(b^2 * d^2) + 1) / d) * e^{((-I * b * c + I * a * d) / d) / (\text{sqrt}(b * d) * (-I * b * d / \text{sqrt}(b^2 * d^2) + 1) * b^3)} - 2 * I * (4 * I * (d * x + c)^{(5/2)} * b^2 * d - 8 * I * (d * x + c)^{(3/2)} * b^2 * c * d + 4 * I * \text{sqrt}(d * x + c) * b^2 * c^2 * d - 10 * (d * x + c)^{(3/2)} * b * d^2 + 12 * \text{sqrt}(d * x + c) * b * c * d^2 - 15 * I * \text{sqrt}(d * x + c) * d^3) * e^{((I * (d * x + c) * b - I * b * c + I * a * d) / d) / b^3} / d^2 + (I * \text{sqrt}(6) * \text{sqrt}(\pi) * (-12 * I * b^2 * c^2 * d - 12 * b * c * d^2 + 5 * I * d^3) * d * \text{erf}(-1/2 * \text{sqrt}(6) * \text{sqrt}(b * d) * \text{sqrt}(d * x + c) * (-I * b * d / \text{sqrt}(b^2 * d^2) + 1) / d) * e^{((-3 * I * b * c + 3 * I * a * d) / d) / (\text{sqrt}(b * d) * (-I * b * d / \text{sqrt}(b^2 * d^2) + 1) * b^3)} - 6 * I * (12 * I * (d * x + c)^{(5/2)} * b^2 * d - 24 * I * (d * x + c)^{(3/2)} * b^2 * c * d + 12 * I * \text{sqrt}(d * x + c) * b^2 * c^2 * d - 10 * (d * x + c)^{(3/2)} * b * d^2 + 12 * \text{sqrt}(d * x + c) * b * c * d^2 - 5 * I * \text{sqrt}(d * x + c) * d^3) * e^{((3 * I * (d * x + c) * b - 3 * I * b * c + 3 * I * a * d) / d) / b^3} / d^2 + 12 * (I * \text{sqrt}(6) * \text{sqrt}(\pi) * (2 * I * b * c * d - d^2) * d * \text{erf}(-1/2 * \text{sqrt}(6) * \text{sqrt}(b * d) * \text{sqrt}(d * x + c) * (I * b * d / \text{sqrt}(b^2 * d^2) + 1) / d) * e^{((3 * I * b * c - 3 * I * a * d) / d) / (\text{sqrt}(b * d) * (I * b * d / \text{sqrt}(b^2 * d^2) + 1) * b^2)} + 9 * I * \text{sqrt}(2) * \text{sqrt}(\pi) * (2 * I * b * c * d - 3 * d^2) * d * \text{erf}(-1/2 * \text{sqrt}(2) * \text{sqrt}(b * d) * \text{sqrt}(d * x + c) * (I * b * d / \text{sqrt}(b^2 * d^2) + 1) / d) * e^{((I * b * c - I * a * d) / d) / (\text{sqrt}(b * d) * (I * b * d / \text{sqrt}(b^2 * d^2) + 1) * b^2)} + 9 * I * \text{sqrt}(2) * \text{sqrt}(\pi) * (2 * I * b * c * d + 3 * d^2) * d * \text{erf}(-1/2 * \text{sqrt}(2) * \text{sqrt}(b * d) * \text{sqrt}(d * x + c) * (-I * b * d / \text{sqrt}(b^2 * d^2) + 1) / d) * e^{((-I * b * c + I * a * d) / d) / (\text{sqrt}(b * d) * (-I * b * d / \text{sqrt}(b^2 * d^2) + 1) * b^2)} + I * \text{sqrt}(6) * \text{sqrt}(\pi) * (2 * I * b * c * d + d^2) * d * \text{erf}(-1/2 * \text{sqrt}(6) * \text{sqrt}(b * d) * \text{sqrt}(d * x + c) * (-I * b * d / \text{sqrt}(b^2 * d^2) + 1) / d) * e^{((-3 * I * b * c + 3 * I * a * d) / d) / (\text{sqrt}(b * d) * (-I * b * d / \text{sqrt}(b^2 * d^2) + 1) * b^2)} - 6 * I * (2 * I * (d * x + c)^{(3/2)} * b * d - 2 * I * \text{sqrt}(d * x + c) * b * c * d - \text{sqrt}(d * x + c) * d^2) * e^{((3 * I * (d * x + c) * b - 3 * I * b * c + 3 * I * a * d) / d) / b^2} - 18 * I * (2 * I * (d * x + c)^{(3/2)} * b * d - 2 * I * \text{sqrt}(d * x + c) * b * c * d - 3 * \text{sqrt}(d * x + c) * d^2) * e^{((I * (d * x + c) * b - I * b * c + I * a * d) / d) / b^2} - 18 * I * (2 * I * (d * x + c)^{(3/2)} * b * d - 2 * I * \text{sqrt}(d * x + c) * b * c * d + 3 * \text{sqrt}(d * x + c) * d^2) * e^{((-I * (d * x + c) * b + I * b * c - I * a * d) / d) / b^2} - 6 * I * (2 * I * (d * x + c)^{(3/2)} * b * d - 2 * I * \text{sqrt}(d * x + c) * b * c * d + \text{sqrt}(d * x + c) * d^2) * e^{((-3 * I * (d * x + c) * b + 3 * I * b * c - 3 * I * a * d) / d) / b^2} * c) / d
\end{aligned}$$

3.119 $\int (c + dx)^{3/2} \cos^2(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=353

$$\frac{\sqrt{\frac{\pi}{6}} d^{3/2} \sin\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}} - \frac{3\sqrt{\frac{\pi}{2}} d^{3/2} \sin\left(a - \frac{bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} - \frac{3\sqrt{\frac{\pi}{2}} d^{3/2} \cos\left(a - \frac{bc}{d}\right)}{8b^{5/2}}$$

```
[Out] -((c + d*x)^(3/2)*Cos[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/(
(12*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b
*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/2)) -
(d^(3/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[3*a - (3*b*c)/d])/(24*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + (3*d*Sq
rt[c + d*x]*Sin[a + b*x])/(8*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(24*
b^2)
```

Rubi [A] time = 0.5262, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{6}} d^{3/2} \sin\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}} - \frac{3\sqrt{\frac{\pi}{2}} d^{3/2} \sin\left(a - \frac{bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} - \frac{3\sqrt{\frac{\pi}{2}} d^{3/2} \cos\left(a - \frac{bc}{d}\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x], x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/(
(12*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b
*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/2)) -
(d^(3/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[3*a - (3*b*c)/d])/(24*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + (3*d*Sq
rt[c + d*x]*Sin[a + b*x])/(8*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(24*
b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
```

$*e - c*f)/d]$, $\text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /;$ $\text{FreeQ}\{c, d, e, f\}, x]$ && $\text{ComplexFreeQ}[f]$ && $\text{NeQ}[d*e - c*f, 0]$

Rule 3305

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] := \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Sin}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /;$ $\text{FreeQ}\{c, d, e, f\}, x]$ && $\text{ComplexFreeQ}[f]$ && $\text{EqQ}[d*e - c*f, 0]$

Rule 3351

$\text{Int}[\text{Sin}[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] := \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /;$ $\text{FreeQ}\{d, e, f\}, x]$

Rule 3304

$\text{Int}[\sin[\text{Pi}/2 + (e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] := \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Cos}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /;$ $\text{FreeQ}\{c, d, e, f\}, x]$ && $\text{ComplexFreeQ}[f]$ && $\text{EqQ}[d*e - c*f, 0]$

Rule 3352

$\text{Int}[\text{Cos}[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] := \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /;$ $\text{FreeQ}\{d, e, f\}, x]$

Rubi steps

$$\begin{aligned} \int (c + dx)^{3/2} \cos^2(a + bx) \sin(a + bx) dx &= \int \left(\frac{1}{4}(c + dx)^{3/2} \sin(a + bx) + \frac{1}{4}(c + dx)^{3/2} \sin(3a + 3bx) \right) dx \\ &= \frac{1}{4} \int (c + dx)^{3/2} \sin(a + bx) dx + \frac{1}{4} \int (c + dx)^{3/2} \sin(3a + 3bx) dx \\ &= -\frac{(c + dx)^{3/2} \cos(a + bx)}{4b} - \frac{(c + dx)^{3/2} \cos(3a + 3bx)}{12b} + \frac{d \int \sqrt{c + dx} \cos(3a + 3bx) dx}{8b} \\ &= -\frac{(c + dx)^{3/2} \cos(a + bx)}{4b} - \frac{(c + dx)^{3/2} \cos(3a + 3bx)}{12b} + \frac{3d\sqrt{c + dx} \sin(a + bx)}{8b^2} \\ &= -\frac{(c + dx)^{3/2} \cos(a + bx)}{4b} - \frac{(c + dx)^{3/2} \cos(3a + 3bx)}{12b} + \frac{3d\sqrt{c + dx} \sin(a + bx)}{8b^2} \\ &= -\frac{(c + dx)^{3/2} \cos(a + bx)}{4b} - \frac{(c + dx)^{3/2} \cos(3a + 3bx)}{12b} + \frac{3d\sqrt{c + dx} \sin(a + bx)}{8b^2} \\ &= -\frac{(c + dx)^{3/2} \cos(a + bx)}{4b} - \frac{(c + dx)^{3/2} \cos(3a + 3bx)}{12b} - \frac{3d^{3/2} \sqrt{\frac{\pi}{2}} \cos\left(a - \frac{bc}{d}\right)}{8b^{5/2}} \end{aligned}$$

Mathematica [C] time = 9.26121, size = 676, normalized size = 1.92

$$\frac{c\sqrt{c + dx} e^{-\frac{i(ad+bc)}{d}} \left(-\frac{e^{2ia} \text{Gamma}\left(\frac{3}{2}, -\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}} \text{Gamma}\left(\frac{3}{2}, \frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}} \right)}{8b} - \frac{d\sqrt{\frac{b}{d}} \left(\sqrt{2\pi} \text{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c + dx}\right) \right) \left(3d \sin\left(a - \frac{bc}{d}\right) \right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]

[Out] (c*Sqrt[c + d*x]*(-(E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)*b*(c + d*x))/d]) - (E^((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[(I*b*(c + d*x))/d])/(8*b*E^((I*(b*c + a*d))/d)) - (c*(2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(24*Sqrt[3]*b*Sqrt[b/d]) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(3*d*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a + b*x])))/(16*b^3) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(48*Sqrt[3]*b^3)

Maple [A] time = 0.031, size = 384, normalized size = 1.1

$$2 \frac{1}{d} \left(-1/8 \frac{d(dx+c)^{3/2}}{b} \cos\left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d}\right) + 3/8 \frac{d}{b} \left(1/2 \frac{d\sqrt{dx+c}}{b} \sin\left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d}\right) - 1/4 \frac{d\sqrt{2}\sqrt{\pi}}{b} \left(\cos\left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x)

[Out] 2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/8/b*d*(1/2/b*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))-1/24/b*d*(d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/8/b*d*(1/6/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))

Maxima [C] time = 2.52107, size = 1793, normalized size = 5.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")

[Out] -1/576*sqrt(3)*(16*sqrt(3)*(d*x + c)^(3/2)*b*d*abs(b)*cos(3*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 48*sqrt(3)*(d*x + c)^(3/2)*b*d*abs(b)*cos(((d*x + c)*b - b*c + a*d)/d)/abs(d) - 8*sqrt(3)*sqrt(d*x + c)*d^2*abs(b)*sin(3*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 72*sqrt(3)*sqrt(d*x + c)*d^2*abs(b)*sin(((d*x + c)*b - b*c + a*d)/d)/abs(d) - ((-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,

```

b) + 1/2*arctan2(0, d/sqrt(d^2))) * d^2 * sqrt(abs(b)/abs(d)) * cos(-3*(b*c - a*d)/d) - (sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) * d^2 * sqrt(abs(b)/abs(d)) * sin(-3*(b*c - a*d)/d) * erf(sqrt(d*x + c) * sqrt(3*I*b/d)) - (sqrt(3)*(-9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * cos(-(b*c - a*d)/d) - sqrt(3)*(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * sin(-(b*c - a*d)/d) * erf(sqrt(d*x + c) * sqrt(I*b/d)) - (sqrt(3)*(9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * cos(-(b*c - a*d)/d) - sqrt(3)*(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * sin(-(b*c - a*d)/d) * erf(sqrt(d*x + c) * sqrt(-I*b/d)) - ((I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * cos(-3*(b*c - a*d)/d) - (sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * sin(-3*(b*c - a*d)/d) * erf(sqrt(d*x + c) * sqrt(-3*I*b/d))) * abs(d)/(b^2*d*abs(b))

```

Fricas [A] time = 0.624486, size = 716, normalized size = 2.03

$$\sqrt{6}\pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{3(bc-ad)}{d}\right) S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) + 27\sqrt{2}\pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{bc-ad}{d}\right) S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) + 27\sqrt{2}\pi d^2 \sqrt{\frac{b}{\pi d}} C$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) * sin(-(b*c - a*d)/d) + sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) * sin(-3*(b*c - a*d)/d) + 24*(2*(b^2*d*x + b^2*c) * cos(b*x + a)^3 - (b*d*cos(b*x + a)^2 + 2*b*d)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a),x)

[Out] Timed out

Giac [C] time = 1.35446, size = 1513, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/288*(2*(\sqrt{6}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x+c})*(I \\ & *b*d/\sqrt{b^2*d^2+1})/d)*e^{((3*I*b*c-3*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{ \\ & (b^2*d^2+1)*b)+9*\sqrt{2}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{ \\ & d*x+c}*(I*b*d/\sqrt{b^2*d^2+1})/d)*e^{((I*b*c-I*a*d)/d)/(\sqrt{b*d}*(I*b \\ & *d/\sqrt{b^2*d^2+1)*b)+9*\sqrt{2}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d} \\ &)*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2+1})/d)*e^{((-I*b*c+I*a*d)/d)/(\sqrt{ \\ & b*d}*(-I*b*d/\sqrt{b^2*d^2+1)*b)+\sqrt{6}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{6})* \\ & \sqrt{b*d}*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2+1})/d)*e^{((-3*I*b*c+3*I*a* \\ & d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2+1)*b)+6*\sqrt{d*x+c}*d*e^{((3*I* \\ & (d*x+c)*b-3*I*b*c+3*I*a*d)/d)/b+18*\sqrt{d*x+c}*d*e^{((I*(d*x+c)* \\ & b-I*b*c+I*a*d)/d)/b+18*\sqrt{d*x+c}*d*e^{((-I*(d*x+c)*b+I*b*c-I \\ & *a*d)/d)/b+6*\sqrt{d*x+c}*d*e^{((-3*I*(d*x+c)*b+3*I*b*c-3*I*a*d)/d} \\ & /b)*c+I*\sqrt{6}*\sqrt{\pi})*(2*I*b*c*d-d^2)*d*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d})* \\ & \sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2+1})/d)*e^{((3*I*b*c-3*I*a*d)/d)/(\sqrt{b \\ & *d}*(I*b*d/\sqrt{b^2*d^2+1})*b^2)+9*I*\sqrt{2}*\sqrt{\pi})*(2*I*b*c*d-3*d^ \\ & 2)*d*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2+1})/d)* \\ & e^{((I*b*c-I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2+1})*b^2)+9*I*\sqrt{2} \\ & *\sqrt{\pi})*(2*I*b*c*d+3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x+c})* \\ & (-I*b*d/\sqrt{b^2*d^2+1})/d)*e^{((-I*b*c+I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{ \\ & t(b^2*d^2+1)*b^2)+I*\sqrt{6}*\sqrt{\pi})*(2*I*b*c*d+d^2)*d*\operatorname{erf}(-1/2*\sqrt{ \\ & 6}*\sqrt{b*d}*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2+1})/d)*e^{((-3*I*b*c+3* \\ & I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2+1})*b^2)-6*I*(2*I*(d*x+c)^(\\ & 3/2)*b*d-2*I*\sqrt{d*x+c})*b*c*d-\sqrt{d*x+c}*d^2)*e^{((3*I*(d*x+c)*b \\ & -3*I*b*c+3*I*a*d)/d)/b^2-18*I*(2*I*(d*x+c)^(3/2)*b*d-2*I*\sqrt{d*x \\ & +c})*b*c*d-3*\sqrt{d*x+c}*d^2)*e^{((I*(d*x+c)*b-I*b*c+I*a*d)/d)/b^ \\ & 2-18*I*(2*I*(d*x+c)^(3/2)*b*d-2*I*\sqrt{d*x+c})*b*c*d+3*\sqrt{d*x+c} \\ & *d^2)*e^{((-I*(d*x+c)*b+I*b*c-I*a*d)/d)/b^2-6*I*(2*I*(d*x+c)^(3/ \\ & 2)*b*d-2*I*\sqrt{d*x+c})*b*c*d+\sqrt{d*x+c}*d^2)*e^{((-3*I*(d*x+c)*b \\ & +3*I*b*c-3*I*a*d)/d)/b^2)/d \end{aligned}$$

3.120 $\int \sqrt{c + dx} \cos^2(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=304

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(a - \frac{bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{\frac{\pi}{6}} \sqrt{d} \cos\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} - \frac{\sqrt{\frac{\pi}{6}} \sqrt{d} \sin\left(3a - \frac{3bc}{d}\right) S\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}$$

[Out] $-(\text{Sqrt}[c + d*x]*\text{Cos}[a + b*x])/(4*b) - (\text{Sqrt}[c + d*x]*\text{Cos}[3*a + 3*b*x])/(12*b) + (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a - (b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(4*b^{(3/2)}) + (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(12*b^{(3/2)}) - (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}/6]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[3*a - (3*b*c)/d])/(12*b^{(3/2)}) - (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[a - (b*c)/d])/(4*b^{(3/2)})$

Rubi [A] time = 0.41614, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(a - \frac{bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{\frac{\pi}{6}} \sqrt{d} \cos\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} - \frac{\sqrt{\frac{\pi}{6}} \sqrt{d} \sin\left(3a - \frac{3bc}{d}\right) S\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[c + d*x]*\text{Cos}[a + b*x]^2*\text{Sin}[a + b*x], x]$

[Out] $-(\text{Sqrt}[c + d*x]*\text{Cos}[a + b*x])/(4*b) - (\text{Sqrt}[c + d*x]*\text{Cos}[3*a + 3*b*x])/(12*b) + (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a - (b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(4*b^{(3/2)}) + (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(12*b^{(3/2)}) - (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}/6]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[3*a - (3*b*c)/d])/(12*b^{(3/2)}) - (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[a - (b*c)/d])/(4*b^{(3/2)})$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)*((c_.) + (d_.)*(x_.))^{(m_.)*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] :> \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] :> -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{GtQ}[m, 0]$

Rule 3306

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] :> \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{ComplexFreeQ}[f] \&\& \text{NeQ}[d*e - c*f, 0]$

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d,
Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f},
x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d,
Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{c+dx} \cos^2(a+bx) \sin(a+bx) dx &= \int \left(\frac{1}{4} \sqrt{c+dx} \sin(a+bx) + \frac{1}{4} \sqrt{c+dx} \sin(3a+3bx) \right) dx \\
&= \frac{1}{4} \int \sqrt{c+dx} \sin(a+bx) dx + \frac{1}{4} \int \sqrt{c+dx} \sin(3a+3bx) dx \\
&= -\frac{\sqrt{c+dx} \cos(a+bx)}{4b} - \frac{\sqrt{c+dx} \cos(3a+3bx)}{12b} + \frac{d \int \frac{\cos(3a+3bx)}{\sqrt{c+dx}} dx}{24b} + \frac{d \int \frac{\cos(a+bx)}{\sqrt{c+dx}} dx}{24b} \\
&= -\frac{\sqrt{c+dx} \cos(a+bx)}{4b} - \frac{\sqrt{c+dx} \cos(3a+3bx)}{12b} + \frac{\left(d \cos\left(3a - \frac{3bc}{d}\right) \right) \int \frac{\cos\left(\frac{3bx}{\sqrt{c+dx}}\right) dx}{24b}}{24b} \\
&= -\frac{\sqrt{c+dx} \cos(a+bx)}{4b} - \frac{\sqrt{c+dx} \cos(3a+3bx)}{12b} + \frac{\cos\left(3a - \frac{3bc}{d}\right) \text{Subst}\left(\int \frac{\cos\left(\frac{3bx}{\sqrt{c+dx}}\right) dx}{24b}\right)}{24b} \\
&= -\frac{\sqrt{c+dx} \cos(a+bx)}{4b} - \frac{\sqrt{c+dx} \cos(3a+3bx)}{12b} + \frac{\sqrt{d} \sqrt{\frac{\pi}{2}} \cos\left(a - \frac{bc}{d}\right) C\left(\frac{\sqrt{b}}{\sqrt{d}} \sqrt{\frac{c+dx}{d}}\right)}{4b^{3/2}}
\end{aligned}$$

Mathematica [C] time = 6.64042, size = 278, normalized size = 0.91

$$\frac{-\sqrt{2\pi} \cos\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\sqrt{\frac{6}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c+dx}\right) + \sqrt{2\pi} \sin\left(3a - \frac{3bc}{d}\right) S\left(\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right) + 2\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left(3a - \frac{3bc}{d}\right)}{24\sqrt{3}b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x], x]
```

```
[Out] (Sqrt[c + d*x]*(-(E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[(-I)*b*(c + d*x)/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[(I*b*(c + d*x))/d]))/(8*b*E^((I*(b*c + a*d))/d)) - (2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt
```

$[b/d] \cdot \sqrt{6/\pi} \cdot \sqrt{c + d \cdot x}] + \sqrt{2 \cdot \pi} \cdot \text{FresnelS}[\sqrt{b/d} \cdot \sqrt{6/\pi} \cdot \sqrt{c + d \cdot x}] \cdot \sin[3 \cdot a - (3 \cdot b \cdot c)/d]/(24 \cdot \sqrt{3} \cdot b \cdot \sqrt{b/d})$

Maple [A] time = 0.03, size = 296, normalized size = 1.

$$2 \frac{1}{d} \left(-\frac{1}{8} \frac{d\sqrt{dx+c}}{b} \cos\left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d}\right) + \frac{1}{16} \frac{d\sqrt{2}\sqrt{\pi}}{b} \left(\cos\left(\frac{ad-bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{2}\sqrt{dx+cb}}{\sqrt{\pi d}} \frac{1}{\sqrt{\frac{b}{d}}}\right) - \sin\left(\frac{ad-bc}{d}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x)

[Out] $2/d * (-1/8/b * d * (d*x+c)^{(1/2)} * \cos(1/d * (d*x+c) * b + (a*d-b*c)/d) + 1/16/b * d * 2^{(1/2)} * \pi^{(1/2)} / (b/d)^{(1/2)} * (\cos((a*d-b*c)/d) * \text{FresnelC}(2^{(1/2)}/\pi^{(1/2)} / (b/d)^{(1/2)} * (d*x+c)^{(1/2)} * b/d) - \sin((a*d-b*c)/d) * \text{FresnelS}(2^{(1/2)}/\pi^{(1/2)} / (b/d)^{(1/2)} * (d*x+c)^{(1/2)} * b/d)) - 1/24/b * d * (d*x+c)^{(1/2)} * \cos(3/d * (d*x+c) * b + 3 * (a*d-b*c)/d) + 1/144/b * d * 2^{(1/2)} * \pi^{(1/2)} * 3^{(1/2)} / (b/d)^{(1/2)} * (\cos(3 * (a*d-b*c)/d) * \text{FresnelC}(2^{(1/2)}/\pi^{(1/2)} * 3^{(1/2)} / (b/d)^{(1/2)} * (d*x+c)^{(1/2)} * b/d) - \sin(3 * (a*d-b*c)/d) * \text{FresnelS}(2^{(1/2)}/\pi^{(1/2)} * 3^{(1/2)} / (b/d)^{(1/2)} * (d*x+c)^{(1/2)} * b/d))$

Maxima [C] time = 2.40068, size = 1656, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")

[Out] $-1/288 * \sqrt{3} * (8 * \sqrt{3} * \sqrt{d*x+c} * d * \text{abs}(b) * \cos(3 * ((d*x+c) * b - b*c + a*d)/d) / \text{abs}(d) + 24 * \sqrt{3} * \sqrt{d*x+c} * d * \text{abs}(b) * \cos(((d*x+c) * b - b*c + a*d)/d) / \text{abs}(d) - ((\sqrt{\pi} * \cos(1/4 * \pi + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\sqrt{d^2})) + \sqrt{\pi} * \cos(-1/4 * \pi + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\sqrt{d^2})) - I * \sqrt{\pi} * \sin(1/4 * \pi + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\sqrt{d^2})) + I * \sqrt{\pi} * \sin(-1/4 * \pi + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\sqrt{d^2}))) * d * \sqrt{\text{abs}(b) / \text{abs}(d)} * \cos(-3 * (b*c - a*d)/d) - (I * \sqrt{\pi} * \cos(1/4 * \pi + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\sqrt{d^2})) + I * \sqrt{\pi} * \cos(-1/4 * \pi + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\sqrt{d^2})) + \sqrt{\pi} * \sin(1/4 * \pi + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\sqrt{d^2})) - \sqrt{\pi} * \sin(-1/4 * \pi + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\sqrt{d^2}))) * d * \sqrt{\text{abs}(b) / \text{abs}(d)} * \sin(-3 * (b*c - a*d)/d) * \text{erf}(\sqrt{d*x+c} * \sqrt{3 * I * b/d}) - (\sqrt{3} * (3 * \sqrt{\pi} * \cos(1/4 * \pi + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\sqrt{d^2})) + 3 * \sqrt{\pi} * \cos(-1/4 * \pi + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\sqrt{d^2})) - 3 * I * \sqrt{\pi} * \sin(1/4 * \pi + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\sqrt{d^2})) + 3 * I * \sqrt{\pi} * \sin(-1/4 * \pi + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\sqrt{d^2}))) * d * \sqrt{\text{abs}(b) / \text{abs}(d)} * \cos(-(b*c - a*d)/d) - \sqrt{3} * (3 * I * \sqrt{\pi} * \cos(1/4 * \pi + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\sqrt{d^2})) + 3 * I * \sqrt{\pi} * \cos(-1/4 * \pi + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\sqrt{d^2})) + 3 * \sqrt{\pi} * \sin(1/4 * \pi + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\sqrt{d^2})) - 3 * \sqrt{\pi} * \sin(-1/4 * \pi + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\sqrt{d^2}))) * d * \sqrt{\text{abs}(b) / \text{abs}(d)} * \sin(-(b*c - a*d)/d) * \text{erf}(\sqrt{d*x+c} * \sqrt{I * b/d}) - (\sqrt{3} * (3 * \sqrt{\pi} * \cos(1/4 * \pi + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\sqrt{d^2})) + 3 * \sqrt{\pi} * \cos(-1/4 * \pi + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\sqrt{d^2})) + 3 * I * \sqrt{\pi} * \sin(1/4 * \pi + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\sqrt{d^2})) + 3 * I * \sqrt{\pi} * \sin(-1/4 * \pi + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\sqrt{d^2}))) * d * \sqrt{\text{abs}(b) / \text{abs}(d)}$

```
) - 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
)*d*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) - sqrt(3)*(-3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
)*d*sqrt(abs(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
)*d*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
)*d*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*abs(d)/(b*d*abs(b))
```

Fricas [A] time = 0.59076, size = 613, normalized size = 2.02

$$\frac{\sqrt{6}\pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{3(bc-ad)}{d}\right) C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) + 9\sqrt{2}\pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{bc-ad}{d}\right) C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 9\sqrt{2}\pi d \sqrt{\frac{b}{\pi d}} S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)}{72b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a), x, algorithm="fricas")

[Out] 1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*sqrt(d*x + c)*b*cos(b*x + a)^3/b^2

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a), x)

[Out] Timed out

Giac [C] time = 1.22731, size = 659, normalized size = 2.17

$$\frac{\sqrt{6}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{2d}\right) e^{\left(\frac{3ibc-3iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} + \frac{9\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{2d}\right) e^{\left(\frac{ibc-iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} + \frac{9\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{2d}\right) e^{\left(-\frac{ibc-3iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/144*(sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 6*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b/d
```

3.121 $\int \sqrt{c + dx} \cos^2(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=304

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(a - \frac{bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{\frac{\pi}{6}} \sqrt{d} \cos\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} - \frac{\sqrt{\frac{\pi}{6}} \sqrt{d} \sin\left(3a - \frac{3bc}{d}\right)}{12b^{3/2}}$$

```
[Out] -(Sqrt[c + d*x]*Cos[a + b*x])/(4*b) - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(12*
b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]
*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) - (Sqrt
[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a
- (3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2))
```

Rubi [A] time = 0.419121, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(a - \frac{bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{\frac{\pi}{6}} \sqrt{d} \cos\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} - \frac{\sqrt{\frac{\pi}{6}} \sqrt{d} \sin\left(3a - \frac{3bc}{d}\right)}{12b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[a + b*x])/(4*b) - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(12*
b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]
*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) - (Sqrt
[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a
- (3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2))
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]
^n*cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3304

Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned} \int \sqrt{c+dx} \cos^2(a+bx) \sin(a+bx) dx &= \int \left(\frac{1}{4} \sqrt{c+dx} \sin(a+bx) + \frac{1}{4} \sqrt{c+dx} \sin(3a+3bx) \right) dx \\ &= \frac{1}{4} \int \sqrt{c+dx} \sin(a+bx) dx + \frac{1}{4} \int \sqrt{c+dx} \sin(3a+3bx) dx \\ &= -\frac{\sqrt{c+dx} \cos(a+bx)}{4b} - \frac{\sqrt{c+dx} \cos(3a+3bx)}{12b} + \frac{d \int \frac{\cos(3a+3bx)}{\sqrt{c+dx}} dx}{24b} + \frac{d \int \frac{\cos(a+bx)}{\sqrt{c+dx}} dx}{8b} \\ &= -\frac{\sqrt{c+dx} \cos(a+bx)}{4b} - \frac{\sqrt{c+dx} \cos(3a+3bx)}{12b} + \frac{\left(d \cos\left(3a - \frac{3bc}{d}\right) \right) \int \frac{\cos\left(\frac{3bc}{d} + \sqrt{c+dx}\right)}{\sqrt{c+dx}} dx}{24b} \\ &= -\frac{\sqrt{c+dx} \cos(a+bx)}{4b} - \frac{\sqrt{c+dx} \cos(3a+3bx)}{12b} + \frac{\cos\left(3a - \frac{3bc}{d}\right) \text{Subst}\left(\int \cos\left(\frac{3bc}{d} + \sqrt{c+dx}\right) dx\right)}{12b} \\ &= -\frac{\sqrt{c+dx} \cos(a+bx)}{4b} - \frac{\sqrt{c+dx} \cos(3a+3bx)}{12b} + \frac{\sqrt{d} \sqrt{\frac{\pi}{2}} \cos\left(a - \frac{bc}{d}\right) C\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} \end{aligned}$$

Mathematica [C] time = 6.64312, size = 278, normalized size = 0.91

$$\frac{-\sqrt{2\pi} \cos\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\sqrt{\frac{6}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c+dx}\right) + \sqrt{2\pi} \sin\left(3a - \frac{3bc}{d}\right) S\left(\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right) + 2\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos(3a)}{24\sqrt{3}b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x], x]

[Out] (Sqrt[c + d*x]*(-(E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[(I*b*(c + d*x))/d])/(8*b*E^((I*(b*c + a*d))/d)) - (2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt

$[b/d] \cdot \text{Sqrt}[6/\text{Pi}] \cdot \text{Sqrt}[c + d \cdot x] + \text{Sqrt}[2 \cdot \text{Pi}] \cdot \text{FresnelS}[\text{Sqrt}[b/d] \cdot \text{Sqrt}[6/\text{Pi}] \cdot \text{Sqrt}[c + d \cdot x]] \cdot \text{Sin}[3 \cdot a - (3 \cdot b \cdot c)/d] / (24 \cdot \text{Sqrt}[3] \cdot b \cdot \text{Sqrt}[b/d])$

Maple [A] time = 0.03, size = 296, normalized size = 1.

$$2 \frac{1}{d} \left(-\frac{1}{8} \frac{d\sqrt{dx+c}}{b} \cos\left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d}\right) + \frac{1}{16} \frac{d\sqrt{2}\sqrt{\pi}}{b} \left(\cos\left(\frac{ad-bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{2}\sqrt{dx+cb}}{\sqrt{\pi d}} \frac{1}{\sqrt{\frac{b}{d}}}\right) - \sin\left(\frac{ad-bc}{d}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x)

[Out] $2/d * (-1/8/b*d*(d*x+c)^(1/2)*\cos(1/d*(d*x+c)*b+(a*d-b*c)/d) + 1/16/b*d*2^(1/2)*\text{Pi}^(1/2)/(b/d)^(1/2)*(\cos((a*d-b*c)/d)*\text{FresnelC}(2^(1/2)/\text{Pi}^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d) - \sin((a*d-b*c)/d)*\text{FresnelS}(2^(1/2)/\text{Pi}^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)) - 1/24/b*d*(d*x+c)^(1/2)*\cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d) + 1/144/b*d*2^(1/2)*\text{Pi}^(1/2)*3^(1/2)/(b/d)^(1/2)*(\cos(3*(a*d-b*c)/d)*\text{FresnelC}(2^(1/2)/\text{Pi}^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d) - \sin(3*(a*d-b*c)/d)*\text{FresnelS}(2^(1/2)/\text{Pi}^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))$

Maxima [C] time = 2.21505, size = 1656, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")

[Out] $-1/288*\text{sqrt}(3)*(8*\text{sqrt}(3)*\text{sqrt}(d*x + c)*d*\text{abs}(b)*\cos(3*((d*x + c)*b - b*c + a*d)/d)/\text{abs}(d) + 24*\text{sqrt}(3)*\text{sqrt}(d*x + c)*d*\text{abs}(b)*\cos(((d*x + c)*b - b*c + a*d)/d)/\text{abs}(d) - ((\text{sqrt}(\text{pi})*\cos(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + \text{sqrt}(\text{pi})*\cos(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) - I*\text{sqrt}(\text{pi})*\sin(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + I*\text{sqrt}(\text{pi})*\sin(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2)))))*d*\text{sqrt}(\text{abs}(b)/\text{abs}(d))*\cos(-3*(b*c - a*d)/d) - (I*\text{sqrt}(\text{pi})*\cos(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + I*\text{sqrt}(\text{pi})*\cos(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + \text{sqrt}(\text{pi})*\sin(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) - \text{sqrt}(\text{pi})*\sin(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2)))))*d*\text{sqrt}(\text{abs}(b)/\text{abs}(d))*\sin(-3*(b*c - a*d)/d))*\text{erf}(\text{sqrt}(d*x + c)*\text{sqrt}(3*I*b/d)) - (\text{sqrt}(3)*3*\text{sqrt}(\text{pi})*\cos(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + 3*\text{sqrt}(\text{pi})*\cos(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) - 3*I*\text{sqrt}(\text{pi})*\sin(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + 3*I*\text{sqrt}(\text{pi})*\sin(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2)))))*d*\text{sqrt}(\text{abs}(b)/\text{abs}(d))*\cos(-(b*c - a*d)/d) - \text{sqrt}(3)*3*I*\text{sqrt}(\text{pi})*\cos(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + 3*I*\text{sqrt}(\text{pi})*\cos(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + 3*\text{sqrt}(\text{pi})*\sin(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) - 3*\text{sqrt}(\text{pi})*\sin(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2)))))*d*\text{sqrt}(\text{abs}(b)/\text{abs}(d))*\sin(-(b*c - a*d)/d))*\text{erf}(\text{sqrt}(d*x + c)*\text{sqrt}(I*b/d)) - (\text{sqrt}(3)*3*\text{sqrt}(\text{pi})*\cos(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + 3*\text{sqrt}(\text{pi})*\cos(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + 3*I*\text{sqrt}(\text{pi})*\sin(1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))) + 3*I*\text{sqrt}(\text{pi})*\sin(-1/4*\text{pi} + 1/2*\text{arctan2}(0, b) + 1/2*\text{arctan2}(0, d/\text{sqrt}(d^2))))$

) - 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) * d*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) - sqrt(3)*(-3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d*sqrt(abs(b)/abs(d))*sin(-(b*c - a*d)/d)*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d))*abs(d)/(b*d*abs(b))

Fricas [A] time = 0.5916, size = 613, normalized size = 2.02

$$\frac{\sqrt{6}\pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{3(bc-ad)}{d}\right) C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) + 9\sqrt{2}\pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{bc-ad}{d}\right) C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 9\sqrt{2}\pi d \sqrt{\frac{b}{\pi d}} S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)}{72b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a), x, algorithm="fricas")

[Out] 1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*sqrt(d*x + c)*b*cos(b*x + a)^3/b^2

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a), x)

[Out] Timed out

Giac [C] time = 1.25722, size = 659, normalized size = 2.17

$$\frac{\sqrt{6}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{2d}\right) e^{\left(\frac{3ibc-3iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} + \frac{9\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{2d}\right) e^{\left(\frac{ibc-iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} + \frac{9\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{2d}\right) e^{\left(\frac{-ibc+iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/144*(sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 6*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b/d
```

3.122 $\int (c + dx)^{3/2} \cos^2(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=353

$$\frac{\sqrt{\frac{\pi}{6}} d^{3/2} \sin\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}} - \frac{3\sqrt{\frac{\pi}{2}} d^{3/2} \sin\left(a - \frac{bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} - \frac{3\sqrt{\frac{\pi}{2}} d^{3/2} \cos\left(a - \frac{bc}{d}\right)}{8b^{5/2}}$$

[Out] $-\left((c + d*x)^{(3/2)}*\text{Cos}[a + b*x]\right)/(4*b) - \left((c + d*x)^{(3/2)}*\text{Cos}[3*a + 3*b*x]\right)/(12*b) - \left(3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a - (b*c)/d]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]\right)/(8*b^{(5/2)}) - \left(d^{(3/2)}*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]\right)/(24*b^{(5/2)}) - \left(d^{(3/2)}*\text{Sqrt}[\text{Pi}/6]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[3*a - (3*b*c)/d]\right)/(24*b^{(5/2)}) - \left(3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[a - (b*c)/d]\right)/(8*b^{(5/2)}) + \left(3*d*\text{Sqrt}[c + d*x]*\text{Sin}[a + b*x]\right)/(8*b^2) + \left(d*\text{Sqrt}[c + d*x]*\text{Sin}[3*a + 3*b*x]\right)/(24*b^2)$

Rubi [A] time = 0.52797, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{6}} d^{3/2} \sin\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}} - \frac{3\sqrt{\frac{\pi}{2}} d^{3/2} \sin\left(a - \frac{bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} - \frac{3\sqrt{\frac{\pi}{2}} d^{3/2} \cos\left(a - \frac{bc}{d}\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^{(3/2)}*\text{Cos}[a + b*x]^2*\text{Sin}[a + b*x], x]$

[Out] $-\left((c + d*x)^{(3/2)}*\text{Cos}[a + b*x]\right)/(4*b) - \left((c + d*x)^{(3/2)}*\text{Cos}[3*a + 3*b*x]\right)/(12*b) - \left(3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a - (b*c)/d]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]\right)/(8*b^{(5/2)}) - \left(d^{(3/2)}*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]\right)/(24*b^{(5/2)}) - \left(d^{(3/2)}*\text{Sqrt}[\text{Pi}/6]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[3*a - (3*b*c)/d]\right)/(24*b^{(5/2)}) - \left(3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[a - (b*c)/d]\right)/(8*b^{(5/2)}) + \left(3*d*\text{Sqrt}[c + d*x]*\text{Sin}[a + b*x]\right)/(8*b^2) + \left(d*\text{Sqrt}[c + d*x]*\text{Sin}[3*a + 3*b*x]\right)/(24*b^2)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3306

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[c*f/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d$

*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]

Rule 3305

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3304

Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}
 \int (c + dx)^{3/2} \cos^2(a + bx) \sin(a + bx) dx &= \int \left(\frac{1}{4}(c + dx)^{3/2} \sin(a + bx) + \frac{1}{4}(c + dx)^{3/2} \sin(3a + 3bx) \right) dx \\
 &= \frac{1}{4} \int (c + dx)^{3/2} \sin(a + bx) dx + \frac{1}{4} \int (c + dx)^{3/2} \sin(3a + 3bx) dx \\
 &= -\frac{(c + dx)^{3/2} \cos(a + bx)}{4b} - \frac{(c + dx)^{3/2} \cos(3a + 3bx)}{12b} + \frac{d \int \sqrt{c + dx} \cos(3a + 3bx) dx}{8b} \\
 &= -\frac{(c + dx)^{3/2} \cos(a + bx)}{4b} - \frac{(c + dx)^{3/2} \cos(3a + 3bx)}{12b} + \frac{3d\sqrt{c + dx} \sin(a + bx)}{8b^2} \\
 &= -\frac{(c + dx)^{3/2} \cos(a + bx)}{4b} - \frac{(c + dx)^{3/2} \cos(3a + 3bx)}{12b} + \frac{3d\sqrt{c + dx} \sin(a + bx)}{8b^2} \\
 &= -\frac{(c + dx)^{3/2} \cos(a + bx)}{4b} - \frac{(c + dx)^{3/2} \cos(3a + 3bx)}{12b} + \frac{3d\sqrt{c + dx} \sin(a + bx)}{8b^2} \\
 &= -\frac{(c + dx)^{3/2} \cos(a + bx)}{4b} - \frac{(c + dx)^{3/2} \cos(3a + 3bx)}{12b} - \frac{3d^{3/2} \sqrt{\frac{\pi}{2}} \cos\left(a - \frac{b}{d}\sqrt{c + dx}\right)}{8b^5}
 \end{aligned}$$

Mathematica [C] time = 9.21484, size = 676, normalized size = 1.92

$$\frac{c\sqrt{c + dx} e^{-\frac{i(ad+bc)}{d}} \left(-\frac{e^{2ia} \Gamma\left(\frac{3}{2}, -\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}} \Gamma\left(\frac{3}{2}, \frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}} \right)}{8b} - \frac{d\sqrt{\frac{b}{d}} \left(\sqrt{2\pi} \text{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c + dx}\right) \right) (3d \sin(a + bx))}{8b^5}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x], x]

[Out] (c*Sqrt[c + d*x]*(-(E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)*b*(c + d*x))/d]) - (E^((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[(I*b*(c + d*x))/d])/(8*b*E^((I*(b*c + a*d))/d)) - (c*(2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(24*Sqrt[3]*b*Sqrt[b/d]) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(3*d*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a + b*x])))/(16*b^3) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(48*Sqrt[3]*b^3)

Maple [A] time = 0.029, size = 384, normalized size = 1.1

$$2 \frac{1}{d} \left(-1/8 \frac{d(dx+c)^{3/2}}{b} \cos\left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d}\right) + 3/8 \frac{d}{b} \left(1/2 \frac{d\sqrt{dx+c}}{b} \sin\left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d}\right) - 1/4 \frac{d\sqrt{2}\sqrt{\pi}}{b} \left(\cos\left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a), x)

[Out] 2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/8/b*d*(1/2/b*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))-1/24/b*d*(d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/8/b*d*(1/6/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))

Maxima [C] time = 2.49685, size = 1793, normalized size = 5.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a), x, algorithm="maxima")

[Out] -1/576*sqrt(3)*(16*sqrt(3)*(d*x + c)^(3/2)*b*d*abs(b)*cos(3*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 48*sqrt(3)*(d*x + c)^(3/2)*b*d*abs(b)*cos(((d*x + c)*b - b*c + a*d)/d)/abs(d) - 8*sqrt(3)*sqrt(d*x + c)*d^2*abs(b)*sin(3*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 72*sqrt(3)*sqrt(d*x + c)*d^2*abs(b)*sin(((d*x + c)*b - b*c + a*d)/d)/abs(d) - ((-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))

$b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^2*\sqrt{\text{abs}(b)/\text{abs}(d)}*\cos(-3*(b*c - a*d)/d) - (\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + \sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^2*\sqrt{\text{abs}(b)/\text{abs}(d)}*\sin(-3*(b*c - a*d)/d)*\text{erf}(\sqrt{d*x + c})*\sqrt{3*I*b/d) - (\sqrt{3})*(-9*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 9*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 9*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 9*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^2*\sqrt{\text{abs}(b)/\text{abs}(d)}*\cos(-(b*c - a*d)/d) - \sqrt{3}*(9*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 9*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 9*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 9*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^2*\sqrt{\text{abs}(b)/\text{abs}(d)}*\sin(-(b*c - a*d)/d)*\text{erf}(\sqrt{d*x + c})*\sqrt{I*b/d) - (\sqrt{3})*(9*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 9*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 9*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 9*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^2*\sqrt{\text{abs}(b)/\text{abs}(d)}*\cos(-(b*c - a*d)/d) - \sqrt{3}*(9*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 9*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 9*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 9*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^2*\sqrt{\text{abs}(b)/\text{abs}(d)}*\sin(-(b*c - a*d)/d)*\text{erf}(\sqrt{d*x + c})*\sqrt{-I*b/d) - ((I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - \sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + \sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^2*\sqrt{\text{abs}(b)/\text{abs}(d)}*\cos(-3*(b*c - a*d)/d) - (\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + \sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^2*\sqrt{\text{abs}(b)/\text{abs}(d)}*\sin(-3*(b*c - a*d)/d)*\text{erf}(\sqrt{d*x + c})*\sqrt{-3*I*b/d))*\text{abs}(d)/(b^2*d*\text{abs}(b))$

Fricas [A] time = 0.610789, size = 716, normalized size = 2.03

$$\sqrt{6\pi d^2} \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{3(bc-ad)}{d}\right) S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) + 27\sqrt{2}\pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{bc-ad}{d}\right) S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) + 27\sqrt{2}\pi d^2 \sqrt{\frac{b}{\pi d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")

[Out] $-1/144*(\sqrt{6}*\pi*d^2*\sqrt{b/(\pi*d)}*\cos(-3*(b*c - a*d)/d)*\text{fresnel_sin}(\sqrt{6}*\sqrt{d*x + c}*\sqrt{b/(\pi*d)}) + 27*\sqrt{2}*\pi*d^2*\sqrt{b/(\pi*d)}*\cos(-(b*c - a*d)/d)*\text{fresnel_sin}(\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/(\pi*d)}) + 27*\sqrt{2}*\pi*d^2*\sqrt{b/(\pi*d)}*\text{fresnel_cos}(\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/(\pi*d)})*\sin(-(b*c - a*d)/d) + \sqrt{6}*\pi*d^2*\sqrt{b/(\pi*d)}*\text{fresnel_cos}(\sqrt{6}*\sqrt{d*x + c}*\sqrt{b/(\pi*d)})*\sin(-3*(b*c - a*d)/d) + 24*(2*(b^2*d*x + b^2*c)*\cos(b*x + a)^3 - (b*d*\cos(b*x + a)^2 + 2*b*d)*\sin(b*x + a))*\sqrt{d*x + c})/b^3$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a),x)

[Out] Timed out

Giac [C] time = 1.38964, size = 1513, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/288*(2*(\sqrt{6}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x+c}*(I \\ & *b*d/\sqrt{b^2*d^2+1})/d)*e^{((3*I*b*c-3*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{ \\ & (b^2*d^2+1)*b)+9*\sqrt{2}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{ \\ & d*x+c}*(I*b*d/\sqrt{b^2*d^2+1})/d)*e^{((I*b*c-I*a*d)/d)/(\sqrt{b*d}*(I*b \\ & *d/\sqrt{b^2*d^2+1)*b)+9*\sqrt{2}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d} \\ &)*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2+1})/d)*e^{((-I*b*c+I*a*d)/d)/(\sqrt{ \\ & b*d}*(-I*b*d/\sqrt{b^2*d^2+1)*b)+\sqrt{6}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{6} \\ & *\sqrt{b*d}*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2+1})/d)*e^{((-3*I*b*c+3*I*a \\ & d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2+1)*b)+6*\sqrt{d*x+c}*d*e^{((3*I \\ & (d*x+c)*b-3*I*b*c+3*I*a*d)/d)/b+18*\sqrt{d*x+c}*d*e^{((I*(d*x+c)* \\ & b-I*b*c+I*a*d)/d)/b+18*\sqrt{d*x+c}*d*e^{((-I*(d*x+c)*b+I*b*c-I \\ & *a*d)/d)/b+6*\sqrt{d*x+c}*d*e^{((-3*I*(d*x+c)*b+3*I*b*c-3*I*a*d)/d} \\ & /b)*c+I*\sqrt{6}*\sqrt{\pi}*(2*I*b*c*d-d^2)*d*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{ \\ & d*x+c}*(I*b*d/\sqrt{b^2*d^2+1})/d)*e^{((3*I*b*c-3*I*a*d)/d)/(\sqrt{b \\ & *d}*(I*b*d/\sqrt{b^2*d^2+1)*b^2)+9*I*\sqrt{2}*\sqrt{\pi}*(2*I*b*c*d-3*d^ \\ & 2)*d*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2+1})/d)* \\ & e^{((I*b*c-I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2+1)*b^2)+9*I*\sqrt{2} \\ & *\sqrt{\pi}*(2*I*b*c*d+3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x+c} \\ &)*(-I*b*d/\sqrt{b^2*d^2+1})/d)*e^{((-I*b*c+I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{ \\ & (b^2*d^2+1)*b^2)+I*\sqrt{6}*\sqrt{\pi}*(2*I*b*c*d+d^2)*d*\operatorname{erf}(-1/2*\sqrt{6} \\ & *\sqrt{b*d}*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2+1})/d)*e^{((-3*I*b*c+3 \\ & I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2+1)*b^2)-6*I*(2*I*(d*x+c)^(\\ & 3/2)*b*d-2*I*\sqrt{d*x+c}*b*c*d-\sqrt{d*x+c}*d^2)*e^{((3*I*(d*x+c)*b \\ & -3*I*b*c+3*I*a*d)/d)/b^2-18*I*(2*I*(d*x+c)^(3/2)*b*d-2*I*\sqrt{d*x \\ & +c}*b*c*d-3*\sqrt{d*x+c}*d^2)*e^{((I*(d*x+c)*b-I*b*c+I*a*d)/d)/b^ \\ & 2-18*I*(2*I*(d*x+c)^(3/2)*b*d-2*I*\sqrt{d*x+c}*b*c*d+3*\sqrt{d*x+c} \\ & *d^2)*e^{((-I*(d*x+c)*b+I*b*c-I*a*d)/d)/b^2-6*I*(2*I*(d*x+c)^(3/ \\ & 2)*b*d-2*I*\sqrt{d*x+c}*b*c*d+\sqrt{d*x+c}*d^2)*e^{((-3*I*(d*x+c)*b \\ & +3*I*b*c-3*I*a*d)/d)/b^2)/d \end{aligned}$$

3.123 $\int (c + dx)^{5/2} \cos^2(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=406

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} - \frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}} + \frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a - \frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}$$

[Out] $(15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[a + b*x])/(16*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[a + b*x])/ (4*b) + (5*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[3*a + 3*b*x])/(144*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[3*a + 3*b*x])/(12*b) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a - (b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(16*b^{(7/2)}) - (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(144*b^{(7/2)}) + (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[3*a - (3*b*c)/d])/(144*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[a - (b*c)/d])/(16*b^{(7/2)}) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[a + b*x])/(8*b^2) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[3*a + 3*b*x])/(72*b^2)$

Rubi [A] time = 0.627123, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} - \frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}} + \frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a - \frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^{(5/2)}*\text{Cos}[a + b*x]^2*\text{Sin}[a + b*x], x]$

[Out] $(15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[a + b*x])/(16*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[a + b*x])/ (4*b) + (5*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[3*a + 3*b*x])/(144*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[3*a + 3*b*x])/(12*b) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a - (b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(16*b^{(7/2)}) - (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(144*b^{(7/2)}) + (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[3*a - (3*b*c)/d])/(144*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[a - (b*c)/d])/(16*b^{(7/2)}) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[a + b*x])/(8*b^2) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[3*a + 3*b*x])/(72*b^2)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[p, 0]$

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{GtQ}[m, 0]$

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^{5/2} \cos^2(a+bx) \sin(a+bx) dx &= \int \left(\frac{1}{4}(c+dx)^{5/2} \sin(a+bx) + \frac{1}{4}(c+dx)^{5/2} \sin(3a+3bx) \right) dx \\
&= \frac{1}{4} \int (c+dx)^{5/2} \sin(a+bx) dx + \frac{1}{4} \int (c+dx)^{5/2} \sin(3a+3bx) dx \\
&= -\frac{(c+dx)^{5/2} \cos(a+bx)}{4b} - \frac{(c+dx)^{5/2} \cos(3a+3bx)}{12b} + \frac{(5d) \int (c+dx)^{3/2} \cos(3a+3bx) dx}{24b} \\
&= -\frac{(c+dx)^{5/2} \cos(a+bx)}{4b} - \frac{(c+dx)^{5/2} \cos(3a+3bx)}{12b} + \frac{5d(c+dx)^{3/2} \sin(a+bx)}{8b^2} \\
&= \frac{15d^2 \sqrt{c+dx} \cos(a+bx)}{16b^3} - \frac{(c+dx)^{5/2} \cos(a+bx)}{4b} + \frac{5d^2 \sqrt{c+dx} \cos(3a+3bx)}{144b^3} \\
&= \frac{15d^2 \sqrt{c+dx} \cos(a+bx)}{16b^3} - \frac{(c+dx)^{5/2} \cos(a+bx)}{4b} + \frac{5d^2 \sqrt{c+dx} \cos(3a+3bx)}{144b^3} \\
&= \frac{15d^2 \sqrt{c+dx} \cos(a+bx)}{16b^3} - \frac{(c+dx)^{5/2} \cos(a+bx)}{4b} + \frac{5d^2 \sqrt{c+dx} \cos(3a+3bx)}{144b^3} \\
&= \frac{15d^2 \sqrt{c+dx} \cos(a+bx)}{16b^3} - \frac{(c+dx)^{5/2} \cos(a+bx)}{4b} + \frac{5d^2 \sqrt{c+dx} \cos(3a+3bx)}{144b^3}
\end{aligned}$$

Mathematica [C] time = 16.2415, size = 1168, normalized size = 2.88

$$\frac{e^{-\frac{i(bc+ad)}{d}} \sqrt{c+dx} \left(-\frac{e^{2ia} \Gamma\left(\frac{3}{2}, -\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}} \Gamma\left(\frac{3}{2}, \frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}} \right) c^2}{8b} - \left(2\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos(3(a+bx)) - \sqrt{2\pi} \cos\left(3a - \frac{3bc}{d}\right) \right)$$

Maxima [C] time = 2.42005, size = 1874, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")

[Out]
$$\begin{aligned} & 1/3456*\sqrt{3}*(80*\sqrt{3}*(d*x + c)^{(3/2)}*b*d^2*\text{abs}(b)*\sin(3*((d*x + c)*b \\ & - b*c + a*d)/d)/\text{abs}(d) + 720*\sqrt{3}*(d*x + c)^{(3/2)}*b*d^2*\text{abs}(b)*\sin(((d*x \\ & + c)*b - b*c + a*d)/d)/\text{abs}(d) - 8*(12*\sqrt{3}*(d*x + c)^{(5/2)}*b^2*d*\text{abs}(b) \\ & / \text{abs}(d) - 5*\sqrt{3}*\sqrt{d*x + c}*d^3*\text{abs}(b)/\text{abs}(d))*\cos(3*((d*x + c)*b - b \\ & *c + a*d)/d) - 72*(4*\sqrt{3}*(d*x + c)^{(5/2)}*b^2*d*\text{abs}(b)/\text{abs}(d) - 15*\sqrt{3} \\ & *\sqrt{d*x + c}*d^3*\text{abs}(b)/\text{abs}(d))*\cos(((d*x + c)*b - b*c + a*d)/d) - ((5* \\ & \sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 5* \\ & \sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 5 \\ & *I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + \\ & 5*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) \\ &))*d^3*\sqrt{\text{abs}(b)/\text{abs}(d)}*\cos(-3*(b*c - a*d)/d) - (5*I*\sqrt{\pi}*\cos(1/4*\pi \\ & + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 5*I*\sqrt{\pi}*\cos(-1/4 \\ & *\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 5*\sqrt{\pi}*\sin(1/4 \\ & *\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 5*\sqrt{\pi}*\sin(-1/ \\ & 4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^3*\sqrt{\text{abs}(b)/\text{ab} \\ & \text{s}(d)}*\sin(-3*(b*c - a*d)/d)*\text{erf}(\sqrt{d*x + c}*\sqrt{3*I*b/d}) - (\sqrt{3}*(1 \\ & 35*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + \\ & 135*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) \\ &) - 135*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d \\ & ^2})) + 135*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/s \\ & \sqrt{d^2})))*d^3*\sqrt{\text{abs}(b)/\text{abs}(d)}*\cos(-(b*c - a*d)/d) - \sqrt{3}*(135*I*\sqrt{ \\ & \pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 135* \\ & I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + \\ & 135*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) \\ & - 135*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2} \\ &)))*d^3*\sqrt{\text{abs}(b)/\text{abs}(d)}*\sin(-(b*c - a*d)/d)*\text{erf}(\sqrt{d*x + c}*\sqrt{I* \\ & b/d}) - (\sqrt{3}*(135*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2 \\ & (0, d/\sqrt{d^2})) + 135*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arct \\ & an2(0, d/\sqrt{d^2})) + 135*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2* \\ & arctan2(0, d/\sqrt{d^2})) - 135*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + \\ & 1/2*\arctan2(0, d/\sqrt{d^2})))*d^3*\sqrt{\text{abs}(b)/\text{abs}(d)}*\cos(-(b*c - a*d)/d) \\ & - \sqrt{3}*(-135*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, \\ & d/\sqrt{d^2})) - 135*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan \\ & 2(0, d/\sqrt{d^2})) + 135*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arct \\ & an2(0, d/\sqrt{d^2})) - 135*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*a \\ & rctan2(0, d/\sqrt{d^2})))*d^3*\sqrt{\text{abs}(b)/\text{abs}(d)}*\sin(-(b*c - a*d)/d)*\text{erf}(s \\ & \sqrt{d*x + c}*\sqrt{-I*b/d}) - ((5*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + \\ & 1/2*\arctan2(0, d/\sqrt{d^2})) + 5*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + \\ & 1/2*\arctan2(0, d/\sqrt{d^2})) + 5*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) \\ & + 1/2*\arctan2(0, d/\sqrt{d^2})) - 5*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, \\ & b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^3*\sqrt{\text{abs}(b)/\text{abs}(d)}*\cos(-3*(b*c - a \\ & *d)/d) - (-5*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/s \\ & \sqrt{d^2})) - 5*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, \\ & d/\sqrt{d^2})) + 5*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, \\ & d/\sqrt{d^2})) - 5*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, \\ & d/\sqrt{d^2})))*d^3*\sqrt{\text{abs}(b)/\text{abs}(d)}*\sin(-3*(b*c - a*d)/d)*\text{erf}(\sqrt{d*x \\ & + c}*\sqrt{-3*I*b/d}))*\text{abs}(d)/(b^3*d*\text{abs}(b)) \end{aligned}$$

Fricas [A] time = 0.671895, size = 856, normalized size = 2.11

$$\frac{5\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\sin\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+405\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\sin\left(-\frac{3(bc-ad)}{d}\right)C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/864*(5*\sqrt{6}*\pi*d^3*\sqrt{b/(pi*d)}*\cos(-3*(b*c - a*d)/d)*\text{fresnel_cos}(\sqrt{6}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) \\ & + 405*\sqrt{2}*\pi*d^3*\sqrt{b/(pi*d)}*\cos(-3*(b*c - a*d)/d)*\text{fresnel_cos}(\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) \\ & - 405*\sqrt{2}*\pi*d^3*\sqrt{b/(pi*d)}*\text{fresnel_sin}(\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) \\ & + 405*\sqrt{6}*\pi*d^3*\sqrt{b/(pi*d)}*\text{fresnel_sin}(\sqrt{6}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) \\ & - 5*\sqrt{6}*\pi*d^3*\sqrt{b/(pi*d)}*\text{fresnel_sin}(\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) \\ & + 5*\sqrt{2}*\pi*d^3*\sqrt{b/(pi*d)}*\text{fresnel_sin}(\sqrt{6}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) \\ & - 24*(30*b*d^2*\cos(b*x + a) - (12*b^3*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*\cos(b*x + a)^3 \\ & + 10*(2*b^2*d^2*x + 2*b^2*c*d + (b^2*d^2*x + b^2*c*d)*\cos(b*x + a)^2)*\sin(b*x + a)*\sqrt{d*x + c})/b^4 \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a),x)

[Out] Timed out

Giac [C] time = 1.53231, size = 2724, normalized size = 6.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/1728*(12*(\sqrt{6}*\sqrt{\pi})*d^2*\text{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x + c})* \\ & (I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((3*I*b*c - 3*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b)} \\ & + 9*\sqrt{2}*\sqrt{\pi})*d^2*\text{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c})* \\ & (I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((I*b*c - I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b)} \\ & + 9*\sqrt{2}*\sqrt{\pi})*d^2*\text{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c})* \\ & (-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-I*b*c + I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b)} \\ & + \sqrt{6}*\sqrt{\pi})*d^2*\text{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x + c})* \\ & (-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-3*I*b*c + 3*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b)} \\ & + 6*\sqrt{d*x + c})*d*e^{((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b} \\ & + 18*\sqrt{d*x + c})*d*e^{((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b} \\ & + 18*\sqrt{d*x + c})*d*e^{((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b} \\ & + 6*\sqrt{d*x + c})*d*e^{((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b} \\ & *c^2 + d^2*((I*\sqrt{6}*\sqrt{\pi})*(-12*I*b^2*c^2*d + 12*b*c*d^2 + 5*I*d^3)* \\ & d*\text{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x + c})* \\ & (I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((3*I*b*c - 3*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^3)} \\ & - 6*I*(12*I*(d*x + c)^{(5/2)}*b^2*d - 24*I*(d*x + c)^{(3/2)}*b^2*c*d + 12*I*\sqrt{d*x + c}) \end{aligned}$$

$$\begin{aligned}
& c) * b^2 * c^2 * d + 10 * (d * x + c)^{(3/2)} * b * d^2 - 12 * \text{sqrt}(d * x + c) * b * c * d^2 - 5 * I * \text{sqrt}(d * x + c) * d^3 * e^{((-3 * I * (d * x + c) * b + 3 * I * b * c - 3 * I * a * d) / d) / b^3} / d^2 + 2 \\
& 7 * (I * \text{sqrt}(2) * \text{sqrt}(\pi) * (-4 * I * b^2 * c^2 * d + 12 * b * c * d^2 + 15 * I * d^3) * d * \text{erf}(-1/2 * \text{sqrt}(2) * \text{sqrt}(b * d) * \text{sqrt}(d * x + c) * (I * b * d / \text{sqrt}(b^2 * d^2) + 1) / d) * e^{((I * b * c - I * a * d) / d) / (\text{sqrt}(b * d) * (I * b * d / \text{sqrt}(b^2 * d^2) + 1) * b^3)} - 2 * I * (4 * I * (d * x + c)^{(5/2)} * b^2 * d - 8 * I * (d * x + c)^{(3/2)} * b^2 * c * d + 4 * I * \text{sqrt}(d * x + c) * b^2 * c^2 * d + 10 * (d * x + c)^{(3/2)} * b * d^2 - 12 * \text{sqrt}(d * x + c) * b * c * d^2 - 15 * I * \text{sqrt}(d * x + c) * d^3) * e^{((-I * (d * x + c) * b + I * b * c - I * a * d) / d) / b^3} / d^2 + 27 * (I * \text{sqrt}(2) * \text{sqrt}(\pi) * (-4 * I * b^2 * c^2 * d - 12 * b * c * d^2 + 15 * I * d^3) * d * \text{erf}(-1/2 * \text{sqrt}(2) * \text{sqrt}(b * d) * \text{sqrt}(d * x + c) * (-I * b * d / \text{sqrt}(b^2 * d^2) + 1) / d) * e^{((-I * b * c + I * a * d) / d) / (\text{sqrt}(b * d) * (-I * b * d / \text{sqrt}(b^2 * d^2) + 1) * b^3)} - 2 * I * (4 * I * (d * x + c)^{(5/2)} * b^2 * d - 8 * I * (d * x + c)^{(3/2)} * b^2 * c * d + 4 * I * \text{sqrt}(d * x + c) * b^2 * c^2 * d - 10 * (d * x + c)^{(3/2)} * b * d^2 + 12 * \text{sqrt}(d * x + c) * b * c * d^2 - 15 * I * \text{sqrt}(d * x + c) * d^3) * e^{((I * (d * x + c) * b - I * b * c + I * a * d) / d) / b^3} / d^2 + (I * \text{sqrt}(6) * \text{sqrt}(\pi) * (-12 * I * b^2 * c^2 * d - 12 * b * c * d^2 + 5 * I * d^3) * d * \text{erf}(-1/2 * \text{sqrt}(6) * \text{sqrt}(b * d) * \text{sqrt}(d * x + c) * (-I * b * d / \text{sqrt}(b^2 * d^2) + 1) / d) * e^{((-3 * I * b * c + 3 * I * a * d) / d) / (\text{sqrt}(b * d) * (-I * b * d / \text{sqrt}(b^2 * d^2) + 1) * b^3)} - 6 * I * (12 * I * (d * x + c)^{(5/2)} * b^2 * d - 24 * I * (d * x + c)^{(3/2)} * b^2 * c * d + 12 * I * \text{sqrt}(d * x + c) * b^2 * c^2 * d - 10 * (d * x + c)^{(3/2)} * b * d^2 + 12 * \text{sqrt}(d * x + c) * b * c * d^2 - 5 * I * \text{sqrt}(d * x + c) * d^3) * e^{((3 * I * (d * x + c) * b - 3 * I * b * c + 3 * I * a * d) / d) / b^3} / d^2 + 12 * (I * \text{sqrt}(6) * \text{sqrt}(\pi) * (2 * I * b * c * d - d^2) * d * \text{erf}(-1/2 * \text{sqrt}(6) * \text{sqrt}(b * d) * \text{sqrt}(d * x + c) * (I * b * d / \text{sqrt}(b^2 * d^2) + 1) / d) * e^{((3 * I * b * c - 3 * I * a * d) / d) / (\text{sqrt}(b * d) * (I * b * d / \text{sqrt}(b^2 * d^2) + 1) * b^2)} + 9 * I * \text{sqrt}(2) * \text{sqrt}(\pi) * (2 * I * b * c * d - 3 * d^2) * d * \text{erf}(-1/2 * \text{sqrt}(2) * \text{sqrt}(b * d) * \text{sqrt}(d * x + c) * (I * b * d / \text{sqrt}(b^2 * d^2) + 1) / d) * e^{((I * b * c - I * a * d) / d) / (\text{sqrt}(b * d) * (I * b * d / \text{sqrt}(b^2 * d^2) + 1) * b^2)} + 9 * I * \text{sqrt}(2) * \text{sqrt}(\pi) * (2 * I * b * c * d + 3 * d^2) * d * \text{erf}(-1/2 * \text{sqrt}(2) * \text{sqrt}(b * d) * \text{sqrt}(d * x + c) * (-I * b * d / \text{sqrt}(b^2 * d^2) + 1) / d) * e^{((-I * b * c + I * a * d) / d) / (\text{sqrt}(b * d) * (-I * b * d / \text{sqrt}(b^2 * d^2) + 1) * b^2)} + I * \text{sqrt}(6) * \text{sqrt}(\pi) * (2 * I * b * c * d + d^2) * d * \text{erf}(-1/2 * \text{sqrt}(6) * \text{sqrt}(b * d) * \text{sqrt}(d * x + c) * (-I * b * d / \text{sqrt}(b^2 * d^2) + 1) / d) * e^{((-3 * I * b * c + 3 * I * a * d) / d) / (\text{sqrt}(b * d) * (-I * b * d / \text{sqrt}(b^2 * d^2) + 1) * b^2)} - 6 * I * (2 * I * (d * x + c)^{(3/2)} * b * d - 2 * I * \text{sqrt}(d * x + c) * b * c * d - \text{sqrt}(d * x + c) * d^2) * e^{((3 * I * (d * x + c) * b - 3 * I * b * c + 3 * I * a * d) / d) / b^2} - 18 * I * (2 * I * (d * x + c)^{(3/2)} * b * d - 2 * I * \text{sqrt}(d * x + c) * b * c * d - 3 * \text{sqrt}(d * x + c) * d^2) * e^{((I * (d * x + c) * b - I * b * c + I * a * d) / d) / b^2} - 18 * I * (2 * I * (d * x + c)^{(3/2)} * b * d - 2 * I * \text{sqrt}(d * x + c) * b * c * d + 3 * \text{sqrt}(d * x + c) * d^2) * e^{((-I * (d * x + c) * b + I * b * c - I * a * d) / d) / b^2} - 6 * I * (2 * I * (d * x + c)^{(3/2)} * b * d - 2 * I * \text{sqrt}(d * x + c) * b * c * d + \text{sqrt}(d * x + c) * d^2) * e^{((-3 * I * (d * x + c) * b + 3 * I * b * c - 3 * I * a * d) / d) / b^2} * c) / d
\end{aligned}$$

3.124 $\int (c + dx)^{5/2} \cos^2(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=228

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} - \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a - \frac{4bc}{d}\right)\text{S}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} + \frac{15d^2\sqrt{c+dx}\sin(4a)}{2048b^3}$$

[Out] $(c + dx)^{(7/2)}/(28*d) - (5*d*(c + dx)^{(3/2)}*\text{Cos}[4*a + 4*b*x])/(256*b^2) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[4*a - (4*b*c)/d]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(4096*b^{(7/2)}) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[4*a - (4*b*c)/d])/(4096*b^{(7/2)}) + (15*d^2*\text{Sqrt}[c + d*x]*\text{Sin}[4*a + 4*b*x])/(2048*b^3) - ((c + d*x)^{(5/2)}*\text{Sin}[4*a + 4*b*x])/(32*b)$

Rubi [A] time = 0.398063, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} - \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a - \frac{4bc}{d}\right)\text{S}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} + \frac{15d^2\sqrt{c+dx}\sin(4a)}{2048b^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^{(5/2)}*\text{Cos}[a + b*x]^2*\text{Sin}[a + b*x]^2, x]$

[Out] $(c + d*x)^{(7/2)}/(28*d) - (5*d*(c + d*x)^{(3/2)}*\text{Cos}[4*a + 4*b*x])/(256*b^2) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[4*a - (4*b*c)/d]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(4096*b^{(7/2)}) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[4*a - (4*b*c)/d])/(4096*b^{(7/2)}) + (15*d^2*\text{Sqrt}[c + d*x]*\text{Sin}[4*a + 4*b*x])/(2048*b^3) - ((c + d*x)^{(5/2)}*\text{Sin}[4*a + 4*b*x])/(32*b)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.))^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3306

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)]]/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)]]/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^{5/2} \cos^2(a + bx) \sin^2(a + bx) dx &= \int \left(\frac{1}{8}(c + dx)^{5/2} - \frac{1}{8}(c + dx)^{5/2} \cos(4a + 4bx) \right) dx \\
&= \frac{(c + dx)^{7/2}}{28d} - \frac{1}{8} \int (c + dx)^{5/2} \cos(4a + 4bx) dx \\
&= \frac{(c + dx)^{7/2}}{28d} - \frac{(c + dx)^{5/2} \sin(4a + 4bx)}{32b} + \frac{(5d) \int (c + dx)^{3/2} \sin(4a + 4bx) dx}{64b} \\
&= \frac{(c + dx)^{7/2}}{28d} - \frac{5d(c + dx)^{3/2} \cos(4a + 4bx)}{256b^2} - \frac{(c + dx)^{5/2} \sin(4a + 4bx)}{32b} + \frac{(15d^2 \sqrt{c + dx} \sin(4a + 4bx))}{2048b^3} \\
&= \frac{(c + dx)^{7/2}}{28d} - \frac{5d(c + dx)^{3/2} \cos(4a + 4bx)}{256b^2} + \frac{15d^2 \sqrt{c + dx} \sin(4a + 4bx)}{2048b^3} - \frac{(c + dx)^{5/2} \sin(4a + 4bx)}{32b} \\
&= \frac{(c + dx)^{7/2}}{28d} - \frac{5d(c + dx)^{3/2} \cos(4a + 4bx)}{256b^2} + \frac{15d^2 \sqrt{c + dx} \sin(4a + 4bx)}{2048b^3} - \frac{(c + dx)^{5/2} \sin(4a + 4bx)}{32b} \\
&= \frac{(c + dx)^{7/2}}{28d} - \frac{5d(c + dx)^{3/2} \cos(4a + 4bx)}{256b^2} + \frac{15d^2 \sqrt{c + dx} \sin(4a + 4bx)}{2048b^3} - \frac{(c + dx)^{5/2} \sin(4a + 4bx)}{32b} \\
&= \frac{(c + dx)^{7/2}}{28d} - \frac{5d(c + dx)^{3/2} \cos(4a + 4bx)}{256b^2} - \frac{15d^{5/2} \sqrt{\frac{\pi}{2}} \cos\left(4a - \frac{4bc}{d}\right) S\left(\frac{2\sqrt{b}}{d} \sqrt{c + dx}\right)}{4096b^{7/2}}
\end{aligned}$$

Mathematica [A] time = 3.65687, size = 206, normalized size = 0.9

$$\frac{\sqrt{\frac{b}{d}} \left(4\sqrt{\frac{b}{d}} \sqrt{c + dx} (-7d \sin(4(a + bx))) (64b^2(c + dx)^2 - 15d^2) - 280bd^2(c + dx) \cos(4(a + bx)) + 512b^3(c + dx)^3 \right) - 105\sqrt{b} d^2}{57344b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(-105*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 105*d^3*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x])*Sin[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(512*b^3
```

$$\frac{(c + dx)^3 - 280bd^2(c + dx)\cos[4(a + bx)] - 7d(-15d^2 + 64b^2(c + dx)^2)\sin[4(a + bx)]}{(57344b^4)}$$

Maple [A] time = 0.037, size = 251, normalized size = 1.1

$$2\frac{1}{d}\left(\frac{(dx+c)^{7/2}}{56} - \frac{d(dx+c)^{5/2}}{64b}\sin\left(4\frac{(dx+c)b}{d} + 4\frac{ad-bc}{d}\right) + \frac{5d}{64b}\left(-\frac{1}{8}\frac{d(dx+c)^{3/2}}{b}\cos\left(4\frac{(dx+c)b}{d} + 4\frac{ad-bc}{d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)

[Out] $2/d*(1/56*(d*x+c)^{(7/2)}-1/64/b*d*(d*x+c)^{(5/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+5/64/b*d*(-1/8/b*d*(d*x+c)^{(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+3/8/b*d*(1/8/b*d*(d*x+c)^{(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^{(1/2)*Pi^{(1/2)/(b/d)^{(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^{(1/2)/Pi^{(1/2)/(b/d)^{(1/2)*(d*x+c)^{(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^{(1/2)/Pi^{(1/2)/(b/d)^{(1/2)*(d*x+c)^{(1/2)*b/d))}}$

Maxima [C] time = 2.08834, size = 922, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")

[Out] $1/229376*(8192*(d*x + c)^{(7/2)*b^3*\sqrt{abs(b)/abs(d)} - 4480*(d*x + c)^{(3/2)*b*d^2*\sqrt{abs(b)/abs(d)}*\cos(4*((d*x + c)*b - b*c + a*d)/d) - ((105*I*\sqrt{pi}*\cos(1/4*pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 105*I*\sqrt{pi}*\cos(-1/4*pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 105*\sqrt{pi}*\sin(1/4*pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 105*\sqrt{pi}*\sin(-1/4*pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^3*\cos(-4*(b*c - a*d)/d) + (105*\sqrt{pi}*\cos(1/4*pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 105*\sqrt{pi}*\cos(-1/4*pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 105*I*\sqrt{pi}*\sin(1/4*pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 105*I*\sqrt{pi}*\sin(-1/4*pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^3*\sin(-4*(b*c - a*d)/d) * erf(2*\sqrt{d*x + c}*\sqrt{I*b/d}) - ((-105*I*\sqrt{pi}*\cos(1/4*pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 105*I*\sqrt{pi}*\cos(-1/4*pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 105*\sqrt{pi}*\sin(1/4*pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 105*\sqrt{pi}*\sin(-1/4*pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^3*\cos(-4*(b*c - a*d)/d) + (105*\sqrt{pi}*\cos(1/4*pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 105*\sqrt{pi}*\cos(-1/4*pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 105*I*\sqrt{pi}*\sin(1/4*pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 105*I*\sqrt{pi}*\sin(-1/4*pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^3*\sin(-4*(b*c - a*d)/d) * erf(2*\sqrt{d*x + c}*\sqrt{-I*b/d}) - 112*(64*(d*x + c)^{(5/2)*b^2*d*\sqrt{abs(b)/abs(d)} - 15*\sqrt{d*x + c} * d^3*\sqrt{abs(b)/abs(d)}) * sin(4*((d*x + c)*b - b*c + a*d)/d) / (b^3*d*\sqrt{abs(b)/abs(d)})$

Fricas [A] time = 0.651637, size = 840, normalized size = 3.68

$$105 \sqrt{2} \pi d^4 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{4(bc-ad)}{d}\right) S\left(2 \sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) + 105 \sqrt{2} \pi d^4 \sqrt{\frac{b}{\pi d}} C\left(2 \sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{4(bc-ad)}{d}\right) - 16 (128$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")

[Out] -1/57344*(105*sqrt(2)*pi*d^4*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_s
in(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 105*sqrt(2)*pi*d^4*sqrt(b/(pi*
d))*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/
d) - 16*(128*b^4*d^3*x^3 + 384*b^4*c*d^2*x^2 + 128*b^4*c^3 - 70*b^2*c*d^2 -
560*(b^2*d^3*x + b^2*c*d^2)*cos(b*x + a)^4 + 560*(b^2*d^3*x + b^2*c*d^2)*c
os(b*x + a)^2 + 2*(192*b^4*c^2*d - 35*b^2*d^3)*x - 7*(2*(64*b^3*d^3*x^2 + 1
28*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a)^3 - (64*b^3*d^3*x^2
+ 128*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a))*sin(b*x + a))*sq
rt(d*x + c))/(b^4*d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)

[Out] Timed out

Giac [C] time = 1.68562, size = 1467, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")

[Out] -1/1720320*(2240*(3*I*sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) - 3*I*sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sq
rt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 64*(d*x + c)^(3/2) - 12*I*sqrt(d*x +
c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 12*I*sqrt(d*x + c)*d*
e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c^2 - d^2*(4096*(15*(d*x +
c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2)/d^2 + 105*(sqrt(2)
)*sqrt(pi)*(-64*I*b^2*c^2*d + 48*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 4*(64*I*(d*x + c)^(5/2)*b^2*d - 12
8*I*(d*x + c)^(3/2)*b^2*c*d + 64*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(
3/2)*b*d^2 - 48*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-4*I*(d
*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^3)/d^2 + 105*(sqrt(2)*sqrt(pi)*(64*I*b^
2*c^2*d + 48*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq

$$\begin{aligned}
& \text{rt}(b^2*d^2 + 1)*b^3) - 4*(-64*I*(d*x + c)^{(5/2)}*b^2*d + 128*I*(d*x + c)^{(3/2)}*b^2*c*d - 64*I*\text{sqrt}(d*x + c)*b^2*c^2*d + 40*(d*x + c)^{(3/2)}*b*d^2 - 48*\text{sqrt}(d*x + c)*b*c*d^2 + 15*I*\text{sqrt}(d*x + c)*d^3)*e^{((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^3}/d^2) - 112*(1536*(d*x + c)^{(5/2)} - 2560*(d*x + c)^{(3/2)}*c + 15*\text{sqrt}(2)*\text{sqrt}(\pi)*(8*I*b*c*d - 3*d^2)*d*\text{erf}(-\text{sqrt}(2)*\text{sqrt}(b*d)*\text{sqrt}(d*x + c)*(I*b*d/\text{sqrt}(b^2*d^2) + 1)/d)*e^{((4*I*b*c - 4*I*a*d)/d)/(\text{sqrt}(b*d)*(I*b*d/\text{sqrt}(b^2*d^2) + 1)*b^2) + 15*\text{sqrt}(2)*\text{sqrt}(\pi)*(-8*I*b*c*d - 3*d^2)*d*\text{erf}(-\text{sqrt}(2)*\text{sqrt}(b*d)*\text{sqrt}(d*x + c)*(-I*b*d/\text{sqrt}(b^2*d^2) + 1)/d)*e^{((-4*I*b*c + 4*I*a*d)/d)/(\text{sqrt}(b*d)*(-I*b*d/\text{sqrt}(b^2*d^2) + 1)*b^2) - 60*(-8*I*(d*x + c)^{(3/2)}*b*d + 8*I*\text{sqrt}(d*x + c)*b*c*d + 3*\text{sqrt}(d*x + c)*d^2)*e^{((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2} - 60*(8*I*(d*x + c)^{(3/2)}*b*d - 8*I*\text{sqrt}(d*x + c)*b*c*d + 3*\text{sqrt}(d*x + c)*d^2)*e^{((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2}*c)/d
\end{aligned}$$

3.125 $\int (c + dx)^{3/2} \cos^2(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=200

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a - \frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2}$$

[Out] $(c + d*x)^{(5/2)}/(20*d) - (3*d*\text{Sqrt}[c + d*x]*\text{Cos}[4*a + 4*b*x])/(256*b^2) + (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[4*a - (4*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(512*b^{(5/2)}) - (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[4*a - (4*b*c)/d])/(512*b^{(5/2)}) - ((c + d*x)^{(3/2)}*\text{Sin}[4*a + 4*b*x])/(32*b)$

Rubi [A] time = 0.328737, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a - \frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^{(3/2)}*\text{Cos}[a + b*x]^2*\text{Sin}[a + b*x]^2, x]$

[Out] $(c + d*x)^{(5/2)}/(20*d) - (3*d*\text{Sqrt}[c + d*x]*\text{Cos}[4*a + 4*b*x])/(256*b^2) + (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[4*a - (4*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(512*b^{(5/2)}) - (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[4*a - (4*b*c)/d])/(512*b^{(5/2)}) - ((c + d*x)^{(3/2)}*\text{Sin}[4*a + 4*b*x])/(32*b)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{GtQ}[m, 0]$

Rule 3306

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{ComplexFreeQ}[f] \&\& \text{NeQ}[d*e - c*f, 0]$

Rule 3305

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Sin}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}\{c, d, e, f\}$

, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))²], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3304

Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x²)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))²], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}
 \int (c + dx)^{3/2} \cos^2(a + bx) \sin^2(a + bx) dx &= \int \left(\frac{1}{8}(c + dx)^{3/2} - \frac{1}{8}(c + dx)^{3/2} \cos(4a + 4bx) \right) dx \\
 &= \frac{(c + dx)^{5/2}}{20d} - \frac{1}{8} \int (c + dx)^{3/2} \cos(4a + 4bx) dx \\
 &= \frac{(c + dx)^{5/2}}{20d} - \frac{(c + dx)^{3/2} \sin(4a + 4bx)}{32b} + \frac{(3d) \int \sqrt{c + dx} \sin(4a + 4bx) dx}{64b} \\
 &= \frac{(c + dx)^{5/2}}{20d} - \frac{3d\sqrt{c + dx} \cos(4a + 4bx)}{256b^2} - \frac{(c + dx)^{3/2} \sin(4a + 4bx)}{32b} + \frac{(3d)^2 \int \sqrt{c + dx} dx}{64b^2} \\
 &= \frac{(c + dx)^{5/2}}{20d} - \frac{3d\sqrt{c + dx} \cos(4a + 4bx)}{256b^2} - \frac{(c + dx)^{3/2} \sin(4a + 4bx)}{32b} + \frac{(3d)^2 \sqrt{c + dx}}{64b^2} \\
 &= \frac{(c + dx)^{5/2}}{20d} - \frac{3d\sqrt{c + dx} \cos(4a + 4bx)}{256b^2} - \frac{(c + dx)^{3/2} \sin(4a + 4bx)}{32b} + \frac{(3d)^2 \sqrt{c + dx}}{64b^2} \\
 &= \frac{(c + dx)^{5/2}}{20d} - \frac{3d\sqrt{c + dx} \cos(4a + 4bx)}{256b^2} + \frac{3d^{3/2} \sqrt{\frac{\pi}{2}} \cos\left(4a - \frac{4bc}{d}\right) C\left(\frac{2\sqrt{b}}{d}\sqrt{c + dx}\right)}{512b^{5/2}}
 \end{aligned}$$

Mathematica [A] time = 3.15495, size = 187, normalized size = 0.94

$$\frac{\sqrt{\frac{b}{d}} \left(15\sqrt{2\pi}d^2 \cos\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c + dx}\right) - 15\sqrt{2\pi}d^2 \sin\left(4a - \frac{4bc}{d}\right) S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c + dx}\right) + 4\sqrt{\frac{b}{d}}\sqrt{c + dx} \right)}{5120b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]

[Out] (Sqrt[b/d]*(15*d²*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 15*d²*Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(-15*d²*Cos[4*(a + b*x)] + 8*b*(c + d*x)*(8*b*(c + d*x) - 5*d*Sin[4*(a + b*x)])))/(5120*b³)

Maple [A] time = 0.036, size = 206, normalized size = 1.

$$2 \frac{1}{d} \left(\frac{1}{40} (dx+c)^{5/2} - \frac{d(dx+c)^{3/2}}{64b} \sin \left(4 \frac{(dx+c)b}{d} + 4 \frac{ad-bc}{d} \right) + \frac{3d}{64b} \left(-\frac{1}{8} \frac{d\sqrt{dx+c}}{b} \cos \left(4 \frac{(dx+c)b}{d} + 4 \frac{ad-bc}{d} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)

[Out] 2/d*(1/40*(d*x+c)^(5/2)-1/64/b*d*(d*x+c)^(3/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+3/64/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))

Maxima [C] time = 1.94746, size = 886, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")

[Out] 1/20480*(1024*(d*x + c)^(5/2)*b^2*sqrt(abs(b)/abs(d)) - 640*(d*x + c)^(3/2)*b*d*sqrt(abs(b)/abs(d))*sin(4*((d*x + c)*b - b*c + a*d)/d) - 240*sqrt(d*x + c)*d^2*sqrt(abs(b)/abs(d))*cos(4*((d*x + c)*b - b*c + a*d)/d) + ((15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*cos(-4*(b*c - a*d)/d) - (15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + ((15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*cos(-4*(b*c - a*d)/d) - (-15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/(b^2*d*sqrt(abs(b)/abs(d)))

Fricas [A] time = 0.597456, size = 621, normalized size = 3.1

$$15 \sqrt{2} \pi d^3 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{4(bc-ad)}{d}\right) C\left(2 \sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) - 15 \sqrt{2} \pi d^3 \sqrt{\frac{b}{\pi d}} S\left(2 \sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{4(bc-ad)}{d}\right) + 4(64 b^3 d^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5120*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) +
4*(64*b^3*d^2*x^2 - 120*b*d^2*cos(b*x + a)^4 + 128*b^3*c*d*x + 64*b^3*c^2
+ 120*b*d^2*cos(b*x + a)^2 - 15*b*d^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*cos(b*
x + a)^3 - (b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a)*sqrt(d*x + c))
/(b^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.45801, size = 805, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/30720*(1536*(d*x + c)^(5/2) - 2560*(d*x + c)^(3/2)*c - 40*(3*I*sqrt(2)*sq
rt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 3*I*s
qrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)*b) - 64*(d*x + c)^(3/2) - 12*I*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*
b*c + 4*I*a*d)/d)/b + 12*I*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b + 4*I*b*c -
4*I*a*d)/d)/b*c + 15*sqrt(2)*sqrt(pi)*(8*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)
/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 15*sqrt(2)*sqrt(pi)*(-8*I*b
*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^
2) - 60*(-8*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x +
c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2 - 60*(8*I*(d*x + c)
^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-4*I*(d*x +
c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d
```

3.126 $\int \sqrt{c + dx} \cos^2(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=174

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \sin\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} + \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(4a - \frac{4bc}{d}\right) S\left(\frac{2\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} - \frac{\sqrt{c+dx} \sin(4a + 4bx)}{32b} + \frac{(c + d^2 x^2)^{3/2}}{12d} - \frac{(c + d^2 x^2)^{3/2}}{64b^{3/2}} + \frac{(c + d^2 x^2)^{3/2}}{32b} - \frac{(c + d^2 x^2)^{3/2}}{64b^{3/2}}$$

[Out] (c + d*x)^(3/2)/(12*d) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(32*b)

Rubi [A] time = 0.270369, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \sin\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} + \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(4a - \frac{4bc}{d}\right) S\left(\frac{2\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} - \frac{\sqrt{c+dx} \sin(4a + 4bx)}{32b} + \frac{(c + d^2 x^2)^{3/2}}{12d} - \frac{(c + d^2 x^2)^{3/2}}{64b^{3/2}} + \frac{(c + d^2 x^2)^{3/2}}{32b} - \frac{(c + d^2 x^2)^{3/2}}{64b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]

[Out] (c + d*x)^(3/2)/(12*d) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(32*b)

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3306

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]

Rule 3305

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned} \int \sqrt{c+dx} \cos^2(a+bx) \sin^2(a+bx) dx &= \int \left(\frac{1}{8} \sqrt{c+dx} - \frac{1}{8} \sqrt{c+dx} \cos(4a+4bx) \right) dx \\ &= \frac{(c+dx)^{3/2}}{12d} - \frac{1}{8} \int \sqrt{c+dx} \cos(4a+4bx) dx \\ &= \frac{(c+dx)^{3/2}}{12d} - \frac{\sqrt{c+dx} \sin(4a+4bx)}{32b} + \frac{d \int \frac{\sin(4a+4bx)}{\sqrt{c+dx}} dx}{64b} \\ &= \frac{(c+dx)^{3/2}}{12d} - \frac{\sqrt{c+dx} \sin(4a+4bx)}{32b} + \frac{\left(d \cos\left(4a - \frac{4bc}{d}\right) \right) \int \frac{\sin\left(\frac{4bc}{d} + 4bx\right)}{\sqrt{c+dx}} dx}{64b} \\ &= \frac{(c+dx)^{3/2}}{12d} - \frac{\sqrt{c+dx} \sin(4a+4bx)}{32b} + \frac{\cos\left(4a - \frac{4bc}{d}\right) \text{Subst}\left(\int \sin\left(\frac{4bx^2}{d}\right) dx\right)}{32b} \\ &= \frac{(c+dx)^{3/2}}{12d} + \frac{\sqrt{d} \sqrt{\frac{\pi}{2}} \cos\left(4a - \frac{4bc}{d}\right) S\left(\frac{2\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} + \frac{\sqrt{d} \sqrt{\frac{\pi}{2}} C\left(\frac{2\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.885184, size = 161, normalized size = 0.93

$$\frac{3\sqrt{2\pi}d \sin\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c+dx}\right) + 3\sqrt{2\pi}d \cos\left(4a - \frac{4bc}{d}\right) S\left(2\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right) + 4\sqrt{\frac{b}{d}} \sqrt{c+dx} (8b(c+dx) \text{FresnelS}\left(2\sqrt{\frac{2}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c+dx}\right) - 8b(c+dx) \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c+dx}\right))}{384d^2 \left(\frac{b}{d}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]2*Sin[a + b*x]2, x]
```

```
[Out] (3*d*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] + 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(8*b*(c + d*x) - 3*d*Sin[4*(a + b*x)]))/(384*(b/d)^(3/2)*d^2)
```

Maple [A] time = 0.036, size = 159, normalized size = 0.9

$$2 \frac{1}{d} \left(\frac{1}{24} (dx+c)^{3/2} - \frac{d\sqrt{dx+c}}{64b} \sin\left(4 \frac{(dx+c)b}{d} + 4 \frac{ad-bc}{d}\right) + \frac{d\sqrt{2}\sqrt{\pi}}{256b} \left(\cos\left(4 \frac{ad-bc}{d}\right) \text{FresnelS}\left(2 \frac{\sqrt{2}\sqrt{dx+c}b}{\sqrt{\pi}d}\right) - \sin\left(4 \frac{ad-bc}{d}\right) \text{FresnelC}\left(2 \frac{\sqrt{2}\sqrt{dx+c}b}{\sqrt{\pi}d}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)`

[Out] $2/d*(1/24*(d*x+c)^{(3/2)}-1/64/b*d*(d*x+c)^{(1/2)}*\sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+1/256/b*d*2^{(1/2)}*Pi^{(1/2)}/(b/d)^{(1/2)}*(\cos(4*(a*d-b*c)/d)*\text{FresnelS}(2*2^{(1/2)}/Pi^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d)+\sin(4*(a*d-b*c)/d)*\text{FresnelC}(2*2^{(1/2)}/Pi^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d))$

Maxima [C] time = 1.93622, size = 817, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")`

[Out] $1/1536*(128*(d*x + c)^{(3/2)}*b*\sqrt{\text{abs}(b)/\text{abs}(d)} - 48*\sqrt{d*x + c}*d*\sqrt{\text{abs}(b)/\text{abs}(d)}*\sin(4*((d*x + c)*b - b*c + a*d)/d) - ((-3*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 3*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 3*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d*\cos(-4*(b*c - a*d)/d) - (3*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 3*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d*\sin(-4*(b*c - a*d)/d))*\text{erf}(2*\sqrt{d*x + c}*\sqrt{I*b/d}) - ((3*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 3*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d*\cos(-4*(b*c - a*d)/d) - (3*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 3*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d*\sin(-4*(b*c - a*d)/d))*\text{erf}(2*\sqrt{d*x + c}*\sqrt{-I*b/d}))/ (b*d*\sqrt{\text{abs}(b)/\text{abs}(d)})$

Fricas [A] time = 0.569092, size = 447, normalized size = 2.57

$$\frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+16(2b^2dx+384b^2d)}{384b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")`

[Out] $1/384*(3*\sqrt{2}*\pi*d^2*\sqrt{b/(pi*d)}*\cos(-4*(b*c - a*d)/d)*\text{fresnel_sin}(2*\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) + 3*\sqrt{2}*\pi*d^2*\sqrt{b/(pi*d)}*\text{fresnel_cos}(2*\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/(pi*d)})*\sin(-4*(b*c - a*d)/d) + 16*(2*b^2*d*x + 2*b^2*c - 3*(2*b*d*\cos(b*x + a)^3 - b*d*\cos(b*x + a))*\sin(b*x + a))*\sqrt{d*x + c}))/ (b^2*d)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)

[Out] Timed out

Giac [C] time = 1.29019, size = 347, normalized size = 1.99

$$\frac{3i\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right) e^{\left(\frac{4ibc-4iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} - \frac{3i\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right) e^{\left(\frac{-4ibc+4iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} - 64(dx+c)^{\frac{3}{2}} - \frac{12i\sqrt{dx+c}e^{\left(\frac{4i(d}{b}\right)}}{b}}{768d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")

[Out]
$$-1/768*(3*I*\sqrt{2}*\sqrt{\pi)*d^2*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d}*\sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((4*I*b*c-4*I*a*d)/d)}/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2}+1)*b) - 3*I*\sqrt{2}*\sqrt{\pi)*d^2*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d}*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((-4*I*b*c+4*I*a*d)/d)}/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2}+1)*b) - 64*(d*x+c)^{(3/2)} - 12*I*\sqrt{d*x+c}*d*e^{((4*I*(d*x+c)*b-4*I*b*c+4*I*a*d)/d)/b} + 12*I*\sqrt{d*x+c}*d*e^{((-4*I*(d*x+c)*b+4*I*b*c-4*I*a*d)/d)/b)/d$$

3.127 $\int \sqrt{c + dx} \cos^2(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=174

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \sin\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} + \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(4a - \frac{4bc}{d}\right) S\left(\frac{2\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} - \frac{\sqrt{c+dx} \sin(4a + 4bx)}{32b} + \frac{(c + d^2 x^2)^{3/2}}{12d} - \frac{(c + d^2 x^2)^{3/2}}{64b^{3/2}} + \frac{(c + d^2 x^2)^{3/2}}{32b} - \frac{(c + d^2 x^2)^{3/2}}{64b^{3/2}}$$

[Out] (c + d*x)^(3/2)/(12*d) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(32*b)

Rubi [A] time = 0.249162, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \sin\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} + \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(4a - \frac{4bc}{d}\right) S\left(\frac{2\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} - \frac{\sqrt{c+dx} \sin(4a + 4bx)}{32b} + \frac{(c + d^2 x^2)^{3/2}}{12d} - \frac{(c + d^2 x^2)^{3/2}}{64b^{3/2}} + \frac{(c + d^2 x^2)^{3/2}}{32b} - \frac{(c + d^2 x^2)^{3/2}}{64b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]

[Out] (c + d*x)^(3/2)/(12*d) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(32*b)

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3306

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]

Rule 3305

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{c+dx} \cos^2(a+bx) \sin^2(a+bx) dx &= \int \left(\frac{1}{8} \sqrt{c+dx} - \frac{1}{8} \sqrt{c+dx} \cos(4a+4bx) \right) dx \\
&= \frac{(c+dx)^{3/2}}{12d} - \frac{1}{8} \int \sqrt{c+dx} \cos(4a+4bx) dx \\
&= \frac{(c+dx)^{3/2}}{12d} - \frac{\sqrt{c+dx} \sin(4a+4bx)}{32b} + \frac{d \int \frac{\sin(4a+4bx)}{\sqrt{c+dx}} dx}{64b} \\
&= \frac{(c+dx)^{3/2}}{12d} - \frac{\sqrt{c+dx} \sin(4a+4bx)}{32b} + \frac{\left(d \cos\left(4a - \frac{4bc}{d}\right) \right) \int \frac{\sin\left(\frac{4bc}{d} + 4bx\right)}{\sqrt{c+dx}} dx}{64b} \\
&= \frac{(c+dx)^{3/2}}{12d} - \frac{\sqrt{c+dx} \sin(4a+4bx)}{32b} + \frac{\cos\left(4a - \frac{4bc}{d}\right) \text{Subst}\left(\int \sin\left(\frac{4bx^2}{d}\right) dx\right)}{32b} \\
&= \frac{(c+dx)^{3/2}}{12d} + \frac{\sqrt{d} \sqrt{\frac{\pi}{2}} \cos\left(4a - \frac{4bc}{d}\right) S\left(\frac{2\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} + \frac{\sqrt{d} \sqrt{\frac{\pi}{2}} C\left(\frac{2\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.136948, size = 161, normalized size = 0.93

$$\frac{3\sqrt{2\pi}d \sin\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c+dx}\right) + 3\sqrt{2\pi}d \cos\left(4a - \frac{4bc}{d}\right) S\left(2\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right) + 4\sqrt{\frac{b}{d}} \sqrt{c+dx} (8b(c+dx) \text{FresnelS}\left(2\sqrt{\frac{2}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c+dx}\right) - 8b(c+dx) \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c+dx}\right))}{384d^2 \left(\frac{b}{d}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]2*Sin[a + b*x]2, x]
```

```
[Out] (3*d*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] + 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(8*b*(c + d*x) - 3*d*Sin[4*(a + b*x)]))/(384*(b/d)^(3/2)*d^2)
```

Maple [A] time = 0.036, size = 159, normalized size = 0.9

$$2 \frac{1}{d} \left(\frac{1}{24} (dx+c)^{3/2} - \frac{d\sqrt{dx+c}}{64b} \sin\left(4 \frac{(dx+c)b}{d} + 4 \frac{ad-bc}{d}\right) + \frac{d\sqrt{2}\sqrt{\pi}}{256b} \left(\cos\left(4 \frac{ad-bc}{d}\right) \text{FresnelS}\left(2 \frac{\sqrt{2}\sqrt{dx+c}b}{\sqrt{\pi}d}\right) - \sin\left(4 \frac{ad-bc}{d}\right) \text{FresnelC}\left(2 \frac{\sqrt{2}\sqrt{dx+c}b}{\sqrt{\pi}d}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)`

[Out] $2/d*(1/24*(d*x+c)^{(3/2)}-1/64/b*d*(d*x+c)^{(1/2)}*\sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+1/256/b*d*2^{(1/2)}*Pi^{(1/2)}/(b/d)^{(1/2)}*(\cos(4*(a*d-b*c)/d)*\text{FresnelS}(2*2^{(1/2)}/Pi^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d)+\sin(4*(a*d-b*c)/d)*\text{FresnelC}(2*2^{(1/2)}/Pi^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d))$

Maxima [C] time = 2.01321, size = 817, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")`

[Out] $1/1536*(128*(d*x + c)^{(3/2)}*b*\sqrt{\text{abs}(b)/\text{abs}(d)} - 48*\sqrt{d*x + c}*d*\sqrt{\text{abs}(b)/\text{abs}(d)}*\sin(4*((d*x + c)*b - b*c + a*d)/d) - ((-3*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 3*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 3*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d*\cos(-4*(b*c - a*d)/d) - (3*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 3*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d*\sin(-4*(b*c - a*d)/d))*\text{erf}(2*\sqrt{d*x + c}*\sqrt{I*b/d}) - ((3*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 3*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d*\cos(-4*(b*c - a*d)/d) - (3*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 3*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d*\sin(-4*(b*c - a*d)/d))*\text{erf}(2*\sqrt{d*x + c}*\sqrt{-I*b/d}))/ (b*d*\sqrt{\text{abs}(b)/\text{abs}(d)})$

Fricas [A] time = 0.571745, size = 447, normalized size = 2.57

$$\frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+16(2b^2dx+384b^2d)}{384b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")`

[Out] $1/384*(3*\sqrt{2}*\pi*d^2*\sqrt{b/(pi*d)}*\cos(-4*(b*c - a*d)/d)*\text{fresnel_sin}(2*\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) + 3*\sqrt{2}*\pi*d^2*\sqrt{b/(pi*d)}*\text{fresnel_cos}(2*\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/(pi*d)})*\sin(-4*(b*c - a*d)/d) + 16*(2*b^2*d*x + 2*b^2*c - 3*(2*b*d*\cos(b*x + a))^3 - b*d*\cos(b*x + a))*\sin(b*x + a))*\sqrt{d*x + c}))/ (b^2*d)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)

[Out] Timed out

Giac [C] time = 1.29994, size = 347, normalized size = 1.99

$$\frac{3i\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\left(\frac{4ibc-4iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} - \frac{3i\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\left(\frac{-4ibc+4iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} - 64(dx+c)^{\frac{3}{2}} - \frac{12i\sqrt{dx+c}e^{\left(\frac{4i(d}{b}\right)}}{b}}{768d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/768*(3*I*\sqrt{2}*\sqrt{\pi)*d^2*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d}*\sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((4*I*b*c-4*I*a*d)/d)}/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2}+1)*b) \\ & - 3*I*\sqrt{2}*\sqrt{\pi)*d^2*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d}*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((-4*I*b*c+4*I*a*d)/d)}/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2}+1)*b) \\ & - 64*(d*x+c)^{(3/2)} - 12*I*\sqrt{d*x+c}*d*e^{((4*I*(d*x+c)*b-4*I*b*c+4*I*a*d)/d)/b} + 12*I*\sqrt{d*x+c}*d*e^{((-4*I*(d*x+c)*b+4*I*b*c-4*I*a*d)/d)/b}/d \end{aligned}$$

3.128 $\int (c + dx)^{3/2} \cos^2(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=200

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a - \frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2}$$

[Out] $(c + d*x)^{(5/2)}/(20*d) - (3*d*\text{Sqrt}[c + d*x]*\text{Cos}[4*a + 4*b*x])/(256*b^2) + (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[4*a - (4*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(512*b^{(5/2)}) - (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[4*a - (4*b*c)/d])/(512*b^{(5/2)}) - ((c + d*x)^{(3/2)}*\text{Sin}[4*a + 4*b*x])/(32*b)$

Rubi [A] time = 0.31823, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a - \frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^{(3/2)}*\text{Cos}[a + b*x]^2*\text{Sin}[a + b*x]^2, x]$

[Out] $(c + d*x)^{(5/2)}/(20*d) - (3*d*\text{Sqrt}[c + d*x]*\text{Cos}[4*a + 4*b*x])/(256*b^2) + (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[4*a - (4*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(512*b^{(5/2)}) - (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[4*a - (4*b*c)/d])/(512*b^{(5/2)}) - ((c + d*x)^{(3/2)}*\text{Sin}[4*a + 4*b*x])/(32*b)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[p, 0]$

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{GtQ}[m, 0]$

Rule 3306

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{NeQ}[d*e - c*f, 0]$

Rule 3305

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Sin}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}\{c, d, e, f\}$

, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))²], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3304

Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x²)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))²], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}
 \int (c + dx)^{3/2} \cos^2(a + bx) \sin^2(a + bx) dx &= \int \left(\frac{1}{8}(c + dx)^{3/2} - \frac{1}{8}(c + dx)^{3/2} \cos(4a + 4bx) \right) dx \\
 &= \frac{(c + dx)^{5/2}}{20d} - \frac{1}{8} \int (c + dx)^{3/2} \cos(4a + 4bx) dx \\
 &= \frac{(c + dx)^{5/2}}{20d} - \frac{(c + dx)^{3/2} \sin(4a + 4bx)}{32b} + \frac{(3d) \int \sqrt{c + dx} \sin(4a + 4bx) dx}{64b} \\
 &= \frac{(c + dx)^{5/2}}{20d} - \frac{3d\sqrt{c + dx} \cos(4a + 4bx)}{256b^2} - \frac{(c + dx)^{3/2} \sin(4a + 4bx)}{32b} + \frac{(3d)^2 \int \sqrt{c + dx} dx}{512b^2} \\
 &= \frac{(c + dx)^{5/2}}{20d} - \frac{3d\sqrt{c + dx} \cos(4a + 4bx)}{256b^2} - \frac{(c + dx)^{3/2} \sin(4a + 4bx)}{32b} + \frac{(3d)^2 \sqrt{c + dx}}{512b^2} \\
 &= \frac{(c + dx)^{5/2}}{20d} - \frac{3d\sqrt{c + dx} \cos(4a + 4bx)}{256b^2} - \frac{(c + dx)^{3/2} \sin(4a + 4bx)}{32b} + \frac{(3d)^2 \sqrt{c + dx}}{512b^2} \\
 &= \frac{(c + dx)^{5/2}}{20d} - \frac{3d\sqrt{c + dx} \cos(4a + 4bx)}{256b^2} + \frac{3d^{3/2} \sqrt{\frac{\pi}{2}} \cos\left(4a - \frac{4bc}{d}\right) C\left(\frac{2\sqrt{b}}{d}\sqrt{c + dx}\right)}{512b^{5/2}}
 \end{aligned}$$

Mathematica [A] time = 1.69443, size = 187, normalized size = 0.94

$$\frac{\sqrt{\frac{b}{d}} \left(15\sqrt{2\pi}d^2 \cos\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c + dx}\right) - 15\sqrt{2\pi}d^2 \sin\left(4a - \frac{4bc}{d}\right) S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c + dx}\right) + 4\sqrt{\frac{b}{d}}\sqrt{c + dx} \right)}{5120b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]

[Out] (Sqrt[b/d]*(15*d²*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 15*d²*Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(-15*d²*Cos[4*(a + b*x)] + 8*b*(c + d*x)*(8*b*(c + d*x) - 5*d*Sin[4*(a + b*x)])))/(5120*b³)

Maple [A] time = 0.036, size = 206, normalized size = 1.

$$2 \frac{1}{d} \left(\frac{1}{40} (dx+c)^{5/2} - \frac{d(dx+c)^{3/2}}{64b} \sin \left(4 \frac{(dx+c)b}{d} + 4 \frac{ad-bc}{d} \right) + \frac{3d}{64b} \left(-\frac{1}{8} \frac{d\sqrt{dx+c}}{b} \cos \left(4 \frac{(dx+c)b}{d} + 4 \frac{ad-bc}{d} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)

[Out] 2/d*(1/40*(d*x+c)^(5/2)-1/64/b*d*(d*x+c)^(3/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+3/64/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))

Maxima [C] time = 2.15975, size = 886, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")

[Out] 1/20480*(1024*(d*x + c)^(5/2)*b^2*sqrt(abs(b)/abs(d)) - 640*(d*x + c)^(3/2)*b*d*sqrt(abs(b)/abs(d))*sin(4*((d*x + c)*b - b*c + a*d)/d) - 240*sqrt(d*x + c)*d^2*sqrt(abs(b)/abs(d))*cos(4*((d*x + c)*b - b*c + a*d)/d) + ((15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*cos(-4*(b*c - a*d)/d) - (15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + ((15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*cos(-4*(b*c - a*d)/d) - (-15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/(b^2*d*sqrt(abs(b)/abs(d)))

Fricas [A] time = 0.606359, size = 621, normalized size = 3.1

$$15 \sqrt{2} \pi d^3 \sqrt{\frac{b}{\pi d}} \cos \left(-\frac{4(bc-ad)}{d} \right) C \left(2 \sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}} \right) - 15 \sqrt{2} \pi d^3 \sqrt{\frac{b}{\pi d}} S \left(2 \sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}} \right) \sin \left(-\frac{4(bc-ad)}{d} \right) + 4 (64 b^3 d^2 x$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5120*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) +
4*(64*b^3*d^2*x^2 - 120*b*d^2*cos(b*x + a)^4 + 128*b^3*c*d*x + 64*b^3*c^2
+ 120*b*d^2*cos(b*x + a)^2 - 15*b*d^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*cos(b*
x + a)^3 - (b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a)*sqrt(d*x + c))
/(b^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.43897, size = 805, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/30720*(1536*(d*x + c)^(5/2) - 2560*(d*x + c)^(3/2)*c - 40*(3*I*sqrt(2)*sq
rt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 3*I*s
qrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)*b) - 64*(d*x + c)^(3/2) - 12*I*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*
b*c + 4*I*a*d)/d)/b + 12*I*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b + 4*I*b*c -
4*I*a*d)/d)/b*c + 15*sqrt(2)*sqrt(pi)*(8*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)
/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 15*sqrt(2)*sqrt(pi)*(-8*I*b
*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^
2) - 60*(-8*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x +
c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2 - 60*(8*I*(d*x + c)
^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-4*I*(d*x +
c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d
```

3.129 $\int (c + dx)^{5/2} \cos^2(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=228

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} - \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a - \frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} + \frac{15d^2\sqrt{c+dx}\sin(4a + 4bx)}{2048b^3}$$

[Out] $(c + d*x)^{(7/2)}/(28*d) - (5*d*(c + d*x)^{(3/2)}*\text{Cos}[4*a + 4*b*x])/(256*b^2) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[4*a - (4*b*c)/d]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(4096*b^{(7/2)}) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[4*a - (4*b*c)/d])/(4096*b^{(7/2)}) + (15*d^2*\text{Sqrt}[c + d*x]*\text{Sin}[4*a + 4*b*x])/(2048*b^3) - ((c + d*x)^{(5/2)}*\text{Sin}[4*a + 4*b*x])/(32*b)$

Rubi [A] time = 0.377683, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} - \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a - \frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} + \frac{15d^2\sqrt{c+dx}\sin(4a + 4bx)}{2048b^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^{(5/2)}*\text{Cos}[a + b*x]^2*\text{Sin}[a + b*x]^2, x]$

[Out] $(c + d*x)^{(7/2)}/(28*d) - (5*d*(c + d*x)^{(3/2)}*\text{Cos}[4*a + 4*b*x])/(256*b^2) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[4*a - (4*b*c)/d]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(4096*b^{(7/2)}) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[4*a - (4*b*c)/d])/(4096*b^{(7/2)}) + (15*d^2*\text{Sqrt}[c + d*x]*\text{Sin}[4*a + 4*b*x])/(2048*b^3) - ((c + d*x)^{(5/2)}*\text{Sin}[4*a + 4*b*x])/(32*b)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3306

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d,
Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f},
x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^(2)], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d,
Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^(2)], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^{5/2} \cos^2(a + bx) \sin^2(a + bx) dx &= \int \left(\frac{1}{8}(c + dx)^{5/2} - \frac{1}{8}(c + dx)^{5/2} \cos(4a + 4bx) \right) dx \\
&= \frac{(c + dx)^{7/2}}{28d} - \frac{1}{8} \int (c + dx)^{5/2} \cos(4a + 4bx) dx \\
&= \frac{(c + dx)^{7/2}}{28d} - \frac{(c + dx)^{5/2} \sin(4a + 4bx)}{32b} + \frac{(5d) \int (c + dx)^{3/2} \sin(4a + 4bx) dx}{64b} \\
&= \frac{(c + dx)^{7/2}}{28d} - \frac{5d(c + dx)^{3/2} \cos(4a + 4bx)}{256b^2} - \frac{(c + dx)^{5/2} \sin(4a + 4bx)}{32b} + \frac{(2d) \int (c + dx)^{1/2} \sin(4a + 4bx) dx}{64b} \\
&= \frac{(c + dx)^{7/2}}{28d} - \frac{5d(c + dx)^{3/2} \cos(4a + 4bx)}{256b^2} + \frac{15d^2 \sqrt{c + dx} \sin(4a + 4bx)}{2048b^3} - \frac{(2d) \int (c + dx)^{1/2} \sin(4a + 4bx) dx}{64b} \\
&= \frac{(c + dx)^{7/2}}{28d} - \frac{5d(c + dx)^{3/2} \cos(4a + 4bx)}{256b^2} + \frac{15d^2 \sqrt{c + dx} \sin(4a + 4bx)}{2048b^3} \\
&= \frac{(c + dx)^{7/2}}{28d} - \frac{5d(c + dx)^{3/2} \cos(4a + 4bx)}{256b^2} + \frac{15d^2 \sqrt{c + dx} \sin(4a + 4bx)}{2048b^3} - \frac{(2d) \int (c + dx)^{1/2} \sin(4a + 4bx) dx}{64b} \\
&= \frac{(c + dx)^{7/2}}{28d} - \frac{5d(c + dx)^{3/2} \cos(4a + 4bx)}{256b^2} - \frac{15d^{5/2} \sqrt{\frac{\pi}{2}} \cos\left(4a - \frac{4bc}{d}\right) S\left(\frac{2\sqrt{b(d(c + dx) + c)}}{d}\right)}{4096b^{7/2}}
\end{aligned}$$

Mathematica [A] time = 2.7336, size = 206, normalized size = 0.9

$$\frac{\sqrt{\frac{b}{d}} \left(4 \sqrt{\frac{b}{d}} \sqrt{c + dx} (-7d \sin(4(a + bx))) (64b^2(c + dx)^2 - 15d^2) - 280bd^2(c + dx) \cos(4(a + bx)) + 512b^3(c + dx)^3 \right) - 10}{57344b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(-105*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 105*d^3*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(512*b^3
```

$(c + dx)^3 - 280bd^2(c + dx)\cos[4(a + bx)] - 7d(-15d^2 + 64b^2(c + dx)^2)\sin[4(a + bx)]\bigg)/(57344b^4)$

Maple [A] time = 0.034, size = 251, normalized size = 1.1

$$2\frac{1}{d}\left(\frac{(dx+c)^{7/2}}{56} - \frac{d(dx+c)^{5/2}}{64b}\sin\left(4\frac{(dx+c)b}{d} + 4\frac{ad-bc}{d}\right) + \frac{5d}{64b}\left(-\frac{1}{8}\frac{d(dx+c)^{3/2}}{b}\cos\left(4\frac{(dx+c)b}{d} + 4\frac{ad-bc}{d}\right) + \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)`

[Out] $2/d*(1/56*(d*x+c)^{(7/2)} - 1/64/b*d*(d*x+c)^{(5/2)}*\sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d) + 5/64/b*d*(-1/8/b*d*(d*x+c)^{(3/2)}*\cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d) + 3/8/b*d*(1/8/b*d*(d*x+c)^{(1/2)}*\sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d) - 1/32/b*d*2^{(1/2)}*Pi^{(1/2)}/(b/d)^{(1/2)}*(\cos(4*(a*d-b*c)/d)*FresnelS(2*2^{(1/2)}/Pi^{(1/2)}/(b/d))^{(1/2)}*(d*x+c)^{(1/2)}*b/d + \sin(4*(a*d-b*c)/d)*FresnelC(2*2^{(1/2)}/Pi^{(1/2)}/(b/d))^{(1/2)}*(d*x+c)^{(1/2)}*b/d))$

Maxima [C] time = 2.13834, size = 922, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")`

[Out] $1/229376*(8192*(d*x + c)^{(7/2)}*b^3*\sqrt{\text{abs}(b)/\text{abs}(d)} - 4480*(d*x + c)^{(3/2)}*b*d^2*\sqrt{\text{abs}(b)/\text{abs}(d)}*\cos(4*((d*x + c)*b - b*c + a*d)/d) - ((105*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 105*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 105*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 105*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^3*\cos(-4*(b*c - a*d)/d) + (105*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 105*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 105*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 105*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^3*\sin(-4*(b*c - a*d)/d))*\text{erf}(2*\sqrt{d*x + c}*\sqrt{I*b/d}) - ((-105*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 105*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 105*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 105*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^3*\cos(-4*(b*c - a*d)/d) + (105*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 105*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 105*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 105*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^3*\sin(-4*(b*c - a*d)/d))*\text{erf}(2*\sqrt{d*x + c}*\sqrt{-I*b/d}) - 112*(64*(d*x + c)^{(5/2)}*b^2*d*\sqrt{\text{abs}(b)/\text{abs}(d)} - 15*\sqrt{d*x + c}*d^3*\sqrt{\text{abs}(b)/\text{abs}(d)})*\sin(4*((d*x + c)*b - b*c + a*d)/d)/(b^3*d*\sqrt{\text{abs}(b)/\text{abs}(d)})$

Fricas [A] time = 0.648786, size = 840, normalized size = 3.68

$$\frac{105 \sqrt{2} \pi d^4 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{4(bc-ad)}{d}\right) S\left(2 \sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) + 105 \sqrt{2} \pi d^4 \sqrt{\frac{b}{\pi d}} C\left(2 \sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{4(bc-ad)}{d}\right) - 16}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")

[Out]
$$-1/57344*(105*\sqrt{2}*\pi*d^4*\sqrt{b/(\pi*d)}*\cos(-4*(b*c - a*d)/d)*\text{fresnel_sin}(2*\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/(\pi*d)}) + 105*\sqrt{2}*\pi*d^4*\sqrt{b/(\pi*d)}*\text{fresnel_cos}(2*\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/(\pi*d)})*\sin(-4*(b*c - a*d)/d) - 16*(128*b^4*d^3*x^3 + 384*b^4*c*d^2*x^2 + 128*b^4*c^3 - 70*b^2*c*d^2 - 560*(b^2*d^3*x + b^2*c*d^2)*\cos(b*x + a)^4 + 560*(b^2*d^3*x + b^2*c*d^2)*\cos(b*x + a)^2 + 2*(192*b^4*c^2*d - 35*b^2*d^3)*x - 7*(2*(64*b^3*d^3*x^2 + 128*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*\cos(b*x + a)^3 - (64*b^3*d^3*x^2 + 128*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*\cos(b*x + a))*\sin(b*x + a))*\sqrt{d*x + c})/(b^4*d)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)

[Out] Timed out

Giac [C] time = 1.63911, size = 1467, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")

[Out]
$$-1/1720320*(2240*(3*I*\sqrt{2}*\sqrt{\pi}*d^2*\text{erf}(-\sqrt{2}*\sqrt{b*d})*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((4*I*b*c - 4*I*a*d)/d)}/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b) - 3*I*\sqrt{2}*\sqrt{\pi}*d^2*\text{erf}(-\sqrt{2}*\sqrt{b*d})*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-4*I*b*c + 4*I*a*d)/d)}/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b) - 64*(d*x + c)^{(3/2)} - 12*I*\sqrt{d*x + c}*d*e^{((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b} + 12*I*\sqrt{d*x + c}*d*e^{((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b}*c^2 - d^2*(4096*(15*(d*x + c)^{(7/2)} - 42*(d*x + c)^{(5/2)}*c + 35*(d*x + c)^{(3/2)}*c^2)/d^2 + 105*(\sqrt{2})*\sqrt{\pi}*(-64*I*b^2*c^2*d + 48*b*c*d^2 + 15*I*d^3)*d*\text{erf}(-\sqrt{2}*\sqrt{b*d})*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((4*I*b*c - 4*I*a*d)/d)}/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^3) - 4*(64*I*(d*x + c)^{(5/2)}*b^2*d - 128*I*(d*x + c)^{(3/2)}*b^2*c*d + 64*I*\sqrt{d*x + c}*b^2*c^2*d + 40*(d*x + c)^{(3/2)}*b*d^2 - 48*\sqrt{d*x + c}*b*c*d^2 - 15*I*\sqrt{d*x + c}*d^3)*e^{((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^3}/d^2 + 105*(\sqrt{2})*\sqrt{\pi}*(64*I*b^2*c^2*d + 48*b*c*d^2 - 15*I*d^3)*d*\text{erf}(-\sqrt{2}*\sqrt{b*d})*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-4*I*b*c + 4*I*a*d)/d)}/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)$$

$$\begin{aligned}
& \text{rt}(b^2*d^2 + 1)*b^3) - 4*(-64*I*(d*x + c)^{(5/2)}*b^2*d + 128*I*(d*x + c)^{(3/2)}*b^2*c*d - 64*I*\text{sqrt}(d*x + c)*b^2*c^2*d + 40*(d*x + c)^{(3/2)}*b*d^2 - 48*\text{sqrt}(d*x + c)*b*c*d^2 + 15*I*\text{sqrt}(d*x + c)*d^3)*e^{((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^3}/d^2) - 112*(1536*(d*x + c)^{(5/2)} - 2560*(d*x + c)^{(3/2)}*c + 15*\text{sqrt}(2)*\text{sqrt}(\pi)*(8*I*b*c*d - 3*d^2)*d*\text{erf}(-\text{sqrt}(2)*\text{sqrt}(b*d)*\text{sqrt}(d*x + c)*(I*b*d/\text{sqrt}(b^2*d^2) + 1)/d)*e^{((4*I*b*c - 4*I*a*d)/d)/(\text{sqrt}(b*d)*(I*b*d/\text{sqrt}(b^2*d^2) + 1)*b^2) + 15*\text{sqrt}(2)*\text{sqrt}(\pi)*(-8*I*b*c*d - 3*d^2)*d*\text{erf}(-\text{sqrt}(2)*\text{sqrt}(b*d)*\text{sqrt}(d*x + c)*(-I*b*d/\text{sqrt}(b^2*d^2) + 1)/d)*e^{((-4*I*b*c + 4*I*a*d)/d)/(\text{sqrt}(b*d)*(-I*b*d/\text{sqrt}(b^2*d^2) + 1)*b^2) - 60*(-8*I*(d*x + c)^{(3/2)}*b*d + 8*I*\text{sqrt}(d*x + c)*b*c*d + 3*\text{sqrt}(d*x + c)*d^2)*e^{((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2} - 60*(8*I*(d*x + c)^{(3/2)}*b*d - 8*I*\text{sqrt}(d*x + c)*b*c*d + 3*\text{sqrt}(d*x + c)*d^2)*e^{((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)*c)/d
\end{aligned}$$

3.130 $\int (c + dx)^{5/2} \cos^2(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=615

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}} - \frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}} + \frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\cos\left(5a - \frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}$$

[Out] $(15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[a + b*x])/(32*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[a + b*x])/(8*b) + (5*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[3*a + 3*b*x])/(48*b) - (3*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[5*a + 5*b*x])/(1600*b^3) + ((c + d*x)^{(5/2)}*\text{Cos}[5*a + 5*b*x])/(80*b) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a - (b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(32*b^{(7/2)}) - (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(576*b^{(7/2)}) + (3*d^{(5/2)}*\text{Sqrt}[\text{Pi}/10]*\text{Cos}[5*a - (5*b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(1600*b^{(7/2)}) - (3*d^{(5/2)}*\text{Sqrt}[\text{Pi}/10]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d])*Sin[5*a - (5*b*c)/d])/(1600*b^{(7/2)}) + (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d])*Sin[3*a - (3*b*c)/d])/(576*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d])*Sin[a - (b*c)/d])/(32*b^{(7/2)}) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[a + b*x])/(16*b^2) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[3*a + 3*b*x])/(288*b^2) - (d*(c + d*x)^{(3/2)}*\text{Sin}[5*a + 5*b*x])/(160*b^2)$

Rubi [A] time = 1.153, antiderivative size = 615, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}} - \frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}} + \frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\cos\left(5a - \frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^{(5/2)}*\text{Cos}[a + b*x]^2*\text{Sin}[a + b*x]^3, x]$

[Out] $(15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[a + b*x])/(32*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[a + b*x])/(8*b) + (5*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[3*a + 3*b*x])/(48*b) - (3*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[5*a + 5*b*x])/(1600*b^3) + ((c + d*x)^{(5/2)}*\text{Cos}[5*a + 5*b*x])/(80*b) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a - (b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(32*b^{(7/2)}) - (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(576*b^{(7/2)}) + (3*d^{(5/2)}*\text{Sqrt}[\text{Pi}/10]*\text{Cos}[5*a - (5*b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(1600*b^{(7/2)}) - (3*d^{(5/2)}*\text{Sqrt}[\text{Pi}/10]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d])*Sin[5*a - (5*b*c)/d])/(1600*b^{(7/2)}) + (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d])*Sin[3*a - (3*b*c)/d])/(576*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d])*Sin[a - (b*c)/d])/(32*b^{(7/2)}) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[a + b*x])/(16*b^2) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[3*a + 3*b*x])/(288*b^2) - (d*(c + d*x)^{(3/2)}*\text{Sin}[5*a + 5*b*x])/(160*b^2)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]$

$]^n \cos[a + b x]^p, x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3306

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]

Rule 3305

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3304

Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}
\int (c + dx)^{5/2} \cos^2(a + bx) \sin^3(a + bx) dx &= \int \left(\frac{1}{8}(c + dx)^{5/2} \sin(a + bx) + \frac{1}{16}(c + dx)^{5/2} \sin(3a + 3bx) - \frac{1}{16}(c + dx)^{5/2} \right. \\
&= \frac{1}{16} \int (c + dx)^{5/2} \sin(3a + 3bx) dx - \frac{1}{16} \int (c + dx)^{5/2} \sin(5a + 5bx) dx + \frac{1}{8} \\
&= -\frac{(c + dx)^{5/2} \cos(a + bx)}{8b} - \frac{(c + dx)^{5/2} \cos(3a + 3bx)}{48b} + \frac{(c + dx)^{5/2} \cos(5a)}{80b} \\
&= -\frac{(c + dx)^{5/2} \cos(a + bx)}{8b} - \frac{(c + dx)^{5/2} \cos(3a + 3bx)}{48b} + \frac{(c + dx)^{5/2} \cos(5a)}{80b} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(a + bx)}{32b^3} - \frac{(c + dx)^{5/2} \cos(a + bx)}{8b} + \frac{5d^2 \sqrt{c + dx} \cos(3a)}{576b^3} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(a + bx)}{32b^3} - \frac{(c + dx)^{5/2} \cos(a + bx)}{8b} + \frac{5d^2 \sqrt{c + dx} \cos(3a)}{576b^3} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(a + bx)}{32b^3} - \frac{(c + dx)^{5/2} \cos(a + bx)}{8b} + \frac{5d^2 \sqrt{c + dx} \cos(3a)}{576b^3} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(a + bx)}{32b^3} - \frac{(c + dx)^{5/2} \cos(a + bx)}{8b} + \frac{5d^2 \sqrt{c + dx} \cos(3a)}{576b^3}
\end{aligned}$$

Mathematica [C] time = 25.8253, size = 3348, normalized size = 5.44

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]

[Out] (c^2*Sqrt[c + d*x]*(-(E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[(-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[(I*b*(c + d*x))/d])/(16*b*E^((I*(b*c + a*d))/d) + (c^2*(2*Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d]))/(160*Sqrt[5]*b*Sqrt[b/d]) - (c^2*(2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(96*Sqrt[3]*b*Sqrt[b/d]) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(3*d*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a + b*x])))/(16*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (b*c)/d] + (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) - 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(d*(-15 + 4*b^2*x^2)*Cos[a + b*x] + 2*b*(c - 5*d*x)*Sin[a + b*x]))/(64*b^5) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(96*Sqrt[3]*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[

$$\begin{aligned} & b/d] * d * \text{Sqrt}[c + d*x] * (d * (5 - 12*b^2*x^2) * \text{Cos}[3*(a + b*x)] - 2*b*(c - 5*d*x) \\ & * \text{Sin}[3*(a + b*x)])) / (1152 * \text{Sqrt}[3] * b^5) + (c * \text{Sqrt}[b/d] * d * (\text{Sqrt}[2 * \text{Pi}] * \text{FresnelS} \\ & \text{S}[\text{Sqrt}[b/d] * \text{Sqrt}[10/\text{Pi}] * \text{Sqrt}[c + d*x]] * (3*d * \text{Cos}[5*a - (5*b*c)/d] - 10*b*c * \\ & \text{Sin}[5*a - (5*b*c)/d]) + \text{Sqrt}[2 * \text{Pi}] * \text{FresnelC}[\text{Sqrt}[b/d] * \text{Sqrt}[10/\text{Pi}] * \text{Sqrt}[c + \\ & d*x]] * (10*b*c * \text{Cos}[5*a - (5*b*c)/d] + 3*d * \text{Sin}[5*a - (5*b*c)/d]) + 2 * \text{Sqrt}[5] * \\ & \text{Sqrt}[b/d] * d * \text{Sqrt}[c + d*x] * (10*b*x * \text{Cos}[5*(a + b*x)] - 3 * \text{Sin}[5*(a + b*x)])) / \\ & (800 * \text{Sqrt}[5] * b^3) - (d^2 * (\text{Sin}[5*a] * ((c^2 * (-\text{Sqrt}[5] * \text{Sqrt}[b/d] * \text{Sqrt}[c + d*x] \\ & * \text{Cos}[(5*b*(c + d*x))/d]) + \text{Sqrt}[\text{Pi}/2] * \text{FresnelC}[\text{Sqrt}[b/d] * \text{Sqrt}[10/\text{Pi}] * \text{Sqrt}[c \\ & + d*x]] * \text{Sin}[(5*b*c)/d]) / (5 * \text{Sqrt}[5] * (b/d)^(3/2) * d^3) + (c^2 * \text{Cos}[(5*b*c)/d] \\ & * (-\text{Sqrt}[\text{Pi}/2] * \text{FresnelS}[\text{Sqrt}[b/d] * \text{Sqrt}[10/\text{Pi}] * \text{Sqrt}[c + d*x]])) + \text{Sqrt}[5] * \text{Sqr} \\ & \text{t}[b/d] * \text{Sqrt}[c + d*x] * \text{Sin}[(5*b*(c + d*x))/d]) / (5 * \text{Sqrt}[5] * (b/d)^(3/2) * d^3) - \\ & (2*c * \text{Cos}[(5*b*c)/d] * (-3 * (-\text{Sqrt}[5] * \text{Sqrt}[b/d] * \text{Sqrt}[c + d*x] * \text{Cos}[(5*b*(c + \\ & d*x))/d]) + \text{Sqrt}[\text{Pi}/2] * \text{FresnelC}[\text{Sqrt}[b/d] * \text{Sqrt}[10/\text{Pi}] * \text{Sqrt}[c + d*x]])) / 2 + \\ & 5 * \text{Sqrt}[5] * (b/d)^(3/2) * (c + d*x)^(3/2) * \text{Sin}[(5*b*(c + d*x))/d]) / (25 * \text{Sqrt}[5] * \\ & (b/d)^(5/2) * d^3) - (2*c * \text{Sin}[(5*b*c)/d] * (-5 * \text{Sqrt}[5] * (b/d)^(3/2) * (c + d*x)^(3 \\ & /2) * \text{Cos}[(5*b*(c + d*x))/d] + (3 * (-\text{Sqrt}[\text{Pi}/2] * \text{FresnelS}[\text{Sqrt}[b/d] * \text{Sqrt}[10/\text{Pi}] \\ &] * \text{Sqrt}[c + d*x]])) + \text{Sqrt}[5] * \text{Sqrt}[b/d] * \text{Sqrt}[c + d*x] * \text{Sin}[(5*b*(c + d*x))/d] \\ &) / 2) / (25 * \text{Sqrt}[5] * (b/d)^(5/2) * d^3) + (\text{Sin}[(5*b*c)/d] * (-25 * \text{Sqrt}[5] * (b/d)^(5 \\ & /2) * (c + d*x)^(5/2) * \text{Cos}[(5*b*(c + d*x))/d] + (5 * ((-3 * (-\text{Sqrt}[5] * \text{Sqrt}[b/d] * \text{Sqr} \\ & \text{t}[c + d*x] * \text{Cos}[(5*b*(c + d*x))/d]) + \text{Sqrt}[\text{Pi}/2] * \text{FresnelC}[\text{Sqrt}[b/d] * \text{Sqrt}[10 \\ & / \text{Pi}] * \text{Sqrt}[c + d*x]])) / 2 + 5 * \text{Sqrt}[5] * (b/d)^(3/2) * (c + d*x)^(3/2) * \text{Sin}[(5*b*(c \\ & + d*x))/d]) / 2) / (125 * \text{Sqrt}[5] * (b/d)^(7/2) * d^3) + (\text{Cos}[(5*b*c)/d] * (25 * \text{Sqrt}[\\ & 5] * (b/d)^(5/2) * (c + d*x)^(5/2) * \text{Sin}[(5*b*(c + d*x))/d] - (5 * (-5 * \text{Sqrt}[5] * (b/d \\ &)^(3/2) * (c + d*x)^(3/2) * \text{Cos}[(5*b*(c + d*x))/d] + (3 * (-\text{Sqrt}[\text{Pi}/2] * \text{FresnelS}[\\ & \text{Sqrt}[b/d] * \text{Sqrt}[10/\text{Pi}] * \text{Sqrt}[c + d*x]])) + \text{Sqrt}[5] * \text{Sqrt}[b/d] * \text{Sqrt}[c + d*x] * \text{Sin} \\ & [(5*b*(c + d*x))/d]) / 2) / (125 * \text{Sqrt}[5] * (b/d)^(7/2) * d^3) + \text{Cos}[5*a] * ((c \\ & ^2 * \text{Cos}[(5*b*c)/d] * (-\text{Sqrt}[5] * \text{Sqrt}[b/d] * \text{Sqrt}[c + d*x] * \text{Cos}[(5*b*(c + d*x))/d] \\ &) + \text{Sqrt}[\text{Pi}/2] * \text{FresnelC}[\text{Sqrt}[b/d] * \text{Sqrt}[10/\text{Pi}] * \text{Sqrt}[c + d*x]])) / (5 * \text{Sqrt}[5] * \\ & (b/d)^(3/2) * d^3) - (c^2 * \text{Sin}[(5*b*c)/d] * (-\text{Sqrt}[\text{Pi}/2] * \text{FresnelS}[\text{Sqrt}[b/d] * \text{Sqrt} \\ & [10/\text{Pi}] * \text{Sqrt}[c + d*x]])) + \text{Sqrt}[5] * \text{Sqrt}[b/d] * \text{Sqrt}[c + d*x] * \text{Sin}[(5*b*(c + d*x) \\ &) / d]) / (5 * \text{Sqrt}[5] * (b/d)^(3/2) * d^3) + (2*c * \text{Sin}[(5*b*c)/d] * ((-3 * (-\text{Sqrt}[5] * \text{S} \\ & \text{qrt}[b/d] * \text{Sqrt}[c + d*x] * \text{Cos}[(5*b*(c + d*x))/d]) + \text{Sqrt}[\text{Pi}/2] * \text{FresnelC}[\text{Sqrt}[b \\ & /d] * \text{Sqrt}[10/\text{Pi}] * \text{Sqrt}[c + d*x]])) / 2 + 5 * \text{Sqrt}[5] * (b/d)^(3/2) * (c + d*x)^(3/2) * \\ & \text{Sin}[(5*b*(c + d*x))/d]) / (25 * \text{Sqrt}[5] * (b/d)^(5/2) * d^3) - (2*c * \text{Cos}[(5*b*c)/d] \\ & * (-5 * \text{Sqrt}[5] * (b/d)^(3/2) * (c + d*x)^(3/2) * \text{Cos}[(5*b*(c + d*x))/d] + (3 * (-\text{Sqr} \\ & \text{t}[\text{Pi}/2] * \text{FresnelS}[\text{Sqrt}[b/d] * \text{Sqrt}[10/\text{Pi}] * \text{Sqrt}[c + d*x]])) + \text{Sqrt}[5] * \text{Sqrt}[b/d] * \\ & \text{Sqrt}[c + d*x] * \text{Sin}[(5*b*(c + d*x))/d]) / 2) / (25 * \text{Sqrt}[5] * (b/d)^(5/2) * d^3) + (\\ & \text{Cos}[(5*b*c)/d] * (-25 * \text{Sqrt}[5] * (b/d)^(5/2) * (c + d*x)^(5/2) * \text{Cos}[(5*b*(c + d*x)) \\ & /d] + (5 * ((-3 * (-\text{Sqrt}[5] * \text{Sqrt}[b/d] * \text{Sqrt}[c + d*x] * \text{Cos}[(5*b*(c + d*x))/d]) + \\ & \text{Sqrt}[\text{Pi}/2] * \text{FresnelC}[\text{Sqrt}[b/d] * \text{Sqrt}[10/\text{Pi}] * \text{Sqrt}[c + d*x]])) / 2 + 5 * \text{Sqrt}[5] * (b \\ & /d)^(3/2) * (c + d*x)^(3/2) * \text{Sin}[(5*b*(c + d*x))/d]) / 2) / (125 * \text{Sqrt}[5] * (b/d)^(\\ & 7/2) * d^3) - (\text{Sin}[(5*b*c)/d] * (25 * \text{Sqrt}[5] * (b/d)^(5/2) * (c + d*x)^(5/2) * \text{Sin}[(5* \\ & b*(c + d*x))/d] - (5 * (-5 * \text{Sqrt}[5] * (b/d)^(3/2) * (c + d*x)^(3/2) * \text{Cos}[(5*b*(c + \\ & d*x))/d] + (3 * (-\text{Sqrt}[\text{Pi}/2] * \text{FresnelS}[\text{Sqrt}[b/d] * \text{Sqrt}[10/\text{Pi}] * \text{Sqrt}[c + d*x]])) \\ & + \text{Sqrt}[5] * \text{Sqrt}[b/d] * \text{Sqrt}[c + d*x] * \text{Sin}[(5*b*(c + d*x))/d]) / 2) / 2) / (125 * \text{Sqr} \\ & \text{t}[5] * (b/d)^(7/2) * d^3))) / 16 \end{aligned}$$

Maple [A] time = 0.043, size = 719, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d*x+c)^(5/2)*\cos(b*x+a)^2*\sin(b*x+a)^3,x)$

[Out] $2/d*(-1/16/b*d*(d*x+c)^(5/2)*\cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+5/16/b*d*(1/2/b$
 $*d*(d*x+c)^(3/2)*\sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^($

$$\begin{aligned} & 1/2) * \cos(1/d * (d*x+c) * b + (a*d-b*c)/d) + 1/4/b*d*2^{(1/2)} * \text{Pi}^{(1/2)} / (b/d)^{(1/2)} * (\cos((a*d-b*c)/d) * \text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)}/(b/d)^{(1/2)} * (d*x+c)^{(1/2)} * b/d) - \sin((a*d-b*c)/d) * \text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}/(b/d)^{(1/2)} * (d*x+c)^{(1/2)} * b/d)) - \\ & 1/96/b*d*(d*x+c)^{(5/2)} * \cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d) + 5/96/b*d*(1/6/b*d*(d*x+c)^{(3/2)} * \sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d) - 1/2/b*d*(-1/6/b*d*(d*x+c)^{(1/2)} * \cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d) + 1/36/b*d*2^{(1/2)} * \text{Pi}^{(1/2)} * 3^{(1/2)} / (b/d)^{(1/2)} * (\cos(3*(a*d-b*c)/d) * \text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)} * 3^{(1/2)} / (b/d)^{(1/2)} * (d*x+c)^{(1/2)} * b/d) - \sin(3*(a*d-b*c)/d) * \text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)} * 3^{(1/2)} / (b/d)^{(1/2)} * (d*x+c)^{(1/2)} * b/d))) + 1/160/b*d*(d*x+c)^{(5/2)} * \cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d) - 1/32/b*d*(1/10/b*d*(d*x+c)^{(3/2)} * \sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d) - 3/10/b*d*(-1/10/b*d*(d*x+c)^{(1/2)} * \cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d) + 1/100/b*d*2^{(1/2)} * \text{Pi}^{(1/2)} * 5^{(1/2)} / (b/d)^{(1/2)} * (\cos(5*(a*d-b*c)/d) * \text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)} * 5^{(1/2)} / (b/d)^{(1/2)} * (d*x+c)^{(1/2)} * b/d) - \sin(5*(a*d-b*c)/d) * \text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)} * 5^{(1/2)} / (b/d)^{(1/2)} * (d*x+c)^{(1/2)} * b/d))) \end{aligned}$$

Maxima [C] time = 3.00513, size = 2943, normalized size = 4.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/1728000 * \text{sqrt}(5) * \text{sqrt}(3) * (720 * \text{sqrt}(5) * \text{sqrt}(3) * (d*x + c)^{(3/2)} * b * d^2 * \text{sqrt}(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) * \sin(5 * ((d*x + c) * b - b*c + a*d)/d) / \text{abs}(d) - 2000 * \text{sqrt}(5) * \text{sqrt}(3) * (d*x + c)^{(3/2)} * b * d^2 * \text{sqrt}(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) * \sin(3 * ((d*x + c) * b - b*c + a*d)/d) / \text{abs}(d) - 36000 * \text{sqrt}(5) * \text{sqrt}(3) * (d*x + c)^{(3/2)} * b * d^2 * \text{sqrt}(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) * \sin(((d*x + c) * b - b*c + a*d)/d) / \text{abs}(d) - 72 * (20 * \text{sqrt}(5) * \text{sqrt}(3) * (d*x + c)^{(5/2)} * b^2 * d * \text{sqrt}(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) / \text{abs}(d) - 3 * \text{sqrt}(5) * \text{sqrt}(3) * \text{sqrt}(d*x + c) * d^3 * \text{sqrt}(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) / \text{abs}(d)) * \cos(5 * ((d*x + c) * b - b*c + a*d)/d) + 200 * (12 * \text{sqrt}(5) * \text{sqrt}(3) * (d*x + c)^{(5/2)} * b^2 * d * \text{sqrt}(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) / \text{abs}(d) - 5 * \text{sqrt}(5) * \text{sqrt}(3) * \text{sqrt}(d*x + c) * d^3 * \text{sqrt}(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) / \text{abs}(d)) * \cos(3 * ((d*x + c) * b - b*c + a*d)/d) + 3600 * (4 * \text{sqrt}(5) * \text{sqrt}(3) * (d*x + c)^{(5/2)} * b^2 * d * \text{sqrt}(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) / \text{abs}(d) - 15 * \text{sqrt}(5) * \text{sqrt}(3) * \text{sqrt}(d*x + c) * d^3 * \text{sqrt}(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) / \text{abs}(d)) * \cos(((d*x + c) * b - b*c + a*d)/d) - (\text{sqrt}(3) * (27 * \text{sqrt}(\text{pi}) * \cos(1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) + 27 * \text{sqrt}(\text{pi}) * \cos(-1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) - 27 * I * \text{sqrt}(\text{pi}) * \sin(1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) + 27 * I * \text{sqrt}(\text{pi}) * \sin(-1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2)))) * d^3 * \text{abs}(b) * \cos(-5 * (b*c - a*d)/d) / \text{abs}(d) - \text{sqrt}(3) * (27 * I * \text{sqrt}(\text{pi}) * \cos(1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) + 27 * I * \text{sqrt}(\text{pi}) * \cos(-1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) + 27 * \text{sqrt}(\text{pi}) * \sin(1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) - 27 * \text{sqrt}(\text{pi}) * \sin(-1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2)))) * d^3 * \text{abs}(b) * \sin(-5 * (b*c - a*d)/d) / \text{abs}(d)) * \text{erf}(\text{sqrt}(d*x + c) * \text{sqrt}(5 * I * b/d)) + (\text{sqrt}(5) * (125 * \text{sqrt}(\text{pi}) * \cos(1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) + 125 * \text{sqrt}(\text{pi}) * \cos(-1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) - 125 * I * \text{sqrt}(\text{pi}) * \sin(1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) + 125 * I * \text{sqrt}(\text{pi}) * \sin(-1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2)))) * d^3 * \text{abs}(b) * \cos(-3 * (b*c - a*d)/d) / \text{abs}(d) + \text{sqrt}(5) * (-125 * I * \text{sqrt}(\text{pi}) * \cos(1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) - 125 * I * \text{sqrt}(\text{pi}) * \cos(-1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) - 125 * \text{sqrt}(\text{pi}) * \sin(1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) + 125 * \text{sqrt}(\text{pi}) * \sin(-1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2)))) * d^3 * \text{abs}(b) * \sin(-3 * (b*c - a*d)/d) / \text{abs}(d)) * \text{erf}(\text{sqrt}(d*x + c) * \text{sqrt}(3 * I * b/d)) + (\text{sqrt}(5) * \text{sqrt}(3) * (6750 * \text{sqrt}(\text{pi}) * \cos(1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) + \end{aligned}$$


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qrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d
))) *sin(-3*(b*c - a*d)/d) - 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(s
qrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) *sin(-5*(b*c - a*d)/d) + 480*(9*(20*b^
3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*cos(b*x + a)^5 + 390*b*d^2
*cos(b*x + a) - 5*(60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 - 13*b*d^2)*
*cos(b*x + a)^3 + 10*(26*b^2*d^2*x - 9*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^4
+ 26*b^2*c*d + 13*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(
d*x + c))/b^4

```

Sympy [F-1] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 2.23538, size = 4084, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```

[Out] 1/864000*(60*(9*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*d^2*er
f(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I
*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt
(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) +
9*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)*b) + 90*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)
/d)/b - 150*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b -
900*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 900*sqrt(d*x
+ c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b - 150*sqrt(d*x + c)*d*e^((
-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*sqrt(d*x + c)*d*e^((-5*I*(d
*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)*c^2 - d^2*(27*(I*sqrt(10)*sqrt(pi))*(20
*I*b^2*c^2*d - 12*b*c*d^2 - 3*I*d^3)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b^3) - 10*I*(-20*I*(d*x + c)^(5/2)*b^2*d + 40*I*(d*x
+ c)^(3/2)*b^2*c*d - 20*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d
^2 + 12*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*x + c)*d^3)*e^((-5*I*(d*x + c)*b
+ 5*I*b*c - 5*I*a*d)/d)/b^3)/d^2 + 125*(I*sqrt(6)*sqrt(pi))*(-12*I*b^2*c^2*
d + 12*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2

```

$$\begin{aligned}
& *d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^{(5/2)}*b^2*d - 24*I*(d*x + c)^{(3/2)}*b^2*c*d + 12*I*\sqrt{d*x + c}*b^2*c^2*d + 10*(d*x + c)^{(3/2)}*b*d^2 - 12*\sqrt{d*x + c}*b*c*d^2 - 5*I*\sqrt{d*x + c}*d^3)*e^{((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^3)/d^2 + 6750*(I*\sqrt{2}*\sqrt{\pi})*(-4*I*b^2*c^2*d + 12*b*c*d^2 + 15*I*d^3)*d*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((I*b*c - I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^3) - 2*I*(4*I*(d*x + c)^{(5/2)}*b^2*d - 8*I*(d*x + c)^{(3/2)}*b^2*c*d + 4*I*\sqrt{d*x + c}*b^2*c^2*d + 10*(d*x + c)^{(3/2)}*b*d^2 - 12*\sqrt{d*x + c}*b*c*d^2 - 15*I*\sqrt{d*x + c}*d^3)*e^{((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^2 + 6750*(I*\sqrt{2}*\sqrt{\pi})*(-4*I*b^2*c^2*d - 12*b*c*d^2 + 15*I*d^3)*d*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-I*b*c + I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^3) - 2*I*(4*I*(d*x + c)^{(5/2)}*b^2*d - 8*I*(d*x + c)^{(3/2)}*b^2*c*d + 4*I*\sqrt{d*x + c}*b^2*c^2*d - 10*(d*x + c)^{(3/2)}*b*d^2 + 12*\sqrt{d*x + c}*b*c*d^2 - 15*I*\sqrt{d*x + c}*d^3)*e^{((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^2 + 125*(I*\sqrt{6}*\sqrt{\pi})*(-12*I*b^2*c^2*d - 12*b*c*d^2 + 5*I*d^3)*d*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-3*I*b*c + 3*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^3) - 6*I*(12*I*(d*x + c)^{(5/2)}*b^2*d - 24*I*(d*x + c)^{(3/2)}*b^2*c*d + 12*I*\sqrt{d*x + c}*b^2*c^2*d - 10*(d*x + c)^{(3/2)}*b*d^2 + 12*\sqrt{d*x + c}*b*c*d^2 - 5*I*\sqrt{d*x + c}*d^3)*e^{((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/d^2 + 27*(I*\sqrt{10}*\sqrt{\pi})*(20*I*b^2*c^2*d + 12*b*c*d^2 - 3*I*d^3)*d*\operatorname{erf}(-1/2*\sqrt{10}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-5*I*b*c + 5*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^3) - 10*I*(-20*I*(d*x + c)^{(5/2)}*b^2*d + 40*I*(d*x + c)^{(3/2)}*b^2*c*d - 20*I*\sqrt{d*x + c}*b^2*c^2*d + 10*(d*x + c)^{(3/2)}*b*d^2 - 12*\sqrt{d*x + c}*b*c*d^2 + 3*I*\sqrt{d*x + c}*d^3)*e^{((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^3)/d^2 - 12*(9*I*\sqrt{10}*\sqrt{\pi})*(-10*I*b*c*d + 3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{10}*\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((5*I*b*c - 5*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2) + 125*I*\sqrt{6}*\sqrt{\pi}*(2*I*b*c*d - d^2)*d*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((3*I*b*c - 3*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2) + 2250*I*\sqrt{2}*\sqrt{\pi}*(2*I*b*c*d - 3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((I*b*c - I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2) + 2250*I*\sqrt{2}*\sqrt{\pi}*(2*I*b*c*d + 3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-I*b*c + I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2) + 125*I*\sqrt{6}*\sqrt{\pi}*(2*I*b*c*d + d^2)*d*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-3*I*b*c + 3*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2) + 9*I*\sqrt{10}*\sqrt{\pi})*(-10*I*b*c*d - 3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{10}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-5*I*b*c + 5*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2) - 90*I*(-10*I*(d*x + c)^{(3/2)}*b*d + 10*I*\sqrt{d*x + c}*b*c*d + 3*\sqrt{d*x + c}*d^2)*e^{((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^2 - 750*I*(2*I*(d*x + c)^{(3/2)}*b*d - 2*I*\sqrt{d*x + c}*b*c*d - \sqrt{d*x + c}*d^2)*e^{((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2 - 4500*I*(2*I*(d*x + c)^{(3/2)}*b*d - 2*I*\sqrt{d*x + c}*b*c*d - 3*\sqrt{d*x + c}*d^2)*e^{((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2 - 4500*I*(2*I*(d*x + c)^{(3/2)}*b*d - 2*I*\sqrt{d*x + c}*b*c*d + 3*\sqrt{d*x + c}*d^2)*e^{((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2 - 750*I*(2*I*(d*x + c)^{(3/2)}*b*d - 2*I*\sqrt{d*x + c}*b*c*d + \sqrt{d*x + c}*d^2)*e^{((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2 - 90*I*(-10*I*(d*x + c)^{(3/2)}*b*d + 10*I*\sqrt{d*x + c}*b*c*d - 3*\sqrt{d*x + c}*d^2)*e^{((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^2)*c)/d
\end{aligned}$$

3.131 $\int (c + dx)^{3/2} \cos^2(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=534

$$\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\sin\left(5a - \frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}} - \frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} - \frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{5/2}}$$

[Out] $-\frac{(c + dx)^{3/2}\cos[a + bx]}{8b} - \frac{(c + dx)^{3/2}\cos[3a + 3bx]}{48b} + \frac{(c + dx)^{3/2}\cos[5a + 5bx]}{80b} - \frac{3d^{3/2}\sqrt{\pi/2}\cos[a - (b*c)/d]\text{FresnelS}[(\sqrt{b}\sqrt{2/\pi}\sqrt{c + dx})/\sqrt{d}]}{(16*b^{5/2})} - \frac{d^{3/2}\sqrt{\pi/6}\cos[3a - (3*b*c)/d]\text{FresnelS}[(\sqrt{b}\sqrt{6/\pi}\sqrt{c + dx})/\sqrt{d}]}{(96*b^{5/2})} + \frac{3d^{3/2}\sqrt{\pi/10}\cos[5a - (5*b*c)/d]\text{FresnelS}[(\sqrt{b}\sqrt{10/\pi}\sqrt{c + dx})/\sqrt{d}]}{(800*b^{5/2})} + \frac{3d^{3/2}\sqrt{\pi/10}\text{FresnelC}[(\sqrt{b}\sqrt{10/\pi}\sqrt{c + dx})/\sqrt{d}]\sin[5a - (5*b*c)/d]}{(800*b^{5/2})} - \frac{d^{3/2}\sqrt{\pi/6}\text{FresnelC}[(\sqrt{b}\sqrt{6/\pi}\sqrt{c + dx})/\sqrt{d}]\sin[3a - (3*b*c)/d]}{(96*b^{5/2})} - \frac{3d^{3/2}\sqrt{\pi/2}\text{FresnelC}[(\sqrt{b}\sqrt{2/\pi}\sqrt{c + dx})/\sqrt{d}]\sin[a - (b*c)/d]}{(16*b^{5/2})} + \frac{3d\sqrt{c + dx}\sin[a + bx]}{(16*b^2)} + \frac{d\sqrt{c + dx}\sin[3a + 3bx]}{(96*b^2)} - \frac{3d\sqrt{c + dx}\sin[5a + 5bx]}{(800*b^2)}$

Rubi [A] time = 0.879348, antiderivative size = 534, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\sin\left(5a - \frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}} - \frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} - \frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{5/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + dx)^{3/2}\cos[a + bx]^2\sin[a + bx]^3, x]$

[Out] $-\frac{(c + dx)^{3/2}\cos[a + bx]}{8b} - \frac{(c + dx)^{3/2}\cos[3a + 3bx]}{48b} + \frac{(c + dx)^{3/2}\cos[5a + 5bx]}{80b} - \frac{3d^{3/2}\sqrt{\pi/2}\cos[a - (b*c)/d]\text{FresnelS}[(\sqrt{b}\sqrt{2/\pi}\sqrt{c + dx})/\sqrt{d}]}{(16*b^{5/2})} - \frac{d^{3/2}\sqrt{\pi/6}\cos[3a - (3*b*c)/d]\text{FresnelS}[(\sqrt{b}\sqrt{6/\pi}\sqrt{c + dx})/\sqrt{d}]}{(96*b^{5/2})} + \frac{3d^{3/2}\sqrt{\pi/10}\cos[5a - (5*b*c)/d]\text{FresnelS}[(\sqrt{b}\sqrt{10/\pi}\sqrt{c + dx})/\sqrt{d}]}{(800*b^{5/2})} + \frac{3d^{3/2}\sqrt{\pi/10}\text{FresnelC}[(\sqrt{b}\sqrt{10/\pi}\sqrt{c + dx})/\sqrt{d}]\sin[5a - (5*b*c)/d]}{(800*b^{5/2})} - \frac{d^{3/2}\sqrt{\pi/6}\text{FresnelC}[(\sqrt{b}\sqrt{6/\pi}\sqrt{c + dx})/\sqrt{d}]\sin[3a - (3*b*c)/d]}{(96*b^{5/2})} - \frac{3d^{3/2}\sqrt{\pi/2}\text{FresnelC}[(\sqrt{b}\sqrt{2/\pi}\sqrt{c + dx})/\sqrt{d}]\sin[a - (b*c)/d]}{(16*b^{5/2})} + \frac{3d\sqrt{c + dx}\sin[a + bx]}{(16*b^2)} + \frac{d\sqrt{c + dx}\sin[3a + 3bx]}{(96*b^2)} - \frac{3d\sqrt{c + dx}\sin[5a + 5bx]}{(800*b^2)}$

Rule 4406

$\text{Int}[\cos[(a_.) + (b_.)(x_.)]^{(p_.)}((c_.) + (d_.)(x_.))^{(m_.)}\sin[(a_.) + (b_.)(x_.)]^{(n_.)}, x_Symbol] := \text{Int}[\text{ExpandTrigReduce}[(c + dx)^m, \sin[a + bx]^n \cos[a + bx]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)]/(f*Rt[d, 2])), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)]/(f*Rt[d, 2])), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^{3/2} \cos^2(a + bx) \sin^3(a + bx) dx &= \int \left(\frac{1}{8}(c + dx)^{3/2} \sin(a + bx) + \frac{1}{16}(c + dx)^{3/2} \sin(3a + 3bx) - \frac{1}{16}(c + dx)^{3/2} \sin(5a + 5bx) \right) dx \\
&= \frac{1}{16} \int (c + dx)^{3/2} \sin(3a + 3bx) dx - \frac{1}{16} \int (c + dx)^{3/2} \sin(5a + 5bx) dx + \frac{1}{8} \int (c + dx)^{3/2} \sin(a + bx) dx \\
&= -\frac{(c + dx)^{3/2} \cos(a + bx)}{8b} - \frac{(c + dx)^{3/2} \cos(3a + 3bx)}{48b} + \frac{(c + dx)^{3/2} \cos(5a + 5bx)}{80b} \\
&= -\frac{(c + dx)^{3/2} \cos(a + bx)}{8b} - \frac{(c + dx)^{3/2} \cos(3a + 3bx)}{48b} + \frac{(c + dx)^{3/2} \cos(5a + 5bx)}{80b} \\
&= -\frac{(c + dx)^{3/2} \cos(a + bx)}{8b} - \frac{(c + dx)^{3/2} \cos(3a + 3bx)}{48b} + \frac{(c + dx)^{3/2} \cos(5a + 5bx)}{80b} \\
&= -\frac{(c + dx)^{3/2} \cos(a + bx)}{8b} - \frac{(c + dx)^{3/2} \cos(3a + 3bx)}{48b} + \frac{(c + dx)^{3/2} \cos(5a + 5bx)}{80b} \\
&= -\frac{(c + dx)^{3/2} \cos(a + bx)}{8b} - \frac{(c + dx)^{3/2} \cos(3a + 3bx)}{48b} + \frac{(c + dx)^{3/2} \cos(5a + 5bx)}{80b}
\end{aligned}$$

Mathematica [C] time = 12.2837, size = 1041, normalized size = 1.95

$$\frac{ce^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(-\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b} + \frac{c\left(2\sqrt{5}\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(5(a+bx))-\sqrt{2\pi}\cos(5a-\right.$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]

[Out] (c*Sqrt[c + d*x]*(-(E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[(I*b*(c + d*x))/d])/(16*b*E^((I*(b*c + a*d))/d) + (c*(2*Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d]))/(160*Sqrt[5]*b*Sqrt[b/d]) - (c*(2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(96*Sqrt[3]*b*Sqrt[b/d]) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(3*d*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d] + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a + b*x])))/(32*b^3) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d] + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(192*Sqrt[3]*b^3) + (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(3*d*Cos[5*a - (5*b*c)/d] - 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*Sin[5*a - (5*b*c)/d] + 2*Sqrt[5]*Sqrt[b/d]*d*Sqrt[c + d*x]*(10*b*x*Cos[5*(a + b*x)] - 3*Sin[5*(a + b*x)])))/(1600*Sqrt[5]*b^3)

Maple [A] time = 0.042, size = 580, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)

[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/16/b*d*(1/2/b*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))-1/96/b*d*(d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/32/b*d*(1/6/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))+1/160/b*d*(d*x+c)^(3/2)*cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-3/160/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))

$c)^{(1/2)*b/d))$

Maxima [C] time = 2.90146, size = 2790, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")

[Out] $\frac{1}{288000}\sqrt{5}\sqrt{3}(240\sqrt{5}\sqrt{3}(d*x + c)^{3/2}b*d*\sqrt{\frac{abs(b)}{abs(d)}}*abs(b)*\cos(5*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 400\sqrt{5}\sqrt{3}(d*x + c)^{3/2}b*d*\sqrt{\frac{abs(b)}{abs(d)}}*abs(b)*\cos(3*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 2400\sqrt{5}\sqrt{3}(d*x + c)^{3/2}b*d*\sqrt{\frac{abs(b)}{abs(d)}}*abs(b)*\cos(((d*x + c)*b - b*c + a*d)/d)/abs(d) - 72\sqrt{5}\sqrt{3})*\sqrt{d*x + c}*d^2*\sqrt{\frac{abs(b)}{abs(d)}}*abs(b)*\sin(5*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 200\sqrt{5}\sqrt{3}\sqrt{d*x + c}*d^2*\sqrt{\frac{abs(b)}{abs(d)}}*abs(b)*\sin(3*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 3600\sqrt{5}\sqrt{3}\sqrt{d*x + c}*d^2*\sqrt{\frac{abs(b)}{abs(d)}}*abs(b)*\sin(((d*x + c)*b - b*c + a*d)/d)/abs(d) - (\sqrt{3})*(-9*I*\sqrt{\pi})*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 9*I*\sqrt{\pi})*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 9*\sqrt{\pi})*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 9*\sqrt{\pi})*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^2*abs(b)*\cos(-5*(b*c - a*d)/d)/abs(d) - \sqrt{3}*(9*\sqrt{\pi})*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 9*\sqrt{\pi})*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 9*I*\sqrt{\pi})*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 9*I*\sqrt{\pi})*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^2*abs(b)*\sin(-5*(b*c - a*d)/d)/abs(d))*erf(\sqrt{d*x + c}*\sqrt{5*I*b/d}) - (\sqrt{5}*(25*I*\sqrt{\pi})*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 25*I*\sqrt{\pi})*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 25*\sqrt{\pi})*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 25*\sqrt{\pi})*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^2*abs(b)*\cos(-3*(b*c - a*d)/d)/abs(d) + \sqrt{5}*(25*\sqrt{\pi})*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 25*\sqrt{\pi})*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 25*I*\sqrt{\pi})*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 25*I*\sqrt{\pi})*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^2*abs(b)*\sin(-3*(b*c - a*d)/d)/abs(d))*erf(\sqrt{d*x + c}*\sqrt{3*I*b/d}) - (\sqrt{5}*\sqrt{3}*(450*I*\sqrt{\pi})*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 450*I*\sqrt{\pi})*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 450*\sqrt{\pi})*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 450*\sqrt{\pi})*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^2*abs(b)*\cos(-(b*c - a*d)/d)/abs(d) + \sqrt{5}*\sqrt{3}*(450*\sqrt{\pi})*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 450*\sqrt{\pi})*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 450*I*\sqrt{\pi})*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 450*I*\sqrt{\pi})*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^2*abs(b)*\sin(-(b*c - a*d)/d)/abs(d))*erf(\sqrt{d*x + c}*\sqrt{I*b/d}) - (\sqrt{5}*\sqrt{3}*(-450*I*\sqrt{\pi})*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 450*I*\sqrt{\pi})*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 450*\sqrt{\pi})*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 450*\sqrt{\pi})*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d^2*abs(b)*\cos(-(b*c - a*d)/d)/abs(d) + \sqrt{5}*\sqrt{3}*(450*\sqrt{\pi})*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 450*\sqrt{\pi})*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 450*I*\sqrt{\pi})*\sin(1/4$

```

*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 450*I*sqrt(pi)*sin
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^2*abs(b)*sin
(-(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - (sqrt(5)*(-25*I*
sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 25
*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 25*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
- 25*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
))) *d^2*abs(b)*cos(-3*(b*c - a*d)/d)/abs(d) + sqrt(5)*(25*sqrt(pi)*cos(1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 25*sqrt(pi)*cos(-1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 25*I*sqrt(pi)*sin
(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 25*I*sqrt(pi)*
sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^2*abs(b)*
sin(-3*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - (sqrt(3)*
(9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) *d^2*abs(b)*cos(-5*(b*c - a*d)/d)/abs(d) - sqrt(3)*(9*sqrt(pi)*cos(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*si
n(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^2*abs(b)*si
n(-5*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)) *abs(d)/(b^2*
d*sqrt(abs(b)/abs(d))*abs(b))

```

Fricas [A] time = 0.780473, size = 1118, normalized size = 2.09

$$27 \sqrt{10} \pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{5(bc-ad)}{d}\right) S\left(\sqrt{10} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) - 125 \sqrt{6} \pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{3(bc-ad)}{d}\right) S\left(\sqrt{6} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) - 6750 \sqrt{2} \pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{b*c-a*d}{d}\right) S\left(\sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) - 6750 \sqrt{2} \pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{b*c-a*d}{d}\right) S\left(\sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) - 6750 \sqrt{2} \pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{b*c-a*d}{d}\right) S\left(\sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{b*c-a*d}{d}\right) - 125 \sqrt{6} \pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{3(bc-ad)}{d}\right) S\left(\sqrt{6} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{3(bc-ad)}{d}\right) + 27 \sqrt{10} \pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{5(bc-ad)}{d}\right) S\left(\sqrt{10} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{5(bc-ad)}{d}\right) + 480 \cdot (30 \cdot (b^2 \cdot dx + b^2 \cdot c) \cos(b \cdot x + a)^5 - 50 \cdot (b^2 \cdot dx + b^2 \cdot c) \cos(b \cdot x + a)^3 - (9 \cdot b \cdot d \cdot \cos(b \cdot x + a)^4 - 13 \cdot b \cdot d \cdot \cos(b \cdot x + a)^2 - 26 \cdot b \cdot d) \sin(b \cdot x + a) \sqrt{dx+c}) / b^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_si
n(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 125*sqrt(6)*pi*d^2*sqrt(b/(pi*d)
)*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) -
6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d))) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel
_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*sqrt(6
)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-3*(b*c - a*d)/d) + 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(1
0)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(30*(b^2*d*x +
b^2*c)*cos(b*x + a)^5 - 50*(b^2*d*x + b^2*c)*cos(b*x + a)^3 - (9*b*d*cos(b
*x + a)^4 - 13*b*d*cos(b*x + a)^2 - 26*b*d)*sin(b*x + a)*sqrt(d*x + c))/b^
3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

[Out] Timed out

Giac [C] time = 1.80713, size = 2269, normalized size = 4.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")

[Out]
$$\frac{1}{144000} \left(10 \cdot (9 \sqrt{10}) \sqrt{\pi} d^2 \operatorname{erf}\left(-\frac{1}{2} \sqrt{10} \sqrt{b d} \sqrt{d x + c}\right) \sqrt{\frac{I b d}{\sqrt{b^2 d^2} + 1}} \sqrt{\frac{I b^2 d^2 + 1}{d}} e^{\left(\frac{5 I b^2 c - 5 I a^2 d}{d}\right)} \sqrt{\frac{I b^2 d^2 + 1}{d}} \sqrt{\frac{I b^2 d^2 + 1}{d}} - 25 \sqrt{6} \sqrt{\pi} d^2 \operatorname{erf}\left(-\frac{1}{2} \sqrt{6} \sqrt{b d} \sqrt{d x + c}\right) \sqrt{\frac{I b d}{\sqrt{b^2 d^2} + 1}} \sqrt{\frac{I b^2 d^2 + 1}{d}} e^{\left(\frac{3 I b^2 c - 3 I a^2 d}{d}\right)} \sqrt{\frac{I b^2 d^2 + 1}{d}} \sqrt{\frac{I b^2 d^2 + 1}{d}} - 450 \sqrt{2} \sqrt{\pi} d^2 \operatorname{erf}\left(-\frac{1}{2} \sqrt{2} \sqrt{b d} \sqrt{d x + c}\right) \sqrt{\frac{I b d}{\sqrt{b^2 d^2} + 1}} \sqrt{\frac{I b^2 d^2 + 1}{d}} e^{\left(\frac{I b^2 c - I a^2 d}{d}\right)} \sqrt{\frac{I b^2 d^2 + 1}{d}} \sqrt{\frac{I b^2 d^2 + 1}{d}} - 450 \sqrt{2} \sqrt{\pi} d^2 \operatorname{erf}\left(-\frac{1}{2} \sqrt{2} \sqrt{b d} \sqrt{d x + c}\right) \sqrt{\frac{-I b d}{\sqrt{b^2 d^2} + 1}} \sqrt{\frac{-I b^2 d^2 + 1}{d}} e^{\left(\frac{-I b^2 c + I a^2 d}{d}\right)} \sqrt{\frac{-I b^2 d^2 + 1}{d}} \sqrt{\frac{-I b^2 d^2 + 1}{d}} - 25 \sqrt{6} \sqrt{\pi} d^2 \operatorname{erf}\left(-\frac{1}{2} \sqrt{6} \sqrt{b d} \sqrt{d x + c}\right) \sqrt{\frac{-I b d}{\sqrt{b^2 d^2} + 1}} \sqrt{\frac{-I b^2 d^2 + 1}{d}} e^{\left(\frac{-3 I b^2 c + 3 I a^2 d}{d}\right)} \sqrt{\frac{-I b^2 d^2 + 1}{d}} \sqrt{\frac{-I b^2 d^2 + 1}{d}} + 9 \sqrt{10} \sqrt{\pi} d^2 \operatorname{erf}\left(-\frac{1}{2} \sqrt{10} \sqrt{b d} \sqrt{d x + c}\right) \sqrt{\frac{-I b d}{\sqrt{b^2 d^2} + 1}} \sqrt{\frac{-I b^2 d^2 + 1}{d}} e^{\left(\frac{-5 I b^2 c + 5 I a^2 d}{d}\right)} \sqrt{\frac{-I b^2 d^2 + 1}{d}} \sqrt{\frac{-I b^2 d^2 + 1}{d}} + 90 \sqrt{d x + c} d e^{\left(\frac{5 I (d x + c) b - 5 I b^2 c + 5 I a^2 d}{d}\right)} \sqrt{\frac{5 I (d x + c) b - 5 I b^2 c + 5 I a^2 d}{d}} \sqrt{\frac{5 I (d x + c) b - 5 I b^2 c + 5 I a^2 d}{d}} - 150 \sqrt{d x + c} d e^{\left(\frac{3 I (d x + c) b - 3 I b^2 c + 3 I a^2 d}{d}\right)} \sqrt{\frac{3 I (d x + c) b - 3 I b^2 c + 3 I a^2 d}{d}} \sqrt{\frac{3 I (d x + c) b - 3 I b^2 c + 3 I a^2 d}{d}} - 900 \sqrt{d x + c} d e^{\left(\frac{I (d x + c) b - I b^2 c + I a^2 d}{d}\right)} \sqrt{\frac{I (d x + c) b - I b^2 c + I a^2 d}{d}} \sqrt{\frac{I (d x + c) b - I b^2 c + I a^2 d}{d}} - 900 \sqrt{d x + c} d e^{\left(\frac{-I (d x + c) b + I b^2 c - I a^2 d}{d}\right)} \sqrt{\frac{-I (d x + c) b + I b^2 c - I a^2 d}{d}} \sqrt{\frac{-I (d x + c) b + I b^2 c - I a^2 d}{d}} - 150 \sqrt{d x + c} d e^{\left(\frac{-3 I (d x + c) b + 3 I b^2 c - 3 I a^2 d}{d}\right)} \sqrt{\frac{-3 I (d x + c) b + 3 I b^2 c - 3 I a^2 d}{d}} \sqrt{\frac{-3 I (d x + c) b + 3 I b^2 c - 3 I a^2 d}{d}} + 90 \sqrt{d x + c} d e^{\left(\frac{-5 I (d x + c) b + 5 I b^2 c - 5 I a^2 d}{d}\right)} \sqrt{\frac{-5 I (d x + c) b + 5 I b^2 c - 5 I a^2 d}{d}} \sqrt{\frac{-5 I (d x + c) b + 5 I b^2 c - 5 I a^2 d}{d}} / b^2 - 9 I \sqrt{10} \sqrt{\pi} (-10 I b^2 c d + 3 d^2) d \operatorname{erf}\left(-\frac{1}{2} \sqrt{10} \sqrt{b d} \sqrt{d x + c}\right) \sqrt{\frac{I b d}{\sqrt{b^2 d^2} + 1}} \sqrt{\frac{I b^2 d^2 + 1}{d}} e^{\left(\frac{5 I b^2 c - 5 I a^2 d}{d}\right)} \sqrt{\frac{I b^2 d^2 + 1}{d}} \sqrt{\frac{I b^2 d^2 + 1}{d}} - 125 I \sqrt{6} \sqrt{\pi} (2 I b^2 c d - d^2) d \operatorname{erf}\left(-\frac{1}{2} \sqrt{6} \sqrt{b d} \sqrt{d x + c}\right) \sqrt{\frac{I b d}{\sqrt{b^2 d^2} + 1}} \sqrt{\frac{I b^2 d^2 + 1}{d}} e^{\left(\frac{3 I b^2 c - 3 I a^2 d}{d}\right)} \sqrt{\frac{I b^2 d^2 + 1}{d}} \sqrt{\frac{I b^2 d^2 + 1}{d}} - 2250 I \sqrt{2} \sqrt{\pi} (2 I b^2 c d - 3 d^2) d \operatorname{erf}\left(-\frac{1}{2} \sqrt{2} \sqrt{b d} \sqrt{d x + c}\right) \sqrt{\frac{I b d}{\sqrt{b^2 d^2} + 1}} \sqrt{\frac{I b^2 d^2 + 1}{d}} e^{\left(\frac{I b^2 c - I a^2 d}{d}\right)} \sqrt{\frac{I b^2 d^2 + 1}{d}} \sqrt{\frac{I b^2 d^2 + 1}{d}} - 2250 I \sqrt{2} \sqrt{\pi} (2 I b^2 c d + 3 d^2) d \operatorname{erf}\left(-\frac{1}{2} \sqrt{2} \sqrt{b d} \sqrt{d x + c}\right) \sqrt{\frac{-I b d}{\sqrt{b^2 d^2} + 1}} \sqrt{\frac{-I b^2 d^2 + 1}{d}} e^{\left(\frac{-I b^2 c + I a^2 d}{d}\right)} \sqrt{\frac{-I b^2 d^2 + 1}{d}} \sqrt{\frac{-I b^2 d^2 + 1}{d}} - 125 I \sqrt{6} \sqrt{\pi} (2 I b^2 c d + d^2) d \operatorname{erf}\left(-\frac{1}{2} \sqrt{6} \sqrt{b d} \sqrt{d x + c}\right) \sqrt{\frac{-I b d}{\sqrt{b^2 d^2} + 1}} \sqrt{\frac{-I b^2 d^2 + 1}{d}} e^{\left(\frac{-3 I b^2 c + 3 I a^2 d}{d}\right)} \sqrt{\frac{-I b^2 d^2 + 1}{d}} \sqrt{\frac{-I b^2 d^2 + 1}{d}} - 9 I \sqrt{10} \sqrt{\pi} (-10 I b^2 c d - 3 d^2) d \operatorname{erf}\left(-\frac{1}{2} \sqrt{10} \sqrt{b d} \sqrt{d x + c}\right) \sqrt{\frac{-I b d}{\sqrt{b^2 d^2} + 1}} \sqrt{\frac{-I b^2 d^2 + 1}{d}} e^{\left(\frac{-5 I b^2 c + 5 I a^2 d}{d}\right)} \sqrt{\frac{-I b^2 d^2 + 1}{d}} \sqrt{\frac{-I b^2 d^2 + 1}{d}} + 90 I (-10 I (d x + c)^{3/2} b d + 10 I \sqrt{d x + c} b^2 c d + 3 \sqrt{d x + c} d^2) e^{\left(\frac{5 I (d x + c) b - 5 I b^2 c + 5 I a^2 d}{d}\right)} \sqrt{\frac{5 I (d x + c) b - 5 I b^2 c + 5 I a^2 d}{d}} \sqrt{\frac{5 I (d x + c) b - 5 I b^2 c + 5 I a^2 d}{d}} / b^2 + 750 I (2 I (d x + c)^{3/2} b d - 2 I \sqrt{d x + c} b^2 c d - \sqrt{d x + c} d^2) e^{\left(\frac{3 I (d x + c) b - 3 I b^2 c + 3 I a^2 d}{d}\right)} \sqrt{\frac{3 I (d x + c) b - 3 I b^2 c + 3 I a^2 d}{d}} \sqrt{\frac{3 I (d x + c) b - 3 I b^2 c + 3 I a^2 d}{d}} / b^2 + 4500 I (2 I (d x + c)^{3/2} b d - 2 I \sqrt{d x + c} b^2 c d - 3 \sqrt{d x + c} d^2) e^{\left(\frac{I (d x + c) b - I b^2 c + I a^2 d}{d}\right)} \sqrt{\frac{I (d x + c) b - I b^2 c + I a^2 d}{d}} \sqrt{\frac{I (d x + c) b - I b^2 c + I a^2 d}{d}} / b^2 + 4500 I (2 I (d x + c)^{3/2} b d - 2 I \sqrt{d x + c} b^2 c d + 3 \sqrt{d x + c} d^2) e^{\left(\frac{-I (d x + c) b + I b^2 c - I a^2 d}{d}\right)} \sqrt{\frac{-I (d x + c) b + I b^2 c - I a^2 d}{d}} \sqrt{\frac{-I (d x + c) b + I b^2 c - I a^2 d}{d}} / b^2 + 750 I (2 I (d x + c)^{3/2} b d - 2 I \sqrt{d x + c} b^2 c d + \sqrt{d x + c} d^2) e^{\left(\frac{-3 I (d x + c) b + 3 I b^2 c - 3 I a^2 d}{d}\right)} \sqrt{\frac{-3 I (d x + c) b + 3 I b^2 c - 3 I a^2 d}{d}} \sqrt{\frac{-3 I (d x + c) b + 3 I b^2 c - 3 I a^2 d}{d}} / b^2 + 90 I (-10 I (d x + c)^{3/2} b d + 10 I \sqrt{d x + c} b^2 c d - 3 \sqrt{d x + c} d^2) e^{\left(\frac{-5 I (d x + c) b + 5 I b^2 c - 5 I a^2 d}{d}\right)} \sqrt{\frac{-5 I (d x + c) b + 5 I b^2 c - 5 I a^2 d}{d}} \sqrt{\frac{-5 I (d x + c) b + 5 I b^2 c - 5 I a^2 d}{d}} / b^2 / d$$

3.132 $\int \sqrt{c + dx} \cos^2(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=459

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(a - \frac{bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}} + \frac{\sqrt{\frac{\pi}{6}} \sqrt{d} \cos\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} - \frac{\sqrt{\frac{\pi}{10}} \sqrt{d} \cos\left(5a - \frac{5bc}{d}\right)}{80}$$

```
[Out] -(Sqrt[c + d*x]*Cos[a + b*x])/(8*b) - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(48*
b) + (Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(80*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a -
(b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(3/2))
+ (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqr
t[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/10]*Cos[5*a - (5*b*c)
/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(80*b^(3/2)) + (
Sqrt[d]*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[5*a - (5*b*c)/d])/(80*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(48*b^(3/2)) - (Sqr
t[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a
- (b*c)/d])/(8*b^(3/2))
```

Rubi [A] time = 0.670548, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(a - \frac{bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}} + \frac{\sqrt{\frac{\pi}{6}} \sqrt{d} \cos\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} - \frac{\sqrt{\frac{\pi}{10}} \sqrt{d} \cos\left(5a - \frac{5bc}{d}\right)}{80}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[a + b*x])/(8*b) - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(48*
b) + (Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(80*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a -
(b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(3/2))
+ (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqr
t[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/10]*Cos[5*a - (5*b*c)
/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(80*b^(3/2)) + (
Sqrt[d]*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[5*a - (5*b*c)/d])/(80*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(48*b^(3/2)) - (Sqr
t[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a
- (b*c)/d])/(8*b^(3/2))
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_.)]^(p_.)*((c_.) + (d_.)*(x_.))^(m_.)*Sin[(a_.) + (b
_.)*(x_.)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

$e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{GtQ}[m, 0]$

Rule 3306

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{NeQ}[d*e - c*f, 0]$

Rule 3305

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Sin}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

Rule 3351

$\text{Int}[\text{Sin}[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /; \text{FreeQ}\{d, e, f\}, x]$

Rule 3304

$\text{Int}[\sin[\text{Pi}/2 + (e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Cos}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

Rule 3352

$\text{Int}[\text{Cos}[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /; \text{FreeQ}\{d, e, f\}, x]$

Rubi steps

$$\begin{aligned} \int \sqrt{c+dx} \cos^2(a+bx) \sin^3(a+bx) dx &= \int \left(\frac{1}{8} \sqrt{c+dx} \sin(a+bx) + \frac{1}{16} \sqrt{c+dx} \sin(3a+3bx) - \frac{1}{16} \sqrt{c+dx} \sin(5a+5bx) \right) dx \\ &= \frac{1}{16} \int \sqrt{c+dx} \sin(3a+3bx) dx - \frac{1}{16} \int \sqrt{c+dx} \sin(5a+5bx) dx + \frac{1}{8} \int \sqrt{c+dx} \sin(a+bx) dx \\ &= -\frac{\sqrt{c+dx} \cos(a+bx)}{8b} - \frac{\sqrt{c+dx} \cos(3a+3bx)}{48b} + \frac{\sqrt{c+dx} \cos(5a+5bx)}{80b} - \frac{d}{80b} \int \sqrt{c+dx} dx \\ &= -\frac{\sqrt{c+dx} \cos(a+bx)}{8b} - \frac{\sqrt{c+dx} \cos(3a+3bx)}{48b} + \frac{\sqrt{c+dx} \cos(5a+5bx)}{80b} - \frac{d}{80b} \left(\frac{2}{3} \sqrt{c+dx} + \frac{2}{3} \int \sqrt{c+dx} dx \right) \\ &= -\frac{\sqrt{c+dx} \cos(a+bx)}{8b} - \frac{\sqrt{c+dx} \cos(3a+3bx)}{48b} + \frac{\sqrt{c+dx} \cos(5a+5bx)}{80b} - \frac{cd}{40b} \\ &= -\frac{\sqrt{c+dx} \cos(a+bx)}{8b} - \frac{\sqrt{c+dx} \cos(3a+3bx)}{48b} + \frac{\sqrt{c+dx} \cos(5a+5bx)}{80b} + \frac{d}{80b} \int \sqrt{c+dx} dx \end{aligned}$$

Mathematica [C] time = 7.40148, size = 432, normalized size = 0.94

$$\frac{\sqrt{c+dx} e^{-\frac{i(ad+bc)}{d}} \left(-\frac{e^{2ia} \text{Gamma}\left(\frac{3}{2}, -\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}} \text{Gamma}\left(\frac{3}{2}, \frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}} \right)}{16b} + \frac{-\sqrt{2\pi} \cos\left(5a - \frac{5bc}{d}\right) \text{FresnelC}\left(\sqrt{\frac{10}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c+dx}\right) + \sqrt{c+dx}}{80b}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]

[Out] (Sqrt[c + d*x]*(-(E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[(I*b*(c + d*x))/d])/(16*b*E^((I*(b*c + a*d))/d)) + (2*Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d])/(160*Sqrt[5]*b*Sqrt[b/d]) - (2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d])/(96*Sqrt[3]*b*Sqrt[b/d])

Maple [A] time = 0.041, size = 447, normalized size = 1.

$$2 \frac{1}{d} \left(-1/16 \frac{d\sqrt{dx+c}}{b} \cos\left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d}\right) + 1/32 \frac{d\sqrt{2}\sqrt{\pi}}{b} \left(\cos\left(\frac{ad-bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{2}\sqrt{dx+cb}}{\sqrt{\pi d}} \frac{1}{\sqrt{\frac{b}{d}}}\right) - \sin\left(\frac{ad-bc}{d}\right) \text{FresnelS}\left(\frac{\sqrt{2}\sqrt{dx+cb}}{\sqrt{\pi d}} \frac{1}{\sqrt{\frac{b}{d}}}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)

[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))-1/96/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))+1/160/b*d*(d*x+c)^(1/2)*cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-1/1600/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))

Maxima [C] time = 2.84933, size = 2531, normalized size = 5.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")

[Out] 1/28800*sqrt(5)*sqrt(3)*(24*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d*sqrt(abs(b)/abs(d))*abs(b)*cos(5*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 40*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d*sqrt(abs(b)/abs(d))*abs(b)*cos(3*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 240*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d*sqrt(abs(b)/abs(d))*abs(b)*cos(((d*x + c)*b - b*c + a*d)/d)/abs(d) - (sqrt(3)*(3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d*abs(b)*cos(-5*(b

```

*c - a*d)/d)/abs(d) - sqrt(3)*(3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-5*(b*c - a*d)/d)/abs(d))*
erf(sqrt(d*x + c)*sqrt(5*I*b/d)) + (sqrt(5)*(5*sqrt(pi)*cos(1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*cos(-1/4*pi + 1/2*a
rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*sin(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*sin(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(-3*(b*c - a*
d)/d)/abs(d) + sqrt(5)*(-5*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-3*(b*c - a*d)/d)/abs(d))*erf(sq
rt(d*x + c)*sqrt(3*I*b/d)) + (sqrt(5)*sqrt(3)*(30*sqrt(pi)*cos(1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*cos(-1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*I*sqrt(pi)*sin(1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*I*sqrt(pi)*sin(-1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(-(b*c
- a*d)/d)/abs(d) + sqrt(5)*sqrt(3)*(-30*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*I*sqrt(pi)*cos(-1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*sqrt(pi)*sin(1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*sin(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-(b*c - a*d)/d)/
abs(d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (sqrt(5)*sqrt(3)*(30*sqrt(pi)*cos(
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*cos
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*I*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*I*sqrt(
pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b
)*cos(-(b*c - a*d)/d)/abs(d) + sqrt(5)*sqrt(3)*(30*I*sqrt(pi)*cos(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*I*sqrt(pi)*cos(-1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*sqrt(pi)*sin(1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*sin(-1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-(b*c
- a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (sqrt(5)*(5*sqrt(pi)*c
os(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*c
os(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(
pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b
)*cos(-3*(b*c - a*d)/d)/abs(d) + sqrt(5)*(5*I*sqrt(pi)*cos(1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*cos(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)*sin(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*sin(-1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-3*(b*c - a*d)/
d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - (sqrt(3)*(3*sqrt(pi)*cos(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*si
n(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(
-5*(b*c - a*d)/d)/abs(d) - sqrt(3)*(-3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(-1/4*pi + 1/2*arct
an2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-5*(b*c - a*d)/d)/ab
s(d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d))*abs(d)/(b*d*sqrt(abs(b)/abs(d))*ab
s(b))

```


Fricas [A] time = 0.712261, size = 952, normalized size = 2.07

$$\frac{9\sqrt{10}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)C\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 25\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 450\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")

[Out] -1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_cos(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^5 - 5*b*cos(b*x + a)^3)*sqrt(d*x + c))/b^2

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)

[Out] Timed out

Giac [C] time = 1.42523, size = 988, normalized size = 2.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")

[Out] 1/14400*(9*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 90*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b - 150*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b - 900*

$$\begin{aligned} & \sqrt{d*x + c} * d * e^{(I*(d*x + c)*b - I*b*c + I*a*d)/d} / b - 90 * \sqrt{d*x + c} \\ & * d * e^{(-I*(d*x + c)*b + I*b*c - I*a*d)/d} / b - 150 * \sqrt{d*x + c} * d * e^{(-3*I* \\ & (d*x + c)*b + 3*I*b*c - 3*I*a*d)/d} / b + 90 * \sqrt{d*x + c} * d * e^{(-5*I*(d*x + \\ & c)*b + 5*I*b*c - 5*I*a*d)/d} / b) / d \end{aligned}$$

3.133 $\int \sqrt{c + dx} \cos^2(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=459

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(a - \frac{bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}} + \frac{\sqrt{\frac{\pi}{6}} \sqrt{d} \cos\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} - \frac{\sqrt{\frac{\pi}{10}} \sqrt{d} \cos\left(5a - \frac{5bc}{d}\right)}{80}$$

```
[Out] -(Sqrt[c + d*x]*Cos[a + b*x])/(8*b) - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(48*
b) + (Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(80*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a -
(b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(3/2))
+ (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqr
t[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/10]*Cos[5*a - (5*b*c)
/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(80*b^(3/2)) + (
Sqrt[d]*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[5*a - (5*b*c)/d])/(80*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(48*b^(3/2)) - (Sqr
t[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a
- (b*c)/d])/(8*b^(3/2))
```

Rubi [A] time = 0.658185, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(a - \frac{bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}} + \frac{\sqrt{\frac{\pi}{6}} \sqrt{d} \cos\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} - \frac{\sqrt{\frac{\pi}{10}} \sqrt{d} \cos\left(5a - \frac{5bc}{d}\right)}{80}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[a + b*x])/(8*b) - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(48*
b) + (Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(80*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a -
(b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(3/2))
+ (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqr
t[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/10]*Cos[5*a - (5*b*c)
/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(80*b^(3/2)) + (
Sqrt[d]*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[5*a - (5*b*c)/d])/(80*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(48*b^(3/2)) - (Sqr
t[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a
- (b*c)/d])/(8*b^(3/2))
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_.)]^(p_.)*((c_.) + (d_.)*(x_.))^(m_.)*Sin[(a_.) + (b
_.)*(x_.)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

$e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{GtQ}[m, 0]$

Rule 3306

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{ComplexFreeQ}[f] \&\& \text{NeQ}[d*e - c*f, 0]$

Rule 3305

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Sin}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{ComplexFreeQ}[f] \&\& \text{EqQ}[d*e - c*f, 0]$

Rule 3351

$\text{Int}[\text{Sin}[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /; \text{FreeQ}\{d, e, f\}, x]$

Rule 3304

$\text{Int}[\sin[\text{Pi}/2 + (e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Cos}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{ComplexFreeQ}[f] \&\& \text{EqQ}[d*e - c*f, 0]$

Rule 3352

$\text{Int}[\text{Cos}[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /; \text{FreeQ}\{d, e, f\}, x]$

Rubi steps

$$\begin{aligned} \int \sqrt{c+dx} \cos^2(a+bx) \sin^3(a+bx) dx &= \int \left(\frac{1}{8} \sqrt{c+dx} \sin(a+bx) + \frac{1}{16} \sqrt{c+dx} \sin(3a+3bx) - \frac{1}{16} \sqrt{c+dx} \sin(5a+5bx) \right) dx \\ &= \frac{1}{16} \int \sqrt{c+dx} \sin(3a+3bx) dx - \frac{1}{16} \int \sqrt{c+dx} \sin(5a+5bx) dx + \frac{1}{8} \int \sqrt{c+dx} \sin(a+bx) dx \\ &= -\frac{\sqrt{c+dx} \cos(a+bx)}{8b} - \frac{\sqrt{c+dx} \cos(3a+3bx)}{48b} + \frac{\sqrt{c+dx} \cos(5a+5bx)}{80b} - \frac{d}{80b} \int \sqrt{c+dx} dx \\ &= -\frac{\sqrt{c+dx} \cos(a+bx)}{8b} - \frac{\sqrt{c+dx} \cos(3a+3bx)}{48b} + \frac{\sqrt{c+dx} \cos(5a+5bx)}{80b} - \frac{d}{80b} \left(\frac{2}{3} \sqrt{c+dx} + \frac{2}{3} \int \sqrt{c+dx} dx \right) \\ &= -\frac{\sqrt{c+dx} \cos(a+bx)}{8b} - \frac{\sqrt{c+dx} \cos(3a+3bx)}{48b} + \frac{\sqrt{c+dx} \cos(5a+5bx)}{80b} - \frac{cd}{40b} \\ &= -\frac{\sqrt{c+dx} \cos(a+bx)}{8b} - \frac{\sqrt{c+dx} \cos(3a+3bx)}{48b} + \frac{\sqrt{c+dx} \cos(5a+5bx)}{80b} + \frac{d}{80b} \int \sqrt{c+dx} dx \end{aligned}$$

Mathematica [C] time = 7.34617, size = 432, normalized size = 0.94

$$\frac{\sqrt{c+dx} e^{-\frac{i(ad+bc)}{d}} \left(-\frac{e^{2ia} \text{Gamma}\left(\frac{3}{2}, -\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}} \text{Gamma}\left(\frac{3}{2}, \frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}} \right)}{16b} + \frac{-\sqrt{2\pi} \cos\left(5a - \frac{5bc}{d}\right) \text{FresnelC}\left(\sqrt{\frac{10}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c+dx}\right) + \sqrt{c+dx}}{80b}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]

[Out] (Sqrt[c + d*x]*(-(E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[(I*b*(c + d*x))/d])/(16*b*E^((I*(b*c + a*d))/d)) + (2*Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d])/(160*Sqrt[5]*b*Sqrt[b/d]) - (2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d])/(96*Sqrt[3]*b*Sqrt[b/d])

Maple [A] time = 0.042, size = 447, normalized size = 1.

$$2 \frac{1}{d} \left(-1/16 \frac{d\sqrt{dx+c}}{b} \cos\left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d}\right) + 1/32 \frac{d\sqrt{2}\sqrt{\pi}}{b} \left(\cos\left(\frac{ad-bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{2}\sqrt{dx+cb}}{\sqrt{\pi d}} \frac{1}{\sqrt{\frac{b}{d}}}\right) - \sin\left(\frac{ad-bc}{d}\right) \text{FresnelS}\left(\frac{\sqrt{2}\sqrt{dx+cb}}{\sqrt{\pi d}} \frac{1}{\sqrt{\frac{b}{d}}}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)

[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))-1/96/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))+1/160/b*d*(d*x+c)^(1/2)*cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-1/1600/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))

Maxima [C] time = 2.75907, size = 2531, normalized size = 5.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")

[Out] 1/28800*sqrt(5)*sqrt(3)*(24*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d*sqrt(abs(b)/abs(d))*abs(b)*cos(5*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 40*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d*sqrt(abs(b)/abs(d))*abs(b)*cos(3*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 240*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d*sqrt(abs(b)/abs(d))*abs(b)*cos(((d*x + c)*b - b*c + a*d)/d)/abs(d) - (sqrt(3)*(3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d*abs(b)*cos(-5*(b

```

*c - a*d)/d)/abs(d) - sqrt(3)*(3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-5*(b*c - a*d)/d)/abs(d))*
erf(sqrt(d*x + c)*sqrt(5*I*b/d)) + (sqrt(5)*(5*sqrt(pi)*cos(1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*cos(-1/4*pi + 1/2*a
rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*sin(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*sin(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(-3*(b*c - a*
d)/d)/abs(d) + sqrt(5)*(-5*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-3*(b*c - a*d)/d)/abs(d))*erf(sq
rt(d*x + c)*sqrt(3*I*b/d)) + (sqrt(5)*sqrt(3)*(30*sqrt(pi)*cos(1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*cos(-1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*I*sqrt(pi)*sin(1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*I*sqrt(pi)*sin(-1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(-(b*c
- a*d)/d)/abs(d) + sqrt(5)*sqrt(3)*(-30*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*I*sqrt(pi)*cos(-1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*sqrt(pi)*sin(1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*sin(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-(b*c - a*d)/d)/
abs(d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (sqrt(5)*sqrt(3)*(30*sqrt(pi)*cos(
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*cos
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*I*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*I*sqrt(
pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b
)*cos(-(b*c - a*d)/d)/abs(d) + sqrt(5)*sqrt(3)*(30*I*sqrt(pi)*cos(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*I*sqrt(pi)*cos(-1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*sqrt(pi)*sin(1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*sin(-1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-(b*c
- a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (sqrt(5)*(5*sqrt(pi)*c
os(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*c
os(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(
pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b
)*cos(-3*(b*c - a*d)/d)/abs(d) + sqrt(5)*(5*I*sqrt(pi)*cos(1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*cos(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)*sin(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*sin(-1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-3*(b*c - a*d)/
d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - (sqrt(3)*(3*sqrt(pi)*cos(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*si
n(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(
-5*(b*c - a*d)/d)/abs(d) - sqrt(3)*(-3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(-1/4*pi + 1/2*arct
an2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-5*(b*c - a*d)/d)/ab
s(d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d))*abs(d)/(b*d*sqrt(abs(b)/abs(d))*ab
s(b))

```

Fricas [A] time = 0.72081, size = 952, normalized size = 2.07

$$\frac{9\sqrt{10}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)C\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 25\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 450\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")

[Out] -1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_cos(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^5 - 5*b*cos(b*x + a)^3)*sqrt(d*x + c))/b^2

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)

[Out] Timed out

Giac [C] time = 1.45461, size = 988, normalized size = 2.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")

[Out] 1/14400*(9*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 90*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b - 150*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b - 900*

$$\begin{aligned} & \sqrt{d*x + c} * d * e^{(I*(d*x + c)*b - I*b*c + I*a*d)/d} / b - 90 * \sqrt{d*x + c} \\ & * d * e^{(-I*(d*x + c)*b + I*b*c - I*a*d)/d} / b - 150 * \sqrt{d*x + c} * d * e^{(-3*I* \\ & (d*x + c)*b + 3*I*b*c - 3*I*a*d)/d} / b + 90 * \sqrt{d*x + c} * d * e^{(-5*I*(d*x + \\ & c)*b + 5*I*b*c - 5*I*a*d)/d} / b) / d \end{aligned}$$

3.134 $\int (c + dx)^{3/2} \cos^2(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=534

$$\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\sin\left(5a - \frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}} - \frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} - \frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{5/2}}$$

```
[Out] -((c + d*x)^(3/2)*Cos[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/(48*b) + ((c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(80*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(800*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(96*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[a + b*x])/(16*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(96*b^2) - (3*d*Sqrt[c + d*x]*Sin[5*a + 5*b*x])/(800*b^2)
```

Rubi [A] time = 0.8014, antiderivative size = 534, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\sin\left(5a - \frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}} - \frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} - \frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/(48*b) + ((c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(80*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(800*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(96*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[a + b*x])/(16*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(96*b^2) - (3*d*Sqrt[c + d*x]*Sin[5*a + 5*b*x])/(800*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n * Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^{3/2} \cos^2(a + bx) \sin^3(a + bx) dx &= \int \left(\frac{1}{8}(c + dx)^{3/2} \sin(a + bx) + \frac{1}{16}(c + dx)^{3/2} \sin(3a + 3bx) - \frac{1}{16}(c + dx)^{3/2} \sin(5a + 5bx) \right) dx \\
&= \frac{1}{16} \int (c + dx)^{3/2} \sin(3a + 3bx) dx - \frac{1}{16} \int (c + dx)^{3/2} \sin(5a + 5bx) dx + \frac{1}{8} \int (c + dx)^{3/2} \sin(a + bx) dx \\
&= -\frac{(c + dx)^{3/2} \cos(a + bx)}{8b} - \frac{(c + dx)^{3/2} \cos(3a + 3bx)}{48b} + \frac{(c + dx)^{3/2} \cos(5a + 5bx)}{80b} \\
&= -\frac{(c + dx)^{3/2} \cos(a + bx)}{8b} - \frac{(c + dx)^{3/2} \cos(3a + 3bx)}{48b} + \frac{(c + dx)^{3/2} \cos(5a + 5bx)}{80b} \\
&= -\frac{(c + dx)^{3/2} \cos(a + bx)}{8b} - \frac{(c + dx)^{3/2} \cos(3a + 3bx)}{48b} + \frac{(c + dx)^{3/2} \cos(5a + 5bx)}{80b} \\
&= -\frac{(c + dx)^{3/2} \cos(a + bx)}{8b} - \frac{(c + dx)^{3/2} \cos(3a + 3bx)}{48b} + \frac{(c + dx)^{3/2} \cos(5a + 5bx)}{80b} \\
&= -\frac{(c + dx)^{3/2} \cos(a + bx)}{8b} - \frac{(c + dx)^{3/2} \cos(3a + 3bx)}{48b} + \frac{(c + dx)^{3/2} \cos(5a + 5bx)}{80b}
\end{aligned}$$

Mathematica [C] time = 12.1833, size = 1041, normalized size = 1.95

$$\frac{ce^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(-\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b} + \frac{c\left(2\sqrt{5}\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(5(a+bx))-\sqrt{2\pi}\cos(5a-\right.$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]

[Out] (c*Sqrt[c + d*x]*(-(E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[(I*b*(c + d*x))/d])/(16*b*E^((I*(b*c + a*d))/d) + (c*(2*Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d]))/(160*Sqrt[5]*b*Sqrt[b/d]) - (c*(2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(96*Sqrt[3]*b*Sqrt[b/d]) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(3*d*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d] + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a + b*x])))/(32*b^3) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d] + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(192*Sqrt[3]*b^3) + (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(3*d*Cos[5*a - (5*b*c)/d] - 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*Sin[5*a - (5*b*c)/d] + 2*Sqrt[5]*Sqrt[b/d]*d*Sqrt[c + d*x]*(10*b*x*Cos[5*(a + b*x)] - 3*Sin[5*(a + b*x)])))/(1600*Sqrt[5]*b^3)

Maple [A] time = 0.038, size = 580, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)

[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/16/b*d*(1/2/b*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))-1/96/b*d*(d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/32/b*d*(1/6/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))+1/160/b*d*(d*x+c)^(3/2)*cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-3/160/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))

$c)^{(1/2)*b/d))))$

Maxima [C] time = 2.93574, size = 2790, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")

[Out] $1/288000*\sqrt{5}*\sqrt{3}*(240*\sqrt{5}*\sqrt{3}*(d*x + c)^{(3/2)*b*d*\sqrt{\text{abs}(b)/\text{abs}(d)}*\text{abs}(b)*\cos(5*((d*x + c)*b - b*c + a*d)/d)/\text{abs}(d) - 400*\sqrt{5}*\sqrt{3}*(d*x + c)^{(3/2)*b*d*\sqrt{\text{abs}(b)/\text{abs}(d)}*\text{abs}(b)*\cos(3*((d*x + c)*b - b*c + a*d)/d)/\text{abs}(d) - 2400*\sqrt{5}*\sqrt{3}*(d*x + c)^{(3/2)*b*d*\sqrt{\text{abs}(b)/\text{abs}(d)}*\text{abs}(b)*\cos(((d*x + c)*b - b*c + a*d)/d)/\text{abs}(d) - 72*\sqrt{5}*\sqrt{3}*\sqrt{d*x + c}*d^2*\sqrt{\text{abs}(b)/\text{abs}(d)}*\text{abs}(b)*\sin(5*((d*x + c)*b - b*c + a*d)/d)/\text{abs}(d) + 200*\sqrt{5}*\sqrt{3}*\sqrt{d*x + c}*d^2*\sqrt{\text{abs}(b)/\text{abs}(d)}*\text{abs}(b)*\sin(3*((d*x + c)*b - b*c + a*d)/d)/\text{abs}(d) + 3600*\sqrt{5}*\sqrt{3}*\sqrt{d*x + c}*d^2*\sqrt{\text{abs}(b)/\text{abs}(d)}*\text{abs}(b)*\sin(((d*x + c)*b - b*c + a*d)/d)/\text{abs}(d) - (\sqrt{3}*(-9*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 9*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 9*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 9*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) *d^2*\text{abs}(b)*\cos(-5*(b*c - a*d)/d)/\text{abs}(d) - \sqrt{3}*(9*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 9*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 9*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 9*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) *d^2*\text{abs}(b)*\sin(-5*(b*c - a*d)/d)/\text{abs}(d))*\text{erf}(\sqrt{d*x + c}*\sqrt{5*I*b/d}) - (\sqrt{5}*(25*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 25*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 25*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 25*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) *d^2*\text{abs}(b)*\cos(-3*(b*c - a*d)/d)/\text{abs}(d) + \sqrt{5}*(25*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 25*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 25*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 25*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) *d^2*\text{abs}(b)*\sin(-3*(b*c - a*d)/d)/\text{abs}(d))*\text{erf}(\sqrt{d*x + c}*\sqrt{3*I*b/d}) - (\sqrt{5}*\sqrt{3}*(450*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 450*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 450*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 450*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) *d^2*\text{abs}(b)*\cos(-(b*c - a*d)/d)/\text{abs}(d) + \sqrt{5}*\sqrt{3}*(450*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 450*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 450*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 450*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) *d^2*\text{abs}(b)*\sin(-(b*c - a*d)/d)/\text{abs}(d))*\text{erf}(\sqrt{d*x + c}*\sqrt{I*b/d}) - (\sqrt{5}*\sqrt{3}*(-450*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 450*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 450*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 450*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) *d^2*\text{abs}(b)*\cos(-(b*c - a*d)/d)/\text{abs}(d) + \sqrt{5}*\sqrt{3}*(450*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 450*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 450*I*\sqrt{\pi}*\sin(1/4$

```

*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 450*I*sqrt(pi)*sin
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))d^2*abs(b)*sin
(-(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - (sqrt(5)*(-25*I*
sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 25
*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 25*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
- 25*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)))d^2*abs(b)*cos(-3*(b*c - a*d)/d)/abs(d) + sqrt(5)*(25*sqrt(pi)*cos(1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 25*sqrt(pi)*cos(-1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 25*I*sqrt(pi)*sin
(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 25*I*sqrt(pi)*
sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))d^2*abs(b)*
sin(-3*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - (sqrt(3)*
(9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2)))))d^2*abs(b)*cos(-5*(b*c - a*d)/d)/abs(d) - sqrt(3)*(9*sqrt(pi)*cos(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*si
n(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))d^2*abs(b)*si
n(-5*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d))*abs(d)/(b^2*
d*sqrt(abs(b)/abs(d))*abs(b))

```

Fricas [A] time = 0.767489, size = 1118, normalized size = 2.09

$$27 \sqrt{10} \pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{5(bc-ad)}{d}\right) S\left(\sqrt{10} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) - 125 \sqrt{6} \pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{3(bc-ad)}{d}\right) S\left(\sqrt{6} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) - 6750 \sqrt{2} \pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{bc-ad}{d}\right) S\left(\sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) - 6750 \sqrt{2} \pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{bc-ad}{d}\right) S\left(\sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) - 6750 \sqrt{2} \pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{bc-ad}{d}\right) S\left(\sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{bc-ad}{d}\right) - 125 \sqrt{6} \pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{3(bc-ad)}{d}\right) S\left(\sqrt{6} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{3(bc-ad)}{d}\right) + 27 \sqrt{10} \pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{5(bc-ad)}{d}\right) S\left(\sqrt{10} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{5(bc-ad)}{d}\right) + 480(30(b^2 dx + b^2 c) \cos(bx+a)^5 - 50(b^2 dx + b^2 c) \cos(bx+a)^3 - (9b dx + b^2 c) \cos(bx+a)^2 - 13b dx \cos(bx+a) - 26b d \sin(bx+a) \sqrt{dx+c}) / b^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")

[Out] 1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_si
n(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 125*sqrt(6)*pi*d^2*sqrt(b/(pi*d)
) *cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) -
6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d))) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel
_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*sqrt(6
) *pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-3*(b*c - a*d)/d) + 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(1
0)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(30*(b^2*d*x +
b^2*c)*cos(b*x + a)^5 - 50*(b^2*d*x + b^2*c)*cos(b*x + a)^3 - (9*b*d*cos(b
*x + a)^4 - 13*b*d*cos(b*x + a)^2 - 26*b*d)*sin(b*x + a)*sqrt(d*x + c))/b^3

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)

[Out] Timed out

Giac [C] time = 1.81472, size = 2269, normalized size = 4.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/144000*(10*(9*\sqrt{10}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{10}*\sqrt{b*d})*\sqrt{d*x} \\ & + c)*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((5*I*b*c - 5*I*a*d)/d)/(\sqrt{b*d}*(I*b \\ & *d/\sqrt{b^2*d^2} + 1)*b) - 25*\sqrt{6}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d} \\ & *\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((3*I*b*c - 3*I*a*d)/d)/(\sqrt{b*d} \\ & *(I*b*d/\sqrt{b^2*d^2} + 1)*b) - 450*\sqrt{2}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d} \\ & *\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((I*b*c - I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} \\ & + 1)*b) - 450*\sqrt{2}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} \\ & + 1)/d)*e^{((-I*b*c + I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b) - 25*\sqrt{6}*\sqrt{\pi} \\ & *d^2*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-3*I*b*c + 3*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} \\ & + 1)*b) + 9*\sqrt{10}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{10}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} \\ & + 1)/d)*e^{((-5*I*b*c + 5*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b) + 90*\sqrt{d*x + c} \\ & *d*e^{((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b} - 150*\sqrt{d*x + c}*d*e^{((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b} \\ & - 900*\sqrt{d*x + c}*d*e^{((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b} - 900*\sqrt{d*x + c}*d*e^{((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b} \\ & - 150*\sqrt{d*x + c}*d*e^{((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b} + 90*\sqrt{d*x + c}*d*e^{((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b} \\ & *c - 9*I*\sqrt{10}*\sqrt{\pi})*(-10*I*b*c*d + 3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{10}*\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} \\ & + 1)/d)*e^{((5*I*b*c - 5*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2) - 125*I*\sqrt{6}*\sqrt{\pi})*(2*I*b*c*d - d^2)*d*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x + c} \\ & *(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((3*I*b*c - 3*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2) - 2250*I*\sqrt{2}*\sqrt{\pi})*(2*I*b*c*d - 3*d^2) \\ & *d*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((I*b*c - I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2) - 2250*I \\ & *\sqrt{2}*\sqrt{\pi})*(2*I*b*c*d + 3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-I*b*c + I*a*d)/d)/(\sqrt{b*d}*(-I*b \\ & *d/\sqrt{b^2*d^2} + 1)*b^2) - 125*I*\sqrt{6}*\sqrt{\pi})*(2*I*b*c*d + d^2)*d*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-3*I \\ & *b*c + 3*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2) - 9*I*\sqrt{10}*\sqrt{\pi})*(-10*I*b*c*d - 3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{10}*\sqrt{b*d}*\sqrt{d*x + c} \\ &)*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-5*I*b*c + 5*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2) + 90*I*(-10*I*(d*x + c)^(3/2)*b*d + 10*I*\sqrt{d*x + c} \\ & *b*c*d + 3*\sqrt{d*x + c}*d^2)*e^{((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^2 + 750*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*\sqrt{d*x + c}*b*c*d - \sqrt{d*x + c} \\ & *d^2)*e^{((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2 + 4500*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*\sqrt{d*x + c}*b*c*d + 3*\sqrt{d*x + c}*d^2)*e^{((I \\ & *(d*x + c)*b - I*b*c + I*a*d)/d)/b^2 + 4500*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*\sqrt{d*x + c}*b*c*d + 3*\sqrt{d*x + c}*d^2)*e^{((-I*(d*x + c)*b + I*b*c - I \\ & *a*d)/d)/b^2 + 750*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*\sqrt{d*x + c}*b*c*d + \sqrt{d*x + c}*d^2)*e^{((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2} \\ & + 90*I*(-10*I*(d*x + c)^(3/2)*b*d + 10*I*\sqrt{d*x + c}*b*c*d - 3*\sqrt{d*x + c}*d^2)*e^{((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^2)/d \end{aligned}$$

3.135 $\int (c + dx)^{5/2} \cos^2(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=615

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}} - \frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}} + \frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\cos\left(5a - \frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}$$

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(32*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x])/(8*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^(5/2)*Cos[3*a + 3*b*x])/(48*b) - (3*d^2*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(1600*b^3) + ((c + d*x)^(5/2)*Cos[5*a + 5*b*x])/(80*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(32*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) + (3*d^(5/2)*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[a + b*x])/(16*b^2) + (5*d*(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(288*b^2) - (d*(c + d*x)^(3/2)*Sin[5*a + 5*b*x])/(160*b^2)
```

Rubi [A] time = 0.952804, antiderivative size = 615, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}} - \frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}} + \frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\cos\left(5a - \frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(32*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x])/(8*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^(5/2)*Cos[3*a + 3*b*x])/(48*b) - (3*d^2*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(1600*b^3) + ((c + d*x)^(5/2)*Cos[5*a + 5*b*x])/(80*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(32*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) + (3*d^(5/2)*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[a + b*x])/(16*b^2) + (5*d*(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(288*b^2) - (d*(c + d*x)^(3/2)*Sin[5*a + 5*b*x])/(160*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]
```

$]^n \cos[a + b*x]^p, x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3306

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]

Rule 3305

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))²], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresnel
S[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)]]/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3304

Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))²], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresnel
C[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)]]/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}
\int (c + dx)^{5/2} \cos^2(a + bx) \sin^3(a + bx) dx &= \int \left(\frac{1}{8}(c + dx)^{5/2} \sin(a + bx) + \frac{1}{16}(c + dx)^{5/2} \sin(3a + 3bx) - \frac{1}{16}(c + dx)^{5/2} \right. \\
&= \frac{1}{16} \int (c + dx)^{5/2} \sin(3a + 3bx) dx - \frac{1}{16} \int (c + dx)^{5/2} \sin(5a + 5bx) dx + \frac{1}{8} \\
&= -\frac{(c + dx)^{5/2} \cos(a + bx)}{8b} - \frac{(c + dx)^{5/2} \cos(3a + 3bx)}{48b} + \frac{(c + dx)^{5/2} \cos(5a)}{80b} \\
&= -\frac{(c + dx)^{5/2} \cos(a + bx)}{8b} - \frac{(c + dx)^{5/2} \cos(3a + 3bx)}{48b} + \frac{(c + dx)^{5/2} \cos(5a)}{80b} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(a + bx)}{32b^3} - \frac{(c + dx)^{5/2} \cos(a + bx)}{8b} + \frac{5d^2 \sqrt{c + dx} \cos(3a)}{576b^3} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(a + bx)}{32b^3} - \frac{(c + dx)^{5/2} \cos(a + bx)}{8b} + \frac{5d^2 \sqrt{c + dx} \cos(3a)}{576b^3} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(a + bx)}{32b^3} - \frac{(c + dx)^{5/2} \cos(a + bx)}{8b} + \frac{5d^2 \sqrt{c + dx} \cos(3a)}{576b^3} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(a + bx)}{32b^3} - \frac{(c + dx)^{5/2} \cos(a + bx)}{8b} + \frac{5d^2 \sqrt{c + dx} \cos(3a)}{576b^3}
\end{aligned}$$

Mathematica [C] time = 25.3137, size = 3348, normalized size = 5.44

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]

[Out] (c^2*Sqrt[c + d*x]*(-(E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[(-I)*b*(c + d*x)/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[(I*b*(c + d*x))/d])/(16*b*E^((I*(b*c + a*d))/d)) + (c^2*(2*Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d]))/(160*Sqrt[5]*b*Sqrt[b/d]) - (c^2*(2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(96*Sqrt[3]*b*Sqrt[b/d]) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(3*d*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a + b*x])))/(16*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (b*c)/d] + (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) - 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(d*(-15 + 4*b^2*x^2)*Cos[a + b*x] + 2*b*(c - 5*d*x)*Sin[a + b*x]))/(64*b^5) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(96*Sqrt[3]*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(96*Sqrt[3]*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(96*Sqrt[3]*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(96*Sqrt[3]*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(96*Sqrt[3]*b^3)

$$\begin{aligned} & b/d * d * \text{Sqrt}[c + d*x] * (d * (5 - 12*b^2*x^2) * \text{Cos}[3*(a + b*x)] - 2*b*(c - 5*d*x) \\ & * \text{Sin}[3*(a + b*x)]) / ((1152 * \text{Sqrt}[3] * b^5) + (c * \text{Sqrt}[b/d] * d * (\text{Sqrt}[2 * \text{Pi}] * \text{FresnelS} \\ & \text{S}[\text{Sqrt}[b/d] * \text{Sqrt}[10/\text{Pi}] * \text{Sqrt}[c + d*x]] * (3*d * \text{Cos}[5*a - (5*b*c)/d] - 10*b*c * \\ & \text{Sin}[5*a - (5*b*c)/d]) + \text{Sqrt}[2 * \text{Pi}] * \text{FresnelC}[\text{Sqrt}[b/d] * \text{Sqrt}[10/\text{Pi}] * \text{Sqrt}[c + \\ & d*x]] * (10*b*c * \text{Cos}[5*a - (5*b*c)/d] + 3*d * \text{Sin}[5*a - (5*b*c)/d]) + 2 * \text{Sqrt}[5] * \\ & \text{Sqrt}[b/d] * d * \text{Sqrt}[c + d*x] * (10*b*x * \text{Cos}[5*(a + b*x)] - 3 * \text{Sin}[5*(a + b*x)])) / \\ & (800 * \text{Sqrt}[5] * b^3) - (d^2 * (\text{Sin}[5*a] * ((c^2 * (-\text{Sqrt}[5] * \text{Sqrt}[b/d] * \text{Sqrt}[c + d*x] \\ & * \text{Cos}[(5*b*(c + d*x))/d]) + \text{Sqrt}[\text{Pi}/2] * \text{FresnelC}[\text{Sqrt}[b/d] * \text{Sqrt}[10/\text{Pi}] * \text{Sqrt}[c \\ & + d*x]] * \text{Sin}[(5*b*c)/d]) / (5 * \text{Sqrt}[5] * (b/d)^{(3/2)} * d^3) + (c^2 * \text{Cos}[(5*b*c)/d] \\ & * (-\text{Sqrt}[\text{Pi}/2] * \text{FresnelS}[\text{Sqrt}[b/d] * \text{Sqrt}[10/\text{Pi}] * \text{Sqrt}[c + d*x]])) + \text{Sqrt}[5] * \text{Sqrt} \\ & [b/d] * \text{Sqrt}[c + d*x] * \text{Sin}[(5*b*(c + d*x))/d]) / (5 * \text{Sqrt}[5] * (b/d)^{(3/2)} * d^3) - \\ & (2*c * \text{Cos}[(5*b*c)/d] * (-3 * (-\text{Sqrt}[5] * \text{Sqrt}[b/d] * \text{Sqrt}[c + d*x] * \text{Cos}[(5*b*(c + \\ & d*x))/d]) + \text{Sqrt}[\text{Pi}/2] * \text{FresnelC}[\text{Sqrt}[b/d] * \text{Sqrt}[10/\text{Pi}] * \text{Sqrt}[c + d*x]])) / 2 + \\ & 5 * \text{Sqrt}[5] * (b/d)^{(3/2)} * (c + d*x)^{(3/2)} * \text{Sin}[(5*b*(c + d*x))/d]) / (25 * \text{Sqrt}[5] * \\ & (b/d)^{(5/2)} * d^3) - (2*c * \text{Sin}[(5*b*c)/d] * (-5 * \text{Sqrt}[5] * (b/d)^{(3/2)} * (c + d*x)^{(3 \\ & /2)} * \text{Cos}[(5*b*(c + d*x))/d] + (3 * (-\text{Sqrt}[\text{Pi}/2] * \text{FresnelS}[\text{Sqrt}[b/d] * \text{Sqrt}[10/\text{Pi}] \\ &] * \text{Sqrt}[c + d*x]])) + \text{Sqrt}[5] * \text{Sqrt}[b/d] * \text{Sqrt}[c + d*x] * \text{Sin}[(5*b*(c + d*x))/d] \\ &) / 2) / (25 * \text{Sqrt}[5] * (b/d)^{(5/2)} * d^3) + (\text{Sin}[(5*b*c)/d] * (-25 * \text{Sqrt}[5] * (b/d)^{(5 \\ & /2)} * (c + d*x)^{(5/2)} * \text{Cos}[(5*b*(c + d*x))/d] + (5 * ((-3 * (-\text{Sqrt}[5] * \text{Sqrt}[b/d] * \text{Sqr} \\ & \text{t}[c + d*x] * \text{Cos}[(5*b*(c + d*x))/d]) + \text{Sqrt}[\text{Pi}/2] * \text{FresnelC}[\text{Sqrt}[b/d] * \text{Sqrt}[10 \\ & /\text{Pi}] * \text{Sqrt}[c + d*x]])) / 2 + 5 * \text{Sqrt}[5] * (b/d)^{(3/2)} * (c + d*x)^{(3/2)} * \text{Sin}[(5*b*(c \\ & + d*x))/d]) / 2) / (125 * \text{Sqrt}[5] * (b/d)^{(7/2)} * d^3) + (\text{Cos}[(5*b*c)/d] * (25 * \text{Sqrt}[\\ & 5] * (b/d)^{(5/2)} * (c + d*x)^{(5/2)} * \text{Sin}[(5*b*(c + d*x))/d] - (5 * (-5 * \text{Sqrt}[5] * (b/d \\ &)^{(3/2)} * (c + d*x)^{(3/2)} * \text{Cos}[(5*b*(c + d*x))/d] + (3 * (-\text{Sqrt}[\text{Pi}/2] * \text{FresnelS}[\\ & \text{Sqrt}[b/d] * \text{Sqrt}[10/\text{Pi}] * \text{Sqrt}[c + d*x]])) + \text{Sqrt}[5] * \text{Sqrt}[b/d] * \text{Sqrt}[c + d*x] * \text{Sin} \\ & [(5*b*(c + d*x))/d]) / 2) / (125 * \text{Sqrt}[5] * (b/d)^{(7/2)} * d^3) + \text{Cos}[5*a] * ((c \\ & ^2 * \text{Cos}[(5*b*c)/d] * (-\text{Sqrt}[5] * \text{Sqrt}[b/d] * \text{Sqrt}[c + d*x] * \text{Cos}[(5*b*(c + d*x))/d] \\ &) + \text{Sqrt}[\text{Pi}/2] * \text{FresnelC}[\text{Sqrt}[b/d] * \text{Sqrt}[10/\text{Pi}] * \text{Sqrt}[c + d*x]])) / (5 * \text{Sqrt}[5] * (\\ & b/d)^{(3/2)} * d^3) - (c^2 * \text{Sin}[(5*b*c)/d] * (-\text{Sqrt}[\text{Pi}/2] * \text{FresnelS}[\text{Sqrt}[b/d] * \text{Sqrt} \\ & [10/\text{Pi}] * \text{Sqrt}[c + d*x]])) + \text{Sqrt}[5] * \text{Sqrt}[b/d] * \text{Sqrt}[c + d*x] * \text{Sin}[(5*b*(c + d*x) \\ &) / d]) / (5 * \text{Sqrt}[5] * (b/d)^{(3/2)} * d^3) + (2*c * \text{Sin}[(5*b*c)/d] * ((-3 * (-\text{Sqrt}[5] * \text{S} \\ & \text{qrt}[b/d] * \text{Sqrt}[c + d*x] * \text{Cos}[(5*b*(c + d*x))/d]) + \text{Sqrt}[\text{Pi}/2] * \text{FresnelC}[\text{Sqrt}[b \\ & /d] * \text{Sqrt}[10/\text{Pi}] * \text{Sqrt}[c + d*x]])) / 2 + 5 * \text{Sqrt}[5] * (b/d)^{(3/2)} * (c + d*x)^{(3/2)} * \\ & \text{Sin}[(5*b*(c + d*x))/d]) / (25 * \text{Sqrt}[5] * (b/d)^{(5/2)} * d^3) - (2*c * \text{Cos}[(5*b*c)/d] \\ & * (-5 * \text{Sqrt}[5] * (b/d)^{(3/2)} * (c + d*x)^{(3/2)} * \text{Cos}[(5*b*(c + d*x))/d] + (3 * (-\text{Sqr} \\ & \text{t}[\text{Pi}/2] * \text{FresnelS}[\text{Sqrt}[b/d] * \text{Sqrt}[10/\text{Pi}] * \text{Sqrt}[c + d*x]])) + \text{Sqrt}[5] * \text{Sqrt}[b/d] * \\ & \text{Sqrt}[c + d*x] * \text{Sin}[(5*b*(c + d*x))/d]) / 2) / (25 * \text{Sqrt}[5] * (b/d)^{(5/2)} * d^3) + (\\ & \text{Cos}[(5*b*c)/d] * (-25 * \text{Sqrt}[5] * (b/d)^{(5/2)} * (c + d*x)^{(5/2)} * \text{Cos}[(5*b*(c + d*x)) \\ & /d] + (5 * ((-3 * (-\text{Sqrt}[5] * \text{Sqrt}[b/d] * \text{Sqrt}[c + d*x] * \text{Cos}[(5*b*(c + d*x))/d]) + \\ & \text{Sqrt}[\text{Pi}/2] * \text{FresnelC}[\text{Sqrt}[b/d] * \text{Sqrt}[10/\text{Pi}] * \text{Sqrt}[c + d*x]])) / 2 + 5 * \text{Sqrt}[5] * (b \\ & /d)^{(3/2)} * (c + d*x)^{(3/2)} * \text{Sin}[(5*b*(c + d*x))/d]) / 2) / (125 * \text{Sqrt}[5] * (b/d)^{(\\ & 7/2)} * d^3) - (\text{Sin}[(5*b*c)/d] * (25 * \text{Sqrt}[5] * (b/d)^{(5/2)} * (c + d*x)^{(5/2)} * \text{Sin}[(5* \\ & b*(c + d*x))/d] - (5 * (-5 * \text{Sqrt}[5] * (b/d)^{(3/2)} * (c + d*x)^{(3/2)} * \text{Cos}[(5*b*(c + \\ & d*x))/d] + (3 * (-\text{Sqrt}[\text{Pi}/2] * \text{FresnelS}[\text{Sqrt}[b/d] * \text{Sqrt}[10/\text{Pi}] * \text{Sqrt}[c + d*x]])) \\ & + \text{Sqrt}[5] * \text{Sqrt}[b/d] * \text{Sqrt}[c + d*x] * \text{Sin}[(5*b*(c + d*x))/d]) / 2) / 2) / (125 * \text{Sqr} \\ & \text{t}[5] * (b/d)^{(7/2)} * d^3))) / 16 \end{aligned}$$

Maple [A] time = 0.043, size = 719, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d*x+c)^{(5/2)} * \cos(b*x+a)^2 * \sin(b*x+a)^3, x)$

[Out] $2/d * (-1/16/b * d * (d*x+c)^{(5/2)} * \cos(1/d * (d*x+c) * b + (a*d-b*c)/d) + 5/16/b * d * (1/2/b * d * (d*x+c)^{(3/2)} * \sin(1/d * (d*x+c) * b + (a*d-b*c)/d) - 3/2/b * d * (-1/2/b * d * (d*x+c)^{($

$$\begin{aligned} & 1/2) * \cos(1/d * (d*x+c) * b + (a*d-b*c)/d) + 1/4/b*d*2^{(1/2)} * \text{Pi}^{(1/2)} / (b/d)^{(1/2)} * (\cos((a*d-b*c)/d) * \text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)}/(b/d)^{(1/2)} * (d*x+c)^{(1/2)} * b/d) - \sin((a*d-b*c)/d) * \text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}/(b/d)^{(1/2)} * (d*x+c)^{(1/2)} * b/d)) - \\ & 1/96/b*d*(d*x+c)^{(5/2)} * \cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d) + 5/96/b*d*(1/6/b*d*(d*x+c)^{(3/2)} * \sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d) - 1/2/b*d*(-1/6/b*d*(d*x+c)^{(1/2)} * \cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d) + 1/36/b*d*2^{(1/2)} * \text{Pi}^{(1/2)} * 3^{(1/2)} / (b/d)^{(1/2)} * (\cos(3*(a*d-b*c)/d) * \text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)} * 3^{(1/2)} / (b/d)^{(1/2)} * (d*x+c)^{(1/2)} * b/d) - \sin(3*(a*d-b*c)/d) * \text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)} * 3^{(1/2)} / (b/d)^{(1/2)} * (d*x+c)^{(1/2)} * b/d))) + 1/160/b*d*(d*x+c)^{(5/2)} * \cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d) - 1/32/b*d*(1/10/b*d*(d*x+c)^{(3/2)} * \sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d) - 3/10/b*d*(-1/10/b*d*(d*x+c)^{(1/2)} * \cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d) + 1/100/b*d*2^{(1/2)} * \text{Pi}^{(1/2)} * 5^{(1/2)} / (b/d)^{(1/2)} * (\cos(5*(a*d-b*c)/d) * \text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)} * 5^{(1/2)} / (b/d)^{(1/2)} * (d*x+c)^{(1/2)} * b/d) - \sin(5*(a*d-b*c)/d) * \text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)} * 5^{(1/2)} / (b/d)^{(1/2)} * (d*x+c)^{(1/2)} * b/d))) \end{aligned}$$

Maxima [C] time = 2.95017, size = 2943, normalized size = 4.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/1728000 * \text{sqrt}(5) * \text{sqrt}(3) * (720 * \text{sqrt}(5) * \text{sqrt}(3) * (d*x + c)^{(3/2)} * b * d^2 * \text{sqrt}(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) * \sin(5 * ((d*x + c) * b - b*c + a*d)/d) / \text{abs}(d) - 2000 * \text{sqrt}(5) * \text{sqrt}(3) * (d*x + c)^{(3/2)} * b * d^2 * \text{sqrt}(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) * \sin(3 * ((d*x + c) * b - b*c + a*d)/d) / \text{abs}(d) - 36000 * \text{sqrt}(5) * \text{sqrt}(3) * (d*x + c)^{(3/2)} * b * d^2 * \text{sqrt}(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) * \sin(((d*x + c) * b - b*c + a*d)/d) / \text{abs}(d) - 72 * (20 * \text{sqrt}(5) * \text{sqrt}(3) * (d*x + c)^{(5/2)} * b^2 * d * \text{sqrt}(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) / \text{abs}(d) - 3 * \text{sqrt}(5) * \text{sqrt}(3) * \text{sqrt}(d*x + c) * d^3 * \text{sqrt}(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) / \text{abs}(d)) * \cos(5 * ((d*x + c) * b - b*c + a*d)/d) + 200 * (12 * \text{sqrt}(5) * \text{sqrt}(3) * (d*x + c)^{(5/2)} * b^2 * d * \text{sqrt}(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) / \text{abs}(d) - 5 * \text{sqrt}(5) * \text{sqrt}(3) * \text{sqrt}(d*x + c) * d^3 * \text{sqrt}(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) / \text{abs}(d)) * \cos(3 * ((d*x + c) * b - b*c + a*d)/d) + 3600 * (4 * \text{sqrt}(5) * \text{sqrt}(3) * (d*x + c)^{(5/2)} * b^2 * d * \text{sqrt}(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) / \text{abs}(d) - 15 * \text{sqrt}(5) * \text{sqrt}(3) * \text{sqrt}(d*x + c) * d^3 * \text{sqrt}(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) / \text{abs}(d)) * \cos(((d*x + c) * b - b*c + a*d)/d) - (\text{sqrt}(3) * (27 * \text{sqrt}(\text{pi}) * \cos(1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) + 27 * \text{sqrt}(\text{pi}) * \cos(-1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) - 27 * I * \text{sqrt}(\text{pi}) * \sin(1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) + 27 * I * \text{sqrt}(\text{pi}) * \sin(-1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2)))) * d^3 * \text{abs}(b) * \cos(-5 * (b*c - a*d)/d) / \text{abs}(d) - \text{sqrt}(3) * (27 * I * \text{sqrt}(\text{pi}) * \cos(1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) + 27 * I * \text{sqrt}(\text{pi}) * \cos(-1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) + 27 * \text{sqrt}(\text{pi}) * \sin(1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) - 27 * \text{sqrt}(\text{pi}) * \sin(-1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2)))) * d^3 * \text{abs}(b) * \sin(-5 * (b*c - a*d)/d) / \text{abs}(d) * \text{erf}(\text{sqrt}(d*x + c) * \text{sqrt}(5 * I * b/d)) + (\text{sqrt}(5) * (125 * \text{sqrt}(\text{pi}) * \cos(1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) + 125 * \text{sqrt}(\text{pi}) * \cos(-1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) - 125 * I * \text{sqrt}(\text{pi}) * \sin(1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) + 125 * I * \text{sqrt}(\text{pi}) * \sin(-1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2)))) * d^3 * \text{abs}(b) * \cos(-3 * (b*c - a*d)/d) / \text{abs}(d) + \text{sqrt}(5) * (-125 * I * \text{sqrt}(\text{pi}) * \cos(1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) - 125 * I * \text{sqrt}(\text{pi}) * \cos(-1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) - 125 * \text{sqrt}(\text{pi}) * \sin(1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) + 125 * \text{sqrt}(\text{pi}) * \sin(-1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2)))) * d^3 * \text{abs}(b) * \sin(-3 * (b*c - a*d)/d) / \text{abs}(d) * \text{erf}(\text{sqrt}(d*x + c) * \text{sqrt}(3 * I * b/d)) + (\text{sqrt}(5) * \text{sqrt}(3) * (6750 * \text{sqrt}(\text{pi}) * \cos(1/4 * \text{pi} + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\text{sqrt}(d^2))) + \end{aligned}$$

```

6750*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
)) - 6750*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt
(d^2))) + 6750*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) *d^3*abs(b)*cos(-(b*c - a*d)/d)/abs(d) + sqrt(5)*sqrt(3)*(-67
50*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
- 6750*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) - 6750*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) + 6750*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) *d^3*abs(b)*sin(-(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqr
t(I*b/d)) + (sqrt(5)*sqrt(3)*(6750*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) + 6750*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + 6750*I*sqrt(pi)*sin(1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 6750*I*sqrt(pi)*sin(-1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^3*abs(b)*cos(-(b*c - a*d)/
d)/abs(d) + sqrt(5)*sqrt(3)*(6750*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) + 6750*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 6750*sqrt(pi)*sin(1/4*pi + 1/2*arct
an2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 6750*sqrt(pi)*sin(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^3*abs(b)*sin(-(b*c - a*d)/d
)/abs(d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (sqrt(5)*(125*sqrt(pi)*cos(1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 125*sqrt(pi)*cos(-1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 125*I*sqrt(pi)*s
in(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 125*I*sqrt(p
i)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^3*abs(
b)*cos(-3*(b*c - a*d)/d)/abs(d) + sqrt(5)*(125*I*sqrt(pi)*cos(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 125*I*sqrt(pi)*cos(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 125*sqrt(pi)*sin(1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 125*sqrt(pi)*sin(-1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^3*abs(b)*sin(-3*(
b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - (sqrt(3)*(27*sqrt
(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*sqrt
(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*I
*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 2
7*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
)*d^3*abs(b)*cos(-5*(b*c - a*d)/d)/abs(d) - sqrt(3)*(-27*I*sqrt(pi)*cos(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 27*I*sqrt(pi)*cos(
-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*sqrt(pi)*si
n(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 27*sqrt(pi)*s
in(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^3*abs(b)*s
in(-5*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d))*abs(d)/(b^3
*d*sqrt(abs(b)/abs(d))*abs(b))

```

Fricas [A] time = 0.832086, size = 1337, normalized size = 2.17

$$81 \sqrt{10} \pi d^3 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{5(bc-ad)}{d}\right) C\left(\sqrt{10} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) - 625 \sqrt{6} \pi d^3 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{3(bc-ad)}{d}\right) C\left(\sqrt{6} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) - 101250 \sqrt{2} \pi d^3 \sqrt{\frac{b}{\pi d}} \sin\left(-\frac{5(bc-ad)}{d}\right) \operatorname{erf}\left(\sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) + 625 \sqrt{6} \pi d^3 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{3(bc-ad)}{d}\right) \operatorname{erf}\left(\sqrt{6} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) + 101250 \sqrt{2} \pi d^3 \sqrt{\frac{b}{\pi d}} \sin\left(-\frac{5(bc-ad)}{d}\right) \operatorname{erf}\left(\sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")

[Out] 1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_c
os(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 625*sqrt(6)*pi*d^3*sqrt(b/(pi*d)
))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))
- 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fr
esnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 625*s

```

qrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d
))) *sin(-3*(b*c - a*d)/d) - 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(s
qrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) *sin(-5*(b*c - a*d)/d) + 480*(9*(20*b^
3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*cos(b*x + a)^5 + 390*b*d^2
*cos(b*x + a) - 5*(60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 - 13*b*d^2)*
*cos(b*x + a)^3 + 10*(26*b^2*d^2*x - 9*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^4
+ 26*b^2*c*d + 13*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(
d*x + c))/b^4

```

Sympy [F-1] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 2.21758, size = 4084, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```

[Out] 1/864000*(60*(9*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*d^2*er
f(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I
*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt
(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) +
9*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)*b) + 90*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)
/d)/b - 150*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b -
900*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 900*sqrt(d*x
+ c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b - 150*sqrt(d*x + c)*d*e^((
-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*sqrt(d*x + c)*d*e^((-5*I*(d
*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)*c^2 - d^2*(27*(I*sqrt(10)*sqrt(pi))*(20
*I*b^2*c^2*d - 12*b*c*d^2 - 3*I*d^3)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b^3) - 10*I*(-20*I*(d*x + c)^(5/2)*b^2*d + 40*I*(d*x
+ c)^(3/2)*b^2*c*d - 20*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d
^2 + 12*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*x + c)*d^3)*e^((-5*I*(d*x + c)*b
+ 5*I*b*c - 5*I*a*d)/d)/b^3)/d^2 + 125*(I*sqrt(6)*sqrt(pi))*(-12*I*b^2*c^2*
d + 12*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2

```

$$\begin{aligned}
& *d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^{(5/2)}*b^2*d - 24*I*(d*x + c)^{(3/2)}*b^2*c*d + 12*I*\sqrt{d*x + c}*b^2*c^2*d + 10*(d*x + c)^{(3/2)}*b*d^2 - 12*\sqrt{d*x + c}*b*c*d^2 - 5*I*\sqrt{d*x + c}*d^3)*e^{((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^3)/d^2 + 6750*(I*\sqrt{2}*\sqrt{\pi})*(-4*I*b^2*c^2*d + 12*b*c*d^2 + 15*I*d^3)*d*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((I*b*c - I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^3) - 2*I*(4*I*(d*x + c)^{(5/2)}*b^2*d - 8*I*(d*x + c)^{(3/2)}*b^2*c*d + 4*I*\sqrt{d*x + c}*b^2*c^2*d + 10*(d*x + c)^{(3/2)}*b*d^2 - 12*\sqrt{d*x + c}*b*c*d^2 - 15*I*\sqrt{d*x + c}*d^3)*e^{((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^2 + 6750*(I*\sqrt{2}*\sqrt{\pi})*(-4*I*b^2*c^2*d - 12*b*c*d^2 + 15*I*d^3)*d*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-I*b*c + I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^3) - 2*I*(4*I*(d*x + c)^{(5/2)}*b^2*d - 8*I*(d*x + c)^{(3/2)}*b^2*c*d + 4*I*\sqrt{d*x + c}*b^2*c^2*d - 10*(d*x + c)^{(3/2)}*b*d^2 + 12*\sqrt{d*x + c}*b*c*d^2 - 15*I*\sqrt{d*x + c}*d^3)*e^{((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^2 + 125*(I*\sqrt{6}*\sqrt{\pi})*(-12*I*b^2*c^2*d - 12*b*c*d^2 + 5*I*d^3)*d*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-3*I*b*c + 3*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^3) - 6*I*(12*I*(d*x + c)^{(5/2)}*b^2*d - 24*I*(d*x + c)^{(3/2)}*b^2*c*d + 12*I*\sqrt{d*x + c}*b^2*c^2*d - 10*(d*x + c)^{(3/2)}*b*d^2 + 12*\sqrt{d*x + c}*b*c*d^2 - 5*I*\sqrt{d*x + c}*d^3)*e^{((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/d^2 + 27*(I*\sqrt{10}*\sqrt{\pi})*(20*I*b^2*c^2*d + 12*b*c*d^2 - 3*I*d^3)*d*\operatorname{erf}(-1/2*\sqrt{10}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-5*I*b*c + 5*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^3) - 10*I*(-20*I*(d*x + c)^{(5/2)}*b^2*d + 40*I*(d*x + c)^{(3/2)}*b^2*c*d - 20*I*\sqrt{d*x + c}*b^2*c^2*d + 10*(d*x + c)^{(3/2)}*b*d^2 - 12*\sqrt{d*x + c}*b*c*d^2 + 3*I*\sqrt{d*x + c}*d^3)*e^{((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^3)/d^2 - 12*(9*I*\sqrt{10}*\sqrt{\pi})*(-10*I*b*c*d + 3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{10}*\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((5*I*b*c - 5*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2) + 125*I*\sqrt{6}*\sqrt{\pi}*(2*I*b*c*d - d^2)*d*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((3*I*b*c - 3*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2) + 2250*I*\sqrt{2}*\sqrt{\pi}*(2*I*b*c*d - 3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((I*b*c - I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2) + 2250*I*\sqrt{2}*\sqrt{\pi}*(2*I*b*c*d + 3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-I*b*c + I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2) + 125*I*\sqrt{6}*\sqrt{\pi}*(2*I*b*c*d + d^2)*d*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-3*I*b*c + 3*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2) + 9*I*\sqrt{10}*\sqrt{\pi})*(-10*I*b*c*d - 3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{10}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-5*I*b*c + 5*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2) - 90*I*(-10*I*(d*x + c)^{(3/2)}*b*d + 10*I*\sqrt{d*x + c}*b*c*d + 3*\sqrt{d*x + c}*d^2)*e^{((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^2 - 750*I*(2*I*(d*x + c)^{(3/2)}*b*d - 2*I*\sqrt{d*x + c}*b*c*d - \sqrt{d*x + c}*d^2)*e^{((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2 - 4500*I*(2*I*(d*x + c)^{(3/2)}*b*d - 2*I*\sqrt{d*x + c}*b*c*d - 3*\sqrt{d*x + c}*d^2)*e^{((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2 - 4500*I*(2*I*(d*x + c)^{(3/2)}*b*d - 2*I*\sqrt{d*x + c}*b*c*d + 3*\sqrt{d*x + c}*d^2)*e^{((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2 - 750*I*(2*I*(d*x + c)^{(3/2)}*b*d - 2*I*\sqrt{d*x + c}*b*c*d + \sqrt{d*x + c}*d^2)*e^{((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2 - 90*I*(-10*I*(d*x + c)^{(3/2)}*b*d + 10*I*\sqrt{d*x + c}*b*c*d - 3*\sqrt{d*x + c}*d^2)*e^{((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^2)*c)/d
\end{aligned}$$

3.136 $\int (c + dx)^m \cos^3(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=273

$$\frac{2^{-m-4} e^{2i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{2ib(c+dx)}{d}\right)}{b} - \frac{2^{-2(m+3)} e^{4i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{2ib(c+dx)}{d}\right)}{b}$$

```
[Out] -((2^(-4 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d])/(b*((-I)*b*(c + d*x))/d)^m) - (2^(-4 - m)*(c + d*x)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - (E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*E^((4*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.288119, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4406, 3308, 2181}

$$\frac{2^{-m-4} e^{2i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{2ib(c+dx)}{d}\right)}{b} - \frac{2^{-2(m+3)} e^{4i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{2ib(c+dx)}{d}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m * Cos[a + b*x]^3 * Sin[a + b*x], x]
```

```
[Out] -((2^(-4 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d])/(b*((-I)*b*(c + d*x))/d)^m) - (2^(-4 - m)*(c + d*x)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - (E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*E^((4*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n * Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, -(f*g*Log[F])/d]*(c + d*x)]/(d*(-(f*g*Log[F])/d)^(IntPart[m] + 1)*(-(f*g*Log[F])*(c + d*x)/d)^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^m \cos^3(a+bx) \sin(a+bx) dx &= \int \left(\frac{1}{4}(c+dx)^m \sin(2a+2bx) + \frac{1}{8}(c+dx)^m \sin(4a+4bx) \right) dx \\
&= \frac{1}{8} \int (c+dx)^m \sin(4a+4bx) dx + \frac{1}{4} \int (c+dx)^m \sin(2a+2bx) dx \\
&= \frac{1}{16} i \int e^{-i(4a+4bx)} (c+dx)^m dx - \frac{1}{16} i \int e^{i(4a+4bx)} (c+dx)^m dx + \frac{1}{8} i \int e^{-i(2a+2bx)} (c+dx)^m dx \\
&\quad - \frac{1}{8} i \int e^{i(2a+2bx)} (c+dx)^m dx \\
&= -\frac{2^{-4-m} e^{2i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{2ib(c+dx)}{d}\right)}{b} - \frac{2^{-4-m} e^{-2i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, \frac{2ib(c+dx)}{d}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.247976, size = 245, normalized size = 0.9

$$4^{-m-3} e^{-\frac{4i(ad+bc)}{d}} (c+dx)^m \left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m} \left(2^{m+2} e^{2i\left(a+\frac{3bc}{d}\right)} \left(-\frac{ib(c+dx)}{d}\right)^m \Gamma\left(m+1, \frac{2ib(c+dx)}{d}\right) + 2^{m+2} e^{2i\left(3a+\frac{bc}{d}\right)} \left(\frac{ib(c+dx)}{d}\right)^m \Gamma\left(m+1, -\frac{2ib(c+dx)}{d}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^m * Cos[a + b*x]^3 * Sin[a + b*x], x]

[Out] -((4^(-3 - m) * (c + d*x)^m * (2^(2 + m) * E^((2*I)*(3*a + (b*c)/d)) * ((I*b*(c + d*x))/d)^m * Gamma[1 + m, ((-2*I)*b*(c + d*x))/d] + 2^(2 + m) * E^((2*I)*(a + (3*b*c)/d)) * (((-I)*b*(c + d*x))/d)^m * Gamma[1 + m, ((2*I)*b*(c + d*x))/d] + E^((8*I)*a) * ((I*b*(c + d*x))/d)^m * Gamma[1 + m, ((-4*I)*b*(c + d*x))/d] + E^(((8*I)*b*c)/d) * (((-I)*b*(c + d*x))/d)^m * Gamma[1 + m, ((4*I)*b*(c + d*x))/d]) / (b * E^(((4*I)*(b*c + a*d))/d) * (b^2*(c + d*x)^2/d^2)^m)

Maple [F] time = 0.266, size = 0, normalized size = 0.

$$\int (dx+c)^m (\cos (bx+a))^3 \sin (bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x)

[Out] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos (bx+a)^3 \sin (bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")

[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a), x)

Fricas [A] time = 0.54418, size = 487, normalized size = 1.78

$$\frac{e^{\left(-\frac{dm \log\left(\frac{4ib}{d}\right) - 4ibc + 4iad}{d}\right)} \Gamma\left(m+1, \frac{4ibdx + 4ibc}{d}\right) + 4e^{\left(-\frac{dm \log\left(\frac{2ib}{d}\right) - 2ibc + 2iad}{d}\right)} \Gamma\left(m+1, \frac{2ibdx + 2ibc}{d}\right) + 4e^{\left(-\frac{dm \log\left(-\frac{2ib}{d}\right) + 2ibc - 2iad}{d}\right)} \Gamma\left(m+1, \frac{-2ibdx - 2ibc}{d}\right) + e^{\left(-\frac{dm \log\left(-\frac{4ib}{d}\right) + 4ibc - 4iad}{d}\right)} \Gamma\left(m+1, \frac{-4ibdx - 4ibc}{d}\right)}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")

[Out] $-1/64*(e^{-(d*m*\log(4*I*b/d) - 4*I*b*c + 4*I*a*d)/d}*gamma(m + 1, (4*I*b*d*x + 4*I*b*c)/d) + 4*e^{-(d*m*\log(2*I*b/d) - 2*I*b*c + 2*I*a*d)/d}*gamma(m + 1, (2*I*b*d*x + 2*I*b*c)/d) + 4*e^{-(d*m*\log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d}*gamma(m + 1, (-2*I*b*d*x - 2*I*b*c)/d) + e^{-(d*m*\log(-4*I*b/d) + 4*I*b*c - 4*I*a*d)/d}*gamma(m + 1, (-4*I*b*d*x - 4*I*b*c)/d))/b$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**m*cos(b*x+a)**3*sin(b*x+a),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a)^3 \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a), x)

3.137 $\int (c + dx)^4 \cos^3(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=260

$$\frac{3d^2(c + dx)^2 \cos^4(a + bx)}{16b^3} + \frac{9d^2(c + dx)^2 \cos^2(a + bx)}{16b^3} - \frac{3d^3(c + dx) \sin(a + bx) \cos^3(a + bx)}{32b^4} - \frac{45d^3(c + dx) \sin(a + bx)}{64b^4}$$

[Out] $(-45*c*d^3*x)/(64*b^3) - (45*d^4*x^2)/(128*b^3) + (3*(c + d*x)^4)/(32*b) - (45*d^4*\text{Cos}[a + b*x]^2)/(128*b^5) + (9*d^2*(c + d*x)^2*\text{Cos}[a + b*x]^2)/(16*b^3) - (3*d^4*\text{Cos}[a + b*x]^4)/(128*b^5) + (3*d^2*(c + d*x)^2*\text{Cos}[a + b*x]^4)/(16*b^3) - ((c + d*x)^4*\text{Cos}[a + b*x]^4)/(4*b) - (45*d^3*(c + d*x)*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/(64*b^4) + (3*d*(c + d*x)^3*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/(8*b^2) - (3*d^3*(c + d*x)*\text{Cos}[a + b*x]^3*\text{Sin}[a + b*x])/(32*b^4) + (d*(c + d*x)^3*\text{Cos}[a + b*x]^3*\text{Sin}[a + b*x])/(4*b^2)$

Rubi [A] time = 0.234001, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4405, 3311, 32, 3310}

$$\frac{3d^2(c + dx)^2 \cos^4(a + bx)}{16b^3} + \frac{9d^2(c + dx)^2 \cos^2(a + bx)}{16b^3} - \frac{3d^3(c + dx) \sin(a + bx) \cos^3(a + bx)}{32b^4} - \frac{45d^3(c + dx) \sin(a + bx)}{64b^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^4*\text{Cos}[a + b*x]^3*\text{Sin}[a + b*x], x]$

[Out] $(-45*c*d^3*x)/(64*b^3) - (45*d^4*x^2)/(128*b^3) + (3*(c + d*x)^4)/(32*b) - (45*d^4*\text{Cos}[a + b*x]^2)/(128*b^5) + (9*d^2*(c + d*x)^2*\text{Cos}[a + b*x]^2)/(16*b^3) - (3*d^4*\text{Cos}[a + b*x]^4)/(128*b^5) + (3*d^2*(c + d*x)^2*\text{Cos}[a + b*x]^4)/(16*b^3) - ((c + d*x)^4*\text{Cos}[a + b*x]^4)/(4*b) - (45*d^3*(c + d*x)*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/(64*b^4) + (3*d*(c + d*x)^3*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/(8*b^2) - (3*d^3*(c + d*x)*\text{Cos}[a + b*x]^3*\text{Sin}[a + b*x])/(32*b^4) + (d*(c + d*x)^3*\text{Cos}[a + b*x]^3*\text{Sin}[a + b*x])/(4*b^2)$

Rule 4405

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_)]^{(n_.)}*((c_.) + (d_.)*(x_))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[a + b*x]^{(n + 1)} / (b*(n + 1)), x] + \text{Dist}[(d*m) / (b*(n + 1)), \text{Int}[(c + d*x)^{(m - 1)}*\text{Cos}[a + b*x]^{(n + 1)}, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 3311

$\text{Int}[(c_.) + (d_.)*(x_)]^{(m_.)}*((b_.)*\text{sin}[(e_.) + (f_.)*(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(d*m*(c + d*x)^{(m - 1)}*(b*\text{Sin}[e + f*x])^n) / (f^2*n^2), x] + (\text{Dist}[(b^2*(n - 1)) / n, \text{Int}[(c + d*x)^m*(b*\text{Sin}[e + f*x])^{(n - 2)}, x], x] - \text{Dist}[(d^2*m*(m - 1)) / (f^2*n^2), \text{Int}[(c + d*x)^{(m - 2)}*(b*\text{Sin}[e + f*x])^n, x], x] - \text{Simp}[(b*(c + d*x)^m*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(n - 1)}) / (f*n), x]) /;$ FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rule 32

$\text{Int}[(a_.) + (b_.)*(x_)]^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)} / (b*(m + 1)), x] /;$ FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 3310

```
Int[((c_.) + (d_.)*(x_.))*((b_.)*sin[(e_.) + (f_.)*(x_.)])^(n_), x_Symbol] :=
Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^4 \cos^3(a + bx) \sin(a + bx) dx &= -\frac{(c + dx)^4 \cos^4(a + bx)}{4b} + \frac{d \int (c + dx)^3 \cos^4(a + bx) dx}{b} \\ &= \frac{3d^2(c + dx)^2 \cos^4(a + bx)}{16b^3} - \frac{(c + dx)^4 \cos^4(a + bx)}{4b} + \frac{d(c + dx)^3 \cos^3(a + bx)}{4b^2} \\ &= \frac{9d^2(c + dx)^2 \cos^2(a + bx)}{16b^3} - \frac{3d^4 \cos^4(a + bx)}{128b^5} + \frac{3d^2(c + dx)^2 \cos^4(a + bx)}{16b^3} \\ &= \frac{3(c + dx)^4}{32b} - \frac{45d^4 \cos^2(a + bx)}{128b^5} + \frac{9d^2(c + dx)^2 \cos^2(a + bx)}{16b^3} - \frac{3d^4 \cos^4(a + bx)}{128b^5} \\ &= -\frac{45cd^3x}{64b^3} - \frac{45d^4x^2}{128b^3} + \frac{3(c + dx)^4}{32b} - \frac{45d^4 \cos^2(a + bx)}{128b^5} + \frac{9d^2(c + dx)^2 \cos^2(a + bx)}{16b^3} \end{aligned}$$

Mathematica [A] time = 1.8831, size = 158, normalized size = 0.61

$$\frac{64 \cos(2(a + bx)) \left(-6b^2d^2(c + dx)^2 + 2b^4(c + dx)^4 + 3d^4\right) + \cos(4(a + bx)) \left(-24b^2d^2(c + dx)^2 + 32b^4(c + dx)^4 + 3d^4\right)}{1024b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x], x]
```

```
[Out] -(64*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)] +
(3*d^4 - 24*b^2*d^2*(c + d*x)^2 + 32*b^4*(c + d*x)^4)*Cos[4*(a + b*x)] - 8
*b*d*(c + d*x)*(16*(-3*d^2 + 2*b^2*(c + d*x)^2) + (-3*d^2 + 8*b^2*(c + d*x)
^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)]/(1024*b^5)
```

Maple [B] time = 0.049, size = 1150, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a), x)
```

```
[Out] 1/b*(1/b^4*d^4*(-1/4*(b*x+a)^4*cos(b*x+a)^4+(b*x+a)^3*(1/4*(cos(b*x+a)^3+3/
2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+3/16*(b*x+a)^2*cos(b*x+a)^4-3/8*(b*
x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+45/128*(b
*x+a)^2-3/128*cos(b*x+a)^4-9/128*cos(b*x+a)^2+9/16*(b*x+a)^2*cos(b*x+a)^2-9
/8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+9/32*sin(b*x+a)^2-9/32
*(b*x+a)^4)-4/b^4*a*d^4*(-1/4*(b*x+a)^3*cos(b*x+a)^4+3/4*(b*x+a)^2*(1/4*(co
s(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)*cos(b*x+a)
^4-3/128*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)-45/256*b*x-45/256*a+9/32
*(b*x+a)*cos(b*x+a)^2-9/64*cos(b*x+a)*sin(b*x+a)-3/16*(b*x+a)^3)+4/b^3*c*d^
3*(-1/4*(b*x+a)^3*cos(b*x+a)^4+3/4*(b*x+a)^2*(1/4*(cos(b*x+a)^3+3/2*cos(b*x
+a))*sin(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)*cos(b*x+a)^4-3/128*(cos(b*x+a)^
```

$$3+3/2*\cos(b*x+a))*\sin(b*x+a)-45/256*b*x-45/256*a+9/32*(b*x+a)*\cos(b*x+a)^2-9/64*\cos(b*x+a)*\sin(b*x+a)-3/16*(b*x+a)^3)+6/b^4*a^2*d^4*(-1/4*(b*x+a)^2*\cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(\cos(b*x+a)^3+3/2*\cos(b*x+a))*\sin(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/32*\cos(b*x+a)^4+3/32*\cos(b*x+a)^2)-12/b^3*a*c*d^3*(-1/4*(b*x+a)^2*\cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(\cos(b*x+a)^3+3/2*\cos(b*x+a))*\sin(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/32*\cos(b*x+a)^4+3/32*\cos(b*x+a)^2)+6/b^2*c^2*d^2*(-1/4*(b*x+a)^2*\cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(\cos(b*x+a)^3+3/2*\cos(b*x+a))*\sin(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/32*\cos(b*x+a)^4+3/32*\cos(b*x+a)^2)-4/b^4*a^3*d^4*(-1/4*(b*x+a)*\cos(b*x+a)^4+1/16*(\cos(b*x+a)^3+3/2*\cos(b*x+a))*\sin(b*x+a)+3/32*b*x+3/32*a)+12/b^3*a^2*c*d^3*(-1/4*(b*x+a)*\cos(b*x+a)^4+1/16*(\cos(b*x+a)^3+3/2*\cos(b*x+a))*\sin(b*x+a)+3/32*b*x+3/32*a)-12/b^2*a*c^2*d^2*(-1/4*(b*x+a)*\cos(b*x+a)^4+1/16*(\cos(b*x+a)^3+3/2*\cos(b*x+a))*\sin(b*x+a)+3/32*b*x+3/32*a)+4/b*c^3*d*(-1/4*(b*x+a)*\cos(b*x+a)^4+1/16*(\cos(b*x+a)^3+3/2*\cos(b*x+a))*\sin(b*x+a)+3/32*b*x+3/32*a)-1/4/b^4*a^4*d^4*\cos(b*x+a)^4+1/b^3*a^3*c*d^3*\cos(b*x+a)^4-3/2/b^2*a^2*c^2*d^2*\cos(b*x+a)^4+1/b*a*c^3*d*\cos(b*x+a)^4-1/4*c^4*\cos(b*x+a)^4)$$

Maxima [B] time = 1.394, size = 1305, normalized size = 5.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")

[Out]
$$-1/1024*(256*c^4*\cos(b*x + a)^4 - 1024*a*c^3*d*\cos(b*x + a)^4/b + 1536*a^2*c^2*d^2*\cos(b*x + a)^4/b^2 - 1024*a^3*c*d^3*\cos(b*x + a)^4/b^3 + 256*a^4*d^4*\cos(b*x + a)^4/b^4 + 32*(4*(b*x + a)*\cos(4*b*x + 4*a) + 16*(b*x + a)*\cos(2*b*x + 2*a) - \sin(4*b*x + 4*a) - 8*\sin(2*b*x + 2*a))*c^3*d/b - 96*(4*(b*x + a)*\cos(4*b*x + 4*a) + 16*(b*x + a)*\cos(2*b*x + 2*a) - \sin(4*b*x + 4*a) - 8*\sin(2*b*x + 2*a))*a*c^2*d^2/b^2 + 96*(4*(b*x + a)*\cos(4*b*x + 4*a) + 16*(b*x + a)*\cos(2*b*x + 2*a) - \sin(4*b*x + 4*a) - 8*\sin(2*b*x + 2*a))*a^2*c*d^3/b^3 - 32*(4*(b*x + a)*\cos(4*b*x + 4*a) + 16*(b*x + a)*\cos(2*b*x + 2*a) - \sin(4*b*x + 4*a) - 8*\sin(2*b*x + 2*a))*a^3*d^4/b^4 + 24*((8*(b*x + a)^2 - 1)*\cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*\cos(2*b*x + 2*a) - 4*(b*x + a)*\sin(4*b*x + 4*a) - 32*(b*x + a)*\sin(2*b*x + 2*a))*c^2*d^2/b^2 - 48*((8*(b*x + a)^2 - 1)*\cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*\cos(2*b*x + 2*a) - 4*(b*x + a)*\sin(4*b*x + 4*a) - 32*(b*x + a)*\sin(2*b*x + 2*a))*a*c*d^3/b^3 + 24*((8*(b*x + a)^2 - 1)*\cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*\cos(2*b*x + 2*a) - 4*(b*x + a)*\sin(4*b*x + 4*a) - 32*(b*x + a)*\sin(2*b*x + 2*a))*a^2*d^4/b^4 + 4*(4*(8*(b*x + a)^3 - 3*b*x - 3*a)*\cos(4*b*x + 4*a) + 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*\cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*\sin(4*b*x + 4*a) - 96*(2*(b*x + a)^2 - 1)*\sin(2*b*x + 2*a))*c*d^3/b^3 - 4*(4*(8*(b*x + a)^3 - 3*b*x - 3*a)*\cos(4*b*x + 4*a) + 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*\cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*\sin(4*b*x + 4*a) - 96*(2*(b*x + a)^2 - 1)*\sin(2*b*x + 2*a))*a*d^4/b^4 + ((32*(b*x + a)^4 - 24*(b*x + a)^2 + 3)*\cos(4*b*x + 4*a) + 64*(2*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*\cos(2*b*x + 2*a) - 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*\sin(4*b*x + 4*a) - 128*(2*(b*x + a)^3 - 3*b*x - 3*a)*\sin(2*b*x + 2*a))*d^4/b^4)/b$$

Fricas [A] time = 0.536091, size = 802, normalized size = 3.08

$$12b^4d^4x^4 + 48b^4cd^3x^3 - (32b^4d^4x^4 + 128b^4cd^3x^3 + 32b^4c^4 - 24b^2c^2d^2 + 3d^4 + 24(8b^4c^2d^2 - b^2d^4)x^2 + 16(8b^4c^3d -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")

[Out] $\frac{1}{128}(12b^4d^4x^4 + 48b^4cd^3x^3 - (32b^4d^4x^4 + 128b^4cd^3x^3 + 32b^4c^4 - 24b^2c^2d^2 + 3d^4 + 24(8b^4c^2d^2 - b^2d^4))x^2 + 16(8b^4c^3d - 3b^2cd^3)x)\cos(bx + a)^4 + 9(8b^4c^2d^2 - 5b^2d^4)x^2 + 9(8b^2d^4x^2 + 16b^2cd^3x + 8b^2c^2d^2 - 5d^4)\cos(bx + a)^2 + 6(8b^4c^3d - 15b^2cd^3)x + 2(2(8b^3d^4x^3 + 24b^3cd^3x^2 + 8b^3c^3d - 3b^2cd^3 + 3(8b^3c^2d^2 - b^2d^4))x)\cos(bx + a)^3 + 3(8b^3d^4x^3 + 24b^3cd^3x^2 + 8b^3c^3d - 15b^2cd^3 + 3(8b^3c^2d^2 - 5b^2d^4))x)\cos(bx + a)\sin(bx + a)/b^5$

Sympy [A] time = 21.9182, size = 976, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**4*cos(b*x+a)**3*sin(b*x+a),x)

[Out] Piecewise(((c**4*sin(a + b*x)**4/(4*b) + c**4*sin(a + b*x)**2*cos(a + b*x)**2/(2*b) + 3*c**3*d*x*sin(a + b*x)**4/(8*b) + 3*c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(4*b) - 5*c**3*d*x*cos(a + b*x)**4/(8*b) + 9*c**2*d**2*x**2*sin(a + b*x)**4/(16*b) + 9*c**2*d**2*x**2*cos(a + b*x)**2/(8*b) - 15*c**2*d**2*x**2*cos(a + b*x)**4/(16*b) + 3*c*d**3*x**3*sin(a + b*x)**4/(8*b) + 3*c*d**3*x**3*cos(a + b*x)**2/(4*b) - 5*c*d**3*x**3*cos(a + b*x)**4/(8*b) + 3*d**4*x**4*sin(a + b*x)**4/(32*b) + 3*d**4*x**4*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 5*d**4*x**4*cos(a + b*x)**4/(32*b) + 3*c**3*d*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 5*c**3*d*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 9*c**2*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 15*c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 9*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 3*d**4*x**3*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 5*d**4*x**3*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) - 3*c**2*d**2*sin(a + b*x)**4/(4*b**3) - 15*c**2*d**2*sin(a + b*x)**2*cos(a + b*x)**2/(16*b**3) - 45*c*d**3*x*sin(a + b*x)**4/(64*b**3) - 9*c*d**3*x*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**3) + 51*c*d**3*x*cos(a + b*x)**4/(64*b**3) - 45*d**4*x**2*sin(a + b*x)**4/(128*b**3) - 9*d**4*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(64*b**3) + 51*d**4*x**2*cos(a + b*x)**4/(128*b**3) - 45*c*d**3*sin(a + b*x)**3*cos(a + b*x)/(64*b**4) - 51*c*d**3*sin(a + b*x)*cos(a + b*x)**3/(64*b**4) - 45*d**4*x*sin(a + b*x)**3*cos(a + b*x)/(64*b**4) - 51*d**4*x*sin(a + b*x)*cos(a + b*x)**3/(64*b**4) + 3*d**4*sin(a + b*x)**4/(8*b**5) + 51*d**4*sin(a + b*x)**2*cos(a + b*x)**2/(128*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*cos(a)**3, True))

Giac [A] time = 1.10955, size = 487, normalized size = 1.87

$$\frac{(32b^4d^4x^4 + 128b^4cd^3x^3 + 192b^4c^2d^2x^2 + 128b^4c^3dx + 32b^4c^4 - 24b^2d^4x^2 - 48b^2cd^3x - 24b^2c^2d^2 + 3d^4)\cos(4bx)}{1024b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")

[Out] $-1/1024(32b^4d^4x^4 + 128b^4cd^3x^3 + 192b^4c^2d^2x^2 + 128b^4c^3dx + 32b^4c^4 - 24b^2d^4x^2 - 48b^2cd^3x - 24b^2c^2d^2 +$

$$\begin{aligned}
& 3*d^4)*\cos(4*b*x + 4*a)/b^5 - 1/16*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3*x \\
& - 6*b^2*c^2*d^2 + 3*d^4)*\cos(2*b*x + 2*a)/b^5 + 1/256*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 24*b^3*c^2*d^2*x + 8*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*\sin(\\
& 4*b*x + 4*a)/b^5 + 1/8*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x + 2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*\sin(2*b*x + 2*a)/b^5
\end{aligned}$$

3.138 $\int (c + dx)^3 \cos^3(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=196

$$\frac{3d^2(c + dx) \cos^4(a + bx)}{32b^3} + \frac{9d^2(c + dx) \cos^2(a + bx)}{32b^3} + \frac{3d(c + dx)^2 \sin(a + bx) \cos^3(a + bx)}{16b^2} + \frac{9d(c + dx)^2 \sin(a + bx)}{32b^2}$$

```
[Out] (-45*d^3*x)/(256*b^3) + (3*(c + d*x)^3)/(32*b) + (9*d^2*(c + d*x)*Cos[a + b*x]^2)/(32*b^3) + (3*d^2*(c + d*x)*Cos[a + b*x]^4)/(32*b^3) - ((c + d*x)^3*Cos[a + b*x]^4)/(4*b) - (45*d^3*Cos[a + b*x]*Sin[a + b*x])/(256*b^4) + (9*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(32*b^2) - (3*d^3*Cos[a + b*x]^3*Sin[a + b*x])/(128*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x])/(16*b^2)
```

Rubi [A] time = 0.160841, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4405, 3311, 32, 2635, 8}

$$\frac{3d^2(c + dx) \cos^4(a + bx)}{32b^3} + \frac{9d^2(c + dx) \cos^2(a + bx)}{32b^3} + \frac{3d(c + dx)^2 \sin(a + bx) \cos^3(a + bx)}{16b^2} + \frac{9d(c + dx)^2 \sin(a + bx)}{32b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x], x]
```

```
[Out] (-45*d^3*x)/(256*b^3) + (3*(c + d*x)^3)/(32*b) + (9*d^2*(c + d*x)*Cos[a + b*x]^2)/(32*b^3) + (3*d^2*(c + d*x)*Cos[a + b*x]^4)/(32*b^3) - ((c + d*x)^3*Cos[a + b*x]^4)/(4*b) - (45*d^3*Cos[a + b*x]*Sin[a + b*x])/(256*b^4) + (9*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(32*b^2) - (3*d^3*Cos[a + b*x]^3*Sin[a + b*x])/(128*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x])/(16*b^2)
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(d*m*(c + d*x)^(m - 1)*(b*Ssin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Ssin[e + f*x])^(n - 2), x], x] - Dist[(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Ssin[e + f*x])^n, x], x] - Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Ssin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[(b_.)*sin[(c_.) + (d_.)*(x_)]^(n_.), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c
```

$+ d*x])^{(n - 2), x], x] /; \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 8

$\text{Int}[a_, x_Symbol] \text{ :> } \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rubi steps

$$\begin{aligned} \int (c + dx)^3 \cos^3(a + bx) \sin(a + bx) dx &= -\frac{(c + dx)^3 \cos^4(a + bx)}{4b} + \frac{(3d) \int (c + dx)^2 \cos^4(a + bx) dx}{4b} \\ &= \frac{3d^2(c + dx) \cos^4(a + bx)}{32b^3} - \frac{(c + dx)^3 \cos^4(a + bx)}{4b} + \frac{3d(c + dx)^2 \cos^3(a + bx)}{16b^2} \\ &= \frac{9d^2(c + dx) \cos^2(a + bx)}{32b^3} + \frac{3d^2(c + dx) \cos^4(a + bx)}{32b^3} - \frac{(c + dx)^3 \cos^4(a + bx)}{4b} \\ &= \frac{3(c + dx)^3}{32b} + \frac{9d^2(c + dx) \cos^2(a + bx)}{32b^3} + \frac{3d^2(c + dx) \cos^4(a + bx)}{32b^3} - \frac{(c + dx)^3}{4b} \\ &= -\frac{45d^3x}{256b^3} + \frac{3(c + dx)^3}{32b} + \frac{9d^2(c + dx) \cos^2(a + bx)}{32b^3} + \frac{3d^2(c + dx) \cos^4(a + bx)}{32b^3} \end{aligned}$$

Mathematica [A] time = 1.01191, size = 135, normalized size = 0.69

$$\frac{-64b(c + dx) \cos(2(a + bx)) (2b^2(c + dx)^2 - 3d^2) - 4b(c + dx) \cos(4(a + bx)) (8b^2(c + dx)^2 - 3d^2) + 6d \sin(2(a + bx)) (c + dx)^3}{1024b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x], x]

[Out] (-64*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] - 4*b*(c + d*x)*(-3*d^2 + 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] + 6*d*(16*(-d^2 + 2*b^2*(c + d*x)^2) + (-d^2 + 8*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)]/(1024*b^4)

Maple [B] time = 0.021, size = 594, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a), x)

[Out] 1/b*(1/b^3*d^3*(-1/4*(b*x+a)^3*cos(b*x+a)^4+3/4*(b*x+a)^2*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)*cos(b*x+a)^4-3/128*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)-45/256*b*x-45/256*a+9/32*(b*x+a)*cos(b*x+a)^2-9/64*cos(b*x+a)*sin(b*x+a)-3/16*(b*x+a)^3)-3/b^3*a*d^3*(-1/4*(b*x+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/32*cos(b*x+a)^4+3/32*cos(b*x+a)^2)+3/b^2*c*d^2*(-1/4*(b*x+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/32*cos(b*x+a)^4+3/32*cos(b*x+a)^2)+3/b^3*a^2*d^3*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)-6/b^2*a*c*d^2*(-1/4*(b*x+a)*cos(b

$$\begin{aligned} & *x+a)^4+1/16*(\cos(b*x+a)^3+3/2*\cos(b*x+a))*\sin(b*x+a)+3/32*b*x+3/32*a)+3/b* \\ & c^2*d*(-1/4*(b*x+a)*\cos(b*x+a)^4+1/16*(\cos(b*x+a)^3+3/2*\cos(b*x+a))*\sin(b*x \\ & +a)+3/32*b*x+3/32*a)+1/4/b^3*a^3*d^3*\cos(b*x+a)^4-3/4/b^2*a^2*c*d^2*\cos(b*x \\ & +a)^4+3/4/b*a*c^2*d*\cos(b*x+a)^4-1/4*c^3*\cos(b*x+a)^4 \end{aligned}$$

Maxima [B] time = 1.2684, size = 741, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/1024*(256*c^3*\cos(b*x + a)^4 - 768*a*c^2*d*\cos(b*x + a)^4/b + 768*a^2*c* \\ & d^2*\cos(b*x + a)^4/b^2 - 256*a^3*d^3*\cos(b*x + a)^4/b^3 + 24*(4*(b*x + a)*c \\ & \cos(4*b*x + 4*a) + 16*(b*x + a)*\cos(2*b*x + 2*a) - \sin(4*b*x + 4*a) - 8*\sin(\\ & 2*b*x + 2*a))*c^2*d/b - 48*(4*(b*x + a)*\cos(4*b*x + 4*a) + 16*(b*x + a)*\cos \\ & (2*b*x + 2*a) - \sin(4*b*x + 4*a) - 8*\sin(2*b*x + 2*a))*a*c*d^2/b^2 + 24*(4* \\ & (b*x + a)*\cos(4*b*x + 4*a) + 16*(b*x + a)*\cos(2*b*x + 2*a) - \sin(4*b*x + 4* \\ & a) - 8*\sin(2*b*x + 2*a))*a^2*d^3/b^3 + 12*((8*(b*x + a)^2 - 1)*\cos(4*b*x + \\ & 4*a) + 16*(2*(b*x + a)^2 - 1)*\cos(2*b*x + 2*a) - 4*(b*x + a)*\sin(4*b*x + 4* \\ & a) - 32*(b*x + a)*\sin(2*b*x + 2*a))*c*d^2/b^2 - 12*((8*(b*x + a)^2 - 1)*\cos \\ & (4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*\cos(2*b*x + 2*a) - 4*(b*x + a)*\sin(4 \\ & *b*x + 4*a) - 32*(b*x + a)*\sin(2*b*x + 2*a))*a*d^3/b^3 + (4*(8*(b*x + a)^3 \\ & - 3*b*x - 3*a)*\cos(4*b*x + 4*a) + 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*\cos(2*b* \\ & x + 2*a) - 3*(8*(b*x + a)^2 - 1)*\sin(4*b*x + 4*a) - 96*(2*(b*x + a)^2 - 1)* \\ & \sin(2*b*x + 2*a))*d^3/b^3)/b \end{aligned}$$

Fricas [A] time = 0.506452, size = 518, normalized size = 2.64

$$24b^3d^3x^3 + 72b^3cd^2x^2 - 8(8b^3d^3x^3 + 24b^3cd^2x^2 + 8b^3c^3 - 3bcd^2 + 3(8b^3c^2d - bd^3)x)\cos(bx + a)^4 + 72(bd^3x + b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & 1/256*(24*b^3*d^3*x^3 + 72*b^3*c*d^2*x^2 - 8*(8*b^3*d^3*x^3 + 24*b^3*c*d^2* \\ & x^2 + 8*b^3*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*\cos(b*x + a)^4 + 7 \\ & 2*(b*d^3*x + b*c*d^2)*\cos(b*x + a)^2 + 9*(8*b^3*c^2*d - 5*b*d^3)*x + 3*(2*(\\ & 8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*\cos(b*x + a)^3 + 3*(8*b \\ & ^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - 5*d^3)*\cos(b*x + a))*\sin(b*x + \\ & a))/b^4 \end{aligned}$$

Sympy [A] time = 11.6901, size = 634, normalized size = 3.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*cos(b*x+a)**3*sin(b*x+a),x)

```
[Out] Piecewise((c**3*sin(a + b*x)**4/(4*b) + c**3*sin(a + b*x)**2*cos(a + b*x)**2/(2*b) + 9*c**2*d*x*sin(a + b*x)**4/(32*b) + 9*c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 15*c**2*d*x*cos(a + b*x)**4/(32*b) + 9*c*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 15*c*d**2*x**2*cos(a + b*x)**4/(32*b) + 3*d**3*x**3*sin(a + b*x)**4/(32*b) + 3*d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 5*d**3*x**3*cos(a + b*x)**4/(32*b) + 9*c**2*d*sin(a + b*x)**3*cos(a + b*x)/(32*b**2) + 15*c**2*d*sin(a + b*x)*cos(a + b*x)**3/(32*b**2) + 9*c*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 15*c*d**2*x*sin(a + b*x)*cos(a + b*x)**3/(16*b**2) + 9*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)/(32*b**2) + 15*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(32*b**2) - 3*c*d**2*sin(a + b*x)**4/(8*b**3) - 15*c*d**2*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**3) - 45*d**3*x*sin(a + b*x)**4/(256*b**3) - 9*d**3*x*sin(a + b*x)**2*cos(a + b*x)**2/(128*b**3) + 51*d**3*x*cos(a + b*x)**4/(256*b**3) - 45*d**3*sin(a + b*x)**3*cos(a + b*x)/(256*b**4) - 51*d**3*sin(a + b*x)*cos(a + b*x)**3/(256*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)*cos(a)**3, True))
```

Giac [A] time = 1.14536, size = 325, normalized size = 1.66

$$\frac{(8b^3d^3x^3 + 24b^3cd^2x^2 + 24b^3c^2dx + 8b^3c^3 - 3bd^3x - 3bcd^2)\cos(4bx + 4a)}{256b^4} - \frac{(2b^3d^3x^3 + 6b^3cd^2x^2 + 6b^3c^2dx + 2b^3c^3 - 3bd^3x - 3bcd^2)\cos(2bx + 2a)}{16b^4} + \frac{3(8b^2d^3x^2 + 16b^2c^2d^2x + 8b^2c^2d - d^3)\sin(4bx + 4a)}{1024b^4} + \frac{3(2b^2d^3x^2 + 4b^2c^2d^2x + 2b^2c^2d - d^3)\sin(2bx + 2a)}{32b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(4*b*x + 4*a)/b^4 - 1/16*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(2*b*x + 2*a)/b^4 + 3/1024*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*sin(4*b*x + 4*a)/b^4 + 3/32*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*sin(2*b*x + 2*a)/b^4
```

3.139 $\int (c + dx)^2 \cos^3(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=134

$$\frac{d(c + dx) \sin(a + bx) \cos^3(a + bx)}{8b^2} + \frac{3d(c + dx) \sin(a + bx) \cos(a + bx)}{16b^2} + \frac{d^2 \cos^4(a + bx)}{32b^3} + \frac{3d^2 \cos^2(a + bx)}{32b^3} - \frac{(c + dx)^2 \cos^4(a + bx)}{4b} + \frac{3d(c + dx) \cos^3(a + bx) \sin(a + bx)}{8b^2}$$

[Out] (3*c*d*x)/(16*b) + (3*d^2*x^2)/(32*b) + (3*d^2*Cos[a + b*x]^2)/(32*b^3) + (d^2*Cos[a + b*x]^4)/(32*b^3) - ((c + d*x)^2*Cos[a + b*x]^4)/(4*b) + (3*d*(c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(16*b^2) + (d*(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x])/(8*b^2)

Rubi [A] time = 0.086674, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4405, 3310}

$$\frac{d(c + dx) \sin(a + bx) \cos^3(a + bx)}{8b^2} + \frac{3d(c + dx) \sin(a + bx) \cos(a + bx)}{16b^2} + \frac{d^2 \cos^4(a + bx)}{32b^3} + \frac{3d^2 \cos^2(a + bx)}{32b^3} - \frac{(c + dx)^2 \cos^4(a + bx)}{4b} + \frac{3d(c + dx) \cos^3(a + bx) \sin(a + bx)}{8b^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x],x]

[Out] (3*c*d*x)/(16*b) + (3*d^2*x^2)/(32*b) + (3*d^2*Cos[a + b*x]^2)/(32*b^3) + (d^2*Cos[a + b*x]^4)/(32*b^3) - ((c + d*x)^2*Cos[a + b*x]^4)/(4*b) + (3*d*(c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(16*b^2) + (d*(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x])/(8*b^2)

Rule 4405

Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 3310

Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \cos^3(a + bx) \sin(a + bx) dx &= -\frac{(c + dx)^2 \cos^4(a + bx)}{4b} + \frac{d \int (c + dx) \cos^4(a + bx) dx}{2b} \\ &= \frac{d^2 \cos^4(a + bx)}{32b^3} - \frac{(c + dx)^2 \cos^4(a + bx)}{4b} + \frac{d(c + dx) \cos^3(a + bx) \sin(a + bx)}{8b^2} \\ &= \frac{3d^2 \cos^2(a + bx)}{32b^3} + \frac{d^2 \cos^4(a + bx)}{32b^3} - \frac{(c + dx)^2 \cos^4(a + bx)}{4b} + \frac{3d(c + dx) \cos^3(a + bx) \sin(a + bx)}{8b^2} \\ &= \frac{3cdx}{16b} + \frac{3d^2x^2}{32b} + \frac{3d^2 \cos^2(a + bx)}{32b^3} + \frac{d^2 \cos^4(a + bx)}{32b^3} - \frac{(c + dx)^2 \cos^4(a + bx)}{4b} \end{aligned}$$

Mathematica [A] time = 0.486177, size = 89, normalized size = 0.66

$$\frac{-16 \cos(2(a + bx)) (2b^2(c + dx)^2 - d^2) + \cos(4(a + bx)) (d^2 - 8b^2(c + dx)^2) + 4bd(c + dx)(8 \sin(2(a + bx)) + \sin(4(a + bx)))}{256b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x], x]

[Out] (-16*(-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + (d^2 - 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] + 4*b*d*(c + d*x)*(8*Sin[2*(a + b*x)] + Sin[4*(a + b*x)])/(256*b^3)

Maple [B] time = 0.019, size = 260, normalized size = 1.9

$$\frac{1}{b} \left(\frac{d^2}{b^2} \left(-\frac{(bx+a)^2 (\cos(bx+a))^4}{4} + \frac{bx+a}{2} \left(\frac{\sin(bx+a)}{4} \left((\cos(bx+a))^3 + \frac{3 \cos(bx+a)}{2} \right) + \frac{3bx}{8} + \frac{3a}{8} \right) - \frac{3(bx+a)^2}{32} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a), x)

[Out] 1/b*(1/b^2*d^2*(-1/4*(b*x+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/32*cos(b*x+a)^4+3/32*cos(b*x+a)^2)-2/b^2*a*d^2*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)+2/b*c*d*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)-1/4*d^2/2/b^2*a^2*cos(b*x+a)^4+1/2*c*d/b*a*cos(b*x+a)^4-1/4*c^2*cos(b*x+a)^4)

Maxima [B] time = 1.15485, size = 355, normalized size = 2.65

$$\frac{64c^2 \cos(bx+a)^4 - \frac{128acd \cos(bx+a)^4}{b} + \frac{64a^2d^2 \cos(bx+a)^4}{b^2} + \frac{4(4(bx+a) \cos(4bx+4a) + 16(bx+a) \cos(2bx+2a) - \sin(4bx+4a) - 8 \sin(2bx+2a))c}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a), x, algorithm="maxima")

[Out] -1/256*(64*c^2*cos(b*x + a)^4 - 128*a*c*d*cos(b*x + a)^4/b + 64*a^2*d^2*cos(b*x + a)^4/b^2 + 4*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*c*d/b - 4*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*a*d^2/b^2 + ((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*d^2/b^2)/b

Fricas [A] time = 0.49275, size = 294, normalized size = 2.19

$$\frac{3b^2d^2x^2 + 6b^2cdx - (8b^2d^2x^2 + 16b^2cdx + 8b^2c^2 - d^2) \cos(bx+a)^4 + 3d^2 \cos(bx+a)^2 + 2(2(bd^2x + bcd) \cos(bx+a))}{32b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")

[Out] $\frac{1}{32}*(3*b^2*d^2*x^2 + 6*b^2*c*d*x - (8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*\cos(b*x + a)^4 + 3*d^2*\cos(b*x + a)^2 + 2*(2*(b*d^2*x + b*c*d)*\cos(b*x + a)^3 + 3*(b*d^2*x + b*c*d)*\cos(b*x + a))*\sin(b*x + a))/b^3$

Sympy [A] time = 13.5359, size = 350, normalized size = 2.61

$$\left\{ \begin{array}{l} \frac{c^2 \sin^4(a+bx)}{4b} + \frac{c^2 \sin^2(a+bx) \cos^2(a+bx)}{2b} + \frac{3cdx \sin^4(a+bx)}{16b} + \frac{3cdx \sin^2(a+bx) \cos^2(a+bx)}{8b} - \frac{5cdx \cos^4(a+bx)}{16b} + \frac{3d^2x^2 \sin^4(a+bx)}{32b} + \frac{3d^2x^2 \sin^2(a+bx) \cos^2(a+bx)}{16b} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3} \right) \sin(a) \cos^3(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*cos(b*x+a)**3*sin(b*x+a),x)

[Out] Piecewise((c**2*sin(a + b*x)**4/(4*b) + c**2*sin(a + b*x)**2*cos(a + b*x)**2/(2*b) + 3*c*d*x*sin(a + b*x)**4/(16*b) + 3*c*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(8*b) - 5*c*d*x*cos(a + b*x)**4/(16*b) + 3*d**2*x**2*sin(a + b*x)**4/(32*b) + 3*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 5*d**2*x**2*cos(a + b*x)**4/(32*b) + 3*c*d*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 5*c*d*sin(a + b*x)*cos(a + b*x)**3/(16*b**2) + 3*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 5*d**2*x*sin(a + b*x)*cos(a + b*x)**3/(16*b**2) - d**2*sin(a + b*x)**4/(8*b**3) - 5*d**2*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)*cos(a)**3, True))

Giac [A] time = 1.10493, size = 196, normalized size = 1.46

$$\frac{(8b^2d^2x^2 + 16b^2cdx + 8b^2c^2 - d^2)\cos(4bx + 4a)}{256b^3} - \frac{(2b^2d^2x^2 + 4b^2cdx + 2b^2c^2 - d^2)\cos(2bx + 2a)}{16b^3} + \frac{(bd^2x + b^2c^2 - d^2)\cos(bx + a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")

[Out] $-\frac{1}{256}*(8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*\cos(4*b*x + 4*a)/b^3 - \frac{1}{16}*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*\cos(2*b*x + 2*a)/b^3 + \frac{1}{64}*(b*d^2*x + b*c*d)*\sin(4*b*x + 4*a)/b^3 + \frac{1}{8}*(b*d^2*x + b*c*d)*\sin(2*b*x + 2*a)/b^3$

3.140 $\int (c + dx) \cos^3(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=72

$$\frac{d \sin(a + bx) \cos^3(a + bx)}{16b^2} + \frac{3d \sin(a + bx) \cos(a + bx)}{32b^2} - \frac{(c + dx) \cos^4(a + bx)}{4b} + \frac{3dx}{32b}$$

[Out] (3*d*x)/(32*b) - ((c + d*x)*Cos[a + b*x]^4)/(4*b) + (3*d*Cos[a + b*x]*Sin[a + b*x])/(32*b^2) + (d*Cos[a + b*x]^3*Sin[a + b*x])/(16*b^2)

Rubi [A] time = 0.0470773, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {4405, 2635, 8}

$$\frac{d \sin(a + bx) \cos^3(a + bx)}{16b^2} + \frac{3d \sin(a + bx) \cos(a + bx)}{32b^2} - \frac{(c + dx) \cos^4(a + bx)}{4b} + \frac{3dx}{32b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x],x]

[Out] (3*d*x)/(32*b) - ((c + d*x)*Cos[a + b*x]^4)/(4*b) + (3*d*Cos[a + b*x]*Sin[a + b*x])/(32*b^2) + (d*Cos[a + b*x]^3*Sin[a + b*x])/(16*b^2)

Rule 4405

Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int (c + dx) \cos^3(a + bx) \sin(a + bx) dx &= -\frac{(c + dx) \cos^4(a + bx)}{4b} + \frac{d \int \cos^4(a + bx) dx}{4b} \\ &= -\frac{(c + dx) \cos^4(a + bx)}{4b} + \frac{d \cos^3(a + bx) \sin(a + bx)}{16b^2} + \frac{(3d) \int \cos^2(a + bx) dx}{16b} \\ &= -\frac{(c + dx) \cos^4(a + bx)}{4b} + \frac{3d \cos(a + bx) \sin(a + bx)}{32b^2} + \frac{d \cos^3(a + bx) \sin(a + bx)}{16b^2} \\ &= \frac{3dx}{32b} - \frac{(c + dx) \cos^4(a + bx)}{4b} + \frac{3d \cos(a + bx) \sin(a + bx)}{32b^2} + \frac{d \cos^3(a + bx) \sin(a + bx)}{16b^2} \end{aligned}$$

Mathematica [A] time = 0.151772, size = 75, normalized size = 1.04

$$\frac{d(\sin(2(a+bx)) - 2bx \cos(2(a+bx)))}{16b^2} + \frac{d(\sin(4(a+bx)) - 4bx \cos(4(a+bx)))}{128b^2} - \frac{c \cos^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x], x]

[Out] -(c*cos[a + b*x]^4)/(4*b) + (d*(-2*b*x*cos[2*(a + b*x)] + Sin[2*(a + b*x)]))/(16*b^2) + (d*(-4*b*x*cos[4*(a + b*x)] + Sin[4*(a + b*x)]))/(128*b^2)

Maple [A] time = 0.02, size = 85, normalized size = 1.2

$$\frac{1}{b} \left(\frac{d}{b} \left(-\frac{(bx+a) \cos(bx+a)^4}{4} + \frac{\sin(bx+a)}{16} \left((\cos(bx+a))^3 + \frac{3 \cos(bx+a)}{2} \right) + \frac{3bx}{32} + \frac{3a}{32} \right) + \frac{ad \cos(bx+a)^4}{4b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*cos(b*x+a)^3*sin(b*x+a), x)

[Out] 1/b*(d/b*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)+1/4/b*d*a*cos(b*x+a)^4-1/4*cos(b*x+a)^4*c)

Maxima [A] time = 1.11205, size = 124, normalized size = 1.72

$$\frac{32 c \cos(bx+a)^4 - \frac{32 ad \cos(bx+a)^4}{b} + \frac{(4(bx+a) \cos(4bx+4a)+16(bx+a) \cos(2bx+2a)-\sin(4bx+4a)-8 \sin(2bx+2a))d}{b}}{128 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a), x, algorithm="maxima")

[Out] -1/128*(32*c*cos(b*x + a)^4 - 32*a*d*cos(b*x + a)^4/b + (4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*d/b)/b

Fricas [A] time = 0.48835, size = 147, normalized size = 2.04

$$\frac{8(bdx+bc) \cos(bx+a)^4 - 3bdx - (2d \cos(bx+a)^3 + 3d \cos(bx+a)) \sin(bx+a)}{32b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a), x, algorithm="fricas")

[Out] -1/32*(8*(b*d*x + b*c)*cos(b*x + a)^4 - 3*b*d*x - (2*d*cos(b*x + a)^3 + 3*d*cos(b*x + a))*sin(b*x + a))/b^2

Sympy [A] time = 2.81213, size = 160, normalized size = 2.22

$$\left\{ \begin{array}{l} \frac{c \sin^4(a+bx)}{4b} + \frac{c \sin^2(a+bx) \cos^2(a+bx)}{2b} + \frac{3dx \sin^4(a+bx)}{32b} + \frac{3dx \sin^2(a+bx) \cos^2(a+bx)}{16b} - \frac{5dx \cos^4(a+bx)}{32b} + \frac{3d \sin^3(a+bx) \cos(a+bx)}{32b^2} + \frac{5d \sin(a+bx) \cos^3(a+bx)}{32b^2} \\ \left(cx + \frac{dx^2}{2} \right) \sin(a) \cos^3(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)**3*sin(b*x+a),x)

[Out] Piecewise((c*sin(a + b*x)**4/(4*b) + c*sin(a + b*x)**2*cos(a + b*x)**2/(2*b) + 3*d*x*sin(a + b*x)**4/(32*b) + 3*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 5*d*x*cos(a + b*x)**4/(32*b) + 3*d*sin(a + b*x)**3*cos(a + b*x)/(32*b**2) + 5*d*sin(a + b*x)*cos(a + b*x)**3/(32*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*sin(a)*cos(a)**3, True))

Giac [A] time = 1.11548, size = 101, normalized size = 1.4

$$-\frac{(bdx + bc) \cos(4bx + 4a)}{32b^2} - \frac{(bdx + bc) \cos(2bx + 2a)}{8b^2} + \frac{d \sin(4bx + 4a)}{128b^2} + \frac{d \sin(2bx + 2a)}{16b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")

[Out] -1/32*(b*d*x + b*c)*cos(4*b*x + 4*a)/b^2 - 1/8*(b*d*x + b*c)*cos(2*b*x + 2*a)/b^2 + 1/128*d*sin(4*b*x + 4*a)/b^2 + 1/16*d*sin(2*b*x + 2*a)/b^2

$$3.141 \quad \int \frac{\cos^3(a+bx) \sin(a+bx)}{c+dx} dx$$

Optimal. Leaf size=129

$$\frac{\sin\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{8d} + \frac{\sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{4d} + \frac{\cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{4d} +$$

[Out] (CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(8*d) + (CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(4*d) + (Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(4*d) + (Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(8*d)

Rubi [A] time = 0.211813, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4406, 3303, 3299, 3302}

$$\frac{\sin\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{8d} + \frac{\sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{4d} + \frac{\cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{4d} +$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x),x]

[Out] (CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(8*d) + (CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(4*d) + (Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(4*d) + (Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(8*d)

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx} dx &= \int \left(\frac{\sin(2a+2bx)}{4(c+dx)} + \frac{\sin(4a+4bx)}{8(c+dx)} \right) dx \\
&= \frac{1}{8} \int \frac{\sin(4a+4bx)}{c+dx} dx + \frac{1}{4} \int \frac{\sin(2a+2bx)}{c+dx} dx \\
&= \frac{1}{8} \cos\left(4a - \frac{4bc}{d}\right) \int \frac{\sin\left(\frac{4bc}{d} + 4bx\right)}{c+dx} dx + \frac{1}{4} \cos\left(2a - \frac{2bc}{d}\right) \int \frac{\sin\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx + \frac{1}{8} \sin\left(4a - \frac{4bc}{d}\right) \frac{c+dx}{d} \\
&= \frac{\text{Ci}\left(\frac{4bc}{d} + 4bx\right) \sin\left(4a - \frac{4bc}{d}\right)}{8d} + \frac{\text{Ci}\left(\frac{2bc}{d} + 2bx\right) \sin\left(2a - \frac{2bc}{d}\right)}{4d} + \frac{\cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{4d} + \frac{\cos\left(4a - \frac{4bc}{d}\right) (c+dx)}{8d}
\end{aligned}$$

Mathematica [A] time = 0.344294, size = 110, normalized size = 0.85

$$\frac{\sin\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right) + 2 \sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) + 2 \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2b(c+dx)}{d}\right) + \cos\left(4a - \frac{4bc}{d}\right) (c+dx)}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x), x]

[Out] (CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + 2*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + 2*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(8*d)

Maple [A] time = 0.021, size = 178, normalized size = 1.4

$$\frac{1}{b} \left(\frac{b}{8} \left(2 \frac{1}{d} \text{Si} \left(2bx + 2a + 2 \frac{-ad+bc}{d} \right) \cos \left(2 \frac{-ad+bc}{d} \right) - 2 \frac{1}{d} \text{Ci} \left(2bx + 2a + 2 \frac{-ad+bc}{d} \right) \sin \left(2 \frac{-ad+bc}{d} \right) \right) + \frac{b}{32} \left(4 \frac{1}{d} \text{Si} \left(2bx + 2a + 2 \frac{-ad+bc}{d} \right) \cos \left(2 \frac{-ad+bc}{d} \right) - 2 \frac{1}{d} \text{Ci} \left(2bx + 2a + 2 \frac{-ad+bc}{d} \right) \sin \left(2 \frac{-ad+bc}{d} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c), x)

[Out] 1/b*(1/8*b*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+1/32*b*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d))

Maxima [C] time = 1.4869, size = 370, normalized size = 2.87

$$\frac{b \left(2i E_1 \left(\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) - 2i E_1 \left(-\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) \right) \cos \left(-\frac{2(bc-ad)}{d} \right) + b \left(i E_1 \left(\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) - i E_1 \left(-\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) \right) \cos \left(-\frac{2(bc-ad)}{d} \right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c), x, algorithm="maxima")

[Out] -1/16*(b*(2*I*exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) - 2*I*exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c - a*d)/d) + b*(I*exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) - I*exp_integral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-2*(b*c - a*d)/d)

d) - I*exp_integral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*(b*c - a*d)/d) + 2*b*(exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) + exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2*(b*c - a*d)/d) + b*(exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) + exp_integral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*sin(-4*(b*c - a*d)/d))/(b*d)

Fricas [A] time = 0.481013, size = 421, normalized size = 3.26

$$\frac{2 \left(\operatorname{Ci} \left(\frac{2(bdx+bc)}{d} \right) + \operatorname{Ci} \left(-\frac{2(bdx+bc)}{d} \right) \right) \sin \left(-\frac{2(bc-ad)}{d} \right) + \left(\operatorname{Ci} \left(\frac{4(bdx+bc)}{d} \right) + \operatorname{Ci} \left(-\frac{4(bdx+bc)}{d} \right) \right) \sin \left(-\frac{4(bc-ad)}{d} \right) + 2 \cos \left(-\frac{4(bc-ad)}{d} \right)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x, algorithm="fricas")

[Out] 1/16*(2*(cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d) + (cos_integral(4*(b*d*x + b*c)/d) + cos_integral(-4*(b*d*x + b*c)/d))*sin(-4*(b*c - a*d)/d) + 2*cos(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + 4*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d))/d

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a + bx) \cos^3(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c),x)

[Out] Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x), x)

Giac [C] time = 1.75453, size = 8162, normalized size = 63.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x, algorithm="giac")

[Out] 1/16*(imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 2*real_

$$\begin{aligned}
& \text{part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)^2*\tan(a)^2*\tan(2*b*c/d)*\tan(b \\
& *c/d)^2 - 4*\text{real_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(2*a)^2*\tan(a)*\tan(\\
& 2*b*c/d)^2*\tan(b*c/d)^2 - 4*\text{real_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(2 \\
& *a)^2*\tan(a)*\tan(2*b*c/d)^2*\tan(b*c/d)^2 - 2*\text{real_part}(\cos_integral(4*b*x + \\
& 4*b*c/d))*\tan(2*a)*\tan(a)^2*\tan(2*b*c/d)^2*\tan(b*c/d)^2 - 2*\text{real_part}(\cos_ \\
& integral(-4*b*x - 4*b*c/d))*\tan(2*a)*\tan(a)^2*\tan(2*b*c/d)^2*\tan(b*c/d)^2 + \\
& \text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a)^2*\tan(a)^2*\tan(2*b*c/d)^ \\
& 2 - 2*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(2*a)^2*\tan(a)^2*\tan(2*b* \\
& c/d)^2 + 2*\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(2*a)^2*\tan(a)^2*ta \\
& n(2*b*c/d)^2 - \text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)^2*\tan(a)^ \\
& 2*\tan(2*b*c/d)^2 + 2*\text{sin_integral}(4*(b*d*x + b*c)/d)*\tan(2*a)^2*\tan(a)^2*ta \\
& n(2*b*c/d)^2 - 4*\text{sin_integral}(2*(b*d*x + b*c)/d)*\tan(2*a)^2*\tan(a)^2*\tan(2* \\
& b*c/d)^2 + 8*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(2*a)^2*\tan(a)*\tan \\
& (2*b*c/d)^2*\tan(b*c/d) - 8*\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(2* \\
& a)^2*\tan(a)*\tan(2*b*c/d)^2*\tan(b*c/d) + 16*\text{sin_integral}(2*(b*d*x + b*c)/d)* \\
& \tan(2*a)^2*\tan(a)*\tan(2*b*c/d)^2*\tan(b*c/d) - \text{imag_part}(\cos_integral(4*b*x \\
& + 4*b*c/d))*\tan(2*a)^2*\tan(a)^2*\tan(b*c/d)^2 + 2*\text{imag_part}(\cos_integral(2*b \\
& *x + 2*b*c/d))*\tan(2*a)^2*\tan(a)^2*\tan(b*c/d)^2 - 2*\text{imag_part}(\cos_integral(\\
& -2*b*x - 2*b*c/d))*\tan(2*a)^2*\tan(a)^2*\tan(b*c/d)^2 + \text{imag_part}(\cos_integra \\
& l(-4*b*x - 4*b*c/d))*\tan(2*a)^2*\tan(a)^2*\tan(b*c/d)^2 - 2*\text{sin_integral}(4*(b \\
& *d*x + b*c)/d)*\tan(2*a)^2*\tan(a)^2*\tan(b*c/d)^2 + 4*\text{sin_integral}(2*(b*d*x + \\
& b*c)/d)*\tan(2*a)^2*\tan(a)^2*\tan(b*c/d)^2 + 4*\text{imag_part}(\cos_integral(4*b*x \\
& + 4*b*c/d))*\tan(2*a)*\tan(a)^2*\tan(2*b*c/d)*\tan(b*c/d)^2 - 4*\text{imag_part}(\cos_i \\
& ntegral(-4*b*x - 4*b*c/d))*\tan(2*a)*\tan(a)^2*\tan(2*b*c/d)*\tan(b*c/d)^2 + 8* \\
& \text{sin_integral}(4*(b*d*x + b*c)/d)*\tan(2*a)*\tan(a)^2*\tan(2*b*c/d)*\tan(b*c/d)^2 \\
& + \text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a)^2*\tan(2*b*c/d)^2*\tan(b \\
& *c/d)^2 - 2*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(2*a)^2*\tan(2*b*c/d \\
&)^2*\tan(b*c/d)^2 + 2*\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(2*a)^2*t \\
& an(2*b*c/d)^2*\tan(b*c/d)^2 - \text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(\\
& 2*a)^2*\tan(2*b*c/d)^2*\tan(b*c/d)^2 + 2*\text{sin_integral}(4*(b*d*x + b*c)/d)*\tan(\\
& 2*a)^2*\tan(2*b*c/d)^2*\tan(b*c/d)^2 - 4*\text{sin_integral}(2*(b*d*x + b*c)/d)*\tan(\\
& 2*a)^2*\tan(2*b*c/d)^2*\tan(b*c/d)^2 - \text{imag_part}(\cos_integral(4*b*x + 4*b*c/d \\
&))*\tan(a)^2*\tan(2*b*c/d)^2*\tan(b*c/d)^2 + 2*\text{imag_part}(\cos_integral(2*b*x + \\
& 2*b*c/d))*\tan(a)^2*\tan(2*b*c/d)^2*\tan(b*c/d)^2 - 2*\text{imag_part}(\cos_integral(- \\
& 2*b*x - 2*b*c/d))*\tan(a)^2*\tan(2*b*c/d)^2*\tan(b*c/d)^2 + \text{imag_part}(\cos_inte \\
& gral(-4*b*x - 4*b*c/d))*\tan(a)^2*\tan(2*b*c/d)^2*\tan(b*c/d)^2 - 2*\text{sin_integr \\
& al}(4*(b*d*x + b*c)/d)*\tan(a)^2*\tan(2*b*c/d)^2*\tan(b*c/d)^2 + 4*\text{sin_integral} \\
& (2*(b*d*x + b*c)/d)*\tan(a)^2*\tan(2*b*c/d)^2*\tan(b*c/d)^2 + 2*\text{real_part}(\cos_ \\
& integral(4*b*x + 4*b*c/d))*\tan(2*a)^2*\tan(a)^2*\tan(2*b*c/d) + 2*\text{real_part}(c \\
& os_integral(-4*b*x - 4*b*c/d))*\tan(2*a)^2*\tan(a)^2*\tan(2*b*c/d) + 4*\text{real_pa} \\
& rt(\cos_integral(2*b*x + 2*b*c/d))*\tan(2*a)^2*\tan(a)*\tan(2*b*c/d)^2 + 4*\text{real} \\
& _part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(2*a)^2*\tan(a)*\tan(2*b*c/d)^2 - 2* \\
& \text{real_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a)*\tan(a)^2*\tan(2*b*c/d)^2 - \\
& 2*\text{real_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)*\tan(a)^2*\tan(2*b*c/d) \\
& ^2 + 4*\text{real_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(2*a)^2*\tan(a)^2*\tan(b*c \\
& /d) + 4*\text{real_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(2*a)^2*\tan(a)^2*\tan(b \\
& *c/d) - 4*\text{real_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(2*a)^2*\tan(2*b*c/d)^ \\
& 2*\tan(b*c/d) - 4*\text{real_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(2*a)^2*\tan(2 \\
& *b*c/d)^2*\tan(b*c/d) + 4*\text{real_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(a)^2* \\
& \tan(2*b*c/d)^2*\tan(b*c/d) + 4*\text{real_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan \\
& (a)^2*\tan(2*b*c/d)^2*\tan(b*c/d) - 4*\text{real_part}(\cos_integral(2*b*x + 2*b*c/d \\
&))*\tan(2*a)^2*\tan(a)*\tan(b*c/d)^2 - 4*\text{real_part}(\cos_integral(-2*b*x - 2*b*c/d \\
&))*\tan(2*a)^2*\tan(a)*\tan(b*c/d)^2 + 2*\text{real_part}(\cos_integral(4*b*x + 4*b*c \\
& /d))*\tan(2*a)*\tan(a)^2*\tan(b*c/d)^2 + 2*\text{real_part}(\cos_integral(-4*b*x - 4*b \\
& *c/d))*\tan(2*a)*\tan(a)^2*\tan(b*c/d)^2 + 2*\text{real_part}(\cos_integral(4*b*x + 4* \\
& b*c/d))*\tan(2*a)^2*\tan(2*b*c/d)*\tan(b*c/d)^2 + 2*\text{real_part}(\cos_integral(-4* \\
& b*x - 4*b*c/d))*\tan(2*a)^2*\tan(2*b*c/d)*\tan(b*c/d)^2 - 2*\text{real_part}(\cos_inte \\
& gral(4*b*x + 4*b*c/d))*\tan(a)^2*\tan(2*b*c/d)*\tan(b*c/d)^2 - 2*\text{real_part}(\cos \\
& _integral(-4*b*x - 4*b*c/d))*\tan(a)^2*\tan(2*b*c/d)*\tan(b*c/d)^2 - 2*\text{real_pa}
\end{aligned}$$

$$\begin{aligned}
& \text{rt}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a)*\tan(2*b*c/d)^2*\tan(b*c/d)^2 - 2* \\
& \text{real_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)*\tan(2*b*c/d)^2*\tan(b*c/d) \\
&)^2 - 4*\text{real_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(a)*\tan(2*b*c/d)^2*\tan(\\
& b*c/d)^2 - 4*\text{real_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(a)*\tan(2*b*c/d)^ \\
& 2*\tan(b*c/d)^2 - \text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a)^2*\tan(a) \\
& ^2 - 2*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(2*a)^2*\tan(a)^2 + 2*\text{ima} \\
& \text{g_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(2*a)^2*\tan(a)^2 + \text{imag_part}(\cos_ \\
& \text{integral}(-4*b*x - 4*b*c/d))*\tan(2*a)^2*\tan(a)^2 - 2*\text{sin_integral}(4*(b*d*x + \\
& b*c)/d)*\tan(2*a)^2*\tan(a)^2 - 4*\text{sin_integral}(2*(b*d*x + b*c)/d)*\tan(2*a)^2 \\
& *\tan(a)^2 + 4*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a)*\tan(a)^2*\text{ta} \\
& \text{n}(2*b*c/d) - 4*\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)*\tan(a)^2* \\
& \text{tan}(2*b*c/d) + 8*\text{sin_integral}(4*(b*d*x + b*c)/d)*\tan(2*a)*\tan(a)^2*\text{tan}(2*b* \\
& c/d) + \text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a)^2*\tan(2*b*c/d)^2 + \\
& 2*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(2*a)^2*\tan(2*b*c/d)^2 - 2*i \\
& \text{mag_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(2*a)^2*\tan(2*b*c/d)^2 - \text{imag_p} \\
& \text{art}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)^2*\tan(2*b*c/d)^2 + 2*\text{sin_integ} \\
& \text{ral}(4*(b*d*x + b*c)/d)*\tan(2*a)^2*\tan(2*b*c/d)^2 + 4*\text{sin_integral}(2*(b*d*x \\
& + b*c)/d)*\tan(2*a)^2*\tan(2*b*c/d)^2 - \text{imag_part}(\cos_integral(4*b*x + 4*b*c/ \\
& d))*\tan(a)^2*\tan(2*b*c/d)^2 - 2*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d))*\text{ta} \\
& \text{n}(a)^2*\tan(2*b*c/d)^2 + 2*\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(a)^ \\
& 2*\tan(2*b*c/d)^2 + \text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(a)^2*\tan(2 \\
& *b*c/d)^2 - 2*\text{sin_integral}(4*(b*d*x + b*c)/d)*\tan(a)^2*\tan(2*b*c/d)^2 - 4*s \\
& \text{in_integral}(2*(b*d*x + b*c)/d)*\tan(a)^2*\tan(2*b*c/d)^2 + 8*\text{imag_part}(\cos_in \\
& \text{tegral}(2*b*x + 2*b*c/d))*\tan(2*a)^2*\tan(a)*\tan(b*c/d) - 8*\text{imag_part}(\cos_int \\
& \text{egral}(-2*b*x - 2*b*c/d))*\tan(2*a)^2*\tan(a)*\tan(b*c/d) + 16*\text{sin_integral}(2*(\\
& b*d*x + b*c)/d)*\tan(2*a)^2*\tan(a)*\tan(b*c/d) + 8*\text{imag_part}(\cos_integral(2*b \\
& *x + 2*b*c/d))*\tan(a)*\tan(2*b*c/d)^2*\tan(b*c/d) - 8*\text{imag_part}(\cos_integral(\\
& -2*b*x - 2*b*c/d))*\tan(a)*\tan(2*b*c/d)^2*\tan(b*c/d) + 16*\text{sin_integral}(2*(b* \\
& d*x + b*c)/d)*\tan(a)*\tan(2*b*c/d)^2*\tan(b*c/d) - \text{imag_part}(\cos_integral(4*b \\
& *x + 4*b*c/d))*\tan(2*a)^2*\tan(b*c/d)^2 - 2*\text{imag_part}(\cos_integral(2*b*x + 2 \\
& *b*c/d))*\tan(2*a)^2*\tan(b*c/d)^2 + 2*\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/ \\
& d))*\tan(2*a)^2*\tan(b*c/d)^2 + \text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan \\
& (2*a)^2*\tan(b*c/d)^2 - 2*\text{sin_integral}(4*(b*d*x + b*c)/d)*\tan(2*a)^2*\tan(b*c \\
& /d)^2 - 4*\text{sin_integral}(2*(b*d*x + b*c)/d)*\tan(2*a)^2*\tan(b*c/d)^2 + \text{imag_pa} \\
& \text{rt}(\cos_integral(4*b*x + 4*b*c/d))*\tan(a)^2*\tan(b*c/d)^2 + 2*\text{imag_part}(\cos_i \\
& \text{ntegral}(2*b*x + 2*b*c/d))*\tan(a)^2*\tan(b*c/d)^2 - 2*\text{imag_part}(\cos_integral(\\
& -2*b*x - 2*b*c/d))*\tan(a)^2*\tan(b*c/d)^2 - \text{imag_part}(\cos_integral(-4*b*x - \\
& 4*b*c/d))*\tan(a)^2*\tan(b*c/d)^2 + 2*\text{sin_integral}(4*(b*d*x + b*c)/d)*\tan(a)^ \\
& 2*\tan(b*c/d)^2 + 4*\text{sin_integral}(2*(b*d*x + b*c)/d)*\tan(a)^2*\tan(b*c/d)^2 + \\
& 4*\text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a)*\tan(2*b*c/d)*\tan(b*c/d) \\
& ^2 - 4*\text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)*\tan(2*b*c/d)*\tan(\\
& b*c/d)^2 + 8*\text{sin_integral}(4*(b*d*x + b*c)/d)*\tan(2*a)*\tan(2*b*c/d)*\tan(b*c/ \\
& d)^2 - \text{imag_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*b*c/d)^2*\tan(b*c/d)^2 \\
& - 2*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(2*b*c/d)^2*\tan(b*c/d)^2 + \\
& 2*\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(2*b*c/d)^2*\tan(b*c/d)^2 + \\
& \text{imag_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*b*c/d)^2*\tan(b*c/d)^2 - 2*s \\
& \text{in_integral}(4*(b*d*x + b*c)/d)*\tan(2*b*c/d)^2*\tan(b*c/d)^2 - 4*\text{sin_integral} \\
& (2*(b*d*x + b*c)/d)*\tan(2*b*c/d)^2*\tan(b*c/d)^2 + 4*\text{real_part}(\cos_integral(\\
& 2*b*x + 2*b*c/d))*\tan(2*a)^2*\tan(a) + 4*\text{real_part}(\cos_integral(-2*b*x - 2*b \\
& *c/d))*\tan(2*a)^2*\tan(a) + 2*\text{real_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2 \\
& *a)*\tan(a)^2 + 2*\text{real_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)*\tan(a)^ \\
& 2 + 2*\text{real_part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(2*a)^2*\tan(2*b*c/d) + 2* \\
& \text{real_part}(\cos_integral(-4*b*x - 4*b*c/d))*\tan(2*a)^2*\tan(2*b*c/d) - 2*\text{real_} \\
& \text{part}(\cos_integral(4*b*x + 4*b*c/d))*\tan(a)^2*\tan(2*b*c/d) - 2*\text{real_part}(\cos \\
& _integral(-4*b*x - 4*b*c/d))*\tan(a)^2*\tan(2*b*c/d) - 2*\text{real_part}(\cos_integr \\
& al(4*b*x + 4*b*c/d))*\tan(2*a)*\tan(2*b*c/d)^2 - 2*\text{real_part}(\cos_integral(-4* \\
& b*x - 4*b*c/d))*\tan(2*a)*\tan(2*b*c/d)^2 + 4*\text{real_part}(\cos_integral(2*b*x + \\
& 2*b*c/d))*\tan(a)*\tan(2*b*c/d)^2 + 4*\text{real_part}(\cos_integral(-2*b*x - 2*b*c/d) \\
&))*\tan(a)*\tan(2*b*c/d)^2 - 4*\text{real_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(2
\end{aligned}$$

$$\begin{aligned}
& *a)^2 \tan(b*c/d) - 4*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(2*a)^2 * \tan(b*c/d) + 4*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(a)^2 * \tan(b*c/d) \\
& + 4*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(a)^2 * \tan(b*c/d) - 4*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(2*b*c/d)^2 * \tan(b*c/d) - 4*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(2*b*c/d)^2 * \tan(b*c/d) + 2*\text{real_part}(\text{cos_integral}(4*b*x + 4*b*c/d))*\tan(2*a) * \tan(b*c/d)^2 + 2*\text{real_part}(\text{cos_integral}(-4*b*x - 4*b*c/d))*\tan(2*a) * \tan(b*c/d)^2 - 4*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(a) * \tan(b*c/d)^2 - 4*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(a) * \tan(b*c/d)^2 - 2*\text{real_part}(\text{cos_integral}(4*b*x + 4*b*c/d))*\tan(2*b*c/d) * \tan(b*c/d)^2 - 2*\text{real_part}(\text{cos_integral}(-4*b*x - 4*b*c/d))*\tan(2*b*c/d) * \tan(b*c/d)^2 - \text{imag_part}(\text{cos_integral}(4*b*x + 4*b*c/d))*\tan(2*a)^2 + 2 * \text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(2*a)^2 - 2*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(2*a)^2 + \text{imag_part}(\text{cos_integral}(-4*b*x - 4*b*c/d))*\tan(2*a)^2 - 2*\text{sin_integral}(4*(b*d*x + b*c)/d)*\tan(2*a)^2 + 4*\text{sin_integral}(2*(b*d*x + b*c)/d)*\tan(2*a)^2 + \text{imag_part}(\text{cos_integral}(4*b*x + 4*b*c/d))*\tan(a)^2 - 2*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(a)^2 + 2*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(a)^2 - \text{imag_part}(\text{cos_integral}(-4*b*x - 4*b*c/d))*\tan(a)^2 + 2*\text{sin_integral}(4*(b*d*x + b*c)/d)*\tan(a)^2 - 4*\text{sin_integral}(2*(b*d*x + b*c)/d)*\tan(a)^2 + 4*\text{imag_part}(\text{cos_integral}(4*b*x + 4*b*c/d))*\tan(2*a) * \tan(2*b*c/d) - 4*\text{imag_part}(\text{cos_integral}(-4*b*x - 4*b*c/d))*\tan(2*a) * \tan(2*b*c/d) + 8*\text{sin_integral}(4*(b*d*x + b*c)/d)*\tan(2*a) * \tan(2*b*c/d) - \text{imag_part}(\text{cos_integral}(4*b*x + 4*b*c/d))*\tan(2*b*c/d)^2 + 2*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(2*b*c/d)^2 - 2*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(2*b*c/d)^2 + \text{imag_part}(\text{cos_integral}(-4*b*x - 4*b*c/d))*\tan(2*b*c/d)^2 - 2*\text{sin_integral}(4*(b*d*x + b*c)/d)*\tan(2*b*c/d)^2 + 4*\text{sin_integral}(2*(b*d*x + b*c)/d)*\tan(2*b*c/d)^2 + 8*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(a) * \tan(b*c/d) - 8*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(a) * \tan(b*c/d) + 16*\text{sin_integral}(2*(b*d*x + b*c)/d)*\tan(a) * \tan(b*c/d) + \text{imag_part}(\text{cos_integral}(4*b*x + 4*b*c/d))*\tan(b*c/d)^2 - 2*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*c/d)^2 + 2*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*c/d)^2 - \text{imag_part}(\text{cos_integral}(-4*b*x - 4*b*c/d))*\tan(b*c/d)^2 + 2*\text{sin_integral}(4*(b*d*x + b*c)/d)*\tan(b*c/d)^2 - 4*\text{sin_integral}(2*(b*d*x + b*c)/d)*\tan(b*c/d)^2 + 2*\text{real_part}(\text{cos_integral}(4*b*x + 4*b*c/d))*\tan(2*a) + 2*\text{real_part}(\text{cos_integral}(-4*b*x - 4*b*c/d))*\tan(2*a) + 4*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(a) + 4*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(a) - 2*\text{real_part}(\text{cos_integral}(4*b*x + 4*b*c/d))*\tan(2*b*c/d) - 2*\text{real_part}(\text{cos_integral}(-4*b*x - 4*b*c/d))*\tan(2*b*c/d) - 4*\text{real_part}(\text{cos_integral}(2*b*x + 2*b*c/d))*\tan(b*c/d) - 4*\text{real_part}(\text{cos_integral}(-2*b*x - 2*b*c/d))*\tan(b*c/d) + \text{imag_part}(\text{cos_integral}(4*b*x + 4*b*c/d)) + 2*\text{imag_part}(\text{cos_integral}(2*b*x + 2*b*c/d)) - 2*\text{imag_part}(\text{cos_integral}(-2*b*x - 2*b*c/d)) - \text{imag_part}(\text{cos_integral}(-4*b*x - 4*b*c/d)) + 2*\text{sin_integral}(4*(b*d*x + b*c)/d) + 4*\text{sin_integral}(2*(b*d*x + b*c)/d))/(d*\tan(2*a)^2 * \tan(a)^2 * \tan(2*b*c/d)^2 * \tan(b*c/d)^2 + d*\tan(2*a)^2 * \tan(a)^2 * \tan(2*b*c/d)^2 + d*\tan(2*a)^2 * \tan(a)^2 * \tan(b*c/d)^2 + d*\tan(2*a)^2 * \tan(2*b*c/d)^2 * \tan(b*c/d)^2 + d*\tan(a)^2 * \tan(2*b*c/d)^2 * \tan(b*c/d)^2 + d*\tan(2*a)^2 * \tan(a)^2 + d*\tan(2*a)^2 * \tan(2*b*c/d)^2 + d*\tan(a)^2 * \tan(2*b*c/d)^2 + d*\tan(2*a)^2 * \tan(b*c/d)^2 + d*\tan(a)^2 * \tan(b*c/d)^2 + d*\tan(2*b*c/d)^2 * \tan(b*c/d)^2 + d*\tan(2*a)^2 + d*\tan(a)^2 + d*\tan(2*b*c/d)^2 + d*\tan(b*c/d)^2 + d
\end{aligned}$$

$$3.142 \quad \int \frac{\cos^3(a+bx) \sin(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=179

$$\frac{b \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{2d^2} + \frac{b \cos\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{2d^2} - \frac{b \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{2d^2}$$

[Out] (b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) + (b*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(2*d^2) - Sin[2*a + 2*b*x]/(4*d*(c + d*x)) - Sin[4*a + 4*b*x]/(8*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) - (b*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(2*d^2)

Rubi [A] time = 0.268476, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{2d^2} + \frac{b \cos\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{2d^2} - \frac{b \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{2d^2}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^2, x]

[Out] (b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) + (b*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(2*d^2) - Sin[2*a + 2*b*x]/(4*d*(c + d*x)) - Sin[4*a + 4*b*x]/(8*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) - (b*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(2*d^2)

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n * Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cos^3(a + bx) \sin(a + bx)}{(c + dx)^2} dx &= \int \left(\frac{\sin(2a + 2bx)}{4(c + dx)^2} + \frac{\sin(4a + 4bx)}{8(c + dx)^2} \right) dx \\ &= \frac{1}{8} \int \frac{\sin(4a + 4bx)}{(c + dx)^2} dx + \frac{1}{4} \int \frac{\sin(2a + 2bx)}{(c + dx)^2} dx \\ &= -\frac{\sin(2a + 2bx)}{4d(c + dx)} - \frac{\sin(4a + 4bx)}{8d(c + dx)} + \frac{b \int \frac{\cos(2a + 2bx)}{c + dx} dx}{2d} + \frac{b \int \frac{\cos(4a + 4bx)}{c + dx} dx}{2d} \\ &= -\frac{\sin(2a + 2bx)}{4d(c + dx)} - \frac{\sin(4a + 4bx)}{8d(c + dx)} + \frac{\left(b \cos\left(4a - \frac{4bc}{d}\right) \int \frac{\cos\left(\frac{4bc}{d} + 4bx\right)}{c + dx} dx \right)}{2d} + \frac{\left(b \cos\left(2a - \frac{2bc}{d}\right) \int \frac{\cos\left(\frac{2bc}{d} + 2bx\right)}{c + dx} dx \right)}{2d} \\ &= \frac{b \cos\left(2a - \frac{2bc}{d}\right) \text{Ci}\left(\frac{2bc}{d} + 2bx\right)}{2d^2} + \frac{b \cos\left(4a - \frac{4bc}{d}\right) \text{Ci}\left(\frac{4bc}{d} + 4bx\right)}{2d^2} - \frac{\sin(2a + 2bx)}{4d(c + dx)} - \frac{\sin(4a + 4bx)}{8d(c + dx)} \end{aligned}$$

Mathematica [A] time = 1.67309, size = 151, normalized size = 0.84

$$\frac{-4b \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) - 4b \cos\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right) + 4b \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2b(c+dx)}{d}\right) + 4b \sin\left(4a - \frac{4bc}{d}\right) \text{Si}\left(\frac{4b(c+dx)}{d}\right)}{8d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^2,x]

[Out] $-\frac{(-4*b*\text{Cos}[2*a - (2*b*c)/d]*\text{CosIntegral}[(2*b*(c + d*x))/d] - 4*b*\text{Cos}[4*a - (4*b*c)/d]*\text{CosIntegral}[(4*b*(c + d*x))/d] + (2*d*\text{Sin}[2*(a + b*x)])/(c + d*x) + (d*\text{Sin}[4*(a + b*x)])/(c + d*x) + 4*b*\text{Sin}[2*a - (2*b*c)/d]*\text{SinIntegral}[(2*b*(c + d*x))/d] + 4*b*\text{Sin}[4*a - (4*b*c)/d]*\text{SinIntegral}[(4*b*(c + d*x))/d])/(8*d^2)}$

Maple [A] time = 0.023, size = 256, normalized size = 1.4

$$\frac{1}{b} \left(\frac{b^2}{8} \left(-2 \frac{\sin(2bx + 2a)}{((bx + a)d - ad + bc)d} + 2 \frac{1}{d} \left(2 \frac{1}{d} \text{Si} \left(2bx + 2a + 2 \frac{-ad + bc}{d} \right) \sin \left(2 \frac{-ad + bc}{d} \right) + 2 \frac{1}{d} \text{Ci} \left(2bx + 2a + 2 \frac{-ad + bc}{d} \right) \cos \left(2 \frac{-ad + bc}{d} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x)

[Out] $\frac{1}{b} \left(\frac{1}{8} b^2 \left(-2 \sin(2bx + 2a) / ((bx + a)d - ad + bc) / d + 2 \left(2 \text{Si}(2bx + 2a + 2 \frac{-ad + bc}{d}) \sin(2 \frac{-ad + bc}{d}) / d + 2 \text{Ci}(2bx + 2a + 2 \frac{-ad + bc}{d}) \cos(2 \frac{-ad + bc}{d}) / d \right) \right) + \frac{1}{32} b^2 \left(-4 \sin(4bx + 4a) / ((bx + a)d - ad + bc) / d + 4 \left(4 \text{Si}(4bx + 4a + 4 \frac{-ad + bc}{d}) \sin(4 \frac{-ad + bc}{d}) / d + 4 \text{Ci}(4bx + 4a + 4 \frac{-ad + bc}{d}) \cos(4 \frac{-ad + bc}{d}) / d \right) \right) \right)$

Maxima [C] time = 1.74053, size = 406, normalized size = 2.27

$$\frac{b^2 \left(2i E_2 \left(\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) - 2i E_2 \left(\frac{-2i bc + 2i (bx+a)d - 2i ad}{d} \right) \right) \cos \left(-\frac{2(bc-ad)}{d} \right) + b^2 \left(i E_2 \left(\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) - i E_2 \left(\frac{-4i bc + 4i (bx+a)d - 4i ad}{d} \right) \right) \sin \left(-\frac{2(bc-ad)}{d} \right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")

[Out] -1/16*(b^2*(2*I*exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) - 2*I*exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c - a*d)/d) + b^2*(I*exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) - I*exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*(b*c - a*d)/d) + 2*b^2*(exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) + exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2*(b*c - a*d)/d) + b^2*(exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) + exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*sin(-4*(b*c - a*d)/d)/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)

Fricas [A] time = 0.557949, size = 597, normalized size = 3.34

$$\frac{4d \cos(bx + a)^3 \sin(bx + a) + 2(bdx + bc) \sin\left(-\frac{4(bc-ad)}{d}\right) \operatorname{Si}\left(\frac{4(bdx+bc)}{d}\right) + 2(bdx + bc) \sin\left(-\frac{2(bc-ad)}{d}\right) \operatorname{Si}\left(\frac{2(bdx+bc)}{d}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")

[Out] -1/4*(4*d*cos(b*x + a)^3*sin(b*x + a) + 2*(b*d*x + b*c)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + 2*(b*d*x + b*c)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - ((b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d) - ((b*d*x + b*c)*cos_integral(4*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-4*(b*d*x + b*c)/d))*cos(-4*(b*c - a*d)/d))/(d^3*x + c*d^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a + bx) \cos^3(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c)**2,x)

[Out] Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx + a)^3 \sin(bx + a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3*sin(b*x + a)/(d*x + c)^2, x)
```

3.143 $\int \frac{\cos^3(a+bx) \sin(a+bx)}{(c+dx)^3} dx$

Optimal. Leaf size=231

$$\frac{b^2 \sin\left(4a - \frac{4bc}{d}\right) \operatorname{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{d^3} - \frac{b^2 \sin\left(2a - \frac{2bc}{d}\right) \operatorname{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{2d^3} - \frac{b^2 \cos\left(2a - \frac{2bc}{d}\right) \operatorname{Si}\left(\frac{2bc}{d}\right)}{2d^3}$$

[Out] $-(b \operatorname{Cos}[2a + 2bx]) / (4d^2(c + dx)) - (b \operatorname{Cos}[4a + 4bx]) / (4d^2(c + dx)) - (b^2 \operatorname{CosIntegral}[(4bc)/d + 4bx] \operatorname{Sin}[4a - (4bc)/d]) / d^3 - (b^2 \operatorname{CosIntegral}[(2bc)/d + 2bx] \operatorname{Sin}[2a - (2bc)/d]) / (2d^3) - \operatorname{Sin}[2a + 2bx] / (8d(c + dx)^2) - \operatorname{Sin}[4a + 4bx] / (16d(c + dx)^2) - (b^2 \operatorname{Cos}[2a - (2bc)/d] \operatorname{SinIntegral}[(2bc)/d + 2bx]) / (2d^3) - (b^2 \operatorname{Cos}[4a - (4bc)/d] \operatorname{SinIntegral}[(4bc)/d + 4bx]) / d^3$

Rubi [A] time = 0.32919, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b^2 \sin\left(4a - \frac{4bc}{d}\right) \operatorname{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{d^3} - \frac{b^2 \sin\left(2a - \frac{2bc}{d}\right) \operatorname{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{2d^3} - \frac{b^2 \cos\left(2a - \frac{2bc}{d}\right) \operatorname{Si}\left(\frac{2bc}{d}\right)}{2d^3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Cos}[a + bx])^3 \operatorname{Sin}[a + bx] / (c + dx)^3, x]$

[Out] $-(b \operatorname{Cos}[2a + 2bx]) / (4d^2(c + dx)) - (b \operatorname{Cos}[4a + 4bx]) / (4d^2(c + dx)) - (b^2 \operatorname{CosIntegral}[(4bc)/d + 4bx] \operatorname{Sin}[4a - (4bc)/d]) / d^3 - (b^2 \operatorname{CosIntegral}[(2bc)/d + 2bx] \operatorname{Sin}[2a - (2bc)/d]) / (2d^3) - \operatorname{Sin}[2a + 2bx] / (8d(c + dx)^2) - \operatorname{Sin}[4a + 4bx] / (16d(c + dx)^2) - (b^2 \operatorname{Cos}[2a - (2bc)/d] \operatorname{SinIntegral}[(2bc)/d + 2bx]) / (2d^3) - (b^2 \operatorname{Cos}[4a - (4bc)/d] \operatorname{SinIntegral}[(4bc)/d + 4bx]) / d^3$

Rule 4406

$\operatorname{Int}[\operatorname{Cos}[(a_.) + (b_.)(x_.)]^{(p_.)} ((c_.) + (d_.)(x_.))^{(m_.)} \operatorname{Sin}[(a_.) + (b_.)(x_.)]^{(n_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandTrigReduce}[(c + dx)^m, \operatorname{Sin}[a + bx]]^n \operatorname{Cos}[a + bx]^p, x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3297

$\operatorname{Int}(((c_.) + (d_.)(x_.))^{(m_.)} \operatorname{sin}[(e_.) + (f_.)(x_.)], x_Symbol] \rightarrow \operatorname{Simp}[(c + dx)^{(m+1)} \operatorname{Sin}[e + fx] / (d(m+1)), x] - \operatorname{Dist}[f / (d(m+1)), \operatorname{Int}[(c + dx)^{(m+1)} \operatorname{Cos}[e + fx], x], x] /;$ FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

$\operatorname{Int}[\operatorname{sin}[(e_.) + (f_.)(x_.)] / ((c_.) + (d_.)(x_.)), x_Symbol] \rightarrow \operatorname{Dist}[\operatorname{Cos}[(d e - c f) / d], \operatorname{Int}[\operatorname{Sin}[(c f) / d + f x] / (c + dx), x], x] + \operatorname{Dist}[\operatorname{Sin}[(d e - c f) / d], \operatorname{Int}[\operatorname{Cos}[(c f) / d + f x] / (c + dx), x], x] /;$ FreeQ[{c, d, e, f}, x] && NeQ[d e - c f, 0]

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3} dx &= \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^3} + \frac{\sin(4a+4bx)}{8(c+dx)^3} \right) dx \\
&= \frac{1}{8} \int \frac{\sin(4a+4bx)}{(c+dx)^3} dx + \frac{1}{4} \int \frac{\sin(2a+2bx)}{(c+dx)^3} dx \\
&= -\frac{\sin(2a+2bx)}{8d(c+dx)^2} - \frac{\sin(4a+4bx)}{16d(c+dx)^2} + \frac{b \int \frac{\cos(2a+2bx)}{(c+dx)^2} dx}{4d} + \frac{b \int \frac{\cos(4a+4bx)}{(c+dx)^2} dx}{4d} \\
&= -\frac{b \cos(2a+2bx)}{4d^2(c+dx)} - \frac{b \cos(4a+4bx)}{4d^2(c+dx)} - \frac{\sin(2a+2bx)}{8d(c+dx)^2} - \frac{\sin(4a+4bx)}{16d(c+dx)^2} - \frac{b^2 \int \frac{\sin(2a+2bx)}{c+dx}}{2d^2} \\
&= -\frac{b \cos(2a+2bx)}{4d^2(c+dx)} - \frac{b \cos(4a+4bx)}{4d^2(c+dx)} - \frac{\sin(2a+2bx)}{8d(c+dx)^2} - \frac{\sin(4a+4bx)}{16d(c+dx)^2} - \frac{b^2 \cos\left(4a - \frac{4bc}{d}\right)}{2d^2} \\
&= -\frac{b \cos(2a+2bx)}{4d^2(c+dx)} - \frac{b \cos(4a+4bx)}{4d^2(c+dx)} - \frac{b^2 \operatorname{Ci}\left(\frac{4bc}{d} + 4bx\right) \sin\left(4a - \frac{4bc}{d}\right)}{d^3} - \frac{b^2 \operatorname{Ci}\left(\frac{2bc}{d} + 2bx\right)}{2d^2}
\end{aligned}$$

Mathematica [A] time = 3.79172, size = 197, normalized size = 0.85

$$\frac{16b^2 \sin\left(4a - \frac{4bc}{d}\right) \operatorname{CosIntegral}\left(\frac{4b(c+dx)}{d}\right) + 8b^2 \sin\left(2a - \frac{2bc}{d}\right) \operatorname{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) + 8b^2 \cos\left(2a - \frac{2bc}{d}\right) \operatorname{Si}\left(\frac{2b(c+dx)}{d}\right)}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^3, x]
```

```
[Out] -(16*b^2*CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + 8*b^2*CosInte
gral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (2*d*(2*b*(c + d*x)*Cos[2*(
a + b*x)] + d*Sin[2*(a + b*x)]))/(c + d*x)^2 + (d*(4*b*(c + d*x)*Cos[4*(a +
b*x)] + d*Sin[4*(a + b*x)]))/(c + d*x)^2 + 8*b^2*Cos[2*a - (2*b*c)/d]*SinI
ntegral[(2*b*(c + d*x))/d] + 16*b^2*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(
c + d*x))/d])/(16*d^3)
```

Maple [A] time = 0.023, size = 329, normalized size = 1.4

$$\frac{1}{b} \left(\frac{b^3}{8} \left(-\frac{\sin(2bx+2a)}{((bx+a)d-ad+bc)^2 d} + \frac{1}{d} \left(-2 \frac{\cos(2bx+2a)}{((bx+a)d-ad+bc)d} - 2 \frac{1}{d} \operatorname{Si}\left(2bx+2a+2\frac{-ad+bc}{d}\right) \cos\left(2\frac{-ad+bc}{d}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3, x)
```

[Out] $1/b*(1/8*b^3*(-\sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)^2/d+(-2*\cos(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*\cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*\sin(2*(-a*d+b*c)/d)/d)/d)+1/32*b^3*(-2*\sin(4*b*x+4*a)/((b*x+a)*d-a*d+b*c)^2/d+2*(-4*\cos(4*b*x+4*a)/((b*x+a)*d-a*d+b*c)/d-4*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*\cos(4*(-a*d+b*c)/d)/d-4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*\sin(4*(-a*d+b*c)/d)/d)/d)$

Maxima [C] time = 2.2536, size = 454, normalized size = 1.97

$$\frac{b^3 \left(2i E_3 \left(\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) - 2i E_3 \left(-\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) \right) \cos \left(-\frac{2(bc-ad)}{d} \right) + b^3 \left(i E_3 \left(\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) - i E_3 \left(-\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) \right) \sin \left(-\frac{4(bc-ad)}{d} \right)}{16 \left((b^2 c^2 d - 2 a b c d^2 + (b x + a)^2 d^3 + a^2 d^3 + 2 (b c d^2 - a d^3) (b x + a) \right) b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")`

[Out] $-1/16*(b^3*(2*I*\exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) - 2*I*\exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*\cos(-2*(b*c - a*d)/d) + b^3*(I*\exp_integral_e(3, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) - I*\exp_integral_e(3, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*\cos(-4*(b*c - a*d)/d) + 2*b^3*(\exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) + \exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*\sin(-2*(b*c - a*d)/d) + b^3*(\exp_integral_e(3, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) + \exp_integral_e(3, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*\sin(-4*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)$

Fricas [A] time = 0.624906, size = 918, normalized size = 3.97

$$2 d^2 \cos(bx + a)^3 \sin(bx + a) + 8 (bd^2x + bcd) \cos(bx + a)^4 - 6 (bd^2x + bcd) \cos(bx + a)^2 + 4 (b^2d^2x^2 + 2b^2cdx + b^2c^2) \cos(-4(b*c - a*d)/d) \sin_integral(4*(b*d*x + b*c)/d) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(-2*(b*c - a*d)/d)*\sin_integral(2*(b*d*x + b*c)/d) + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos_integral(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos_integral(-2*(b*d*x + b*c)/d))*\sin(-2*(b*c - a*d)/d) + 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos_integral(4*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos_integral(-4*(b*d*x + b*c)/d))*\sin(-4*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")`

[Out] $-1/4*(2*d^2*\cos(b*x + a)^3*\sin(b*x + a) + 8*(b*d^2*x + b*c*d)*\cos(b*x + a)^4 - 6*(b*d^2*x + b*c*d)*\cos(b*x + a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(-4*(b*c - a*d)/d)*\sin_integral(4*(b*d*x + b*c)/d) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(-2*(b*c - a*d)/d)*\sin_integral(2*(b*d*x + b*c)/d) + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos_integral(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos_integral(-2*(b*d*x + b*c)/d))*\sin(-2*(b*c - a*d)/d) + 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos_integral(4*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos_integral(-4*(b*d*x + b*c)/d))*\sin(-4*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a + bx) \cos^3(a + bx)}{(c + dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.144 \quad \int \frac{\cos^3(a+bx) \sin(a+bx)}{(c+dx)^4} dx$$

Optimal. Leaf size=287

$$\frac{b^3 \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} - \frac{4b^3 \cos\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{3d^4} + \frac{b^3 \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d}\right)}{3d^4}$$

[Out] $-(b \cos[2a + 2bx]) / (12d^2(c + dx)^2) - (b \cos[4a + 4bx]) / (12d^2(c + dx)^2) - (b^3 \cos[2a - (2bc)/d] \text{CosIntegral}[(2bc)/d + 2bx]) / (3d^4) - (4b^3 \cos[4a - (4bc)/d] \text{CosIntegral}[(4bc)/d + 4bx]) / (3d^4) - \text{Sin}[2a + 2bx] / (12d(c + dx)^3) + (b^2 \text{Sin}[2a + 2bx]) / (6d^3(c + dx)) - \text{Sin}[4a + 4bx] / (24d(c + dx)^3) + (b^2 \text{Sin}[4a + 4bx]) / (3d^3(c + dx)) + (b^3 \text{Sin}[2a - (2bc)/d] \text{SinIntegral}[(2bc)/d + 2bx]) / (3d^4) + (4b^3 \text{Sin}[4a - (4bc)/d] \text{SinIntegral}[(4bc)/d + 4bx]) / (3d^4)$

Rubi [A] time = 0.451177, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b^3 \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} - \frac{4b^3 \cos\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4bc}{d} + 4bx\right)}{3d^4} + \frac{b^3 \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d}\right)}{3d^4}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^4,x]

[Out] $-(b \cos[2a + 2bx]) / (12d^2(c + dx)^2) - (b \cos[4a + 4bx]) / (12d^2(c + dx)^2) - (b^3 \cos[2a - (2bc)/d] \text{CosIntegral}[(2bc)/d + 2bx]) / (3d^4) - (4b^3 \cos[4a - (4bc)/d] \text{CosIntegral}[(4bc)/d + 4bx]) / (3d^4) - \text{Sin}[2a + 2bx] / (12d(c + dx)^3) + (b^2 \text{Sin}[2a + 2bx]) / (6d^3(c + dx)) - \text{Sin}[4a + 4bx] / (24d(c + dx)^3) + (b^2 \text{Sin}[4a + 4bx]) / (3d^3(c + dx)) + (b^3 \text{Sin}[2a - (2bc)/d] \text{SinIntegral}[(2bc)/d + 2bx]) / (3d^4) + (4b^3 \text{Sin}[4a - (4bc)/d] \text{SinIntegral}[(4bc)/d + 4bx]) / (3d^4)$

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x]) / (d*(m + 1)), x] - Dist[f / (d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)] / ((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f) / d], Int[Sin[(c*f) / d + f*x] / (c + d*x), x], x] + Dist[Sin[(d*e - c*f) / d], Int[Cos[(c*f) / d + f*x] / (c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\cos^3(a + bx) \sin(a + bx)}{(c + dx)^4} dx &= \int \left(\frac{\sin(2a + 2bx)}{4(c + dx)^4} + \frac{\sin(4a + 4bx)}{8(c + dx)^4} \right) dx \\
 &= \frac{1}{8} \int \frac{\sin(4a + 4bx)}{(c + dx)^4} dx + \frac{1}{4} \int \frac{\sin(2a + 2bx)}{(c + dx)^4} dx \\
 &= -\frac{\sin(2a + 2bx)}{12d(c + dx)^3} - \frac{\sin(4a + 4bx)}{24d(c + dx)^3} + \frac{b \int \frac{\cos(2a+2bx)}{(c+dx)^3} dx}{6d} + \frac{b \int \frac{\cos(4a+4bx)}{(c+dx)^3} dx}{6d} \\
 &= -\frac{b \cos(2a + 2bx)}{12d^2(c + dx)^2} - \frac{b \cos(4a + 4bx)}{12d^2(c + dx)^2} - \frac{\sin(2a + 2bx)}{12d(c + dx)^3} - \frac{\sin(4a + 4bx)}{24d(c + dx)^3} - \frac{b^2 \int \frac{\sin(2a+2bx)}{(c+dx)^2} dx}{6d^2} \\
 &= -\frac{b \cos(2a + 2bx)}{12d^2(c + dx)^2} - \frac{b \cos(4a + 4bx)}{12d^2(c + dx)^2} - \frac{\sin(2a + 2bx)}{12d(c + dx)^3} + \frac{b^2 \sin(2a + 2bx)}{6d^3(c + dx)} - \frac{\sin(4a + 4bx)}{24d(c + dx)^3} \\
 &= -\frac{b \cos(2a + 2bx)}{12d^2(c + dx)^2} - \frac{b \cos(4a + 4bx)}{12d^2(c + dx)^2} - \frac{\sin(2a + 2bx)}{12d(c + dx)^3} + \frac{b^2 \sin(2a + 2bx)}{6d^3(c + dx)} - \frac{\sin(4a + 4bx)}{24d(c + dx)^3} \\
 &= -\frac{b \cos(2a + 2bx)}{12d^2(c + dx)^2} - \frac{b \cos(4a + 4bx)}{12d^2(c + dx)^2} - \frac{b^3 \cos\left(2a - \frac{2bc}{d}\right) \text{Ci}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} - \frac{4b^3 \cos\left(4a - \frac{4bc}{d}\right) \text{Si}\left(\frac{4bc}{d} + 4bx\right)}{3d^4}
 \end{aligned}$$

Mathematica [A] time = 2.54981, size = 316, normalized size = 1.1

$$8b^3(c + dx)^3 \left(\cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) - \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2b(c+dx)}{d}\right) \right) + 32b^3(c + dx)^3 \left(\cos\left(4a - \frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right) - \sin\left(4a - \frac{4bc}{d}\right) \text{Si}\left(\frac{4b(c+dx)}{d}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^4, x]

[Out] $-(2*d*\text{Cos}[2*b*x]*(b*d*(c + d*x)*\text{Cos}[2*a] + (d^2 - 2*b^2*(c + d*x)^2)*\text{Sin}[2*a]) + d*\text{Cos}[4*b*x]*(2*b*d*(c + d*x)*\text{Cos}[4*a] + (d^2 - 8*b^2*(c + d*x)^2)*\text{Sin}[4*a]) - 2*d*((-d^2 + 2*b^2*(c + d*x)^2)*\text{Cos}[2*a] + b*d*(c + d*x)*\text{Sin}[2*a])*\text{Sin}[2*b*x] - d*((-d^2 + 8*b^2*(c + d*x)^2)*\text{Cos}[4*a] + 2*b*d*(c + d*x)*\text{Sin}[4*a])*\text{Sin}[4*b*x] + 8*b^3*(c + d*x)^3*(\text{Cos}[2*a - (2*b*c)/d]*\text{CosIntegral}[(2*b*(c + d*x))/d] - \text{Sin}[2*a - (2*b*c)/d]*\text{SinIntegral}[(2*b*(c + d*x))/d]) + 32*b^3*(c + d*x)^3*(\text{Cos}[4*a - (4*b*c)/d]*\text{CosIntegral}[(4*b*(c + d*x))/d] - \text{Sin}[4*a - (4*b*c)/d]*\text{SinIntegral}[(4*b*(c + d*x))/d]))/(24*d^4*(c + d*x)^3)$

Maple [A] time = 0.024, size = 404, normalized size = 1.4

$$\frac{1}{b} \left(\frac{b^4}{8} \left(-\frac{2 \sin(2bx + 2a)}{3((bx + a)d - ad + bc)^3 d} + \frac{2}{3d} \left(-\frac{\cos(2bx + 2a)}{((bx + a)d - ad + bc)^2 d} - \frac{1}{d} \left(-2 \frac{\sin(2bx + 2a)}{((bx + a)d - ad + bc)d} + 2 \frac{1}{d} \text{Si}\left(2bx + \frac{2a}{d}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x)`

[Out] $\frac{1}{b} \left(\frac{1}{8} b^4 \left(-\frac{2}{3} \sin(2bx+2a) \right) / \left((bx+a)d-ad+bc \right)^3 / d + \frac{2}{3} \left(-\cos(2bx+2a) \right) / \left((bx+a)d-ad+bc \right)^2 / d - \left(-2 \sin(2bx+2a) \right) / \left((bx+a)d-ad+bc \right) / d + 2 \left(2 \operatorname{Si}(2bx+2a+2(-ad+bc)/d) \right) \sin(2(-ad+bc)/d) / d + 2 \operatorname{Ci}(2bx+2a+2(-ad+bc)/d) \cos(2(-ad+bc)/d) / d \right) / d + \frac{1}{32} b^4 \left(-\frac{4}{3} \sin(4bx+4a) \right) / \left((bx+a)d-ad+bc \right)^3 / d + \frac{4}{3} \left(-2 \cos(4bx+4a) \right) / \left((bx+a)d-ad+bc \right)^2 / d - 2 \left(-4 \sin(4bx+4a) \right) / \left((bx+a)d-ad+bc \right) / d + 4 \left(4 \operatorname{Si}(4bx+4a+4(-ad+bc)/d) \right) \sin(4(-ad+bc)/d) / d + 4 \operatorname{Ci}(4bx+4a+4(-ad+bc)/d) \cos(4(-ad+bc)/d) / d \right) / d$

Maxima [C] time = 2.91, size = 521, normalized size = 1.82

$$\frac{b^4 \left(2i E_4 \left(\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) - 2i E_4 \left(-\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) \right) \cos \left(-\frac{2(bc-ad)}{d} \right) + b^4 \left(i E_4 \left(\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) - i E_4 \left(-\frac{4i bc + 4i (bx+a)d - 4i ad}{d} \right) \right) \cos \left(-\frac{4(bc-ad)}{d} \right)}{16 (b^3 c^3 d - 3 a b^2 c^2 d^2 + 3 a^2 b c^2 d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x, algorithm="maxima")`

[Out] $-\frac{1}{16} b^4 \left(2 \operatorname{I} \exp_{\text{integral_e}}(4, (2 \operatorname{I} b^* c + 2 \operatorname{I} (b^* x + a) d - 2 \operatorname{I} a^* d) / d) - 2 \operatorname{I} \exp_{\text{integral_e}}(4, -(2 \operatorname{I} b^* c + 2 \operatorname{I} (b^* x + a) d - 2 \operatorname{I} a^* d) / d) \right) \cos(-2 * (b^* c - a^* d) / d) + b^4 \left(\operatorname{I} \exp_{\text{integral_e}}(4, (4 \operatorname{I} b^* c + 4 \operatorname{I} (b^* x + a) d - 4 \operatorname{I} a^* d) / d) - \operatorname{I} \exp_{\text{integral_e}}(4, -(4 \operatorname{I} b^* c + 4 \operatorname{I} (b^* x + a) d - 4 \operatorname{I} a^* d) / d) \right) \cos(-4 * (b^* c - a^* d) / d) + 2 b^4 \left(\exp_{\text{integral_e}}(4, (2 \operatorname{I} b^* c + 2 \operatorname{I} (b^* x + a) d - 2 \operatorname{I} a^* d) / d) + \exp_{\text{integral_e}}(4, -(2 \operatorname{I} b^* c + 2 \operatorname{I} (b^* x + a) d - 2 \operatorname{I} a^* d) / d) \right) \sin(-2 * (b^* c - a^* d) / d) + b^4 \left(\exp_{\text{integral_e}}(4, (4 \operatorname{I} b^* c + 4 \operatorname{I} (b^* x + a) d - 4 \operatorname{I} a^* d) / d) + \exp_{\text{integral_e}}(4, -(4 \operatorname{I} b^* c + 4 \operatorname{I} (b^* x + a) d - 4 \operatorname{I} a^* d) / d) \right) \sin(-4 * (b^* c - a^* d) / d) \right) / \left((b^3 c^3 d - 3 a b^2 c^2 d^2 + 3 a^2 b c^2 d^3 + (b^* x + a)^3 d^4 - a^3 d^4 + 3 (b^* c d^3 - a^* d^4) (b^* x + a)^2 + 3 (b^2 c^2 d^2 - 2 a b^* c d^3 + a^2 d^4) (b^* x + a)) * b \right)$

Fricas [B] time = 0.682596, size = 1256, normalized size = 4.38

$$\frac{4 (bd^3x + bcd^2) \cos(bx+a)^4 - 3 (bd^3x + bcd^2) \cos(bx+a)^2 - 8 (b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + b^3c^3) \sin\left(-\frac{4(bc-a)}{d}\right)}{16 (b^3 c^3 d - 3 a b^2 c^2 d^2 + 3 a^2 b c^2 d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x, algorithm="fricas")`

[Out] $-\frac{1}{6} \left(4 (b^3 d^3 x + b^3 c d^2) \cos(bx+a)^4 - 3 (b^3 d^3 x + b^3 c d^2) \cos(bx+a)^2 - 8 (b^3 d^3 x^3 + 3 b^3 c d^2 x^2 + 3 b^3 c^2 d x + b^3 c^3) \sin(-4 * (b^* c - a^* d) / d) \right) \sin_{\text{integral}}(4 * (b^* d^* x + b^* c) / d) - 2 (b^3 d^3 x^3 + 3 b^3 c d^2 x^2 + 3 b^3 c^2 d x + b^3 c^3) \sin(-2 * (b^* c - a^* d) / d) \sin_{\text{integral}}(2 * (b^* d^* x + b^* c) / d) + ((b^3 d^3 x^3 + 3 b^3 c d^2 x^2 + 3 b^3 c^2 d x + b^3 c^3) * \cos_{\text{integral}}(2 * (b^* d^* x + b^* c) / d) + (b^3 d^3 x^3 + 3 b^3 c d^2 x^2 + 3 b^3 c^2 d x + b^3 c^3) * \cos_{\text{integral}}(-2 * (b^* d^* x + b^* c) / d)) \cos(-2 * (b^* c - a^* d) / d) + 4 ((b^3 d^3 x^3 + 3 b^3 c d^2 x^2 + 3 b^3 c^2 d x + b^3 c^3) * \cos_{\text{integral}}(4 * (b^* d^* x + b^* c) / d) + (b^3 d^3 x^3 + 3 b^3 c d^2 x^2 + 3 b^3 c^2 d x + b^3 c^3) * \cos_{\text{integral}}(-4 * (b^* d^* x + b^* c) / d)) \sin(-4 * (b^* c - a^* d) / d) \right) / \left((b^3 c^3 d - 3 a b^2 c^2 d^2 + 3 a^2 b c^2 d^3 + (b^* x + a)^3 d^4 - a^3 d^4 + 3 (b^* c d^3 - a^* d^4) (b^* x + a)^2 + 3 (b^2 c^2 d^2 - 2 a b^* c d^3 + a^2 d^4) (b^* x + a)) * b \right)$

$$3) \cos_integral(-4*(b*d*x + b*c)/d) * \cos(-4*(b*c - a*d)/d) - 2*((8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3) * \cos(b*x + a)^3 - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d) * \cos(b*x + a)) * \sin(b*x + a) / (d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c)**4,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")

[Out] Exception raised: NotImplementedError

3.145 $\int (c + dx)^m \cos^3(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=419

$$\frac{i e^{i\left(a - \frac{bc}{d}\right)} (c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, -\frac{ib(c+dx)}{d}\right)}{16b} + \frac{i 3^{-m-1} e^{3i\left(a - \frac{bc}{d}\right)} (c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, -\frac{ib(c+dx)}{d}\right)}{32b}$$

[Out] $((-I/16)*E^{(I*(a - (b*c)/d))}*(c + d*x)^m*\Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/16)*(c + d*x)^m*\Gamma[1 + m, (I*b*(c + d*x))/d])/(b*E^{(I*(a - (b*c)/d))}*((I*b*(c + d*x))/d)^m) + ((I/32)*3^{(-1 - m)}*E^{((3*I)*(a - (b*c)/d))}*(c + d*x)^m*\Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) - ((I/32)*3^{(-1 - m)}*(c + d*x)^m*\Gamma[1 + m, ((3*I)*b*(c + d*x))/d])/(b*E^{((3*I)*(a - (b*c)/d))}*((I*b*(c + d*x))/d)^m) + ((I/32)*5^{(-1 - m)}*E^{((5*I)*(a - (b*c)/d))}*(c + d*x)^m*\Gamma[1 + m, ((-5*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) - ((I/32)*5^{(-1 - m)}*(c + d*x)^m*\Gamma[1 + m, ((5*I)*b*(c + d*x))/d])/(b*E^{((5*I)*(a - (b*c)/d))}*((I*b*(c + d*x))/d)^m)$

Rubi [A] time = 0.434733, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3307, 2181}

$$\frac{i e^{i\left(a - \frac{bc}{d}\right)} (c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, -\frac{ib(c+dx)}{d}\right)}{16b} + \frac{i 3^{-m-1} e^{3i\left(a - \frac{bc}{d}\right)} (c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, -\frac{ib(c+dx)}{d}\right)}{32b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^m * Cos[a + b*x]^3 * Sin[a + b*x]^2, x]

[Out] $((-I/16)*E^{(I*(a - (b*c)/d))}*(c + d*x)^m*\Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/16)*(c + d*x)^m*\Gamma[1 + m, (I*b*(c + d*x))/d])/(b*E^{(I*(a - (b*c)/d))}*((I*b*(c + d*x))/d)^m) + ((I/32)*3^{(-1 - m)}*E^{((3*I)*(a - (b*c)/d))}*(c + d*x)^m*\Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) - ((I/32)*3^{(-1 - m)}*(c + d*x)^m*\Gamma[1 + m, ((3*I)*b*(c + d*x))/d])/(b*E^{((3*I)*(a - (b*c)/d))}*((I*b*(c + d*x))/d)^m) + ((I/32)*5^{(-1 - m)}*E^{((5*I)*(a - (b*c)/d))}*(c + d*x)^m*\Gamma[1 + m, ((-5*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) - ((I/32)*5^{(-1 - m)}*(c + d*x)^m*\Gamma[1 + m, ((5*I)*b*(c + d*x))/d])/(b*E^{((5*I)*(a - (b*c)/d))}*((I*b*(c + d*x))/d)^m)$

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n * Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3307

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m * E^(I*k*Pi) * E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]

Rule 2181

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^m \cos^3(a + bx) \sin^2(a + bx) dx &= \int \left(\frac{1}{8} (c + dx)^m \cos(a + bx) - \frac{1}{16} (c + dx)^m \cos(3a + 3bx) - \frac{1}{16} (c + dx)^m \cos(5a + 5bx) \right) dx \\ &= -\left(\frac{1}{16} \int (c + dx)^m \cos(3a + 3bx) dx \right) - \frac{1}{16} \int (c + dx)^m \cos(5a + 5bx) dx + \frac{1}{8} \int (c + dx)^m \cos(a + bx) dx \\ &= -\left(\frac{1}{32} \int e^{-i(3a+3bx)} (c + dx)^m dx \right) - \frac{1}{32} \int e^{i(3a+3bx)} (c + dx)^m dx - \frac{1}{32} \int e^{-i(5a+5bx)} (c + dx)^m dx + \frac{1}{8} \int e^{i(a+bx)} (c + dx)^m dx \\ &= -\frac{ie^{i\left(a-\frac{bc}{d}\right)} (c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, -\frac{ib(c+dx)}{d}\right)}{16b} + \frac{ie^{-i\left(a-\frac{bc}{d}\right)} (c + dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, \frac{ib(c+dx)}{d}\right)}{16b} \end{aligned}$$

Mathematica [A] time = 0.605986, size = 409, normalized size = 0.98

$$\frac{i3^{-m-1} e^{-\frac{3i(ad+bc)}{d}} (c + dx)^m \left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m} \left(e^{\frac{6ibc}{d}} \left(-\frac{ib(c+dx)}{d}\right)^m \Gamma\left(m + 1, \frac{3ib(c+dx)}{d}\right) - e^{6ia} \left(\frac{ib(c+dx)}{d}\right)^m \Gamma\left(m + 1, -\frac{3ib(c+dx)}{d}\right)}{32b}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^m * Cos[a + b*x]^3 * Sin[a + b*x]^2, x]

[Out] $((-I/16)*(c + d*x)^m*((E^((2*I)*a))*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(((-I)*b*(c + d*x))/d)^m - (E^(((2*I)*b*c)/d))*Gamma[1 + m, (I*b*(c + d*x))/d])/((I*b*(c + d*x))/d)^m)/(b*E^((I*(b*c + a*d))/d)) - ((I/32)*3^(-1 - m)*(c + d*x)^m*(-(E^(((6*I)*a))*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d]) + E^(((6*I)*b*c)/d)*(((I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((3*I)*b*(c + d*x))/d]))/(b*E^(((3*I)*(b*c + a*d))/d))*((b^2*(c + d*x)^2)/d^2)^m - ((I/32)*5^(-1 - m)*(c + d*x)^m*(-(E^(((10*I)*a))*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-5*I)*b*(c + d*x))/d]) + E^(((10*I)*b*c)/d)*(((I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((5*I)*b*(c + d*x))/d]))/(b*E^(((5*I)*(b*c + a*d))/d))*((b^2*(c + d*x)^2)/d^2)^m)$

Maple [F] time = 0.331, size = 0, normalized size = 0.

$$\int (dx + c)^m (\cos(bx + a))^3 (\sin(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x)

[Out] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a)^3 \sin(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")

[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^2, x)

Fricas [A] time = 0.577308, size = 726, normalized size = 1.73

$$-3ie^{\left(\frac{dm \log\left(\frac{5ib}{d}\right) - 5ibc + 5iad}{d}\right)} \Gamma\left(m + 1, \frac{5ibdx + 5ibc}{d}\right) - 5ie^{\left(\frac{dm \log\left(\frac{3ib}{d}\right) - 3ibc + 3iad}{d}\right)} \Gamma\left(m + 1, \frac{3ibdx + 3ibc}{d}\right) + 30ie^{\left(\frac{dm \log\left(\frac{ib}{d}\right) - ibc + iad}{d}\right)} \Gamma\left(m + 1, \frac{ibdx + ibc}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")

[Out] $\frac{1}{480} \left(-3Ie^{-(d*m*\log(5*I*b/d) - 5*I*b*c + 5*I*a*d)/d} * \text{gamma}(m + 1, (5*I*b*d*x + 5*I*b*c)/d) - 5Ie^{-(d*m*\log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d} * \text{gamma}(m + 1, (3*I*b*d*x + 3*I*b*c)/d) + 30Ie^{-(d*m*\log(I*b/d) - I*b*c + I*a*d)/d} * \text{gamma}(m + 1, (I*b*d*x + I*b*c)/d) - 30Ie^{-(d*m*\log(-I*b/d) + I*b*c - I*a*d)/d} * \text{gamma}(m + 1, (-I*b*d*x - I*b*c)/d) + 5Ie^{-(d*m*\log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d} * \text{gamma}(m + 1, (-3*I*b*d*x - 3*I*b*c)/d) + 3Ie^{-(d*m*\log(-5*I*b/d) + 5*I*b*c - 5*I*a*d)/d} * \text{gamma}(m + 1, (-5*I*b*d*x - 5*I*b*c)/d) \right) / b$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**m*cos(b*x+a)**3*sin(b*x+a)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a)^3 \sin(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^2, x)

3.146 $\int (c + dx)^4 \cos^3(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=330

$$-\frac{3d^2(c + dx)^2 \sin(a + bx)}{2b^3} + \frac{d^2(c + dx)^2 \sin(3a + 3bx)}{36b^3} + \frac{3d^2(c + dx)^2 \sin(5a + 5bx)}{500b^3} - \frac{3d^3(c + dx) \cos(a + bx)}{b^4} + \frac{d^3(c + dx)^3 \cos(a + bx)}{54b^4}$$

[Out] $(-3*d^3*(c + d*x)*Cos[a + b*x])/b^4 + (d*(c + d*x)^3*Cos[a + b*x])/(2*b^2) + (d^3*(c + d*x)*Cos[3*a + 3*b*x])/(54*b^4) - (d*(c + d*x)^3*Cos[3*a + 3*b*x])/(36*b^2) + (3*d^3*(c + d*x)*Cos[5*a + 5*b*x])/(1250*b^4) - (d*(c + d*x)^3*Cos[5*a + 5*b*x])/(100*b^2) + (3*d^4*Sin[a + b*x])/b^5 - (3*d^2*(c + d*x)^2*Sin[a + b*x])/(2*b^3) + ((c + d*x)^4*Sin[a + b*x])/(8*b) - (d^4*Sin[3*a + 3*b*x])/(162*b^5) + (d^2*(c + d*x)^2*Sin[3*a + 3*b*x])/(36*b^3) - ((c + d*x)^4*Sin[3*a + 3*b*x])/(48*b) - (3*d^4*Sin[5*a + 5*b*x])/(6250*b^5) + (3*d^2*(c + d*x)^2*Sin[5*a + 5*b*x])/(500*b^3) - ((c + d*x)^4*Sin[5*a + 5*b*x])/(80*b)$

Rubi [A] time = 0.368317, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3296, 2637}

$$-\frac{3d^2(c + dx)^2 \sin(a + bx)}{2b^3} + \frac{d^2(c + dx)^2 \sin(3a + 3bx)}{36b^3} + \frac{3d^2(c + dx)^2 \sin(5a + 5bx)}{500b^3} - \frac{3d^3(c + dx) \cos(a + bx)}{b^4} + \frac{d^3(c + dx)^3 \cos(a + bx)}{54b^4}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^2,x]

[Out] $(-3*d^3*(c + d*x)*Cos[a + b*x])/b^4 + (d*(c + d*x)^3*Cos[a + b*x])/(2*b^2) + (d^3*(c + d*x)*Cos[3*a + 3*b*x])/(54*b^4) - (d*(c + d*x)^3*Cos[3*a + 3*b*x])/(36*b^2) + (3*d^3*(c + d*x)*Cos[5*a + 5*b*x])/(1250*b^4) - (d*(c + d*x)^3*Cos[5*a + 5*b*x])/(100*b^2) + (3*d^4*Sin[a + b*x])/b^5 - (3*d^2*(c + d*x)^2*Sin[a + b*x])/(2*b^3) + ((c + d*x)^4*Sin[a + b*x])/(8*b) - (d^4*Sin[3*a + 3*b*x])/(162*b^5) + (d^2*(c + d*x)^2*Sin[3*a + 3*b*x])/(36*b^3) - ((c + d*x)^4*Sin[3*a + 3*b*x])/(48*b) - (3*d^4*Sin[5*a + 5*b*x])/(6250*b^5) + (3*d^2*(c + d*x)^2*Sin[5*a + 5*b*x])/(500*b^3) - ((c + d*x)^4*Sin[5*a + 5*b*x])/(80*b)$

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_.)]^(p_.)*((c_.) + (d_.)*(x_.))^(m_.)*Sin[(a_.) + (b_.)*(x_.)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] :> -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (c + dx)^4 \cos^3(a + bx) \sin^2(a + bx) dx &= \int \left(\frac{1}{8} (c + dx)^4 \cos(a + bx) - \frac{1}{16} (c + dx)^4 \cos(3a + 3bx) - \frac{1}{16} (c + dx)^4 \cos(5a + 5bx) \right) dx \\
&= - \left(\frac{1}{16} \int (c + dx)^4 \cos(3a + 3bx) dx \right) - \frac{1}{16} \int (c + dx)^4 \cos(5a + 5bx) dx + \frac{1}{8} \int (c + dx)^4 \cos(a + bx) dx \\
&= \frac{(c + dx)^4 \sin(a + bx)}{8b} - \frac{(c + dx)^4 \sin(3a + 3bx)}{48b} - \frac{(c + dx)^4 \sin(5a + 5bx)}{80b} \\
&= \frac{d(c + dx)^3 \cos(a + bx)}{2b^2} - \frac{d(c + dx)^3 \cos(3a + 3bx)}{36b^2} - \frac{d(c + dx)^3 \cos(5a + 5bx)}{100b^2} \\
&= \frac{d(c + dx)^3 \cos(a + bx)}{2b^2} - \frac{d(c + dx)^3 \cos(3a + 3bx)}{36b^2} - \frac{d(c + dx)^3 \cos(5a + 5bx)}{100b^2} \\
&= -\frac{3d^3(c + dx) \cos(a + bx)}{b^4} + \frac{d(c + dx)^3 \cos(a + bx)}{2b^2} + \frac{d^3(c + dx) \cos(3a + 3bx)}{54b^4} \\
&= -\frac{3d^3(c + dx) \cos(a + bx)}{b^4} + \frac{d(c + dx)^3 \cos(a + bx)}{2b^2} + \frac{d^3(c + dx) \cos(3a + 3bx)}{54b^4}
\end{aligned}$$

Mathematica [A] time = 3.75681, size = 563, normalized size = 1.71

$$\frac{-3037500b^2c^2d^2 \left((b^2x^2 - 2) \sin(a + bx) + 2bx \cos(a + bx) \right) + 56250b^2c^2d^2 \left((9b^2x^2 - 2) \sin(3(a + bx)) + 6bx \cos(3(a + bx)) \right)}{(4050000b^5)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^4*cos[a + b*x]^3*Sin[a + b*x]^2,x]

[Out]
$$\begin{aligned}
& -(-506250*b^4*c^4*\sin[a + b*x] - 2025000*b^3*c^3*d*(\cos[a + b*x] + b*x*\sin[a + b*x]) - 2025000*b*c*d^3*(3*(-2 + b^2*x^2)*\cos[a + b*x] + b*x*(-6 + b^2*x^2)*\sin[a + b*x]) - 3037500*b^2*c^2*d^2*(2*b*x*\cos[a + b*x] + (-2 + b^2*x^2)*\sin[a + b*x]) - 506250*d^4*(4*b*x*(-6 + b^2*x^2)*\cos[a + b*x] + (24 - 12*b^2*x^2 + b^4*x^4)*\sin[a + b*x]) + 84375*b^4*c^4*\sin[3*(a + b*x)] + 112500*b^3*c^3*d*(\cos[3*(a + b*x)] + 3*b*x*\sin[3*(a + b*x)]) + 37500*b*c*d^3*((-2 + 9*b^2*x^2)*\cos[3*(a + b*x)] + 3*b*x*(-2 + 3*b^2*x^2)*\sin[3*(a + b*x)]) + 56250*b^2*c^2*d^2*(6*b*x*\cos[3*(a + b*x)] + (-2 + 9*b^2*x^2)*\sin[3*(a + b*x)]) + 3125*d^4*(12*b*x*(-2 + 3*b^2*x^2)*\cos[3*(a + b*x)] + (8 - 36*b^2*x^2 + 27*b^4*x^4)*\sin[3*(a + b*x)]) + 50625*b^4*c^4*\sin[5*(a + b*x)] + 40500*b^3*c^3*d*(\cos[5*(a + b*x)] + 5*b*x*\sin[5*(a + b*x)]) + 1620*b*c*d^3*((-6 + 75*b^2*x^2)*\cos[5*(a + b*x)] + 5*b*x*(-6 + 25*b^2*x^2)*\sin[5*(a + b*x)]) + 12150*b^2*c^2*d^2*(10*b*x*\cos[5*(a + b*x)] + (-2 + 25*b^2*x^2)*\sin[5*(a + b*x)]) + 81*d^4*(20*b*x*(-6 + 25*b^2*x^2)*\cos[5*(a + b*x)] + (24 - 300*b^2*x^2 + 625*b^4*x^4)*\sin[5*(a + b*x)])/(4050000*b^5)
\end{aligned}$$

Maple [B] time = 0.052, size = 1842, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x)

[Out]
$$\begin{aligned}
& 1/b*(1/b^4*d^4*(1/3*(b*x+a)^4*(2+\cos(b*x+a)^2)*\sin(b*x+a)+8/15*(b*x+a)^3*\cos(b*x+a)-8/5*(b*x+a)^2*\sin(b*x+a)+3424/1125*\sin(b*x+a)-3424/1125*(b*x+a)*\cos(b*x+a)+4/45*(b*x+a)^3*\cos(b*x+a)^3-4/45*(b*x+a)^2*(2+\cos(b*x+a)^2)*\sin(b*x+a)+88/3375*(b*x+a)*\cos(b*x+a)^3-88/10125*(2+\cos(b*x+a)^2)*\sin(b*x+a)-1/5*
\end{aligned}$$

$$\begin{aligned}
& (b*x+a)^4*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+a)^2)*\sin(b*x+a)-4/25*(b*x+a)^3*\cos \\
& (b*x+a)^5+12/125*(b*x+a)^2*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+a)^2)*\sin(b*x+a)+ \\
& 4/625*(b*x+a)*\cos(b*x+a)^5-24/3125*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+a)^2)*\sin(\\
& b*x+a))-4/b^4*a*d^4*(1/3*(b*x+a)^3*(2+\cos(b*x+a)^2)*\sin(b*x+a)+2/5*(b*x+a)^ \\
& 2*\cos(b*x+a)-856/1125*\cos(b*x+a)-4/5*(b*x+a)*\sin(b*x+a)+1/15*(b*x+a)^2*\cos(\\
& b*x+a)^3-2/45*(b*x+a)*(2+\cos(b*x+a)^2)*\sin(b*x+a)+22/3375*\cos(b*x+a)^3-1/5* \\
& (b*x+a)^3*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+a)^2)*\sin(b*x+a)-3/25*(b*x+a)^2*\cos \\
& (b*x+a)^5+6/125*(b*x+a)*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+a)^2)*\sin(b*x+a)+6/62 \\
& 5*\cos(b*x+a)^5)+4/b^3*c*d^3*(1/3*(b*x+a)^3*(2+\cos(b*x+a)^2)*\sin(b*x+a)+2/5* \\
& (b*x+a)^2*\cos(b*x+a)-856/1125*\cos(b*x+a)-4/5*(b*x+a)*\sin(b*x+a)+1/15*(b*x+a) \\
&)^2*\cos(b*x+a)^3-2/45*(b*x+a)*(2+\cos(b*x+a)^2)*\sin(b*x+a)+22/3375*\cos(b*x+a) \\
&)^3-1/5*(b*x+a)^3*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+a)^2)*\sin(b*x+a)-3/25*(b*x+ \\
& a)^2*\cos(b*x+a)^5+6/125*(b*x+a)*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+a)^2)*\sin(b*x \\
& +a)+6/625*\cos(b*x+a)^5)+6/b^4*a^2*d^4*(1/3*(b*x+a)^2*(2+\cos(b*x+a)^2)*\sin(b \\
& *x+a)-4/15*\sin(b*x+a)+4/15*(b*x+a)*\cos(b*x+a)+2/45*(b*x+a)*\cos(b*x+a)^3-2/1 \\
& 35*(2+\cos(b*x+a)^2)*\sin(b*x+a)-1/5*(b*x+a)^2*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+ \\
& a)^2)*\sin(b*x+a)-2/25*(b*x+a)*\cos(b*x+a)^5+2/125*(8/3+\cos(b*x+a)^4+4/3*\cos(\\
& b*x+a)^2)*\sin(b*x+a))-12/b^3*a*c*d^3*(1/3*(b*x+a)^2*(2+\cos(b*x+a)^2)*\sin(b* \\
& x+a)-4/15*\sin(b*x+a)+4/15*(b*x+a)*\cos(b*x+a)+2/45*(b*x+a)*\cos(b*x+a)^3-2/13 \\
& 5*(2+\cos(b*x+a)^2)*\sin(b*x+a)-1/5*(b*x+a)^2*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+a) \\
&)^2)*\sin(b*x+a)-2/25*(b*x+a)*\cos(b*x+a)^5+2/125*(8/3+\cos(b*x+a)^4+4/3*\cos(b \\
& *x+a)^2)*\sin(b*x+a))+6/b^2*c^2*d^2*(1/3*(b*x+a)^2*(2+\cos(b*x+a)^2)*\sin(b*x+ \\
& a)-4/15*\sin(b*x+a)+4/15*(b*x+a)*\cos(b*x+a)+2/45*(b*x+a)*\cos(b*x+a)^3-2/135* \\
& (2+\cos(b*x+a)^2)*\sin(b*x+a)-1/5*(b*x+a)^2*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+a)^ \\
& 2)*\sin(b*x+a)-2/25*(b*x+a)*\cos(b*x+a)^5+2/125*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x \\
& +a)^2)*\sin(b*x+a))-4/b^4*a^3*d^4*(1/3*(b*x+a)*(2+\cos(b*x+a)^2)*\sin(b*x+a)+1 \\
& /45*\cos(b*x+a)^3+2/15*\cos(b*x+a)-1/5*(b*x+a)*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+ \\
& a)^2)*\sin(b*x+a)-1/25*\cos(b*x+a)^5)+12/b^3*a^2*c*d^3*(1/3*(b*x+a)*(2+\cos(b* \\
& x+a)^2)*\sin(b*x+a)+1/45*\cos(b*x+a)^3+2/15*\cos(b*x+a)-1/5*(b*x+a)*(8/3+\cos(b \\
& *x+a)^4+4/3*\cos(b*x+a)^2)*\sin(b*x+a)-1/25*\cos(b*x+a)^5)-12/b^2*a*c^2*d^2*(1 \\
& /3*(b*x+a)*(2+\cos(b*x+a)^2)*\sin(b*x+a)+1/45*\cos(b*x+a)^3+2/15*\cos(b*x+a)-1/ \\
& 5*(b*x+a)*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+a)^2)*\sin(b*x+a)-1/25*\cos(b*x+a)^5) \\
& +4/b*c^3*d*(1/3*(b*x+a)*(2+\cos(b*x+a)^2)*\sin(b*x+a)+1/45*\cos(b*x+a)^3+2/15* \\
& \cos(b*x+a)-1/5*(b*x+a)*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+a)^2)*\sin(b*x+a)-1/25* \\
& \cos(b*x+a)^5)+1/b^4*a^4*d^4*(-1/5*\sin(b*x+a)*\cos(b*x+a)^4+1/15*(2+\cos(b*x+a) \\
&)^2)*\sin(b*x+a))-4/b^3*a^3*c*d^3*(-1/5*\sin(b*x+a)*\cos(b*x+a)^4+1/15*(2+\cos(\\
& b*x+a)^2)*\sin(b*x+a))+6/b^2*a^2*c^2*d^2*(-1/5*\sin(b*x+a)*\cos(b*x+a)^4+1/15* \\
& (2+\cos(b*x+a)^2)*\sin(b*x+a))-4/b*a*c^3*d*(-1/5*\sin(b*x+a)*\cos(b*x+a)^4+1/15 \\
& *(2+\cos(b*x+a)^2)*\sin(b*x+a))+c^4*(-1/5*\sin(b*x+a)*\cos(b*x+a)^4+1/15*(2+\cos \\
& (b*x+a)^2)*\sin(b*x+a)))
\end{aligned}$$

Maxima [B] time = 1.48933, size = 1808, normalized size = 5.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")

[Out] $-1/4050000*(270000*(3*\sin(b*x + a)^5 - 5*\sin(b*x + a)^3)*c^4 - 1080000*(3*\sin(b*x + a)^5 - 5*\sin(b*x + a)^3)*a*c^3*d/b + 1620000*(3*\sin(b*x + a)^5 - 5*\sin(b*x + a)^3)*a^2*c^2*d^2/b^2 - 1080000*(3*\sin(b*x + a)^5 - 5*\sin(b*x + a)^3)*a^3*c*d^3/b^3 + 270000*(3*\sin(b*x + a)^5 - 5*\sin(b*x + a)^3)*a^4*d^4/b^4 + 4500*(45*(b*x + a)*\sin(5*b*x + 5*a) + 75*(b*x + a)*\sin(3*b*x + 3*a) - 450*(b*x + a)*\sin(b*x + a) + 9*\cos(5*b*x + 5*a) + 25*\cos(3*b*x + 3*a) - 450*\cos(b*x + a))*c^3*d/b - 13500*(45*(b*x + a)*\sin(5*b*x + 5*a) + 75*(b*x + a)*\sin(3*b*x + 3*a) - 450*(b*x + a)*\sin(b*x + a) + 9*\cos(5*b*x + 5*a) + 25*$

$$\begin{aligned} & \cos(3bx + 3a) - 450\cos(bx + a)) * a^2 d^2 / b^2 + 13500(45(bx + a) \sin(5bx + 5a) \\ & + 75(bx + a) \sin(3bx + 3a) - 450(bx + a) \sin(bx + a) + 9\cos(5bx + 5a) + 25\cos(3bx + 3a) \\ & - 450\cos(bx + a)) * a^2 c d^3 / b^3 - 4500(45(bx + a) \sin(5bx + 5a) + 75(bx + a) \sin(3bx + 3a) \\ & - 450(bx + a) \sin(bx + a) + 9\cos(5bx + 5a) + 25\cos(3bx + 3a) - 450\cos(bx + a)) * a^3 d^4 / b^4 \\ & + 450(270(bx + a) \cos(5bx + 5a) + 750(bx + a) \cos(3bx + 3a) - 13500(bx + a) \cos(bx + a) \\ & + 27(25(bx + a)^2 - 2) \sin(5bx + 5a) + 125(9(bx + a)^2 - 2) \sin(3bx + 3a) - 6750((bx + a)^2 - 2) \\ & \sin(bx + a)) * c^2 d^2 / b^2 - 900(270(bx + a) \cos(5bx + 5a) + 750(bx + a) \cos(3bx + 3a) \\ & - 13500(bx + a) \cos(bx + a) + 27(25(bx + a)^2 - 2) \sin(5bx + 5a) + 125(9(bx + a)^2 - 2) \sin(3bx + 3a) \\ & - 6750((bx + a)^2 - 2) \sin(bx + a)) * a c d^3 / b^3 + 450(270(bx + a) \cos(5bx + 5a) + 750(bx + a) \cos(3bx + 3a) \\ & - 13500(bx + a) \cos(bx + a) + 27(25(bx + a)^2 - 2) \sin(5bx + 5a) + 125(9(bx + a)^2 - 2) \sin(3bx + 3a) \\ & - 6750((bx + a)^2 - 2) \sin(bx + a)) * a^2 d^4 / b^4 + 60(81(25(bx + a)^2 - 2) \cos(5bx + 5a) \\ & + 625(9(bx + a)^2 - 2) \cos(3bx + 3a) - 101250((bx + a)^2 - 2) \cos(bx + a) + 135(25(bx + a)^3 - 6bx - 6a) \\ & \sin(5bx + 5a) + 1875(3(bx + a)^3 - 2bx - 2a) \sin(3bx + 3a) - 33750((bx + a)^3 - 6bx - 6a) \sin(bx + a)) \\ & * c d^3 / b^3 - 60(81(25(bx + a)^2 - 2) \cos(5bx + 5a) + 625(9(bx + a)^2 - 2) \cos(3bx + 3a) - 101250((bx + a)^2 - 2) \cos(bx + a) \\ & + 135(25(bx + a)^3 - 6bx - 6a) \sin(5bx + 5a) + 1875(3(bx + a)^3 - 2bx - 2a) \sin(3bx + 3a) - 33750((bx + a)^3 - 6bx - 6a) \sin(bx + a)) \\ & * a d^4 / b^4 + (1620(25(bx + a)^3 - 6bx - 6a) \cos(5bx + 5a) + 37500(3(bx + a)^3 - 2bx - 2a) \cos(3bx + 3a) \\ & - 2025000((bx + a)^3 - 6bx - 6a) \cos(bx + a) + 81(625(bx + a)^4 - 300(bx + a)^2 + 24) \sin(5bx + 5a) + 3125(27(bx + a)^4 \\ & - 36(bx + a)^2 + 8) \sin(3bx + 3a) - 506250((bx + a)^4 - 12(bx + a)^2 + 24) \sin(bx + a)) * d^4 / b^4) / b \end{aligned}$$

Fricas [A] time = 0.568267, size = 1245, normalized size = 3.77

$$\frac{1620(25b^3d^4x^3 + 75b^3cd^3x^2 + 25b^3c^3d - 6bcd^3 + 3(25b^3c^2d^2 - 2bd^4)x) \cos(bx + a)^5 - 300(75b^3d^4x^3 + 225b^3cd^3x^2 + 75b^3c^3d - 6bcd^3 + 3(25b^3c^2d^2 - 2bd^4)x) \sin(bx + a)^5}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/253125*(1620*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 25*b^3*c^3*d - 6*b*c*d^3 \\ & + 3*(25*b^3*c^2*d^2 - 2*b*d^4)*x)*\cos(b*x + a)^5 - 300*(75*b^3*d^4*x^3 + 225*b^3*c*d^3*x^2 \\ & + 75*b^3*c^3*d + 22*b*c*d^3 + (225*b^3*c^2*d^2 + 22*b*d^4)*x)*\cos(b*x + a)^3 - 1800*(75*b^3*d^4*x^3 \\ & + 225*b^3*c*d^3*x^2 + 75*b^3*c^3*d - 428*b*c*d^3 + (225*b^3*c^2*d^2 - 428*b*d^4)*x)*\cos(b*x + a) - (33750*b^4*d^4*x^4 \\ & + 135000*b^4*c*d^3*x^3 + 33750*b^4*c^4 - 385200*b^2*c^2*d^2 - 81*(625*b^4*d^4*x^4 + 2500*b^4*c*d^3*x^3 \\ & + 625*b^4*c^4 - 300*b^2*c^2*d^2 + 24*d^4 + 150*(25*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 100*(25*b^4*c^3*d - 6*b^2*c*d^3)*x) \\ & *\cos(b*x + a)^4 + 760816*d^4 + 900*(225*b^4*c^2*d^2 - 428*b^2*d^4)*x^2 + (16875*b^4*d^4*x^4 + 67500*b^4*c*d^3*x^3 \\ & + 16875*b^4*c^4 + 9900*b^2*c^2*d^2 - 4792*d^4 + 450*(225*b^4*c^2*d^2 + 22*b^2*d^4)*x^2 + 900*(75*b^4*c^3*d \\ & + 22*b^2*c*d^3)*x)*\cos(b*x + a)^2 + 1800*(75*b^4*c^3*d - 428*b^2*c*d^3)*x*\sin(b*x + a))/b^5 \end{aligned}$$

Sympy [A] time = 48.0104, size = 1098, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**4*cos(b*x+a)**3*sin(b*x+a)**2,x)

[Out] Piecewise((2*c**4*sin(a + b*x)**5/(15*b) + c**4*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 8*c**3*d*x*sin(a + b*x)**5/(15*b) + 4*c**3*d*x*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 4*c**2*d**2*x**2*sin(a + b*x)**5/(5*b) + 2*c**2*d**2*x**2*sin(a + b*x)**3*cos(a + b*x)**2/b + 8*c*d**3*x**3*sin(a + b*x)**5/(15*b) + 4*c*d**3*x**3*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 2*d**4*x**4*sin(a + b*x)**5/(15*b) + d**4*x**4*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 8*c**3*d*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 52*c**3*d*sin(a + b*x)**2*cos(a + b*x)**3/(45*b**2) + 104*c**3*d*cos(a + b*x)**5/(225*b**2) + 8*c**2*d**2*x*sin(a + b*x)**4*cos(a + b*x)/(5*b**2) + 52*c**2*d**2*x*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 104*c**2*d**2*x*cos(a + b*x)**5/(75*b**2) + 8*c*d**3*x**2*sin(a + b*x)**4*cos(a + b*x)/(5*b**2) + 52*c*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 104*c*d**3*x**2*cos(a + b*x)**5/(75*b**2) + 8*d**4*x**3*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 52*d**4*x**3*sin(a + b*x)**2*cos(a + b*x)**3/(45*b**2) + 104*d**4*x**3*cos(a + b*x)**5/(225*b**2) - 1712*c**2*d**2*sin(a + b*x)**5/(1125*b**3) - 676*c**2*d**2*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 104*c**2*d**2*sin(a + b*x)*cos(a + b*x)**4/(75*b**3) - 3424*c*d**3*x*sin(a + b*x)**5/(1125*b**3) - 1352*c*d**3*x*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 208*c*d**3*x*sin(a + b*x)*cos(a + b*x)**4/(75*b**3) - 1712*d**4*x**2*sin(a + b*x)**5/(1125*b**3) - 676*d**4*x**2*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 104*d**4*x**2*sin(a + b*x)*cos(a + b*x)**4/(75*b**3) - 3424*c*d**3*sin(a + b*x)**4*cos(a + b*x)/(1125*b**4) - 20456*c*d**3*sin(a + b*x)**2*cos(a + b*x)**3/(3375*b**4) - 50272*c*d**3*cos(a + b*x)**5/(16875*b**4) - 3424*d**4*x*sin(a + b*x)**4*cos(a + b*x)/(1125*b**4) - 20456*d**4*x*sin(a + b*x)**2*cos(a + b*x)**3/(3375*b**4) - 50272*d**4*x*cos(a + b*x)**5/(16875*b**4) + 760816*d**4*sin(a + b*x)**5/(253125*b**5) + 303368*d**4*sin(a + b*x)**3*cos(a + b*x)**2/(50625*b**5) + 50272*d**4*sin(a + b*x)*cos(a + b*x)**4/(16875*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**2*cos(a)**3, True))

Giac [A] time = 1.14666, size = 717, normalized size = 2.17

$$\frac{(25b^3d^4x^3 + 75b^3cd^3x^2 + 75b^3c^2d^2x + 25b^3c^3d - 6bd^4x - 6bcd^3)\cos(5bx + 5a)}{2500b^5} - \frac{(3b^3d^4x^3 + 9b^3cd^3x^2 + 9b^3c^2d^2x + 3b^3c^3d - 6bd^4x - 6bcd^3)\cos(5bx + 5a)}{2500b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")

[Out] -1/2500*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 75*b^3*c^2*d^2*x + 25*b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*cos(5*b*x + 5*a)/b^5 - 1/108*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*cos(3*b*x + 3*a)/b^5 + 1/2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*cos(b*x + a)/b^5 - 1/50000*(625*b^4*d^4*x^4 + 2500*b^4*c*d^3*x^3 + 3750*b^4*c^2*d^2*x^2 + 2500*b^4*c^3*d*x + 625*b^4*c^4 - 300*b^2*d^4*x^2 - 600*b^2*c*d^3*x - 300*b^2*c^2*d^2 + 24*d^4)*sin(5*b*x + 5*a)/b^5 - 1/1296*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8*d^4)*sin(3*b*x + 3*a)/b^5 + 1/8*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^2*c^2*d^2 + 24*d^4)*sin(b*x + a)/b^5

3.147 $\int (c + dx)^3 \cos^3(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=259

$$-\frac{3d^2(c + dx) \sin(a + bx)}{4b^3} + \frac{d^2(c + dx) \sin(3a + 3bx)}{72b^3} + \frac{3d^2(c + dx) \sin(5a + 5bx)}{1000b^3} + \frac{3d(c + dx)^2 \cos(a + bx)}{8b^2} - \frac{d(c + dx)^3 \cos^3(a + bx)}{216b^4}$$

```
[Out] (-3*d^3*Cos[a + b*x])/(4*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x])/(8*b^2) + (d^3*Cos[3*a + 3*b*x])/(216*b^4) - (d*(c + d*x)^2*Cos[3*a + 3*b*x])/(48*b^2) + (3*d^3*Cos[5*a + 5*b*x])/(5000*b^4) - (3*d*(c + d*x)^2*Cos[5*a + 5*b*x])/(400*b^2) - (3*d^2*(c + d*x)*Sin[a + b*x])/(4*b^3) + ((c + d*x)^3*SIN[a + b*x])/(8*b) + (d^2*(c + d*x)*Sin[3*a + 3*b*x])/(72*b^3) - ((c + d*x)^3*SIN[3*a + 3*b*x])/(48*b) + (3*d^2*(c + d*x)*Sin[5*a + 5*b*x])/(1000*b^3) - ((c + d*x)^3*SIN[5*a + 5*b*x])/(80*b)
```

Rubi [A] time = 0.272605, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3296, 2638}

$$-\frac{3d^2(c + dx) \sin(a + bx)}{4b^3} + \frac{d^2(c + dx) \sin(3a + 3bx)}{72b^3} + \frac{3d^2(c + dx) \sin(5a + 5bx)}{1000b^3} + \frac{3d(c + dx)^2 \cos(a + bx)}{8b^2} - \frac{d(c + dx)^3 \cos^3(a + bx)}{216b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^3*SIN[a + b*x]^2,x]
```

```
[Out] (-3*d^3*Cos[a + b*x])/(4*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x])/(8*b^2) + (d^3*Cos[3*a + 3*b*x])/(216*b^4) - (d*(c + d*x)^2*Cos[3*a + 3*b*x])/(48*b^2) + (3*d^3*Cos[5*a + 5*b*x])/(5000*b^4) - (3*d*(c + d*x)^2*Cos[5*a + 5*b*x])/(400*b^2) - (3*d^2*(c + d*x)*Sin[a + b*x])/(4*b^3) + ((c + d*x)^3*SIN[a + b*x])/(8*b) + (d^2*(c + d*x)*Sin[3*a + 3*b*x])/(72*b^3) - ((c + d*x)^3*SIN[3*a + 3*b*x])/(48*b) + (3*d^2*(c + d*x)*Sin[5*a + 5*b*x])/(1000*b^3) - ((c + d*x)^3*SIN[5*a + 5*b*x])/(80*b)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3 \cos^3(a+bx) \sin^2(a+bx) dx &= \int \left(\frac{1}{8}(c+dx)^3 \cos(a+bx) - \frac{1}{16}(c+dx)^3 \cos(3a+3bx) - \frac{1}{16}(c+dx)^3 \cos(5a+5bx) \right) dx \\
&= -\left(\frac{1}{16} \int (c+dx)^3 \cos(3a+3bx) dx \right) - \frac{1}{16} \int (c+dx)^3 \cos(5a+5bx) dx + \frac{1}{8} \int (c+dx)^3 \cos(a+bx) dx \\
&= \frac{(c+dx)^3 \sin(a+bx)}{8b} - \frac{(c+dx)^3 \sin(3a+3bx)}{48b} - \frac{(c+dx)^3 \sin(5a+5bx)}{80b} + \frac{1}{8} \int (c+dx)^3 \cos(a+bx) dx \\
&= \frac{3d(c+dx)^2 \cos(a+bx)}{8b^2} - \frac{d(c+dx)^2 \cos(3a+3bx)}{48b^2} - \frac{3d(c+dx)^2 \cos(5a+5bx)}{400b^2} + \frac{1}{8} \int (c+dx)^3 \cos(a+bx) dx \\
&= \frac{3d(c+dx)^2 \cos(a+bx)}{8b^2} - \frac{d(c+dx)^2 \cos(3a+3bx)}{48b^2} - \frac{3d(c+dx)^2 \cos(5a+5bx)}{400b^2} + \frac{1}{8} \int (c+dx)^3 \cos(a+bx) dx \\
&= -\frac{3d^3 \cos(a+bx)}{4b^4} + \frac{3d(c+dx)^2 \cos(a+bx)}{8b^2} + \frac{d^3 \cos(3a+3bx)}{216b^4} - \frac{d(c+dx)^2 \cos(5a+5bx)}{400b^2} + \frac{1}{8} \int (c+dx)^3 \cos(a+bx) dx
\end{aligned}$$

Mathematica [A] time = 2.2165, size = 195, normalized size = 0.75

$$\frac{30b(c+dx) \sin(a+bx) (8 \cos(2(a+bx)) (75b^2(c+dx)^2 - 38d^2) + 9 \cos(4(a+bx)) (25b^2(c+dx)^2 - 6d^2) - 825b^2c^2 - 1}{1}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^2,x]

[Out]
$$\begin{aligned}
& -(-101250*d*(-2*d^2 + b^2*(c + d*x)^2)*\text{Cos}[a + b*x] + 625*d*(-2*d^2 + 9*b^2 \\
& *(c + d*x)^2)*\text{Cos}[3*(a + b*x)] + 81*d*(-2*d^2 + 25*b^2*(c + d*x)^2)*\text{Cos}[5*(\\
& a + b*x)] + 30*b*(c + d*x)*(-825*b^2*c^2 + 6598*d^2 - 1650*b^2*c*d*x - 825* \\
& b^2*d^2*x^2 + 8*(-38*d^2 + 75*b^2*(c + d*x)^2)*\text{Cos}[2*(a + b*x)] + 9*(-6*d^2 \\
& + 25*b^2*(c + d*x)^2)*\text{Cos}[4*(a + b*x)])*\text{Sin}[a + b*x]/(270000*b^4)
\end{aligned}$$

Maple [B] time = 0.021, size = 1016, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x)

[Out]
$$\begin{aligned}
& 1/b*(1/b^3*d^3*(1/3*(b*x+a)^3*(2+\cos(b*x+a))^2*\sin(b*x+a)+2/5*(b*x+a)^2*\cos \\
& (b*x+a)-856/1125*\cos(b*x+a)-4/5*(b*x+a)*\sin(b*x+a)+1/15*(b*x+a)^2*\cos(b*x+a) \\
&)^3-2/45*(b*x+a)*(2+\cos(b*x+a))^2*\sin(b*x+a)+22/3375*\cos(b*x+a)^3-1/5*(b*x+ \\
& a)^3*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+a)^2)*\sin(b*x+a)-3/25*(b*x+a)^2*\cos(b*x+ \\
& a)^5+6/125*(b*x+a)*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+a)^2)*\sin(b*x+a)+6/625*\cos \\
& (b*x+a)^5-3/b^3*a*d^3*(1/3*(b*x+a)^2*(2+\cos(b*x+a))^2*\sin(b*x+a)-4/15*\sin(\\
& b*x+a)+4/15*(b*x+a)*\cos(b*x+a)+2/45*(b*x+a)*\cos(b*x+a)^3-2/135*(2+\cos(b*x+a) \\
&)^2*\sin(b*x+a)-1/5*(b*x+a)^2*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+a)^2)*\sin(b*x+a) \\
&)-2/25*(b*x+a)*\cos(b*x+a)^5+2/125*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+a)^2)*\sin(b \\
& *x+a))+3/b^2*c*d^2*(1/3*(b*x+a)^2*(2+\cos(b*x+a))^2*\sin(b*x+a)-4/15*\sin(b*x+ \\
& a)+4/15*(b*x+a)*\cos(b*x+a)+2/45*(b*x+a)*\cos(b*x+a)^3-2/135*(2+\cos(b*x+a)^2) \\
& *\sin(b*x+a)-1/5*(b*x+a)^2*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+a)^2)*\sin(b*x+a)-2/ \\
& 25*(b*x+a)*\cos(b*x+a)^5+2/125*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+a)^2)*\sin(b*x+a) \\
&))+3/b^3*a^2*d^3*(1/3*(b*x+a)*(2+\cos(b*x+a))^2*\sin(b*x+a)+1/45*\cos(b*x+a)^3 \\
& +2/15*\cos(b*x+a)-1/5*(b*x+a)*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+a)^2)*\sin(b*x+a) \\
& -1/25*\cos(b*x+a)^5)-6/b^2*a*c*d^2*(1/3*(b*x+a)*(2+\cos(b*x+a))^2*\sin(b*x+a)+
\end{aligned}$$

$$\frac{1}{45}\cos(bx+a)^3 + \frac{2}{15}\cos(bx+a) - \frac{1}{5}(bx+a) \cdot \left(\frac{8}{3} + \cos(bx+a)^4 + \frac{4}{3}\cos(bx+a)^2\right) \sin(bx+a) - \frac{1}{25}\cos(bx+a)^5 + \frac{3}{b^3c^2d} \left(\frac{1}{3}(bx+a) \cdot (2 + \cos(bx+a)^2) \sin(bx+a) + \frac{1}{45}\cos(bx+a)^3 + \frac{2}{15}\cos(bx+a) - \frac{1}{5}(bx+a) \cdot \left(\frac{8}{3} + \cos(bx+a)^4 + \frac{4}{3}\cos(bx+a)^2\right) \sin(bx+a) - \frac{1}{25}\cos(bx+a)^5\right) - \frac{1}{b^3a^3d^3} \left(-\frac{1}{5}\sin(bx+a) \cos(bx+a)^4 + \frac{1}{15}(2 + \cos(bx+a)^2) \sin(bx+a)\right) + \frac{3}{b^2a^2cd^2} \left(-\frac{1}{5}\sin(bx+a) \cos(bx+a)^4 + \frac{1}{15}(2 + \cos(bx+a)^2) \sin(bx+a)\right) - \frac{3}{b^2ac^2d} \left(-\frac{1}{5}\sin(bx+a) \cos(bx+a)^4 + \frac{1}{15}(2 + \cos(bx+a)^2) \sin(bx+a)\right) + c^3 \left(-\frac{1}{5}\sin(bx+a) \cos(bx+a)^4 + \frac{1}{15}(2 + \cos(bx+a)^2) \sin(bx+a)\right)$$

Maxima [B] time = 1.26809, size = 1034, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")

[Out]
$$-\frac{1}{270000} \cdot (18000 \cdot (3 \sin(bx+a)^5 - 5 \sin(bx+a)^3) \cdot c^3 - 54000 \cdot (3 \sin(bx+a)^5 - 5 \sin(bx+a)^3) \cdot a \cdot c^2 \cdot d / b + 54000 \cdot (3 \sin(bx+a)^5 - 5 \sin(bx+a)^3) \cdot a^2 \cdot c \cdot d^2 / b^2 - 18000 \cdot (3 \sin(bx+a)^5 - 5 \sin(bx+a)^3) \cdot a^3 \cdot d^3 / b^3 + 225 \cdot (45 \cdot (bx+a) \cdot \sin(5bx+5a) + 75 \cdot (bx+a) \cdot \sin(3bx+3a)) - 450 \cdot (bx+a) \cdot \sin(bx+a) + 9 \cdot \cos(5bx+5a) + 25 \cdot \cos(3bx+3a) - 450 \cdot \cos(bx+a)) \cdot c^2 \cdot d / b - 450 \cdot (45 \cdot (bx+a) \cdot \sin(5bx+5a) + 75 \cdot (bx+a) \cdot \sin(3bx+3a) - 450 \cdot (bx+a) \cdot \sin(bx+a) + 9 \cdot \cos(5bx+5a) + 25 \cdot \cos(3bx+3a) - 450 \cdot \cos(bx+a)) \cdot a \cdot c \cdot d^2 / b^2 + 225 \cdot (45 \cdot (bx+a) \cdot \sin(5bx+5a) + 75 \cdot (bx+a) \cdot \sin(3bx+3a) - 450 \cdot (bx+a) \cdot \sin(bx+a) + 9 \cdot \cos(5bx+5a) + 25 \cdot \cos(3bx+3a) - 450 \cdot \cos(bx+a)) \cdot a^2 \cdot d^3 / b^3 + 15 \cdot (270 \cdot (bx+a) \cdot \cos(5bx+5a) + 750 \cdot (bx+a) \cdot \cos(3bx+3a) - 13500 \cdot (bx+a) \cdot \cos(bx+a) + 27 \cdot (25 \cdot (bx+a)^2 - 2) \cdot \sin(5bx+5a) + 125 \cdot (9 \cdot (bx+a)^2 - 2) \cdot \sin(3bx+3a) - 6750 \cdot ((bx+a)^2 - 2) \cdot \sin(bx+a)) \cdot c \cdot d^2 / b^2 - 15 \cdot (270 \cdot (bx+a) \cdot \cos(5bx+5a) + 750 \cdot (bx+a) \cdot \cos(3bx+3a) - 13500 \cdot (bx+a) \cdot \cos(bx+a) + 27 \cdot (25 \cdot (bx+a)^2 - 2) \cdot \sin(5bx+5a) + 125 \cdot (9 \cdot (bx+a)^2 - 2) \cdot \sin(3bx+3a) - 6750 \cdot ((bx+a)^2 - 2) \cdot \sin(bx+a)) \cdot a \cdot d^3 / b^3 + (81 \cdot (25 \cdot (bx+a)^2 - 2) \cdot \cos(5bx+5a) + 625 \cdot (9 \cdot (bx+a)^2 - 2) \cdot \cos(3bx+3a) - 101250 \cdot ((bx+a)^2 - 2) \cdot \cos(bx+a) + 135 \cdot (25 \cdot (bx+a)^3 - 6 \cdot bx - 6 \cdot a) \cdot \sin(5bx+5a) + 1875 \cdot (3 \cdot (bx+a)^3 - 2 \cdot bx - 2 \cdot a) \cdot \sin(3bx+3a) - 33750 \cdot ((bx+a)^3 - 6 \cdot bx - 6 \cdot a) \cdot \sin(bx+a)) \cdot d^3 / b^3) / b$$

Fricas [A] time = 0.539504, size = 798, normalized size = 3.08

$$\frac{81 \left(25 b^2 d^3 x^2 + 50 b^2 c d^2 x + 25 b^2 c^2 d - 2 d^3\right) \cos(bx+a)^5 - 5 \left(225 b^2 d^3 x^2 + 450 b^2 c d^2 x + 225 b^2 c^2 d + 22 d^3\right) \cos(bx+a)^3}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")

[Out]
$$-\frac{1}{16875} \cdot (81 \cdot (25 \cdot b^2 \cdot d^3 \cdot x^2 + 50 \cdot b^2 \cdot c \cdot d^2 \cdot x + 25 \cdot b^2 \cdot c^2 \cdot d - 2 \cdot d^3) \cdot \cos(bx+a)^5 - 5 \cdot (225 \cdot b^2 \cdot d^3 \cdot x^2 + 450 \cdot b^2 \cdot c \cdot d^2 \cdot x + 225 \cdot b^2 \cdot c^2 \cdot d + 22 \cdot d^3) \cdot \cos(bx+a)^3 - 30 \cdot (225 \cdot b^2 \cdot d^3 \cdot x^2 + 450 \cdot b^2 \cdot c \cdot d^2 \cdot x + 225 \cdot b^2 \cdot c^2 \cdot d - 42 \cdot 8 \cdot d^3) \cdot \cos(bx+a) - 15 \cdot (150 \cdot b^3 \cdot d^3 \cdot x^3 + 450 \cdot b^3 \cdot c \cdot d^2 \cdot x^2 + 150 \cdot b^3 \cdot c^3 - 9 \cdot (25 \cdot b^3 \cdot d^3 \cdot x^3 + 75 \cdot b^3 \cdot c \cdot d^2 \cdot x^2 + 25 \cdot b^3 \cdot c^3 - 6 \cdot b \cdot c \cdot d^2 + 3 \cdot (25 \cdot b^3 \cdot c^2 \cdot d - 2 \cdot b \cdot d^3) \cdot x) \cdot \cos(bx+a)^4 - 856 \cdot b \cdot c \cdot d^2 + (75 \cdot b^3 \cdot d^3 \cdot x^3 + 225 \cdot b^3 \cdot c \cdot d^2 \cdot x^2 + 75 \cdot b^3 \cdot c^3 + 22 \cdot b \cdot c \cdot d^2 + (225 \cdot b^3 \cdot c^2 \cdot d + 22 \cdot b \cdot d^3) \cdot x) \cdot \cos$$

$$(b*x + a)^2 + 2*(225*b^3*c^2*d - 428*b*d^3)*x*\sin(b*x + a))/b^4$$

Sympy [A] time = 53.5758, size = 690, normalized size = 2.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*cos(b*x+a)**3*sin(b*x+a)**2,x)

[Out] Piecewise((2*c**3*sin(a + b*x)**5/(15*b) + c**3*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 2*c**2*d*x*sin(a + b*x)**5/(5*b) + c**2*d*x*sin(a + b*x)**3*cos(a + b*x)**2/b + 2*c*d**2*x**2*sin(a + b*x)**5/(5*b) + c*d**2*x**2*sin(a + b*x)**3*cos(a + b*x)**2/b + 2*d**3*x**3*sin(a + b*x)**5/(15*b) + d**3*x**3*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 2*c**2*d*sin(a + b*x)**4*cos(a + b*x)/(5*b**2) + 13*c**2*d*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 26*c**2*d*cos(a + b*x)**5/(75*b**2) + 4*c*d**2*x*sin(a + b*x)**4*cos(a + b*x)/(5*b**2) + 26*c*d**2*x*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 52*c*d**2*x*cos(a + b*x)**5/(75*b**2) + 2*d**3*x**2*sin(a + b*x)**4*cos(a + b*x)/(5*b**2) + 13*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 26*d**3*x**2*cos(a + b*x)**5/(75*b**2) - 856*c*d**2*sin(a + b*x)**5/(1125*b**3) - 338*c*d**2*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 52*c*d**2*sin(a + b*x)*cos(a + b*x)**4/(75*b**3) - 856*d**3*x*sin(a + b*x)**5/(1125*b**3) - 338*d**3*x*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 52*d**3*x*sin(a + b*x)*cos(a + b*x)**4/(75*b**3) - 856*d**3*sin(a + b*x)**4*cos(a + b*x)/(1125*b**4) - 5114*d**3*sin(a + b*x)**2*cos(a + b*x)**3/(3375*b**4) - 12568*d**3*cos(a + b*x)**5/(16875*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**2*cos(a)**3, True))

Giac [A] time = 1.11776, size = 474, normalized size = 1.83

$$\frac{3(25b^2d^3x^2 + 50b^2cd^2x + 25b^2c^2d - 2d^3)\cos(5bx + 5a)}{10000b^4} - \frac{(9b^2d^3x^2 + 18b^2cd^2x + 9b^2c^2d - 2d^3)\cos(3bx + 3a)}{432b^4} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")

[Out] -3/10000*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*cos(5*b*x + 5*a)/b^4 - 1/432*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(3*b*x + 3*a)/b^4 + 3/8*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a)/b^4 - 1/2000*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 75*b^3*c^2*d*x + 25*b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(5*b*x + 5*a)/b^4 - 1/144*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^3*x - 2*b*c*d^2)*sin(3*b*x + 3*a)/b^4 + 1/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(b*x + a)/b^4

3.148 $\int (c + dx)^2 \cos^3(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=184

$$\frac{d(c + dx) \cos(a + bx)}{4b^2} - \frac{d(c + dx) \cos(3a + 3bx)}{72b^2} - \frac{d(c + dx) \cos(5a + 5bx)}{200b^2} - \frac{d^2 \sin(a + bx)}{4b^3} + \frac{d^2 \sin(3a + 3bx)}{216b^3} + \frac{d^2 \sin(5a + 5bx)}{80b^3}$$

```
[Out] (d*(c + d*x)*Cos[a + b*x])/(4*b^2) - (d*(c + d*x)*Cos[3*a + 3*b*x])/(72*b^2) - (d*(c + d*x)*Cos[5*a + 5*b*x])/(200*b^2) - (d^2*Sin[a + b*x])/(4*b^3) + ((c + d*x)^2*Sin[a + b*x])/(8*b) + (d^2*Sin[3*a + 3*b*x])/(216*b^3) - ((c + d*x)^2*Sin[3*a + 3*b*x])/(48*b) + (d^2*Sin[5*a + 5*b*x])/(1000*b^3) - ((c + d*x)^2*Sin[5*a + 5*b*x])/(80*b)
```

Rubi [A] time = 0.189954, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3296, 2637}

$$\frac{d(c + dx) \cos(a + bx)}{4b^2} - \frac{d(c + dx) \cos(3a + 3bx)}{72b^2} - \frac{d(c + dx) \cos(5a + 5bx)}{200b^2} - \frac{d^2 \sin(a + bx)}{4b^3} + \frac{d^2 \sin(3a + 3bx)}{216b^3} + \frac{d^2 \sin(5a + 5bx)}{80b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (d*(c + d*x)*Cos[a + b*x])/(4*b^2) - (d*(c + d*x)*Cos[3*a + 3*b*x])/(72*b^2) - (d*(c + d*x)*Cos[5*a + 5*b*x])/(200*b^2) - (d^2*Sin[a + b*x])/(4*b^3) + ((c + d*x)^2*Sin[a + b*x])/(8*b) + (d^2*Sin[3*a + 3*b*x])/(216*b^3) - ((c + d*x)^2*Sin[3*a + 3*b*x])/(48*b) + (d^2*Sin[5*a + 5*b*x])/(1000*b^3) - ((c + d*x)^2*Sin[5*a + 5*b*x])/(80*b)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2 \cos^3(a+bx) \sin^2(a+bx) dx &= \int \left(\frac{1}{8}(c+dx)^2 \cos(a+bx) - \frac{1}{16}(c+dx)^2 \cos(3a+3bx) - \frac{1}{16}(c+dx)^2 \cos(5a+5bx) \right) dx \\
&= -\left(\frac{1}{16} \int (c+dx)^2 \cos(3a+3bx) dx \right) - \frac{1}{16} \int (c+dx)^2 \cos(5a+5bx) dx + \frac{1}{8} \int (c+dx)^2 \cos(a+bx) dx \\
&= \frac{(c+dx)^2 \sin(a+bx)}{8b} - \frac{(c+dx)^2 \sin(3a+3bx)}{48b} - \frac{(c+dx)^2 \sin(5a+5bx)}{80b} + \frac{1}{8} \int (c+dx)^2 \cos(a+bx) dx \\
&= \frac{d(c+dx) \cos(a+bx)}{4b^2} - \frac{d(c+dx) \cos(3a+3bx)}{72b^2} - \frac{d(c+dx) \cos(5a+5bx)}{200b^2} + \frac{1}{8} \int (c+dx)^2 \cos(a+bx) dx \\
&= \frac{d(c+dx) \cos(a+bx)}{4b^2} - \frac{d(c+dx) \cos(3a+3bx)}{72b^2} - \frac{d(c+dx) \cos(5a+5bx)}{200b^2} - \frac{1}{8} \int (c+dx)^2 \cos(a+bx) dx
\end{aligned}$$

Mathematica [A] time = 1.01095, size = 252, normalized size = 1.37

$$\frac{-6750b^2c^2 \sin(a+bx) + 1125b^2c^2 \sin(3(a+bx)) + 675b^2c^2 \sin(5(a+bx)) - 13500b^2cdx \sin(a+bx) + 2250b^2cdx \sin(3(a+bx))}{54000b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^2,x]

[Out]
$$\begin{aligned}
& -(-13500*b*d*(c + d*x)*Cos[a + b*x] + 750*b*d*(c + d*x)*Cos[3*(a + b*x)] + \\
& 270*b*c*d*Cos[5*(a + b*x)] + 270*b*d^2*x*Cos[5*(a + b*x)] - 6750*b^2*c^2*Sin[a + b*x] + 13500*d^2*Sin[a + b*x] - \\
& 13500*b^2*c*d*x*Sin[a + b*x] - 6750*b^2*d^2*x^2*Sin[a + b*x] + 1125*b^2*c^2*Sin[3*(a + b*x)] - 250*d^2*Sin[3*(a + b*x)] + \\
& 2250*b^2*c*d*x*Sin[3*(a + b*x)] + 1125*b^2*d^2*x^2*Sin[3*(a + b*x)] + 675*b^2*c^2*Sin[5*(a + b*x)] - \\
& 54*d^2*Sin[5*(a + b*x)] + 1350*b^2*c*d*x*Sin[5*(a + b*x)] + 675*b^2*d^2*x^2*Sin[5*(a + b*x)])/(54000*b^3)
\end{aligned}$$

Maple [B] time = 0.02, size = 484, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x)

[Out]
$$\begin{aligned}
& 1/b*(1/b^2*d^2*(1/3*(b*x+a)^2*(2+\cos(b*x+a)^2)*\sin(b*x+a)-4/15*\sin(b*x+a)+4/15*(b*x+a)*\cos(b*x+a)+2/45*(b*x+a)*\cos(b*x+a)^3-2/135*(2+\cos(b*x+a)^2)*\sin(b*x+a)-1/5*(b*x+a)^2*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+a)^2)*\sin(b*x+a)-2/25*(b*x+a)*\cos(b*x+a)^5+2/125*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+a)^2)*\sin(b*x+a))-2/b^2*a*d^2*(1/3*(b*x+a)*(2+\cos(b*x+a)^2)*\sin(b*x+a)+1/45*\cos(b*x+a)^3+2/15*\cos(b*x+a)-1/5*(b*x+a)*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+a)^2)*\sin(b*x+a)-1/25*\cos(b*x+a)^5)+2/b*c*d*(1/3*(b*x+a)*(2+\cos(b*x+a)^2)*\sin(b*x+a)+1/45*\cos(b*x+a)^3+2/15*\cos(b*x+a)-1/5*(b*x+a)*(8/3+\cos(b*x+a)^4+4/3*\cos(b*x+a)^2)*\sin(b*x+a)-1/25*\cos(b*x+a)^5)+d^2/b^2*a^2*(-1/5*\sin(b*x+a)*\cos(b*x+a)^4+1/15*(2+\cos(b*x+a)^2)*\sin(b*x+a))-2*c*d/b*a*(-1/5*\sin(b*x+a)*\cos(b*x+a)^4+1/15*(2+\cos(b*x+a)^2)*\sin(b*x+a))+c^2*(-1/5*\sin(b*x+a)*\cos(b*x+a)^4+1/15*(2+\cos(b*x+a)^2)*\sin(b*x+a)))
\end{aligned}$$

Maxima [B] time = 1.17677, size = 506, normalized size = 2.75

$$\frac{3600(3 \sin(bx+a)^5 - 5 \sin(bx+a)^3)c^2 - \frac{7200(3 \sin(bx+a)^5 - 5 \sin(bx+a)^3)acd}{b} + \frac{3600(3 \sin(bx+a)^5 - 5 \sin(bx+a)^3)a^2d^2}{b^2} + \frac{30(45(bx+a)^5 - 75(bx+a)^3)cd}{b}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")

[Out] $-1/54000*(3600*(3*\sin(b*x + a)^5 - 5*\sin(b*x + a)^3)*c^2 - 7200*(3*\sin(b*x + a)^5 - 5*\sin(b*x + a)^3)*a*c*d/b + 3600*(3*\sin(b*x + a)^5 - 5*\sin(b*x + a)^3)*a^2*d^2/b^2 + 30*(45*(b*x + a)*\sin(5*b*x + 5*a) + 75*(b*x + a)*\sin(3*b*x + 3*a) - 450*(b*x + a)*\sin(b*x + a) + 9*\cos(5*b*x + 5*a) + 25*\cos(3*b*x + 3*a) - 450*\cos(b*x + a))*c*d/b - 30*(45*(b*x + a)*\sin(5*b*x + 5*a) + 75*(b*x + a)*\sin(3*b*x + 3*a) - 450*(b*x + a)*\sin(b*x + a) + 9*\cos(5*b*x + 5*a) + 25*\cos(3*b*x + 3*a) - 450*\cos(b*x + a))*a*d^2/b^2 + (270*(b*x + a)*\cos(5*b*x + 5*a) + 750*(b*x + a)*\cos(3*b*x + 3*a) - 13500*(b*x + a)*\cos(b*x + a) + 27*(25*(b*x + a)^2 - 2)*\sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 - 2)*\sin(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*\sin(b*x + a))*d^2/b^2)/b$

Fricas [A] time = 0.499786, size = 470, normalized size = 2.55

$$\frac{270(bd^2x + bcd)\cos(bx + a)^5 - 150(bd^2x + bcd)\cos(bx + a)^3 - 900(bd^2x + bcd)\cos(bx + a) - (450b^2d^2x^2 + 900bd^2x + 450b^2d^2)\cos(bx + a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")

[Out] $-1/3375*(270*(b*d^2*x + b*c*d)*\cos(b*x + a)^5 - 150*(b*d^2*x + b*c*d)*\cos(b*x + a)^3 - 900*(b*d^2*x + b*c*d)*\cos(b*x + a) - (450*b^2*d^2*x^2 + 900*b^2*d^2*c*d*x - 27*(25*b^2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*\cos(b*x + a)^4 + 450*b^2*c^2 + (225*b^2*d^2*x^2 + 450*b^2*c*d*x + 225*b^2*c^2 + 22*d^2)*\cos(b*x + a)^2 - 856*d^2)*\sin(b*x + a))/b^3$

Sympy [A] time = 11.5424, size = 382, normalized size = 2.08

$$\left\{ \begin{array}{l} \frac{2c^2 \sin^5(a+bx)}{15b} + \frac{c^2 \sin^3(a+bx) \cos^2(a+bx)}{3b} + \frac{4cdx \sin^5(a+bx)}{15b} + \frac{2cdx \sin^3(a+bx) \cos^2(a+bx)}{3b} + \frac{2d^2x^2 \sin^5(a+bx)}{15b} + \frac{d^2x^2 \sin^3(a+bx) \cos^2(a+bx)}{3b} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3} \right) \sin^2(a) \cos^3(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*cos(b*x+a)**3*sin(b*x+a)**2,x)

[Out] $\text{Piecewise}((2*c**2*\sin(a + b*x)**5/(15*b) + c**2*\sin(a + b*x)**3*\cos(a + b*x)**2/(3*b) + 4*c*d*x*\sin(a + b*x)**5/(15*b) + 2*c*d*x*\sin(a + b*x)**3*\cos(a + b*x)**2/(3*b) + 2*d**2*x**2*\sin(a + b*x)**5/(15*b) + d**2*x**2*\sin(a + b*x)**3*\cos(a + b*x)**2/(3*b) + 4*c*d*\sin(a + b*x)**4*\cos(a + b*x)/(15*b**2) + 26*c*d*\sin(a + b*x)**2*\cos(a + b*x)**3/(45*b**2) + 52*c*d*\cos(a + b*x)**5/(225*b**2) + 4*d**2*x*\sin(a + b*x)**4*\cos(a + b*x)/(15*b**2) + 26*d**2*x*\sin(a + b*x)**2*\cos(a + b*x)**3/(45*b**2) + 52*d**2*x*\cos(a + b*x)**5/(225*b**2) - 856*d**2*\sin(a + b*x)**5/(3375*b**3) - 338*d**2*\sin(a + b*x)**3*\cos(a + b*x)**2/(675*b**3) - 52*d**2*\sin(a + b*x)*\cos(a + b*x)**4/(225*b**3), \text{Ne}(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*\sin(a)**2*\cos(a)**3, \text{True}))$

Giac [A] time = 1.11354, size = 282, normalized size = 1.53

$$-\frac{(bd^2x + bcd)\cos(5bx + 5a)}{200b^3} - \frac{(bd^2x + bcd)\cos(3bx + 3a)}{72b^3} + \frac{(bd^2x + bcd)\cos(bx + a)}{4b^3} - \frac{(25b^2d^2x^2 + 50b^2cdx + 25b^2c^2)\sin(5bx + 5a)}{200b^3} - \frac{(9b^2d^2x^2 + 18b^2cdx + 9b^2c^2 - 2d^2)\sin(3bx + 3a)}{432b^3} + \frac{(b^2d^2x^2 + 2b^2cdx + b^2c^2 - 2d^2)\sin(bx + a)}{432b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")

[Out] -1/200*(b*d^2*x + b*c*d)*cos(5*b*x + 5*a)/b^3 - 1/72*(b*d^2*x + b*c*d)*cos(3*b*x + 3*a)/b^3 + 1/4*(b*d^2*x + b*c*d)*cos(b*x + a)/b^3 - 1/2000*(25*b^2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*sin(5*b*x + 5*a)/b^3 - 1/432*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*sin(3*b*x + 3*a)/b^3 + 1/432*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b*x + a)/b^3

3.149 $\int (c + dx) \cos^3(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=109

$$\frac{d \cos(a + bx)}{8b^2} - \frac{d \cos(3a + 3bx)}{144b^2} - \frac{d \cos(5a + 5bx)}{400b^2} + \frac{(c + dx) \sin(a + bx)}{8b} - \frac{(c + dx) \sin(3a + 3bx)}{48b} - \frac{(c + dx) \sin(5a + 5bx)}{80b}$$

[Out] (d*Cos[a + b*x])/(8*b^2) - (d*Cos[3*a + 3*b*x])/(144*b^2) - (d*Cos[5*a + 5*b*x])/(400*b^2) + ((c + d*x)*Sin[a + b*x])/(8*b) - ((c + d*x)*Sin[3*a + 3*b*x])/(48*b) - ((c + d*x)*Sin[5*a + 5*b*x])/(80*b)

Rubi [A] time = 0.0939269, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4406, 3296, 2638}

$$\frac{d \cos(a + bx)}{8b^2} - \frac{d \cos(3a + 3bx)}{144b^2} - \frac{d \cos(5a + 5bx)}{400b^2} + \frac{(c + dx) \sin(a + bx)}{8b} - \frac{(c + dx) \sin(3a + 3bx)}{48b} - \frac{(c + dx) \sin(5a + 5bx)}{80b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]

[Out] (d*Cos[a + b*x])/(8*b^2) - (d*Cos[3*a + 3*b*x])/(144*b^2) - (d*Cos[5*a + 5*b*x])/(400*b^2) + ((c + d*x)*Sin[a + b*x])/(8*b) - ((c + d*x)*Sin[3*a + 3*b*x])/(48*b) - ((c + d*x)*Sin[5*a + 5*b*x])/(80*b)

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (c + dx) \cos^3(a + bx) \sin^2(a + bx) dx &= \int \left(\frac{1}{8}(c + dx) \cos(a + bx) - \frac{1}{16}(c + dx) \cos(3a + 3bx) - \frac{1}{16}(c + dx) \cos(5a + 5bx) \right) \sin^2(a + bx) dx \\ &= -\left(\frac{1}{16} \int (c + dx) \cos(3a + 3bx) dx \right) - \frac{1}{16} \int (c + dx) \cos(5a + 5bx) dx + \frac{1}{8} \int (c + dx) \cos(a + bx) dx \\ &= \frac{(c + dx) \sin(a + bx)}{8b} - \frac{(c + dx) \sin(3a + 3bx)}{48b} - \frac{(c + dx) \sin(5a + 5bx)}{80b} + \frac{d \cos(a + bx)}{8b^2} \\ &= \frac{d \cos(a + bx)}{8b^2} - \frac{d \cos(3a + 3bx)}{144b^2} - \frac{d \cos(5a + 5bx)}{400b^2} + \frac{(c + dx) \sin(a + bx)}{8b} - \frac{(c + dx) \sin(3a + 3bx)}{48b} - \frac{(c + dx) \sin(5a + 5bx)}{80b} \end{aligned}$$

Mathematica [A] time = 0.365338, size = 110, normalized size = 1.01

$$\frac{-450bc \sin(a + bx) + 75bc \sin(3(a + bx)) + 45bc \sin(5(a + bx)) - 450bdx \sin(a + bx) + 75bdx \sin(3(a + bx)) + 45bdx \sin(5(a + bx))}{3600b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]

[Out]
$$\frac{-(-450*d*\text{Cos}[a + b*x] + 25*d*\text{Cos}[3*(a + b*x)] + 9*d*\text{Cos}[5*(a + b*x)] - 450*b*c*\text{Sin}[a + b*x] - 450*b*d*x*\text{Sin}[a + b*x] + 75*b*c*\text{Sin}[3*(a + b*x)] + 75*b*d*x*\text{Sin}[3*(a + b*x)] + 45*b*c*\text{Sin}[5*(a + b*x)] + 45*b*d*x*\text{Sin}[5*(a + b*x)])}{(3600*b^2)}$$

Maple [A] time = 0.023, size = 175, normalized size = 1.6

$$\frac{1}{b} \left(\frac{d}{b} \left(\frac{(bx+a)(2+(\cos(bx+a))^2) \sin(bx+a)}{3} + \frac{(\cos(bx+a))^3}{45} + \frac{2 \cos(bx+a)}{15} - \frac{(bx+a) \sin(bx+a)}{5} \right) \left(\frac{8}{3} + (\cos(bx+a))^2 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x)

[Out]
$$\frac{1}{b} \left(\frac{d}{b} \left(\frac{1}{3} (bx+a) (2 + \cos(bx+a)^2) \sin(bx+a) + \frac{1}{45} \cos(bx+a)^3 + \frac{2}{15} \cos(bx+a) - \frac{1}{5} (bx+a) \sin(bx+a) \right) \left(\frac{8}{3} + \cos(bx+a)^2 \right) + c \left(-\frac{1}{5} \sin(bx+a) \cos(bx+a)^4 + \frac{1}{15} (2 + \cos(bx+a)^2) \sin(bx+a) \right) \right)$$

Maxima [A] time = 1.05287, size = 188, normalized size = 1.72

$$\frac{240 \left(3 \sin(bx+a)^5 - 5 \sin(bx+a)^3 \right) c - \frac{240 \left(3 \sin(bx+a)^5 - 5 \sin(bx+a)^3 \right) ad}{b} + \frac{(45(bx+a) \sin(5bx+5a) + 75(bx+a) \sin(3bx+3a) - 450(bx+a) \sin(bx+a)) d}{3600b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")

[Out]
$$\frac{-1}{3600} \left(240 \left(3 \sin(bx+a)^5 - 5 \sin(bx+a)^3 \right) c - 240 \left(3 \sin(bx+a)^5 - 5 \sin(bx+a)^3 \right) a d / b + (45 (bx+a) \sin(5bx+5a) + 75 (bx+a) \sin(3bx+3a) - 450 (bx+a) \sin(bx+a) + 9 \cos(5bx+5a) + 25 \cos(3bx+3a) - 450 \cos(bx+a)) d / b \right)$$

Fricas [A] time = 0.488231, size = 235, normalized size = 2.16

$$\frac{9d \cos(bx+a)^5 - 5d \cos(bx+a)^3 - 30d \cos(bx+a) + 15 \left(3(bdx+bc) \cos(bx+a)^4 - 2bdx - (bdx+bc) \cos(bx+a) \right) d}{225b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")

[Out] $-1/225*(9*d*\cos(b*x + a)^5 - 5*d*\cos(b*x + a)^3 - 30*d*\cos(b*x + a) + 15*(3*(b*d*x + b*c)*\cos(b*x + a)^4 - 2*b*d*x - (b*d*x + b*c)*\cos(b*x + a)^2 - 2*b*c)*\sin(b*x + a))/b^2$

Sympy [A] time = 5.46708, size = 163, normalized size = 1.5

$$\left\{ \begin{array}{l} \frac{2c \sin^5(a+bx)}{15b} + \frac{c \sin^3(a+bx) \cos^2(a+bx)}{3b} + \frac{2dx \sin^5(a+bx)}{15b} + \frac{dx \sin^3(a+bx) \cos^2(a+bx)}{3b} + \frac{2d \sin^4(a+bx) \cos(a+bx)}{15b^2} + \frac{13d \sin^2(a+bx) \cos^3(a+bx)}{45b^2} \\ \left(cx + \frac{dx^2}{2} \right) \sin^2(a) \cos^3(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*cos(b*x+a)**3*sin(b*x+a)**2,x)`

[Out] `Piecewise((2*c*sin(a + b*x)**5/(15*b) + c*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 2*d*x*sin(a + b*x)**5/(15*b) + d*x*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 2*d*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 13*d*sin(a + b*x)**2*cos(a + b*x)**3/(45*b**2) + 26*d*cos(a + b*x)**5/(225*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*sin(a)**2*cos(a)**3, True))`

Giac [A] time = 1.11593, size = 143, normalized size = 1.31

$$-\frac{d \cos(5bx + 5a)}{400b^2} - \frac{d \cos(3bx + 3a)}{144b^2} + \frac{d \cos(bx + a)}{8b^2} - \frac{(bdx + bc) \sin(5bx + 5a)}{80b^2} - \frac{(bdx + bc) \sin(3bx + 3a)}{48b^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")`

[Out] $-1/400*d*\cos(5*b*x + 5*a)/b^2 - 1/144*d*\cos(3*b*x + 3*a)/b^2 + 1/8*d*\cos(b*x + a)/b^2 - 1/80*(b*d*x + b*c)*\sin(5*b*x + 5*a)/b^2 - 1/48*(b*d*x + b*c)*\sin(3*b*x + 3*a)/b^2 + 1/8*(b*d*x + b*c)*\sin(b*x + a)/b^2$

$$3.150 \quad \int \frac{\cos^3(a+bx) \sin^2(a+bx)}{c+dx} dx$$

Optimal. Leaf size=185

$$\frac{\cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{8d} - \frac{\cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{16d} - \frac{\cos\left(5a - \frac{5bc}{d}\right) \text{CosIntegral}\left(\frac{5bc}{d} + 5bx\right)}{16d}$$

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d) - (Cos[3*a - (3*b*c)/d]
*CosIntegral[(3*b*c)/d + 3*b*x])/(16*d) - (Cos[5*a - (5*b*c)/d]*CosIntegral
[(5*b*c)/d + 5*b*x])/(16*d) - (Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x]
)/(8*d) + (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d) + (Si
n[5*a - (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d)
```

Rubi [A] time = 0.279852, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4406, 3303, 3299, 3302}

$$\frac{\cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{8d} - \frac{\cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{16d} - \frac{\cos\left(5a - \frac{5bc}{d}\right) \text{CosIntegral}\left(\frac{5bc}{d} + 5bx\right)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x), x]
```

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d) - (Cos[3*a - (3*b*c)/d]
*CosIntegral[(3*b*c)/d + 3*b*x])/(16*d) - (Cos[5*a - (5*b*c)/d]*CosIntegral
[(5*b*c)/d + 5*b*x])/(16*d) - (Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x]
)/(8*d) + (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d) + (Si
n[5*a - (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx} dx &= \int \left(\frac{\cos(a+bx)}{8(c+dx)} - \frac{\cos(3a+3bx)}{16(c+dx)} - \frac{\cos(5a+5bx)}{16(c+dx)} \right) dx \\
&= -\left(\frac{1}{16} \int \frac{\cos(3a+3bx)}{c+dx} dx \right) - \frac{1}{16} \int \frac{\cos(5a+5bx)}{c+dx} dx + \frac{1}{8} \int \frac{\cos(a+bx)}{c+dx} dx \\
&= -\left(\frac{1}{16} \cos\left(5a - \frac{5bc}{d}\right) \int \frac{\cos\left(\frac{5bc}{d} + 5bx\right)}{c+dx} dx \right) - \frac{1}{16} \cos\left(3a - \frac{3bc}{d}\right) \int \frac{\cos\left(\frac{3bc}{d} + 3bx\right)}{c+dx} dx \\
&= \frac{\cos\left(a - \frac{bc}{d}\right) \text{Ci}\left(\frac{bc}{d} + bx\right)}{8d} - \frac{\cos\left(3a - \frac{3bc}{d}\right) \text{Ci}\left(\frac{3bc}{d} + 3bx\right)}{16d} - \frac{\cos\left(5a - \frac{5bc}{d}\right) \text{Ci}\left(\frac{5bc}{d} + 5bx\right)}{16d}
\end{aligned}$$

Mathematica [A] time = 0.520874, size = 154, normalized size = 0.83

$$\frac{2 \cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(b\left(\frac{c}{d} + x\right)\right) - \cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right) - \cos\left(5a - \frac{5bc}{d}\right) \text{CosIntegral}\left(\frac{5b(c+dx)}{d}\right)}{16d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x),x]

[Out] (2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] - Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*(c + d*x))/d] - Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b*(c + d*x))/d] - 2*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] + Sin[5*a - (5*b*c)/d]*SinIntegral[(5*b*(c + d*x))/d])/(16*d)

Maple [A] time = 0.023, size = 252, normalized size = 1.4

$$\frac{1}{b} \left(\frac{1}{8} \left(\frac{1}{d} \text{Si}\left(bx + a + \frac{-ad + bc}{d}\right) \sin\left(\frac{-ad + bc}{d}\right) + \frac{1}{d} \text{Ci}\left(bx + a + \frac{-ad + bc}{d}\right) \cos\left(\frac{-ad + bc}{d}\right) \right) - \frac{b}{80} \left(5 \frac{1}{d} \text{Si}\left(5bx + 5a + \frac{-5ad + 5bc}{d}\right) \sin\left(\frac{-5ad + 5bc}{d}\right) + \frac{1}{d} \text{Ci}\left(5bx + 5a + \frac{-5ad + 5bc}{d}\right) \cos\left(\frac{-5ad + 5bc}{d}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x)

[Out] 1/b*(1/8*b*(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-1/80*b*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d-1/48*b*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d))

Maxima [C] time = 1.62652, size = 551, normalized size = 2.98

$$\frac{2b \left(E_1\left(\frac{ibc+i(bx+a)d-id}{d}\right) + E_1\left(-\frac{ibc+i(bx+a)d-id}{d}\right) \right) \cos\left(-\frac{bc-ad}{d}\right) - b \left(E_1\left(\frac{3ibc+3i(bx+a)d-3iad}{d}\right) + E_1\left(-\frac{3ibc+3i(bx+a)d-3iad}{d}\right) \right) \cos\left(-\frac{3bc-3ad}{d}\right) - \frac{b}{80} \left(5 E_1\left(\frac{5ibc+5i(bx+a)d-5iad}{d}\right) + E_1\left(-\frac{5ibc+5i(bx+a)d-5iad}{d}\right) \right) \cos\left(-\frac{5bc-5ad}{d}\right) }{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")

[Out]
$$-1/32*(2*b*(\exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + \exp_integral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*\cos(-(b*c - a*d)/d) - b*(\exp_integral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + \exp_integral_e(1, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*\cos(-3*(b*c - a*d)/d) - b*(\exp_integral_e(1, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) + \exp_integral_e(1, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*\cos(-5*(b*c - a*d)/d) + b*(-2*I*\exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 2*I*\exp_integral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*\sin(-(b*c - a*d)/d) + b*(I*\exp_integral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*\exp_integral_e(1, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*\sin(-3*(b*c - a*d)/d) + b*(I*\exp_integral_e(1, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) - I*\exp_integral_e(1, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*\sin(-5*(b*c - a*d)/d))/(b*d)$$

Fricas [A] time = 0.49701, size = 612, normalized size = 3.31

$$2 \left(\operatorname{Ci} \left(\frac{bdx+bc}{d} \right) + \operatorname{Ci} \left(-\frac{bdx+bc}{d} \right) \right) \cos \left(-\frac{bc-ad}{d} \right) - \left(\operatorname{Ci} \left(\frac{3(bdx+bc)}{d} \right) + \operatorname{Ci} \left(-\frac{3(bdx+bc)}{d} \right) \right) \cos \left(-\frac{3(bc-ad)}{d} \right) - \left(\operatorname{Ci} \left(\frac{5(bdx+bc)}{d} \right) + \operatorname{Ci} \left(-\frac{5(bdx+bc)}{d} \right) \right) \cos \left(-\frac{5(bc-ad)}{d} \right) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")

[Out]
$$1/32*(2*(\cos_integral((b*d*x + b*c)/d) + \cos_integral(-(b*d*x + b*c)/d))*\cos(-(b*c - a*d)/d) - (\cos_integral(3*(b*d*x + b*c)/d) + \cos_integral(-3*(b*d*x + b*c)/d))*\cos(-3*(b*c - a*d)/d) - (\cos_integral(5*(b*d*x + b*c)/d) + \cos_integral(-5*(b*d*x + b*c)/d))*\cos(-5*(b*c - a*d)/d) + 2*\sin(-5*(b*c - a*d)/d)*\sin_integral(5*(b*d*x + b*c)/d) + 2*\sin(-3*(b*c - a*d)/d)*\sin_integral(3*(b*d*x + b*c)/d) - 4*\sin(-(b*c - a*d)/d)*\sin_integral((b*d*x + b*c)/d))/d$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a + bx) \cos^3(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c),x)

[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x), x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.151 \quad \int \frac{\cos^3(a+bx) \sin^2(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=257

$$\frac{5b \sin\left(5a - \frac{5bc}{d}\right) \operatorname{CosIntegral}\left(\frac{5bc}{d} + 5bx\right)}{16d^2} + \frac{3b \sin\left(3a - \frac{3bc}{d}\right) \operatorname{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{16d^2} - \frac{b \sin\left(a - \frac{bc}{d}\right) \operatorname{CosIntegral}\left(\frac{bc}{d} + bx\right)}{8d^2}$$

```
[Out] -Cos[a + b*x]/(8*d*(c + d*x)) + Cos[3*a + 3*b*x]/(16*d*(c + d*x)) + Cos[5*a + 5*b*x]/(16*d*(c + d*x)) + (5*b*CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a - (5*b*c)/d])/(16*d^2) + (3*b*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(16*d^2) - (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(8*d^2) - (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^2) + (3*b*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) + (5*b*Cos[5*a - (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d^2)
```

Rubi [A] time = 0.344944, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{5b \sin\left(5a - \frac{5bc}{d}\right) \operatorname{CosIntegral}\left(\frac{5bc}{d} + 5bx\right)}{16d^2} + \frac{3b \sin\left(3a - \frac{3bc}{d}\right) \operatorname{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{16d^2} - \frac{b \sin\left(a - \frac{bc}{d}\right) \operatorname{CosIntegral}\left(\frac{bc}{d} + bx\right)}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -Cos[a + b*x]/(8*d*(c + d*x)) + Cos[3*a + 3*b*x]/(16*d*(c + d*x)) + Cos[5*a + 5*b*x]/(16*d*(c + d*x)) + (5*b*CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a - (5*b*c)/d])/(16*d^2) + (3*b*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(16*d^2) - (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(8*d^2) - (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^2) + (3*b*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) + (5*b*Cos[5*a - (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2} dx &= \int \left(\frac{\cos(a+bx)}{8(c+dx)^2} - \frac{\cos(3a+3bx)}{16(c+dx)^2} - \frac{\cos(5a+5bx)}{16(c+dx)^2} \right) dx \\ &= -\left(\frac{1}{16} \int \frac{\cos(3a+3bx)}{(c+dx)^2} dx \right) - \frac{1}{16} \int \frac{\cos(5a+5bx)}{(c+dx)^2} dx + \frac{1}{8} \int \frac{\cos(a+bx)}{(c+dx)^2} dx \\ &= -\frac{\cos(a+bx)}{8d(c+dx)} + \frac{\cos(3a+3bx)}{16d(c+dx)} + \frac{\cos(5a+5bx)}{16d(c+dx)} - \frac{b \int \frac{\sin(a+bx)}{c+dx} dx}{8d} + \frac{(3b) \int \frac{\sin(3a+3bx)}{c+dx} dx}{16d} \\ &= -\frac{\cos(a+bx)}{8d(c+dx)} + \frac{\cos(3a+3bx)}{16d(c+dx)} + \frac{\cos(5a+5bx)}{16d(c+dx)} + \frac{\left(5b \cos\left(5a - \frac{5bc}{d}\right)\right) \int \frac{\sin\left(\frac{5bc}{d} + 5bx\right)}{c+dx} dx}{16d} \\ &= -\frac{\cos(a+bx)}{8d(c+dx)} + \frac{\cos(3a+3bx)}{16d(c+dx)} + \frac{\cos(5a+5bx)}{16d(c+dx)} + \frac{5b \operatorname{Ci}\left(\frac{5bc}{d} + 5bx\right) \sin\left(5a - \frac{5bc}{d}\right)}{16d^2} + \dots \end{aligned}$$

Mathematica [A] time = 2.13917, size = 212, normalized size = 0.82

$$-2 \left(b \sin\left(a - \frac{bc}{d}\right) \operatorname{CosIntegral}\left(b\left(\frac{c}{d} + x\right)\right) + b \cos\left(a - \frac{bc}{d}\right) \operatorname{Si}\left(b\left(\frac{c}{d} + x\right)\right) + \frac{d \cos(a+bx)}{c+dx} \right) + 5b \sin\left(5a - \frac{5bc}{d}\right) \operatorname{CosIntegral}\left(b\left(\frac{c}{d} + x\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^2,x]

[Out] ((d*Cos[3*(a + b*x)])/(c + d*x) + (d*Cos[5*(a + b*x)])/(c + d*x) + 5*b*CosIntegral[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d] + 3*b*CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] - 2*((d*Cos[a + b*x])/(c + d*x) + b*CosIntegral[b*(c/d + x)]*Sin[a - (b*c)/d] + b*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)]) + 3*b*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] + 5*b*Cos[5*a - (5*b*c)/d]*SinIntegral[(5*b*(c + d*x))/d])/(16*d^2)

Maple [A] time = 0.026, size = 367, normalized size = 1.4

$$\frac{1}{b} \left(\frac{b^2}{8} \left(-\frac{\cos(bx+a)}{((bx+a)d - ad + bc)d} - \frac{1}{d} \left(\frac{1}{d} \operatorname{Si}\left(bx+a + \frac{-ad+bc}{d}\right) \cos\left(\frac{-ad+bc}{d}\right) - \frac{1}{d} \operatorname{Ci}\left(bx+a + \frac{-ad+bc}{d}\right) \sin\left(\frac{-ad+bc}{d}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x)

[Out] 1/b*(1/8*b^2*(-cos(b*x+a)/((b*x+a)*d-a*d+b*c)/d-(Si(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d-1/80*b^2*(-

$$5*\cos(5*b*x+5*a)/((b*x+a)*d-a*d+b*c)/d-5*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*\cos(5*(-a*d+b*c)/d)/d-5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*\sin(5*(-a*d+b*c)/d)/d)/d-1/48*b^2*(-3*\cos(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)/d-3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*\cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*\sin(3*(-a*d+b*c)/d)/d)/d)$$

Maxima [C] time = 1.96772, size = 593, normalized size = 2.31

$$1073741824 b^2 \left(E_2 \left(\frac{i bc+i(bx+a)d-i ad}{d} \right) + E_2 \left(-\frac{i bc+i(bx+a)d-i ad}{d} \right) \right) \cos \left(-\frac{bc-ad}{d} \right) - 536870912 b^2 \left(E_2 \left(\frac{3i bc+3i(bx+a)d-3i ad}{d} \right) + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")

[Out] -1/17179869184*(1073741824*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - 536870912*b^2*(exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d) - 536870912*b^2*(exp_integral_e(2, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) + exp_integral_e(2, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*cos(-5*(b*c - a*d)/d) + b^2*(-1073741824*I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 1073741824*I*exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^2*(536870912*I*exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - 536870912*I*exp_integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d) + b^2*(536870912*I*exp_integral_e(2, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) - 536870912*I*exp_integral_e(2, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*sin(-5*(b*c - a*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)

Fricas [A] time = 0.63162, size = 867, normalized size = 3.37

$$32 d \cos(bx + a)^5 - 32 d \cos(bx + a)^3 + 10(bdx + bc) \cos\left(-\frac{5(bc-ad)}{d}\right) \text{Si}\left(\frac{5(bdx+bc)}{d}\right) + 6(bdx + bc) \cos\left(-\frac{3(bc-ad)}{d}\right) \text{Si}\left(\frac{3(bdx+bc)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")

[Out] 1/32*(32*d*cos(b*x + a)^5 - 32*d*cos(b*x + a)^3 + 10*(b*d*x + b*c)*cos(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) + 6*(b*d*x + b*c)*cos(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - 4*(b*d*x + b*c)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) - 2*((b*d*x + b*c)*cos_integral((b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/d) + 3*((b*d*x + b*c)*cos_integral(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-3*(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d) + 5*((b*d*x + b*c)*cos_integral(5*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-5*(b*d*x + b*c)/d))*sin(-5*(b*c - a*d)/d))/(d^3*x + c*d^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a + bx) \cos^3(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c)**2,x)

[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx + a)^3 \sin(bx + a)^2}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(cos(b*x + a)^3*sin(b*x + a)^2/(d*x + c)^2, x)

$$3.152 \quad \int \frac{\cos^3(a+bx) \sin^2(a+bx)}{(c+dx)^3} dx$$

Optimal. Leaf size=338

$$\frac{b^2 \cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{16d^3} + \frac{9b^2 \cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{32d^3} + \frac{25b^2 \cos\left(5a - \frac{5bc}{d}\right) \text{CosIntegral}\left(\frac{5bc}{d} + 5bx\right)}{32d^3}$$

```
[Out] -Cos[a + b*x]/(16*d*(c + d*x)^2) + Cos[3*a + 3*b*x]/(32*d*(c + d*x)^2) + Cos[5*a + 5*b*x]/(32*d*(c + d*x)^2) - (b^2*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(16*d^3) + (9*b^2*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(32*d^3) + (25*b^2*Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b*c)/d + 5*b*x])/(32*d^3) + (b*Sin[a + b*x])/(16*d^2*(c + d*x)) - (3*b*Sin[3*a + 3*b*x])/(32*d^2*(c + d*x)) - (5*b*Sin[5*a + 5*b*x])/(32*d^2*(c + d*x)) + (b^2*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(16*d^3) - (9*b^2*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(32*d^3) - (25*b^2*Sin[5*a - (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(32*d^3)
```

Rubi [A] time = 0.4378, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b^2 \cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{16d^3} + \frac{9b^2 \cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{32d^3} + \frac{25b^2 \cos\left(5a - \frac{5bc}{d}\right) \text{CosIntegral}\left(\frac{5bc}{d} + 5bx\right)}{32d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] -Cos[a + b*x]/(16*d*(c + d*x)^2) + Cos[3*a + 3*b*x]/(32*d*(c + d*x)^2) + Cos[5*a + 5*b*x]/(32*d*(c + d*x)^2) - (b^2*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(16*d^3) + (9*b^2*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(32*d^3) + (25*b^2*Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b*c)/d + 5*b*x])/(32*d^3) + (b*Sin[a + b*x])/(16*d^2*(c + d*x)) - (3*b*Sin[3*a + 3*b*x])/(32*d^2*(c + d*x)) - (5*b*Sin[5*a + 5*b*x])/(32*d^2*(c + d*x)) + (b^2*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(16*d^3) - (9*b^2*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(32*d^3) - (25*b^2*Sin[5*a - (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(32*d^3)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a + bx) \sin^2(a + bx)}{(c + dx)^3} dx &= \int \left(\frac{\cos(a + bx)}{8(c + dx)^3} - \frac{\cos(3a + 3bx)}{16(c + dx)^3} - \frac{\cos(5a + 5bx)}{16(c + dx)^3} \right) dx \\
&= -\left(\frac{1}{16} \int \frac{\cos(3a + 3bx)}{(c + dx)^3} dx \right) - \frac{1}{16} \int \frac{\cos(5a + 5bx)}{(c + dx)^3} dx + \frac{1}{8} \int \frac{\cos(a + bx)}{(c + dx)^3} dx \\
&= -\frac{\cos(a + bx)}{16d(c + dx)^2} + \frac{\cos(3a + 3bx)}{32d(c + dx)^2} + \frac{\cos(5a + 5bx)}{32d(c + dx)^2} - \frac{b \int \frac{\sin(a+bx)}{(c+dx)^2} dx}{16d} + \frac{(3b) \int \frac{\sin(3a+3bx)}{(c+dx)^2} dx}{32d} \\
&= -\frac{\cos(a + bx)}{16d(c + dx)^2} + \frac{\cos(3a + 3bx)}{32d(c + dx)^2} + \frac{\cos(5a + 5bx)}{32d(c + dx)^2} + \frac{b \sin(a + bx)}{16d^2(c + dx)} - \frac{3b \sin(3a + 3bx)}{32d^2(c + dx)} \\
&= -\frac{\cos(a + bx)}{16d(c + dx)^2} + \frac{\cos(3a + 3bx)}{32d(c + dx)^2} + \frac{\cos(5a + 5bx)}{32d(c + dx)^2} + \frac{b \sin(a + bx)}{16d^2(c + dx)} - \frac{3b \sin(3a + 3bx)}{32d^2(c + dx)} \\
&= -\frac{\cos(a + bx)}{16d(c + dx)^2} + \frac{\cos(3a + 3bx)}{32d(c + dx)^2} + \frac{\cos(5a + 5bx)}{32d(c + dx)^2} - \frac{b^2 \cos\left(a - \frac{bc}{d}\right) \text{Ci}\left(\frac{bc}{d} + bx\right)}{16d^3} + \frac{9b^2}{16d^3}
\end{aligned}$$

Mathematica [A] time = 3.29578, size = 283, normalized size = 0.84

$$-2b^2 \cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(b\left(\frac{c}{d} + x\right)\right) + 9b^2 \cos\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right) + 25b^2 \cos\left(5a - \frac{5bc}{d}\right) \text{CosIntegral}\left(\frac{5b(c+dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] ((d^2*Cos[3*(a + b*x)])/(c + d*x)^2 + (d^2*Cos[5*(a + b*x)])/(c + d*x)^2 -
2*b^2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] + 9*b^2*Cos[3*a - (3*b*c)/d
]*CosIntegral[(3*b*(c + d*x))/d] + 25*b^2*Cos[5*a - (5*b*c)/d]*CosIntegral[
(5*b*(c + d*x))/d] + (2*d*(-(d*Cos[a + b*x]) + b*(c + d*x)*Sin[a + b*x]))/(
c + d*x)^2 - (3*b*d*Sin[3*(a + b*x)])/(c + d*x) - (5*b*d*Sin[5*(a + b*x)])/(
c + d*x) + 2*b^2*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 9*b^2*Sin[3*a
- (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] - 25*b^2*Sin[5*a - (5*b*c)/d]*
SinIntegral[(5*b*(c + d*x))/d])/(32*d^3)
```

Maple [A] time = 0.026, size = 473, normalized size = 1.4

$$\frac{1}{b} \left(\frac{b^3}{8} \left(-\frac{\cos(bx+a)}{2((bx+a)d-ad+bc)^2 d} - \frac{1}{2d} \left(-\frac{\sin(bx+a)}{((bx+a)d-ad+bc)d} + \frac{1}{d} \left(\frac{1}{d} \operatorname{Si} \left(bx+a + \frac{-ad+bc}{d} \right) \sin \left(\frac{-ad+bc}{d} \right) \right) \right) \right) + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x)

[Out] 1/b*(1/8*b^3*(-1/2*cos(b*x+a)/((b*x+a)*d-a*d+b*c)^2/d-1/2*(-sin(b*x+a)/((b*x+a)*d-a*d+b*c)/d+(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)-1/80*b^3*(-5/2*cos(5*b*x+5*a)/((b*x+a)*d-a*d+b*c)^2/d-5/2*(-5*sin(5*b*x+5*a)/((b*x+a)*d-a*d+b*c)/d+5*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d)/d)-1/48*b^3*(-3/2*cos(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)^2/d-3/2*(-3*sin(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)/d+3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d)/d)

Maxima [C] time = 2.60331, size = 640, normalized size = 1.89

$$\frac{1073741824 b^3 \left(E_3 \left(\frac{ibc+i(bx+a)d-iad}{d} \right) + E_3 \left(-\frac{ibc+i(bx+a)d-iad}{d} \right) \right) \cos \left(-\frac{bc-ad}{d} \right) - 536870912 b^3 \left(E_3 \left(\frac{3ibc+3i(bx+a)d-3iad}{d} \right) + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")

[Out] -1/17179869184*(1073741824*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - 536870912*b^3*(exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d) - 536870912*b^3*(exp_integral_e(3, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) + exp_integral_e(3, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*cos(-5*(b*c - a*d)/d) + b^3*(-1073741824*I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 1073741824*I*exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^3*(536870912*I*exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - 536870912*I*exp_integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d) + b^3*(536870912*I*exp_integral_e(3, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) - 536870912*I*exp_integral_e(3, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*sin(-5*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)

Fricas [A] time = 0.723896, size = 1312, normalized size = 3.88

$$32 d^2 \cos(bx+a)^5 - 32 d^2 \cos(bx+a)^3 - 50 (b^2 d^2 x^2 + 2 b^2 c d x + b^2 c^2) \sin \left(-\frac{5(bc-ad)}{d} \right) \operatorname{Si} \left(\frac{5(bdx+bc)}{d} \right) - 18 (b^2 d^2 x^2 + 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")

```
[Out] 1/64*(32*d^2*cos(b*x + a)^5 - 32*d^2*cos(b*x + a)^3 - 50*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) - 18*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-(b*d*x + b*c)/d))*cos(-(b*c - a*d)/d) + 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)/d) + 25*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(5*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-5*(b*d*x + b*c)/d))*cos(-5*(b*c - a*d)/d) - 32*(5*(b*d^2*x + b*c*d)*cos(b*x + a)^4 - 3*(b*d^2*x + b*c*d)*cos(b*x + a)^2)*sin(b*x + a)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a + bx) \cos^3(a + bx)}{(c + dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```


$$3.153 \quad \int \frac{\cos^3(a+bx) \sin^2(a+bx)}{(c+dx)^4} dx$$

Optimal. Leaf size=413

$$\frac{125b^3 \sin\left(5a - \frac{5bc}{d}\right) \text{CosIntegral}\left(\frac{5bc}{d} + 5bx\right)}{96d^4} - \frac{9b^3 \sin\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{32d^4} + \frac{b^3 \sin\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{48d^4}$$

[Out] $-\text{Cos}[a + b*x]/(24*d*(c + d*x)^3) + (b^2*\text{Cos}[a + b*x])/(48*d^3*(c + d*x)) + \text{Cos}[3*a + 3*b*x]/(48*d*(c + d*x)^3) - (3*b^2*\text{Cos}[3*a + 3*b*x])/(32*d^3*(c + d*x)) + \text{Cos}[5*a + 5*b*x]/(48*d*(c + d*x)^3) - (25*b^2*\text{Cos}[5*a + 5*b*x])/(96*d^3*(c + d*x)) - (125*b^3*\text{CosIntegral}[(5*b*c)/d + 5*b*x]*\text{Sin}[5*a - (5*b*c)/d])/(96*d^4) - (9*b^3*\text{CosIntegral}[(3*b*c)/d + 3*b*x]*\text{Sin}[3*a - (3*b*c)/d])/(32*d^4) + (b^3*\text{CosIntegral}[(b*c)/d + b*x]*\text{Sin}[a - (b*c)/d])/(48*d^4) + (b*\text{Sin}[a + b*x])/(48*d^2*(c + d*x)^2) - (b*\text{Sin}[3*a + 3*b*x])/(32*d^2*(c + d*x)^2) - (5*b*\text{Sin}[5*a + 5*b*x])/(96*d^2*(c + d*x)^2) + (b^3*\text{Cos}[a - (b*c)/d]*\text{SinIntegral}[(b*c)/d + b*x])/(48*d^4) - (9*b^3*\text{Cos}[3*a - (3*b*c)/d]*\text{SinIntegral}[(3*b*c)/d + 3*b*x])/(32*d^4) - (125*b^3*\text{Cos}[5*a - (5*b*c)/d]*\text{SinIntegral}[(5*b*c)/d + 5*b*x])/(96*d^4)$

Rubi [A] time = 0.538389, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{125b^3 \sin\left(5a - \frac{5bc}{d}\right) \text{CosIntegral}\left(\frac{5bc}{d} + 5bx\right)}{96d^4} - \frac{9b^3 \sin\left(3a - \frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d} + 3bx\right)}{32d^4} + \frac{b^3 \sin\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{48d^4}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^4,x]

[Out] $-\text{Cos}[a + b*x]/(24*d*(c + d*x)^3) + (b^2*\text{Cos}[a + b*x])/(48*d^3*(c + d*x)) + \text{Cos}[3*a + 3*b*x]/(48*d*(c + d*x)^3) - (3*b^2*\text{Cos}[3*a + 3*b*x])/(32*d^3*(c + d*x)) + \text{Cos}[5*a + 5*b*x]/(48*d*(c + d*x)^3) - (25*b^2*\text{Cos}[5*a + 5*b*x])/(96*d^3*(c + d*x)) - (125*b^3*\text{CosIntegral}[(5*b*c)/d + 5*b*x]*\text{Sin}[5*a - (5*b*c)/d])/(96*d^4) - (9*b^3*\text{CosIntegral}[(3*b*c)/d + 3*b*x]*\text{Sin}[3*a - (3*b*c)/d])/(32*d^4) + (b^3*\text{CosIntegral}[(b*c)/d + b*x]*\text{Sin}[a - (b*c)/d])/(48*d^4) + (b*\text{Sin}[a + b*x])/(48*d^2*(c + d*x)^2) - (b*\text{Sin}[3*a + 3*b*x])/(32*d^2*(c + d*x)^2) - (5*b*\text{Sin}[5*a + 5*b*x])/(96*d^2*(c + d*x)^2) + (b^3*\text{Cos}[a - (b*c)/d]*\text{SinIntegral}[(b*c)/d + b*x])/(48*d^4) - (9*b^3*\text{Cos}[3*a - (3*b*c)/d]*\text{SinIntegral}[(3*b*c)/d + 3*b*x])/(32*d^4) - (125*b^3*\text{Cos}[5*a - (5*b*c)/d]*\text{SinIntegral}[(5*b*c)/d + 5*b*x])/(96*d^4)$

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4} dx &= \int \left(\frac{\cos(a+bx)}{8(c+dx)^4} - \frac{\cos(3a+3bx)}{16(c+dx)^4} - \frac{\cos(5a+5bx)}{16(c+dx)^4} \right) dx \\
&= -\left(\frac{1}{16} \int \frac{\cos(3a+3bx)}{(c+dx)^4} dx \right) - \frac{1}{16} \int \frac{\cos(5a+5bx)}{(c+dx)^4} dx + \frac{1}{8} \int \frac{\cos(a+bx)}{(c+dx)^4} dx \\
&= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} + \frac{\cos(5a+5bx)}{48d(c+dx)^3} - \frac{b \int \frac{\sin(a+bx)}{(c+dx)^3} dx}{24d} + \frac{b \int \frac{\sin(3a+3bx)}{(c+dx)^3} dx}{16d} \\
&= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} + \frac{\cos(5a+5bx)}{48d(c+dx)^3} + \frac{b \sin(a+bx)}{48d^2(c+dx)^2} - \frac{b \sin(3a+3bx)}{32d^2(c+dx)^2} \\
&= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{b^2 \cos(a+bx)}{48d^3(c+dx)} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} - \frac{3b^2 \cos(3a+3bx)}{32d^3(c+dx)} + \frac{\cos(5a+5bx)}{48d(c+dx)^3} \\
&= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{b^2 \cos(a+bx)}{48d^3(c+dx)} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} - \frac{3b^2 \cos(3a+3bx)}{32d^3(c+dx)} + \frac{\cos(5a+5bx)}{48d(c+dx)^3} \\
&= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{b^2 \cos(a+bx)}{48d^3(c+dx)} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} - \frac{3b^2 \cos(3a+3bx)}{32d^3(c+dx)} + \frac{\cos(5a+5bx)}{48d(c+dx)^3}
\end{aligned}$$

Mathematica [A] time = 3.47955, size = 451, normalized size = 1.09

$$-2 \left(b^3(c+dx)^3 \left(\sin \left(a - \frac{bc}{d} \right) \text{CosIntegral} \left(b \left(\frac{c}{d} + x \right) \right) + \cos \left(a - \frac{bc}{d} \right) \text{Si} \left(b \left(\frac{c}{d} + x \right) \right) \right) + d \cos(bx) \left(\cos(a) \left(b^2(c+dx)^2 - 2 \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^4,x]

```
[Out] -(d*Cos[3*b*x]*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*x)*Sin
[3*a]) + d*Cos[5*b*x]*((-2*d^2 + 25*b^2*(c + d*x)^2)*Cos[5*a] + 5*b*d*(c +
d*x)*Sin[5*a]) + d*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)
*Sin[3*a])*Sin[3*b*x] + d*(5*b*d*(c + d*x)*Cos[5*a] - (-2*d^2 + 25*b^2*(c +
d*x)^2)*Sin[5*a])*Sin[5*b*x] - 2*(d*Cos[b*x]*((-2*d^2 + b^2*(c + d*x)^2)*C
```

$$\begin{aligned} & \cos[a] + b*d*(c + d*x)*\sin[a] + d*(b*d*(c + d*x)*\cos[a] - (-2*d^2 + b^2*(c \\ & + d*x)^2)*\sin[a])* \sin[b*x] + b^3*(c + d*x)^3*(\cos\text{Integral}[b*(c/d + x)]*\sin[\\ & a - (b*c)/d] + \cos[a - (b*c)/d]*\sin\text{Integral}[b*(c/d + x)]) + 27*b^3*(c + d* \\ & x)^3*(\cos\text{Integral}[(3*b*(c + d*x))/d]*\sin[3*a - (3*b*c)/d] + \cos[3*a - (3*b* \\ & c)/d]*\sin\text{Integral}[(3*b*(c + d*x))/d]) + 125*b^3*(c + d*x)^3*(\cos\text{Integral}[(5 \\ & *b*(c + d*x))/d]*\sin[5*a - (5*b*c)/d] + \cos[5*a - (5*b*c)/d]*\sin\text{Integral}[(5 \\ & *b*(c + d*x))/d]))/(96*d^4*(c + d*x)^3) \end{aligned}$$

Maple [A] time = 0.024, size = 583, normalized size = 1.4

$$\frac{1}{b} \left(\frac{b^4}{8} \left(-\frac{\cos(bx+a)}{3((bx+a)d-ad+bc)^3 d} - \frac{1}{3d} \left(-\frac{\sin(bx+a)}{2((bx+a)d-ad+bc)^2 d} + \frac{1}{2d} \left(-\frac{\cos(bx+a)}{((bx+a)d-ad+bc)d} - \frac{1}{d} \left(\frac{1}{d} \text{Si} \left(bx \right. \right. \right. \right. \right. \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x)

[Out] $\frac{1}{b} \left(\frac{1}{8} b^4 \left(-\frac{1}{3} \cos(b*x+a) / ((b*x+a)*d-a*d+b*c)^3 / d - \frac{1}{3} \left(-\frac{1}{2} \sin(b*x+a) / ((b*x+a)*d-a*d+b*c)^2 / d + \frac{1}{2} \left(-\cos(b*x+a) / ((b*x+a)*d-a*d+b*c) / d - \text{Si}(b*x+a+(-a*d+b*c)/d) * \cos((-a*d+b*c)/d) / d - \text{Ci}(b*x+a+(-a*d+b*c)/d) * \sin((-a*d+b*c)/d) / d \right) / d \right) - \frac{1}{80} b^4 \left(-\frac{5}{3} \cos(5*b*x+5*a) / ((b*x+a)*d-a*d+b*c)^3 / d - \frac{5}{3} \left(-\frac{5}{2} \sin(5*b*x+5*a) / ((b*x+a)*d-a*d+b*c)^2 / d + \frac{5}{2} \left(-5 \cos(5*b*x+5*a) / ((b*x+a)*d-a*d+b*c) / d - 5 \left(5 \text{Si}(5*b*x+5*a+5*(-a*d+b*c)/d) * \cos(5*(-a*d+b*c)/d) / d - 5 \text{Ci}(5*b*x+5*a+5*(-a*d+b*c)/d) * \sin(5*(-a*d+b*c)/d) / d \right) / d \right) - \frac{1}{48} b^4 \left(-\cos(3*b*x+3*a) / ((b*x+a)*d-a*d+b*c)^3 / d - \left(-\frac{3}{2} \sin(3*b*x+3*a) / ((b*x+a)*d-a*d+b*c)^2 / d + \frac{3}{2} \left(-3 \cos(3*b*x+3*a) / ((b*x+a)*d-a*d+b*c) / d - 3 \left(3 \text{Si}(3*b*x+3*a+3*(-a*d+b*c)/d) * \cos(3*(-a*d+b*c)/d) / d - 3 \text{Ci}(3*b*x+3*a+3*(-a*d+b*c)/d) * \sin(3*(-a*d+b*c)/d) / d \right) / d \right) / d \right) \right)$

Maxima [C] time = 3.97539, size = 707, normalized size = 1.71

$$\frac{1073741824 b^4 \left(E_4 \left(\frac{i bc+i(bx+a)d-i ad}{d} \right) + E_4 \left(-\frac{i bc+i(bx+a)d-i ad}{d} \right) \right) \cos \left(-\frac{bc-ad}{d} \right) - 536870912 b^4 \left(E_4 \left(\frac{3i bc+3i(bx+a)d-3i ad}{d} \right) + E_4 \left(-\frac{3i bc+3i(bx+a)d-3i ad}{d} \right) \right) \cos \left(-\frac{bc-ad}{d} \right) + \dots}{(b^3 c^3 d - 3 a b^2 c^2 d^2 + 3 a^2 b c d^3 + (b x + a)^3 d^4 - a^3 d^4 + 3 (b c d^3 - a d^4) (b x + a)^2 + 3 (b^2 c^2 d^2 - a^2 d^4) (b x + a) + a^2 d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")

[Out] $-1/17179869184*(1073741824*b^4*(\exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + \exp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*\cos(-(b*c - a*d)/d) - 536870912*b^4*(\exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + \exp_integral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*\cos(-3*(b*c - a*d)/d) - 536870912*b^4*(\exp_integral_e(4, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) + \exp_integral_e(4, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*\cos(-5*(b*c - a*d)/d) + b^4*(-1073741824*I*\exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 1073741824*I*\exp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*\sin(-(b*c - a*d)/d) + b^4*(536870912*I*\exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - 536870912*I*\exp_integral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*\sin(-3*(b*c - a*d)/d) + b^4*(536870912*I*\exp_integral_e(4, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) - 536870912*I*\exp_integral_e(4, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*\sin(-5*(b*c - a*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - a^2*d^4)*(b*x + a) + a^2*d^4))$

$$2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)$$

Fricas [B] time = 0.844789, size = 1814, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/192*(32*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*\cos(b*x \\ & + a)^5 - 32*(29*b^2*d^3*x^2 + 58*b^2*c*d^2*x + 29*b^2*c^2*d - 2*d^3)*\cos(b \\ & *x + a)^3 + 250*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*c \\ & \cos(-5*(b*c - a*d)/d)*\sin_integral(5*(b*d*x + b*c)/d) + 54*(b^3*d^3*x^3 + 3* \\ & b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\cos(-3*(b*c - a*d)/d)*\sin_integral \\ & (3*(b*d*x + b*c)/d) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^ \\ & 3*c^3)*\cos(-(b*c - a*d)/d)*\sin_integral((b*d*x + b*c)/d) + 192*(b^2*d^3*x^2 \\ & + 2*b^2*c*d^2*x + b^2*c^2*d)*\cos(b*x + a) + 32*(5*(b*d^3*x + b*c*d^2)*\cos(\\ & b*x + a)^4 - 3*(b*d^3*x + b*c*d^2)*\cos(b*x + a)^2)*\sin(b*x + a) - 2*((b^3*d \\ & ^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\cos_integral((b*d*x + b \\ & *c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\cos_inte \\ & gral(-(b*d*x + b*c)/d))*\sin(-(b*c - a*d)/d) + 27*((b^3*d^3*x^3 + 3*b^3*c*d^ \\ & 2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\cos_integral(3*(b*d*x + b*c)/d) + (b^3*d^3 \\ & *x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\cos_integral(-3*(b*d*x + \\ & b*c)/d))*\sin(-3*(b*c - a*d)/d) + 125*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^ \\ & 3*c^2*d*x + b^3*c^3)*\cos_integral(5*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^ \\ & 3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\cos_integral(-5*(b*d*x + b*c)/d))*\sin \\ & (-5*(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c)**4,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")

[Out] Exception raised: NotImplementedError

3.154 $\int (c + dx)^m \cos^3(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=285

$$\frac{3 \cdot 2^{-m-7} e^{2i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{2ib(c+dx)}{d}\right)}{b} + \frac{2^{-m-7} 3^{-m-1} e^{6i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{6ib(c+dx)}{d}\right)}{b}$$

```
[Out] (-3*2^(-7 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d])/(b*((-I)*b*(c + d*x))/d)^m - (3*2^(-7 - m)*(c + d*x)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + (2^(-7 - m)*3^(-1 - m)*E^((6*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-6*I)*b*(c + d*x))/d])/(b*((-I)*b*(c + d*x))/d)^m + (2^(-7 - m)*3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((6*I)*b*(c + d*x))/d])/(b*E^((6*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.316992, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3308, 2181}

$$\frac{3 \cdot 2^{-m-7} e^{2i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{2ib(c+dx)}{d}\right)}{b} + \frac{2^{-m-7} 3^{-m-1} e^{6i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{6ib(c+dx)}{d}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m * Cos[a + b*x]^3 * Sin[a + b*x]^3, x]
```

```
[Out] (-3*2^(-7 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d])/(b*((-I)*b*(c + d*x))/d)^m - (3*2^(-7 - m)*(c + d*x)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + (2^(-7 - m)*3^(-1 - m)*E^((6*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-6*I)*b*(c + d*x))/d])/(b*((-I)*b*(c + d*x))/d)^m + (2^(-7 - m)*3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((6*I)*b*(c + d*x))/d])/(b*E^((6*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n * Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, -(f*g*Log[F])/d]*(c + d*x)]/(d*(-(f*g*Log[F])/d)^(IntPart[m] + 1)*(-(f*g*Log[F])*(c + d*x)/d)^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^m \cos^3(a + bx) \sin^3(a + bx) dx &= \int \left(\frac{3}{32} (c + dx)^m \sin(2a + 2bx) - \frac{1}{32} (c + dx)^m \sin(6a + 6bx) \right) dx \\
&= -\left(\frac{1}{32} \int (c + dx)^m \sin(6a + 6bx) dx \right) + \frac{3}{32} \int (c + dx)^m \sin(2a + 2bx) dx \\
&= -\left(\frac{1}{64} i \int e^{-i(6a+6bx)} (c + dx)^m dx \right) + \frac{1}{64} i \int e^{i(6a+6bx)} (c + dx)^m dx + \frac{3}{64} i \int e^{-i(2a+2bx)} (c + dx)^m dx \\
&= -\frac{3 \cdot 2^{-7-m} e^{2i\left(a-\frac{bc}{d}\right)} (c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, -\frac{2ib(c+dx)}{d}\right)}{b} - \frac{3 \cdot 2^{-7-m} e^{-2i\left(a+\frac{bc}{d}\right)} (c + dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, \frac{2ib(c+dx)}{d}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 3.41225, size = 255, normalized size = 0.89

$$2^{-m-7} 3^{-m-1} e^{-\frac{6i(ad+bc)}{d}} (c + dx)^m \left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m} \left(-3^{m+2} e^{4ia + \frac{8ibc}{d}} \left(-\frac{ib(c+dx)}{d}\right)^m \Gamma\left(m + 1, \frac{2ib(c+dx)}{d}\right) - 3^{m+2} e^{4i\left(2a + \frac{bc}{d}\right)} \left(\frac{ib(c+dx)}{d}\right)^m \Gamma\left(m + 1, -\frac{2ib(c+dx)}{d}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^m * Cos[a + b*x]^3 * Sin[a + b*x]^3, x]

[Out] (2^(-7 - m) * 3^(-1 - m) * (c + d*x)^m * (-3^(2 + m) * E^(((4*I)*(2*a + (b*c)/d)) * (I*b*(c + d*x))/d)^m * Gamma[1 + m, ((-2*I)*b*(c + d*x))/d]) - 3^(2 + m) * E^(((4*I)*a + ((8*I)*b*c)/d) * (((-I)*b*(c + d*x))/d)^m * Gamma[1 + m, ((2*I)*b*(c + d*x))/d] + E^(((12*I)*a) * ((I*b*(c + d*x))/d)^m * Gamma[1 + m, ((-6*I)*b*(c + d*x))/d] + E^(((12*I)*b*c)/d) * (((-I)*b*(c + d*x))/d)^m * Gamma[1 + m, ((6*I)*b*(c + d*x))/d]) / (b * E^(((6*I)*(b*c + a*d))/d) * ((b^2*(c + d*x)^2)/d^2)^m)

Maple [F] time = 0.264, size = 0, normalized size = 0.

$$\int (dx + c)^m (\cos(bx + a))^3 (\sin(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x)

[Out] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a)^3 \sin(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")

[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^3, x)

Fricas [A] time = 0.556983, size = 487, normalized size = 1.71

$$\frac{e^{\left(-\frac{dm \log\left(\frac{6i b}{d}\right) - 6i bc + 6i ad}{d}\right)} \Gamma\left(m + 1, \frac{6i b dx + 6i bc}{d}\right) - 9 e^{\left(-\frac{dm \log\left(\frac{2i b}{d}\right) - 2i bc + 2i ad}{d}\right)} \Gamma\left(m + 1, \frac{2i b dx + 2i bc}{d}\right) - 9 e^{\left(-\frac{dm \log\left(-\frac{2i b}{d}\right) + 2i bc - 2i ad}{d}\right)} \Gamma\left(m + 1, \frac{-2i b dx - 2i bc}{d}\right) + e^{\left(-\frac{dm \log\left(-\frac{6i b}{d}\right) + 6i bc - 6i ad}{d}\right)} \Gamma\left(m + 1, \frac{-6i b dx - 6i bc}{d}\right)}{384 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")

[Out] $\frac{1}{384} \left(e^{-\frac{dm \log(6I*b/d) - 6I*b*c + 6I*a*d}{d}} \Gamma(m + 1, (6I*b*d*x + 6I*b*c)/d) - 9 e^{-\frac{dm \log(2I*b/d) - 2I*b*c + 2I*a*d}{d}} \Gamma(m + 1, (2I*b*d*x + 2I*b*c)/d} - 9 e^{-\frac{dm \log(-2I*b/d) + 2I*b*c - 2I*a*d}{d}} \Gamma(m + 1, (-2I*b*d*x - 2I*b*c)/d} + e^{-\frac{dm \log(-6I*b/d) + 6I*b*c - 6I*a*d}{d}} \Gamma(m + 1, (-6I*b*d*x - 6I*b*c)/d} \right) / b$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**m*cos(b*x+a)**3*sin(b*x+a)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a)^3 \sin(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")

[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^3, x)

3.155 $\int (c + dx)^4 \cos^3(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=233

$$-\frac{9d^3(c + dx) \sin(2a + 2bx)}{64b^4} + \frac{d^3(c + dx) \sin(6a + 6bx)}{1728b^4} + \frac{9d^2(c + dx)^2 \cos(2a + 2bx)}{64b^3} - \frac{d^2(c + dx)^2 \cos(6a + 6bx)}{576b^3} + \frac{3d(c + dx)^3 \sin(2a + 2bx)}{32b^2} - \frac{d^3(c + dx) \sin(6a + 6bx)}{288b^2}$$

```
[Out] (-9*d^4*Cos[2*a + 2*b*x])/(128*b^5) + (9*d^2*(c + d*x)^2*Cos[2*a + 2*b*x])/(64*b^3) - (3*(c + d*x)^4*Cos[2*a + 2*b*x])/(64*b) + (d^4*Cos[6*a + 6*b*x])/(10368*b^5) - (d^2*(c + d*x)^2*Cos[6*a + 6*b*x])/(576*b^3) + ((c + d*x)^4*Cos[6*a + 6*b*x])/(192*b) - (9*d^3*(c + d*x)*Sin[2*a + 2*b*x])/(64*b^4) + (3*d*(c + d*x)^3*SIN[2*a + 2*b*x])/(32*b^2) + (d^3*(c + d*x)*Sin[6*a + 6*b*x])/(1728*b^4) - (d*(c + d*x)^3*SIN[6*a + 6*b*x])/(288*b^2)
```

Rubi [A] time = 0.265973, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3296, 2638}

$$-\frac{9d^3(c + dx) \sin(2a + 2bx)}{64b^4} + \frac{d^3(c + dx) \sin(6a + 6bx)}{1728b^4} + \frac{9d^2(c + dx)^2 \cos(2a + 2bx)}{64b^3} - \frac{d^2(c + dx)^2 \cos(6a + 6bx)}{576b^3} + \frac{3d(c + dx)^3 \sin(2a + 2bx)}{32b^2} - \frac{d^3(c + dx) \sin(6a + 6bx)}{288b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^3*SIN[a + b*x]^3,x]
```

```
[Out] (-9*d^4*Cos[2*a + 2*b*x])/(128*b^5) + (9*d^2*(c + d*x)^2*Cos[2*a + 2*b*x])/(64*b^3) - (3*(c + d*x)^4*Cos[2*a + 2*b*x])/(64*b) + (d^4*Cos[6*a + 6*b*x])/(10368*b^5) - (d^2*(c + d*x)^2*Cos[6*a + 6*b*x])/(576*b^3) + ((c + d*x)^4*Cos[6*a + 6*b*x])/(192*b) - (9*d^3*(c + d*x)*Sin[2*a + 2*b*x])/(64*b^4) + (3*d*(c + d*x)^3*SIN[2*a + 2*b*x])/(32*b^2) + (d^3*(c + d*x)*Sin[6*a + 6*b*x])/(1728*b^4) - (d*(c + d*x)^3*SIN[6*a + 6*b*x])/(288*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^4 \cos^3(a+bx) \sin^3(a+bx) dx &= \int \left(\frac{3}{32}(c+dx)^4 \sin(2a+2bx) - \frac{1}{32}(c+dx)^4 \sin(6a+6bx) \right) dx \\
&= -\left(\frac{1}{32} \int (c+dx)^4 \sin(6a+6bx) dx \right) + \frac{3}{32} \int (c+dx)^4 \sin(2a+2bx) dx \\
&= -\frac{3(c+dx)^4 \cos(2a+2bx)}{64b} + \frac{(c+dx)^4 \cos(6a+6bx)}{192b} - \frac{d \int (c+dx)^3 \cos(6a+6bx) dx}{48b} \\
&= -\frac{3(c+dx)^4 \cos(2a+2bx)}{64b} + \frac{(c+dx)^4 \cos(6a+6bx)}{192b} + \frac{3d(c+dx)^3 \sin(2a+2bx)}{32b^2} \\
&= \frac{9d^2(c+dx)^2 \cos(2a+2bx)}{64b^3} - \frac{3(c+dx)^4 \cos(2a+2bx)}{64b} - \frac{d^2(c+dx)^2 \cos(6a+6bx)}{576b^3} \\
&= \frac{9d^2(c+dx)^2 \cos(2a+2bx)}{64b^3} - \frac{3(c+dx)^4 \cos(2a+2bx)}{64b} - \frac{d^2(c+dx)^2 \cos(6a+6bx)}{576b^3} \\
&= -\frac{9d^4 \cos(2a+2bx)}{128b^5} + \frac{9d^2(c+dx)^2 \cos(2a+2bx)}{64b^3} - \frac{3(c+dx)^4 \cos(2a+2bx)}{64b}
\end{aligned}$$

Mathematica [A] time = 1.5526, size = 153, normalized size = 0.66

$$\frac{-243 \cos(2(a+bx)) \left(-6b^2 d^2 (c+dx)^2 + 2b^4 (c+dx)^4 + 3d^4 \right) + \cos(6(a+bx)) \left(-18b^2 d^2 (c+dx)^2 + 54b^4 (c+dx)^4 + d^4 \right)}{10368b^5}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^4 * Cos[a + b*x]^3 * Sin[a + b*x]^3, x]

[Out] (-243*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)] + (d^4 - 18*b^2*d^2*(c + d*x)^2 + 54*b^4*(c + d*x)^4)*Cos[6*(a + b*x)] - 12*b*d*(c + d*x)*(121*d^2 - 78*b^2*(c + d*x)^2 + (-d^2 + 6*b^2*(c + d*x)^2)*Cos[4*(a + b*x)])*Sin[2*(a + b*x)]/(10368*b^5)

Maple [B] time = 0.074, size = 2061, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x)

[Out] $\frac{1}{b} \left(\frac{1}{b^4 d^4} \left(\frac{1}{4} (b*x+a)^4 \sin(b*x+a)^4 - (b*x+a)^3 \left(-\frac{1}{4} (\sin(b*x+a))^3 + 3/2 \sin(b*x+a) \right) \cos(b*x+a) + 3/8 b*x + 3/8 a \right) - \frac{1}{12} (b*x+a)^2 \sin(b*x+a)^4 + \frac{1}{6} (b*x+a) \left(-\frac{1}{4} (\sin(b*x+a))^3 + 3/2 \sin(b*x+a) \right) \cos(b*x+a) + 3/8 b*x + 3/8 a \right) + \frac{1}{9} (b*x+a)^2 + \frac{1}{216} \sin(b*x+a)^4 + \frac{5}{36} \sin(b*x+a)^2 + \frac{1}{4} (b*x+a)^2 \cos(b*x+a)^2 - \frac{1}{2} (b*x+a) \left(\frac{1}{2} \cos(b*x+a) \sin(b*x+a) + \frac{1}{2} b*x + \frac{1}{2} a \right) + \frac{1}{8} (b*x+a)^4 - \frac{1}{6} (b*x+a)^4 \sin(b*x+a)^6 + \frac{2}{3} (b*x+a)^3 \left(-\frac{1}{6} (\sin(b*x+a))^5 + \frac{5}{4} \sin(b*x+a)^3 + \frac{15}{8} \sin(b*x+a) \right) \cos(b*x+a) + \frac{5}{16} b*x + \frac{5}{16} a \right) + \frac{1}{18} (b*x+a)^2 \sin(b*x+a)^6 - \frac{1}{9} (b*x+a) \left(-\frac{1}{6} (\sin(b*x+a))^5 + \frac{5}{4} \sin(b*x+a)^3 + \frac{15}{8} \sin(b*x+a) \right) \cos(b*x+a) + \frac{5}{16} b*x + \frac{5}{16} a \right) - \frac{1}{324} \sin(b*x+a)^6 - \frac{4}{b^4} a d^4 \left(\frac{1}{4} (b*x+a)^3 \sin(b*x+a)^4 - \frac{3}{4} (b*x+a)^2 \left(-\frac{1}{4} (\sin(b*x+a))^3 + 3/2 \sin(b*x+a) \right) \cos(b*x+a) + 3/8 b*x + 3/8 a \right) - \frac{1}{24} (b*x+a) \sin(b*x+a)^4 - \frac{1}{96} (\sin(b*x+a))^3 + 3/2 \sin(b*x+a) \right) \cos(b*x+a) - \frac{1}{18} b*x - \frac{1}{18} a + \frac{1}{8} (b*x+a) \cos(b*x+a)^2 - \frac{1}{16} \cos(b*x+a) \sin(b*x+a) + \frac{1}{12} (b*x+a)^3 - \frac{1}{6} (b*x+a)^3 \sin(b*x+a)^6 + \frac{1}{2} (b*x+a)^2 \left(-\frac{1}{6} (\sin(b*x+a))^5 + \frac{5}{4} \sin(b*x+a)^3 + \frac{15}{8} \sin(b*x+a) \right) \cos(b*x+a) + \frac{5}{16} b*x + \frac{5}{16} a \right) + \frac{1}{36} (b*x+a) \sin(b*x+a)^6 + \frac{1}{216} (\sin(b*x+a))^5 + \frac{5}{4} \sin(b*x+a)^3 + \frac{15}{8} \sin(b*x+a) \right) \cos(b*x+a) \right) + \frac{4}{b^3} c d^3$

```

*(1/4*(b*x+a)^3*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))
*cos(b*x+a)+3/8*b*x+3/8*a)-1/24*(b*x+a)*sin(b*x+a)^4-1/96*(sin(b*x+a)^3+3/2*
sin(b*x+a))*cos(b*x+a)-1/18*b*x-1/18*a+1/8*(b*x+a)*cos(b*x+a)^2-1/16*cos(b*x+a)
*sin(b*x+a)+1/12*(b*x+a)^3-1/6*(b*x+a)^3*sin(b*x+a)^6+1/2*(b*x+a)^2*(-1/6*(sin(b*x+a)^5+5/4*
sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/36*(b*x+a)*sin(b*x+a)^6+1/216*(sin(b*x+a)^5+5/4*
sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+6/b^4*a^2*d^4*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*
sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+1/24*(b*x+a)^2-1/72*sin(b*x+a)^4-1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*sin(b*x+a)^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*
sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/108*sin(b*x+a)^6)-12/b^3*a*c*d^3*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*
sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+1/24*(b*x+a)^2-1/72*sin(b*x+a)^4-1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*sin(b*x+a)^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*
sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/108*sin(b*x+a)^6)+6/b^2*c^2*d^2*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*
sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+1/24*(b*x+a)^2-1/72*sin(b*x+a)^4-1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*sin(b*x+a)^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*
sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/108*sin(b*x+a)^6)-4/b^4*a^3*d^4*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*
sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*
sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+12/b^3*a^2*c*d^3*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*
sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*
sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))-12/b^2*a*c^2*d^2*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*
sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*
sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+4/b*c^3*d*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*
sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*
sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+1/b^4*a^4*d^4*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)-4/b^3*a^3*c*d^3*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+6/b^2*a^2*c^2*d^2*(-1/6*
sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)-4/b*a*c^3*d*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+c^4*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4))

```

Maxima [B] time = 1.44756, size = 1395, normalized size = 5.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")

```

[Out] -1/10368*(864*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c^4 - 3456*(2*sin(b*x +
a)^6 - 3*sin(b*x + a)^4)*a*c^3*d/b + 5184*(2*sin(b*x + a)^6 - 3*sin(b*x +
a)^4)*a^2*c^2*d^2/b^2 - 3456*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*a^3*c*d^3/b^3 + 864*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*a^4*d^4/b^4 - 36*(6*(b*x
+ a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) +
27*sin(2*b*x + 2*a))*c^3*d/b + 108*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x
+ a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*a*c^2*d^2/b^2 - 108*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - si
n(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*a^2*c*d^3/b^3 + 36*(6*(b*x + a)*cos(6
*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b
*x + 2*a))*a^3*d^4/b^4 - 18*((18*(b*x + a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*
(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*sin(6*b*x + 6*a) + 162*(b*x
+ a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 + 36*((18*(b*x + a)^2 - 1)*cos(6*b*x +
6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*sin(6*b*x + 6*

```

$$a) + 162*(b*x + a)*\sin(2*b*x + 2*a))*a*c*d^3/b^3 - 18*((18*(b*x + a)^2 - 1) * \cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*\cos(2*b*x + 2*a) - 6*(b*x + a)*\sin(6*b*x + 6*a) + 162*(b*x + a)*\sin(2*b*x + 2*a))*a^2*d^4/b^4 - 6*(6*(6*(b*x + a)^3 - b*x - a)*\cos(6*b*x + 6*a) - 162*(2*(b*x + a)^3 - 3*b*x - 3*a)*\cos(2*b*x + 2*a) - (18*(b*x + a)^2 - 1)*\sin(6*b*x + 6*a) + 243*(2*(b*x + a)^2 - 1)*\sin(2*b*x + 2*a))*c*d^3/b^3 + 6*(6*(6*(b*x + a)^3 - b*x - a)*\cos(6*b*x + 6*a) - 162*(2*(b*x + a)^3 - 3*b*x - 3*a)*\cos(2*b*x + 2*a) - (18*(b*x + a)^2 - 1)*\sin(6*b*x + 6*a) + 243*(2*(b*x + a)^2 - 1)*\sin(2*b*x + 2*a))*a*d^4/b^4 - ((54*(b*x + a)^4 - 18*(b*x + a)^2 + 1)*\cos(6*b*x + 6*a) - 243*(2*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*\cos(2*b*x + 2*a) - 6*(6*(b*x + a)^3 - b*x - a)*\sin(6*b*x + 6*a) + 486*(2*(b*x + a)^3 - 3*b*x - 3*a)*\sin(2*b*x + 2*a))*d^4/b^4)/b$$

Fricas [B] time = 0.562245, size = 1153, normalized size = 4.95

$$27b^4d^4x^4 + 108b^4cd^3x^3 + 2(54b^4d^4x^4 + 216b^4cd^3x^3 + 54b^4c^4 - 18b^2c^2d^2 + d^4 + 18(18b^4c^2d^2 - b^2d^4)x^2 + 36(6b^4c^3d - b^2cd^3)x)\cos(bx + a)^6 - 3(54b^4d^4x^4 + 216b^4cd^3x^3 + 54b^4c^4 - 18b^2c^2d^2 + d^4 + 18(18b^4c^2d^2 - b^2d^4)x^2 + 36(6b^4c^3d - b^2cd^3)x)\cos(bx + a)^4 + 18(9b^4c^2d^2 - 5b^2d^4)x^2 + 18(9b^2d^4x^2 + 18b^2cd^3x + 9b^2c^2d^2 - 5d^4)\cos(bx + a)^2 + 36(3b^4c^3d - 5b^2cd^3)x - 12((6b^3d^4x^3 + 18b^3cd^3x^2 + 6b^3c^3d - bcd^3 + (18b^3c^2d^2 - b^2d^4)x)\cos(bx + a)^5 - (6b^3d^4x^3 + 18b^3cd^3x^2 + 6b^3c^3d - bcd^3 + (18b^3c^2d^2 - b^2d^4)x)\cos(bx + a)^3 - 3(3b^3d^4x^3 + 9b^3cd^3x^2 + 3b^3c^3d - 5bcd^3 + (9b^3c^2d^2 - 5b^2d^4)x)\cos(bx + a))\sin(bx + a))/b^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")

[Out] 1/648*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 2*(54*b^4*d^4*x^4 + 216*b^4*c*d^3*x^3 + 54*b^4*c^4 - 18*b^2*c^2*d^2 + d^4 + 18*(18*b^4*c^2*d^2 - b^2*d^4)*x^2 + 36*(6*b^4*c^3*d - b^2*c*d^3)*x)*cos(b*x + a)^6 - 3*(54*b^4*d^4*x^4 + 216*b^4*c*d^3*x^3 + 54*b^4*c^4 - 18*b^2*c^2*d^2 + d^4 + 18*(18*b^4*c^2*d^2 - b^2*d^4)*x^2 + 36*(6*b^4*c^3*d - b^2*c*d^3)*x)*cos(b*x + a)^4 + 18*(9*b^4*c^2*d^2 - 5*b^2*d^4)*x^2 + 18*(9*b^2*d^4*x^2 + 18*b^2*c*d^3*x + 9*b^2*c^2*d^2 - 5*d^4)*cos(b*x + a)^2 + 36*(3*b^4*c^3*d - 5*b^2*c*d^3)*x - 12*((6*b^3*d^4*x^3 + 18*b^3*c*d^3*x^2 + 6*b^3*c^3*d - b*c*d^3 + (18*b^3*c^2*d^2 - b^2*d^4)*x)*cos(b*x + a)^5 - (6*b^3*d^4*x^3 + 18*b^3*c*d^3*x^2 + 6*b^3*c^3*d - b*c*d^3 + (18*b^3*c^2*d^2 - b^2*d^4)*x)*cos(b*x + a)^3 - 3*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d - 5*b*c*d^3 + (9*b^3*c^2*d^2 - 5*b^2*d^4)*x)*cos(b*x + a))*sin(b*x + a))/b^5

Sympy [A] time = 46.2417, size = 1334, normalized size = 5.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**4*cos(b*x+a)**3*sin(b*x+a)**3,x)

[Out] Piecewise((-c**4*sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - c**4*cos(a + b*x)**6/(12*b) + c**3*d*x*sin(a + b*x)**6/(6*b) + c**3*d*x*sin(a + b*x)**4*cos(a + b*x)**2/(2*b) - c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**4/(2*b) - c**3*d*x*cos(a + b*x)**6/(6*b) + c**2*d**2*x**2*sin(a + b*x)**6/(4*b) + 3*c**2*d**2*x**2*sin(a + b*x)**4*cos(a + b*x)**2/(4*b) - 3*c**2*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - c**2*d**2*x**2*cos(a + b*x)**6/(4*b) + c*d**3*x**3*sin(a + b*x)**6/(6*b) + c*d**3*x**3*sin(a + b*x)**4*cos(a + b*x)**2/(2*b) - c*d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**4/(2*b) - c*d**3*x**3*cos(a + b*x)**6/(6*b) + d**4*x**4*sin(a + b*x)**6/(24*b) + d**4*x**4*sin(a + b*x)**4*cos(a + b*x)**2/(8*b) - d**4*x**4*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) - d**4*x**4*cos(a + b*x)**6/(24*b) + c**3*d*sin(a + b*x)**5*cos(a + b*x)/

```
(6*b**2) + 4*c**3*d*sin(a + b*x)**3*cos(a + b*x)**3/(9*b**2) + c**3*d*sin(a
+ b*x)*cos(a + b*x)**5/(6*b**2) + c**2*d**2*x*sin(a + b*x)**5*cos(a + b*x)
/(2*b**2) + 4*c**2*d**2*x*sin(a + b*x)**3*cos(a + b*x)**3/(3*b**2) + c**2*d
**2*x*sin(a + b*x)*cos(a + b*x)**5/(2*b**2) + c*d**3*x**2*sin(a + b*x)**5*c
os(a + b*x)/(2*b**2) + 4*c*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)**3/(3*b**
2) + c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**5/(2*b**2) + d**4*x**3*sin(a +
b*x)**5*cos(a + b*x)/(6*b**2) + 4*d**4*x**3*sin(a + b*x)**3*cos(a + b*x)**3
/(9*b**2) + d**4*x**3*sin(a + b*x)*cos(a + b*x)**5/(6*b**2) - c**2*d**2*sin
(a + b*x)**6/(12*b**3) + c**2*d**2*sin(a + b*x)**2*cos(a + b*x)**4/(3*b**3)
+ 7*c**2*d**2*cos(a + b*x)**6/(36*b**3) - 5*c*d**3*x*sin(a + b*x)**6/(18*b
**3) - c*d**3*x*sin(a + b*x)**4*cos(a + b*x)**2/(3*b**3) + c*d**3*x*sin(a +
b*x)**2*cos(a + b*x)**4/(3*b**3) + 5*c*d**3*x*cos(a + b*x)**6/(18*b**3) -
5*d**4*x**2*sin(a + b*x)**6/(36*b**3) - d**4*x**2*sin(a + b*x)**4*cos(a + b
*x)**2/(6*b**3) + d**4*x**2*sin(a + b*x)**2*cos(a + b*x)**4/(6*b**3) + 5*d*
**4*x**2*cos(a + b*x)**6/(36*b**3) - 5*c*d**3*sin(a + b*x)**5*cos(a + b*x)/(
18*b**4) - 31*c*d**3*sin(a + b*x)**3*cos(a + b*x)**3/(54*b**4) - 5*c*d**3*s
in(a + b*x)*cos(a + b*x)**5/(18*b**4) - 5*d**4*x*sin(a + b*x)**5*cos(a + b*
x)/(18*b**4) - 31*d**4*x*sin(a + b*x)**3*cos(a + b*x)**3/(54*b**4) - 5*d**4
*x*sin(a + b*x)*cos(a + b*x)**5/(18*b**4) + 5*d**4*sin(a + b*x)**6/(108*b**
5) - 31*d**4*sin(a + b*x)**2*cos(a + b*x)**4/(216*b**5) - 61*d**4*cos(a + b
*x)**6/(648*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 +
c*d**3*x**4 + d**4*x**5/5)*sin(a)**3*cos(a)**3, True))
```

Giac [A] time = 1.13064, size = 485, normalized size = 2.08

$$\frac{(54b^4d^4x^4 + 216b^4cd^3x^3 + 324b^4c^2d^2x^2 + 216b^4c^3dx + 54b^4c^4 - 18b^2d^4x^2 - 36b^2cd^3x - 18b^2c^2d^2 + d^4)\cos(6bx + 6a)}{10368b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/10368*(54*b^4*d^4*x^4 + 216*b^4*c*d^3*x^3 + 324*b^4*c^2*d^2*x^2 + 216*b^4
*c^3*d*x + 54*b^4*c^4 - 18*b^2*d^4*x^2 - 36*b^2*c*d^3*x - 18*b^2*c^2*d^2 +
d^4)*cos(6*b*x + 6*a)/b^5 - 3/128*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4
*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3*x -
6*b^2*c^2*d^2 + 3*d^4)*cos(2*b*x + 2*a)/b^5 - 1/1728*(6*b^3*d^4*x^3 + 18*b
^3*c*d^3*x^2 + 18*b^3*c^2*d^2*x + 6*b^3*c^3*d - b*d^4*x - b*c*d^3)*sin(6*b*
x + 6*a)/b^5 + 3/64*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x + 2*
b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

3.156 $\int (c + dx)^3 \cos^3(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=181

$$\frac{9d^2(c + dx) \cos(2a + 2bx)}{128b^3} - \frac{d^2(c + dx) \cos(6a + 6bx)}{1152b^3} + \frac{9d(c + dx)^2 \sin(2a + 2bx)}{128b^2} - \frac{d(c + dx)^2 \sin(6a + 6bx)}{384b^2} - \frac{9d^3}{128b^3}$$

[Out] $(9*d^2*(c + d*x)*Cos[2*a + 2*b*x])/(128*b^3) - (3*(c + d*x)^3*Cos[2*a + 2*b*x])/(64*b) - (d^2*(c + d*x)*Cos[6*a + 6*b*x])/(1152*b^3) + ((c + d*x)^3*Cos[6*a + 6*b*x])/(192*b) - (9*d^3*Sin[2*a + 2*b*x])/(256*b^4) + (9*d*(c + d*x)^2*Sin[2*a + 2*b*x])/(128*b^2) + (d^3*Sin[6*a + 6*b*x])/(6912*b^4) - (d*(c + d*x)^2*Sin[6*a + 6*b*x])/(384*b^2)$

Rubi [A] time = 0.218944, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3296, 2637}

$$\frac{9d^2(c + dx) \cos(2a + 2bx)}{128b^3} - \frac{d^2(c + dx) \cos(6a + 6bx)}{1152b^3} + \frac{9d(c + dx)^2 \sin(2a + 2bx)}{128b^2} - \frac{d(c + dx)^2 \sin(6a + 6bx)}{384b^2} - \frac{9d^3}{128b^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^3,x]

[Out] $(9*d^2*(c + d*x)*Cos[2*a + 2*b*x])/(128*b^3) - (3*(c + d*x)^3*Cos[2*a + 2*b*x])/(64*b) - (d^2*(c + d*x)*Cos[6*a + 6*b*x])/(1152*b^3) + ((c + d*x)^3*Cos[6*a + 6*b*x])/(192*b) - (9*d^3*Sin[2*a + 2*b*x])/(256*b^4) + (9*d*(c + d*x)^2*Sin[2*a + 2*b*x])/(128*b^2) + (d^3*Sin[6*a + 6*b*x])/(6912*b^4) - (d*(c + d*x)^2*Sin[6*a + 6*b*x])/(384*b^2)$

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 \cos^3(a + bx) \sin^3(a + bx) dx &= \int \left(\frac{3}{32} (c + dx)^3 \sin(2a + 2bx) - \frac{1}{32} (c + dx)^3 \sin(6a + 6bx) \right) dx \\
&= -\left(\frac{1}{32} \int (c + dx)^3 \sin(6a + 6bx) dx \right) + \frac{3}{32} \int (c + dx)^3 \sin(2a + 2bx) dx \\
&= -\frac{3(c + dx)^3 \cos(2a + 2bx)}{64b} + \frac{(c + dx)^3 \cos(6a + 6bx)}{192b} - \frac{d \int (c + dx)^2 \cos(6a + 6bx) dx}{64b} \\
&= -\frac{3(c + dx)^3 \cos(2a + 2bx)}{64b} + \frac{(c + dx)^3 \cos(6a + 6bx)}{192b} + \frac{9d(c + dx)^2 \sin(2a + 2bx)}{128b^2} \\
&= \frac{9d^2(c + dx) \cos(2a + 2bx)}{128b^3} - \frac{3(c + dx)^3 \cos(2a + 2bx)}{64b} - \frac{d^2(c + dx) \cos(6a + 6bx)}{1152b^3} \\
&= \frac{9d^2(c + dx) \cos(2a + 2bx)}{128b^3} - \frac{3(c + dx)^3 \cos(2a + 2bx)}{64b} - \frac{d^2(c + dx) \cos(6a + 6bx)}{1152b^3}
\end{aligned}$$

Mathematica [A] time = 2.38983, size = 132, normalized size = 0.73

$$\frac{-324b(c + dx) \cos(2(a + bx)) (2b^2(c + dx)^2 - 3d^2) + 12b(c + dx) \cos(6(a + bx)) (6b^2(c + dx)^2 - d^2) - 4d \sin(2(a + bx)) (c + dx)^3}{13824b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^3,x]

[Out] (-324*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 12*b*(c + d*x)*(-d^2 + 6*b^2*(c + d*x)^2)*Cos[6*(a + b*x)] - 4*d*(121*d^2 - 234*b^2*(c + d*x)^2 + (-d^2 + 18*b^2*(c + d*x)^2)*Cos[4*(a + b*x)])*Sin[2*(a + b*x)]/(13824*b^4)

Maple [B] time = 0.027, size = 1100, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x)

[Out] 1/b*(1/b^3*d^3*(1/4*(b*x+a)^3*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a))^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/24*(b*x+a)*sin(b*x+a)^4-1/96*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/18*b*x-1/18*a+1/8*(b*x+a)*cos(b*x+a)^2-1/16*cos(b*x+a)*sin(b*x+a)+1/12*(b*x+a)^3-1/6*(b*x+a)^3*sin(b*x+a)^6+1/2*(b*x+a)^2*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/36*(b*x+a)*sin(b*x+a)^6+1/216*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))-3/b^3*a*d^3*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a))^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+1/24*(b*x+a)^2-1/72*sin(b*x+a)^4-1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*sin(b*x+a)^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/108*sin(b*x+a)^6)+3/b^2*c*d^2*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a))^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+1/24*(b*x+a)^2-1/72*sin(b*x+a)^4-1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*sin(b*x+a)^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/108*sin(b*x+a)^6)+3/b^3*a^2*d^3*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b

$$\begin{aligned} & x+a))\cos(b*x+a))-6/b^2*a*c*d^2*(1/4*(b*x+a)*\sin(b*x+a)^4+1/16*(\sin(b*x+a)^3+3/2*\sin(b*x+a))*\cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*\sin(b*x+a)^6-1/36*(\sin(b*x+a)^5+5/4*\sin(b*x+a)^3+15/8*\sin(b*x+a))*\cos(b*x+a))+3/b*c^2*d*(1/4*(b*x+a)*\sin(b*x+a)^4+1/16*(\sin(b*x+a)^3+3/2*\sin(b*x+a))*\cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*\sin(b*x+a)^6-1/36*(\sin(b*x+a)^5+5/4*\sin(b*x+a)^3+15/8*\sin(b*x+a))*\cos(b*x+a))-1/b^3*a^3*d^3*(-1/6*\sin(b*x+a)^2*\cos(b*x+a)^4-1/12*\cos(b*x+a)^4)+3/b^2*a^2*c*d^2*(-1/6*\sin(b*x+a)^2*\cos(b*x+a)^4-1/12*\cos(b*x+a)^4)-3/b*a*c^2*d*(-1/6*\sin(b*x+a)^2*\cos(b*x+a)^4-1/12*\cos(b*x+a)^4)+c^3*(-1/6*\sin(b*x+a)^2*\cos(b*x+a)^4-1/12*\cos(b*x+a)^4) \end{aligned}$$

Maxima [B] time = 1.33376, size = 813, normalized size = 4.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/6912*(576*(2*\sin(b*x + a)^6 - 3*\sin(b*x + a)^4)*c^3 - 1728*(2*\sin(b*x + a)^6 - 3*\sin(b*x + a)^4)*a*c^2*d/b + 1728*(2*\sin(b*x + a)^6 - 3*\sin(b*x + a)^4)*a^2*c*d^2/b^2 - 576*(2*\sin(b*x + a)^6 - 3*\sin(b*x + a)^4)*a^3*d^3/b^3 - 18*(6*(b*x + a)*\cos(6*b*x + 6*a) - 54*(b*x + a)*\cos(2*b*x + 2*a) - \sin(6*b*x + 6*a) + 27*\sin(2*b*x + 2*a))*c^2*d/b + 36*(6*(b*x + a)*\cos(6*b*x + 6*a) - 54*(b*x + a)*\cos(2*b*x + 2*a) - \sin(6*b*x + 6*a) + 27*\sin(2*b*x + 2*a))*a*c*d^2/b^2 - 18*(6*(b*x + a)*\cos(6*b*x + 6*a) - 54*(b*x + a)*\cos(2*b*x + 2*a) - \sin(6*b*x + 6*a) + 27*\sin(2*b*x + 2*a))*a^2*d^3/b^3 - 6*((18*(b*x + a)^2 - 1)*\cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*\cos(2*b*x + 2*a) - 6*(b*x + a)*\sin(6*b*x + 6*a) + 162*(b*x + a)*\sin(2*b*x + 2*a))*c*d^2/b^2 + 6*((18*(b*x + a)^2 - 1)*\cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*\cos(2*b*x + 2*a) - 6*(b*x + a)*\sin(6*b*x + 6*a) + 162*(b*x + a)*\sin(2*b*x + 2*a))*a*d^3/b^3 - (6*(6*(b*x + a)^3 - b*x - a)*\cos(6*b*x + 6*a) - 162*(2*(b*x + a)^3 - 3*b*x - 3*a)*\cos(2*b*x + 2*a) - (18*(b*x + a)^2 - 1)*\sin(6*b*x + 6*a) + 243*(2*(b*x + a)^2 - 1)*\sin(2*b*x + 2*a))*d^3/b^3)/b \end{aligned}$$

Fricas [B] time = 0.533301, size = 747, normalized size = 4.13

$$9b^3d^3x^3 + 27b^3cd^2x^2 + 6(6b^3d^3x^3 + 18b^3cd^2x^2 + 6b^3c^3 - bcd^2 + (18b^3c^2d - bd^3)x)\cos(bx + a)^6 - 9(6b^3d^3x^3 + 18b^3cd^2x^2 + 6b^3c^3 - bcd^2 + (18b^3c^2d - bd^3)x)\sin(bx + a)^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & 1/216*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x^2 + 6*(6*b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 6*b^3*c^3 - b*c*d^2 + (18*b^3*c^2*d - b*d^3)*x)*\cos(b*x + a)^6 - 9*(6*b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 6*b^3*c^3 - b*c*d^2 + (18*b^3*c^2*d - b*d^3)*x)*\cos(b*x + a)^4 + 27*(b*d^3*x + b*c*d^2)*\cos(b*x + a)^2 + 3*(9*b^3*c^2*d - 5*b*d^3)*x - ((18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*\cos(b*x + a)^5 - (18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*\cos(b*x + a)^3 - 3*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 5*d^3)*\cos(b*x + a))*\sin(b*x + a))/b^4 \end{aligned}$$

Sympy [A] time = 60.5033, size = 867, normalized size = 4.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*cos(b*x+a)**3*sin(b*x+a)**3,x)

[Out] Piecewise((c**3*sin(a + b*x)**6/(12*b) + c**3*sin(a + b*x)**4*cos(a + b*x)*
 *2/(4*b) + c**2*d*x*sin(a + b*x)**6/(8*b) + 3*c**2*d*x*sin(a + b*x)**4*cos(a + b*x)*
 a + b*x)**2/(8*b) - 3*c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) - c**2
 *d*x*cos(a + b*x)**6/(8*b) + c*d**2*x**2*sin(a + b*x)**6/(8*b) + 3*c*d**2*x
 2*sin(a + b*x)4*cos(a + b*x)**2/(8*b) - 3*c*d**2*x**2*sin(a + b*x)**2*c
 os(a + b*x)**4/(8*b) - c*d**2*x**2*cos(a + b*x)**6/(8*b) + d**3*x**3*sin(a
 + b*x)**6/(24*b) + d**3*x**3*sin(a + b*x)**4*cos(a + b*x)**2/(8*b) - d**3*x
 3*sin(a + b*x)2*cos(a + b*x)**4/(8*b) - d**3*x**3*cos(a + b*x)**6/(24*b
) + c**2*d*sin(a + b*x)**5*cos(a + b*x)/(8*b**2) + c**2*d*sin(a + b*x)**3*c
 os(a + b*x)**3/(3*b**2) + c**2*d*sin(a + b*x)*cos(a + b*x)**5/(8*b**2) + c*
 d**2*x*sin(a + b*x)**5*cos(a + b*x)/(4*b**2) + 2*c*d**2*x*sin(a + b*x)**3*c
 os(a + b*x)**3/(3*b**2) + c*d**2*x*sin(a + b*x)*cos(a + b*x)**5/(4*b**2) +
 d**3*x**2*sin(a + b*x)**5*cos(a + b*x)/(8*b**2) + d**3*x**2*sin(a + b*x)**3
 *cos(a + b*x)**3/(3*b**2) + d**3*x**2*sin(a + b*x)*cos(a + b*x)**5/(8*b**2)
 - 5*c*d**2*sin(a + b*x)**6/(36*b**3) - 7*c*d**2*sin(a + b*x)**4*cos(a + b*
 x)**2/(24*b**3) - c*d**2*sin(a + b*x)**2*cos(a + b*x)**4/(8*b**3) - 5*d**3*
 x*sin(a + b*x)**6/(72*b**3) - d**3*x*sin(a + b*x)**4*cos(a + b*x)**2/(12*b*
 *3) + d**3*x*sin(a + b*x)**2*cos(a + b*x)**4/(12*b**3) + 5*d**3*x*cos(a + b
 *x)**6/(72*b**3) - 5*d**3*sin(a + b*x)**5*cos(a + b*x)/(72*b**4) - 31*d**3*
 sin(a + b*x)**3*cos(a + b*x)**3/(216*b**4) - 5*d**3*sin(a + b*x)*cos(a + b*
 x)**5/(72*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3
 *x**4/4)*sin(a)**3*cos(a)**3, True))

Giac [A] time = 1.13246, size = 325, normalized size = 1.8

$$\frac{(6b^3d^3x^3 + 18b^3cd^2x^2 + 18b^3c^2dx + 6b^3c^3 - bd^3x - bcd^2)\cos(6bx + 6a)}{1152b^4} - \frac{3(2b^3d^3x^3 + 6b^3cd^2x^2 + 6b^3c^2dx + 2b^3c^3)}{128b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")

[Out] 1/1152*(6*b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 18*b^3*c^2*d*x + 6*b^3*c^3 - b*d
 ^3*x - b*c*d^2)*cos(6*b*x + 6*a)/b^4 - 3/128*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x
 ^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(2*b*x + 2*a)/b^
 4 - 1/6912*(18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*sin(6*b*x
 + 6*a)/b^4 + 9/256*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*sin
 (2*b*x + 2*a)/b^4

3.157 $\int (c + dx)^2 \cos^3(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=129

$$\frac{3d(c + dx) \sin(2a + 2bx)}{64b^2} - \frac{d(c + dx) \sin(6a + 6bx)}{576b^2} + \frac{3d^2 \cos(2a + 2bx)}{128b^3} - \frac{d^2 \cos(6a + 6bx)}{3456b^3} - \frac{3(c + dx)^2 \cos(2a + 2bx)}{64b}$$

[Out] (3*d^2*Cos[2*a + 2*b*x])/(128*b^3) - (3*(c + d*x)^2*Cos[2*a + 2*b*x])/(64*b^2) - (d^2*Cos[6*a + 6*b*x])/(3456*b^3) + ((c + d*x)^2*Cos[6*a + 6*b*x])/(192*b) + (3*d*(c + d*x)*Sin[2*a + 2*b*x])/(64*b^2) - (d*(c + d*x)*Sin[6*a + 6*b*x])/(576*b^2)

Rubi [A] time = 0.143795, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3296, 2638}

$$\frac{3d(c + dx) \sin(2a + 2bx)}{64b^2} - \frac{d(c + dx) \sin(6a + 6bx)}{576b^2} + \frac{3d^2 \cos(2a + 2bx)}{128b^3} - \frac{d^2 \cos(6a + 6bx)}{3456b^3} - \frac{3(c + dx)^2 \cos(2a + 2bx)}{64b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^3,x]

[Out] (3*d^2*Cos[2*a + 2*b*x])/(128*b^3) - (3*(c + d*x)^2*Cos[2*a + 2*b*x])/(64*b^2) - (d^2*Cos[6*a + 6*b*x])/(3456*b^3) + ((c + d*x)^2*Cos[6*a + 6*b*x])/(192*b) + (3*d*(c + d*x)*Sin[2*a + 2*b*x])/(64*b^2) - (d*(c + d*x)*Sin[6*a + 6*b*x])/(576*b^2)

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (c+dx)^2 \cos^3(a+bx) \sin^3(a+bx) dx &= \int \left(\frac{3}{32}(c+dx)^2 \sin(2a+2bx) - \frac{1}{32}(c+dx)^2 \sin(6a+6bx) \right) dx \\
&= -\left(\frac{1}{32} \int (c+dx)^2 \sin(6a+6bx) dx \right) + \frac{3}{32} \int (c+dx)^2 \sin(2a+2bx) dx \\
&= -\frac{3(c+dx)^2 \cos(2a+2bx)}{64b} + \frac{(c+dx)^2 \cos(6a+6bx)}{192b} - \frac{d \int (c+dx) \cos(6a+6bx) dx}{96b} \\
&= -\frac{3(c+dx)^2 \cos(2a+2bx)}{64b} + \frac{(c+dx)^2 \cos(6a+6bx)}{192b} + \frac{3d(c+dx) \sin(2a+2bx)}{64b^2} \\
&= \frac{3d^2 \cos(2a+2bx)}{128b^3} - \frac{3(c+dx)^2 \cos(2a+2bx)}{64b} - \frac{d^2 \cos(6a+6bx)}{3456b^3} + \frac{(c+dx)^2 \cos(6a+6bx)}{192b}
\end{aligned}$$

Mathematica [A] time = 0.544526, size = 91, normalized size = 0.71

$$\frac{-81 \cos(2(a+bx)) (2b^2(c+dx)^2 - d^2) + \cos(6(a+bx)) (18b^2(c+dx)^2 - d^2) - 6bd(c+dx)(\sin(6(a+bx))) - 27 \sin(2(a+bx))}{3456b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^3,x]

[Out] (-81*(-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + (-d^2 + 18*b^2*(c + d*x)^2)*Cos[6*(a + b*x)] - 6*b*d*(c + d*x)*(-27*Sin[2*(a + b*x)] + Sin[6*(a + b*x)]))/(3456*b^3)

Maple [B] time = 0.024, size = 498, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x)

[Out] 1/b*(1/b^2*d^2*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+1/24*(b*x+a)^2-1/72*sin(b*x+a)^4-1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*sin(b*x+a)^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/108*sin(b*x+a)^6)-2/b^2*a*d^2*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+2/b*c*d*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+d^2/b^2*a^2*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)-2*c*d/b*a*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+c^2*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4))

Maxima [B] time = 1.23834, size = 409, normalized size = 3.17

$$\frac{288 \left(2 \sin^6(bx+a) - 3 \sin^4(bx+a) \right) c^2 - \frac{576 \left(2 \sin^6(bx+a) - 3 \sin^4(bx+a) \right) a c d}{b} + \frac{288 \left(2 \sin^6(bx+a) - 3 \sin^4(bx+a) \right) a^2 d^2}{b^2} - \frac{6(6(bx+a) \cos(6(a+bx)))}{3456b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")

[Out]
$$-1/3456*(288*(2*\sin(b*x + a)^6 - 3*\sin(b*x + a)^4)*c^2 - 576*(2*\sin(b*x + a)^6 - 3*\sin(b*x + a)^4)*a*c*d/b + 288*(2*\sin(b*x + a)^6 - 3*\sin(b*x + a)^4)*a^2*d^2/b^2 - 6*(6*(b*x + a)*\cos(6*b*x + 6*a) - 54*(b*x + a)*\cos(2*b*x + 2*a) - \sin(6*b*x + 6*a) + 27*\sin(2*b*x + 2*a))*c*d/b + 6*(6*(b*x + a)*\cos(6*b*x + 6*a) - 54*(b*x + a)*\cos(2*b*x + 2*a) - \sin(6*b*x + 6*a) + 27*\sin(2*b*x + 2*a))*a*d^2/b^2 - ((18*(b*x + a)^2 - 1)*\cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*\cos(2*b*x + 2*a) - 6*(b*x + a)*\sin(6*b*x + 6*a) + 162*(b*x + a)*\sin(2*b*x + 2*a))*d^2/b^2)/b$$

Fricas [A] time = 0.503551, size = 447, normalized size = 3.47

$$2(18b^2d^2x^2 + 36b^2cdx + 18b^2c^2 - d^2)\cos(bx + a)^6 + 9b^2d^2x^2 + 18b^2cdx - 3(18b^2d^2x^2 + 36b^2cdx + 18b^2c^2 - d^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")

[Out]
$$1/216*(2*(18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*c^2 - d^2)*\cos(b*x + a)^6 + 9*b^2*d^2*x^2 + 18*b^2*c*d*x - 3*(18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*c^2 - d^2)*\cos(b*x + a)^4 + 9*d^2*\cos(b*x + a)^2 - 6*(2*(b*d^2*x + b*c*d)*\cos(b*x + a)^5 - 2*(b*d^2*x + b*c*d)*\cos(b*x + a)^3 - 3*(b*d^2*x + b*c*d)*\cos(b*x + a))*\sin(b*x + a))/b^3$$

Sympy [A] time = 22.629, size = 471, normalized size = 3.65

$$\left\{ \frac{c^2 \sin^6(a+bx)}{12b} + \frac{c^2 \sin^4(a+bx) \cos^2(a+bx)}{4b} + \frac{cdx \sin^6(a+bx)}{12b} + \frac{cdx \sin^4(a+bx) \cos^2(a+bx)}{4b} - \frac{cdx \sin^2(a+bx) \cos^4(a+bx)}{4b} - \frac{cdx \cos^6(a+bx)}{12b} + \frac{d^2}{3} \right\} \sin^3(a) \cos^3(a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*cos(b*x+a)**3*sin(b*x+a)**3,x)

[Out]
$$\text{Piecewise}((c**2*\sin(a + b*x)**6/(12*b) + c**2*\sin(a + b*x)**4*\cos(a + b*x)**2/(4*b) + c*d*x*\sin(a + b*x)**6/(12*b) + c*d*x*\sin(a + b*x)**4*\cos(a + b*x)**2/(4*b) - c*d*x*\sin(a + b*x)**2*\cos(a + b*x)**4/(4*b) - c*d*x*\cos(a + b*x)**6/(12*b) + d**2*x**2*\sin(a + b*x)**6/(24*b) + d**2*x**2*\sin(a + b*x)**4*\cos(a + b*x)**2/(8*b) - d**2*x**2*\sin(a + b*x)**2*\cos(a + b*x)**4/(8*b) - d**2*x**2*\cos(a + b*x)**6/(24*b) + c*d*\sin(a + b*x)**5*\cos(a + b*x)/(12*b**2) + 2*c*d*\sin(a + b*x)**3*\cos(a + b*x)**3/(9*b**2) + c*d*\sin(a + b*x)*\cos(a + b*x)**5/(12*b**2) + d**2*x*\sin(a + b*x)**5*\cos(a + b*x)/(12*b**2) + 2*d**2*x*\sin(a + b*x)**3*\cos(a + b*x)**3/(9*b**2) + d**2*x*\sin(a + b*x)*\cos(a + b*x)**5/(12*b**2) - 5*d**2*\sin(a + b*x)**6/(108*b**3) - 7*d**2*\sin(a + b*x)**4*\cos(a + b*x)**2/(72*b**3) - d**2*\sin(a + b*x)**2*\cos(a + b*x)**4/(24*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*\sin(a)**3*\cos(a)**3, True))$$

Giac [A] time = 1.14459, size = 196, normalized size = 1.52

$$\frac{(18b^2d^2x^2 + 36b^2cdx + 18b^2c^2 - d^2)\cos(6bx + 6a)}{3456b^3} - \frac{3(2b^2d^2x^2 + 4b^2cdx + 2b^2c^2 - d^2)\cos(2bx + 2a)}{128b^3} - \frac{(bd^2x^2 + 3b^2cdx + 3b^2c^2 - d^2)\cos(2bx + 2a)}{128b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/3456*(18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*c^2 - d^2)*cos(6*b*x + 6*a)/  
b^3 - 3/128*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a  
) / b^3 - 1/576*(b*d^2*x + b*c*d)*sin(6*b*x + 6*a) / b^3 + 3/64*(b*d^2*x + b*c*  
d)*sin(2*b*x + 2*a) / b^3
```

3.158 $\int (c + dx) \cos^3(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=77

$$\frac{3d \sin(2a + 2bx)}{128b^2} - \frac{d \sin(6a + 6bx)}{1152b^2} - \frac{3(c + dx) \cos(2a + 2bx)}{64b} + \frac{(c + dx) \cos(6a + 6bx)}{192b}$$

[Out] $(-3*(c + d*x)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)*Cos[6*a + 6*b*x])/(192*b) + (3*d*Sin[2*a + 2*b*x])/(128*b^2) - (d*Sin[6*a + 6*b*x])/(1152*b^2)$

Rubi [A] time = 0.0744242, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4406, 3296, 2637}

$$\frac{3d \sin(2a + 2bx)}{128b^2} - \frac{d \sin(6a + 6bx)}{1152b^2} - \frac{3(c + dx) \cos(2a + 2bx)}{64b} + \frac{(c + dx) \cos(6a + 6bx)}{192b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]

[Out] $(-3*(c + d*x)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)*Cos[6*a + 6*b*x])/(192*b) + (3*d*Sin[2*a + 2*b*x])/(128*b^2) - (d*Sin[6*a + 6*b*x])/(1152*b^2)$

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n * Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m * Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1) * Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (c + dx) \cos^3(a + bx) \sin^3(a + bx) dx &= \int \left(\frac{3}{32} (c + dx) \sin(2a + 2bx) - \frac{1}{32} (c + dx) \sin(6a + 6bx) \right) dx \\ &= - \left(\frac{1}{32} \int (c + dx) \sin(6a + 6bx) dx \right) + \frac{3}{32} \int (c + dx) \sin(2a + 2bx) dx \\ &= - \frac{3(c + dx) \cos(2a + 2bx)}{64b} + \frac{(c + dx) \cos(6a + 6bx)}{192b} - \frac{d \int \cos(6a + 6bx) dx}{192b} \\ &= - \frac{3(c + dx) \cos(2a + 2bx)}{64b} + \frac{(c + dx) \cos(6a + 6bx)}{192b} + \frac{3d \sin(2a + 2bx)}{128b^2} - \frac{d}{128b^2} \end{aligned}$$

Mathematica [A] time = 0.222521, size = 63, normalized size = 0.82

$$\frac{-54b(c + dx) \cos(2(a + bx)) + 6b(c + dx) \cos(6(a + bx)) + d(27 \sin(2(a + bx)) - \sin(6(a + bx)))}{1152b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]

[Out] (-54*b*(c + d*x)*Cos[2*(a + b*x)] + 6*b*(c + d*x)*Cos[6*(a + b*x)] + d*(27*Sin[2*(a + b*x)] - Sin[6*(a + b*x)]))/(1152*b^2)

Maple [B] time = 0.022, size = 176, normalized size = 2.3

$$\frac{1}{b} \left(\frac{d}{b} \left(\frac{(bx + a) (\sin(bx + a))^4}{4} + \frac{\cos(bx + a)}{16} \left((\sin(bx + a))^3 + \frac{3 \sin(bx + a)}{2} \right) - \frac{bx}{24} - \frac{a}{24} - \frac{(bx + a) (\sin(bx + a))^6}{6} - \dots \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x)

[Out] 1/b*(d/b*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))-1/b*d*a*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+c*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4))

Maxima [A] time = 1.17966, size = 161, normalized size = 2.09

$$\frac{96 \left(2 \sin(bx + a)^6 - 3 \sin(bx + a)^4 \right) c - \frac{96 \left(2 \sin(bx + a)^6 - 3 \sin(bx + a)^4 \right) ad}{b} - \frac{(6(bx + a) \cos(6bx + 6a) - 54(bx + a) \cos(2bx + 2a) - \sin(6bx + 6a) + \dots)}{b}}{1152b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")

[Out] -1/1152*(96*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c - 96*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*a*d/b - (6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*d/b)/b

Fricas [A] time = 0.486378, size = 221, normalized size = 2.87

$$\frac{12(bdx + bc) \cos(bx + a)^6 - 18(bdx + bc) \cos(bx + a)^4 + 3bdx - (2d \cos(bx + a)^5 - 2d \cos(bx + a)^3 - 3d \cos(bx + a))}{72b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")

[Out] 1/72*(12*(b*d*x + b*c)*cos(b*x + a)^6 - 18*(b*d*x + b*c)*cos(b*x + a)^4 + 3*b*d*x - (2*d*cos(b*x + a)^5 - 2*d*cos(b*x + a)^3 - 3*d*cos(b*x + a))*sin(b

$(x + a)/b^2$

Sympy [A] time = 11.1778, size = 201, normalized size = 2.61

$$\left\{ \begin{array}{l} \frac{c \sin^6(a+bx)}{12b} + \frac{c \sin^4(a+bx) \cos^2(a+bx)}{4b} + \frac{dx \sin^6(a+bx)}{24b} + \frac{dx \sin^4(a+bx) \cos^2(a+bx)}{8b} - \frac{dx \sin^2(a+bx) \cos^4(a+bx)}{8b} - \frac{dx \cos^6(a+bx)}{24b} + \frac{d \sin^5(a+bx)}{24b} \\ \left(cx + \frac{dx^2}{2} \right) \sin^3(a) \cos^3(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)**3*sin(b*x+a)**3,x)

[Out] Piecewise((c*sin(a + b*x)**6/(12*b) + c*sin(a + b*x)**4*cos(a + b*x)**2/(4*b) + d*x*sin(a + b*x)**6/(24*b) + d*x*sin(a + b*x)**4*cos(a + b*x)**2/(8*b) - d*x*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) - d*x*cos(a + b*x)**6/(24*b) + d*sin(a + b*x)**5*cos(a + b*x)/(24*b**2) + d*sin(a + b*x)**3*cos(a + b*x)**3/(9*b**2) + d*sin(a + b*x)*cos(a + b*x)**5/(24*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*sin(a)**3*cos(a)**3, True))

Giac [A] time = 1.12175, size = 101, normalized size = 1.31

$$\frac{(bdx + bc) \cos(6bx + 6a)}{192b^2} - \frac{3(bdx + bc) \cos(2bx + 2a)}{64b^2} - \frac{d \sin(6bx + 6a)}{1152b^2} + \frac{3d \sin(2bx + 2a)}{128b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")

[Out] 1/192*(b*d*x + b*c)*cos(6*b*x + 6*a)/b^2 - 3/64*(b*d*x + b*c)*cos(2*b*x + 2*a)/b^2 - 1/1152*d*sin(6*b*x + 6*a)/b^2 + 3/128*d*sin(2*b*x + 2*a)/b^2

$$3.159 \quad \int \frac{\cos^3(a+bx) \sin^3(a+bx)}{c+dx} dx$$

Optimal. Leaf size=129

$$\frac{\sin\left(6a - \frac{6bc}{d}\right) \text{CosIntegral}\left(\frac{6bc}{d} + 6bx\right)}{32d} + \frac{3 \sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{32d} + \frac{3 \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{32d}$$

[Out] -(CosIntegral[(6*b*c)/d + 6*b*x]*Sin[6*a - (6*b*c)/d])/(32*d) + (3*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(32*d) + (3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(32*d) - (Cos[6*a - (6*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(32*d)

Rubi [A] time = 0.246071, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4406, 3303, 3299, 3302}

$$\frac{\sin\left(6a - \frac{6bc}{d}\right) \text{CosIntegral}\left(\frac{6bc}{d} + 6bx\right)}{32d} + \frac{3 \sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{32d} + \frac{3 \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{32d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x),x]

[Out] -(CosIntegral[(6*b*c)/d + 6*b*x]*Sin[6*a - (6*b*c)/d])/(32*d) + (3*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(32*d) + (3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(32*d) - (Cos[6*a - (6*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(32*d)

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx} dx &= \int \left(\frac{3\sin(2a+2bx)}{32(c+dx)} - \frac{\sin(6a+6bx)}{32(c+dx)} \right) dx \\
&= -\left(\frac{1}{32} \int \frac{\sin(6a+6bx)}{c+dx} dx \right) + \frac{3}{32} \int \frac{\sin(2a+2bx)}{c+dx} dx \\
&= -\left(\frac{1}{32} \cos\left(6a - \frac{6bc}{d}\right) \int \frac{\sin\left(\frac{6bc}{d} + 6bx\right)}{c+dx} dx \right) + \frac{1}{32} \left(3 \cos\left(2a - \frac{2bc}{d}\right) \right) \int \frac{\sin\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx \\
&= -\frac{\text{Ci}\left(\frac{6bc}{d} + 6bx\right) \sin\left(6a - \frac{6bc}{d}\right)}{32d} + \frac{3\text{Ci}\left(\frac{2bc}{d} + 2bx\right) \sin\left(2a - \frac{2bc}{d}\right)}{32d} + \frac{3 \cos\left(2a - \frac{2bc}{d}\right)}{32}
\end{aligned}$$

Mathematica [A] time = 0.309157, size = 110, normalized size = 0.85

$$\frac{\sin\left(6a - \frac{6bc}{d}\right) \text{CosIntegral}\left(\frac{6b(c+dx)}{d}\right) - 3 \sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) - 3 \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2b(c+dx)}{d}\right) + \cos\left(2a - \frac{2bc}{d}\right)}{32d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x),x]

[Out] -(CosIntegral[(6*b*(c + d*x))/d]*Sin[6*a - (6*b*c)/d] - 3*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - 3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + Cos[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c + d*x))/d])/(32*d)

Maple [A] time = 0.024, size = 178, normalized size = 1.4

$$\frac{1}{b} \left(-\frac{b}{192} \left(6 \frac{1}{d} \text{Si} \left(6bx + 6a + 6 \frac{-ad+bc}{d} \right) \cos \left(6 \frac{-ad+bc}{d} \right) - 6 \frac{1}{d} \text{Ci} \left(6bx + 6a + 6 \frac{-ad+bc}{d} \right) \sin \left(6 \frac{-ad+bc}{d} \right) \right) + \frac{3}{6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x)

[Out] 1/b*(-1/192*b*(6*Si(6*b*x+6*a+6*(-a*d+b*c)/d)*cos(6*(-a*d+b*c)/d)/d-6*Ci(6*b*x+6*a+6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d+3/64*b*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d))

Maxima [C] time = 1.5355, size = 370, normalized size = 2.87

$$b \left(-3i E_1 \left(\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) + 3i E_1 \left(-\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) \right) \cos \left(-\frac{2(bc-ad)}{d} \right) + b \left(i E_1 \left(\frac{6i bc + 6i (bx+a)d - 6i ad}{d} \right) - i E_1 \left(-\frac{6i bc + 6i (bx+a)d - 6i ad}{d} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")

[Out] 1/64*(b*(-3*I*exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) + 3*I*exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*

$$c - a*d)/d) + b*(I*\exp_integral_e(1, (6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d) - I*\exp_integral_e(1, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))*\cos(-6*(b*c - a*d)/d) - 3*b*(\exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) + \exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*\sin(-2*(b*c - a*d)/d) + b*(\exp_integral_e(1, (6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d) + \exp_integral_e(1, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))*\sin(-6*(b*c - a*d)/d))/(b*d)$$

Fricas [A] time = 0.490778, size = 421, normalized size = 3.26

$$\frac{3 \left(\operatorname{Ci} \left(\frac{2(bdx+bc)}{d} \right) + \operatorname{Ci} \left(-\frac{2(bdx+bc)}{d} \right) \right) \sin \left(-\frac{2(bc-ad)}{d} \right) - \left(\operatorname{Ci} \left(\frac{6(bdx+bc)}{d} \right) + \operatorname{Ci} \left(-\frac{6(bdx+bc)}{d} \right) \right) \sin \left(-\frac{6(bc-ad)}{d} \right) - 2 \cos \left(-\frac{6(bc-ad)}{d} \right)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")

[Out] 1/64*(3*(cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d) - (cos_integral(6*(b*d*x + b*c)/d) + cos_integral(-6*(b*d*x + b*c)/d))*sin(-6*(b*c - a*d)/d) - 2*cos(-6*(b*c - a*d)/d)*sin_integral(6*(b*d*x + b*c)/d) + 6*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d))/d

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3(a + bx) \cos^3(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c),x)

[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**3/(c + d*x), x)

Giac [C] time = 1.78309, size = 8162, normalized size = 63.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")

[Out] -1/64*(imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) - 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) + 2*real_part(cos_integr

$$\begin{aligned}
& \cos_integral(-6*b*x - 6*b*c/d)*\tan(a)^2*\tan(3*b*c/d)*\tan(b*c/d)^2 - 2*\text{real_part}(\cos_integral(6*b*x + 6*b*c/d))*\tan(3*a)*\tan(3*b*c/d)^2*\tan(b*c/d)^2 - \\
& 2*\text{real_part}(\cos_integral(-6*b*x - 6*b*c/d))*\tan(3*a)*\tan(3*b*c/d)^2*\tan(b*c/d)^2 + 6*\text{real_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(a)*\tan(3*b*c/d)^2*\tan(b*c/d)^2 + \\
& 6*\text{real_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(a)*\tan(3*b*c/d)^2*\tan(b*c/d)^2 - \text{imag_part}(\cos_integral(6*b*x + 6*b*c/d))*\tan(3*a)^2*\tan(a)^2 + \\
& 3*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(3*a)^2*\tan(a)^2 - 3*\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(3*a)^2*\tan(a)^2 + \text{imag_part}(\cos_integral(-6*b*x - 6*b*c/d))*\tan(3*a)^2*\tan(a)^2 - \\
& 2*\sin_integral(6*(b*d*x + b*c)/d)*\tan(3*a)^2*\tan(a)^2 + 6*\sin_integral(2*(b*d*x + b*c)/d)*\tan(3*a)^2*\tan(a)^2 + 4*\text{imag_part}(\cos_integral(6*b*x + 6*b*c/d))*\tan(3*a)*\tan(a)^2*\tan(3*b*c/d) - \\
& 4*\text{imag_part}(\cos_integral(-6*b*x - 6*b*c/d))*\tan(3*a)*\tan(a)^2*\tan(3*b*c/d) + 8*\sin_integral(6*(b*d*x + b*c)/d)*\tan(3*a)*\tan(a)^2*\tan(3*b*c/d) + \text{imag_part}(\cos_integral(6*b*x + 6*b*c/d))*\tan(3*a)^2*\tan(3*b*c/d)^2 - \\
& 3*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(3*a)^2*\tan(3*b*c/d)^2 + 3*\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(3*a)^2*\tan(3*b*c/d)^2 - \text{imag_part}(\cos_integral(-6*b*x - 6*b*c/d))*\tan(3*a)^2*\tan(3*b*c/d)^2 + \\
& 2*\sin_integral(6*(b*d*x + b*c)/d)*\tan(3*a)^2*\tan(3*b*c/d)^2 - 6*\sin_integral(2*(b*d*x + b*c)/d)*\tan(3*a)^2*\tan(3*b*c/d)^2 - \text{imag_part}(\cos_integral(6*b*x + 6*b*c/d))*\tan(a)^2*\tan(3*b*c/d)^2 + \\
& 3*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(a)^2*\tan(3*b*c/d)^2 - 3*\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(a)^2*\tan(3*b*c/d)^2 + \text{imag_part}(\cos_integral(-6*b*x - 6*b*c/d))*\tan(a)^2*\tan(3*b*c/d)^2 - \\
& 2*\sin_integral(6*(b*d*x + b*c)/d)*\tan(a)^2*\tan(3*b*c/d)^2 + 6*\sin_integral(2*(b*d*x + b*c)/d)*\tan(a)^2*\tan(3*b*c/d)^2 - 12*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(3*a)^2*\tan(a)*\tan(b*c/d) + \\
& 12*\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(3*a)^2*\tan(a)*\tan(b*c/d) - 24*\sin_integral(2*(b*d*x + b*c)/d)*\tan(3*a)^2*\tan(a)*\tan(b*c/d) - 12*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(a)*\tan(3*b*c/d)^2*\tan(b*c/d) + \\
& 12*\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(a)*\tan(3*b*c/d)^2*\tan(b*c/d) - 24*\sin_integral(2*(b*d*x + b*c)/d)*\tan(a)*\tan(3*b*c/d)^2*\tan(b*c/d) - \text{imag_part}(\cos_integral(6*b*x + 6*b*c/d))*\tan(3*a)^2*\tan(b*c/d)^2 + \\
& 3*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(3*a)^2*\tan(b*c/d)^2 - 3*\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(3*a)^2*\tan(b*c/d)^2 + \text{imag_part}(\cos_integral(-6*b*x - 6*b*c/d))*\tan(3*a)^2*\tan(b*c/d)^2 - \\
& 2*\sin_integral(6*(b*d*x + b*c)/d)*\tan(3*a)^2*\tan(b*c/d)^2 + 6*\sin_integral(2*(b*d*x + b*c)/d)*\tan(3*a)^2*\tan(b*c/d)^2 + \text{imag_part}(\cos_integral(6*b*x + 6*b*c/d))*\tan(a)^2*\tan(b*c/d)^2 - 3*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(a)^2*\tan(b*c/d)^2 + \\
& 3*\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(a)^2*\tan(b*c/d)^2 - \text{imag_part}(\cos_integral(-6*b*x - 6*b*c/d))*\tan(a)^2*\tan(b*c/d)^2 + 2*\sin_integral(6*(b*d*x + b*c)/d)*\tan(a)^2*\tan(b*c/d)^2 - \\
& 6*\sin_integral(2*(b*d*x + b*c)/d)*\tan(a)^2*\tan(b*c/d)^2 + 4*\text{imag_part}(\cos_integral(6*b*x + 6*b*c/d))*\tan(3*a)*\tan(3*b*c/d)*\tan(b*c/d)^2 - 4*\text{imag_part}(\cos_integral(-6*b*x - 6*b*c/d))*\tan(3*a)*\tan(3*b*c/d)*\tan(b*c/d)^2 + \\
& 8*\sin_integral(6*(b*d*x + b*c)/d)*\tan(3*a)*\tan(3*b*c/d)*\tan(b*c/d)^2 - \text{imag_part}(\cos_integral(6*b*x + 6*b*c/d))*\tan(3*b*c/d)^2*\tan(b*c/d)^2 + 3*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(3*b*c/d)^2*\tan(b*c/d)^2 - \\
& 3*\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(3*b*c/d)^2*\tan(b*c/d)^2 + \text{imag_part}(\cos_integral(-6*b*x - 6*b*c/d))*\tan(3*b*c/d)^2*\tan(b*c/d)^2 - 2*\sin_integral(6*(b*d*x + b*c)/d)*\tan(3*b*c/d)^2*\tan(b*c/d)^2 + 6*\sin_integral(2*(b*d*x + b*c)/d)*\tan(3*b*c/d)^2*\tan(b*c/d)^2 - \\
& 6*\text{real_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(3*a)^2*\tan(a) - 6*\text{real_part}(\cos_integral(-2*b*x - 2*b*c/d))*\tan(3*a)^2*\tan(a) + 2*\text{real_part}(\cos_integral(6*b*x + 6*b*c/d))*\tan(3*a)*\tan(a)^2 + 2*\text{real_part}(\cos_integral(-6*b*x - 6*b*c/d))*\tan(3*a)*\tan(a)^2 + 2*\text{real_part}(\cos_integral(6*b*x + 6*b*c/d))*\tan(3*a)^2*\tan(3*b*c/d) + 2*\text{real_part}(\cos_integral(-6*b*x - 6*b*c/d))*\tan(3*a)^2*\tan(3*b*c/d) - 2*\text{real_part}(\cos_integral(6*b*x + 6*b*c/d))*\tan(a)^2*\tan(3*b*c/d) - 2*\text{real_part}(\cos_integral(-6*b*x - 6*b*c/d))*\tan(a)^2*\tan(3*b*c/d) - 2*\text{real_part}(\cos_integral(6*b*x + 6*b*c/d))*\tan(3*a)*\tan(3*b*c/d)^2 - 2*\text{real_part}(\cos_integral(-6*b*x - 6*b*c/d))*\tan(3*a)*\tan(3*b*c/d)^2 - 6*\text{real_part}(\cos_integral(2*b*x + 2*b*c/d))*\tan(a)*\tan(3*b*c/d)^2 - 6*\text{real_part}(\cos_integral(-2*b*x -
\end{aligned}$$

$$\begin{aligned}
& 2*b*c/d)) * \tan(a) * \tan(3*b*c/d)^2 + 6*\text{real_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(3*a)^2 * \tan(b*c/d) + 6*\text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(3*a)^2 * \tan(b*c/d) - 6*\text{real_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(a)^2 * \tan(b*c/d) - 6*\text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(a)^2 * \tan(b*c/d) + 6*\text{real_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(3*b*c/d)^2 * \tan(b*c/d) + 6*\text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(3*b*c/d)^2 * \tan(b*c/d) + 2*\text{real_part}(\cos_integral(6*b*x + 6*b*c/d)) * \tan(3*a) * \tan(b*c/d)^2 + 2*\text{real_part}(\cos_integral(-6*b*x - 6*b*c/d)) * \tan(3*a) * \tan(b*c/d)^2 + 6*\text{real_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(a) * \tan(b*c/d)^2 + 6*\text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(a) * \tan(b*c/d)^2 - 2*\text{real_part}(\cos_integral(6*b*x + 6*b*c/d)) * \tan(3*b*c/d) * \tan(b*c/d)^2 - 2*\text{real_part}(\cos_integral(-6*b*x - 6*b*c/d)) * \tan(3*b*c/d) * \tan(b*c/d)^2 - \text{imag_part}(\cos_integral(6*b*x + 6*b*c/d)) * \tan(3*a)^2 - 3*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(3*a)^2 + 3*\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(3*a)^2 + \text{imag_part}(\cos_integral(-6*b*x - 6*b*c/d)) * \tan(3*a)^2 - 2*\sin_integral(6*(b*d*x + b*c)/d) * \tan(3*a)^2 - 6*\sin_integral(2*(b*d*x + b*c)/d) * \tan(3*a)^2 + \text{imag_part}(\cos_integral(6*b*x + 6*b*c/d)) * \tan(a)^2 + 3*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(a)^2 - 3*\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(a)^2 - \text{imag_part}(\cos_integral(-6*b*x - 6*b*c/d)) * \tan(a)^2 + 2*\sin_integral(6*(b*d*x + b*c)/d) * \tan(a)^2 + 6*\sin_integral(2*(b*d*x + b*c)/d) * \tan(a)^2 + 4*\text{imag_part}(\cos_integral(6*b*x + 6*b*c/d)) * \tan(3*a) * \tan(3*b*c/d) - 4*\text{imag_part}(\cos_integral(-6*b*x - 6*b*c/d)) * \tan(3*a) * \tan(3*b*c/d) + 8*\sin_integral(6*(b*d*x + b*c)/d) * \tan(3*a) * \tan(3*b*c/d) - \text{imag_part}(\cos_integral(6*b*x + 6*b*c/d)) * \tan(3*b*c/d)^2 - 3*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(3*b*c/d)^2 + 3*\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(3*b*c/d)^2 + \text{imag_part}(\cos_integral(-6*b*x - 6*b*c/d)) * \tan(3*b*c/d)^2 - 2*\sin_integral(6*(b*d*x + b*c)/d) * \tan(3*b*c/d)^2 - 6*\sin_integral(2*(b*d*x + b*c)/d) * \tan(3*b*c/d)^2 - 12*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(a) * \tan(b*c/d) + 12*\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(a) * \tan(b*c/d) - 24*\sin_integral(2*(b*d*x + b*c)/d) * \tan(a) * \tan(b*c/d) + \text{imag_part}(\cos_integral(6*b*x + 6*b*c/d)) * \tan(b*c/d)^2 + 3*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(b*c/d)^2 - 3*\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(b*c/d)^2 - \text{imag_part}(\cos_integral(-6*b*x - 6*b*c/d)) * \tan(b*c/d)^2 + 2*\sin_integral(6*(b*d*x + b*c)/d) * \tan(b*c/d)^2 + 6*\sin_integral(2*(b*d*x + b*c)/d) * \tan(b*c/d)^2 + 2*\text{real_part}(\cos_integral(6*b*x + 6*b*c/d)) * \tan(3*a) + 2*\text{real_part}(\cos_integral(-6*b*x - 6*b*c/d)) * \tan(3*a) - 6*\text{real_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(a) - 6*\text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(a) - 2*\text{real_part}(\cos_integral(6*b*x + 6*b*c/d)) * \tan(3*b*c/d) - 2*\text{real_part}(\cos_integral(-6*b*x - 6*b*c/d)) * \tan(3*b*c/d) + 6*\text{real_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(b*c/d) + 6*\text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(b*c/d) + \text{imag_part}(\cos_integral(6*b*x + 6*b*c/d)) - 3*\text{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) + 3*\text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d)) - \text{imag_part}(\cos_integral(-6*b*x - 6*b*c/d)) + 2*\sin_integral(6*(b*d*x + b*c)/d) - 6*\sin_integral(2*(b*d*x + b*c)/d)) / (d*\tan(3*a)^2 * \tan(a)^2 * \tan(3*b*c/d)^2 * \tan(b*c/d)^2 + d*\tan(3*a)^2 * \tan(a)^2 * \tan(3*b*c/d)^2 * \tan(b*c/d)^2 + d*\tan(3*a)^2 * \tan(a)^2 * \tan(b*c/d)^2 + d*\tan(3*a)^2 * \tan(3*b*c/d)^2 * \tan(b*c/d)^2 + d*\tan(3*a)^2 * \tan(a)^2 + d*\tan(3*a)^2 * \tan(3*b*c/d)^2 + d*\tan(a)^2 * \tan(3*b*c/d)^2 + d*\tan(3*a)^2 * \tan(b*c/d)^2 + d*\tan(a)^2 * \tan(b*c/d)^2 + d*\tan(3*b*c/d)^2 * \tan(b*c/d)^2 + d*\tan(3*a)^2 + d*\tan(a)^2 + d*\tan(3*b*c/d)^2 + d*\tan(b*c/d)^2 + d)
\end{aligned}$$

$$3.160 \quad \int \frac{\cos^3(a+bx) \sin^3(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=179

$$\frac{3b \cos\left(2a - \frac{2bc}{d}\right) \operatorname{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{16d^2} - \frac{3b \cos\left(6a - \frac{6bc}{d}\right) \operatorname{CosIntegral}\left(\frac{6bc}{d} + 6bx\right)}{16d^2} - \frac{3b \sin\left(2a - \frac{2bc}{d}\right) \operatorname{Si}\left(\frac{2bc}{d} + 2bx\right)}{16d^2}$$

[Out] (3*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(16*d^2) - (3*b*Cos[6*a - (6*b*c)/d]*CosIntegral[(6*b*c)/d + 6*b*x])/(16*d^2) - (3*Sin[2*a + 2*b*x])/(32*d*(c + d*x)) + Sin[6*a + 6*b*x]/(32*d*(c + d*x)) - (3*b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(16*d^2) + (3*b*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(16*d^2)

Rubi [A] time = 0.297098, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{3b \cos\left(2a - \frac{2bc}{d}\right) \operatorname{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{16d^2} - \frac{3b \cos\left(6a - \frac{6bc}{d}\right) \operatorname{CosIntegral}\left(\frac{6bc}{d} + 6bx\right)}{16d^2} - \frac{3b \sin\left(2a - \frac{2bc}{d}\right) \operatorname{Si}\left(\frac{2bc}{d} + 2bx\right)}{16d^2}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^2,x]

[Out] (3*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(16*d^2) - (3*b*Cos[6*a - (6*b*c)/d]*CosIntegral[(6*b*c)/d + 6*b*x])/(16*d^2) - (3*Sin[2*a + 2*b*x])/(32*d*(c + d*x)) + Sin[6*a + 6*b*x]/(32*d*(c + d*x)) - (3*b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(16*d^2) + (3*b*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(16*d^2)

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

`Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]`

Rubi steps

$$\begin{aligned} \int \frac{\cos^3(a + bx) \sin^3(a + bx)}{(c + dx)^2} dx &= \int \left(\frac{3 \sin(2a + 2bx)}{32(c + dx)^2} - \frac{\sin(6a + 6bx)}{32(c + dx)^2} \right) dx \\ &= -\left(\frac{1}{32} \int \frac{\sin(6a + 6bx)}{(c + dx)^2} dx \right) + \frac{3}{32} \int \frac{\sin(2a + 2bx)}{(c + dx)^2} dx \\ &= -\frac{3 \sin(2a + 2bx)}{32d(c + dx)} + \frac{\sin(6a + 6bx)}{32d(c + dx)} + \frac{(3b) \int \frac{\cos(2a+2bx)}{c+dx} dx}{16d} - \frac{(3b) \int \frac{\cos(6a+6bx)}{c+dx} dx}{16d} \\ &= -\frac{3 \sin(2a + 2bx)}{32d(c + dx)} + \frac{\sin(6a + 6bx)}{32d(c + dx)} - \frac{\left(3b \cos\left(6a - \frac{6bc}{d}\right)\right) \int \frac{\cos\left(\frac{6bc}{d} + 6bx\right)}{c+dx} dx}{16d} + \frac{\left(3b \cos\left(2a - \frac{2bc}{d}\right)\right) \int \frac{\cos\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx}{16d} \\ &= \frac{3b \cos\left(2a - \frac{2bc}{d}\right) \text{Ci}\left(\frac{2bc}{d} + 2bx\right)}{16d^2} - \frac{3b \cos\left(6a - \frac{6bc}{d}\right) \text{Ci}\left(\frac{6bc}{d} + 6bx\right)}{16d^2} - \frac{3 \sin(2a + 2bx)}{32d(c + dx)} \end{aligned}$$

Mathematica [A] time = 0.9808, size = 189, normalized size = 1.06

$$6b(c + dx) \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) - 6b(c + dx) \cos\left(6a - \frac{6bc}{d}\right) \text{CosIntegral}\left(\frac{6b(c+dx)}{d}\right) - 6b(c + dx) \sin\left(2a + 2bx\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^2,x]

[Out] (6*b*(c + d*x)*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - 6*b*(c + d*x)*Cos[6*a - (6*b*c)/d]*CosIntegral[(6*b*(c + d*x))/d] - 3*d*Cos[2*b*x]*Sin[2*a] + d*Cos[6*b*x]*Sin[6*a] - 3*d*Cos[2*a]*Sin[2*b*x] + d*Cos[6*a]*Sin[6*b*x] - 6*b*(c + d*x)*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 6*b*(c + d*x)*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c + d*x))/d])/(32*d^2*(c + d*x))

Maple [A] time = 0.028, size = 256, normalized size = 1.4

$$\frac{1}{b} \left(-\frac{b^2}{192} \left(-6 \frac{\sin(6bx + 6a)}{((bx + a)d - ad + bc)d} + 6 \frac{1}{d} \text{Si}\left(6bx + 6a + 6 \frac{-ad + bc}{d}\right) \sin\left(6 \frac{-ad + bc}{d}\right) + 6 \frac{1}{d} \text{Ci}\left(6bx + 6a + 6 \frac{-ad + bc}{d}\right) \cos\left(6 \frac{-ad + bc}{d}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x)

[Out] 1/b*(-1/192*b^2*(-6*sin(6*b*x+6*a)/((b*x+a)*d-a*d+b*c)/d+6*(6*Si(6*b*x+6*a+6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d+6*Ci(6*b*x+6*a+6*(-a*d+b*c)/d)*cos(6*(-a*d+b*c)/d)/d)/d+3/64*b^2*(-2*sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)/d+2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)

Maxima [C] time = 1.76057, size = 406, normalized size = 2.27

$$b^2 \left(-3i E_2 \left(\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) + 3i E_2 \left(-\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) \right) \cos \left(-\frac{2(bc-ad)}{d} \right) + b^2 \left(i E_2 \left(\frac{6i bc + 6i (bx+a)d - 6i ad}{d} \right) - i E_2 \left(-\frac{6i bc + 6i (bx+a)d - 6i ad}{d} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")

[Out] $\frac{1}{64} b^2 (-3i \exp_{\text{integral}}(2, (2I*b*c + 2I*(b*x + a)*d - 2I*a*d)/d) + 3i \exp_{\text{integral}}(2, -(2I*b*c + 2I*(b*x + a)*d - 2I*a*d)/d)) \cos(-2*(b*c - a*d)/d) + b^2 (I \exp_{\text{integral}}(2, (6I*b*c + 6I*(b*x + a)*d - 6I*a*d)/d) - I \exp_{\text{integral}}(2, -(6I*b*c + 6I*(b*x + a)*d - 6I*a*d)/d)) \cos(-6*(b*c - a*d)/d) - 3*b^2 (\exp_{\text{integral}}(2, (2I*b*c + 2I*(b*x + a)*d - 2I*a*d)/d) + \exp_{\text{integral}}(2, -(2I*b*c + 2I*(b*x + a)*d - 2I*a*d)/d)) \sin(-2*(b*c - a*d)/d) + b^2 (\exp_{\text{integral}}(2, (6I*b*c + 6I*(b*x + a)*d - 6I*a*d)/d) + \exp_{\text{integral}}(2, -(6I*b*c + 6I*(b*x + a)*d - 6I*a*d)/d)) \sin(-6*(b*c - a*d)/d) / ((b*c*d + (b*x + a)*d^2 - a*d^2)*b)$

Fricas [A] time = 0.596735, size = 632, normalized size = 3.53

$$6(bdx + bc) \sin\left(-\frac{6(bc-ad)}{d}\right) \text{Si}\left(\frac{6(bdx+bc)}{d}\right) - 6(bdx + bc) \sin\left(-\frac{2(bc-ad)}{d}\right) \text{Si}\left(\frac{2(bdx+bc)}{d}\right) + 3\left((bdx + bc) \text{Ci}\left(\frac{2(bdx+bc)}{d}\right) + (bdx + bc) \text{Ci}\left(-\frac{2(bc-ad)}{d}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")

[Out] $\frac{1}{32} (6*(b*d*x + b*c) \sin(-6*(b*c - a*d)/d) \sin_{\text{integral}}(6*(b*d*x + b*c)/d) - 6*(b*d*x + b*c) \sin(-2*(b*c - a*d)/d) \sin_{\text{integral}}(2*(b*d*x + b*c)/d) + 3*((b*d*x + b*c) \cos_{\text{integral}}(2*(b*d*x + b*c)/d) + (b*d*x + b*c) \cos_{\text{integral}}(-2*(b*d*x + b*c)/d)) \cos(-2*(b*c - a*d)/d) - 3*((b*d*x + b*c) \cos_{\text{integral}}(6*(b*d*x + b*c)/d) + (b*d*x + b*c) \cos_{\text{integral}}(-6*(b*d*x + b*c)/d)) \cos(-6*(b*c - a*d)/d) + 32*(d*\cos(b*x + a)^5 - d*\cos(b*x + a)^3) \sin(b*x + a) / (d^3*x + c*d^2)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx + a)^3 \sin(bx + a)^3}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3*sin(b*x + a)^3/(d*x + c)^2, x)
```

$$3.161 \quad \int \frac{\cos^3(a+bx) \sin^3(a+bx)}{(c+dx)^3} dx$$

Optimal. Leaf size=235

$$\frac{9b^2 \sin\left(6a - \frac{6bc}{d}\right) \text{CosIntegral}\left(\frac{6bc}{d} + 6bx\right)}{16d^3} - \frac{3b^2 \sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{16d^3} - \frac{3b^2 \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{16d^3}$$

[Out] $(-3*b*\text{Cos}[2*a + 2*b*x])/(32*d^2*(c + d*x)) + (3*b*\text{Cos}[6*a + 6*b*x])/(32*d^2*(c + d*x)) + (9*b^2*\text{CosIntegral}[(6*b*c)/d + 6*b*x]*\text{Sin}[6*a - (6*b*c)/d])/(16*d^3) - (3*b^2*\text{CosIntegral}[(2*b*c)/d + 2*b*x]*\text{Sin}[2*a - (2*b*c)/d])/(16*d^3) - (3*\text{Sin}[2*a + 2*b*x])/(64*d*(c + d*x)^2) + \text{Sin}[6*a + 6*b*x]/(64*d*(c + d*x)^2) - (3*b^2*\text{Cos}[2*a - (2*b*c)/d]*\text{SinIntegral}[(2*b*c)/d + 2*b*x])/(16*d^3) + (9*b^2*\text{Cos}[6*a - (6*b*c)/d]*\text{SinIntegral}[(6*b*c)/d + 6*b*x])/(16*d^3)$

Rubi [A] time = 0.353223, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{9b^2 \sin\left(6a - \frac{6bc}{d}\right) \text{CosIntegral}\left(\frac{6bc}{d} + 6bx\right)}{16d^3} - \frac{3b^2 \sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{16d^3} - \frac{3b^2 \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{16d^3}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^3,x]

[Out] $(-3*b*\text{Cos}[2*a + 2*b*x])/(32*d^2*(c + d*x)) + (3*b*\text{Cos}[6*a + 6*b*x])/(32*d^2*(c + d*x)) + (9*b^2*\text{CosIntegral}[(6*b*c)/d + 6*b*x]*\text{Sin}[6*a - (6*b*c)/d])/(16*d^3) - (3*b^2*\text{CosIntegral}[(2*b*c)/d + 2*b*x]*\text{Sin}[2*a - (2*b*c)/d])/(16*d^3) - (3*\text{Sin}[2*a + 2*b*x])/(64*d*(c + d*x)^2) + \text{Sin}[6*a + 6*b*x]/(64*d*(c + d*x)^2) - (3*b^2*\text{Cos}[2*a - (2*b*c)/d]*\text{SinIntegral}[(2*b*c)/d + 2*b*x])/(16*d^3) + (9*b^2*\text{Cos}[6*a - (6*b*c)/d]*\text{SinIntegral}[(6*b*c)/d + 6*b*x])/(16*d^3)$

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3} dx &= \int \left(\frac{3\sin(2a+2bx)}{32(c+dx)^3} - \frac{\sin(6a+6bx)}{32(c+dx)^3} \right) dx \\
&= -\left(\frac{1}{32} \int \frac{\sin(6a+6bx)}{(c+dx)^3} dx \right) + \frac{3}{32} \int \frac{\sin(2a+2bx)}{(c+dx)^3} dx \\
&= -\frac{3\sin(2a+2bx)}{64d(c+dx)^2} + \frac{\sin(6a+6bx)}{64d(c+dx)^2} + \frac{(3b) \int \frac{\cos(2a+2bx)}{(c+dx)^2} dx}{32d} - \frac{(3b) \int \frac{\cos(6a+6bx)}{(c+dx)^2} dx}{32d} \\
&= -\frac{3b\cos(2a+2bx)}{32d^2(c+dx)} + \frac{3b\cos(6a+6bx)}{32d^2(c+dx)} - \frac{3\sin(2a+2bx)}{64d(c+dx)^2} + \frac{\sin(6a+6bx)}{64d(c+dx)^2} - \frac{(3b^2)}{16d^3} \text{Ci}\left(\frac{6bc}{d} + 6bx\right) \sin\left(6a - \frac{6bc}{d}\right) - \frac{3b^2 \text{Ci}\left(\frac{6bc}{d} + 6bx\right) \cos\left(6a - \frac{6bc}{d}\right)}{16d^3} \\
&= -\frac{3b\cos(2a+2bx)}{32d^2(c+dx)} + \frac{3b\cos(6a+6bx)}{32d^2(c+dx)} - \frac{3\sin(2a+2bx)}{64d(c+dx)^2} + \frac{\sin(6a+6bx)}{64d(c+dx)^2} + \frac{(9b^2)}{16d^3} \text{Ci}\left(\frac{6bc}{d} + 6bx\right) \sin\left(6a - \frac{6bc}{d}\right) - \frac{3b^2 \text{Ci}\left(\frac{6bc}{d} + 6bx\right) \cos\left(6a - \frac{6bc}{d}\right)}{16d^3} \\
&= -\frac{3b\cos(2a+2bx)}{32d^2(c+dx)} + \frac{3b\cos(6a+6bx)}{32d^2(c+dx)} + \frac{9b^2 \text{Ci}\left(\frac{6bc}{d} + 6bx\right) \sin\left(6a - \frac{6bc}{d}\right)}{16d^3} - \frac{3b^2 \text{Ci}\left(\frac{6bc}{d} + 6bx\right) \cos\left(6a - \frac{6bc}{d}\right)}{16d^3}
\end{aligned}$$

Mathematica [A] time = 1.09066, size = 239, normalized size = 1.02

$$6b^2(c+dx)^2 \left(6\sin\left(6a - \frac{6bc}{d}\right) \text{CosIntegral}\left(\frac{6b(c+dx)}{d}\right) - 2\sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) - 2\cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2b(c+dx)}{d}\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] (-3*d*Cos[2*b*x]*(2*b*(c + d*x)*Cos[2*a] + d*Sin[2*a]) + d*Cos[6*b*x]*(6*b*(c + d*x)*Cos[6*a] + d*Sin[6*a]) + 3*d*(-(d*Cos[2*a]) + 2*b*(c + d*x)*Sin[2*a])*Sin[2*b*x] + d*(d*Cos[6*a] - 6*b*(c + d*x)*Sin[6*a])*Sin[6*b*x] + 6*b^2*(c + d*x)^2*(6*CosIntegral[(6*b*(c + d*x))/d]*Sin[6*a - (6*b*c)/d] - 2*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - 2*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 6*Cos[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c + d*x))/d]))/(64*d^3*(c + d*x)^2)
```

Maple [A] time = 0.028, size = 329, normalized size = 1.4

$$\frac{1}{b} \left(-\frac{b^3}{192} \left(-3 \frac{\sin(6bx+6a)}{((bx+a)d-ad+bc)^2 d} + 3 \frac{1}{d} \left(-6 \frac{\cos(6bx+6a)}{((bx+a)d-ad+bc)d} - 6 \frac{1}{d} \text{Si}\left(6bx+6a+6\frac{-ad+bc}{d}\right) \cos\left(6a-\frac{6bc}{d}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x)
```

```
[Out] 1/b*(-1/192*b^3*(-3*sin(6*b*x+6*a)/((b*x+a)*d-a*d+b*c)^2/d+3*(-6*cos(6*b*x+6*a)/((b*x+a)*d-a*d+b*c)/d-6*(6*Si(6*b*x+6*a+6*(-a*d+b*c)/d)*cos(6*(-a*d+b*c)/d)/d-6*Ci(6*b*x+6*a+6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d)/d)+3/64*b^3*(-sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)^2/d+(-2*cos(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)
```

Maxima [C] time = 2.3008, size = 454, normalized size = 1.93

$$\frac{b^3 \left(-3i E_3 \left(\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) + 3i E_3 \left(-\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) \right) \cos \left(-\frac{2(bc-ad)}{d} \right) + b^3 \left(i E_3 \left(\frac{6i bc + 6i (bx+a)d - 6i ad}{d} \right) - i E_3 \left(-\frac{6i bc + 6i (bx+a)d - 6i ad}{d} \right) \right)}{64 (b^2 c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(b^3*(-3*I*exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) + 3*I*exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c - a*d)/d) + b^3*(I*exp_integral_e(3, (6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d) - I*exp_integral_e(3, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))*cos(-6*(b*c - a*d)/d) - 3*b^3*(exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) + exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2*(b*c - a*d)/d) + b^3*(exp_integral_e(3, (6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d) + exp_integral_e(3, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))*sin(-6*(b*c - a*d)/d)/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

Fricas [A] time = 0.705712, size = 1010, normalized size = 4.3

$$96 (bd^2x + bcd) \cos (bx + a)^6 - 144 (bd^2x + bcd) \cos (bx + a)^4 + 48 (bd^2x + bcd) \cos (bx + a)^2 + 18 (b^2d^2x^2 + 2b^2cdx + b^2c^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(96*(b*d^2*x + b*c*d)*cos(b*x + a)^6 - 144*(b*d^2*x + b*c*d)*cos(b*x + a)^4 + 48*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 18*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-6*(b*c - a*d)/d)*sin_integral(6*(b*d*x + b*c)/d) - 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) + 16*(d^2*cos(b*x + a)^5 - d^2*cos(b*x + a)^3)*sin(b*x + a) - 3*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d) + 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(6*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-6*(b*d*x + b*c)/d))*sin(-6*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.162 \quad \int \frac{\cos^3(a+bx) \sin^3(a+bx)}{(c+dx)^4} dx$$

Optimal. Leaf size=287

$$-\frac{b^3 \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{8d^4} + \frac{9b^3 \cos\left(6a - \frac{6bc}{d}\right) \text{CosIntegral}\left(\frac{6bc}{d} + 6bx\right)}{8d^4} + \frac{b^3 \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{8d^4}$$

[Out] $-(b \cos[2a + 2bx]) / (32d^2(c + dx)^2) + (b \cos[6a + 6bx]) / (32d^2(c + dx)^2) - (b^3 \cos[2a - (2bc)/d] \text{CosIntegral}[(2bc)/d + 2bx]) / (8d^4) + (9b^3 \cos[6a - (6bc)/d] \text{CosIntegral}[(6bc)/d + 6bx]) / (8d^4) - \sin[2a + 2bx] / (32d(c + dx)^3) + (b^2 \sin[2a + 2bx]) / (16d^3(c + dx)) + \sin[6a + 6bx] / (96d(c + dx)^3) - (3b^2 \sin[6a + 6bx]) / (16d^3(c + dx)) + (b^3 \sin[2a - (2bc)/d] \text{SinIntegral}[(2bc)/d + 2bx]) / (8d^4) - (9b^3 \sin[6a - (6bc)/d] \text{SinIntegral}[(6bc)/d + 6bx]) / (8d^4)$

Rubi [A] time = 0.419222, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^3 \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{8d^4} + \frac{9b^3 \cos\left(6a - \frac{6bc}{d}\right) \text{CosIntegral}\left(\frac{6bc}{d} + 6bx\right)}{8d^4} + \frac{b^3 \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{8d^4}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^4,x]

[Out] $-(b \cos[2a + 2bx]) / (32d^2(c + dx)^2) + (b \cos[6a + 6bx]) / (32d^2(c + dx)^2) - (b^3 \cos[2a - (2bc)/d] \text{CosIntegral}[(2bc)/d + 2bx]) / (8d^4) + (9b^3 \cos[6a - (6bc)/d] \text{CosIntegral}[(6bc)/d + 6bx]) / (8d^4) - \sin[2a + 2bx] / (32d(c + dx)^3) + (b^2 \sin[2a + 2bx]) / (16d^3(c + dx)) + \sin[6a + 6bx] / (96d(c + dx)^3) - (3b^2 \sin[6a + 6bx]) / (16d^3(c + dx)) + (b^3 \sin[2a - (2bc)/d] \text{SinIntegral}[(2bc)/d + 2bx]) / (8d^4) - (9b^3 \sin[6a - (6bc)/d] \text{SinIntegral}[(6bc)/d + 6bx]) / (8d^4)$

Rule 4406

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n * Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&

NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\cos^3(a + bx) \sin^3(a + bx)}{(c + dx)^4} dx &= \int \left(\frac{3 \sin(2a + 2bx)}{32(c + dx)^4} - \frac{\sin(6a + 6bx)}{32(c + dx)^4} \right) dx \\
 &= -\left(\frac{1}{32} \int \frac{\sin(6a + 6bx)}{(c + dx)^4} dx \right) + \frac{3}{32} \int \frac{\sin(2a + 2bx)}{(c + dx)^4} dx \\
 &= -\frac{\sin(2a + 2bx)}{32d(c + dx)^3} + \frac{\sin(6a + 6bx)}{96d(c + dx)^3} + \frac{b \int \frac{\cos(2a+2bx)}{(c+dx)^3} dx}{16d} - \frac{b \int \frac{\cos(6a+6bx)}{(c+dx)^3} dx}{16d} \\
 &= -\frac{b \cos(2a + 2bx)}{32d^2(c + dx)^2} + \frac{b \cos(6a + 6bx)}{32d^2(c + dx)^2} - \frac{\sin(2a + 2bx)}{32d(c + dx)^3} + \frac{\sin(6a + 6bx)}{96d(c + dx)^3} - \frac{b^2 \int \frac{\sin(2a)}{(c+dx)} dx}{16d} \\
 &= -\frac{b \cos(2a + 2bx)}{32d^2(c + dx)^2} + \frac{b \cos(6a + 6bx)}{32d^2(c + dx)^2} - \frac{\sin(2a + 2bx)}{32d(c + dx)^3} + \frac{b^2 \sin(2a + 2bx)}{16d^3(c + dx)} + \frac{\sin(6a + 6bx)}{96d(c + dx)^3} \\
 &= -\frac{b \cos(2a + 2bx)}{32d^2(c + dx)^2} + \frac{b \cos(6a + 6bx)}{32d^2(c + dx)^2} - \frac{\sin(2a + 2bx)}{32d(c + dx)^3} + \frac{b^2 \sin(2a + 2bx)}{16d^3(c + dx)} + \frac{\sin(6a + 6bx)}{96d(c + dx)^3} \\
 &= -\frac{b \cos(2a + 2bx)}{32d^2(c + dx)^2} + \frac{b \cos(6a + 6bx)}{32d^2(c + dx)^2} - \frac{b^3 \cos\left(2a - \frac{2bc}{d}\right) \text{Ci}\left(\frac{2bc}{d} + 2bx\right)}{8d^4} + \frac{9b^3 \cos\left(6a - \frac{6bc}{d}\right) \text{Ci}\left(\frac{6bc}{d} + 6bx\right)}{8d^4}
 \end{aligned}$$

Mathematica [A] time = 5.13101, size = 554, normalized size = 1.93

$$12b^3c^3 \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2b(c+dx)}{d}\right) + 36b^3c^2 dx \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2b(c+dx)}{d}\right) - 108b^3c^3 \sin\left(6a - \frac{6bc}{d}\right) \text{Si}\left(\frac{6b(c+dx)}{d}\right) - 324b^3c^2 dx \sin\left(6a - \frac{6bc}{d}\right) \text{Si}\left(\frac{6b(c+dx)}{d}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^4,x]

[Out] (-3*b*c*d^2*Cos[2*(a + b*x)] - 3*b*d^3*x*Cos[2*(a + b*x)] + 3*b*c*d^2*Cos[6*(a + b*x)] + 3*b*d^3*x*Cos[6*(a + b*x)] - 12*b^3*(c + d*x)^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + 108*b^3*(c + d*x)^3*Cos[6*a - (6*b*c)/d]*CosIntegral[(6*b*(c + d*x))/d] + 6*b^2*c^2*d*Sin[2*(a + b*x)] - 3*d^3*Sin[2*(a + b*x)] + 12*b^2*c*d^2*x*Sin[2*(a + b*x)] + 6*b^2*d^3*x^2*Sin[2*(a + b*x)] - 18*b^2*c^2*d*Sin[6*(a + b*x)] + d^3*Sin[6*(a + b*x)] - 36*b^2*c*d^2*x*Sin[6*(a + b*x)] - 18*b^2*d^3*x^2*Sin[6*(a + b*x)] + 12*b^3*c^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 36*b^3*c^2*d*x*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 36*b^3*c*d^2*x^2*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 12*b^3*d^3*x^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] - 108*b^3*c^3*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c + d*x))/d] - 324*b^3*c^2*d*x*Sin[6*a - (6*b*c)/d]*SinIntegral[

$$\frac{(6*b*(c + d*x))/d - 324*b^3*c*d^2*x^2*\text{Sin}[6*a - (6*b*c)/d]*\text{SinIntegral}[(6*b*(c + d*x))/d] - 108*b^3*d^3*x^3*\text{Sin}[6*a - (6*b*c)/d]*\text{SinIntegral}[(6*b*(c + d*x))/d]}{(96*d^4*(c + d*x)^3}$$

Maple [A] time = 0.028, size = 404, normalized size = 1.4

$$\frac{1}{b} \left(-\frac{b^4}{192} \left(-2 \frac{\sin(6bx + 6a)}{((bx + a)d - ad + bc)^3 d} + 2 \frac{1}{d} \left(-3 \frac{\cos(6bx + 6a)}{((bx + a)d - ad + bc)^2 d} - 3 \frac{1}{d} \left(-6 \frac{\sin(6bx + 6a)}{((bx + a)d - ad + bc)d} + 6 \frac{1}{d} \text{Si} \left(\frac{6bx + 6a}{d} \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x)

[Out] $\frac{1}{b} \left(-\frac{1}{192} b^4 \left(-2 \frac{\sin(6bx + 6a)}{((bx + a)d - ad + bc)^3 d} + 2 \frac{1}{d} \left(-3 \frac{\cos(6bx + 6a)}{((bx + a)d - ad + bc)^2 d} - 3 \frac{1}{d} \left(-6 \frac{\sin(6bx + 6a)}{((bx + a)d - ad + bc)d} + 6 \frac{1}{d} \text{Si} \left(\frac{6bx + 6a}{d} \right) \right) \right) \right) + 3 \frac{1}{64} b^4 \left(-\frac{2}{3} \frac{\sin(2bx + 2a)}{((bx + a)d - ad + bc)^3 d} + \frac{2}{3} \frac{(-\cos(2bx + 2a))}{((bx + a)d - ad + bc)^2 d} - \frac{2}{3} \frac{\sin(2bx + 2a)}{((bx + a)d - ad + bc)d} + 2 \frac{1}{d} \left(2 \frac{\text{Si}(2bx + 2a)}{d} + 2 \frac{\sin(2bx + 2a)}{d} \right) \right) \right)$

Maxima [C] time = 3.22064, size = 521, normalized size = 1.82

$$\frac{b^4 \left(-3i E_4 \left(\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) + 3i E_4 \left(-\frac{2i bc + 2i (bx+a)d - 2i ad}{d} \right) \right) \cos \left(-\frac{2(bc-ad)}{d} \right) + b^4 \left(i E_4 \left(\frac{6i bc + 6i (bx+a)d - 6i ad}{d} \right) - i E_4 \left(-\frac{6i bc + 6i (bx+a)d - 6i ad}{d} \right) \right)}{64 (b^3 c^3 d - 3 ab^2 c^2 d^2 + 3 a^2 b c d^3 + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="maxima")

[Out] $\frac{1}{64} \left(b^4 \left(-3 \text{I} \exp_integral_e(4, (2 \text{I} b c + 2 \text{I} (b x + a) d - 2 \text{I} a d) / d) + 3 \text{I} \exp_integral_e(4, -(2 \text{I} b c + 2 \text{I} (b x + a) d - 2 \text{I} a d) / d) \right) \cos(-2 (b c - a d) / d) + b^4 \left(\text{I} \exp_integral_e(4, (6 \text{I} b c + 6 \text{I} (b x + a) d - 6 \text{I} a d) / d) - \text{I} \exp_integral_e(4, -(6 \text{I} b c + 6 \text{I} (b x + a) d - 6 \text{I} a d) / d) \right) \cos(-6 (b c - a d) / d) - 3 b^4 \left(\exp_integral_e(4, (2 \text{I} b c + 2 \text{I} (b x + a) d - 2 \text{I} a d) / d) + \exp_integral_e(4, -(2 \text{I} b c + 2 \text{I} (b x + a) d - 2 \text{I} a d) / d) \right) \sin(-2 (b c - a d) / d) + b^4 \left(\exp_integral_e(4, (6 \text{I} b c + 6 \text{I} (b x + a) d - 6 \text{I} a d) / d) + \exp_integral_e(4, -(6 \text{I} b c + 6 \text{I} (b x + a) d - 6 \text{I} a d) / d) \right) \sin(-6 (b c - a d) / d) \right) / ((b^3 c^3 d - 3 a b^2 c^2 d^2 + 3 a^2 b c d^3 + (b x + a)^3 d^4 - a^3 d^4 + 3 (b c d^3 - a d^4) (b x + a)^2 + 3 (b^2 c^2 d^2 - 2 a b c d^3 + a^2 d^4) (b x + a)) b)$

Fricas [B] time = 0.761782, size = 1419, normalized size = 4.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="fricas")

[Out] $\frac{1}{48} \left(48 (b d^3 x + b c d^2) \cos(b x + a)^6 - 72 (b d^3 x + b c d^2) \cos(b x + a)^4 + 24 (b d^3 x + b c d^2) \cos(b x + a)^2 - 54 (b^3 d^3 x^3 + 3 b^3 \dots \right)$

$$c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\sin(-6*(b*c - a*d)/d)*\sin_integral(6*(b*d*x + b*c)/d) + 6*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\sin(-2*(b*c - a*d)/d)*\sin_integral(2*(b*d*x + b*c)/d) - 3*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\cos_integral(2*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\cos_integral(-2*(b*d*x + b*c)/d))*\cos(-2*(b*c - a*d)/d) + 27*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\cos_integral(6*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\cos_integral(-6*(b*d*x + b*c)/d))*\cos(-6*(b*c - a*d)/d) - 16*((18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*\cos(b*x + a)^5 - (18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*\cos(b*x + a)^3 + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*\cos(b*x + a))*\sin(b*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c)**4,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")

[Out] Exception raised: NotImplementedError

3.163 $\int (c + dx)^m \cos^2(a + bx) \cot(a + bx) dx$

Optimal. Leaf size=151

$$\text{Unintegrable}(\cot(a + bx)(c + dx)^m, x) + \frac{2^{-m-3} e^{2i\left(a - \frac{bc}{d}\right)} (c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, -\frac{2ib(c+dx)}{d}\right)}{b} + \frac{2^{-m-3} e^{-2i\left(a - \frac{bc}{d}\right)} (c + dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, \frac{2ib(c+dx)}{d}\right)}{b}$$

[Out] $(2^{(-3 - m)} E^{((2I)*(a - (b*c)/d)}) * (c + d*x)^m * \Gamma[1 + m, ((-2I)*b*(c + d*x))/d]) / (b * (((-I)*b*(c + d*x))/d)^m) + (2^{(-3 - m)} * (c + d*x)^m * \Gamma[1 + m, ((2I)*b*(c + d*x))/d]) / (b * E^{((2I)*(a - (b*c)/d)}) * ((I*b*(c + d*x))/d)^m) + \text{Unintegrable}[(c + d*x)^m * \text{Cot}[a + b*x], x]$

Rubi [A] time = 0.172095, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \cos^2(a + bx) \cot(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m * Cos[a + b*x]^2 * Cot[a + b*x], x]

[Out] $(2^{(-3 - m)} E^{((2I)*(a - (b*c)/d)}) * (c + d*x)^m * \Gamma[1 + m, ((-2I)*b*(c + d*x))/d]) / (b * (((-I)*b*(c + d*x))/d)^m) + (2^{(-3 - m)} * (c + d*x)^m * \Gamma[1 + m, ((2I)*b*(c + d*x))/d]) / (b * E^{((2I)*(a - (b*c)/d)}) * ((I*b*(c + d*x))/d)^m) + \text{Defer}[\text{Int}][(c + d*x)^m * \text{Cot}[a + b*x], x]$

Rubi steps

$$\begin{aligned} \int (c + dx)^m \cos^2(a + bx) \cot(a + bx) dx &= \int (c + dx)^m \cot(a + bx) dx - \int (c + dx)^m \cos(a + bx) \sin(a + bx) dx \\ &= \int (c + dx)^m \cot(a + bx) dx - \int \frac{1}{2} (c + dx)^m \sin(2a + 2bx) dx \\ &= -\left(\frac{1}{2} \int (c + dx)^m \sin(2a + 2bx) dx\right) + \int (c + dx)^m \cot(a + bx) dx \\ &= -\left(\frac{1}{4} i \int e^{-i(2a+2bx)} (c + dx)^m dx\right) + \frac{1}{4} i \int e^{i(2a+2bx)} (c + dx)^m dx + \int (c + dx)^m \cot(a + bx) dx \\ &= \frac{2^{-3-m} e^{2i\left(a - \frac{bc}{d}\right)} (c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, -\frac{2ib(c+dx)}{d}\right)}{b} + \frac{2^{-3-m} e^{-2i\left(a - \frac{bc}{d}\right)} (c + dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, \frac{2ib(c+dx)}{d}\right)}{b} + \int (c + dx)^m \cot(a + bx) dx \end{aligned}$$

Mathematica [A] time = 7.75228, size = 0, normalized size = 0.

$$\int (c + dx)^m \cos^2(a + bx) \cot(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m * Cos[a + b*x]^2 * Cot[a + b*x], x]

[Out] Integrate[(c + d*x)^m * Cos[a + b*x]^2 * Cot[a + b*x], x]

Maple [A] time = 0.192, size = 0, normalized size = 0.

$$\int (dx + c)^m (\cos (bx + a))^2 \cot (bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x)

[Out] int((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos (bx + a)^2 \cot (bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")

[Out] integrate((d*x + c)^m*cos(b*x + a)^2*cot(b*x + a), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx + c)^m \cos (bx + a)^2 \cot (bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")

[Out] integral((d*x + c)^m*cos(b*x + a)^2*cot(b*x + a), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**m*cos(b*x+a)**2*cot(b*x+a),x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos (bx + a)^2 \cot (bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^m*cos(b*x + a)^2*cot(b*x + a), x)

3.164 $\int (c + dx)^4 \cos^2(a + bx) \cot(a + bx) dx$

Optimal. Leaf size=307

$$\frac{3d^2(c + dx)^2 \text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{b^3} + \frac{3id^3(c + dx) \text{PolyLog}\left(4, e^{2i(a+bx)}\right)}{b^4} - \frac{2id(c + dx)^3 \text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^2} - \frac{3d^4 \text{PolyLog}\left(5, e^{2i(a+bx)}\right)}{b^5}$$

[Out] $(-3*c*d^3*x)/(2*b^3) - (3*d^4*x^2)/(4*b^3) + (c + d*x)^4/(4*b) - ((I/5)*(c + d*x)^5)/d + ((c + d*x)^4*\text{Log}[1 - E^((2*I)*(a + b*x))])/b - ((2*I)*d*(c + d*x)^3*\text{PolyLog}[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)^2*\text{PolyLog}[3, E^((2*I)*(a + b*x))])/b^3 + ((3*I)*d^3*(c + d*x)*\text{PolyLog}[4, E^((2*I)*(a + b*x))])/b^4 - (3*d^4*\text{PolyLog}[5, E^((2*I)*(a + b*x))])/(2*b^5) + (3*d^3*(c + d*x)*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/(2*b^4) - (d*(c + d*x)^3*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b^2 - (3*d^4*\text{Sin}[a + b*x]^2)/(4*b^5) + (3*d^2*(c + d*x)^2*\text{Sin}[a + b*x]^2)/(2*b^3) - ((c + d*x)^4*\text{Sin}[a + b*x]^2)/(2*b)$

Rubi [A] time = 0.339897, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {4408, 4404, 3311, 32, 3310, 3717, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c + dx)^2 \text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{b^3} + \frac{3id^3(c + dx) \text{PolyLog}\left(4, e^{2i(a+bx)}\right)}{b^4} - \frac{2id(c + dx)^3 \text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^2} - \frac{3d^4 \text{PolyLog}\left(5, e^{2i(a+bx)}\right)}{b^5}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^4*\text{Cos}[a + b*x]^2*\text{Cot}[a + b*x], x]$

[Out] $(-3*c*d^3*x)/(2*b^3) - (3*d^4*x^2)/(4*b^3) + (c + d*x)^4/(4*b) - ((I/5)*(c + d*x)^5)/d + ((c + d*x)^4*\text{Log}[1 - E^((2*I)*(a + b*x))])/b - ((2*I)*d*(c + d*x)^3*\text{PolyLog}[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)^2*\text{PolyLog}[3, E^((2*I)*(a + b*x))])/b^3 + ((3*I)*d^3*(c + d*x)*\text{PolyLog}[4, E^((2*I)*(a + b*x))])/b^4 - (3*d^4*\text{PolyLog}[5, E^((2*I)*(a + b*x))])/(2*b^5) + (3*d^3*(c + d*x)*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/(2*b^4) - (d*(c + d*x)^3*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b^2 - (3*d^4*\text{Sin}[a + b*x]^2)/(4*b^5) + (3*d^2*(c + d*x)^2*\text{Sin}[a + b*x]^2)/(2*b^3) - ((c + d*x)^4*\text{Sin}[a + b*x]^2)/(2*b)$

Rule 4408

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_)]^{(n_.)}*\text{Cot}[(a_.) + (b_.)*(x_)]^{(p_.)}*((c_.) + (d_.)*(x_))^{(m_.)}, x_Symbol] \rightarrow -\text{Int}[(c + d*x)^m*\text{Cos}[a + b*x]^n*\text{Cot}[a + b*x]^{(p - 2)}, x] + \text{Int}[(c + d*x)^m*\text{Cos}[a + b*x]^{(n - 2)}*\text{Cot}[a + b*x]^p, x] /; \text{FreeQ}\{a, b, c, d, m, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$

Rule 4404

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Sin}[a + b*x]^{(n + 1)}/(b*(n + 1)), x] - \text{Dist}[(d*m)/(b*(n + 1)), \text{Int}[(c + d*x)^{(m - 1)}*\text{Sin}[a + b*x]^{(n + 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, n, x\} \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[n, -1]$

Rule 3311

$\text{Int}[(c_.) + (d_.)*(x_)]^{(m_.)}*((b_.)*\text{sin}[(e_.) + (f_.)*(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(d*m*(c + d*x)^{(m - 1)}*(b*\text{Sin}[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n - 1))/n, \text{Int}[(c + d*x)^m*(b*\text{Sin}[e + f*x])^{(n - 2)}, x], x] - \text{Dist}[(d^2*m*(m - 1))/(f^2*n^2), \text{Int}[(c + d*x)^{(m - 2)}*(b*\text{Sin}[e + f*x])^n, x], x] - \text{Simp}[(b*(c + d*x)^m*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(n - 1)})/(f*n), x]) /;$

FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 3310

Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(d*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)*(b*Sine[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b*Sine[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rule 3717

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m * E^(2*I*k*Pi) * E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi) * E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m * Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1) * Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m * PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1) * PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 6609

Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m * PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1) * PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d

, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned}
 \int (c + dx)^4 \cos^2(a + bx) \cot(a + bx) dx &= \int (c + dx)^4 \cot(a + bx) dx - \int (c + dx)^4 \cos(a + bx) \sin(a + bx) dx \\
 &= \frac{i(c + dx)^5}{5d} - \frac{(c + dx)^4 \sin^2(a + bx)}{2b} - 2i \int \frac{e^{2i(a+bx)}(c + dx)^4}{1 - e^{2i(a+bx)}} dx + \frac{(2d) \int (c + dx)^4 \cos(a + bx) \sin(a + bx) dx}{b} \\
 &= \frac{i(c + dx)^5}{5d} + \frac{(c + dx)^4 \log(1 - e^{2i(a+bx)})}{b} - \frac{d(c + dx)^3 \cos(a + bx) \sin(a + bx)}{b^2} \\
 &= \frac{(c + dx)^4}{4b} - \frac{i(c + dx)^5}{5d} + \frac{(c + dx)^4 \log(1 - e^{2i(a+bx)})}{b} - \frac{2id(c + dx)^3 \operatorname{Li}_2(e^{2i(a+bx)})}{b^2} \\
 &= -\frac{3cd^3x}{2b^3} - \frac{3d^4x^2}{4b^3} + \frac{(c + dx)^4}{4b} - \frac{i(c + dx)^5}{5d} + \frac{(c + dx)^4 \log(1 - e^{2i(a+bx)})}{b} - \frac{2id(c + dx)^3 \operatorname{Li}_2(e^{2i(a+bx)})}{b^2} \\
 &= -\frac{3cd^3x}{2b^3} - \frac{3d^4x^2}{4b^3} + \frac{(c + dx)^4}{4b} - \frac{i(c + dx)^5}{5d} + \frac{(c + dx)^4 \log(1 - e^{2i(a+bx)})}{b} - \frac{2id(c + dx)^3 \operatorname{Li}_2(e^{2i(a+bx)})}{b^2} \\
 &= -\frac{3cd^3x}{2b^3} - \frac{3d^4x^2}{4b^3} + \frac{(c + dx)^4}{4b} - \frac{i(c + dx)^5}{5d} + \frac{(c + dx)^4 \log(1 - e^{2i(a+bx)})}{b} - \frac{2id(c + dx)^3 \operatorname{Li}_2(e^{2i(a+bx)})}{b^2} \\
 &= -\frac{3cd^3x}{2b^3} - \frac{3d^4x^2}{4b^3} + \frac{(c + dx)^4}{4b} - \frac{i(c + dx)^5}{5d} + \frac{(c + dx)^4 \log(1 - e^{2i(a+bx)})}{b} - \frac{2id(c + dx)^3 \operatorname{Li}_2(e^{2i(a+bx)})}{b^2}
 \end{aligned}$$

Mathematica [B] time = 6.52244, size = 2828, normalized size = 9.21

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2*Cot[a + b*x], x]

[Out] $-\left(\frac{c^2 d^2 E^{(I a)} \operatorname{Csc}[a] \left(\frac{2 b^3 x^3}{E^{(2 I) a}} + (3 I) b^2 (1 - E^{(-2 I) a})\right) x^2 \operatorname{Log}[1 - E^{(-I) (a + b x)}] + (3 I) b^2 (1 - E^{(-2 I) a}) x^2 \operatorname{Log}[1 + E^{(-I) (a + b x)}] - (6 (-1 + E^{(2 I) a}) (b x \operatorname{PolyLog}[2, -E^{(-I) (a + b x)}]) - I \operatorname{PolyLog}[3, -E^{(-I) (a + b x)}])}{E^{(2 I) a}} - (6 (-1 + E^{(2 I) a}) (b x \operatorname{PolyLog}[2, E^{(-I) (a + b x)}]) - I \operatorname{PolyLog}[3, E^{(-I) (a + b x)}])}{E^{(2 I) a}}\right) / b^3 - (c d^3 E^{(I a)} \operatorname{Csc}[a] \left(\frac{b^4 x^4}{E^{(2 I) a}} + (2 I) b^3 (1 - E^{(-2 I) a})\right) x^3 \operatorname{Log}[1 - E^{(-I) (a + b x)}] + (2 I) b^3 (1 - E^{(-2 I) a}) x^3 \operatorname{Log}[1 + E^{(-I) (a + b x)}] - (6 (-1 + E^{(2 I) a}) (b^2 x^2 \operatorname{PolyLog}[2, -E^{(-I) (a + b x)}]) - (2 I) b x \operatorname{PolyLog}[3, -E^{(-I) (a + b x)}]) - 2 \operatorname{PolyLog}[4, -E^{(-I) (a + b x)}])}{E^{(2 I) a}} - (6 (-1 + E^{(2 I) a}) (b^2 x^2 \operatorname{PolyLog}[2, E^{(-I) (a + b x)}]) - (2 I) b x \operatorname{PolyLog}[3, E^{(-I) (a + b x)}]) - 2 \operatorname{PolyLog}[4, E^{(-I) (a + b x)}])}{E^{(2 I) a}}\right) / b^4 - (d^4 E^{(I a)} \operatorname{Csc}[a] \left(\frac{2 b^5 x^5}{E^{(2 I) a}} + (5 I) b^4 (1 - E^{(-2 I) a})\right) x^4 \operatorname{Log}[1 - E^{(-I) (a + b x)}] + (5 I) b^4 (1 - E^{(-2 I) a}) x^4 \operatorname{Log}[1 + E^{(-I) (a + b x)}] - (20 (-1 + E^{(2 I) a}) (b^3 x^3 \operatorname{PolyLog}[2, -E^{(-I) (a + b x)}]) - (3 I) b^2 x^2 \operatorname{PolyLog}[3, -E^{(-I) (a + b x)}]) - 6 b x \operatorname{PolyLog}[4, -E^{(-I) (a + b x)}] + (6 I) \operatorname{PolyLog}[5, -E^{(-I) (a + b x)}])}{E^{(2 I) a}} - (20 (-1 + E^{(2 I) a}) (b^3 x^3 \operatorname{PolyLog}[2, E^{(-I) (a + b x)}]) - (3 I) b^2 x^2 \operatorname{PolyLog}[3, E^{(-I) (a + b x)}]) - 6 b x \operatorname{PolyLog}[4, E^{(-I) (a + b x)}] + (6 I) \operatorname{PolyLog}[5, E^{(-I) (a + b x)}])}{E^{(2 I) a}}\right) / (10 b^5) + (c^4 \operatorname{Csc}[a] \left(-b x \operatorname{Cos}[a] + \operatorname{Log}[\operatorname{Cos}[b x] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[b x]] \operatorname{Sin}[a]\right) / (b (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)) + \operatorname{Csc}[a] \left(\frac{\operatorname{Cos}[2 a + 2 b x]}{160 b^5} - \left(\frac{I}{160}\right) \operatorname{Sin}[2 a + 2 b x]\right) / b^5) (80 b^5 c^4 x \operatorname{Cos}[a + 2 b x] + 160 b^5 c^3 d x^2 \operatorname{Cos}[a + 2 b x] + 160 b^5 c^2 d^2 x^3 \operatorname{Cos}[a + 2 b x] + 80 b^5 c d^3 x^4 \operatorname{Cos}[a + 2 b x] + 16 b^5 d^4 x^5 \operatorname{Cos}[a + 2 b x] + 80 b^5 c^4 x \operatorname{Cos}[3 a + 2 b x] + 160$

$$\begin{aligned}
& b^5 c^3 d^2 x^2 \cos[3a + 2bx] + 160 b^5 c^2 d^2 x^3 \cos[3a + 2bx] + 80 b^5 c d^3 x^4 \cos[3a + 2bx] + 16 b^5 d^4 x^5 \cos[3a + 2bx] + (10I) b^4 c^4 \cos[3a + 4bx] - 20 b^3 c^3 d \cos[3a + 4bx] - (30I) b^2 c^2 d^2 \cos[3a + 4bx] + 30 b^2 c d^3 \cos[3a + 4bx] + (15I) d^4 \cos[3a + 4bx] + (40I) b^4 c^3 d x \cos[3a + 4bx] - 60 b^3 c^2 d^2 x \cos[3a + 4bx] - (60I) b^2 c d^3 x \cos[3a + 4bx] + 30 b^2 d^4 x \cos[3a + 4bx] + (60I) b^4 c^2 d^2 x^2 \cos[3a + 4bx] - 60 b^3 c d^3 x^2 \cos[3a + 4bx] - (30I) b^2 d^4 x^2 \cos[3a + 4bx] + (40I) b^4 c d^3 x^3 \cos[3a + 4bx] - 20 b^3 d^4 x^3 \cos[3a + 4bx] + (10I) b^4 d^4 x^4 \cos[3a + 4bx] - (10I) b^4 c^4 \cos[5a + 4bx] + 20 b^3 c^3 d \cos[5a + 4bx] + (30I) b^2 c^2 d^2 \cos[5a + 4bx] - 30 b^2 c d^3 \cos[5a + 4bx] - (15I) d^4 \cos[5a + 4bx] - (40I) b^4 c^3 d x \cos[5a + 4bx] + 60 b^3 c^2 d^2 x \cos[5a + 4bx] + (60I) b^2 c d^3 x \cos[5a + 4bx] - 30 b^2 d^4 x \cos[5a + 4bx] - (60I) b^4 c^2 d^2 x^2 \cos[5a + 4bx] + 60 b^3 c d^3 x^2 \cos[5a + 4bx] + (30I) b^2 d^4 x^2 \cos[5a + 4bx] - (40I) b^4 c d^3 x^3 \cos[5a + 4bx] + 20 b^3 d^4 x^3 \cos[5a + 4bx] - (10I) b^4 d^4 x^4 \cos[5a + 4bx] + 20 b^4 c^4 \sin[a] - (40I) b^3 c^3 d \sin[a] - 60 b^2 c^2 d^2 \sin[a] + (60I) b^2 c d^3 \sin[a] + 30 d^4 \sin[a] + 80 b^4 c^3 d x \sin[a] - (120I) b^3 c^2 d^2 x \sin[a] - 120 b^2 c d^3 x \sin[a] + (60I) b^2 d^4 x \sin[a] + 120 b^4 c^2 d^2 x^2 \sin[a] - (120I) b^3 c d^3 x^2 \sin[a] - 60 b^2 d^4 x^2 \sin[a] + 80 b^4 c d^3 x^3 \sin[a] - (40I) b^3 d^4 x^3 \sin[a] + 20 b^4 d^4 x^4 \sin[a] + (80I) b^5 c^4 x \sin[a + 2bx] + (160I) b^5 c^3 d x^2 \sin[a + 2bx] + (160I) b^5 c^2 d^2 x^3 \sin[a + 2bx] + (80I) b^5 c d^3 x^4 \sin[a + 2bx] + (16I) b^5 d^4 x^5 \sin[a + 2bx] + (80I) b^5 c^4 x \sin[3a + 2bx] + (160I) b^5 c^3 d x^2 \sin[3a + 2bx] + (160I) b^5 c^2 d^2 x^3 \sin[3a + 2bx] + (80I) b^5 c d^3 x^4 \sin[3a + 2bx] + (16I) b^5 d^4 x^5 \sin[3a + 2bx] - 10 b^4 c^4 \sin[3a + 4bx] - (20I) b^3 c^3 d \sin[3a + 4bx] + 30 b^2 c^2 d^2 \sin[3a + 4bx] + (30I) b^2 c d^3 \sin[3a + 4bx] - 15 d^4 \sin[3a + 4bx] - 40 b^4 c^3 d x \sin[3a + 4bx] - (60I) b^3 c^2 d^2 x \sin[3a + 4bx] + 60 b^2 c d^3 x \sin[3a + 4bx] + (30I) b^2 d^4 x \sin[3a + 4bx] - 60 b^4 c^2 d^2 x^2 \sin[3a + 4bx] - (60I) b^3 c d^3 x^2 \sin[3a + 4bx] + 30 b^2 d^4 x^2 \sin[3a + 4bx] - 40 b^4 c d^3 x^3 \sin[3a + 4bx] - (20I) b^3 d^4 x^3 \sin[3a + 4bx] - 10 b^4 d^4 x^4 \sin[3a + 4bx] + 10 b^4 c^4 \sin[5a + 4bx] + (20I) b^3 c^3 d \sin[5a + 4bx] - 30 b^2 c^2 d^2 \sin[5a + 4bx] - (30I) b^2 c d^3 \sin[5a + 4bx] + 15 d^4 \sin[5a + 4bx] + 40 b^4 c^3 d x \sin[5a + 4bx] + (60I) b^3 c^2 d^2 x \sin[5a + 4bx] - 60 b^2 c d^3 x \sin[5a + 4bx] - (30I) b^2 d^4 x \sin[5a + 4bx] + 60 b^4 c^2 d^2 x^2 \sin[5a + 4bx] + (60I) b^3 c d^3 x^2 \sin[5a + 4bx] - 30 b^2 d^4 x^2 \sin[5a + 4bx] + 40 b^4 c d^3 x^3 \sin[5a + 4bx] + (20I) b^3 d^4 x^3 \sin[5a + 4bx] + 10 b^4 d^4 x^4 \sin[5a + 4bx] - (2c^3 d \operatorname{Csc}[a] \operatorname{Sec}[a] (b^2 E^{(I \operatorname{ArcTan}[\operatorname{Tan}[a]])}) x^2 + ((I b^2 x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}[1 + E^{((-2I) b x)}] - 2(b x + \operatorname{ArcTan}[\operatorname{Tan}[a])) \operatorname{Log}[1 - E^{((2I) (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])})}] + \pi \operatorname{Log}[\operatorname{Cos}[b x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]]]) + I \operatorname{PolyLog}[2, E^{((2I) (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])})}]) \operatorname{Tan}[a]) / \operatorname{Sqrt}[1 + \operatorname{Tan}[a]^2]) / (b^2 \operatorname{Sqrt}[\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)])
\end{aligned}$$

Maple [B] time = 0.49, size = 1492, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (d^2 x + c)^4 \cos(bx + a)^2 \cot(bx + a), x$

[Out] $1/b^4 d^4 \ln(1 - \exp(I(bx + a))) x^4 - 1/b^5 d^4 \ln(1 - \exp(I(bx + a))) a^4 + I c^4 x^4 + 4/b^3 c d^3 \ln(1 - \exp(I(bx + a))) x^3 + 1/16 (2 d^4 x^4 b^4 + 4 I b^3 d^4 x^3 + 8 b$

$$\begin{aligned}
&^4*c*d^3*x^3+12*I*b^3*c*d^3*x^2+12*b^4*c^2*d^2*x^2+12*I*b^3*c^2*d^2*x+8*b^4 \\
&*c^3*d*x+4*I*b^3*c^3*d+2*b^4*c^4-6*b^2*d^4*x^2-6*I*b*d^4*x-12*b^2*c*d^3*x-6 \\
&*I*b*c*d^3-6*c^2*d^2*b^2+3*d^4)/b^5*exp(2*I*(b*x+a))+4/b^4*c*d^3*ln(1-exp(I \\
&*(b*x+a)))*a^3+8/5*I/b^5*d^4*a^5-I*c*d^3*x^4-2*I*c^2*d^2*x^3-2*I*c^3*d*x^2+ \\
&4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3+12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x^2 \\
&+12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))+12/b^3*c^2*d^2*polylog(3,-exp(I*(\\
&b*x+a)))+12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2+6/b*c^2*d^2*ln(1-exp(I*(\\
&b*x+a)))*x^2+1/b*d^4*ln(exp(I*(b*x+a))+1)*x^4-6/b^3*c^2*d^2*a^2*ln(1-exp(I*(\\
&(b*x+a)))+4/b*c^3*d*ln(1-exp(I*(b*x+a)))*x+4/b^2*c^3*d*ln(1-exp(I*(b*x+a))) \\
&*a+4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x+24/b^3*c*d^3*polylog(3,-exp(I*(b*x+a))) \\
&*x+6/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2+24/b^3*c*d^3*polylog(3,exp(I*(b*x+a) \\
&))*x+1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)-2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))-1 \\
&/5*I*d^4*x^5+8/b^2*c^3*d*a*ln(exp(I*(b*x+a)))+6/b^3*c^2*d^2*a^2*ln(exp(I*(b \\
&*x+a))-1)-12/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a)))-4/b^4*c*d^3*a^3*ln(exp(I*(b \\
&*x+a))-1)+8/b^4*c*d^3*a^3*ln(exp(I*(b*x+a)))-4/b^2*c^3*d*a*ln(exp(I*(b*x+a) \\
&))-1)-4*I/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))-4*I/b^2*c^3*d*polylog(2,-exp(I \\
&*(b*x+a)))+24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x+2*I/b^4*d^4*a^4*x+8*I/b \\
&^3*c^2*d^2*a^3-4*I/b^2*c^3*d*a^2-6*I/b^4*c*d^3*a^4-4*I/b^2*d^4*polylog(2,ex \\
&p(I*(b*x+a)))*x^3-4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*p \\
&olylog(4,-exp(I*(b*x+a)))*x+24*I/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))+24*I/b \\
&^4*c*d^3*polylog(4,-exp(I*(b*x+a)))-24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-2 \\
&4*d^4*polylog(5,exp(I*(b*x+a)))/b^5-8*I/b*c^3*d*a*x+12*I/b^2*c^2*d^2*a^2*x- \\
&12*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-12*I/b^2*c^2*d^2*polylog(2,ex \\
&p(I*(b*x+a)))*x-12*I/b^2*c^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+1/b*c^4*ln(ex \\
&p(I*(b*x+a))+1)+1/b*c^4*ln(exp(I*(b*x+a))-1)-2/b*c^4*ln(exp(I*(b*x+a)))+1/1 \\
&6*(2*d^4*x^4*b^4-4*I*b^3*d^4*x^3+8*b^4*c*d^3*x^3-12*I*b^3*c*d^3*x^2+12*b^4* \\
&c^2*d^2*x^2-12*I*b^3*c^2*d^2*x+8*b^4*c^3*d*x-4*I*b^3*c^3*d+2*b^4*c^4-6*b^2* \\
&d^4*x^2+6*I*b*d^4*x-12*b^2*c*d^3*x+6*I*b*c*d^3-6*c^2*d^2*b^2+3*d^4)/b^5*exp \\
&(-2*I*(b*x+a))-12*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2-8*I/b^3*c*d^3*a \\
&^3*x
\end{aligned}$$

Maxima [B] time = 2.63538, size = 2207, normalized size = 7.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")

[Out] $-1/40*(20*(\sin(b*x + a)^2 - \log(\sin(b*x + a)^2))*c^4 - 80*(\sin(b*x + a)^2 - \log(\sin(b*x + a)^2)) * a^2 * c^2 * d^2 / b^2 - 80*(\sin(b*x + a)^2 - \log(\sin(b*x + a)^2)) * a^3 * c * d^3 / b^3 + 20*(\sin(b*x + a)^2 - \log(\sin(b*x + a)^2)) * a^4 * d^4 / b^4 - (-8*I*(b*x + a)^5 * d^4 + (-40*I*b*c*d^3 + 40*I*a*d^4)*(b*x + a)^4 - 960*d^4*polylog(5, -e^(I*b*x + I*a)) - 960*d^4*polylog(5, e^(I*b*x + I*a)) + (-80*I*b^2*c^2*d^2 + 160*I*a*b*c*d^3 - 80*I*a^2*d^4)*(b*x + a)^3 + (-80*I*b^3*c^3*d + 240*I*a*b^2*c^2*d^2 - 240*I*a^2*b*c*d^3 + 80*I*a^3*d^4)*(b*x + a)^2 + (40*I*(b*x + a)^4 * d^4 + (160*I*b*c*d^3 - 160*I*a*d^4)*(b*x + a)^3 + (240*I*b^2*c^2*d^2 - 480*I*a*b*c*d^3 + 240*I*a^2*d^4)*(b*x + a)^2 + (160*I*b^3*c^3*d - 480*I*a*b^2*c^2*d^2 + 480*I*a^2*b*c*d^3 - 160*I*a^3*d^4)*(b*x + a))*arctan2(\sin(b*x + a), \cos(b*x + a) + 1) + (-40*I*(b*x + a)^4 * d^4 + (-160*I*b*c*d^3 + 160*I*a*d^4)*(b*x + a)^3 + (-240*I*b^2*c^2*d^2 + 480*I*a*b*c*d^3 - 240*I*a^2*d^4)*(b*x + a)^2 + (-160*I*b^3*c^3*d + 480*I*a*b^2*c^2*d^2 - 480*I*a^2*b*c*d^3 + 160*I*a^3*d^4)*(b*x + a))*arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) + 5*(2*(b*x + a)^4 * d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 3*(2*a^2 - 1)*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(2*b^2*c^2*d^2 - 4*a*b*c*d^3 + (2*a^2 - 1)*d^4)*(b*x + a)^2 + 4*(2*b^3*c^3*d - 6*a*b^2*c^2*d^2 + 3*(2*a^2 - 1)*b*c*d^3 -$

$$\begin{aligned}
& (2*a^3 - 3*a)*d^4*(b*x + a))*\cos(2*b*x + 2*a) + (-160*I*b^3*c^3*d + 480*I* \\
& a*b^2*c^2*d^2 - 480*I*a^2*b*c*d^3 - 160*I*(b*x + a)^3*d^4 + 160*I*a^3*d^4 + \\
& (-480*I*b*c*d^3 + 480*I*a*d^4)*(b*x + a)^2 + (-480*I*b^2*c^2*d^2 + 960*I*a \\
& *b*c*d^3 - 480*I*a^2*d^4)*(b*x + a))*\operatorname{dilog}(-e^{(I*b*x + I*a)}) + (-160*I*b^3* \\
& c^3*d + 480*I*a*b^2*c^2*d^2 - 480*I*a^2*b*c*d^3 - 160*I*(b*x + a)^3*d^4 + 1 \\
& 60*I*a^3*d^4 + (-480*I*b*c*d^3 + 480*I*a*d^4)*(b*x + a)^2 + (-480*I*b^2*c^2 \\
& *d^2 + 960*I*a*b*c*d^3 - 480*I*a^2*d^4)*(b*x + a))*\operatorname{dilog}(e^{(I*b*x + I*a)}) + \\
& 20*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2 \\
& *a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2* \\
& b*c*d^3 - a^3*d^4)*(b*x + a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b \\
& *x + a) + 1) + 20*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b \\
& ^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2 \\
& *d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*\log(\cos(b*x + a)^2 + \sin(b*x + \\
& a)^2 - 2*\cos(b*x + a) + 1) + (960*I*b*c*d^3 + 960*I*(b*x + a)*d^4 - 960*I*a \\
& *d^4)*\operatorname{polylog}(4, -e^{(I*b*x + I*a)}) + (960*I*b*c*d^3 + 960*I*(b*x + a)*d^4 - \\
& 960*I*a*d^4)*\operatorname{polylog}(4, e^{(I*b*x + I*a)}) + 480*(b^2*c^2*d^2 - 2*a*b*c*d^3 \\
& + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*\operatorname{polylog}(3, -e^{ \\
& (I*b*x + I*a)}) + 480*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 \\
& + 2*(b*c*d^3 - a*d^4)*(b*x + a))*\operatorname{polylog}(3, e^{(I*b*x + I*a)}) - 10*(2*b^3*c \\
& ^3*d - 6*a*b^2*c^2*d^2 + 2*(b*x + a)^3*d^4 + 3*(2*a^2 - 1)*b*c*d^3 - (2*a^3 \\
& - 3*a)*d^4 + 6*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(2*b^2*c^2*d^2 - 4*a*b*c* \\
& d^3 + (2*a^2 - 1)*d^4)*(b*x + a))*\sin(2*b*x + 2*a))/b^4)/b
\end{aligned}$$

Fricas [C] time = 0.922434, size = 3460, normalized size = 11.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")

[Out] $-1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 48*d^4*\operatorname{polylog}(5, \cos(b*x + a) + I*\sin(b*x + a)) + 48*d^4*\operatorname{polylog}(5, \cos(b*x + a) - I*\sin(b*x + a)) + 48*d^4*\operatorname{polylog}(5, -\cos(b*x + a) + I*\sin(b*x + a)) + 48*d^4*\operatorname{polylog}(5, -\cos(b*x + a) - I*\sin(b*x + a)) + 3*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 - (2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*\cos(b*x + a)^2 + 2*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*x)*\cos(b*x + a)*\sin(b*x + a) + 2*(2*b^4*c^3*d - 3*b^2*c*d^3)*x - (-8*I*b^3*d^4*x^3 - 24*I*b^3*c*d^3*x^2 - 24*I*b^3*c^2*d^2*x - 8*I*b^3*c^3*d)*\operatorname{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) - (8*I*b^3*d^4*x^3 + 24*I*b^3*c*d^3*x^2 + 24*I*b^3*c^2*d^2*x + 8*I*b^3*c^3*d)*\operatorname{dilog}(\cos(b*x + a) - I*\sin(b*x + a)) - (8*I*b^3*d^4*x^3 + 24*I*b^3*c*d^3*x^2 + 24*I*b^3*c^2*d^2*x + 8*I*b^3*c^3*d)*\operatorname{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) - (-8*I*b^3*d^4*x^3 - 24*I*b^3*c*d^3*x^2 - 24*I*b^3*c^2*d^2*x - 8*I*b^3*c^3*d)*\operatorname{dilog}(-\cos(b*x + a) - I*\sin(b*x + a)) - 2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) - 2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1) - 2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*\log(-1/2*\cos(b*x + a) + 1/2*I*\sin(b*x + a) + 1/2) - 2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*\log(-1/2*\cos(b*x + a) - 1/2*I*\sin(b*x + a) + 1/2) - 2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1) - 2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*\log(-\cos(b*x + a) - I*\sin(b*x + a) + 1) - (48*I*b*d^4*x + 48*I*b*c*d^3)*\operatorname{polylog}(4, \cos(b*x + a) + I$

```
*sin(b*x + a)) - (-48*I*b*d^4*x - 48*I*b*c*d^3)*polylog(4, cos(b*x + a) - I
*sin(b*x + a)) - (-48*I*b*d^4*x - 48*I*b*c*d^3)*polylog(4, -cos(b*x + a) +
I*sin(b*x + a)) - (48*I*b*d^4*x + 48*I*b*c*d^3)*polylog(4, -cos(b*x + a) -
I*sin(b*x + a)) - 24*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3,
cos(b*x + a) + I*sin(b*x + a)) - 24*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2
*d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 24*(b^2*d^4*x^2 + 2*b^2*c
*d^3*x + b^2*c^2*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 24*(b^2*
d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x
+ a)))/b^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**4*cos(b*x+a)**2*cot(b*x+a),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^4 \cos(bx + a)^2 \cot(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^4*cos(b*x + a)^2*cot(b*x + a), x)

3.165 $\int (c + dx)^3 \cos^2(a + bx) \cot(a + bx) dx$

Optimal. Leaf size=246

$$\frac{3d^2(c + dx)\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3} - \frac{3id(c + dx)^2\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2} + \frac{3id^3\text{PolyLog}\left(4, e^{2i(a+bx)}\right)}{4b^4} + \frac{3d^2(c + dx)\sin(a + bx)}{4b^3}$$

```
[Out] (-3*d^3*x)/(8*b^3) + (c + d*x)^3/(4*b) - ((I/4)*(c + d*x)^4)/d + ((c + d*x)^3*Log[1 - E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) + (((3*I)/4)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4 + (3*d^3*Cos[a + b*x]*Sin[a + b*x])/(8*b^4) - (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(4*b^2) + (3*d^2*(c + d*x)*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)^3*Sin[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.278089, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {4408, 4404, 3311, 32, 2635, 8, 3717, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c + dx)\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3} - \frac{3id(c + dx)^2\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2} + \frac{3id^3\text{PolyLog}\left(4, e^{2i(a+bx)}\right)}{4b^4} + \frac{3d^2(c + dx)\sin(a + bx)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^2*Cot[a + b*x], x]
```

```
[Out] (-3*d^3*x)/(8*b^3) + (c + d*x)^3/(4*b) - ((I/4)*(c + d*x)^4)/d + ((c + d*x)^3*Log[1 - E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) + (((3*I)/4)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4 + (3*d^3*Cos[a + b*x]*Sin[a + b*x])/(8*b^4) - (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(4*b^2) + (3*d^2*(c + d*x)*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)^3*Sin[a + b*x]^2)/(2*b)
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3717

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[(c + d*x)^m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n])]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 6609

Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d}

, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned}
 \int (c + dx)^3 \cos^2(a + bx) \cot(a + bx) dx &= \int (c + dx)^3 \cot(a + bx) dx - \int (c + dx)^3 \cos(a + bx) \sin(a + bx) dx \\
 &= -\frac{i(c + dx)^4}{4d} - \frac{(c + dx)^3 \sin^2(a + bx)}{2b} - 2i \int \frac{e^{2i(a+bx)}(c + dx)^3}{1 - e^{2i(a+bx)}} dx + \frac{(3d) \int (c + dx)^3 \cos(a + bx) \sin(a + bx) dx}{4b^2} \\
 &= -\frac{i(c + dx)^4}{4d} + \frac{(c + dx)^3 \log(1 - e^{2i(a+bx)})}{b} - \frac{3d(c + dx)^2 \cos(a + bx) \sin(a + bx)}{4b^2} \\
 &= \frac{(c + dx)^3}{4b} - \frac{i(c + dx)^4}{4d} + \frac{(c + dx)^3 \log(1 - e^{2i(a+bx)})}{b} - \frac{3id(c + dx)^2 \text{Li}_2(e^{2i(a+bx)})}{2b^2} \\
 &= -\frac{3d^3x}{8b^3} + \frac{(c + dx)^3}{4b} - \frac{i(c + dx)^4}{4d} + \frac{(c + dx)^3 \log(1 - e^{2i(a+bx)})}{b} - \frac{3id(c + dx)^2 \text{Li}_2(e^{2i(a+bx)})}{2b^2} \\
 &= -\frac{3d^3x}{8b^3} + \frac{(c + dx)^3}{4b} - \frac{i(c + dx)^4}{4d} + \frac{(c + dx)^3 \log(1 - e^{2i(a+bx)})}{b} - \frac{3id(c + dx)^2 \text{Li}_2(e^{2i(a+bx)})}{2b^2} \\
 &= -\frac{3d^3x}{8b^3} + \frac{(c + dx)^3}{4b} - \frac{i(c + dx)^4}{4d} + \frac{(c + dx)^3 \log(1 - e^{2i(a+bx)})}{b} - \frac{3id(c + dx)^2 \text{Li}_2(e^{2i(a+bx)})}{2b^2}
 \end{aligned}$$

Mathematica [B] time = 6.40242, size = 1918, normalized size = 7.8

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^3*Cos[a + b*x]^2*Cot[a + b*x], x]

[Out] $-(c*d^2*E^{I*a}*Csc[a]*((2*b^3*x^3)/E^{((2*I)*a)} + (3*I)*b^2*(1 - E^{((-2*I)*a)})*x^2*\text{Log}[1 - E^{((-I)*(a + b*x))}] + (3*I)*b^2*(1 - E^{((-2*I)*a)})*x^2*\text{Log}[1 + E^{((-I)*(a + b*x))}] - (6*(-1 + E^{((2*I)*a)})*(b*x*\text{PolyLog}[2, -E^{((-I)*(a + b*x))}] - I*\text{PolyLog}[3, -E^{((-I)*(a + b*x))}]))/E^{((2*I)*a)} - (6*(-1 + E^{((2*I)*a)})*(b*x*\text{PolyLog}[2, E^{((-I)*(a + b*x))}] - I*\text{PolyLog}[3, E^{((-I)*(a + b*x))}]))/E^{((2*I)*a)})/(2*b^3) - (d^3*E^{I*a}*Csc[a]*((b^4*x^4)/E^{((2*I)*a)} + (2*I)*b^3*(1 - E^{((-2*I)*a)})*x^3*\text{Log}[1 - E^{((-I)*(a + b*x))}] + (2*I)*b^3*(1 - E^{((-2*I)*a)})*x^3*\text{Log}[1 + E^{((-I)*(a + b*x))}] - (6*(-1 + E^{((2*I)*a)})*(b^2*x^2*\text{PolyLog}[2, -E^{((-I)*(a + b*x))}] - (2*I)*b*x*\text{PolyLog}[3, -E^{((-I)*(a + b*x))}] - 2*\text{PolyLog}[4, -E^{((-I)*(a + b*x))}]))/E^{((2*I)*a)} - (6*(-1 + E^{((2*I)*a)})*(b^2*x^2*\text{PolyLog}[2, E^{((-I)*(a + b*x))}] - (2*I)*b*x*\text{PolyLog}[3, E^{((-I)*(a + b*x))}] - 2*\text{PolyLog}[4, E^{((-I)*(a + b*x))}]))/E^{((2*I)*a)})/(4*b^4) + (c^3*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + Csc[a]*(Cos[2*a + 2*b*x]/(64*b^4) - ((I/64)*Sin[2*a + 2*b*x])/b^4)*(32*b^4*c^3*x*Cos[a + 2*b*x] + 48*b^4*c^2*d*x^2*Cos[a + 2*b*x] + 32*b^4*c*d^2*x^3*Cos[a + 2*b*x] + 8*b^4*d^3*x^4*Cos[a + 2*b*x] + 32*b^4*c^3*x*Cos[3*a + 2*b*x] + 48*b^4*c^2*d*x^2*Cos[3*a + 2*b*x] + 32*b^4*c*d^2*x^3*Cos[3*a + 2*b*x] + 8*b^4*d^3*x^4*Cos[3*a + 2*b*x] + (4*I)*b^3*c^3*Cos[3*a + 4*b*x] - 6*b^2*c^2*d*Cos[3*a + 4*b*x] - (6*I)*b*c*d^2*Cos[3*a + 4*b*x] + 3*d^3*Cos[3*a + 4*b*x] + (12*I)*b^3*c^2*d*x*Cos[3*a + 4*b*x] - 12*b^2*c*d^2*x*Cos[3*a + 4*b*x] - (6*I)*b*d^3*x*Cos[3*a + 4*b*x] + (12*I)*b^3*c*d^2*x^2*Cos[3*a + 4*b*x] - 6*b^2*d^3*x^2*Cos[3*a + 4*b*x] + (4*I)*b^3*d^3*x^3*Cos[3*a + 4*b*x] - (4*I)*b^3*c^3*Cos[5*a + 4*b*x] + 6*b^2*c^2*d*Cos[5*a + 4*b*x] + (6*I)*b*c*d^2*Cos[5*a + 4*b*x] - 3*d^3*Cos[5*a + 4*b*x] - (12*I)*b^3*c^2*d*x*Cos[5*a + 4*b*x] + 12*b^2*c*d^2*x*Cos[5*a + 4*b*x] + (6*I)*b*d^3*x*Cos[5*a + 4*b*x] - (12*I)*b^3*c*d^2*x^2*Cos[5*a + 4*b*x] + 6*b^2*d^3*x^2*Cos[5*a + 4*b*x] - (4*I)*b^3*d^3*x^3*Cos[5*a + 4*b*x] + 8*b^3*c^3*Sin[a] - (12*I)*b^2*c^2*d*Sin[a] - 12*b*c*d^2*Sin[a] + (6*I)*d^3*Sin[a] + 2$

$$\begin{aligned}
& 4*b^3*c^2*d*x*\sin[a] - (24*I)*b^2*c*d^2*x*\sin[a] - 12*b*d^3*x*\sin[a] + 24*b^3*c*d^2*x^2*\sin[a] - (12*I)*b^2*d^3*x^2*\sin[a] + 8*b^3*d^3*x^3*\sin[a] + (32*I)*b^4*c^3*x*\sin[a + 2*b*x] + (48*I)*b^4*c^2*d*x^2*\sin[a + 2*b*x] + (32*I)*b^4*c*d^2*x^3*\sin[a + 2*b*x] + (8*I)*b^4*d^3*x^4*\sin[a + 2*b*x] + (32*I)*b^4*c^3*x*\sin[3*a + 2*b*x] + (48*I)*b^4*c^2*d*x^2*\sin[3*a + 2*b*x] + (32*I)*b^4*c*d^2*x^3*\sin[3*a + 2*b*x] + (8*I)*b^4*d^3*x^4*\sin[3*a + 2*b*x] - 4*b^3*c^3*\sin[3*a + 4*b*x] - (6*I)*b^2*c^2*d*\sin[3*a + 4*b*x] + 6*b*c*d^2*\sin[3*a + 4*b*x] + (3*I)*d^3*\sin[3*a + 4*b*x] - 12*b^3*c^2*d*x*\sin[3*a + 4*b*x] - (12*I)*b^2*c*d^2*x*\sin[3*a + 4*b*x] + 6*b*d^3*x*\sin[3*a + 4*b*x] - 12*b^3*c*d^2*x^2*\sin[3*a + 4*b*x] - (6*I)*b^2*d^3*x^2*\sin[3*a + 4*b*x] - 4*b^3*d^3*x^3*\sin[3*a + 4*b*x] + 4*b^3*c^3*\sin[5*a + 4*b*x] + (6*I)*b^2*c^2*d*\sin[5*a + 4*b*x] - 6*b*c*d^2*\sin[5*a + 4*b*x] - (3*I)*d^3*\sin[5*a + 4*b*x] + 12*b^3*c^2*d*x*\sin[5*a + 4*b*x] + (12*I)*b^2*c*d^2*x*\sin[5*a + 4*b*x] - 6*b*d^3*x*\sin[5*a + 4*b*x] + 12*b^3*c*d^2*x^2*\sin[5*a + 4*b*x] + (6*I)*b^2*d^3*x^2*\sin[5*a + 4*b*x] + 4*b^3*d^3*x^3*\sin[5*a + 4*b*x]) - (3*c^2*d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]])) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]])])) + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]]) + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]])]))*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
\end{aligned}$$

Maple [B] time = 0.438, size = 1001, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a), x)

[Out] $3/b*c*d^2*\ln(\exp(I*(b*x+a))+1)*x^2+3/b*c*d^2*\ln(1-\exp(I*(b*x+a)))*x^2-3*I/b^2*c^2*d*a^2+4*I/b^3*c*d^2*a^3-2*I/b^3*d^3*a^3*x-3*I/b^2*d^3*\text{polylog}(2, \exp(I*(b*x+a)))*x^2-3*I/b^2*d^3*\text{polylog}(2, -\exp(I*(b*x+a)))*x^2-3*I/b^2*c^2*d*\text{polylog}(2, \exp(I*(b*x+a)))-3*I/b^2*c^2*d*\text{polylog}(2, -\exp(I*(b*x+a)))+6*I*d^3*\text{polylog}(4, \exp(I*(b*x+a)))/b^4+1/b*d^3*\ln(1-\exp(I*(b*x+a)))*x^3+1/b^4*d^3*\ln(1-\exp(I*(b*x+a)))*a^3+1/b*d^3*\ln(\exp(I*(b*x+a))+1)*x^3-6/b^3*c*d^2*a^2*\ln(\exp(I*(b*x+a)))+3/b^3*c*d^2*a^2*\ln(\exp(I*(b*x+a))-1)+6/b^2*c^2*d*a*\ln(\exp(I*(b*x+a)))-3/b^2*c^2*d*a*\ln(\exp(I*(b*x+a))-1)-3/b^3*c*d^2*a^2*\ln(1-\exp(I*(b*x+a)))+3/b*c^2*d*\ln(\exp(I*(b*x+a))+1)*x+3/b*c^2*d*\ln(1-\exp(I*(b*x+a)))*x+3/b^2*c^2*d*\ln(1-\exp(I*(b*x+a)))*a+I*c^3*x-I*c*d^2*x^3-3/2*I*c^2*d*x^2+6/b^3*d^3*\text{polylog}(3, \exp(I*(b*x+a)))*x+6/b^3*d^3*\text{polylog}(3, -\exp(I*(b*x+a)))*x+6/b^3*c*d^2*\text{polylog}(3, \exp(I*(b*x+a)))+6/b^3*c*d^2*\text{polylog}(3, -\exp(I*(b*x+a)))+2/b^4*d^3*a^3*\ln(\exp(I*(b*x+a)))-1/b^4*d^3*a^3*\ln(\exp(I*(b*x+a))-1)-3/2*I/b^4*d^3*a^4+6*I/b^4*d^3*\text{polylog}(4, -\exp(I*(b*x+a)))-6*I/b*c^2*d*a*x+6*I/b^2*c*d^2*a^2*x-6*I/b^2*c*d^2*\text{polylog}(2, \exp(I*(b*x+a)))*x-6*I/b^2*c*d^2*\text{polylog}(2, -\exp(I*(b*x+a)))*x+1/32*(4*d^3*x^3*b^3+6*I*b^2*d^3*x^2+12*b^3*c*d^2*x^2+12*I*b^2*c*d^2*x+12*b^3*c^2*d*x+6*I*b^2*c^2*d+4*b^3*c^3-6*b*d^3*x-3*I*d^3-6*c*d^2*b)/b^4*\exp(2*I*(b*x+a))-2/b*c^3*\ln(\exp(I*(b*x+a)))+1/b*c^3*\ln(\exp(I*(b*x+a))-1)+1/b*c^3*\ln(\exp(I*(b*x+a))+1)-1/4*I*d^3*x^4+1/32*(4*d^3*x^3*b^3-6*I*b^2*d^3*x^2+12*b^3*c*d^2*x^2-12*I*b^2*c*d^2*x+12*b^3*c^2*d*x-6*I*b^2*c^2*d+4*b^3*c^3-6*b*d^3*x+3*I*d^3-6*c*d^2*b)/b^4*\exp(-2*I*(b*x+a))$

Maxima [B] time = 2.01961, size = 1305, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")

[Out]
$$-1/16*(8*(\sin(b*x + a)^2 - \log(\sin(b*x + a)^2))*c^3 - 24*(\sin(b*x + a)^2 - \log(\sin(b*x + a)^2))*a*c^2*d/b + 24*(\sin(b*x + a)^2 - \log(\sin(b*x + a)^2))*a^2*c*d^2/b^2 - 8*(\sin(b*x + a)^2 - \log(\sin(b*x + a)^2))*a^3*d^3/b^3 - (-4*I*(b*x + a)^4*d^3 + (-16*I*b*c*d^2 + 16*I*a*d^3)*(b*x + a)^3 + 96*I*d^3*\text{polylog}(4, -e^{(I*b*x + I*a)}) + 96*I*d^3*\text{polylog}(4, e^{(I*b*x + I*a)}) + (-24*I*b^2*c^2*d + 48*I*a*b*c*d^2 - 24*I*a^2*d^3)*(b*x + a)^2 + (16*I*(b*x + a)^3*d^3 + (48*I*b*c*d^2 - 48*I*a*d^3)*(b*x + a)^2 + (48*I*b^2*c^2*d - 96*I*a*b*c*d^2 + 48*I*a^2*d^3)*(b*x + a))*\text{arctan2}(\sin(b*x + a), \cos(b*x + a) + 1) + (-16*I*(b*x + a)^3*d^3 + (-48*I*b*c*d^2 + 48*I*a*d^3)*(b*x + a)^2 + (-48*I*b^2*c^2*d + 96*I*a*b*c*d^2 - 48*I*a^2*d^3)*(b*x + a))*\text{arctan2}(\sin(b*x + a), -\cos(b*x + a) + 1) + 2*(2*(b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 6*(b*c*d^2 - a*d^3))*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 - 1)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (-48*I*b^2*c^2*d + 96*I*a*b*c*d^2 - 48*I*(b*x + a)^2*d^3 - 48*I*a^2*d^3 + (-96*I*b*c*d^2 + 96*I*a*d^3)*(b*x + a))*\text{dilog}(-e^{(I*b*x + I*a)}) + (-48*I*b^2*c^2*d + 96*I*a*b*c*d^2 - 48*I*(b*x + a)^2*d^3 - 48*I*a^2*d^3 + (-96*I*b*c*d^2 + 96*I*a*d^3)*(b*x + a))*\text{dilog}(e^{(I*b*x + I*a)}) + 8*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) + 8*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) + 96*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\text{polylog}(3, -e^{(I*b*x + I*a)}) + 96*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\text{polylog}(3, e^{(I*b*x + I*a)}) - 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^2*d^3 + (2*a^2 - 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))/b^3)/b$$

Fricas [C] time = 0.825092, size = 2437, normalized size = 9.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")

[Out]
$$-1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 24*I*d^3*\text{polylog}(4, \cos(b*x + a) + I*\sin(b*x + a)) + 24*I*d^3*\text{polylog}(4, \cos(b*x + a) - I*\sin(b*x + a)) + 24*I*d^3*\text{polylog}(4, -\cos(b*x + a) + I*\sin(b*x + a)) - 24*I*d^3*\text{polylog}(4, -\cos(b*x + a) - I*\sin(b*x + a)) - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*\cos(b*x + a)^2 + 3*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*\cos(b*x + a)*\sin(b*x + a) + 3*(2*b^3*c^2*d - b*d^3)*x - (-12*I*b^2*d^3*x^2 - 24*I*b^2*c*d^2*x - 12*I*b^2*c^2*d)*\text{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) - (12*I*b^2*d^3*x^2 + 24*I*b^2*c*d^2*x + 12*I*b^2*c^2*d)*\text{dilog}(\cos(b*x + a) - I*\sin(b*x + a)) - (12*I*b^2*d^3*x^2 + 24*I*b^2*c*d^2*x + 12*I*b^2*c^2*d)*\text{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) - (-12*I*b^2*d^3*x^2 - 24*I*b^2*c*d^2*x - 12*I*b^2*c^2*d)*\text{dilog}(-\cos(b*x + a) - I*\sin(b*x + a)) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1) - 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\log(-1/2*\cos(b*x + a) + 1/2*I*\sin(b*x + a) + 1/2) - 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\log(-1/2*\cos(b*x + a) - 1/2*I*\sin(b*x + a) + 1/2) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*\log(-\cos(b*x + a) - I*\sin(b*x + a) + 1) - 24*(b*d^3*x + b*c*d^2)*\text{polylog}(3, \cos(b*x + a) + I*\sin(b*x + a)) - 24*(b*d^3*x + b*c*d^2)*\text{polylog}(3, \cos(b*x + a) - I*\sin(b*x + a))$$

```
in(b*x + a)) - 24*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x
+ a)) - 24*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/
b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \cos^2(a + bx) \cot(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**2*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*cos(a + b*x)**2*cot(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \cos(bx + a)^2 \cot(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cos(b*x + a)^2*cot(b*x + a), x)
```


3.166 $\int (c + dx)^2 \cos^2(a + bx) \cot(a + bx) dx$

Optimal. Leaf size=181

$$\frac{id(c + dx)\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^2} + \frac{d^2\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3} - \frac{d(c + dx) \sin(a + bx) \cos(a + bx)}{2b^2} + \frac{d^2 \sin^2(a + bx)}{4b^3} +$$

```
[Out] (c*d*x)/(2*b) + (d^2*x^2)/(4*b) - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[
1 - E^((2*I)*(a + b*x))])/b - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
])/b^2 + (d^2*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) - (d*(c + d*x)*Cos[a
+ b*x]*Sin[a + b*x])/(2*b^2) + (d^2*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)^2
*Sin[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.226928, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4408, 4404, 3310, 3717, 2190, 2531, 2282, 6589}

$$\frac{id(c + dx)\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^2} + \frac{d^2\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3} - \frac{d(c + dx) \sin(a + bx) \cos(a + bx)}{2b^2} + \frac{d^2 \sin^2(a + bx)}{4b^3} +$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^2*Cot[a + b*x], x]
```

```
[Out] (c*d*x)/(2*b) + (d^2*x^2)/(4*b) - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[
1 - E^((2*I)*(a + b*x))])/b - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
])/b^2 + (d^2*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) - (d*(c + d*x)*Cos[a
+ b*x]*Sin[a + b*x])/(2*b^2) + (d^2*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)^2
*Sin[a + b*x]^2)/(2*b)
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] := -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
Simp[(d*(b*Sine + f*x))^n/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sine + f*x)^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sine + f*x)^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[(c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x))]/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
```

x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*(a_) + (b_)*(x_)))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^2 \cos^2(a + bx) \cot(a + bx) dx &= \int (c + dx)^2 \cot(a + bx) dx - \int (c + dx)^2 \cos(a + bx) \sin(a + bx) dx \\
&= -\frac{i(c + dx)^3}{3d} - \frac{(c + dx)^2 \sin^2(a + bx)}{2b} - 2i \int \frac{e^{2i(a+bx)}(c + dx)^2}{1 - e^{2i(a+bx)}} dx + \frac{d \int (c + dx)}{b^2} \\
&= -\frac{i(c + dx)^3}{3d} + \frac{(c + dx)^2 \log(1 - e^{2i(a+bx)})}{b} - \frac{d(c + dx) \cos(a + bx) \sin(a + bx)}{2b^2} \\
&= \frac{cdx}{2b} + \frac{d^2x^2}{4b} - \frac{i(c + dx)^3}{3d} + \frac{(c + dx)^2 \log(1 - e^{2i(a+bx)})}{b} - \frac{id(c + dx) \text{Li}_2(e^{2i(a+bx)})}{b^2} \\
&= \frac{cdx}{2b} + \frac{d^2x^2}{4b} - \frac{i(c + dx)^3}{3d} + \frac{(c + dx)^2 \log(1 - e^{2i(a+bx)})}{b} - \frac{id(c + dx) \text{Li}_2(e^{2i(a+bx)})}{b^2} \\
&= \frac{cdx}{2b} + \frac{d^2x^2}{4b} - \frac{i(c + dx)^3}{3d} + \frac{(c + dx)^2 \log(1 - e^{2i(a+bx)})}{b} - \frac{id(c + dx) \text{Li}_2(e^{2i(a+bx)})}{b^2}
\end{aligned}$$

Mathematica [B] time = 2.98839, size = 564, normalized size = 3.12

$-48ibcd \text{PolyLog}\left(2, e^{2i(\tan^{-1}(\tan(a)+bx)}\right) + 96ibd^2x \text{PolyLog}\left(2, -e^{-i(a+bx)}\right) + 96ibd^2x \text{PolyLog}\left(2, e^{-i(a+bx)}\right) + 96d^2 \text{PolyL}$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2*Cot[a + b*x], x]

```
[Out] ((48*I)*b^2*c*d*Pi*x + (16*I)*b^3*d^2*x^3 - (96*I)*b^2*c*d*x*ArcTan[Tan[a]]
+ 48*b^3*c*d*x^2*Cot[a] - 6*b*c*d*Cos[a + 2*b*x]*Csc[a] - 6*b*d^2*x*Cos[a
+ 2*b*x]*Csc[a] + 6*b*c*d*Cos[3*a + 2*b*x]*Csc[a] + 6*b*d^2*x*Cos[3*a + 2*b
*x]*Csc[a] + 48*b*c*d*Pi*Log[1 + E^((-2*I)*b*x)] + 48*b^2*d^2*x^2*Log[1 - E
^((-I)*(a + b*x))] + 48*b^2*d^2*x^2*Log[1 + E^((-I)*(a + b*x))] + 96*b^2*c*
d*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]])]) + 96*b*c*d*ArcTan[Tan[a]]*Log
[1 - E^((2*I)*(b*x + ArcTan[Tan[a]])]) - 48*b*c*d*Pi*Log[Cos[b*x]] + 48*b^2
*c^2*Log[Sin[a + b*x]] - 96*b*c*d*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a
]]]]) + (96*I)*b*d^2*x*PolyLog[2, -E^((-I)*(a + b*x))] + (96*I)*b*d^2*x*Poly
Log[2, E^((-I)*(a + b*x))] - (48*I)*b*c*d*PolyLog[2, E^((2*I)*(b*x + ArcTan
[Tan[a]])]) + 96*d^2*PolyLog[3, -E^((-I)*(a + b*x))] + 96*d^2*PolyLog[3, E^
((-I)*(a + b*x))] - 48*b^3*c*d*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^
2] - 6*b^2*c^2*Csc[a]*Sin[a + 2*b*x] + 3*d^2*Csc[a]*Sin[a + 2*b*x] - 12*b^2
*c*d*x*Csc[a]*Sin[a + 2*b*x] - 6*b^2*d^2*x^2*Csc[a]*Sin[a + 2*b*x] + 6*b^2*
c^2*Csc[a]*Sin[3*a + 2*b*x] - 3*d^2*Csc[a]*Sin[3*a + 2*b*x] + 12*b^2*c*d*x*
Csc[a]*Sin[3*a + 2*b*x] + 6*b^2*d^2*x^2*Csc[a]*Sin[3*a + 2*b*x])/(48*b^3)
```

Maple [B] time = 0.47, size = 590, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a), x)
```

```
[Out] 1/16*(2*d^2*x^2*b^2-2*I*b*d^2*x+4*b^2*c*d*x-2*I*b*c*d+2*b^2*c^2-d^2)/b^3*ex
p(-2*I*(b*x+a))-4*I/b*c*d*a*x-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d^2*ln
(exp(I*(b*x+a))+1)*x^2+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-2*I/b^2*c*d*polylog
(2,exp(I*(b*x+a)))+I*c^2*x+2/b*c*d*ln(exp(I*(b*x+a))+1)*x-2/b*c^2*ln(exp(I*
(b*x+a))+1)/b*c^2*ln(exp(I*(b*x+a))+1)+1/b*c^2*ln(exp(I*(b*x+a))-1)-1/3*I*d
^2*x^3+2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I*(b*x+a)))
/b^3+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+2/b^2*c*d*ln(1-exp(I*(b*x+a)))*a-2*I/b^
2*d^2*polylog(2,exp(I*(b*x+a)))*x+1/16*(2*d^2*x^2*b^2+2*I*b*d^2*x+4*b^2*c*d
*x+2*I*b*c*d+2*b^2*c^2-d^2)/b^3*exp(2*I*(b*x+a))-2/b^3*d^2*a^2*ln(exp(I*(b*
x+a))+1)/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+4/3*I/b^3*d^2*a^3-I*c*d*x^2-2/b^2
*c*d*a*ln(exp(I*(b*x+a))-1)+4/b^2*c*d*a*ln(exp(I*(b*x+a)))-2*I/b^2*c*d*a^2-
2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-2*I/b^2*c*d*polylog(2,-exp(I*(b*x+
a)))+2*I/b^2*d^2*a^2*x
```

Maxima [B] time = 1.67788, size = 705, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a), x, algorithm="maxima")
```

```
[Out] -1/24*(12*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*c^2 - 24*(sin(b*x + a)^2 -
log(sin(b*x + a)^2))*a*c*d/b + 12*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a
^2*d^2/b^2 - (-8*I*(b*x + a)^3*d^2 + (-24*I*b*c*d + 24*I*a*d^2)*(b*x + a)^2
+ 48*d^2*polylog(3, -e^(I*b*x + I*a)) + 48*d^2*polylog(3, e^(I*b*x + I*a))
+ (24*I*(b*x + a)^2*d^2 + (48*I*b*c*d - 48*I*a*d^2)*(b*x + a))*arctan2(sin
(b*x + a), cos(b*x + a) + 1) + (-24*I*(b*x + a)^2*d^2 + (-48*I*b*c*d + 48*I
*a*d^2)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 3*(2*(b*x + a
```

```
)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(2*b*x + 2*a) + (-48*I*b*c*d - 48*I*(b*x + a)*d^2 + 48*I*a*d^2)*dilog(-e^(I*b*x + I*a)) + (-48*I*b*c*d - 48*I*(b*x + a)*d^2 + 48*I*a*d^2)*dilog(e^(I*b*x + I*a)) + 12*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + 12*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 6*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))/b^2)/b
```

Fricas [C] time = 0.689083, size = 1538, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(b*x + a)^2 - 4*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 4*d^2*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 4*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 4*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) - (-4*I*b*d^2*x - 4*I*b*c*d)*dilog(cos(b*x + a) + I*sin(b*x + a)) - (4*I*b*d^2*x + 4*I*b*c*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - (4*I*b*d^2*x + 4*I*b*c*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - (-4*I*b*d^2*x - 4*I*b*c*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \cos^2(a + bx) \cot(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**2*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*cos(a + b*x)**2*cot(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \cos(bx + a)^2 \cot(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cos(b*x + a)^2*cot(b*x + a), x)
```

3.167 $\int (c + dx) \cos^2(a + bx) \cot(a + bx) dx$

Optimal. Leaf size=114

$$\frac{id\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2} - \frac{d \sin(a + bx) \cos(a + bx)}{4b^2} + \frac{(c + dx) \log\left(1 - e^{2i(a+bx)}\right)}{b} - \frac{(c + dx) \sin^2(a + bx)}{2b} + \frac{dx}{4b} - \frac{i(c + dx) \cos^2(a + bx)}{4b}$$

[Out] (d*x)/(4*b) - ((I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d*Cos[a + b*x]*Sin[a + b*x])/(4*b^2) - ((c + d*x)*Sin[a + b*x]^2)/(2*b)

Rubi [A] time = 0.127923, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {4408, 4404, 2635, 8, 3717, 2190, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2} - \frac{d \sin(a + bx) \cos(a + bx)}{4b^2} + \frac{(c + dx) \log\left(1 - e^{2i(a+bx)}\right)}{b} - \frac{(c + dx) \sin^2(a + bx)}{2b} + \frac{dx}{4b} - \frac{i(c + dx) \cos^2(a + bx)}{4b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Cos[a + b*x]^2*Cot[a + b*x], x]

[Out] (d*x)/(4*b) - ((I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d*Cos[a + b*x]*Sin[a + b*x])/(4*b^2) - ((c + d*x)*Sin[a + b*x]^2)/(2*b)

Rule 4408

Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 4404

Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_.), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3717

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[(c + d*x)^m*E^(2*I*k*Pi)*E^(2*I*(e + f*x))]/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],

x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned} \int (c + dx) \cos^2(a + bx) \cot(a + bx) dx &= \int (c + dx) \cot(a + bx) dx - \int (c + dx) \cos(a + bx) \sin(a + bx) dx \\ &= -\frac{i(c + dx)^2}{2d} - \frac{(c + dx) \sin^2(a + bx)}{2b} - 2i \int \frac{e^{2i(a+bx)}(c + dx)}{1 - e^{2i(a+bx)}} dx + \frac{d \int \sin^2(a + bx)}{2b} \\ &= -\frac{i(c + dx)^2}{2d} + \frac{(c + dx) \log(1 - e^{2i(a+bx)})}{b} - \frac{d \cos(a + bx) \sin(a + bx)}{4b^2} - \frac{(c + dx)}{4b^2} \\ &= \frac{dx}{4b} - \frac{i(c + dx)^2}{2d} + \frac{(c + dx) \log(1 - e^{2i(a+bx)})}{b} - \frac{d \cos(a + bx) \sin(a + bx)}{4b^2} - \frac{(c + dx)}{4b^2} \\ &= \frac{dx}{4b} - \frac{i(c + dx)^2}{2d} + \frac{(c + dx) \log(1 - e^{2i(a+bx)})}{b} - \frac{id \operatorname{Li}_2(e^{2i(a+bx)})}{2b^2} - \frac{d \cos(a + bx)}{4b^2} \end{aligned}$$

Mathematica [A] time = 0.352627, size = 131, normalized size = 1.15

$$\frac{d \left((a + bx) \log(1 - e^{2i(a+bx)}) - \frac{1}{2} i \left((a + bx)^2 + \operatorname{PolyLog}(2, e^{2i(a+bx)}) \right) \right)}{b^2} - \frac{d \sin(2(a + bx))}{8b^2} - \frac{ad \log(\sin(a + bx))}{b^2} - \frac{c \sin^2(a + bx)}{2b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Cos[a + b*x]^2*Cot[a + b*x], x]

[Out] (d*x*Cos[2*(a + b*x)])/(4*b) + (c*Log[Sin[a + b*x]])/b - (a*d*Log[Sin[a + b*x]])/b^2 + (d*((a + b*x)*Log[1 - E^((2*I)*(a + b*x))] - (I/2)*((a + b*x)^2 + PolyLog[2, E^((2*I)*(a + b*x))]]))/b^2 - (c*SIN[a + b*x]^2)/(2*b) - (d*SIN[2*(a + b*x)])/(8*b^2)

Maple [B] time = 0.383, size = 271, normalized size = 2.4

$$icx - \frac{i}{2} dx^2 + \frac{(2dxb + id + 2bc)e^{2i(bx+a)}}{16b^2} + \frac{(2dxb - id + 2bc)e^{-2i(bx+a)}}{16b^2} + \frac{c \ln(e^{i(bx+a)} - 1)}{b} - 2 \frac{c \ln(e^{i(bx+a)})}{b} + \frac{c \ln(e^{i(bx+a)})}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x)
```

```
[Out] I*c*x-1/2*I*d*x^2+1/16*(2*d*x*b+I*d+2*b*c)/b^2*exp(2*I*(b*x+a))+1/16*(2*d*x
*b-I*d+2*b*c)/b^2*exp(-2*I*(b*x+a))+1/b*c*ln(exp(I*(b*x+a))-1)-2/b*c*ln(exp
(I*(b*x+a)))+1/b*c*ln(exp(I*(b*x+a))+1)-I/b^2*d*a^2-2*I/b*d*a*x-I*d*polylog
(2,-exp(I*(b*x+a)))/b^2+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(b
x+a)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2+1/b*d*ln(exp(I*(b*x+a))+1)*x-1/b
^2*d*a*ln(exp(I*(b*x+a))-1)+2/b^2*d*a*ln(exp(I*(b*x+a)))
```

Maxima [B] time = 1.53569, size = 300, normalized size = 2.63

$$-4i b^2 dx^2 - 8i b^2 cx - 8i b dx \arctan(\sin(bx + a), -\cos(bx + a) + 1) + 8i bc \arctan(\sin(bx + a), \cos(bx + a) - 1) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(-4*I*b^2*d*x^2 - 8*I*b^2*c*x - 8*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x
+ a) + 1) + 8*I*b*c*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (8*I*b*d*x
+ 8*I*b*c)*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 2*(b*d*x + b*c)*cos(2*
b*x + 2*a) - 8*I*d*dilog(-e^(I*b*x + I*a)) - 8*I*d*dilog(e^(I*b*x + I*a)) +
4*(b*d*x + b*c)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
+ 4*(b*d*x + b*c)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1)
- d*sin(2*b*x + 2*a))/b^2
```

Fricas [B] time = 0.601803, size = 844, normalized size = 7.4

$$b dx - 2(b dx + bc) \cos(bx + a)^2 + d \cos(bx + a) \sin(bx + a) + 2i d \operatorname{Li}_2(\cos(bx + a) + i \sin(bx + a)) - 2i d \operatorname{Li}_2(\cos(bx + a) - i \sin(bx + a))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a)
+ 2*I*d*dilog(cos(b*x + a) + I*sin(b*x + a)) - 2*I*d*dilog(cos(b*x + a) - I
*sin(b*x + a)) - 2*I*d*dilog(-cos(b*x + a) + I*sin(b*x + a)) + 2*I*d*dilog(
-cos(b*x + a) - I*sin(b*x + a)) - 2*(b*d*x + b*c)*log(cos(b*x + a) + I*sin(
b*x + a) + 1) - 2*(b*d*x + b*c)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 2*
(b*c - a*d)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*(b*c - a*
d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 2*(b*d*x + a*d)*log(
-cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b*d*x + a*d)*log(-cos(b*x + a) - I
*sin(b*x + a) + 1))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \cos^2(a + bx) \cot(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)**2*cot(b*x+a),x)

[Out] Integral((c + d*x)*cos(a + b*x)**2*cot(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \cos(bx + a)^2 \cot(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)*cos(b*x + a)^2*cot(b*x + a), x)

$$3.168 \quad \int \frac{\cos^2(a+bx) \cot(a+bx)}{c+dx} dx$$

Optimal. Leaf size=81

$$\text{Unintegrable}\left(\frac{\cot(a+bx)}{c+dx}, x\right) - \frac{\sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{2d} - \frac{\cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{2d}$$

[Out] -(CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d) - (Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Unintegrable[Cot[a + b*x]/(c + d*x), x]

Rubi [A] time = 0.144313, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cos^2(a+bx) \cot(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x), x]

[Out] -(CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d) - (Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Defer[Int][Cot[a + b*x]/(c + d*x), x]

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(a+bx) \cot(a+bx)}{c+dx} dx &= \int \frac{\cot(a+bx)}{c+dx} dx - \int \frac{\cos(a+bx) \sin(a+bx)}{c+dx} dx \\ &= \int \frac{\cot(a+bx)}{c+dx} dx - \int \frac{\sin(2a+2bx)}{2(c+dx)} dx \\ &= -\left(\frac{1}{2} \int \frac{\sin(2a+2bx)}{c+dx} dx\right) + \int \frac{\cot(a+bx)}{c+dx} dx \\ &= -\left(\frac{1}{2} \cos\left(2a - \frac{2bc}{d}\right) \int \frac{\sin\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx\right) - \frac{1}{2} \sin\left(2a - \frac{2bc}{d}\right) \int \frac{\cos\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx \\ &= -\frac{\text{Ci}\left(\frac{2bc}{d} + 2bx\right) \sin\left(2a - \frac{2bc}{d}\right)}{2d} - \frac{\cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{2d} + \int \frac{\cot(a+bx)}{c+dx} dx \end{aligned}$$

Mathematica [A] time = 0.794424, size = 0, normalized size = 0.

$$\int \frac{\cos^2(a+bx) \cot(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x), x]

[Out] Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x), x]

Maple [A] time = 0.355, size = 0, normalized size = 0.

$$\int \frac{(\cos(bx + a))^2 \cot(bx + a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c), x)

[Out] int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(-i E_1\left(\frac{2i b d x + 2i b c}{d}\right) + i E_1\left(-\frac{2i b d x + 2i b c}{d}\right)\right) \cos\left(-\frac{2(bc-ad)}{d}\right) + 4d \int \frac{\sin(bx+a)}{(dx+c)(\cos(bx+a)^2 + \sin(bx+a)^2 + 2 \cos(bx+a) + 1)} dx - 4d \int \frac{1}{(dx+c)(\cos(bx+a)^2 + \sin(bx+a)^2 + 2 \cos(bx+a) + 1)} dx}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c), x, algorithm="maxima")

[Out] -1/4*((-I*exp_integral_e(1, (2*I*b*d*x + 2*I*b*c)/d) + I*exp_integral_e(1, -(2*I*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 4*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)*cos(b*x + a) + c), x) - 4*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) - (exp_integral_e(1, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(1, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c - a*d)/d)/d

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(bx + a)^2 \cot(bx + a)}{dx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c), x, algorithm="fricas")

[Out] integral(cos(b*x + a)^2*cot(b*x + a)/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(a + bx) \cot(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**2*cot(b*x+a)/(d*x+c), x)

[Out] Integral(cos(a + b*x)**2*cot(a + b*x)/(c + d*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (bx + a)^2 \cot (bx + a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2*cot(b*x + a)/(d*x + c), x)
```

$$3.169 \quad \int \frac{\cos^2(a+bx) \cot(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=101

$$\text{Unintegrable}\left(\frac{\cot(a+bx)}{(c+dx)^2}, x\right) - \frac{b \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{d^2} + \frac{b \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^2} + \frac{\sin(2a + 2bx)}{2d(c+dx)}$$

[Out] -((b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2) + Sin[2*a + 2*b*x]/(2*d*(c + d*x)) + (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^2 + Unintegrable[Cot[a + b*x]/(c + d*x)^2, x]

Rubi [A] time = 0.1703, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cos^2(a+bx) \cot(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x)^2,x]

[Out] -((b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2) + Sin[2*a + 2*b*x]/(2*d*(c + d*x)) + (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^2 + Defer[Int][Cot[a + b*x]/(c + d*x)^2, x]

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(a+bx) \cot(a+bx)}{(c+dx)^2} dx &= \int \frac{\cot(a+bx)}{(c+dx)^2} dx - \int \frac{\cos(a+bx) \sin(a+bx)}{(c+dx)^2} dx \\ &= \int \frac{\cot(a+bx)}{(c+dx)^2} dx - \int \frac{\sin(2a+2bx)}{2(c+dx)^2} dx \\ &= -\left(\frac{1}{2} \int \frac{\sin(2a+2bx)}{(c+dx)^2} dx\right) + \int \frac{\cot(a+bx)}{(c+dx)^2} dx \\ &= \frac{\sin(2a+2bx)}{2d(c+dx)} - \frac{b \int \frac{\cos(2a+2bx)}{c+dx} dx}{d} + \int \frac{\cot(a+bx)}{(c+dx)^2} dx \\ &= \frac{\sin(2a+2bx)}{2d(c+dx)} - \frac{\left(b \cos\left(2a - \frac{2bc}{d}\right)\right) \int \frac{\cos\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx}{d} + \frac{\left(b \sin\left(2a - \frac{2bc}{d}\right)\right) \int \frac{\sin\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx}{d} \\ &= -\frac{b \cos\left(2a - \frac{2bc}{d}\right) \text{Ci}\left(\frac{2bc}{d} + 2bx\right)}{d^2} + \frac{\sin(2a+2bx)}{2d(c+dx)} + \frac{b \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^2} + \dots \end{aligned}$$

Mathematica [A] time = 2.49899, size = 0, normalized size = 0.

$$\int \frac{\cos^2(a+bx) \cot(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x)^2,x]

[Out] Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x)^2, x]

Maple [A] time = 0.63, size = 0, normalized size = 0.

$$\int \frac{(\cos(bx + a))^2 \cot(bx + a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x)

[Out] int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(bx + a)^2 \cot(bx + a)}{d^2x^2 + 2cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(cos(b*x + a)^2*cot(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(a + bx) \cot(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**2*cot(b*x+a)/(d*x+c)**2,x)

[Out] Integral(cos(a + b*x)**2*cot(a + b*x)/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx + a)^2 \cot(bx + a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2*cot(b*x + a)/(d*x + c)^2, x)
```

3.170 $\int (c + dx)^m \cos(a + bx) \cot^2(a + bx) dx$

Optimal. Leaf size=153

$$\text{CannotIntegrate}(\cot(a + bx) \csc(a + bx)(c + dx)^m, x) + \frac{ie^{i\left(a - \frac{bc}{d}\right)}(c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m + 1, -\frac{ib(c+dx)}{d}\right)}{2b}$$

[Out] CannotIntegrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x], x] + ((I/2)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(b*(((I)*b*(c + d*x))/d)^m) - ((I/2)*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)

Rubi [A] time = 0.239148, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \cos(a + bx) \cot^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x]^2, x]

[Out] ((I/2)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(b*(((I)*b*(c + d*x))/d)^m) - ((I/2)*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + Defer[Int][(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x], x]

Rubi steps

$$\begin{aligned} \int (c + dx)^m \cos(a + bx) \cot^2(a + bx) dx &= - \int (c + dx)^m \cos(a + bx) dx + \int (c + dx)^m \cot(a + bx) \csc(a + bx) dx \\ &= - \left(\frac{1}{2} \int e^{-i(a+bx)}(c + dx)^m dx \right) - \frac{1}{2} \int e^{i(a+bx)}(c + dx)^m dx + \int (c + dx)^m \cot(a + bx) \csc(a + bx) dx \\ &= \frac{ie^{i\left(a - \frac{bc}{d}\right)}(c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, -\frac{ib(c+dx)}{d}\right)}{2b} - \frac{ie^{-i\left(a - \frac{bc}{d}\right)}(c + dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, \frac{ib(c+dx)}{d}\right)}{2b} \end{aligned}$$

Mathematica [A] time = 3.96008, size = 0, normalized size = 0.

$$\int (c + dx)^m \cos(a + bx) \cot^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x]^2, x]

[Out] Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x]^2, x]

Maple [A] time = 0.164, size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) (\cot(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x)`

[Out] `int((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x)`

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) \cot(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")`

[Out] `integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a)^2, x)`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx + c)^m \cos(bx + a) \cot(bx + a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")`

[Out] `integral((d*x + c)^m*cos(b*x + a)*cot(b*x + a)^2, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)**m*cos(b*x+a)*cot(b*x+a)**2,x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a) \cot(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")`

[Out] `integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a)^2, x)`

3.171 $\int (c + dx)^4 \cos(a + bx) \cot^2(a + bx) dx$

Optimal. Leaf size=299

$$\frac{24d^3(c + dx)\text{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^4} + \frac{24d^3(c + dx)\text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^4} + \frac{12id^2(c + dx)^2\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} - \frac{12id^2(c + dx)^2\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3}$$

```
[Out] (-8*d*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b^2 + (24*d^3*(c + d*x)*Cos[a +
b*x])/b^4 - (4*d*(c + d*x)^3*Cos[a + b*x])/b^2 - ((c + d*x)^4*Csc[a + b*x]
)/b + ((12*I)*d^2*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((12*I)*d
^2*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^3 - (24*d^3*(c + d*x)*PolyLog
[3, -E^(I*(a + b*x))])/b^4 + (24*d^3*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])
/b^4 - ((24*I)*d^4*PolyLog[4, -E^(I*(a + b*x))])/b^5 + ((24*I)*d^4*PolyLog[
4, E^(I*(a + b*x))])/b^5 - (24*d^4*Sin[a + b*x])/b^5 + (12*d^2*(c + d*x)^2*
Sin[a + b*x])/b^3 - ((c + d*x)^4*Sin[a + b*x])/b
```

Rubi [A] time = 0.292642, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {4408, 3296, 2637, 4410, 4183, 2531, 6609, 2282, 6589}

$$\frac{24d^3(c + dx)\text{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^4} + \frac{24d^3(c + dx)\text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^4} + \frac{12id^2(c + dx)^2\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} - \frac{12id^2(c + dx)^2\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] (-8*d*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b^2 + (24*d^3*(c + d*x)*Cos[a +
b*x])/b^4 - (4*d*(c + d*x)^3*Cos[a + b*x])/b^2 - ((c + d*x)^4*Csc[a + b*x]
)/b + ((12*I)*d^2*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((12*I)*d
^2*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^3 - (24*d^3*(c + d*x)*PolyLog
[3, -E^(I*(a + b*x))])/b^4 + (24*d^3*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])
/b^4 - ((24*I)*d^4*PolyLog[4, -E^(I*(a + b*x))])/b^5 + ((24*I)*d^4*PolyLog[
4, E^(I*(a + b*x))])/b^5 - (24*d^4*Sin[a + b*x])/b^5 + (12*d^2*(c + d*x)^2*
Sin[a + b*x])/b^3 - ((c + d*x)^4*Sin[a + b*x])/b
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] := -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^4 \cos(a + bx) \cot^2(a + bx) dx &= - \int (c + dx)^4 \cos(a + bx) dx + \int (c + dx)^4 \cot(a + bx) \csc(a + bx) dx \\
&= - \frac{(c + dx)^4 \csc(a + bx)}{b} - \frac{(c + dx)^4 \sin(a + bx)}{b} + \frac{(4d) \int (c + dx)^3 \csc(a + bx) dx}{b} \\
&= - \frac{8d(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{4d(c + dx)^3 \cos(a + bx)}{b^2} - \frac{(c + dx)^4 \csc(a + bx)}{b} \\
&= - \frac{8d(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{4d(c + dx)^3 \cos(a + bx)}{b^2} - \frac{(c + dx)^4 \csc(a + bx)}{b} \\
&= - \frac{8d(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b^2} + \frac{24d^3(c + dx) \cos(a + bx)}{b^4} - \frac{4d(c + dx)^3 \cos(a + bx)}{b^2} \\
&= - \frac{8d(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b^2} + \frac{24d^3(c + dx) \cos(a + bx)}{b^4} - \frac{4d(c + dx)^3 \cos(a + bx)}{b^2} \\
&= - \frac{8d(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b^2} + \frac{24d^3(c + dx) \cos(a + bx)}{b^4} - \frac{4d(c + dx)^3 \cos(a + bx)}{b^2}
\end{aligned}$$

Mathematica [B] time = 1.72215, size = 798, normalized size = 2.67

$$\frac{\csc(a + bx) \left(-3c^4 b^4 - 3d^4 x^4 b^4 - 12cd^3 x^3 b^4 - 18c^2 d^2 x^2 b^4 - 12c^3 dx b^4 + c^4 \cos(2(a + bx)) b^4 + d^4 x^4 \cos(2(a + bx)) b^4 + \dots \right)}{\dots}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^4 * Cos[a + b*x] * Cot[a + b*x]^2, x]

[Out] (Csc[a + b*x] * (-3*b^4*c^4 + 12*b^2*c^2*d^2 - 24*d^4 - 12*b^4*c^3*d*x + 24*b^2*c*d^3*x - 18*b^4*c^2*d^2*x^2 + 12*b^2*d^4*x^2 - 12*b^4*c*d^3*x^3 - 3*b^4*d^4*x^4 + b^4*c^4 * Cos[2*(a + b*x)] - 12*b^2*c^2*d^2 * Cos[2*(a + b*x)] + 24*d^4 * Cos[2*(a + b*x)] + 4*b^4*c^3*d*x * Cos[2*(a + b*x)] - 24*b^2*c*d^3*x * Cos[2*(a + b*x)] + 6*b^4*c^2*d^2*x^2 * Cos[2*(a + b*x)] - 12*b^2*d^4*x^2 * Cos[2*(a + b*x)] + 4*b^4*c*d^3*x^3 * Cos[2*(a + b*x)] + b^4*d^4*x^4 * Cos[2*(a + b*x)] - 16*b^3*c^3*d * ArcTanh[Cos[a + b*x] + I * Sin[a + b*x]] * Sin[a + b*x] - 48*b^3 * c^2*d^2 * ArcTanh[Cos[a + b*x] + I * Sin[a + b*x]] * Sin[a + b*x] - 48*b^3 * c*d^3 * x^2 * ArcTanh[Cos[a + b*x] + I * Sin[a + b*x]] * Sin[a + b*x] - 16*b^3 * d^4 * x^3 * ArcTanh[Cos[a + b*x] + I * Sin[a + b*x]] * Sin[a + b*x] + (24*I) * b^2 * d^2 * (c + d*x)^2 * PolyLog[2, -Cos[a + b*x] - I * Sin[a + b*x]] * Sin[a + b*x] - (24*I) * b^2 * d^2 * (c + d*x)^2 * PolyLog[2, Cos[a + b*x] + I * Sin[a + b*x]] * Sin[a + b*x] - 48 * b * c * d^3 * PolyLog[3, -Cos[a + b*x] - I * Sin[a + b*x]] * Sin[a + b*x] - 48 * b * d^4 * x * PolyLog[3, -Cos[a + b*x] - I * Sin[a + b*x]] * Sin[a + b*x] + 48 * b * c * d^3 * PolyLog[3, Cos[a + b*x] + I * Sin[a + b*x]] * Sin[a + b*x] + 48 * b * d^4 * x * PolyLog[3, Cos[a + b*x] + I * Sin[a + b*x]] * Sin[a + b*x] - (48*I) * d^4 * PolyLog[4, -Cos[a + b*x] - I * Sin[a + b*x]] * Sin[a + b*x] + (48*I) * d^4 * PolyLog[4, Cos[a + b*x] + I * Sin[a + b*x]] * Sin[a + b*x] - 4 * b^3 * c^3 * d * Sin[2*(a + b*x)] + 24 * b * c * d^3 * Sin[2*(a + b*x)] - 12 * b^3 * c^2 * d^2 * x * Sin[2*(a + b*x)] + 24 * b * d^4 * x * Sin[2*(a + b*x)] - 12 * b^3 * c * d^3 * x^2 * Sin[2*(a + b*x)] - 4 * b^3 * d^4 * x^3 * Sin[2*(a + b*x)])) / (2 * b^5)

Maple [B] time = 0.209, size = 1056, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x)`

[Out] $-4*d^4/b^5*\ln(\exp(I*(b*x+a))+1)*a^3-24*d^3/b^4*c*a^2*\operatorname{arctanh}(\exp(I*(b*x+a)))+24*d^2/b^3*c^2*a*\operatorname{arctanh}(\exp(I*(b*x+a)))-24*I*d^3/b^3*c*\operatorname{polylog}(2,\exp(I*(b*x+a)))*x+8*d^4/b^5*a^3*\operatorname{arctanh}(\exp(I*(b*x+a)))-8*d/b^2*c^3*\operatorname{arctanh}(\exp(I*(b*x+a)))+4*d^4/b^2*\ln(1-\exp(I*(b*x+a)))*x^3+24*d^4/b^4*\operatorname{polylog}(3,\exp(I*(b*x+a)))*x-24*d^4/b^4*\operatorname{polylog}(3,-\exp(I*(b*x+a)))*x-24*d^3/b^4*c*\operatorname{polylog}(3,-\exp(I*(b*x+a)))+24*d^3/b^4*c*\operatorname{polylog}(3,\exp(I*(b*x+a)))+12*d^3/b^4*c*a^2*\ln(\exp(I*(b*x+a))+1)-12*d^2/b^3*c^2*\ln(\exp(I*(b*x+a))+1)*a+12*I*d^2/b^3*c^2*\operatorname{polylog}(2,-\exp(I*(b*x+a)))+12*I*d^4/b^3*\operatorname{polylog}(2,-\exp(I*(b*x+a)))*x^2+24*I*d^4*\operatorname{polylog}(4,\exp(I*(b*x+a)))/b^5-2*I*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*x^2+4*c^3*d*x+c^4)*\exp(I*(b*x+a))/b/(\exp(2*I*(b*x+a))-1)-1/2*I*(d^4*x^4*b^4+4*b^4*c*d^3*x^3+6*b^4*c^2*d^2*x^2+4*b^4*c^3*d*x-4*I*b^3*d^4*x^3+b^4*c^4-12*b^2*d^4*x^2-12*I*b^3*c*d^3*x^2-24*b^2*c*d^3*x-12*I*b^3*c^2*d^2*x-12*c^2*d^2*b^2-4*I*b^3*c^3*d+24*I*b*d^4*x+24*d^4+24*I*b*c*d^3)/b^5*\exp(-I*(b*x+a))-12*d^3/b^2*c*\ln(\exp(I*(b*x+a))+1)*x^2+12*d^3/b^2*c*\ln(1-\exp(I*(b*x+a)))*x^2-12*d^3/b^4*c*\ln(1-\exp(I*(b*x+a)))*a^2-12*I*d^2/b^3*c^2*\operatorname{polylog}(2,\exp(I*(b*x+a)))-12*I*d^4/b^3*\operatorname{polylog}(2,\exp(I*(b*x+a)))*x^2+24*I*d^3/b^3*c*\operatorname{polylog}(2,-\exp(I*(b*x+a)))*x-24*I*d^4*\operatorname{polylog}(4,-\exp(I*(b*x+a)))/b^5+1/2*I*(d^4*x^4*b^4+4*b^4*c*d^3*x^3+6*b^4*c^2*d^2*x^2+4*b^4*c^3*d*x+4*I*b^3*d^4*x^3+b^4*c^4-12*b^2*d^4*x^2+12*I*b^3*c*d^3*x^2-24*b^2*c*d^3*x+12*I*b^3*c^2*d^2*x-12*c^2*d^2*b^2+4*I*b^3*c^3*d-24*I*b*d^4*x+24*d^4-24*I*b*c*d^3)/b^5*\exp(I*(b*x+a))+4*d^4/b^5*\ln(1-\exp(I*(b*x+a)))*a^3-4*d^4/b^2*\ln(\exp(I*(b*x+a))+1)*x^3+12*d^2/b^2*c^2*\ln(1-\exp(I*(b*x+a)))*x+12*d^2/b^3*c^2*\ln(1-\exp(I*(b*x+a)))*a-12*d^2/b^2*c^2*\ln(\exp(I*(b*x+a))+1)*x$

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")`

[Out] Timed out

Fricas [C] time = 0.88748, size = 3009, normalized size = 10.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")`

[Out] $-(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 12*b^2*c^2*d^2 - 12*I*d^4*\operatorname{polylog}(4, \cos(b*x + a) + I*\sin(b*x + a))*\sin(b*x + a) + 12*I*d^4*\operatorname{polylog}(4, \cos(b*x + a) - I*\sin(b*x + a))*\sin(b*x + a) - 12*I*d^4*\operatorname{polylog}(4, -\cos(b*x + a) + I*\sin(b*x + a))*\sin(b*x + a) + 12*I*d^4*\operatorname{polylog}(4, -\cos(b*x + a) - I*\sin(b*x + a))*\sin(b*x + a) + 24*d^4 + 12*(b^4*c^2*d^2 - b^2*d^4)*x^2 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 24*d^4 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*\cos(b*x + a)^2 + 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d - 6*b*c*d^3 + 3*(b^3*c^2*d^2 - 2*b*d^4)*x)*\cos(b*x + a)*\sin(b*x + a) - (-6*I*b^2*d^4*x^2 - 12*I*b^2*c*d^3*x - 6*I*b^2*c^2*d^2)*\operatorname{dilog}(\cos(b*x + a) + I*\sin(b*x + a))*\sin(b*x + a) - (6$

```

*I*b^2*d^4*x^2 + 12*I*b^2*c*d^3*x + 6*I*b^2*c^2*d^2)*dilog(cos(b*x + a) - I
*sin(b*x + a))*sin(b*x + a) - (-6*I*b^2*d^4*x^2 - 12*I*b^2*c*d^3*x - 6*I*b^
2*c^2*d^2)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*b^2*d^
4*x^2 + 12*I*b^2*c*d^3*x + 6*I*b^2*c^2*d^2)*dilog(-cos(b*x + a) - I*sin(b*x
+ a))*sin(b*x + a) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x +
b^3*c^3*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 2*(b^3*d^4
*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*log(cos(b*x + a) - I*
sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c
*d^3 - a^3*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x +
a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*log(-1/2*co
s(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 2*(b^3*d^4*x^3 + 3*b^
3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*
log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*d^4*x^3 + 3*b
^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)
*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 12*(b*d^4*x + b*c*d
^3)*polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 12*(b*d^4*x +
b*c*d^3)*polylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 12*(b*d^4
*x + b*c*d^3)*polylog(3, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*
(b*d^4*x + b*c*d^3)*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a)
+ 8*(b^4*c^3*d - 3*b^2*c*d^3)*x)/(b^5*sin(b*x + a))

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**4*cos(b*x+a)*cot(b*x+a)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^4 \cos(bx + a) \cot(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^4*cos(b*x + a)*cot(b*x + a)^2, x)

3.172 $\int (c + dx)^3 \cos(a + bx) \cot^2(a + bx) dx$

Optimal. Leaf size=216

$$\frac{6id^2(c + dx)\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} - \frac{6id^2(c + dx)\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3} - \frac{6d^3\text{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^4} + \frac{6d^3\text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^4}$$

[Out] $(-6*d*(c + d*x)^2*\text{ArcTanh}[E^{(I*(a + b*x))}])/b^2 + (6*d^3*\text{Cos}[a + b*x])/b^4 - (3*d*(c + d*x)^2*\text{Cos}[a + b*x])/b^2 - ((c + d*x)^3*\text{Csc}[a + b*x])/b + ((6*I)*d^2*(c + d*x)*\text{PolyLog}[2, -E^{(I*(a + b*x))}])/b^3 - ((6*I)*d^2*(c + d*x)*\text{PolyLog}[2, E^{(I*(a + b*x))}])/b^3 - (6*d^3*\text{PolyLog}[3, -E^{(I*(a + b*x))}])/b^4 + (6*d^3*\text{PolyLog}[3, E^{(I*(a + b*x))}])/b^4 + (6*d^2*(c + d*x)*\text{Sin}[a + b*x])/b^3 - ((c + d*x)^3*\text{Sin}[a + b*x])/b$

Rubi [A] time = 0.222293, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4408, 3296, 2638, 4410, 4183, 2531, 2282, 6589}

$$\frac{6id^2(c + dx)\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} - \frac{6id^2(c + dx)\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3} - \frac{6d^3\text{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^4} + \frac{6d^3\text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^3*\text{Cos}[a + b*x]*\text{Cot}[a + b*x]^2, x]$

[Out] $(-6*d*(c + d*x)^2*\text{ArcTanh}[E^{(I*(a + b*x))}])/b^2 + (6*d^3*\text{Cos}[a + b*x])/b^4 - (3*d*(c + d*x)^2*\text{Cos}[a + b*x])/b^2 - ((c + d*x)^3*\text{Csc}[a + b*x])/b + ((6*I)*d^2*(c + d*x)*\text{PolyLog}[2, -E^{(I*(a + b*x))}])/b^3 - ((6*I)*d^2*(c + d*x)*\text{PolyLog}[2, E^{(I*(a + b*x))}])/b^3 - (6*d^3*\text{PolyLog}[3, -E^{(I*(a + b*x))}])/b^4 + (6*d^3*\text{PolyLog}[3, E^{(I*(a + b*x))}])/b^4 + (6*d^2*(c + d*x)*\text{Sin}[a + b*x])/b^3 - ((c + d*x)^3*\text{Sin}[a + b*x])/b$

Rule 4408

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(n_.)}*\text{Cot}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow -\text{Int}[(c + d*x)^m*\text{Cos}[a + b*x]^n*\text{Cot}[a + b*x]^{(p - 2)}, x] + \text{Int}[(c + d*x)^m*\text{Cos}[a + b*x]^{(n - 2)}*\text{Cot}[a + b*x]^p, x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m - 1)}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

$\text{Int}[\text{sin}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 4410

$\text{Int}[\text{Cot}[(a_.) + (b_.)*(x_.)]^{(p_.)}*\text{Csc}[(a_.) + (b_.)*(x_.)]^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Csc}[a + b*x]^n/(b*n), x] + \text{Dist}[(d*m)/(b*n), \text{Int}[(c + d*x)^{(m - 1)}*\text{Csc}[a + b*x]^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/ (b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^3 \cos(a + bx) \cot^2(a + bx) dx &= - \int (c + dx)^3 \cos(a + bx) dx + \int (c + dx)^3 \cot(a + bx) \csc(a + bx) dx \\ &= - \frac{(c + dx)^3 \csc(a + bx)}{b} - \frac{(c + dx)^3 \sin(a + bx)}{b} + \frac{(3d) \int (c + dx)^2 \csc(a + bx) dx}{b} \\ &= - \frac{6d(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{3d(c + dx)^2 \cos(a + bx)}{b^2} - \frac{(c + dx)^3 \csc(a + bx)}{b} \\ &= - \frac{6d(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{3d(c + dx)^2 \cos(a + bx)}{b^2} - \frac{(c + dx)^3 \csc(a + bx)}{b} \\ &= - \frac{6d(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^2} + \frac{6d^3 \cos(a + bx)}{b^4} - \frac{3d(c + dx)^2 \cos(a + bx)}{b^2} \\ &= - \frac{6d(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^2} + \frac{6d^3 \cos(a + bx)}{b^4} - \frac{3d(c + dx)^2 \cos(a + bx)}{b^2} \end{aligned}$$

Mathematica [B] time = 1.46379, size = 539, normalized size = 2.5

$$\csc(a + bx) \left(12ibd^2(c + dx) \sin(a + bx) \text{PolyLog}\left(2, -e^{i(a+bx)}\right) - 12ibd^2(c + dx) \sin(a + bx) \text{PolyLog}\left(2, e^{i(a+bx)}\right) - 12d^3 \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x]^2, x]
```

```
[Out] (Csc[a + b*x]*(-3*b^3*c^3 + 6*b*c*d^2 - 9*b^3*c^2*d*x + 6*b*d^3*x - 9*b^3*c*d^2*x^2 - 3*b^3*d^3*x^3 + b^3*c^3*Cos[2*(a + b*x)] - 6*b*c*d^2*Cos[2*(a +
```

$$b*x)] + 3*b^3*c^2*d*x*\cos[2*(a + b*x)] - 6*b*d^3*x*\cos[2*(a + b*x)] + 3*b^3*c*d^2*x^2*\cos[2*(a + b*x)] + b^3*d^3*x^3*\cos[2*(a + b*x)] + 6*b^2*c^2*d*\log[1 - E^(I*(a + b*x))]*\sin[a + b*x] + 12*b^2*c*d^2*x*\log[1 - E^(I*(a + b*x))]*\sin[a + b*x] + 6*b^2*d^3*x^2*\log[1 - E^(I*(a + b*x))]*\sin[a + b*x] - 6*b^2*c^2*d*\log[1 + E^(I*(a + b*x))]*\sin[a + b*x] - 12*b^2*c*d^2*x*\log[1 + E^(I*(a + b*x))]*\sin[a + b*x] - 6*b^2*d^3*x^2*\log[1 + E^(I*(a + b*x))]*\sin[a + b*x] + (12*I)*b*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))]*\sin[a + b*x] - (12*I)*b*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))]*\sin[a + b*x] - 12*d^3*PolyLog[3, -E^(I*(a + b*x))]*\sin[a + b*x] + 12*d^3*PolyLog[3, E^(I*(a + b*x))]*\sin[a + b*x] - 3*b^2*c^2*d*\sin[2*(a + b*x)] + 6*d^3*\sin[2*(a + b*x)] - 6*b^2*c*d^2*x*\sin[2*(a + b*x)] - 3*b^2*d^3*x^2*\sin[2*(a + b*x)])))/(2*b^4)$$

Maple [B] time = 0.189, size = 649, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x)

[Out] $\frac{1}{2}I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+c^3)*\exp(I*(b*x+a)) - 2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*\exp(I*(b*x+a))/b/(\exp(2*I*(b*x+a)) - 1) - 6*I*d^3/b^3*\text{polylog}(2, \exp(I*(b*x+a))) * x + 3*d^3/b^2*\ln(1 - \exp(I*(b*x+a))) * x^2 - 3*d^3/b^4*\ln(1 - \exp(I*(b*x+a))) * a^2 + 6*I*d^3/b^3*\text{polylog}(2, -\exp(I*(b*x+a))) * x - 3*d^3/b^2*\ln(\exp(I*(b*x+a))+1) * x^2 + 3*d^3/b^4*\ln(\exp(I*(b*x+a))+1) * a^2 + 6*I*d^2/b^3*c*\text{polylog}(2, -\exp(I*(b*x+a))) - 6*d^3/b^4*a^2*\text{arctanh}(\exp(I*(b*x+a)))) - 6*d^2/b^2*c*\ln(\exp(I*(b*x+a))+1) * x - 6*d^2/b^3*c*\ln(\exp(I*(b*x+a))+1) * a - 1/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*\exp(-I*(b*x+a)) - 6*I*d^2/b^3*c*\text{polylog}(2, \exp(I*(b*x+a))) + 6*d^3*\text{polylog}(3, \exp(I*(b*x+a)))/b^4 - 6*d^3*\text{polylog}(3, -\exp(I*(b*x+a)))/b^4 + 6*d^2/b^2*c*\ln(1 - \exp(I*(b*x+a))) * x + 6*d^2/b^3*c*\ln(1 - \exp(I*(b*x+a))) * a + 12*d^2/b^3*c*a*\text{arctanh}(\exp(I*(b*x+a))) - 6*d/b^2*c^2*\text{arctanh}(\exp(I*(b*x+a)))$

Maxima [B] time = 6.75717, size = 14876, normalized size = 68.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")

[Out] $-1/2*(2*c^3*(1/\sin(b*x + a) + \sin(b*x + a)) - 6*a*c^2*d*(1/\sin(b*x + a) + \sin(b*x + a))/b + 6*a^2*c*d^2*(1/\sin(b*x + a) + \sin(b*x + a))/b^2 - 2*a^3*d^3*(1/\sin(b*x + a) + \sin(b*x + a))/b^3 - 3*((b*x + a)*\sin(2*b*x + 2*a) - \cos(2*b*x + 2*a) + 1)*\cos(3*b*x + 3*a)^3 + (b*x - (b*x + a)*\cos(2*b*x + 2*a) + a - \sin(2*b*x + 2*a))*\sin(3*b*x + 3*a)^3 - 6*(b*x + a)*\sin(b*x + a)^3 - 2*(4*(b*x + a)*\cos(b*x + a)*\sin(2*b*x + 2*a) - (3*(b*x + a)*\sin(b*x + a) + \cos(b*x + a))*\cos(2*b*x + 2*a) + 3*(b*x + a)*\sin(b*x + a) + \cos(b*x + a))*\cos(3*b*x + 3*a)^2 - ((b*x + a)*\sin(b*x + a) + \cos(b*x + a))*\cos(2*b*x + 2*a)^2 + (8*(b*x + a)*\cos(2*b*x + 2*a)*\sin(b*x + a) + ((b*x + a)*\sin(2*b*x + 2*a) - \cos(2*b*x + 2*a) + 1)*\cos(3*b*x + 3*a) - 2*(3*(b*x + a)*\cos(b*x + a) - \sin(b*x + a))*\sin(2*b*x + 2*a) - 8*(b*x + a)*\sin(b*x + a))*\sin(3*b*x + 3*a$

$$\begin{aligned}
&)^2 - ((b*x + a)*\sin(b*x + a) + \cos(b*x + a))*\sin(2*b*x + 2*a)^2 + (12*(b*x \\
&+ a)*\cos(b*x + a)*\sin(b*x + a) - (12*(b*x + a)*\cos(b*x + a)*\sin(b*x + a) + \\
&\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2)*\cos(2*b*x + 2*a) + \cos(2*b*x + 2*a)^2 \\
&+ \cos(b*x + a)^2 + (13*(b*x + a)*\cos(b*x + a)^2 + (b*x + a)*\sin(b*x + a)^2 \\
&)*\sin(2*b*x + 2*a) + \sin(2*b*x + 2*a)^2 + \sin(b*x + a)^2 + 1)*\cos(3*b*x + 3 \\
&*a) + 2*(3*(b*x + a)*\sin(b*x + a)^3 + (3*(b*x + a)*\cos(b*x + a)^2 + b*x + a \\
&)*\sin(b*x + a) + \cos(b*x + a))*\cos(2*b*x + 2*a) - ((\cos(2*b*x + 2*a)^2 + \sin \\
&(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\cos(3*b*x + 3*a)^2 + (\cos(b*x + \\
&a)^2 + \sin(b*x + a)^2)*\cos(2*b*x + 2*a)^2 + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x \\
&+ 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\sin(3*b*x + 3*a)^2 + (\cos(b*x + a)^2 + \\
&\sin(b*x + a)^2)*\sin(2*b*x + 2*a)^2 - 2*(\cos(2*b*x + 2*a)^2*\cos(b*x + a) + \cos \\
&(b*x + a)*\sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a)*\cos(b*x + a) + \cos(b*x \\
&+ a))*\cos(3*b*x + 3*a) - 2*(\cos(b*x + a)^2 + \sin(b*x + a)^2)*\cos(2*b*x + 2* \\
&a) + \cos(b*x + a)^2 - 2*(\cos(2*b*x + 2*a)^2*\sin(b*x + a) + \sin(2*b*x + 2*a) \\
&^2*\sin(b*x + a) - 2*\cos(2*b*x + 2*a)*\sin(b*x + a) + \sin(b*x + a))*\sin(3*b*x \\
&+ 3*a) + \sin(b*x + a)^2*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + \\
&a) + 1) + ((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + \\
&1)*\cos(3*b*x + 3*a)^2 + (\cos(b*x + a)^2 + \sin(b*x + a)^2)*\cos(2*b*x + 2*a) \\
&^2 + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\sin \\
&(3*b*x + 3*a)^2 + (\cos(b*x + a)^2 + \sin(b*x + a)^2)*\sin(2*b*x + 2*a)^2 - 2* \\
&(\cos(2*b*x + 2*a)^2*\cos(b*x + a) + \cos(b*x + a)*\sin(2*b*x + 2*a)^2 - 2*\cos(\\
&2*b*x + 2*a)*\cos(b*x + a) + \cos(b*x + a))*\cos(3*b*x + 3*a) - 2*(\cos(b*x + a \\
&)^2 + \sin(b*x + a)^2)*\cos(2*b*x + 2*a) + \cos(b*x + a)^2 - 2*(\cos(2*b*x + 2* \\
&a)^2*\sin(b*x + a) + \sin(2*b*x + 2*a)^2*\sin(b*x + a) - 2*\cos(2*b*x + 2*a)*\sin \\
&(b*x + a) + \sin(b*x + a))*\sin(3*b*x + 3*a) + \sin(b*x + a)^2*\log(\cos(b*x + \\
&a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) + ((b*x - (b*x + a)*\cos(2*b*x \\
&+ 2*a) + a - \sin(2*b*x + 2*a))*\cos(3*b*x + 3*a)^2 + (b*x + a)*\cos(2*b*x + 2 \\
&*a)^2 + (b*x + a)*\cos(b*x + a)^2 + (b*x + a)*\sin(2*b*x + 2*a)^2 + 13*(b*x + \\
&a)*\sin(b*x + a)^2 + b*x + 2*((b*x + a)*\cos(b*x + a) + \sin(b*x + a))*\cos(2 \\
&*b*x + 2*a) - (b*x + a)*\cos(b*x + a) - ((b*x + a)*\sin(b*x + a) - \cos(b*x + \\
&a))*\sin(2*b*x + 2*a) - \sin(b*x + a))*\cos(3*b*x + 3*a) - ((b*x + a)*\cos(b*x \\
&+ a)^2 + 13*(b*x + a)*\sin(b*x + a)^2 + 2*b*x + 2*a)*\cos(2*b*x + 2*a) + (12* \\
&(b*x + a)*\cos(b*x + a)*\sin(b*x + a) - \cos(b*x + a)^2 - \sin(b*x + a)^2)*\sin(\\
&2*b*x + 2*a) + a)*\sin(3*b*x + 3*a) - 6*((b*x + a)*\cos(b*x + a)^3 + (b*x + a \\
&)*\cos(b*x + a)*\sin(b*x + a)^2)*\sin(2*b*x + 2*a) - (6*(b*x + a)*\cos(b*x + a) \\
&^2 + b*x + a)*\sin(b*x + a) - \cos(b*x + a))*c^2*d/(((\cos(2*b*x + 2*a)^2 + \sin \\
&(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\cos(3*b*x + 3*a)^2 + (\cos(b*x + \\
&a)^2 + \sin(b*x + a)^2)*\cos(2*b*x + 2*a)^2 + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x \\
&+ 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\sin(3*b*x + 3*a)^2 + (\cos(b*x + a)^2 + \\
&\sin(b*x + a)^2)*\sin(2*b*x + 2*a)^2 - 2*(\cos(2*b*x + 2*a)^2*\cos(b*x + a) + \cos \\
&(b*x + a)*\sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a)*\cos(b*x + a) + \cos(b*x \\
&+ a))*\cos(3*b*x + 3*a) - 2*(\cos(b*x + a)^2 + \sin(b*x + a)^2)*\cos(2*b*x + 2* \\
&a) + \cos(b*x + a)^2 - 2*(\cos(2*b*x + 2*a)^2*\sin(b*x + a) + \sin(2*b*x + 2*a) \\
&^2*\sin(b*x + a) - 2*\cos(2*b*x + 2*a)*\sin(b*x + a) + \sin(b*x + a))*\sin(3*b*x \\
&+ 3*a) + \sin(b*x + a)^2)*b) + 6*((b*x + a)*\sin(2*b*x + 2*a) - \cos(2*b*x + \\
&2*a) + 1)*\cos(3*b*x + 3*a)^3 + (b*x - (b*x + a)*\cos(2*b*x + 2*a) + a - \sin \\
&(2*b*x + 2*a))*\sin(3*b*x + 3*a)^3 - 6*(b*x + a)*\sin(b*x + a)^3 - 2*(4*(b*x \\
&+ a)*\cos(b*x + a)*\sin(2*b*x + 2*a) - (3*(b*x + a)*\sin(b*x + a) + \cos(b*x + \\
&a))*\cos(2*b*x + 2*a) + 3*(b*x + a)*\sin(b*x + a) + \cos(b*x + a))*\cos(3*b*x + \\
&3*a)^2 - ((b*x + a)*\sin(b*x + a) + \cos(b*x + a))*\cos(2*b*x + 2*a)^2 + (8*(\\
&b*x + a)*\cos(2*b*x + 2*a)*\sin(b*x + a) + ((b*x + a)*\sin(2*b*x + 2*a) - \cos(\\
&2*b*x + 2*a) + 1)*\cos(3*b*x + 3*a) - 2*(3*(b*x + a)*\cos(b*x + a) - \sin(b*x \\
&+ a))*\sin(2*b*x + 2*a) - 8*(b*x + a)*\sin(b*x + a))*\sin(3*b*x + 3*a)^2 - ((b \\
&x + a)*\sin(b*x + a) + \cos(b*x + a))*\sin(2*b*x + 2*a)^2 + (12*(b*x + a)*\cos \\
&(b*x + a)*\sin(b*x + a) - (12*(b*x + a)*\cos(b*x + a)*\sin(b*x + a) + \cos(b*x \\
&+ a)^2 + \sin(b*x + a)^2 + 2)*\cos(2*b*x + 2*a) + \cos(2*b*x + 2*a)^2 + \cos(b*x \\
&+ a)^2 + (13*(b*x + a)*\cos(b*x + a)^2 + (b*x + a)*\sin(b*x + a)^2)*\sin(2*b \\
&*x + 2*a) + \sin(2*b*x + 2*a)^2 + \sin(b*x + a)^2 + 1)*\cos(3*b*x + 3*a) + 2*(\\
&3*(b*x + a)*\sin(b*x + a)^3 + (3*(b*x + a)*\cos(b*x + a)^2 + b*x + a)*\sin(b*x
\end{aligned}$$

$$\begin{aligned}
& + a) + \cos(b*x + a))*\cos(2*b*x + 2*a) - ((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + \\
& 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\cos(3*b*x + 3*a)^2 + (\cos(b*x + a)^2 + \sin(b*x + a)^2)*\cos(2*b*x + 2*a)^2 + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 \\
& - 2*\cos(2*b*x + 2*a) + 1)*\sin(3*b*x + 3*a)^2 + (\cos(b*x + a)^2 + \sin(b*x + a)^2)*\sin(2*b*x + 2*a)^2 - 2*(\cos(2*b*x + 2*a)^2*\cos(b*x + a) + \cos(b*x + \\
& a)*\sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a)*\cos(b*x + a) + \cos(b*x + a))*\cos \\
& (3*b*x + 3*a) - 2*(\cos(b*x + a)^2 + \sin(b*x + a)^2)*\cos(2*b*x + 2*a) + \cos(b*x + a)^2 - 2*(\cos(2*b*x + 2*a)^2*\sin(b*x + a) + \sin(2*b*x + 2*a)^2*\sin(b*x \\
& + a) - 2*\cos(2*b*x + 2*a)*\sin(b*x + a) + \sin(b*x + a))*\sin(3*b*x + 3*a) + \\
& \sin(b*x + a)^2)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) \\
& + ((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\cos(3 \\
& *b*x + 3*a)^2 + (\cos(b*x + a)^2 + \sin(b*x + a)^2)*\cos(2*b*x + 2*a)^2 + (\cos \\
& (2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\sin(3*b*x + \\
& 3*a)^2 + (\cos(b*x + a)^2 + \sin(b*x + a)^2)*\sin(2*b*x + 2*a)^2 - 2*(\cos(2*b* \\
& x + 2*a)^2*\cos(b*x + a) + \cos(b*x + a)*\sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2 \\
& *a)*\cos(b*x + a) + \cos(b*x + a))*\cos(3*b*x + 3*a) - 2*(\cos(b*x + a)^2 + \sin \\
& (b*x + a)^2)*\cos(2*b*x + 2*a) + \cos(b*x + a)^2 - 2*(\cos(2*b*x + 2*a)^2*\sin \\
& (b*x + a) + \sin(2*b*x + 2*a)^2*\sin(b*x + a) - 2*\cos(2*b*x + 2*a)*\sin(b*x + a \\
&) + \sin(b*x + a))*\sin(3*b*x + 3*a) + \sin(b*x + a)^2)*\log(\cos(b*x + a)^2 + \sin \\
& (b*x + a)^2 - 2*\cos(b*x + a) + 1) + ((b*x - (b*x + a)*\cos(2*b*x + 2*a) + \\
& a - \sin(2*b*x + 2*a))*\cos(3*b*x + 3*a)^2 + (b*x + a)*\cos(2*b*x + 2*a)^2 + (\\
& b*x + a)*\cos(b*x + a)^2 + (b*x + a)*\sin(2*b*x + 2*a)^2 + 13*(b*x + a)*\sin(b \\
& *x + a)^2 + b*x + 2*((b*x + a)*\cos(b*x + a) + \sin(b*x + a))*\cos(2*b*x + 2* \\
& a) - (b*x + a)*\cos(b*x + a) - ((b*x + a)*\sin(b*x + a) - \cos(b*x + a))*\sin(2 \\
& *b*x + 2*a) - \sin(b*x + a))*\cos(3*b*x + 3*a) - ((b*x + a)*\cos(b*x + a)^2 + \\
& 13*(b*x + a)*\sin(b*x + a)^2 + 2*b*x + 2*a)*\cos(2*b*x + 2*a) + (12*(b*x + a) \\
& *\cos(b*x + a)*\sin(b*x + a) - \cos(b*x + a)^2 - \sin(b*x + a)^2)*\sin(2*b*x + 2 \\
& *a) + a)*\sin(3*b*x + 3*a) - 6*((b*x + a)*\cos(b*x + a)^3 + (b*x + a)*\cos(b*x \\
& + a)*\sin(b*x + a)^2)*\sin(2*b*x + 2*a) - (6*(b*x + a)*\cos(b*x + a)^2 + b*x \\
& + a)*\sin(b*x + a) - \cos(b*x + a))*a*c*d^2/(((\cos(2*b*x + 2*a)^2 + \sin(2*b*x \\
& + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\cos(3*b*x + 3*a)^2 + (\cos(b*x + a)^2 + \\
& \sin(b*x + a)^2)*\cos(2*b*x + 2*a)^2 + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a) \\
& ^2 - 2*\cos(2*b*x + 2*a) + 1)*\sin(3*b*x + 3*a)^2 + (\cos(b*x + a)^2 + \sin(b*x \\
& + a)^2)*\sin(2*b*x + 2*a)^2 - 2*(\cos(2*b*x + 2*a)^2*\cos(b*x + a) + \cos(b*x \\
& + a)*\sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a)*\cos(b*x + a) + \cos(b*x + a))*c \\
& os(3*b*x + 3*a) - 2*(\cos(b*x + a)^2 + \sin(b*x + a)^2)*\cos(2*b*x + 2*a) + \cos \\
& (b*x + a)^2 - 2*(\cos(2*b*x + 2*a)^2*\sin(b*x + a) + \sin(2*b*x + 2*a)^2*\sin \\
& (b*x + a) - 2*\cos(2*b*x + 2*a)*\sin(b*x + a) + \sin(b*x + a))*\sin(3*b*x + 3*a) \\
& + \sin(b*x + a)^2)*b^2) - 3*((b*x + a)*\sin(2*b*x + 2*a) - \cos(2*b*x + 2*a) \\
& + 1)*\cos(3*b*x + 3*a)^3 + (b*x - (b*x + a)*\cos(2*b*x + 2*a) + a - \sin(2*b* \\
& x + 2*a))*\sin(3*b*x + 3*a)^3 - 6*(b*x + a)*\sin(b*x + a)^3 - 2*(4*(b*x + a)* \\
& \cos(b*x + a)*\sin(2*b*x + 2*a) - (3*(b*x + a)*\sin(b*x + a) + \cos(b*x + a))*c \\
& os(2*b*x + 2*a) + 3*(b*x + a)*\sin(b*x + a) + \cos(b*x + a))*\cos(3*b*x + 3*a) \\
& ^2 - ((b*x + a)*\sin(b*x + a) + \cos(b*x + a))*\cos(2*b*x + 2*a)^2 + (8*(b*x + \\
& a)*\cos(2*b*x + 2*a)*\sin(b*x + a) + ((b*x + a)*\sin(2*b*x + 2*a) - \cos(2*b*x \\
& + 2*a) + 1)*\cos(3*b*x + 3*a) - 2*(3*(b*x + a)*\cos(b*x + a) - \sin(b*x + a)) \\
& *\sin(2*b*x + 2*a) - 8*(b*x + a)*\sin(b*x + a))*\sin(3*b*x + 3*a)^2 - ((b*x + \\
& a)*\sin(b*x + a) + \cos(b*x + a))*\sin(2*b*x + 2*a)^2 + (12*(b*x + a)*\cos(b*x \\
& + a)*\sin(b*x + a) - (12*(b*x + a)*\cos(b*x + a)*\sin(b*x + a) + \cos(b*x + a) \\
& ^2 + \sin(b*x + a)^2 + 2)*\cos(2*b*x + 2*a) + \cos(2*b*x + 2*a)^2 + \cos(b*x + a) \\
&)^2 + (13*(b*x + a)*\cos(b*x + a)^2 + (b*x + a)*\sin(b*x + a)^2)*\sin(2*b*x + \\
& 2*a) + \sin(2*b*x + 2*a)^2 + \sin(b*x + a)^2 + 1)*\cos(3*b*x + 3*a) + 2*(3*(b* \\
& x + a)*\sin(b*x + a)^3 + (3*(b*x + a)*\cos(b*x + a)^2 + b*x + a)*\sin(b*x + a) \\
& + \cos(b*x + a))*\cos(2*b*x + 2*a) - ((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a) \\
& ^2 - 2*\cos(2*b*x + 2*a) + 1)*\cos(3*b*x + 3*a)^2 + (\cos(b*x + a)^2 + \sin(b*x \\
& + a)^2)*\cos(2*b*x + 2*a)^2 + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2* \\
& \cos(2*b*x + 2*a) + 1)*\sin(3*b*x + 3*a)^2 + (\cos(b*x + a)^2 + \sin(b*x + a)^2) \\
&)*\sin(2*b*x + 2*a)^2 - 2*(\cos(2*b*x + 2*a)^2*\cos(b*x + a) + \cos(b*x + a)*\sin \\
& (2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a)*\cos(b*x + a) + \cos(b*x + a))*\cos(3*b*
\end{aligned}$$

$$\begin{aligned}
& x + 3a) - 2*(\cos(b*x + a)^2 + \sin(b*x + a)^2)*\cos(2*b*x + 2*a) + \cos(b*x + \\
& a)^2 - 2*(\cos(2*b*x + 2*a)^2*\sin(b*x + a) + \sin(2*b*x + 2*a)^2*\sin(b*x + a \\
&) - 2*\cos(2*b*x + 2*a)*\sin(b*x + a) + \sin(b*x + a))*\sin(3*b*x + 3*a) + \sin(\\
& b*x + a)^2)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) + ((\cos \\
& (2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\cos(3*b*x \\
& + 3*a)^2 + (\cos(b*x + a)^2 + \sin(b*x + a)^2)*\cos(2*b*x + 2*a)^2 + (\cos(2*b* \\
& x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\sin(3*b*x + 3*a)^2 \\
& + (\cos(b*x + a)^2 + \sin(b*x + a)^2)*\sin(2*b*x + 2*a)^2 - 2*(\cos(2*b*x + 2* \\
& a)^2*\cos(b*x + a) + \cos(b*x + a)*\sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a)*\cos \\
& (b*x + a) + \cos(b*x + a))*\cos(3*b*x + 3*a) - 2*(\cos(b*x + a)^2 + \sin(b*x \\
& + a)^2)*\cos(2*b*x + 2*a) + \cos(b*x + a)^2 - 2*(\cos(2*b*x + 2*a)^2*\sin(b*x + \\
& a) + \sin(2*b*x + 2*a)^2*\sin(b*x + a) - 2*\cos(2*b*x + 2*a)*\sin(b*x + a) + \sin \\
& (b*x + a))*\sin(3*b*x + 3*a) + \sin(b*x + a)^2)*\log(\cos(b*x + a)^2 + \sin(b* \\
& x + a)^2 - 2*\cos(b*x + a) + 1) + ((b*x - (b*x + a)*\cos(2*b*x + 2*a) + a - \sin \\
& (2*b*x + 2*a))*\cos(3*b*x + 3*a)^2 + (b*x + a)*\cos(2*b*x + 2*a)^2 + (b*x + \\
& a)*\cos(b*x + a)^2 + (b*x + a)*\sin(2*b*x + 2*a)^2 + 13*(b*x + a)*\sin(b*x + \\
& a)^2 + b*x + 2*((b*x + a)*\cos(b*x + a) + \sin(b*x + a))*\cos(2*b*x + 2*a) - \\
& (b*x + a)*\cos(b*x + a) - ((b*x + a)*\sin(b*x + a) - \cos(b*x + a))*\sin(2*b*x \\
& + 2*a) - \sin(b*x + a))*\cos(3*b*x + 3*a) - ((b*x + a)*\cos(b*x + a)^2 + 13*(b \\
& *x + a)*\sin(b*x + a)^2 + 2*b*x + 2*a)*\cos(2*b*x + 2*a) + (12*(b*x + a)*\cos(\\
& b*x + a)*\sin(b*x + a) - \cos(b*x + a)^2 - \sin(b*x + a)^2)*\sin(2*b*x + 2*a) + \\
& a)*\sin(3*b*x + 3*a) - 6*((b*x + a)*\cos(b*x + a)^3 + (b*x + a)*\cos(b*x + a) \\
& *\sin(b*x + a)^2)*\sin(2*b*x + 2*a) - (6*(b*x + a)*\cos(b*x + a)^2 + b*x + a)* \\
& \sin(b*x + a) - \cos(b*x + a))*a^2*d^3/(((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2* \\
& a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\cos(3*b*x + 3*a)^2 + (\cos(b*x + a)^2 + \sin(b \\
& *x + a)^2)*\cos(2*b*x + 2*a)^2 + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - \\
& 2*\cos(2*b*x + 2*a) + 1)*\sin(3*b*x + 3*a)^2 + (\cos(b*x + a)^2 + \sin(b*x + a) \\
& ^2)*\sin(2*b*x + 2*a)^2 - 2*(\cos(2*b*x + 2*a)^2*\cos(b*x + a) + \cos(b*x + a)* \\
& \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a)*\cos(b*x + a) + \cos(b*x + a))*\cos(3* \\
& b*x + 3*a) - 2*(\cos(b*x + a)^2 + \sin(b*x + a)^2)*\cos(2*b*x + 2*a) + \cos(b*x \\
& + a)^2 - 2*(\cos(2*b*x + 2*a)^2*\sin(b*x + a) + \sin(2*b*x + 2*a)^2*\sin(b*x + \\
& a) - 2*\cos(2*b*x + 2*a)*\sin(b*x + a) + \sin(b*x + a))*\sin(3*b*x + 3*a) + \sin \\
& (b*x + a)^2)*b^3) - 2*(-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*(I*a - 1)*d^3 \\
& + (-3*I*b*c*d^2 - 3*(-I*a + 1)*d^3)*(b*x + a)^2 + (I*(b*x + a)^3*d^3 - 6*I* \\
& b*c*d^2 - 6*(-I*a - 1)*d^3 + (3*I*b*c*d^2 - 3*(I*a + 1)*d^3)*(b*x + a)^2 - \\
& (6*b*c*d^2 - (6*a - 6*I)*d^3)*(b*x + a))*\cos(3*b*x + 3*a)^2 + (6*I*(b*x + a) \\
&)^3*d^3 - 12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3 + (18*I*b*c*d^2 - \\
& 18*I*a*d^3)*(b*x + a)^2)*\cos(b*x + a)^2 + (-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 \\
& - 6*(I*a + 1)*d^3 + (-3*I*b*c*d^2 - 3*(-I*a - 1)*d^3)*(b*x + a)^2 + (6*b*c \\
& *d^2 - (6*a - 6*I)*d^3)*(b*x + a))*\sin(3*b*x + 3*a)^2 - 12*((b*x + a)^3*d^3 \\
& - 2*b*c*d^2 - 2*(b*x + a)*d^3 + 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2) \\
& *\cos(b*x + a)*\sin(b*x + a) + (-6*I*(b*x + a)^3*d^3 + 12*I*b*c*d^2 + 12*I*(b \\
& *x + a)*d^3 - 12*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2)*\sin(b \\
& x + a)^2 - (6*b*c*d^2 - (6*a + 6*I)*d^3)*(b*x + a) + (((6*I*(b*x + a)^2*d^3 \\
& + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b* \\
& c*d^2 + 12*I*a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b \\
& *c*d^2 - a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\cos(3*b*x + 3*a) + ((6*I*(b*x \\
& + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*\cos(b*x + a) - 6*((b*x \\
& + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\sin(b*x + a))*\cos(2*b*x + 2*a) \\
& + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*\cos(b*x + \\
& a) - (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^ \\
& 3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - (6*I*(b*x + a)^2*d^ \\
& 3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\sin(3*b*x + 3* \\
& a) - (6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(b*x + a) - (- \\
& 6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*\sin(b*x + a)) \\
& *\sin(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\sin \\
& (b*x + a))*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) + (((6*I*(b*x + a)^2*d^3 \\
& + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b* \\
& c*d^2 + 12*I*a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b
\end{aligned}$$

$$\begin{aligned}
& *c*d^2 - a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\cos(3*b*x + 3*a) + ((6*I*(b*x \\
& + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*\cos(b*x + a) - 6*((b*x \\
& + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\sin(b*x + a))*\cos(2*b*x + 2*a) \\
& + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*\cos(b*x + \\
& a) - (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^ \\
& d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - (6*I*(b*x + a)^2*d^ \\
& 3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\sin(3*b*x + 3* \\
& a) - (6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(b*x + a) - (- \\
& 6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*\sin(b*x + a) \\
&)*\sin(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\sin \\
& (b*x + a))*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) + ((-7*I*(b*x + a)^3*d^ \\
& 3 + 18*I*b*c*d^2 - 6*(3*I*a + 1)*d^3 + (-21*I*b*c*d^2 - 3*(-7*I*a - 1)*d^3) \\
& *(b*x + a)^2 + (6*b*c*d^2 - (6*a - 18*I)*d^3)*(b*x + a))*\cos(b*x + a) + (7* \\
& (b*x + a)^3*d^3 - 18*b*c*d^2 + (18*a - 6*I)*d^3 + (21*b*c*d^2 - (21*a - 3*I \\
&)*d^3)*(b*x + a)^2 + (6*I*b*c*d^2 - 6*(I*a + 3)*d^3)*(b*x + a))*\sin(b*x + a \\
&))*\cos(3*b*x + 3*a) + (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 - 6*(-I*a + 1)*d^3 + \\
& (3*I*b*c*d^2 - 3*(I*a - 1)*d^3)*(b*x + a)^2 + (6*b*c*d^2 - (6*a + 6*I)*d^3 \\
&)*(b*x + a))*\cos(2*b*x + 2*a) + ((-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I \\
& *a*d^3 + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3))*\cos(2*b*x + 2*a) \\
& - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3))*\sin(2*b*x + 2*a))*\cos(3*b*x + 3*a) + \\
& ((-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3))*\cos(b*x + a) + 12*(b*c* \\
& d^2 + (b*x + a)*d^3 - a*d^3))*\sin(b*x + a))*\cos(2*b*x + 2*a) + (12*I*b*c*d^2 \\
& + 12*I*(b*x + a)*d^3 - 12*I*a*d^3))*\cos(b*x + a) + (12*b*c*d^2 + 12*(b*x + \\
& a)*d^3 - 12*a*d^3 - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3))*\cos(2*b*x + 2*a) + \\
& (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3))*\sin(2*b*x + 2*a))*\sin(3* \\
& b*x + 3*a) + (12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3))*\cos(b*x + a) + (12*I*b*c \\
& *d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3))*\sin(b*x + a))*\sin(2*b*x + 2*a) - 12 \\
& *(b*c*d^2 + (b*x + a)*d^3 - a*d^3))*\sin(b*x + a))*\operatorname{dilog}(-e^{I*b*x + I*a}) + \\
& ((12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3 + (-12*I*b*c*d^2 - 12*I*(b \\
& *x + a)*d^3 + 12*I*a*d^3))*\cos(2*b*x + 2*a) + 12*(b*c*d^2 + (b*x + a)*d^3 - \\
& a*d^3))*\sin(2*b*x + 2*a))*\cos(3*b*x + 3*a) + ((12*I*b*c*d^2 + 12*I*(b*x + a) \\
& *d^3 - 12*I*a*d^3))*\cos(b*x + a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3))*\sin(\\
& b*x + a))*\cos(2*b*x + 2*a) + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d \\
& ^3))*\cos(b*x + a) - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 - 12*(b*c*d^2 \\
& + (b*x + a)*d^3 - a*d^3))*\cos(2*b*x + 2*a) - (12*I*b*c*d^2 + 12*I*(b*x + a)* \\
& d^3 - 12*I*a*d^3))*\sin(2*b*x + 2*a))*\sin(3*b*x + 3*a) - (12*(b*c*d^2 + (b*x \\
& + a)*d^3 - a*d^3))*\cos(b*x + a) - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I \\
& *a*d^3))*\sin(b*x + a))*\sin(2*b*x + 2*a) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^ \\
& 3))*\sin(b*x + a))*\operatorname{dilog}(e^{I*b*x + I*a}) + ((3*(b*x + a)^2*d^3 + 6*(b*c*d^2 \\
& - a*d^3)*(b*x + a) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\co \\
& s(2*b*x + 2*a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + \\
& a))*\sin(2*b*x + 2*a))*\cos(3*b*x + 3*a) + (3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - \\
& a*d^3)*(b*x + a))*\cos(b*x + a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I \\
& *a*d^3)*(b*x + a))*\sin(b*x + a))*\cos(2*b*x + 2*a) - 3*((b*x + a)^2*d^3 + 2* \\
& (b*c*d^2 - a*d^3)*(b*x + a))*\cos(b*x + a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c \\
& *d^2 - 6*I*a*d^3)*(b*x + a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a \\
& *d^3)*(b*x + a))*\cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3 \\
&)*(b*x + a))*\sin(2*b*x + 2*a))*\sin(3*b*x + 3*a) + ((3*I*(b*x + a)^2*d^3 + (\\
& 6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*\cos(b*x + a) - 3*((b*x + a)^2*d^3 + 2*(\\
& b*c*d^2 - a*d^3)*(b*x + a))*\sin(b*x + a))*\sin(2*b*x + 2*a) + (-3*I*(b*x + a) \\
&)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*\sin(b*x + a))*\log(\cos(b*x + \\
& a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - ((3*(b*x + a)^2*d^3 + 6*(b*c \\
& *d^2 - a*d^3)*(b*x + a) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) \\
&))*\cos(2*b*x + 2*a) - (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x \\
& + a))*\sin(2*b*x + 2*a))*\cos(3*b*x + 3*a) + (3*((b*x + a)^2*d^3 + 2*(b*c*d^ \\
& 2 - a*d^3)*(b*x + a))*\cos(b*x + a) - (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 \\
& + 6*I*a*d^3)*(b*x + a))*\sin(b*x + a))*\cos(2*b*x + 2*a) - 3*((b*x + a)^2*d^3 \\
& + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(b*x + a) - (-3*I*(b*x + a)^2*d^3 + (- \\
& 6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 -
\end{aligned}$$

$$\begin{aligned}
& 6*I*a*d^3*(b*x + a))*\cos(2*b*x + 2*a) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - \\
& a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\sin(3*b*x + 3*a) - ((-3*I*(b*x + a)^2*d^3 + \\
& (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*\cos(b*x + a) + 3*((b*x + a)^2*d^3 + \\
& 2*(b*c*d^2 - a*d^3)*(b*x + a))*\sin(b*x + a))*\sin(2*b*x + 2*a) - (3*I*(b*x + a)^2*d^3 + \\
& (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*\sin(b*x + a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - \\
& 2*\cos(b*x + a) + 1) - (12*d^3*\cos(b*x + a) + 12*I*d^3*\sin(b*x + a) + 12*(d^3*\cos(2*b*x + 2*a) + \\
& I*d^3*\sin(2*b*x + 2*a) - d^3)*\cos(3*b*x + 3*a) - 12*(d^3*\cos(b*x + a) + I*d^3*\sin(b*x + a))*\cos(2*b*x + 2*a) - \\
& (-12*I*d^3*\cos(2*b*x + 2*a) + 12*d^3*\sin(2*b*x + 2*a) + 12*I*d^3*\sin(3*b*x + 3*a) - (12*I*d^3*\cos(b*x + a) - \\
& 12*d^3*\sin(b*x + a))*\sin(2*b*x + 2*a))*\text{polylog}(3, -e^{(I*b*x + I*a)}) + (12*d^3*\cos(b*x + a) + 12*I*d^3*\sin(b*x + a) + \\
& 12*(d^3*\cos(2*b*x + 2*a) + I*d^3*\sin(2*b*x + 2*a) - d^3)*\cos(3*b*x + 3*a) - 12*(d^3*\cos(b*x + a) + I*d^3*\sin(b*x + a))*\cos(2*b*x + 2*a) + \\
& (12*I*d^3*\cos(2*b*x + 2*a) - 12*d^3*\sin(2*b*x + 2*a) - 12*I*d^3*\sin(3*b*x + 3*a) + (-12*I*d^3*\cos(b*x + a) + 12*d^3*\sin(b*x + a))*\sin(2*b*x + 2*a)) \\
& *\text{polylog}(3, e^{(I*b*x + I*a)}) - ((2*(b*x + a)^3*d^3 - 12*b*c*d^2 + (12*a - 12*I)*d^3 + (6*b*c*d^2 - (6*a - 6*I)*d^3)*(b*x + a)^2 - (-12*I*b*c*d^2 - 12*(-I*a - 1)*d^3)*(b*x + a))*\cos(3*b*x + 3*a) - (7*(b*x + a)^3*d^3 - 18*b*c*d^2 + (18*a - 6*I)*d^3 + (21*b*c*d^2 - (21*a - 3*I)*d^3)*(b*x + a)^2 + (6*I*b*c*d^2 - 6*(I*a + 3)*d^3)*(b*x + a))*\cos(b*x + a) - (7*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2 - 6*(-3*I*a - 1)*d^3 + (21*I*b*c*d^2 - 3*(7*I*a + 1)*d^3)*(b*x + a)^2 - (6*b*c*d^2 - (6*a - 18*I)*d^3)*(b*x + a))*\sin(b*x + a))*\sin(3*b*x + 3*a) - ((b*x + a)^3*d^3 - 6*b*c*d^2 + (6*a + 6*I)*d^3 + (3*b*c*d^2 - (3*a + 3*I)*d^3)*(b*x + a)^2 - (6*I*b*c*d^2 - 6*(I*a - 1)*d^3)*(b*x + a))*\sin(2*b*x + 2*a))/(2*b^3*\cos(b*x + a) + 2*I*b^3*\sin(b*x + a) + (2*b^3*\cos(2*b*x + 2*a) + 2*I*b^3*\sin(2*b*x + 2*a) - 2*b^3)*\cos(3*b*x + 3*a) - (2*b^3*\cos(b*x + a) + 2*I*b^3*\sin(b*x + a))*\cos(2*b*x + 2*a) - (-2*I*b^3*\cos(2*b*x + 2*a) + 2*b^3*\sin(2*b*x + 2*a) + 2*I*b^3)*\sin(3*b*x + 3*a) + 2*(-I*b^3*\cos(b*x + a) + b^3*\sin(b*x + a))*\sin(2*b*x + 2*a))/b
\end{aligned}$$

Fricas [C] time = 0.725879, size = 2018, normalized size = 9.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")

[Out] $-1/2*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 4*b^3*c^3 - 6*d^3*\text{polylog}(3, \cos(b*x + a) + I*\sin(b*x + a))*\sin(b*x + a) - 6*d^3*\text{polylog}(3, \cos(b*x + a) - I*\sin(b*x + a))*\sin(b*x + a) + 6*d^3*\text{polylog}(3, -\cos(b*x + a) + I*\sin(b*x + a))*\sin(b*x + a) + 6*d^3*\text{polylog}(3, -\cos(b*x + a) - I*\sin(b*x + a))*\sin(b*x + a) - 12*b*c*d^2 - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*\cos(b*x + a)^2 + 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*\cos(b*x + a)*\sin(b*x + a) - (-6*I*b*d^3*x - 6*I*b*c*d^2)*\text{dilog}(\cos(b*x + a) + I*\sin(b*x + a))*\sin(b*x + a) - (6*I*b*d^3*x + 6*I*b*c*d^2)*\text{dilog}(\cos(b*x + a) - I*\sin(b*x + a))*\sin(b*x + a) - (-6*I*b*d^3*x - 6*I*b*c*d^2)*\text{dilog}(-\cos(b*x + a) + I*\sin(b*x + a))*\sin(b*x + a) - (6*I*b*d^3*x + 6*I*b*c*d^2)*\text{dilog}(-\cos(b*x + a) - I*\sin(b*x + a))*\sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1)*\sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1)*\sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*\log(-1/2*\cos(b*x + a) + 1/2*I*\sin(b*x + a) + 1/2)*\sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*\log(-1/2*\cos(b*x + a) - 1/2*I*\sin(b*x + a) + 1/2)*\sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1)*\sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*\log(-\cos(b*x + a) - I*\sin(b*x + a) + 1)*\sin(b*x + a)$

a) + 1)*sin(b*x + a) + 12*(b^3*c^2*d - b*d^3)*x)/(b^4*sin(b*x + a))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \cos(a + bx) \cot^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*cos(b*x+a)*cot(b*x+a)**2,x)

[Out] Integral((c + d*x)**3*cos(a + b*x)*cot(a + b*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \cos(bx + a) \cot(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^3*cos(b*x + a)*cot(b*x + a)^2, x)

3.173 $\int (c + dx)^2 \cos(a + bx) \cot^2(a + bx) dx$

Optimal. Leaf size=139

$$\frac{2id^2 \text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} - \frac{2id^2 \text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3} - \frac{2d(c + dx) \cos(a + bx)}{b^2} - \frac{4d(c + dx) \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{2d^2}{b^2}$$

[Out] $(-4*d*(c + d*x)*\text{ArcTanh}[E^{(I*(a + b*x))}])/b^2 - (2*d*(c + d*x)*\text{Cos}[a + b*x])/b^2 - ((c + d*x)^2*\text{Csc}[a + b*x])/b + ((2*I)*d^2*\text{PolyLog}[2, -E^{(I*(a + b*x))}])/b^3 - ((2*I)*d^2*\text{PolyLog}[2, E^{(I*(a + b*x))}])/b^3 + (2*d^2*\text{Sin}[a + b*x])/b^3 - ((c + d*x)^2*\text{Sin}[a + b*x])/b$

Rubi [A] time = 0.14899, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {4408, 3296, 2637, 4410, 4183, 2279, 2391}

$$\frac{2id^2 \text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} - \frac{2id^2 \text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3} - \frac{2d(c + dx) \cos(a + bx)}{b^2} - \frac{4d(c + dx) \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{2d^2}{b^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^2*\text{Cos}[a + b*x]*\text{Cot}[a + b*x]^2, x]$

[Out] $(-4*d*(c + d*x)*\text{ArcTanh}[E^{(I*(a + b*x))}])/b^2 - (2*d*(c + d*x)*\text{Cos}[a + b*x])/b^2 - ((c + d*x)^2*\text{Csc}[a + b*x])/b + ((2*I)*d^2*\text{PolyLog}[2, -E^{(I*(a + b*x))}])/b^3 - ((2*I)*d^2*\text{PolyLog}[2, E^{(I*(a + b*x))}])/b^3 + (2*d^2*\text{Sin}[a + b*x])/b^3 - ((c + d*x)^2*\text{Sin}[a + b*x])/b$

Rule 4408

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(n_.)}*\text{Cot}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow -\text{Int}[(c + d*x)^m*\text{Cos}[a + b*x]^n*\text{Cot}[a + b*x]^{(p - 2)}, x] + \text{Int}[(c + d*x)^m*\text{Cos}[a + b*x]^{(n - 2)}*\text{Cot}[a + b*x]^p, x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m - 1)}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

$\text{Int}[\text{sin}[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\text{Sin}[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 4410

$\text{Int}[\text{Cot}[(a_.) + (b_.)*(x_.)]^{(p_.)}*\text{Csc}[(a_.) + (b_.)*(x_.)]^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Csc}[a + b*x]^n/(b*n), x] + \text{Dist}[(d*m)/(b*n), \text{Int}[(c + d*x)^{(m - 1)}*\text{Csc}[a + b*x]^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

Rule 4183

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(-2*(c + d*x)^m*\text{ArcTanh}[E^{(I*(e + f*x))}])/f, x] + (-\text{Dist}[(d*m)/f, \text{Int}[(c + d$

```
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \cos(a + bx) \cot^2(a + bx) dx &= - \int (c + dx)^2 \cos(a + bx) dx + \int (c + dx)^2 \cot(a + bx) \csc(a + bx) dx \\ &= - \frac{(c + dx)^2 \csc(a + bx)}{b} - \frac{(c + dx)^2 \sin(a + bx)}{b} + \frac{(2d) \int (c + dx) \csc(a + bx) dx}{b} \\ &= - \frac{4d(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{2d(c + dx) \cos(a + bx)}{b^2} - \frac{(c + dx)^2 \csc(a + bx)}{b} \\ &= - \frac{4d(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{2d(c + dx) \cos(a + bx)}{b^2} - \frac{(c + dx)^2 \csc(a + bx)}{b} \\ &= - \frac{4d(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{2d(c + dx) \cos(a + bx)}{b^2} - \frac{(c + dx)^2 \csc(a + bx)}{b} \end{aligned}$$

Mathematica [B] time = 3.95586, size = 310, normalized size = 2.23

$$-4d^2 \left(2 \tan^{-1}(\tan(a)) \tanh^{-1} \left(\cos(a) - \sin(a) \tan \left(\frac{bx}{2} \right) \right) + \frac{\sec(a) \left(i \text{PolyLog} \left(2, -e^{i(\tan^{-1}(\tan(a))+bx)} \right) - i \text{PolyLog} \left(2, e^{i(\tan^{-1}(\tan(a))+bx)} \right) \right) + \dots}{\sqrt{\sec(a)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] -(8*b*c*d*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] + 2*b^2*(c + d*x)^2*Csc[a] - 4*d^2*(2*ArcTan[Tan[a]]*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] + ((b*x + ArcTan[Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[Tan[a]])])) - Log[1 + E^(I*(b*x + ArcTan[Tan[a]])])) + I*PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]])])) - I*PolyLog[2, E^(I*(b*x + ArcTan[Tan[a]])]))*Sec[a])/Sqrt[Sec[a]^2] + 2*Cos[b*x]*(2*b*d*(c + d*x)*Cos[a] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a]) - b^2*(c + d*x)^2*Csc[a/2]*Csc[(a + b*x)/2]*Sin[(b*x)/2] + b^2*(c + d*x)^2*Sec[a/2]*Sec[(a + b*x)/2]*Sin[(b*x)/2] + 2*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] - 2*b*d*(c + d*x)*Sin[a])*Sin[b*x]/(2*b^3)
```

Maple [B] time = 0.16, size = 332, normalized size = 2.4

$$\frac{i}{2} \frac{(d^2x^2b^2 + 2b^2cdx + b^2c^2 + 2ibd^2x - 2d^2 + 2ibcd) e^{i(bx+a)}}{b^3} - \frac{i}{2} \frac{(d^2x^2b^2 + 2b^2cdx + b^2c^2 - 2ibd^2x - 2d^2 - 2ibcd) e^{-i(bx+a)}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d*x+c)^2*\cos(b*x+a)*\cot(b*x+a)^2,x)$

[Out] $\frac{1}{2}I*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*\exp(I*(b*x+a))-1/2*I*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)/b^3*\exp(-I*(b*x+a))-2*I*(d^2*x^2+2*c*d*x+c^2)*\exp(I*(b*x+a))/b/(\exp(2*I*(b*x+a))-1)-4*d/b^2*c*\operatorname{arctanh}(\exp(I*(b*x+a)))+2*d^2/b^2*\ln(1-\exp(I*(b*x+a)))*x+2*d^2/b^3*\ln(1-\exp(I*(b*x+a)))*a-2*I*d^2*\operatorname{polylog}(2,\exp(I*(b*x+a)))/b^3-2*d^2/b^2*\ln(\exp(I*(b*x+a))+1)*x-2*d^2/b^3*\ln(\exp(I*(b*x+a))+1)*a+2*I*d^2*\operatorname{polylog}(2,-\exp(I*(b*x+a)))/b^3+4*d^2/b^3*a*\operatorname{arctanh}(\exp(I*(b*x+a)))$

Maxima [B] time = 5.39042, size = 4435, normalized size = 31.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((d*x+c)^2*\cos(b*x+a)*\cot(b*x+a)^2,x, \text{algorithm}=\text{"maxima"})$

[Out] $(b^2*d^2*x^2*(-I*\cos(a) + \sin(a)) + b^2*c^2*(-I*\cos(a) + \sin(a)) - b*c*d*(2*\cos(a) + 2*I*\sin(a)) - 2*d^2*(-I*\cos(a) + \sin(a)) - (2*b^2*c*d*(I*\cos(a) - \sin(a)) + b*d^2*(2*\cos(a) + 2*I*\sin(a)))*x - ((4*b*d^2*x*(-I*\cos(a) + \sin(a)) + 4*b*c*d*(-I*\cos(a) + \sin(a)) - (-4*I*b*d^2*x - 4*I*b*c*d)*\cos(2*b*x + 3*a) - 4*(b*d^2*x + b*c*d)*\sin(2*b*x + 3*a))*\cos(3*b*x + 3*a) - ((4*I*b*d^2*x + 4*I*b*c*d)*\cos(b*x + a) - 4*(b*d^2*x + b*c*d)*\sin(b*x + a))*\cos(2*b*x + 3*a) + 4*(b*d^2*x*(I*\cos(a) - \sin(a)) + b*c*d*(I*\cos(a) - \sin(a)))*\cos(b*x + a) + (b*d^2*x*(4*\cos(a) + 4*I*\sin(a)) + b*c*d*(4*\cos(a) + 4*I*\sin(a)) - 4*(b*d^2*x + b*c*d)*\cos(2*b*x + 3*a) - (4*I*b*d^2*x + 4*I*b*c*d)*\sin(2*b*x + 3*a))*\sin(3*b*x + 3*a) + (4*(b*d^2*x + b*c*d)*\cos(b*x + a) - (-4*I*b*d^2*x - 4*I*b*c*d)*\sin(b*x + a))*\sin(2*b*x + 3*a) - (b*d^2*x*(4*\cos(a) + 4*I*\sin(a)) + b*c*d*(4*\cos(a) + 4*I*\sin(a)))*\sin(b*x + a))*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) - (4*b*c*d*(-I*\cos(a) + \sin(a))*\cos(b*x + a) + b*c*d*(4*\cos(a) + 4*I*\sin(a))*\sin(b*x + a) + (4*b*c*d*(I*\cos(a) - \sin(a)) - 4*I*b*c*d*\cos(2*b*x + 3*a) + 4*b*c*d*\sin(2*b*x + 3*a))*\cos(3*b*x + 3*a) + 4*(I*b*c*d*\cos(b*x + a) - b*c*d*\sin(b*x + a))*\cos(2*b*x + 3*a) - (b*c*d*(4*\cos(a) + 4*I*\sin(a)) - 4*b*c*d*\cos(2*b*x + 3*a) - 4*I*b*c*d*\sin(2*b*x + 3*a))*\sin(3*b*x + 3*a) - (4*b*c*d*\cos(b*x + a) + 4*I*b*c*d*\sin(b*x + a))*\sin(2*b*x + 3*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) - 1) - (4*b*d^2*x*(I*\cos(a) - \sin(a))*\cos(b*x + a) - b*d^2*x*(4*\cos(a) + 4*I*\sin(a))*\sin(b*x + a) + 4*(b*d^2*x*(-I*\cos(a) + \sin(a)) + I*b*d^2*x*\cos(2*b*x + 3*a) - b*d^2*x*\sin(2*b*x + 3*a))*\cos(3*b*x + 3*a) + 4*(-I*b*d^2*x*\cos(b*x + a) + b*d^2*x*\sin(b*x + a))*\cos(2*b*x + 3*a) + (b*d^2*x*(4*\cos(a) + 4*I*\sin(a)) - 4*b*d^2*x*\cos(2*b*x + 3*a) - 4*I*b*d^2*x*\sin(2*b*x + 3*a))*\sin(3*b*x + 3*a) + (4*b*d^2*x*\cos(b*x + a) + 4*I*b*d^2*x*\sin(b*x + a))*\sin(2*b*x + 3*a))*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) + ((I*b^2*d^2*x^2 + I*b^2*c^2 - 2*b*c*d - 2*I*d^2 + (2*I*b^2*c*d - 2*b*d^2)*x)*\cos(3*b*x + 3*a) + (-I*b^2*d^2*x^2 - I*b^2*c^2 + 2*b*c*d + 2*I*d^2 + (-2*I*b^2*c*d + 2*b*d^2)*x)*\cos(b*x + a) - (b^2*d^2*x^2 + b^2*c^2 + 2*I*b*c*d - 2*d^2 + 2*(b^2*c*d + I*b*d^2)*x)*\sin(3*b*x + 3*a) + (b^2*d^2*x^2 + b^2*c^2 + 2*I*b*c*d - 2*d^2 + 2*(b^2*c*d + I*b*d^2)*x)*\sin(b*x + a))*\cos(3*b*x + 4*a) + ((-6*I*b^2*d^2*x^2 - 12*I*b^2*c*d*x - 6*I*b^2*c^2 + 4*I*d^2)*\cos(b*x + 2*a) + 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*\sin(b*x + 2*a))*\cos(3*b*x + 3*a) + (I*b^2*d^2*x^2 + I*b^2*c^2 + 2*b*c*d - 2*I*d^2 + (2*I*b^2*c*d + 2*b*d^2)*x)*\cos(2*b*x + 3*a) + ((6*I*b^2*d^2*x^2 + 12*I*b^2*c*d*x + 6*I*b^2*c^2 - 4*I*d^2)*\cos(b*x + a) - 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*\sin(b*x + a))*\cos(b*x + 2*a) - (4*d^2*(-I*\cos(a) + \sin(a))*\cos(b*x + a) + d^2*(4*\cos(a) + 4*I*\sin(a))*\sin(b*x + a$

```

) + (4*d^2*(I*cos(a) - sin(a)) - 4*I*d^2*cos(2*b*x + 3*a) + 4*d^2*sin(2*b*x
+ 3*a))*cos(3*b*x + 3*a) - (-4*I*d^2*cos(b*x + a) + 4*d^2*sin(b*x + a))*co
s(2*b*x + 3*a) - (d^2*(4*cos(a) + 4*I*sin(a)) - 4*d^2*cos(2*b*x + 3*a) - 4*
I*d^2*sin(2*b*x + 3*a))*sin(3*b*x + 3*a) - 4*(d^2*cos(b*x + a) + I*d^2*sin(
b*x + a))*sin(2*b*x + 3*a))*dilog(-e^(I*b*x + I*a)) - (4*d^2*(I*cos(a) - si
n(a))*cos(b*x + a) - d^2*(4*cos(a) + 4*I*sin(a))*sin(b*x + a) + (4*d^2*(-I*
cos(a) + sin(a)) + 4*I*d^2*cos(2*b*x + 3*a) - 4*d^2*sin(2*b*x + 3*a))*cos(3
*b*x + 3*a) - (4*I*d^2*cos(b*x + a) - 4*d^2*sin(b*x + a))*cos(2*b*x + 3*a)
+ (d^2*(4*cos(a) + 4*I*sin(a)) - 4*d^2*cos(2*b*x + 3*a) - 4*I*d^2*sin(2*b*x
+ 3*a))*sin(3*b*x + 3*a) + 4*(d^2*cos(b*x + a) + I*d^2*sin(b*x + a))*sin(2
*b*x + 3*a))*dilog(e^(I*b*x + I*a)) + ((b*d^2*x*(2*cos(a) + 2*I*sin(a)) + b
*c*d*(2*cos(a) + 2*I*sin(a)) - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 3*a) + (-2*I
*b*d^2*x - 2*I*b*c*d)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + (2*(b*d^2*x + b*
c*d)*cos(b*x + a) + (2*I*b*d^2*x + 2*I*b*c*d)*sin(b*x + a))*cos(2*b*x + 3*a
) - (b*d^2*x*(2*cos(a) + 2*I*sin(a)) + b*c*d*(2*cos(a) + 2*I*sin(a)))*cos(b
*x + a) - (2*b*d^2*x*(-I*cos(a) + sin(a)) + 2*b*c*d*(-I*cos(a) + sin(a)) -
(-2*I*b*d^2*x - 2*I*b*c*d)*cos(2*b*x + 3*a) - 2*(b*d^2*x + b*c*d)*sin(2*b*x
+ 3*a))*sin(3*b*x + 3*a) + ((2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a) - 2*(b*
d^2*x + b*c*d)*sin(b*x + a))*sin(2*b*x + 3*a) - 2*(b*d^2*x*(I*cos(a) - sin(
a)) + b*c*d*(I*cos(a) - sin(a)))*sin(b*x + a))*log(cos(b*x + a)^2 + sin(b*x
+ a)^2 + 2*cos(b*x + a) + 1) - ((b*d^2*x*(2*cos(a) + 2*I*sin(a)) + b*c*d*(
2*cos(a) + 2*I*sin(a)) - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 3*a) - (2*I*b*d^2*
x + 2*I*b*c*d)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + (2*(b*d^2*x + b*c*d)*co
s(b*x + a) - (-2*I*b*d^2*x - 2*I*b*c*d)*sin(b*x + a))*cos(2*b*x + 3*a) - (b
*d^2*x*(2*cos(a) + 2*I*sin(a)) + b*c*d*(2*cos(a) + 2*I*sin(a)))*cos(b*x + a
) + (2*b*d^2*x*(I*cos(a) - sin(a)) + 2*b*c*d*(I*cos(a) - sin(a)) - (2*I*b*d
^2*x + 2*I*b*c*d)*cos(2*b*x + 3*a) + 2*(b*d^2*x + b*c*d)*sin(2*b*x + 3*a))*
sin(3*b*x + 3*a) - ((-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x + a) + 2*(b*d^2*x +
b*c*d)*sin(b*x + a))*sin(2*b*x + 3*a) + 2*(b*d^2*x*(-I*cos(a) + sin(a)) + b
*c*d*(-I*cos(a) + sin(a)))*sin(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^
2 - 2*cos(b*x + a) + 1) - ((b^2*d^2*x^2 + b^2*c^2 + 2*I*b*c*d - 2*d^2 + 2*(
b^2*c*d + I*b*d^2)*x)*cos(3*b*x + 3*a) - (b^2*d^2*x^2 + b^2*c^2 + 2*I*b*c*d
- 2*d^2 + 2*(b^2*c*d + I*b*d^2)*x)*cos(b*x + a) - (-I*b^2*d^2*x^2 - I*b^2*
c^2 + 2*b*c*d + 2*I*d^2 + (-2*I*b^2*c*d + 2*b*d^2)*x)*sin(3*b*x + 3*a) - (I
*b^2*d^2*x^2 + I*b^2*c^2 - 2*b*c*d - 2*I*d^2 + (2*I*b^2*c*d - 2*b*d^2)*x)*s
in(b*x + a))*sin(3*b*x + 4*a) + (2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2
- 2*d^2)*cos(b*x + 2*a) + (6*I*b^2*d^2*x^2 + 12*I*b^2*c*d*x + 6*I*b^2*c^2
- 4*I*d^2)*sin(b*x + 2*a))*sin(3*b*x + 3*a) - (b^2*d^2*x^2 + b^2*c^2 - 2*I*
b*c*d - 2*d^2 + 2*(b^2*c*d - I*b*d^2)*x)*sin(2*b*x + 3*a) - (2*(3*b^2*d^2*x
^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*cos(b*x + a) - (-6*I*b^2*d^2*x^2 - 12
*I*b^2*c*d*x - 6*I*b^2*c^2 + 4*I*d^2)*sin(b*x + a))*sin(b*x + 2*a))/(b^3*(2
*cos(a) + 2*I*sin(a))*cos(b*x + a) + 2*b^3*(I*cos(a) - sin(a))*sin(b*x + a)
- (b^3*(2*cos(a) + 2*I*sin(a)) - 2*b^3*cos(2*b*x + 3*a) - 2*I*b^3*sin(2*b*
x + 3*a))*cos(3*b*x + 3*a) - (2*b^3*cos(b*x + a) + 2*I*b^3*sin(b*x + a))*co
s(2*b*x + 3*a) + (2*b^3*(-I*cos(a) + sin(a)) + 2*I*b^3*cos(2*b*x + 3*a) - 2
*b^3*sin(2*b*x + 3*a))*sin(3*b*x + 3*a) + 2*(-I*b^3*cos(b*x + a) + b^3*sin(
b*x + a))*sin(2*b*x + 3*a))

```

Fricas [B] time = 0.61068, size = 1172, normalized size = 8.43

$$2b^2d^2x^2 + 4b^2cdx + 2b^2c^2 + id^2\text{Li}_2(\cos(bx+a) + i\sin(bx+a))\sin(bx+a) - id^2\text{Li}_2(\cos(bx+a) - i\sin(bx+a))\sin(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + I*d^2*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - I*d^2*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + I*d^2*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - I*d^2*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - (b*c*d - a*d^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - (b*c*d - a*d^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - (b*d^2*x + a*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - (b*d^2*x + a*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 2*d^2)/(b^3*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \cos(a + bx) \cot^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*cos(a + b*x)*cot(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \cos(bx + a) \cot(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cos(b*x + a)*cot(b*x + a)^2, x)
```

3.174 $\int (c + dx) \cos(a + bx) \cot^2(a + bx) dx$

Optimal. Leaf size=58

$$-\frac{d \cos(a + bx)}{b^2} - \frac{d \tanh^{-1}(\cos(a + bx))}{b^2} - \frac{(c + dx) \sin(a + bx)}{b} - \frac{(c + dx) \csc(a + bx)}{b}$$

[Out] $-\left(\frac{d \operatorname{ArcTanh}[\cos[a + b*x]]}{b^2}\right) - \frac{d \cos[a + b*x]}{b^2} - \frac{(c + d*x) \operatorname{Csc}[a + b*x]}{b} - \frac{(c + d*x) \operatorname{Sin}[a + b*x]}{b}$

Rubi [A] time = 0.0631534, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {4408, 3296, 2638, 4410, 3770}

$$-\frac{d \cos(a + bx)}{b^2} - \frac{d \tanh^{-1}(\cos(a + bx))}{b^2} - \frac{(c + dx) \sin(a + bx)}{b} - \frac{(c + dx) \csc(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + d*x) \operatorname{Cos}[a + b*x] \operatorname{Cot}[a + b*x]^2, x]$

[Out] $-\left(\frac{d \operatorname{ArcTanh}[\cos[a + b*x]]}{b^2}\right) - \frac{d \cos[a + b*x]}{b^2} - \frac{(c + d*x) \operatorname{Csc}[a + b*x]}{b} - \frac{(c + d*x) \operatorname{Sin}[a + b*x]}{b}$

Rule 4408

$\operatorname{Int}[\operatorname{Cos}[(a_.) + (b_.)(x_.)]^{(n_.)} \operatorname{Cot}[(a_.) + (b_.)(x_.)]^{(p_.)} ((c_.) + (d_.)(x_.))^{(m_.)}, x_Symbol] \rightarrow -\operatorname{Int}[(c + d*x)^m \operatorname{Cos}[a + b*x]^n \operatorname{Cot}[a + b*x]^{(p-2)}, x] + \operatorname{Int}[(c + d*x)^m \operatorname{Cos}[a + b*x]^{(n-2)} \operatorname{Cot}[a + b*x]^p, x] /;$ $\operatorname{FreeQ}\{a, b, c, d, m\}, x] \ \&\& \ \operatorname{IGtQ}[n, 0] \ \&\& \ \operatorname{IGtQ}[p, 0]$

Rule 3296

$\operatorname{Int}[(c_.) + (d_.)(x_.)]^{(m_.)} \operatorname{Sin}[(e_.) + (f_.)(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[(c + d*x)^m \operatorname{Cos}[e + f*x]/f, x] + \operatorname{Dist}[(d*m)/f, \operatorname{Int}[(c + d*x)^{(m-1)} \operatorname{Cos}[e + f*x], x], x] /;$ $\operatorname{FreeQ}\{c, d, e, f\}, x] \ \&\& \ \operatorname{GtQ}[m, 0]$

Rule 2638

$\operatorname{Int}[\operatorname{Sin}[(c_.) + (d_.)(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{Cos}[c + d*x]/d, x] /;$ $\operatorname{FreeQ}\{c, d\}, x]$

Rule 4410

$\operatorname{Int}[\operatorname{Cot}[(a_.) + (b_.)(x_.)]^{(p_.)} \operatorname{Csc}[(a_.) + (b_.)(x_.)]^{(n_.)} ((c_.) + (d_.)(x_.))^{(m_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(c + d*x)^m \operatorname{Csc}[a + b*x]^n / (b*n), x] + \operatorname{Dist}[(d*m)/(b*n), \operatorname{Int}[(c + d*x)^{(m-1)} \operatorname{Csc}[a + b*x]^n, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \operatorname{EqQ}[p, 1] \ \&\& \ \operatorname{GtQ}[m, 0]$

Rule 3770

$\operatorname{Int}[\operatorname{Csc}[(c_.) + (d_.)(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /;$ $\operatorname{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \int (c + dx) \cos(a + bx) \cot^2(a + bx) dx &= - \int (c + dx) \cos(a + bx) dx + \int (c + dx) \cot(a + bx) \csc(a + bx) dx \\ &= - \frac{(c + dx) \csc(a + bx)}{b} - \frac{(c + dx) \sin(a + bx)}{b} + \frac{d \int \csc(a + bx) dx}{b} + \frac{d \int \sin(a + bx) dx}{b} \\ &= - \frac{d \tanh^{-1}(\cos(a + bx))}{b^2} - \frac{d \cos(a + bx)}{b^2} - \frac{(c + dx) \csc(a + bx)}{b} - \frac{(c + dx) \sin(a + bx)}{b} \end{aligned}$$

Mathematica [A] time = 0.706704, size = 104, normalized size = 1.79

$$\frac{2bc \sin(a + bx) + 2bc \csc(a + bx) + 2bdx \sin(a + bx) + 2d \cos(a + bx) + bdx \tan\left(\frac{1}{2}(a + bx)\right) + bdx \cot\left(\frac{1}{2}(a + bx)\right)}{2b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Cos[a + b*x]*Cot[a + b*x]^2,x]

[Out] $-(2*d*\text{Cos}[a + b*x] + b*d*x*\text{Cot}[(a + b*x)/2] + 2*b*c*\text{Csc}[a + b*x] + 2*d*\text{Log}[\text{Cos}[(a + b*x)/2]] - 2*d*\text{Log}[\text{Sin}[(a + b*x)/2]] + 2*b*c*\text{Sin}[a + b*x] + 2*b*d*x*\text{Sin}[a + b*x] + b*d*x*\text{Tan}[(a + b*x)/2])/(2*b^2)$

Maple [C] time = 0.119, size = 124, normalized size = 2.1

$$\frac{\frac{i}{2}(dxb + bc + id)e^{i(bx+a)}}{b^2} - \frac{\frac{i}{2}(dxb + bc - id)e^{-i(bx+a)}}{b^2} - \frac{2ie^{i(bx+a)}(dx + c)}{b(e^{2i(bx+a)} - 1)} - \frac{d \ln(e^{i(bx+a)} + 1)}{b^2} + \frac{d \ln(e^{i(bx+a)} - 1)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x)

[Out] $\frac{1}{2}I*(d*x*b+b*c+I*d)/b^2*\exp(I*(b*x+a))-1/2*I*(d*x*b+b*c-I*d)/b^2*\exp(-I*(b*x+a))-2*I*\exp(I*(b*x+a))*(d*x+c)/b/(\exp(2*I*(b*x+a))-1)-d/b^2*\ln(\exp(I*(b*x+a))+1)+d/b^2*\ln(\exp(I*(b*x+a))-1)$

Maxima [B] time = 1.18379, size = 2849, normalized size = 49.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")

[Out] $-1/2*(2*c*(1/\sin(b*x + a) + \sin(b*x + a)) - 2*a*d*(1/\sin(b*x + a) + \sin(b*x + a))/b - (((b*x + a)*\sin(2*b*x + 2*a) - \cos(2*b*x + 2*a) + 1)*\cos(3*b*x + 3*a)^3 + (b*x - (b*x + a)*\cos(2*b*x + 2*a) + a - \sin(2*b*x + 2*a))*\sin(3*b*x + 3*a)^3 - 6*(b*x + a)*\sin(b*x + a)^3 - 2*(4*(b*x + a)*\cos(b*x + a)*\sin(2*b*x + 2*a) - (3*(b*x + a)*\sin(b*x + a) + \cos(b*x + a))*\cos(2*b*x + 2*a) + 3*(b*x + a)*\sin(b*x + a) + \cos(b*x + a))*\cos(3*b*x + 3*a)^2 - ((b*x + a)*\sin(b*x + a) + \cos(b*x + a))*\cos(2*b*x + 2*a)^2 + (8*(b*x + a)*\cos(2*b*x + 2*a)*\sin(b*x + a) + ((b*x + a)*\sin(2*b*x + 2*a) - \cos(2*b*x + 2*a) + 1)*\cos(3*b*x + 3*a) - 2*(3*(b*x + a)*\cos(b*x + a) - \sin(b*x + a))*\sin(2*b*x + 2*a)$

$$\begin{aligned}
& - 8*(b*x + a)*\sin(b*x + a))*\sin(3*b*x + 3*a)^2 - ((b*x + a)*\sin(b*x + a) + \\
& \cos(b*x + a))*\sin(2*b*x + 2*a)^2 + (12*(b*x + a)*\cos(b*x + a)*\sin(b*x + a) \\
& - (12*(b*x + a)*\cos(b*x + a)*\sin(b*x + a) + \cos(b*x + a)^2 + \sin(b*x + a)^2 \\
& + 2)*\cos(2*b*x + 2*a) + \cos(2*b*x + 2*a)^2 + \cos(b*x + a)^2 + (13*(b*x + \\
& a)*\cos(b*x + a)^2 + (b*x + a)*\sin(b*x + a)^2)*\sin(2*b*x + 2*a) + \sin(2*b*x \\
& + 2*a)^2 + \sin(b*x + a)^2 + 1)*\cos(3*b*x + 3*a) + 2*(3*(b*x + a)*\sin(b*x + \\
& a)^3 + (3*(b*x + a)*\cos(b*x + a)^2 + b*x + a)*\sin(b*x + a) + \cos(b*x + a))* \\
& \cos(2*b*x + 2*a) - ((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x \\
& + 2*a) + 1)*\cos(3*b*x + 3*a)^2 + (\cos(b*x + a)^2 + \sin(b*x + a)^2)*\cos(2*b*x \\
& + 2*a)^2 + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) \\
& + 1)*\sin(3*b*x + 3*a)^2 + (\cos(b*x + a)^2 + \sin(b*x + a)^2)*\sin(2*b*x + 2*a \\
&)^2 - 2*(\cos(2*b*x + 2*a)^2*\cos(b*x + a) + \cos(b*x + a)*\sin(2*b*x + 2*a)^2 \\
& - 2*\cos(2*b*x + 2*a)*\cos(b*x + a) + \cos(b*x + a))*\cos(3*b*x + 3*a) - 2*(\cos \\
& (b*x + a)^2 + \sin(b*x + a)^2)*\cos(2*b*x + 2*a) + \cos(b*x + a)^2 - 2*(\cos(2* \\
& b*x + 2*a)^2*\sin(b*x + a) + \sin(2*b*x + 2*a)^2*\sin(b*x + a) - 2*\cos(2*b*x + \\
& 2*a)*\sin(b*x + a) + \sin(b*x + a))*\sin(3*b*x + 3*a) + \sin(b*x + a)^2)*\log(c \\
& \cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) + ((\cos(2*b*x + 2*a)^2 \\
& + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\cos(3*b*x + 3*a)^2 + (\cos(b \\
& *x + a)^2 + \sin(b*x + a)^2)*\cos(2*b*x + 2*a)^2 + (\cos(2*b*x + 2*a)^2 + \sin(\\
& 2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1)*\sin(3*b*x + 3*a)^2 + (\cos(b*x + a) \\
& ^2 + \sin(b*x + a)^2)*\sin(2*b*x + 2*a)^2 - 2*(\cos(2*b*x + 2*a)^2*\cos(b*x + a) \\
&) + \cos(b*x + a)*\sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a)*\cos(b*x + a) + \cos \\
& (b*x + a))*\cos(3*b*x + 3*a) - 2*(\cos(b*x + a)^2 + \sin(b*x + a)^2)*\cos(2*b*x \\
& + 2*a) + \cos(b*x + a)^2 - 2*(\cos(2*b*x + 2*a)^2*\sin(b*x + a) + \sin(2*b*x + \\
& 2*a)^2*\sin(b*x + a) - 2*\cos(2*b*x + 2*a)*\sin(b*x + a) + \sin(b*x + a))*\sin(\\
& 3*b*x + 3*a) + \sin(b*x + a)^2)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos \\
& (b*x + a) + 1) + ((b*x - (b*x + a)*\cos(2*b*x + 2*a) + a - \sin(2*b*x + 2*a))* \\
& \cos(3*b*x + 3*a)^2 + (b*x + a)*\cos(2*b*x + 2*a)^2 + (b*x + a)*\cos(b*x + a)^2 \\
& + (b*x + a)*\sin(2*b*x + 2*a)^2 + 13*(b*x + a)*\sin(b*x + a)^2 + b*x + 2*((\\
& (b*x + a)*\cos(b*x + a) + \sin(b*x + a))*\cos(2*b*x + 2*a) - (b*x + a)*\cos(b*x \\
& + a) - ((b*x + a)*\sin(b*x + a) - \cos(b*x + a))*\sin(2*b*x + 2*a) - \sin(b*x \\
& + a))*\cos(3*b*x + 3*a) - ((b*x + a)*\cos(b*x + a)^2 + 13*(b*x + a)*\sin(b*x + \\
& a)^2 + 2*b*x + 2*a)*\cos(2*b*x + 2*a) + (12*(b*x + a)*\cos(b*x + a)*\sin(b*x \\
& + a) - \cos(b*x + a)^2 - \sin(b*x + a)^2)*\sin(2*b*x + 2*a) + a)*\sin(3*b*x + 3 \\
& *a) - 6*((b*x + a)*\cos(b*x + a)^3 + (b*x + a)*\cos(b*x + a)*\sin(b*x + a)^2)* \\
& \sin(2*b*x + 2*a) - (6*(b*x + a)*\cos(b*x + a)^2 + b*x + a)*\sin(b*x + a) - \co \\
& s(b*x + a))*d/(((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2* \\
& a) + 1)*\cos(3*b*x + 3*a)^2 + (\cos(b*x + a)^2 + \sin(b*x + a)^2)*\cos(2*b*x + \\
& 2*a)^2 + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 - 2*\cos(2*b*x + 2*a) + 1) \\
& *\sin(3*b*x + 3*a)^2 + (\cos(b*x + a)^2 + \sin(b*x + a)^2)*\sin(2*b*x + 2*a)^2 \\
& - 2*(\cos(2*b*x + 2*a)^2*\cos(b*x + a) + \cos(b*x + a)*\sin(2*b*x + 2*a)^2 - 2* \\
& \cos(2*b*x + 2*a)*\cos(b*x + a) + \cos(b*x + a))*\cos(3*b*x + 3*a) - 2*(\cos(b*x \\
& + a)^2 + \sin(b*x + a)^2)*\cos(2*b*x + 2*a) + \cos(b*x + a)^2 - 2*(\cos(2*b*x \\
& + 2*a)^2*\sin(b*x + a) + \sin(2*b*x + 2*a)^2*\sin(b*x + a) - 2*\cos(2*b*x + 2*a) \\
&)*\sin(b*x + a) + \sin(b*x + a))*\sin(3*b*x + 3*a) + \sin(b*x + a)^2)*b))/b
\end{aligned}$$

Fricas [A] time = 0.521035, size = 269, normalized size = 4.64

$$\frac{4bdx - 2(bdx + bc)\cos^2(bx + a) + 2d\cos(bx + a)\sin(bx + a) + d\log\left(\frac{1}{2}\cos(bx + a) + \frac{1}{2}\right)\sin(bx + a) - d\log\left(-\frac{1}{2}\cos(bx + a) + \frac{1}{2}\right)\sin(bx + a)}{2b^2\sin(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")

[Out] -1/2*(4*b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + 2*d*cos(b*x + a)*sin(b*x + a) + d*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - d*log(-1/2*cos(b*x + a)

$$+ 1/2) \sin(b*x + a) + 4*b*c)/(b^2*\sin(b*x + a))$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \cos(a + bx) \cot^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)**2,x)

[Out] Integral((c + d*x)*cos(a + b*x)*cot(a + b*x)**2, x)

Giac [B] time = 2.22117, size = 2655, normalized size = 45.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")

[Out] $\frac{1}{2}*(b*d*x*\tan(1/2*b*x)^4*\tan(1/2*a)^4 + b*c*\tan(1/2*b*x)^4*\tan(1/2*a)^4 + 6*b*d*x*\tan(1/2*b*x)^4*\tan(1/2*a)^2 + 8*b*d*x*\tan(1/2*b*x)^3*\tan(1/2*a)^3 - d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 - 2*\tan(1/2*b*x)*\tan(1/2*a) + 1))*\tan(1/2*b*x)^4*\tan(1/2*a)^3 + d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 + 2*\tan(1/2*b*x)*\tan(1/2*a) + \tan(1/2*a)^2))*\tan(1/2*b*x)^4*\tan(1/2*a)^3 + 6*b*d*x*\tan(1/2*b*x)^2*\tan(1/2*a)^4 - d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 - 2*\tan(1/2*b*x)*\tan(1/2*a) + 1))*\tan(1/2*b*x)^3*\tan(1/2*a)^4 + d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 + 2*\tan(1/2*b*x)*\tan(1/2*a) + \tan(1/2*a)^2))*\tan(1/2*b*x)^3*\tan(1/2*a)^4 + 6*b*c*\tan(1/2*b*x)^4*\tan(1/2*a)^2 + 8*b*c*\tan(1/2*b*x)^3*\tan(1/2*a)^3 - 2*d*\tan(1/2*b*x)^4*\tan(1/2*a)^3 + 6*b*c*\tan(1/2*b*x)^2*\tan(1/2*a)^4 - 2*d*\tan(1/2*b*x)^3*\tan(1/2*a)^4 + b*d*x*\tan(1/2*b*x)^4 - 8*b*d*x*\tan(1/2*b*x)^3*\tan(1/2*a) - d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 - 2*\tan(1/2*b*x)*\tan(1/2*a) + 1))*\tan(1/2*b*x)^4*\tan(1/2*a) + d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 + 2*\tan(1/2*b*x)*\tan(1/2*a) + \tan(1/2*a)^2))*\tan(1/2*b*x)^4*\tan(1/2*a) - 12*b*d*x*\tan(1/2*b*x)^2*\tan(1/2*a)^2 - 8*b*d*x*\tan(1/2*b*x)*\tan(1/2*a)^3 + b*d*x*\tan(1/2*a)^4 - d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 - 2*\tan(1/2*b*x)*\tan(1/2*a) + 1))*\tan(1/2*b*x)*\tan(1/2*a)^4 + d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 + 2*\tan(1/2*b*x)*\tan(1/2*a) + \tan(1/2*a)^2))*\tan(1/2*b*x)*\tan(1/2*a)^4 + b*c*\tan(1/2*b*x)^4 - 8*b*c*\tan(1/2*b*x)^3*\tan(1/2*a) + 2*d*\tan(1/2*b*x)^4*\tan(1/2*a) - 12*b*c*\tan(1/2*b*x)^2*\tan(1/2*a)^2 + 12*d*\tan(1/2*b*x)^3*\tan(1/2*a)^2 - 8*b*c*\tan(1/2*b*x)*\tan(1/2*a)^3 + 12*d*\tan(1/2*b*x)^2*\tan(1/2*a)^3 + b*c*\tan(1/2*a)^4 + 2*d*\tan(1/2*b*x)*\tan(1/2*a)^4 + 6*b*d*x*\tan(1/2*b*x)^2 + d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 - 2*\tan(1/2*b*x)*\tan(1/2*a) + 1))$

$$\begin{aligned}
& * \tan(1/2*b*x)^3 - d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 + 2*\tan(1/2*b*x)*\tan(1/2*a) + \tan(1/2*a)^2)) * \tan(1/2*b*x)^3 + 8*b*d*x*\tan(1/2*b*x)*\tan(1/2*a) + 6*b*d*x*\tan(1/2*a)^2 + d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 - 2*\tan(1/2*b*x)*\tan(1/2*a) + 1)) * \tan(1/2*a)^3 - d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 + 2*\tan(1/2*b*x)*\tan(1/2*a) + \tan(1/2*a)^2)) * \tan(1/2*a)^3 + 6*b*c*\tan(1/2*b*x)^2 - 2*d*\tan(1/2*b*x)^3 + 8*b*c*\tan(1/2*b*x)*\tan(1/2*a) - 12*d*\tan(1/2*b*x)^2*\tan(1/2*a) + 6*b*c*\tan(1/2*a)^2 - 12*d*\tan(1/2*b*x)*\tan(1/2*a)^2 - 2*d*\tan(1/2*a)^3 + b*d*x + d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 - 2*\tan(1/2*b*x)*\tan(1/2*a) + 1)) * \tan(1/2*b*x) - d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 + 2*\tan(1/2*b*x)*\tan(1/2*a) + \tan(1/2*a)^2)) * \tan(1/2*b*x) + d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 - 2*\tan(1/2*b*x)*\tan(1/2*a) + 1)) * \tan(1/2*a) - d*\log(4*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^3*\tan(1/2*a) + \tan(1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*b*x)^2 + 2*\tan(1/2*b*x)*\tan(1/2*a) + \tan(1/2*a)^2)) * \tan(1/2*a) + b*c + 2*d*\tan(1/2*b*x) + 2*d*\tan(1/2*a))/(b^2*\tan(1/2*b*x)^4*\tan(1/2*a)^3 + b^2*\tan(1/2*b*x)^3*\tan(1/2*a)^4 + b^2*\tan(1/2*b*x)^4*\tan(1/2*a) + b^2*\tan(1/2*b*x)*\tan(1/2*a)^4 - b^2*\tan(1/2*b*x)^3 - b^2*\tan(1/2*a)^3 - b^2*\tan(1/2*b*x) - b^2*\tan(1/2*a))
\end{aligned}$$

$$3.175 \quad \int \frac{\cos(a+bx) \cot^2(a+bx)}{c+dx} dx$$

Optimal. Leaf size=74

$$\text{CannotIntegrate}\left(\frac{\cot(a+bx) \csc(a+bx)}{c+dx}, x\right) - \frac{\cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{d} + \frac{\sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d}$$

[Out] CannotIntegrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x), x] - (Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d + (Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d

Rubi [A] time = 0.213677, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cos(a+bx) \cot^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x), x]

[Out] -((Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d) + (Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d + Defer[Int][(Cot[a + b*x]*Csc[a + b*x])/(c + d*x), x]

Rubi steps

$$\begin{aligned} \int \frac{\cos(a+bx) \cot^2(a+bx)}{c+dx} dx &= - \int \frac{\cos(a+bx)}{c+dx} dx + \int \frac{\cot(a+bx) \csc(a+bx)}{c+dx} dx \\ &= - \left(\cos\left(a - \frac{bc}{d}\right) \int \frac{\cos\left(\frac{bc}{d} + bx\right)}{c+dx} dx \right) + \sin\left(a - \frac{bc}{d}\right) \int \frac{\sin\left(\frac{bc}{d} + bx\right)}{c+dx} dx + \int \frac{\cot(a+bx) \csc(a+bx)}{c+dx} dx \\ &= - \frac{\cos\left(a - \frac{bc}{d}\right) \text{Ci}\left(\frac{bc}{d} + bx\right)}{d} + \frac{\sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d} + \int \frac{\cot(a+bx) \csc(a+bx)}{c+dx} dx \end{aligned}$$

Mathematica [A] time = 3.70494, size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx) \cot^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x), x]

[Out] Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x), x]

Maple [A] time = 0.346, size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a) (\cot(bx+a))^2}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x)`

[Out] `int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x)`

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x, algorithm="maxima")`

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(bx+a)\cot(bx+a)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x, algorithm="fricas")`

[Out] `integral(cos(b*x + a)*cot(b*x + a)^2/(d*x + c), x)`

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)*cot(b*x+a)**2/(d*x+c),x)`

[Out] `Integral(cos(a + b*x)*cot(a + b*x)**2/(c + d*x), x)`

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)\cot(bx+a)^2}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x, algorithm="giac")`

[Out] `integrate(cos(b*x + a)*cot(b*x + a)^2/(d*x + c), x)`

$$3.176 \quad \int \frac{\cos(a+bx) \cot^2(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=92

$$\text{CannotIntegrate}\left(\frac{\cot(a+bx) \csc(a+bx)}{(c+dx)^2}, x\right) + \frac{b \sin\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{d^2} + \frac{b \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d^2}$$

[Out] CannotIntegrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x] + Cos[a + b*x]/(d*(c + d*x)) + (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d^2 + (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2

Rubi [A] time = 0.256355, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cos(a+bx) \cot^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x)^2, x]

[Out] Cos[a + b*x]/(d*(c + d*x)) + (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d^2 + (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Defer[Int][(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x]

Rubi steps

$$\begin{aligned} \int \frac{\cos(a+bx) \cot^2(a+bx)}{(c+dx)^2} dx &= - \int \frac{\cos(a+bx)}{(c+dx)^2} dx + \int \frac{\cot(a+bx) \csc(a+bx)}{(c+dx)^2} dx \\ &= \frac{\cos(a+bx)}{d(c+dx)} + \frac{b \int \frac{\sin(a+bx)}{c+dx} dx}{d} + \int \frac{\cot(a+bx) \csc(a+bx)}{(c+dx)^2} dx \\ &= \frac{\cos(a+bx)}{d(c+dx)} + \frac{\left(b \cos\left(a - \frac{bc}{d}\right)\right) \int \frac{\sin\left(\frac{bc}{d} + bx\right)}{c+dx} dx}{d} + \frac{\left(b \sin\left(a - \frac{bc}{d}\right)\right) \int \frac{\cos\left(\frac{bc}{d} + bx\right)}{c+dx} dx}{d} + \int \frac{\cot(a+bx) \csc(a+bx)}{(c+dx)^2} dx \\ &= \frac{\cos(a+bx)}{d(c+dx)} + \frac{b \text{Ci}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{d^2} + \frac{b \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d^2} + \int \frac{\cot(a+bx) \csc(a+bx)}{(c+dx)^2} dx \end{aligned}$$

Mathematica [A] time = 4.00792, size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx) \cot^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x)^2, x]

[Out] Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x)^2, x]

Maple [A] time = 0.492, size = 0, normalized size = 0.

$$\int \frac{\cos (bx+a) (\cot (bx+a))^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x)

[Out] int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos (bx+a) \cot (bx+a)^2}{d^2 x^2+2 c d x+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(cos(b*x + a)*cot(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (a+bx) \cot ^2(a+bx)}{(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*cot(b*x+a)**2/(d*x+c)**2,x)

[Out] Integral(cos(a + b*x)*cot(a + b*x)**2/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (bx+a) \cot (bx+a)^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*cot(b*x + a)^2/(d*x + c)^2, x)
```

3.177 $\int (c + dx)^m \cot^3(a + bx) dx$

Optimal. Leaf size=18

Unintegrable $(\cot^3(a + bx)(c + dx)^m, x)$

[Out] Unintegrable[(c + d*x)^m*Cot[a + b*x]^3, x]

Rubi [A] time = 0.0356774, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \cot^3(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Cot[a + b*x]^3, x]

[Out] Defer[Int] [(c + d*x)^m*Cot[a + b*x]^3, x]

Rubi steps

$$\int (c + dx)^m \cot^3(a + bx) dx = \int (c + dx)^m \cot^3(a + bx) dx$$

Mathematica [A] time = 5.53587, size = 0, normalized size = 0.

$$\int (c + dx)^m \cot^3(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Cot[a + b*x]^3, x]

[Out] Integrate[(c + d*x)^m*Cot[a + b*x]^3, x]

Maple [A] time = 0.178, size = 0, normalized size = 0.

$$\int (dx + c)^m (\cot(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*cot(b*x+a)^3, x)

[Out] int((d*x+c)^m*cot(b*x+a)^3, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cot(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cot(b*x + a)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx + c)^m \cot(bx + a)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cot(b*x + a)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cot(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cot(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cot(b*x + a)^3, x)
```

3.178 $\int (c + dx)^4 \cot^3(a + bx) dx$

Optimal. Leaf size=302

$$\frac{3d^2(c + dx)^2 \text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{b^3} - \frac{6id^3(c + dx) \text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^4} - \frac{3id^3(c + dx) \text{PolyLog}\left(4, e^{2i(a+bx)}\right)}{b^4} + \frac{2id(c + dx)^4}{b^4}$$

[Out] $((-2*I)*d*(c + d*x)^3)/b^2 - (c + d*x)^4/(2*b) + ((I/5)*(c + d*x)^5)/d - (2*d*(c + d*x)^3*\text{Cot}[a + b*x])/b^2 - ((c + d*x)^4*\text{Cot}[a + b*x]^2)/(2*b) + (6*d^2*(c + d*x)^2*\text{Log}[1 - E^((2*I)*(a + b*x))])/b^3 - ((c + d*x)^4*\text{Log}[1 - E^((2*I)*(a + b*x))])/b - ((6*I)*d^3*(c + d*x)*\text{PolyLog}[2, E^((2*I)*(a + b*x))])/b^4 + ((2*I)*d*(c + d*x)^3*\text{PolyLog}[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^4*\text{PolyLog}[3, E^((2*I)*(a + b*x))])/b^5 - (3*d^2*(c + d*x)^2*\text{PolyLog}[3, E^((2*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(c + d*x)*\text{PolyLog}[4, E^((2*I)*(a + b*x))])/b^4 + (3*d^4*\text{PolyLog}[5, E^((2*I)*(a + b*x))])/(2*b^5)$

Rubi [A] time = 0.462334, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3720, 3717, 2190, 2531, 2282, 6589, 32, 6609}

$$\frac{3d^2(c + dx)^2 \text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{b^3} - \frac{6id^3(c + dx) \text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^4} - \frac{3id^3(c + dx) \text{PolyLog}\left(4, e^{2i(a+bx)}\right)}{b^4} + \frac{2id(c + dx)^4}{b^4}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^4*Cot[a + b*x]^3,x]

[Out] $((-2*I)*d*(c + d*x)^3)/b^2 - (c + d*x)^4/(2*b) + ((I/5)*(c + d*x)^5)/d - (2*d*(c + d*x)^3*\text{Cot}[a + b*x])/b^2 - ((c + d*x)^4*\text{Cot}[a + b*x]^2)/(2*b) + (6*d^2*(c + d*x)^2*\text{Log}[1 - E^((2*I)*(a + b*x))])/b^3 - ((c + d*x)^4*\text{Log}[1 - E^((2*I)*(a + b*x))])/b - ((6*I)*d^3*(c + d*x)*\text{PolyLog}[2, E^((2*I)*(a + b*x))])/b^4 + ((2*I)*d*(c + d*x)^3*\text{PolyLog}[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^4*\text{PolyLog}[3, E^((2*I)*(a + b*x))])/b^5 - (3*d^2*(c + d*x)^2*\text{PolyLog}[3, E^((2*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(c + d*x)*\text{PolyLog}[4, E^((2*I)*(a + b*x))])/b^4 + (3*d^4*\text{PolyLog}[5, E^((2*I)*(a + b*x))])/(2*b^5)$

Rule 3720

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 3717

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[(c + d*x)^m*E^(2*I*k*Pi)*E^(2*I*(e + f*x))]/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 2190

Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp[(c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x]

)^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*(a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 6609

Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int (c+dx)^4 \cot^3(a+bx) dx &= -\frac{(c+dx)^4 \cot^2(a+bx)}{2b} + \frac{(2d) \int (c+dx)^3 \cot^2(a+bx) dx}{b} - \int (c+dx)^4 \cot(a+bx) dx \\
&= \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3 \cot(a+bx)}{b^2} - \frac{(c+dx)^4 \cot^2(a+bx)}{2b} + 2i \int \frac{e^{2i(a+bx)}(c+dx)^4}{1-e^{2i(a+bx)}} dx \\
&= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3 \cot(a+bx)}{b^2} - \frac{(c+dx)^4 \cot^2(a+bx)}{2b} \\
&= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3 \cot(a+bx)}{b^2} - \frac{(c+dx)^4 \cot^2(a+bx)}{2b} \\
&= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3 \cot(a+bx)}{b^2} - \frac{(c+dx)^4 \cot^2(a+bx)}{2b} \\
&= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3 \cot(a+bx)}{b^2} - \frac{(c+dx)^4 \cot^2(a+bx)}{2b} \\
&= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3 \cot(a+bx)}{b^2} - \frac{(c+dx)^4 \cot^2(a+bx)}{2b} \\
&= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3 \cot(a+bx)}{b^2} - \frac{(c+dx)^4 \cot^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [B] time = 7.10651, size = 1534, normalized size = 5.08

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^4*Cot[a + b*x]^3,x]

[Out] $-(x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4)*\text{Cot}[a])/5 - ((c + d*x)^4*\text{Csc}[a + b*x]^2)/(2*b) + (c^2*d^2*E^{(I*a)}*\text{Csc}[a]*((2*b^3*x^3)/E^{(2*I)*a} + (3*I)*b^2*(1 - E^{(-2*I)*a})*x^2*\text{Log}[1 - E^{(-I)*(a + b*x)}]) + (3*I)*b^2*(1 - E^{(-2*I)*a})*x^2*\text{Log}[1 + E^{(-I)*(a + b*x)}]) - (6*(-1 + E^{(2*I)*a})*(b*x*\text{PolyLog}[2, -E^{(-I)*(a + b*x)}]) - I*\text{PolyLog}[3, -E^{(-I)*(a + b*x)}]))/E^{(2*I)*a} - (6*(-1 + E^{(2*I)*a})*(b*x*\text{PolyLog}[2, E^{(-I)*(a + b*x)}]) - I*\text{PolyLog}[3, E^{(-I)*(a + b*x)}]))/E^{(2*I)*a})/b^3 - (d^4*E^{(I*a)}*\text{Csc}[a]*((2*b^3*x^3)/E^{(2*I)*a} + (3*I)*b^2*(1 - E^{(-2*I)*a})*x^2*\text{Log}[1 - E^{(-I)*(a + b*x)}]) + (3*I)*b^2*(1 - E^{(-2*I)*a})*x^2*\text{Log}[1 + E^{(-I)*(a + b*x)}]) - (6*(-1 + E^{(2*I)*a})*(b*x*\text{PolyLog}[2, -E^{(-I)*(a + b*x)}]) - I*\text{PolyLog}[3, -E^{(-I)*(a + b*x)}]))/E^{(2*I)*a} - (6*(-1 + E^{(2*I)*a})*(b*x*\text{PolyLog}[2, E^{(-I)*(a + b*x)}]) - I*\text{PolyLog}[3, E^{(-I)*(a + b*x)}]))/E^{(2*I)*a})/b^5 + (c*d^3*E^{(I*a)}*\text{Csc}[a]*((b^4*x^4)/E^{(2*I)*a} + (2*I)*b^3*(1 - E^{(-2*I)*a})*x^3*\text{Log}[1 - E^{(-I)*(a + b*x)}]) + (2*I)*b^3*(1 - E^{(-2*I)*a})*x^3*\text{Log}[1 + E^{(-I)*(a + b*x)}]) - (6*(-1 + E^{(2*I)*a})*(b^2*x^2*\text{PolyLog}[2, -E^{(-I)*(a + b*x)}]) - (2*I)*b*x*\text{PolyLog}[3, -E^{(-I)*(a + b*x)}]) - 2*\text{PolyLog}[4, -E^{(-I)*(a + b*x)}]))/E^{(2*I)*a} - (6*(-1 + E^{(2*I)*a})*(b^2*x^2*\text{PolyLog}[2, E^{(-I)*(a + b*x)}]) - (2*I)*b*x*\text{PolyLog}[3, E^{(-I)*(a + b*x)}]))/E^{(2*I)*a} - (6*(-1 + E^{(2*I)*a})*(b^2*x^2*\text{PolyLog}[2, E^{(-I)*(a + b*x)}]) - (2*I)*b*x*\text{PolyLog}[3, E^{(-I)*(a + b*x)}]))/E^{(2*I)*a})/b^4 + (d^4*E^{(I*a)}*\text{Csc}[a]*((2*b^5*x^5)/E^{(2*I)*a} + (5*I)*b^4*(1 - E^{(-2*I)*a})*x^4*\text{Log}[1 - E^{(-I)*(a + b*x)}]) + (5*I)*b^4*(1 - E^{(-2*I)*a})*x^4*\text{Log}[1 + E^{(-I)*(a + b*x)}]) - (20*(-1 + E^{(2*I)*a})*(b^3*x^3*\text{PolyLog}[2, -E^{(-I)*(a + b*x)}]) - (3*I)*b^2*x^2*\text{PolyLog}[3, -E^{(-I)*(a + b*x)}]) - 6*b*x*\text{PolyLog}[4, -E^{(-I)*(a + b*x)}]) + (6*I)*\text{PolyLog}[5, -E^{(-I)*(a + b*x)}]))/E^{(2*I)*a} - (20*(-1 + E^{(2*I)*a})*(b^3*x^3*\text{PolyLog}[2, E^{(-I)*(a + b*x)}]) - (3*I)*b^2*x^2*\text{PolyLog}[3, E^{(-I)*(a + b*x)}]) - 6*b*x*\text{PolyLog}[4, E^{(-I)*(a + b*x)}]) + (6*I)*\text{PolyLog}[5, E^{(-I)*(a + b*x)}]))/E^{(2*I)*a})/(10*b^5) - (c^4*\text{Csc}[a]*(-b*x*\text{Cos}[a]) + \text{Log}[\text{Cos}[b*x]*\text{Sin}[a] + \text{Cos}[a]*\text{Sin}[b*x]]*\text{Sin}[a]))/(b*(\text{Cos}[a]^2 + \text{S$

$$\begin{aligned} & \ln[a^2]) + (6c^2d^2\text{Csc}[a](-b^x\text{Cos}[a]) + \text{Log}[\text{Cos}[b^x]\text{Sin}[a] + \text{Cos}[a] \\ & * \text{Sin}[b^x]] * \text{Sin}[a])) / (b^3(\text{Cos}[a]^2 + \text{Sin}[a]^2)) + (2\text{Csc}[a]\text{Csc}[a + b^x](c \\ & ^3d\text{Sin}[b^x] + 3c^2d^2x\text{Sin}[b^x] + 3c^2d^3x^2\text{Sin}[b^x] + d^4x^3\text{Sin}[b \\ & ^x])) / b^2 + (2c^3d\text{Csc}[a]\text{Sec}[a](b^2E^{(I\text{ArcTan}[\text{Tan}[a]])}x^2 + ((Ib^x * \\ & (-\text{Pi} + 2\text{ArcTan}[\text{Tan}[a]]) - \text{Pi})\text{Log}[1 + E^{((-2I)b^x)}] - 2(b^x + \text{ArcTan}[\text{Tan} \\ & [a]]) * \text{Log}[1 - E^{((2I)(b^x + \text{ArcTan}[\text{Tan}[a])}]]) + \text{Pi})\text{Log}[\text{Cos}[b^x]] + 2\text{ArcT} \\ & \text{an}[\text{Tan}[a]] * \text{Log}[\text{Sin}[b^x + \text{ArcTan}[\text{Tan}[a]]]]) + I\text{PolyLog}[2, E^{((2I)(b^x + \text{Ar} \\ & \text{cTan}[\text{Tan}[a])}]]) * \text{Tan}[a]) / \text{Sqrt}[1 + \text{Tan}[a]^2])) / (b^2\text{Sqrt}[\text{Sec}[a]^2(\text{Cos}[a]^2 \\ & + \text{Sin}[a]^2)]) - (6c^3d\text{Csc}[a]\text{Sec}[a](b^2E^{(I\text{ArcTan}[\text{Tan}[a]])}x^2 + ((Ib^x * \\ & (-\text{Pi} + 2\text{ArcTan}[\text{Tan}[a]]) - \text{Pi})\text{Log}[1 + E^{((-2I)b^x)}] - 2(b^x + \text{ArcTan} \\ & [\text{Tan}[a]]) * \text{Log}[1 - E^{((2I)(b^x + \text{ArcTan}[\text{Tan}[a])}]]) + \text{Pi})\text{Log}[\text{Cos}[b^x]] + 2 \\ & \text{ArcTan}[\text{Tan}[a]] * \text{Log}[\text{Sin}[b^x + \text{ArcTan}[\text{Tan}[a]]]]) + I\text{PolyLog}[2, E^{((2I)(b^x \\ & + \text{ArcTan}[\text{Tan}[a])}]]) * \text{Tan}[a]) / \text{Sqrt}[1 + \text{Tan}[a]^2])) / (b^4\text{Sqrt}[\text{Sec}[a]^2(\text{Cos}[a] \\ &]^2 + \text{Sin}[a]^2)]) \end{aligned}$$

Maple [B] time = 0.403, size = 1868, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d^x+c)^4*\cot(b^x+a)^3,x)$

[Out]
$$\begin{aligned} & -1/b^4d^4\ln(1-\exp(I(b^x+a)))x^4+1/b^5d^4\ln(1-\exp(I(b^x+a)))a^4+1/5I* \\ & d^4x^5+Ic^3d^3x^4-4/b^3c^3d^3\ln(1-\exp(I(b^x+a)))x^3+6d^4/b^5a^2\ln(\exp \\ & (I(b^x+a))-1)-12d^4/b^5a^2\ln(\exp(I(b^x+a)))x^2+6d^4/b^5a^2\ln(\exp(I(b \\ & ^x+a))+1)+6d^4/b^3c^2\ln(\exp(I(b^x+a))+1)+6d^4/b^3c^2\ln(\exp(I(b^x+a))- \\ & 1)+6d^4/b^3\ln(1-\exp(I(b^x+a)))x^2-6d^4/b^5\ln(1-\exp(I(b^x+a)))a^2+6* \\ & d^4/b^3\ln(\exp(I(b^x+a))+1)x^2-4I^2d^4/b^2x^3+8I^2d^4/b^5a^3-4/b^4c^3d^ \\ & 3\ln(1-\exp(I(b^x+a)))a^3-Ic^4x+12d^4\text{polylog}(3,-\exp(I(b^x+a)))/b^5+12 \\ & *d^4\text{polylog}(3,\exp(I(b^x+a)))/b^5-4/b^3c^3d^3\ln(\exp(I(b^x+a))+1)x^3-12I/ \\ & b^2a^2c^2d^2x+12I/b^2\text{polylog}(2,\exp(I(b^x+a)))c^2d^2x+12I/b^2\text{pol} \\ & \text{ylog}(2,-\exp(I(b^x+a)))c^2d^2x+12I/b^2c^3d^3\text{polylog}(2,-\exp(I(b^x+a))) \\ & *x^2+8I/b^3c^3d^3a^3x+12I/b^2c^3d^3\text{polylog}(2,\exp(I(b^x+a)))x^2+2*(b \\ & ^4x^4\exp(2I(b^x+a))+4b^3c^3d^3x^3\exp(2I(b^x+a))+6b^3c^2d^2x^2\exp \\ & (2I(b^x+a))+4b^3c^3d^3x\exp(2I(b^x+a))-2I^2d^4x^3\exp(2I(b^x+a))+b^3c \\ & ^4\exp(2I(b^x+a))-6I^2c^3d^3x^2\exp(2I(b^x+a))-6I^2c^2d^2x\exp(2I(b \\ & ^x+a))+2I^2d^4x^3-2I^2c^3d^3\exp(2I(b^x+a))+6I^2c^3d^3x^2+6I^2c^2d^2x+2 \\ & *I^2c^3d^3)/b^2/(\exp(2I(b^x+a))-1)^2-12/b^3d^4\text{polylog}(3,\exp(I(b^x+a)))x \\ & ^2-12/b^3c^2d^2\text{polylog}(3,\exp(I(b^x+a)))-12/b^3c^2d^2\text{polylog}(3,-\exp(I \\ & (b^x+a)))-12/b^3d^4\text{polylog}(3,-\exp(I(b^x+a)))x^2-8/5I/b^5d^4a^5-6/b^3 \\ & c^2d^2\ln(1-\exp(I(b^x+a)))x^2-1/b^4d^4\ln(\exp(I(b^x+a))+1)x^4+6/b^3c^2 \\ & *d^2a^2\ln(1-\exp(I(b^x+a)))-4/b^3c^3d^3\ln(1-\exp(I(b^x+a)))x-4/b^2c^3d^3 \\ & \ln(1-\exp(I(b^x+a)))a-4/b^3c^3d^3\ln(\exp(I(b^x+a))+1)x-24/b^3c^3d^3\text{polylo} \\ & \text{g}(3,-\exp(I(b^x+a)))x-6/b^3c^2d^2\ln(\exp(I(b^x+a))+1)x^2-24/b^3c^3d^3\text{po} \\ & \text{lylog}(3,\exp(I(b^x+a)))x+12d^3/b^3c^3\ln(1-\exp(I(b^x+a)))x+12d^3/b^4c^3 \\ & \ln(1-\exp(I(b^x+a)))a+12d^3/b^3c^3\ln(\exp(I(b^x+a))+1)x-12I^2d^4/b^4\text{pol} \\ & \text{ylog}(2,\exp(I(b^x+a)))x-12I^2d^4/b^4\text{polylog}(2,-\exp(I(b^x+a)))x-12I^2d^3 \\ & /b^4c^3\text{polylog}(2,\exp(I(b^x+a)))-12I^2d^3/b^4c^3\text{polylog}(2,-\exp(I(b^x+a)))+ \\ & 12I^2d^4/b^4a^2x-12I^2d^3/b^4c^3a^2-12I^2d^3/b^2c^3x^2-12d^3/b^4c^3a^2\ln(\\ & \exp(I(b^x+a))-1)+24d^3/b^4c^3a^2\ln(\exp(I(b^x+a)))+8I/b^4a^3c^3d^3x-2I/b^4 \\ & *d^4a^4x+4I/b^2c^3d^3\text{polylog}(2,-\exp(I(b^x+a)))+4I/b^2d^4\text{polylog}(2,- \\ & \exp(I(b^x+a)))x^3-24I/b^4d^4\text{polylog}(4,\exp(I(b^x+a)))x-24I/b^4d^4\text{p} \\ & \text{olylog}(4,-\exp(I(b^x+a)))x+4I/b^2d^4\text{polylog}(2,\exp(I(b^x+a)))x^3-24I/ \\ & b^4c^3d^3\text{polylog}(4,\exp(I(b^x+a)))-8I/b^3a^3c^2d^2-24I/b^4c^3d^3\text{poly} \\ & \text{log}(4,-\exp(I(b^x+a)))+4I/b^2a^2c^3d^3+6I/b^4c^3d^3a^4+4I/b^2c^3d^3\text{po} \\ & \text{lylog}(2,\exp(I(b^x+a)))+2I^2c^2d^2x^3+2I^2c^3d^3x^2-1/b^5d^4a^4\ln(\exp(\end{aligned}$$

$$I*(b*x+a))-1)+2/b^5*d^4*a^4*\ln(\exp(I*(b*x+a)))-8/b^2*c^3*d*a*\ln(\exp(I*(b*x+a))) - 6/b^3*c^2*d^2*a^2*\ln(\exp(I*(b*x+a))-1)+12/b^3*c^2*d^2*a^2*\ln(\exp(I*(b*x+a))) + 4/b^4*c*d^3*a^3*\ln(\exp(I*(b*x+a))-1) - 8/b^4*c*d^3*a^3*\ln(\exp(I*(b*x+a))) + 4/b^2*c^3*d*a*\ln(\exp(I*(b*x+a))-1) + 24*d^4*\text{polylog}(5, -\exp(I*(b*x+a)))/b^5 + 24*d^4*\text{polylog}(5, \exp(I*(b*x+a)))/b^5 - 1/b*c^4*\ln(\exp(I*(b*x+a))+1) - 1/b*c^4*\ln(\exp(I*(b*x+a))-1) + 2/b*c^4*\ln(\exp(I*(b*x+a))) - 24*I*d^3/b^3*c*a*x$$

Maxima [B] time = 19.4093, size = 9600, normalized size = 31.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cot(b*x+a)^3,x, algorithm="maxima")

[Out]
$$-1/2*(c^4*(1/\sin(b*x + a)^2 + \log(\sin(b*x + a)^2)) - 4*a*c^3*d*(1/\sin(b*x + a)^2 + \log(\sin(b*x + a)^2))/b + 6*a^2*c^2*d^2*(1/\sin(b*x + a)^2 + \log(\sin(b*x + a)^2))/b^2 - 4*a^3*c*d^3*(1/\sin(b*x + a)^2 + \log(\sin(b*x + a)^2))/b^3 + a^4*d^4*(1/\sin(b*x + a)^2 + \log(\sin(b*x + a)^2))/b^4 - 2*(2*(b*x + a)^5*d^4 + 40*b^3*c^3*d - 120*a*b^2*c^2*d^2 + 120*a^2*b*c*d^3 - 40*a^3*d^4 + 10*(b*c*d^3 - a*d^4)*(b*x + a)^4 + 20*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^3 + 20*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a)^2 - (10*(b*x + a)^4*d^4 - 60*b^2*c^2*d^2 + 120*a*b*c*d^3 - 60*a^2*d^4 + 40*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 40*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a) + 10*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*\cos(4*b*x + 4*a) - 20*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*\cos(2*b*x + 2*a) + (10*I*(b*x + a)^4*d^4 - 60*I*b^2*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*a^2*d^4 + (40*I*b*c*d^3 - 40*I*a*d^4)*(b*x + a)^3 + (60*I*b^2*c^2*d^2 - 120*I*a*b*c*d^3 + (60*I*a^2 - 60*I)*d^4)*(b*x + a)^2 + (40*I*b^3*c^3*d - 120*I*a*b^2*c^2*d^2 + (120*I*a^2 - 120*I)*b*c*d^3 + (-40*I*a^3 + 120*I*a)*d^4)*(b*x + a))*\sin(4*b*x + 4*a) + (-20*I*(b*x + a)^4*d^4 + 120*I*b^2*c^2*d^2 - 240*I*a*b*c*d^3 + 120*I*a^2*d^4 + (-80*I*b*c*d^3 + 80*I*a*d^4)*(b*x + a)^3 + (-120*I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 + (-120*I*a^2 + 120*I)*d^4)*(b*x + a)^2 + (-80*I*b^3*c^3*d + 240*I*a*b^2*c^2*d^2 + (-240*I*a^2 + 240*I)*b*c*d^3 + (80*I*a^3 - 240*I*a)*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) + (60*b^2*c^2*d^2 - 120*a*b*c*d^3 + 60*a^2*d^4 + 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*\cos(4*b*x + 4*a) - 120*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*\cos(2*b*x + 2*a) - (-60*I*b^2*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*a^2*d^4)*\sin(4*b*x + 4*a) - (120*I*b^2*c^2*d^2 - 240*I*a*b*c*d^3 + 120*I*a^2*d^4)*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) - 1) + (10*(b*x + a)^4*d^4 + 40*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 40*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a) + 10*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*\cos(4*b*x + 4*a) - 20*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*\cos(2*b*x + 2*a) - (-10*I*(b*x + a)^4*d^4 + (-40*I*b*c*d^3 + 40*I*a*d^4)*(b*x + a)^3 + (-60*I*b^2*c^2*d^2 + 120*I*a*b*c*d^3 + (-60*I*a^2 + 60*I)*d^4)*(b*x + a)^2 + (-40*I*b^3*c^3*d$$

$$\begin{aligned}
& d + 120Iab^2c^2d^2 + (-120Ia^2 + 120I)bc^2d^3 + (40Ia^3 - 120Ia) \\
& a^4d^4)(bx + a)\sin(4bx + 4a) - (20I(bx + a)^4d^4 + (80Ib^3c^3d^3 \\
& - 80Ia^3d^4)(bx + a)^3 + (120Ib^2c^2d^2 - 240Iab^2c^2d^3 + (120Ia^2 \\
& - 120I)d^4)(bx + a)^2 + (80Ib^3c^3d - 240Iab^2c^2d^2 + (240Ia^2 - 240I) \\
& bc^2d^3 + (-80Ia^3 + 240Ia)d^4)(bx + a)\sin(2bx + 2a))\arctan2(\sin(bx + a), \\
& -\cos(bx + a) + 1) + 2((bx + a)^5d^4 + 5(b^2c^2d^3 - a^2d^4)(bx + a)^4 \\
& + 10(b^2c^2d^2 - 2ab^2c^2d^3 + (a^2 - 2)d^4)(bx + a)^3 + 10(b^3c^3d - 3ab^2c^2d^2 \\
& + 3(a^2 - 2)bc^2d^3 - (a^3 - 6a)d^4)(bx + a)^2 - 60(b^2c^2d^2 - 2ab^2c^2d^3 \\
& + a^2d^4)(bx + a))\cos(4bx + 4a) - (4(bx + a)^5d^4 + 40b^3c^3d - 120ab^2c^2d^2 \\
& + 120a^2bc^2d^3 - 40a^3d^4 + (20bc^2d^3 - (20a - 20I)d^4)(bx + a)^4 \\
& + (40b^2c^2d^2 - (80a - 80I)bc^2d^3 + 40(a^2 - 2Ia - 1)d^4)(bx + a)^3 \\
& + (40b^3c^3d - (120a - 120I)b^2c^2d^2 + 120(a^2 - 2Ia - 1)bc^2d^3 - (40a^3 - 120Ia^2 \\
& - 120a)d^4)(bx + a)^2 + (80Ib^3c^3d - 120(2Ia + 1)b^2c^2d^2 + (240Ia^2 + 240a) \\
& bc^2d^3 + (-80Ia^3 - 120a^2)d^4)(bx + a))\cos(2bx + 2a) + (40b^3c^3d - 120ab^2c^2d^2 \\
& + 40(bx + a)^3d^4 + 120(a^2 - 1)bc^2d^3 - 40(a^3 - 3a)d^4 + 120(bc^2d^3 - a^2d^4) \\
& (bx + a)^2 + 120(b^2c^2d^2 - 2ab^2c^2d^3 + (a^2 - 1)d^4)(bx + a) + 40(b^3c^3d - 3ab^2c^2d^2 \\
& + (bx + a)^3d^4 + 3(a^2 - 1)bc^2d^3 - (a^3 - 3a)d^4 + 3(bc^2d^3 - a^2d^4)(bx + a)^2 \\
& + 3(b^2c^2d^2 - 2ab^2c^2d^3 + (a^2 - 1)d^4)(bx + a))\cos(4bx + 4a) - 80(b^3c^3d - 3ab^2c^2d^2 \\
& + (bx + a)^3d^4 + 3(a^2 - 1)bc^2d^3 - (a^3 - 3a)d^4 + 3(bc^2d^3 - a^2d^4)(bx + a)^2 \\
& + 3(b^2c^2d^2 - 2ab^2c^2d^3 + (a^2 - 1)d^4)(bx + a))\cos(2bx + 2a) - (-40Ib^3c^3d + 120I \\
& ab^2c^2d^2 - 40I(bx + a)^3d^4 + (-120Ia^2 + 120I)bc^2d^3 + (40Ia^3 - 120Ia) \\
& d^4 + (-120Ib^2c^2d^2 + 240Iab^2c^2d^3 + (-120Ia^2 + 120I)d^4)(bx + a))\sin(4bx + 4a) \\
& - (80Ib^3c^3d - 240Iab^2c^2d^2 + 80I(bx + a)^3d^4 + (240Ia^2 - 240I)bc^2d^3 + (-80Ia^3 + 240Ia) \\
& d^4 + (240Ib^2c^2d^2 - 480Iab^2c^2d^3 + (240Ia^2 - 240I)d^4)(bx + a))\sin(2bx + 2a))\operatorname{dilog}(-e^{Ibx + Ia}) \\
& + (40b^3c^3d - 120ab^2c^2d^2 + 40(bx + a)^3d^4 + 120(a^2 - 1)bc^2d^3 - 40(a^3 - 3a) \\
& d^4 + 120(bc^2d^3 - a^2d^4)(bx + a)^2 + 120(b^2c^2d^2 - 2ab^2c^2d^3 + (a^2 - 1)d^4)(bx + a) \\
& + 40(b^3c^3d - 3ab^2c^2d^2 + (bx + a)^3d^4 + 3(a^2 - 1)bc^2d^3 - (a^3 - 3a)d^4 + 3(bc^2d^3 \\
& - a^2d^4)(bx + a)^2 + 3(b^2c^2d^2 - 2ab^2c^2d^3 + (a^2 - 1)d^4)(bx + a))\cos(4bx + 4a) \\
& - 80(b^3c^3d - 3ab^2c^2d^2 + (bx + a)^3d^4 + 3(a^2 - 1)bc^2d^3 - (a^3 - 3a)d^4 + 3(bc^2d^3 \\
& - a^2d^4)(bx + a)^2 + 3(b^2c^2d^2 - 2ab^2c^2d^3 + (a^2 - 1)d^4)(bx + a))\cos(2bx + 2a) \\
& - (-40Ib^3c^3d + 120Iab^2c^2d^2 - 40I(bx + a)^3d^4 + (-120Ia^2 + 120I)bc^2d^3 + (40Ia^3 - 120Ia) \\
& d^4 + (-120Ib^2c^2d^2 + 240Iab^2c^2d^3 + (-120Ia^2 + 120I)d^4)(bx + a))\sin(4bx + 4a) \\
& - (80Ib^3c^3d - 240Iab^2c^2d^2 + 80I(bx + a)^3d^4 + (240Ia^2 - 240I)bc^2d^3 + (-80Ia^3 + 240Ia) \\
& d^4 + (240Ib^2c^2d^2 - 480Iab^2c^2d^3 + (240Ia^2 - 240I)d^4)(bx + a))\sin(2bx + 2a))\operatorname{dilog}(e^{Ibx + Ia}) \\
& - (-5I(bx + a)^4d^4 + 30Ib^2c^2d^2 - 60Iab^2c^2d^3 + 30Ia^2d^4 + (-20Ib^2c^2d^3 + 20Ia^2d^4) \\
& (bx + a)^3 + (-30Ib^2c^2d^2 + 60Iab^2c^2d^3 + (-30Ia^2 + 30I)d^4)(bx + a)^2 + (-20Ib^3c^3d \\
& + 60Iab^2c^2d^2 + (-60Ia^2 + 60I)bc^2d^3 + (20Ia^3 - 60Ia)d^4)(bx + a) + (-5I(bx + a)^4d^4 \\
& + 30Ib^2c^2d^2 - 60Iab^2c^2d^3 + 30Ia^2d^4 + (-20Ib^2c^2d^3 + 20Ia^2d^4)(bx + a)^3 + (-30Ib^2c^2d^2 \\
& + 60Iab^2c^2d^3 + (-30Ia^2 + 30I)d^4)(bx + a)^2 + (-20Ib^3c^3d + 60Iab^2c^2d^2 + (-60Ia^2 + 60I) \\
& bc^2d^3 + (20Ia^3 - 60Ia)d^4)(bx + a))\cos(4bx + 4a) + (10I(bx + a)^4d^4 - 60Ib^2c^2d^2 \\
& + 120Iab^2c^2d^3 - 60Ia^2d^4 + (40Ib^2c^2d^3 - 40Ia^2d^4)(bx + a)^3 + (60Ib^2c^2d^2 - 120Iab^2c^2d^3 \\
& + (60Ia^2 - 60I)d^4)(bx + a)^2 + (40Ib^3c^3d - 120Iab^2c^2d^2 + (120Ia^2 - 120I)bc^2d^3 \\
& + (-40Ia^3 + 120Ia)d^4)(bx + a))\cos(2bx + 2a) + 5((bx + a)^
\end{aligned}$$

$$\begin{aligned}
& 4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x \\
& + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d \\
& - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a) \\
&)*\sin(4*b*x + 4*a) - 10*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - \\
& 6*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 \\
& + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1) \\
& *b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 \\
& + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - (-5*I*(b*x + a)^4*d^4 + 30*I*b^2* \\
& c^2*d^2 - 60*I*a*b*c*d^3 + 30*I*a^2*d^4 + (-20*I*b*c*d^3 + 20*I*a*d^4)*(b*x \\
& + a)^3 + (-30*I*b^2*c^2*d^2 + 60*I*a*b*c*d^3 + (-30*I*a^2 + 30*I)*d^4)*(b* \\
& x + a)^2 + (-20*I*b^3*c^3*d + 60*I*a*b^2*c^2*d^2 + (-60*I*a^2 + 60*I)*b*c*d \\
& ^3 + (20*I*a^3 - 60*I*a)*d^4)*(b*x + a) + (-5*I*(b*x + a)^4*d^4 + 30*I*b^2* \\
& c^2*d^2 - 60*I*a*b*c*d^3 + 30*I*a^2*d^4 + (-20*I*b*c*d^3 + 20*I*a*d^4)*(b*x \\
& + a)^3 + (-30*I*b^2*c^2*d^2 + 60*I*a*b*c*d^3 + (-30*I*a^2 + 30*I)*d^4)*(b* \\
& x + a)^2 + (-20*I*b^3*c^3*d + 60*I*a*b^2*c^2*d^2 + (-60*I*a^2 + 60*I)*b*c*d \\
& ^3 + (20*I*a^3 - 60*I*a)*d^4)*(b*x + a))*\cos(4*b*x + 4*a) + (10*I*(b*x + a) \\
& ^4*d^4 - 60*I*b^2*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*a^2*d^4 + (40*I*b*c*d^3 \\
& - 40*I*a*d^4)*(b*x + a)^3 + (60*I*b^2*c^2*d^2 - 120*I*a*b*c*d^3 + (60*I*a^2 \\
& - 60*I)*d^4)*(b*x + a)^2 + (40*I*b^3*c^3*d - 120*I*a*b^2*c^2*d^2 + (120*I* \\
& a^2 - 120*I)*b*c*d^3 + (-40*I*a^3 + 120*I*a)*d^4)*(b*x + a))*\cos(2*b*x + 2* \\
& a) + 5*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c \\
& *d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(\\
& b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - \\
& 3*a)*d^4)*(b*x + a))*\sin(4*b*x + 4*a) - 10*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 \\
& + 12*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2* \\
& d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2 \\
& *d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*\sin(2*b*x + 2*a))* \\
& \log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) - (240*I*d^4*\cos(\\
& 4*b*x + 4*a) - 480*I*d^4*\cos(2*b*x + 2*a) - 240*d^4*\sin(4*b*x + 4*a) + 480* \\
& d^4*\sin(2*b*x + 2*a) + 240*I*d^4)*\text{polylog}(5, -e^{(I*b*x + I*a)}) - (240*I*d^4 \\
& *\cos(4*b*x + 4*a) - 480*I*d^4*\cos(2*b*x + 2*a) - 240*d^4*\sin(4*b*x + 4*a) + \\
& 480*d^4*\sin(2*b*x + 2*a) + 240*I*d^4)*\text{polylog}(5, e^{(I*b*x + I*a)}) - (240*b \\
& *c*d^3 + 240*(b*x + a)*d^4 - 240*a*d^4 + 240*(b*c*d^3 + (b*x + a)*d^4 - a*d \\
& ^4)*\cos(4*b*x + 4*a) - 480*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*\cos(2*b*x + 2* \\
& a) + (240*I*b*c*d^3 + 240*I*(b*x + a)*d^4 - 240*I*a*d^4)*\sin(4*b*x + 4*a) + \\
& (-480*I*b*c*d^3 - 480*I*(b*x + a)*d^4 + 480*I*a*d^4)*\sin(2*b*x + 2*a))*\text{pol} \\
& \text{ylog}(4, -e^{(I*b*x + I*a)}) - (240*b*c*d^3 + 240*(b*x + a)*d^4 - 240*a*d^4 + \\
& 240*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*\cos(4*b*x + 4*a) - 480*(b*c*d^3 + (b* \\
& x + a)*d^4 - a*d^4)*\cos(2*b*x + 2*a) + (240*I*b*c*d^3 + 240*I*(b*x + a)*d^4 \\
& - 240*I*a*d^4)*\sin(4*b*x + 4*a) + (-480*I*b*c*d^3 - 480*I*(b*x + a)*d^4 + \\
& 480*I*a*d^4)*\sin(2*b*x + 2*a))*\text{polylog}(4, e^{(I*b*x + I*a)}) - (-120*I*b^2*c^ \\
& 2*d^2 + 240*I*a*b*c*d^3 - 120*I*(b*x + a)^2*d^4 + (-120*I*a^2 + 120*I)*d^4 \\
& + (-240*I*b*c*d^3 + 240*I*a*d^4)*(b*x + a) + (-120*I*b^2*c^2*d^2 + 240*I*a* \\
& b*c*d^3 - 120*I*(b*x + a)^2*d^4 + (-120*I*a^2 + 120*I)*d^4 + (-240*I*b*c*d^ \\
& 3 + 240*I*a*d^4)*(b*x + a))*\cos(4*b*x + 4*a) + (240*I*b^2*c^2*d^2 - 480*I*a \\
& *b*c*d^3 + 240*I*(b*x + a)^2*d^4 + (240*I*a^2 - 240*I)*d^4 + (480*I*b*c*d^3 \\
& - 480*I*a*d^4)*(b*x + a))*\cos(2*b*x + 2*a) + 120*(b^2*c^2*d^2 - 2*a*b*c*d^ \\
& 3 + (b*x + a)^2*d^4 + (a^2 - 1)*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*\sin(4* \\
& b*x + 4*a) - 240*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + (a^2 - 1)*d \\
& ^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\text{polylog}(3, -e^{(I*b*x \\
& + I*a)}) - (-120*I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 - 120*I*(b*x + a)^2*d^4 + (\\
& -120*I*a^2 + 120*I)*d^4 + (-240*I*b*c*d^3 + 240*I*a*d^4)*(b*x + a) + (-120* \\
& I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 - 120*I*(b*x + a)^2*d^4 + (-120*I*a^2 + 120 \\
& *I)*d^4 + (-240*I*b*c*d^3 + 240*I*a*d^4)*(b*x + a))*\cos(4*b*x + 4*a) + (240 \\
& *I*b^2*c^2*d^2 - 480*I*a*b*c*d^3 + 240*I*(b*x + a)^2*d^4 + (240*I*a^2 - 240 \\
& *I)*d^4 + (480*I*b*c*d^3 - 480*I*a*d^4)*(b*x + a))*\cos(2*b*x + 2*a) + 120*(\\
& b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + (a^2 - 1)*d^4 + 2*(b*c*d^3 - \\
& a*d^4)*(b*x + a))*\sin(4*b*x + 4*a) - 240*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x \\
& + a)^2*d^4 + (a^2 - 1)*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*\sin(2*b*x + 2*a
\end{aligned}$$

```

)) * polylog(3, e^(I*b*x + I*a)) - (-2*I*(b*x + a)^5*d^4 + (-10*I*b*c*d^3 + 1
0*I*a*d^4)*(b*x + a)^4 + (-20*I*b^2*c^2*d^2 + 40*I*a*b*c*d^3 + (-20*I*a^2 +
40*I)*d^4)*(b*x + a)^3 + (-20*I*b^3*c^3*d + 60*I*a*b^2*c^2*d^2 + (-60*I*a^
2 + 120*I)*b*c*d^3 + (20*I*a^3 - 120*I*a)*d^4)*(b*x + a)^2 + (120*I*b^2*c^2
*d^2 - 240*I*a*b*c*d^3 + 120*I*a^2*d^4)*(b*x + a))*sin(4*b*x + 4*a) - (4*I*
(b*x + a)^5*d^4 + 40*I*b^3*c^3*d - 120*I*a*b^2*c^2*d^2 + 120*I*a^2*b*c*d^3
- 40*I*a^3*d^4 + (20*I*b*c*d^3 - 20*(I*a + 1)*d^4)*(b*x + a)^4 + (40*I*b^2*
c^2*d^2 - 80*(I*a + 1)*b*c*d^3 + (40*I*a^2 + 80*a - 40*I)*d^4)*(b*x + a)^3
+ (40*I*b^3*c^3*d - 120*(I*a + 1)*b^2*c^2*d^2 + (120*I*a^2 + 240*a - 120*I)
*b*c*d^3 + (-40*I*a^3 - 120*a^2 + 120*I*a)*d^4)*(b*x + a)^2 - (80*b^3*c^3*d
- (240*a - 120*I)*b^2*c^2*d^2 + 240*(a^2 - I*a)*b*c*d^3 - 40*(2*a^3 - 3*I*
a^2)*d^4)*(b*x + a))*sin(2*b*x + 2*a))/(-10*I*b^4*cos(4*b*x + 4*a) + 20*I*b
^4*cos(2*b*x + 2*a) + 10*b^4*sin(4*b*x + 4*a) - 20*b^4*sin(2*b*x + 2*a) - 1
0*I*b^4))/b

```

Fricas [C] time = 0.707253, size = 3915, normalized size = 12.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^3,x, algorithm="fricas")
```

```

[Out] 1/4*(4*b^4*d^4*x^4 + 16*b^4*c*d^3*x^3 + 24*b^4*c^2*d^2*x^2 + 16*b^4*c^3*d*x
+ 4*b^4*c^4 + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d + 12*
I*b*c*d^3 - 12*I*(b^3*c^2*d^2 - b*d^4)*x + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^
3*x^2 + 4*I*b^3*c^3*d - 12*I*b*c*d^3 + 12*I*(b^3*c^2*d^2 - b*d^4)*x)*cos(2*
b*x + 2*a))*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + (4*I*b^3*d^4*x^3
+ 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d - 12*I*b*c*d^3 + 12*I*(b^3*c^2*d^2 -
b*d^4)*x + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d + 12*I*b*
c*d^3 - 12*I*(b^3*c^2*d^2 - b*d^4)*x)*cos(2*b*x + 2*a))*dilog(cos(2*b*x + 2
*a) - I*sin(2*b*x + 2*a)) + 2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 1)*b^2*c^
2*d^2 - 4*(a^3 - 3*a)*b*c*d^3 + (a^4 - 6*a^2)*d^4 - (b^4*c^4 - 4*a*b^3*c^3*
d + 6*(a^2 - 1)*b^2*c^2*d^2 - 4*(a^3 - 3*a)*b*c*d^3 + (a^4 - 6*a^2)*d^4)*co
s(2*b*x + 2*a))*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) +
2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 1)*b^2*c^2*d^2 - 4*(a^3 - 3*a)*b*c*d
^3 + (a^4 - 6*a^2)*d^4 - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 1)*b^2*c^2*d^2
- 4*(a^3 - 3*a)*b*c*d^3 + (a^4 - 6*a^2)*d^4)*cos(2*b*x + 2*a))*log(-1/2*co
s(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b^4*d^4*x^4 + 4*b^4*c*d
^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 3*a)*b*c*d^3 - (a^4 -
6*a^2)*d^4 + 6*(b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(b^4*c^3*d - 3*b^2*c*d^3)*x
- (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(
a^3 - 3*a)*b*c*d^3 - (a^4 - 6*a^2)*d^4 + 6*(b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*
(b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) + I*si
n(2*b*x + 2*a) + 1) + 2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*
a^2*b^2*c^2*d^2 + 4*(a^3 - 3*a)*b*c*d^3 - (a^4 - 6*a^2)*d^4 + 6*(b^4*c^2*d^
2 - b^2*d^4)*x^2 + 4*(b^4*c^3*d - 3*b^2*c*d^3)*x - (b^4*d^4*x^4 + 4*b^4*c*d
^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 3*a)*b*c*d^3 - (a^4 -
6*a^2)*d^4 + 6*(b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(b^4*c^3*d - 3*b^2*c*d^3)*x
)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) + 3*(d^
4*cos(2*b*x + 2*a) - d^4)*polylog(5, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))
+ 3*(d^4*cos(2*b*x + 2*a) - d^4)*polylog(5, cos(2*b*x + 2*a) - I*sin(2*b*x
+ 2*a)) + (6*I*b*d^4*x + 6*I*b*c*d^3 + (-6*I*b*d^4*x - 6*I*b*c*d^3)*cos(2*
b*x + 2*a))*polylog(4, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + (-6*I*b*d^4
*x - 6*I*b*c*d^3 + (6*I*b*d^4*x + 6*I*b*c*d^3)*cos(2*b*x + 2*a))*polylog(4,
cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x +
b^2*c^2*d^2 - d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - d^4)*cos(2

```

```
*b*x + 2*a))*polylog(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + 6*(b^2*d^4
*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b
^2*c^2*d^2 - d^4)*cos(2*b*x + 2*a))*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b
*x + 2*a)) + 8*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d
)*sin(2*b*x + 2*a))/(b^5*cos(2*b*x + 2*a) - b^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^4 \cot^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**4*cot(b*x+a)**3,x)

[Out] Integral((c + d*x)**4*cot(a + b*x)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^4 \cot(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*cot(b*x+a)^3,x, algorithm="giac")

[Out] integrate((d*x + c)^4*cot(b*x + a)^3, x)

3.179 $\int (c + dx)^3 \cot^3(a + bx) dx$

Optimal. Leaf size=256

$$\frac{3d^2(c + dx)\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3} + \frac{3id(c + dx)^2\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2} - \frac{3id^3\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^4} - \frac{3id^3\text{PolyLog}\left(4, e^{2i(a+bx)}\right)}{4b^4}$$

[Out] $\left(\frac{-3I}{2}\right)d*(c + d*x)^2/b^2 - (c + d*x)^3/(2*b) + ((I/4)*(c + d*x)^4)/d - (3*d*(c + d*x)^2*\text{Cot}[a + b*x])/(2*b^2) - ((c + d*x)^3*\text{Cot}[a + b*x]^2)/(2*b) + (3*d^2*(c + d*x)*\text{Log}[1 - E^{((2*I)*(a + b*x))}])/b^3 - ((c + d*x)^3*\text{Log}[1 - E^{((2*I)*(a + b*x))}])/b - (((3*I)/2)*d^3*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^4 + (((3*I)/2)*d*(c + d*x)^2*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^2 - (3*d^2*(c + d*x)*\text{PolyLog}[3, E^{((2*I)*(a + b*x))}])/(2*b^3) - (((3*I)/4)*d^3*\text{PolyLog}[4, E^{((2*I)*(a + b*x))}])/b^4$

Rubi [A] time = 0.368376, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3720, 3717, 2190, 2279, 2391, 32, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c + dx)\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3} + \frac{3id(c + dx)^2\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2} - \frac{3id^3\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^4} - \frac{3id^3\text{PolyLog}\left(4, e^{2i(a+bx)}\right)}{4b^4}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^3*Cot[a + b*x]^3, x]

[Out] $\left(\frac{-3I}{2}\right)d*(c + d*x)^2/b^2 - (c + d*x)^3/(2*b) + ((I/4)*(c + d*x)^4)/d - (3*d*(c + d*x)^2*\text{Cot}[a + b*x])/(2*b^2) - ((c + d*x)^3*\text{Cot}[a + b*x]^2)/(2*b) + (3*d^2*(c + d*x)*\text{Log}[1 - E^{((2*I)*(a + b*x))}])/b^3 - ((c + d*x)^3*\text{Log}[1 - E^{((2*I)*(a + b*x))}])/b - (((3*I)/2)*d^3*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^4 + (((3*I)/2)*d*(c + d*x)^2*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^2 - (3*d^2*(c + d*x)*\text{PolyLog}[3, E^{((2*I)*(a + b*x))}])/(2*b^3) - (((3*I)/4)*d^3*\text{PolyLog}[4, E^{((2*I)*(a + b*x))}])/b^4$

Rule 3720

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 3717

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^(m)*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] :> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 32

```
Int[((a_) + (b_)*(x_)^(m_)), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)
*(x_)^(m_)), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_) + (f_)*(x_)^(m_))*PolyLog[n_, (d_)*((F_)^((c_)*((a_) + (b_
)*(x_))))^(p_)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_)^(p_))]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 \cot^3(a + bx) dx &= -\frac{(c + dx)^3 \cot^2(a + bx)}{2b} + \frac{(3d) \int (c + dx)^2 \cot^2(a + bx) dx}{2b} - \int (c + dx)^3 \cot(a + bx) dx \\
&= \frac{i(c + dx)^4}{4d} - \frac{3d(c + dx)^2 \cot(a + bx)}{2b^2} - \frac{(c + dx)^3 \cot^2(a + bx)}{2b} + 2i \int \frac{e^{2i(a+bx)}(c + dx)^3}{1 - e^{2i(a+bx)}} \\
&= -\frac{3id(c + dx)^2}{2b^2} - \frac{(c + dx)^3}{2b} + \frac{i(c + dx)^4}{4d} - \frac{3d(c + dx)^2 \cot(a + bx)}{2b^2} - \frac{(c + dx)^3 \cot^2(a + bx)}{2b} \\
&= -\frac{3id(c + dx)^2}{2b^2} - \frac{(c + dx)^3}{2b} + \frac{i(c + dx)^4}{4d} - \frac{3d(c + dx)^2 \cot(a + bx)}{2b^2} - \frac{(c + dx)^3 \cot^2(a + bx)}{2b} \\
&= -\frac{3id(c + dx)^2}{2b^2} - \frac{(c + dx)^3}{2b} + \frac{i(c + dx)^4}{4d} - \frac{3d(c + dx)^2 \cot(a + bx)}{2b^2} - \frac{(c + dx)^3 \cot^2(a + bx)}{2b} \\
&= -\frac{3id(c + dx)^2}{2b^2} - \frac{(c + dx)^3}{2b} + \frac{i(c + dx)^4}{4d} - \frac{3d(c + dx)^2 \cot(a + bx)}{2b^2} - \frac{(c + dx)^3 \cot^2(a + bx)}{2b} \\
&= -\frac{3id(c + dx)^2}{2b^2} - \frac{(c + dx)^3}{2b} + \frac{i(c + dx)^4}{4d} - \frac{3d(c + dx)^2 \cot(a + bx)}{2b^2} - \frac{(c + dx)^3 \cot^2(a + bx)}{2b}
\end{aligned}$$

Mathematica [B] time = 6.88083, size = 994, normalized size = 3.88

$$\frac{\csc(a)(\log(\cos(bx) \sin(a) + \cos(a) \sin(bx)) \sin(a) - bx \cos(a))c^3}{b(\cos^2(a) + \sin^2(a))} + \frac{3d \csc(a) \sec(a) \left(b^2 e^{i \tan^{-1}(\tan(a))} x^2 + \frac{ibx(2 \tan^{-1}(t))}{\dots} \right)}{\dots}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^3*Cot[a + b*x]^3,x]

[Out] $-(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*\text{Cot}[a])/4 - ((c + d*x)^3*\text{Csc}[a + b*x]^2)/(2*b) + (c*d^2*E^{(I*a)}*\text{Csc}[a]*((2*b^3*x^3)/E^{((2*I)*a)} + (3*I)*b^2*(1 - E^{((-2*I)*a)})*x^2*\text{Log}[1 - E^{((-I)*(a + b*x))}] + (3*I)*b^2*(1 - E^{((-2*I)*a)})*x^2*\text{Log}[1 + E^{((-I)*(a + b*x))}] - (6*(-1 + E^{((2*I)*a)})*(b*x*\text{PolyLog}[2, -E^{((-I)*(a + b*x))}] - I*\text{PolyLog}[3, -E^{((-I)*(a + b*x))}]))/E^{((2*I)*a)} - (6*(-1 + E^{((2*I)*a)})*(b*x*\text{PolyLog}[2, E^{((-I)*(a + b*x))}] - I*\text{PolyLog}[3, E^{((-I)*(a + b*x))}]))/E^{((2*I)*a)})/(2*b^3) + (d^3*E^{(I*a)}*\text{Csc}[a]*((b^4*x^4)/E^{((2*I)*a)} + (2*I)*b^3*(1 - E^{((-2*I)*a)})*x^3*\text{Log}[1 - E^{((-I)*(a + b*x))}] + (2*I)*b^3*(1 - E^{((-2*I)*a)})*x^3*\text{Log}[1 + E^{((-I)*(a + b*x))}] - (6*(-1 + E^{((2*I)*a)})*(b^2*x^2*\text{PolyLog}[2, -E^{((-I)*(a + b*x))}] - (2*I)*b*x*\text{PolyLog}[3, -E^{((-I)*(a + b*x))}] - 2*\text{PolyLog}[4, -E^{((-I)*(a + b*x))}]))/E^{((2*I)*a)} - (6*(-1 + E^{((2*I)*a)})*(b^2*x^2*\text{PolyLog}[2, E^{((-I)*(a + b*x))}] - (2*I)*b*x*\text{PolyLog}[3, E^{((-I)*(a + b*x))}] - 2*\text{PolyLog}[4, E^{((-I)*(a + b*x))}]))/E^{((2*I)*a)})/(4*b^4) - (c^3*\text{Csc}[a]*(-(b*x*\text{Cos}[a]) + \text{Log}[\text{Cos}[b*x]*\text{Sin}[a] + \text{Cos}[a]*\text{Sin}[b*x]]*\text{Sin}[a]))/(b*(\text{Cos}[a]^2 + \text{Sin}[a]^2)) + (3*c*d^2*\text{Csc}[a]*(-(b*x*\text{Cos}[a]) + \text{Log}[\text{Cos}[b*x]*\text{Sin}[a] + \text{Cos}[a]*\text{Sin}[b*x]]*\text{Sin}[a]))/(b^3*(\text{Cos}[a]^2 + \text{Sin}[a]^2)) + (3*\text{Csc}[a]*\text{Csc}[a + b*x]*(c^2*d*\text{Sin}[b*x] + 2*c*d^2*x*\text{Sin}[b*x] + d^3*x^2*\text{Sin}[b*x]))/(2*b^2) + (3*c^2*d*\text{Csc}[a]*\text{Sec}[a]*(b^2*E^{(I*\text{ArcTan}[\text{Tan}[a]])}*x^2 + ((I*b*x*(-\text{Pi} + 2*\text{ArcTan}[\text{Tan}[a])) - \text{Pi}*\text{Log}[1 + E^{((-2*I)*b*x)] - 2*(b*x + \text{ArcTan}[\text{Tan}[a]))*\text{Log}[1 - E^{((2*I)*(b*x + \text{ArcTan}[\text{Tan}[a])}])}] + \text{Pi}*\text{Log}[\text{Cos}[b*x]] + 2*\text{ArcTan}[\text{Tan}[a]]*\text{Log}[\text{Sin}[b*x + \text{ArcTan}[\text{Tan}[a]]]]) + I*\text{PolyLog}[2, E^{((2*I)*(b*x + \text{ArcTan}[\text{Tan}[a])}])}))*\text{Tan}[a])/Sqrt[1 + \text{Tan}[a]^2]))/(2*b^2*Sqrt[\text{Sec}[a]^2*(\text{Cos}[a]^2 + \text{Sin}[a]^2)) - (3*d^3*\text{Csc}[a]*\text{Sec}[a]*(b^2*E^{(I*\text{ArcTan}[\text{Tan}[a])}))*x^2 + ((I*b*x*(-\text{Pi} + 2*\text{ArcTan}[\text{Tan}[a])) - \text{Pi}*\text{Log}[1 + E^{((-2*I)*b*x)] - 2*(b*x + \text{ArcTan}[\text{Tan}[a]))*\text{Log}[1 - E^{((2*I)*(b*x + \text{ArcTan}[\text{Tan}[a])}])}] + \text{Pi}*\text{Log}[\text{Cos}[b*x]] + 2*\text{ArcTan}[\text{Tan}[a]]*\text{Log}[\text{Sin}[b*x + \text{ArcTan}[\text{Tan}[a]]]]) + I*\text{PolyLog}$

$[2, E^{((2*I)*(b*x + \text{ArcTan}[\text{Tan}[a]]))}*\text{Tan}[a])/\text{Sqrt}[1 + \text{Tan}[a]^2)]/(2*b^4*\text{Sqrt}[\text{Sec}[a]^2*(\text{Cos}[a]^2 + \text{Sin}[a]^2)])$

Maple [B] time = 0.363, size = 1194, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d*x+c)^3*\cot(b*x+a)^3,x)$

[Out]
$$\begin{aligned} & -3/b*c*d^2*\ln(\exp(I*(b*x+a))+1)*x^2-3/b*c*d^2*\ln(1-\exp(I*(b*x+a)))*x^2+3/b^3*d^3*\ln(\exp(I*(b*x+a))+1)*x+3/b^3*d^3*\ln(1-\exp(I*(b*x+a)))*x+3/b^4*d^3*\ln(1-\exp(I*(b*x+a)))*a-6*I/b^4*d^3*\text{polylog}(4,\exp(I*(b*x+a)))-3*I/b^2*d^3*x^2+3/2*I/b^4*d^3*a^4-3*I/b^4*d^3*\text{polylog}(2,-\exp(I*(b*x+a)))-3*I/b^4*d^3*a^2+1/4*I*d^3*x^4+I*c*d^2*x^3-I*c^3*x+3/2*I*c^2*d*x^2-6*I*d^3*\text{polylog}(4,-\exp(I*(b*x+a)))/b^4-3/b^4*d^3*a*\ln(\exp(I*(b*x+a))-1)+6/b^4*d^3*a*\ln(\exp(I*(b*x+a)))+3/b^3*d^2*c*\ln(\exp(I*(b*x+a))+1)-6/b^3*d^2*c*\ln(\exp(I*(b*x+a)))+3/b^3*d^2*c*\ln(\exp(I*(b*x+a))-1)-1/b*d^3*\ln(1-\exp(I*(b*x+a)))*x^3-1/b^4*d^3*\ln(1-\exp(I*(b*x+a)))*a^3-1/b*d^3*\ln(\exp(I*(b*x+a))+1)*x^3+6/b^3*c*d^2*a^2*\ln(\exp(I*(b*x+a)))-3/b^3*c*d^2*a^2*\ln(\exp(I*(b*x+a))-1)-6/b^2*c^2*d*a*\ln(\exp(I*(b*x+a)))+3/b^2*c^2*d*a*\ln(\exp(I*(b*x+a))-1)+3/b^3*c*d^2*a^2*\ln(1-\exp(I*(b*x+a)))-3/b*c^2*d*\ln(\exp(I*(b*x+a))+1)*x-3/b*c^2*d*\ln(1-\exp(I*(b*x+a)))*x-3/b^2*c^2*d*\ln(1-\exp(I*(b*x+a)))*a+3*I/b^2*d^3*\text{polylog}(2,\exp(I*(b*x+a)))*x^2-6*I/b^3*d^3*a*x-4*I/b^3*a^3*c*d^2+2*I/b^3*d^3*a^3*x+3*I/b^2*c^2*d*\text{polylog}(2,\exp(I*(b*x+a)))+3*I/b^2*a^2*c^2*d+3*I/b^2*c^2*d*\text{polylog}(2,-\exp(I*(b*x+a)))+3*I/b^2*d^3*\text{polylog}(2,-\exp(I*(b*x+a)))*x^2+(2*b*d^3*x^3*\exp(2*I*(b*x+a))-3*I*d^3*x^2*\exp(2*I*(b*x+a))+6*b*c*d^2*x^2*\exp(2*I*(b*x+a))-6*I*c*d^2*x*\exp(2*I*(b*x+a))+6*b*c^2*d*x*\exp(2*I*(b*x+a))-3*I*c^2*d*\exp(2*I*(b*x+a))+3*I*d^3*x^2+2*b*c^3*\exp(2*I*(b*x+a))+6*I*c*d^2*x+3*I*c^2*d)/b^2/(\exp(2*I*(b*x+a))-1)^2-6/b^3*d^3*\text{polylog}(3,\exp(I*(b*x+a)))*x-6/b^3*d^3*\text{polylog}(3,-\exp(I*(b*x+a)))*x-6/b^3*c*d^2*\text{polylog}(3,\exp(I*(b*x+a)))-6/b^3*c*d^2*\text{polylog}(3,-\exp(I*(b*x+a)))-2/b^4*d^3*a^3*\ln(\exp(I*(b*x+a)))+1/b^4*d^3*a^3*\ln(\exp(I*(b*x+a))-1)+6*I/b*a*c^2*d*x-6*I/b^2*a^2*c*d^2*x+6*I/b^2*\text{polylog}(2,-\exp(I*(b*x+a)))*c*d^2*x+6*I/b^2*\text{polylog}(2,\exp(I*(b*x+a)))*c*d^2*x+2/b*c^3*\ln(\exp(I*(b*x+a)))-1/b*c^3*\ln(\exp(I*(b*x+a))-1)-1/b*c^3*\ln(\exp(I*(b*x+a))+1)-3*I*d^3*\text{polylog}(2,\exp(I*(b*x+a)))/b^4 \end{aligned}$$

Maxima [B] time = 6.14236, size = 5335, normalized size = 20.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((d*x+c)^3*\cot(b*x+a)^3,x, \text{algorithm}=\text{"maxima"})$

[Out]
$$\begin{aligned} & -1/2*(c^3*(1/\sin(b*x + a)^2 + \log(\sin(b*x + a)^2)) - 3*a*c^2*d*(1/\sin(b*x + a)^2 + \log(\sin(b*x + a)^2))/b + 3*a^2*c*d^2*(1/\sin(b*x + a)^2 + \log(\sin(b*x + a)^2))/b^2 - a^3*d^3*(1/\sin(b*x + a)^2 + \log(\sin(b*x + a)^2))/b^3 - 2*((b*x + a)^4*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^2*d^3 + 4*(b*c*d^2 - a*d^3))*(b*x + a)^3 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a)^2 - (4*(b*x + a)^3*d^3 - 12*b*c*d^2 + 12*a*d^3 + 12*(b*c*d^2 - a*d^3))*(b*x + a)^2 + 12*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a) + 4*((b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3))*(b*x + a)^2 + 3*(b^2*c^2*d - \end{aligned}$$

$$\begin{aligned}
& 2*a*b*c*d^2 + (a^2 - 1)*d^3*(b*x + a))*\cos(4*b*x + 4*a) - 8*((b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (4*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a)^2 + (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + (12*I*a^2 - 12*I)*d^3)*(b*x + a))*\sin(4*b*x + 4*a) + (-8*I*(b*x + a)^3*d^3 + 24*I*b*c*d^2 - 24*I*a*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a)^2 + (-24*I*b^2*c^2*d + 48*I*a*b*c*d^2 + (-24*I*a^2 + 24*I)*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) + (12*b*c*d^2 - 12*a*d^3 + 12*(b*c*d^2 - a*d^3))*\cos(4*b*x + 4*a) - 24*(b*c*d^2 - a*d^3))*\cos(2*b*x + 2*a) - (-12*I*b*c*d^2 + 12*I*a*d^3)*\sin(4*b*x + 4*a) - (24*I*b*c*d^2 - 24*I*a*d^3)*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) - 1) + (4*(b*x + a)^3*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 12*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a) + 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*\cos(4*b*x + 4*a) - 8*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - (-4*I*(b*x + a)^3*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 + (-12*I*a^2 + 12*I)*d^3)*(b*x + a))*\sin(4*b*x + 4*a) - (8*I*(b*x + a)^3*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a)^2 + (24*I*b^2*c^2*d - 48*I*a*b*c*d^2 + (24*I*a^2 - 24*I)*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) + ((b*x + a)^4*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a)^3 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a)^2 - 24*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) - (2*(b*x + a)^4*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^2*d^3 + (8*b*c*d^2 - (8*a - 8*I)*d^3)*(b*x + a)^3 + (12*b^2*c^2*d - (24*a - 24*I)*b*c*d^2 + 12*(a^2 - 2*I*a - 1)*d^3)*(b*x + a)^2 + (24*I*b^2*c^2*d - 24*(2*I*a + 1)*b*c*d^2 + (24*I*a^2 + 24*a)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (12*b^2*c^2*d - 24*a*b*c*d^2 + 12*(b*x + a)^2*d^3 + 12*(a^2 - 1)*d^3 + 24*(b*c*d^2 - a*d^3)*(b*x + a) + 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) - 24*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 + (-12*I*a^2 + 12*I)*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*\sin(4*b*x + 4*a) - (24*I*b^2*c^2*d - 48*I*a*b*c*d^2 + 24*I*(b*x + a)^2*d^3 + (24*I*a^2 - 24*I)*d^3 + (48*I*b*c*d^2 - 48*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{dilog}(e^{I*b*x + I*a}) + (12*b^2*c^2*d - 24*a*b*c*d^2 + 12*(b*x + a)^2*d^3 + 12*(a^2 - 1)*d^3 + 24*(b*c*d^2 - a*d^3)*(b*x + a) + 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) - 24*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 + (-12*I*a^2 + 12*I)*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*\sin(4*b*x + 4*a) - (24*I*b^2*c^2*d - 48*I*a*b*c*d^2 + 24*I*(b*x + a)^2*d^3 + (24*I*a^2 - 24*I)*d^3 + (48*I*b*c*d^2 - 48*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{dilog}(e^{I*b*x + I*a}) - (-2*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 6*I)*d^3)*(b*x + a) + (-2*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 6*I)*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + (4*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a)^2 + (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + (12*I*a^2 - 12*I)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + 2*((b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*\sin(4*b*x + 4*a) - 4*((b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - (-2*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 6*I)*d^3)*(b*x + a) + (-2*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(
\end{aligned}$$

b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 6*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (4*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a)^2 + (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + (12*I*a^2 - 12*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 2*((b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 4*((b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (24*d^3*cos(4*b*x + 4*a) - 48*d^3*cos(2*b*x + 2*a) + 24*I*d^3*sin(4*b*x + 4*a) - 48*I*d^3*sin(2*b*x + 2*a) + 24*d^3)*polylog(4, -e^(I*b*x + I*a)) - (24*d^3*cos(4*b*x + 4*a) - 48*d^3*cos(2*b*x + 2*a) + 24*I*d^3*sin(4*b*x + 4*a) - 48*I*d^3*sin(2*b*x + 2*a) + 24*d^3)*polylog(4, e^(I*b*x + I*a)) - (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3 + (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*cos(4*b*x + 4*a) + (48*I*b*c*d^2 + 48*I*(b*x + a)*d^3 - 48*I*a*d^3)*cos(2*b*x + 2*a) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) - 48*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x + I*a)) - (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3 + (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*cos(4*b*x + 4*a) + (48*I*b*c*d^2 + 48*I*(b*x + a)*d^3 - 48*I*a*d^3)*cos(2*b*x + 2*a) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) - 48*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) - (-I*(b*x + a)^4*d^3 + (-4*I*b*c*d^2 + 4*I*a*d^3)*(b*x + a)^3 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 12*I)*d^3)*(b*x + a)^2 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (2*I*(b*x + a)^4*d^3 + 12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^3 + (8*I*b*c*d^2 - 8*(I*a + 1)*d^3)*(b*x + a)^3 + (12*I*b^2*c^2*d - 24*(I*a + 1)*b*c*d^2 + (12*I*a^2 + 24*a - 12*I)*d^3)*(b*x + a)^2 - (24*b^2*c^2*d - (48*a - 24*I)*b*c*d^2 + 24*(a^2 - I*a)*d^3)*(b*x + a))*sin(2*b*x + 2*a))/(-4*I*b^3*cos(4*b*x + 4*a) + 8*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin(4*b*x + 4*a) - 8*b^3*sin(2*b*x + 2*a) - 4*I*b^3))/b

Fricas [C] time = 0.621331, size = 2637, normalized size = 10.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cot(b*x+a)^3,x, algorithm="fricas")

[Out] 1/8*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d + 6*I*d^3 + (6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d - 6*I*d^3))*cos(2*b*x + 2*a))*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + (6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d - 6*I*d^3 + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d + 6*I*d^3))*cos(2*b*x + 2*a))*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3)*cos(2*b*x + 2*a))*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3)*cos(2*b*x + 2*a))*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 3*a)*d^3 + 3*(b^3*c^2*d - b*d^3))*x - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 3*a)*d^3 + 3*(b^3*c^2*d - b*d^3))*x*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 3*a)*d^3 + 3*(b^3*c^2*d - b*d^3))*x - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 -

```

3*a)*d^3 + 3*(b^3*c^2*d - b*d^3)*x)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a
) - I*sin(2*b*x + 2*a) + 1) + (-3*I*d^3*cos(2*b*x + 2*a) + 3*I*d^3)*polylog
(4, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + (3*I*d^3*cos(2*b*x + 2*a) - 3*
I*d^3)*polylog(4, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 6*(b*d^3*x + b*c
*d^2 - (b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*polylog(3, cos(2*b*x + 2*a) +
I*sin(2*b*x + 2*a)) + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(2*b*x
+ 2*a))*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 12*(b^2*d^3*x^2
+ 2*b^2*c*d^2*x + b^2*c^2*d)*sin(2*b*x + 2*a))/(b^4*cos(2*b*x + 2*a) - b^4
)

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \cot^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cot(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**3*cot(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \cot(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cot(b*x + a)^3, x)
```

3.180 $\int (c + dx)^2 \cot^3(a + bx) dx$

Optimal. Leaf size=168

$$\frac{id(c + dx)\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^2} - \frac{d^2\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3} - \frac{d(c + dx)\cot(a + bx)}{b^2} + \frac{d^2 \log(\sin(a + bx))}{b^3} - \frac{(c + dx)^2 \log(\sin(a + bx))}{b^3}$$

[Out] $-\frac{(c*d*x)}{b} - \frac{d^2*x^2}{2*b} + \frac{(I/3)*(c + d*x)^3}{d} - \frac{d*(c + d*x)*\text{Cot}[a + b*x]}{b^2} - \frac{(c + d*x)^2*\text{Cot}[a + b*x]^2}{2*b} - \frac{(c + d*x)^2*\text{Log}[1 - E^{((2*I)*(a + b*x))}]}{b} + \frac{d^2*\text{Log}[\text{Sin}[a + b*x]]}{b^3} + \frac{I*d*(c + d*x)*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}]}{b^2} - \frac{d^2*\text{PolyLog}[3, E^{((2*I)*(a + b*x))}]}{2*b^3}$

Rubi [A] time = 0.265509, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3720, 3475, 3717, 2190, 2531, 2282, 6589}

$$\frac{id(c + dx)\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^2} - \frac{d^2\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3} - \frac{d(c + dx)\cot(a + bx)}{b^2} + \frac{d^2 \log(\sin(a + bx))}{b^3} - \frac{(c + dx)^2 \log(\sin(a + bx))}{b^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2*Cot[a + b*x]^3,x]

[Out] $-\frac{(c*d*x)}{b} - \frac{d^2*x^2}{2*b} + \frac{(I/3)*(c + d*x)^3}{d} - \frac{d*(c + d*x)*\text{Cot}[a + b*x]}{b^2} - \frac{(c + d*x)^2*\text{Cot}[a + b*x]^2}{2*b} - \frac{(c + d*x)^2*\text{Log}[1 - E^{((2*I)*(a + b*x))}]}{b} + \frac{d^2*\text{Log}[\text{Sin}[a + b*x]]}{b^3} + \frac{I*d*(c + d*x)*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}]}{b^2} - \frac{d^2*\text{PolyLog}[3, E^{((2*I)*(a + b*x))}]}{2*b^3}$

Rule 3720

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3717

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[(c + d*x)^m*E^(2*I*k*Pi)*E^(2*I*(e + f*x))]/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]], x]

)^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*(a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \cot^3(a + bx) dx &= -\frac{(c + dx)^2 \cot^2(a + bx)}{2b} + \frac{d \int (c + dx) \cot^2(a + bx) dx}{b} - \int (c + dx)^2 \cot(a + bx) dx \\ &= \frac{i(c + dx)^3}{3d} - \frac{d(c + dx) \cot(a + bx)}{b^2} - \frac{(c + dx)^2 \cot^2(a + bx)}{2b} + 2i \int \frac{e^{2i(a+bx)}(c + dx)^2}{1 - e^{2i(a+bx)}} dx \\ &= -\frac{cdx}{b} - \frac{d^2x^2}{2b} + \frac{i(c + dx)^3}{3d} - \frac{d(c + dx) \cot(a + bx)}{b^2} - \frac{(c + dx)^2 \cot^2(a + bx)}{2b} - \frac{(c + dx)^2}{b^2} \\ &= -\frac{cdx}{b} - \frac{d^2x^2}{2b} + \frac{i(c + dx)^3}{3d} - \frac{d(c + dx) \cot(a + bx)}{b^2} - \frac{(c + dx)^2 \cot^2(a + bx)}{2b} - \frac{(c + dx)^2}{b^2} \\ &= -\frac{cdx}{b} - \frac{d^2x^2}{2b} + \frac{i(c + dx)^3}{3d} - \frac{d(c + dx) \cot(a + bx)}{b^2} - \frac{(c + dx)^2 \cot^2(a + bx)}{2b} - \frac{(c + dx)^2}{b^2} \\ &= -\frac{cdx}{b} - \frac{d^2x^2}{2b} + \frac{i(c + dx)^3}{3d} - \frac{d(c + dx) \cot(a + bx)}{b^2} - \frac{(c + dx)^2 \cot^2(a + bx)}{2b} - \frac{(c + dx)^2}{b^2} \end{aligned}$$

Mathematica [B] time = 6.67168, size = 540, normalized size = 3.21

$$cd \csc(a) \sec(a) \left(\frac{\tan(a) \left(i \operatorname{PolyLog} \left(2, e^{2i(\tan^{-1}(\tan(a)+bx)} \right) \right) + ibx(2 \tan^{-1}(\tan(a)) - \pi) - 2(\tan^{-1}(\tan(a)+bx) \log \left(1 - e^{2i(\tan^{-1}(\tan(a)+bx)} \right) \right) + 2 \tan^{-1}(\tan(a)) \right)}{\sqrt{\tan^2(a)+1}} \right)$$

$$b^2 \sqrt{\sec^2(a) (\sin^2(a) + \cos^2(a))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^2*Cot[a + b*x]^3,x]

[Out] -(x*(3*c^2 + 3*c*d*x + d^2*x^2)*Cot[a])/3 - ((c + d*x)^2*Csc[a + b*x]^2)/(2*b) + (d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*L

```
og[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)
*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))])/E^((2*I)*a) - (6*(-1 + E
^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a +
b*x))])/E^((2*I)*a))/(6*b^3) - (c^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]
*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (d^2*Csc[a]
*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos
[a]^2 + Sin[a]^2)) + (Csc[a]*Csc[a + b*x]*(c*d*Sin[b*x] + d^2*x*Sin[b*x]))/
b^2 + (c*d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*A
rcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x]) - 2*(b*x + ArcTan[Tan[a]])*Log[
1 - E^((2*I)*(b*x + ArcTan[Tan[a]])]) + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]
*Log[Sin[b*x + ArcTan[Tan[a]]]]) + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]
]])))*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2
])])
```

Maple [B] time = 0.313, size = 635, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*cot(b*x+a)^3,x)

```
[Out] -4/3*I/b^3*a^3*d^2+1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2-1/b*d^2*ln(exp(I*(b*x
+a))+1)*x^2+1/3*I*d^2*x^3+I*c*d*x^2-I*c^2*x-1/b*d^2*ln(1-exp(I*(b*x+a)))*x^
2+2*I/b^2*a^2*c*d+2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))+2*I/b^2*polylog(2,e
xp(I*(b*x+a)))*d^2*x+2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+2*I/b^2*c*d*p
olylog(2,-exp(I*(b*x+a)))-2/b^3*d^2*ln(exp(I*(b*x+a)))+1/b^3*d^2*ln(exp(I*(
b*x+a))+1)+1/b^3*d^2*ln(exp(I*(b*x+a))-1)+2*(b*d^2*x^2*exp(2*I*(b*x+a))+2*b
*c*d*x*exp(2*I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^2*x*exp(2*I*(b*x+a))-I*c
*d*exp(2*I*(b*x+a))+I*d^2*x+I*d*c)/b^2/(exp(2*I*(b*x+a))-1)^2+4*I/b*a*c*d*x
-2/b*c*d*ln(exp(I*(b*x+a))+1)*x+2/b*c^2*ln(exp(I*(b*x+a)))-1/b*c^2*ln(exp(I
*(b*x+a))+1)-1/b*c^2*ln(exp(I*(b*x+a))-1)-2*d^2*polylog(3,-exp(I*(b*x+a)))/
b^3-2*d^2*polylog(3,exp(I*(b*x+a)))/b^3-2/b*c*d*ln(1-exp(I*(b*x+a)))*x-2/b^
2*c*d*ln(1-exp(I*(b*x+a)))*a-2*I/b^2*a^2*d^2*x+2/b^3*d^2*a^2*ln(exp(I*(b*x+
a)))-1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+2/b^2*c*d*a*ln(exp(I*(b*x+a))-1)-4/
b^2*c*d*a*ln(exp(I*(b*x+a)))
```

Maxima [B] time = 2.50918, size = 2654, normalized size = 15.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cot(b*x+a)^3,x, algorithm="maxima")

```
[Out] -1/2*(c^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) - 2*a*c*d*(1/sin(b*x + a)
)^2 + log(sin(b*x + a)^2))/b + a^2*d^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)
^2))/b^2 - 2*(2*(b*x + a)^3*d^2 + 6*(b*c*d - a*d^2)*(b*x + a)^2 + 12*b*c*d
- 12*a*d^2 - (6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2)*(b*x + a) - 6*d^2 + 6*
((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(4*b*x + 4*a) - 12
*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(2*b*x + 2*a) + (
6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a) - 6*I*d^2)*sin(4*
b*x + 4*a) + (-12*I*(b*x + a)^2*d^2 + (-24*I*b*c*d + 24*I*a*d^2)*(b*x + a)
+ 12*I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (6*
```

$$\begin{aligned}
& d^2 \cos(4bx + 4a) - 12d^2 \cos(2bx + 2a) + 6I d^2 \sin(4bx + 4a) - \\
& 12I d^2 \sin(2bx + 2a) + 6d^2 \arctan2(\sin(bx + a), \cos(bx + a) - 1) \\
& + (6(bx + a)^2 d^2 + 12(bcd - ad^2)(bx + a) + 6((bx + a)^2 d^2 + \\
& 2(bcd - ad^2)(bx + a)) \cos(4bx + 4a) - 12((bx + a)^2 d^2 + 2(b \\
& cd - ad^2)(bx + a)) \cos(2bx + 2a) - (-6I(bx + a)^2 d^2 + (-12I \\
& bcd + 12I ad^2)(bx + a)) \sin(4bx + 4a) - (12I(bx + a)^2 d^2 + (\\
& 24I bcd - 24I ad^2)(bx + a)) \sin(2bx + 2a) \arctan2(\sin(bx + a), \\
& -\cos(bx + a) + 1) + 2((bx + a)^3 d^2 + 3(bcd - ad^2)(bx + a)^2 - \\
& 6(bx + a)d^2) \cos(4bx + 4a) - (4(bx + a)^3 d^2 + (12bcd - (12a \\
& - 12I)d^2)(bx + a)^2 + 12bcd - 12ad^2 + (24I bcd - 12(2Ia + \\
& 1)d^2)(bx + a)) \cos(2bx + 2a) + (12bcd + 12(bx + a)d^2 - 12ad \\
& ^2 + 12(bcd + (bx + a)d^2 - ad^2) \cos(4bx + 4a) - 24(bcd + (bx \\
& + a)d^2 - ad^2) \cos(2bx + 2a) - (-12I bcd - 12I(bx + a)d^2 + 1 \\
& 2I ad^2) \sin(4bx + 4a) - (24I bcd + 24I(bx + a)d^2 - 24I ad^2) \\
&) \sin(2bx + 2a) \operatorname{dilog}(-e^{I(bx + a)}) + (12bcd + 12(bx + a)d^2 \\
& - 12ad^2 + 12(bcd + (bx + a)d^2 - ad^2) \cos(4bx + 4a) - 24(bcd \\
& + (bx + a)d^2 - ad^2) \cos(2bx + 2a) - (-12I bcd - 12I(bx + a) \\
&)d^2 + 12I ad^2) \sin(4bx + 4a) - (24I bcd + 24I(bx + a)d^2 - 24 \\
& I ad^2) \sin(2bx + 2a) \operatorname{dilog}(e^{I(bx + a)}) - (-3I(bx + a)^2 d^2 \\
& + (-6I bcd + 6I ad^2)(bx + a) + 3I d^2 + (-3I(bx + a)^2 d^2 + (- \\
& 6I bcd + 6I ad^2)(bx + a) + 3I d^2) \cos(4bx + 4a) + (6I(bx + \\
& a)^2 d^2 + (12I bcd - 12I ad^2)(bx + a) - 6I d^2) \cos(2bx + 2a) \\
& + 3((bx + a)^2 d^2 + 2(bcd - ad^2)(bx + a) - d^2) \sin(4bx + 4a) \\
& - 6((bx + a)^2 d^2 + 2(bcd - ad^2)(bx + a) - d^2) \sin(2bx + 2a) \\
&) \log(\cos(bx + a)^2 + \sin(bx + a)^2 + 2\cos(bx + a) + 1) - (-3I(bx + a) \\
&)^2 d^2 + (-6I bcd + 6I ad^2)(bx + a) + 3I d^2 + (-3I(bx + a)^2 d^2 \\
& + (-6I bcd + 6I ad^2)(bx + a) + 3I d^2) \cos(4bx + 4a) + (6I \\
& (bx + a)^2 d^2 + (12I bcd - 12I ad^2)(bx + a) - 6I d^2) \cos(2bx \\
& + 2a) + 3((bx + a)^2 d^2 + 2(bcd - ad^2)(bx + a) - d^2) \sin(4bx \\
& + 4a) - 6((bx + a)^2 d^2 + 2(bcd - ad^2)(bx + a) - d^2) \sin(2bx \\
& + 2a) \log(\cos(bx + a)^2 + \sin(bx + a)^2 - 2\cos(bx + a) + 1) - (-12I \\
& d^2 \cos(4bx + 4a) + 24I d^2 \cos(2bx + 2a) + 12d^2 \sin(4bx + 4a) \\
& - 24d^2 \sin(2bx + 2a) - 12I d^2) \operatorname{polylog}(3, -e^{I(bx + a)}) - (-12I \\
& d^2 \cos(4bx + 4a) + 24I d^2 \cos(2bx + 2a) + 12d^2 \sin(4bx + 4a) \\
&) - 24d^2 \sin(2bx + 2a) - 12I d^2) \operatorname{polylog}(3, e^{I(bx + a)}) - (-2I \\
& (bx + a)^3 d^2 + (-6I bcd + 6I ad^2)(bx + a)^2 + 12I(bx + a)d^2 \\
&) \sin(4bx + 4a) - (4I(bx + a)^3 d^2 + (12I bcd - 12(Ia + 1)d^2) \\
&) (bx + a)^2 + 12I bcd - 12I ad^2 - (24bcd - (24a - 12I)d^2)(bx \\
& + a) \sin(2bx + 2a) / (-6I b^2 \cos(4bx + 4a) + 12I b^2 \cos(2bx \\
& + 2a) + 6b^2 \sin(4bx + 4a) - 12b^2 \sin(2bx + 2a) - 6I b^2) / b
\end{aligned}$$

Fricas [C] time = 0.564182, size = 1581, normalized size = 9.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((dx+c)^2*cot(bx+a)^3,x, algorithm="fricas")

[Out] $1/4(4b^2d^2x^2 + 8b^2cdx + 4b^2c^2 + (-2Ibd^2x - 2Ib^2cd + (2Ibd^2x + 2Ib^2cd)\cos(2bx + 2a))\operatorname{dilog}(\cos(2bx + 2a) + I\sin(2bx + 2a)) + (2Ibd^2x + 2Ib^2cd + (-2Ibd^2x - 2Ib^2cd)\cos(2bx + 2a))\operatorname{dilog}(\cos(2bx + 2a) - I\sin(2bx + 2a)) + 2(b^2c^2 - 2ab^2cd + (a^2 - 1)d^2 - (b^2c^2 - 2ab^2cd + (a^2 - 1)d^2)\cos(2bx + 2a))\log(-1/2\cos(2bx + 2a) + 1/2I\sin(2bx + 2a) + 1/2) + 2(b^2c^2 - 2ab^2cd + (a^2 - 1)d^2 - (b^2c^2 - 2ab^2cd + (a^2 - 1)d^2)\cos(2bx + 2a))\log(-1/2\cos(2bx + 2a) - 1/2I\sin(2bx + 2a) + 1/2) + 2$

```

*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*
d*x + 2*a*b*c*d - a^2*d^2)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) + I*sin(
2*b*x + 2*a) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b
^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(2*b*x + 2*a))*log(-cos(
2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) - (d^2*cos(2*b*x + 2*a) - d^2)*polyl
og(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) - (d^2*cos(2*b*x + 2*a) - d^2)
*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 4*(b*d^2*x + b*c*d)*si
n(2*b*x + 2*a))/(b^3*cos(2*b*x + 2*a) - b^3)

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \cot^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*cot(b*x+a)**3,x)

[Out] Integral((c + d*x)**2*cot(a + b*x)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \cot^3(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cot(b*x+a)^3,x, algorithm="giac")

[Out] integrate((d*x + c)^2*cot(b*x + a)^3, x)

3.181 $\int (c + dx) \cot^3(a + bx) dx$

Optimal. Leaf size=109

$$\frac{id\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2} - \frac{d \cot(a + bx)}{2b^2} - \frac{(c + dx) \log\left(1 - e^{2i(a+bx)}\right)}{b} - \frac{(c + dx) \cot^2(a + bx)}{2b} - \frac{dx}{2b} + \frac{i(c + dx)^2}{2d}$$

[Out] $-(d*x)/(2*b) + ((I/2)*(c + d*x)^2)/d - (d*\text{Cot}[a + b*x])/(2*b^2) - ((c + d*x)*\text{Cot}[a + b*x]^2)/(2*b) - ((c + d*x)*\text{Log}[1 - E^((2*I)*(a + b*x))])/b + ((I/2)*d*\text{PolyLog}[2, E^((2*I)*(a + b*x))])/b^2$

Rubi [A] time = 0.128258, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3720, 3473, 8, 3717, 2190, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2} - \frac{d \cot(a + bx)}{2b^2} - \frac{(c + dx) \log\left(1 - e^{2i(a+bx)}\right)}{b} - \frac{(c + dx) \cot^2(a + bx)}{2b} - \frac{dx}{2b} + \frac{i(c + dx)^2}{2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)*\text{Cot}[a + b*x]^3, x]$

[Out] $-(d*x)/(2*b) + ((I/2)*(c + d*x)^2)/d - (d*\text{Cot}[a + b*x])/(2*b^2) - ((c + d*x)*\text{Cot}[a + b*x]^2)/(2*b) - ((c + d*x)*\text{Log}[1 - E^((2*I)*(a + b*x))])/b + ((I/2)*d*\text{PolyLog}[2, E^((2*I)*(a + b*x))])/b^2$

Rule 3720

$\text{Int}[(c + d*x)^m * (b * \tan[e + f*x])^n, x] \rightarrow \text{Simp}[(b * (c + d*x)^m * (b * \tan[e + f*x])^{n-1}) / (f * (n-1)), x] + (-\text{Dist}[(b * d * m) / (f * (n-1)), \text{Int}[(c + d*x)^{m-1} * (b * \tan[e + f*x])^{n-1}, x], x] - \text{Dist}[b^2, \text{Int}[(c + d*x)^m * (b * \tan[e + f*x])^{n-2}, x], x]) / ; \text{FreeQ}\{b, c, d, e, f, x\} \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{GtQ}[m, 0]$

Rule 3473

$\text{Int}[(b * \tan[c + d*x])^n, x] \rightarrow \text{Simp}[(b * (b * \tan[c + d*x])^{n-1}) / (d * (n-1)), x] - \text{Dist}[b^2, \text{Int}[(b * \tan[c + d*x])^{n-2}, x], x] / ; \text{FreeQ}\{b, c, d, x\} \ \&\& \ \text{GtQ}[n, 1]$

Rule 8

$\text{Int}[a * x, x] \rightarrow \text{Simp}[a * x, x] / ; \text{FreeQ}[a, x]$

Rule 3717

$\text{Int}[(c + d*x)^m * \tan[(e + \text{Pi} * k) + (f * x)], x] \rightarrow \text{Simp}[(I * (c + d*x)^{m+1}) / (d * (m+1)), x] - \text{Dist}[2 * I, \text{Int}[(c + d*x)^m * E^{(2 * I * k * \text{Pi})} * E^{(2 * I * (e + f * x))} / (1 + E^{(2 * I * k * \text{Pi})} * E^{(2 * I * (e + f * x))}), x], x] / ; \text{FreeQ}\{c, d, e, f, x\} \ \&\& \ \text{IntegerQ}[4 * k] \ \&\& \ \text{IGtQ}[m, 0]$

Rule 2190

$\text{Int}[(F^{(g * (e + f * x))})^n * (c + d*x)^m, x] \rightarrow \text{Simp}[(c + d*x)^m * \text{Log}[1 + (b * (F^{(g * (e + f * x))})^n) / a] / (b * f * g * n * \text{Log}[F]), x] - \text{Dist}[(d * m) / (b * f * g * n * \text{Log}[F]), \text{Int}[(c + d*x)^{m-1} * \text{Log}[1 + (b * (F^{(g * (e + f * x))})^n) / a], x]$

)^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned} \int (c + dx) \cot^3(a + bx) dx &= -\frac{(c + dx) \cot^2(a + bx)}{2b} + \frac{d \int \cot^2(a + bx) dx}{2b} - \int (c + dx) \cot(a + bx) dx \\ &= \frac{i(c + dx)^2}{2d} - \frac{d \cot(a + bx)}{2b^2} - \frac{(c + dx) \cot^2(a + bx)}{2b} + 2i \int \frac{e^{2i(a+bx)}(c + dx)}{1 - e^{2i(a+bx)}} dx - \frac{d \int 1 dx}{2b} \\ &= -\frac{dx}{2b} + \frac{i(c + dx)^2}{2d} - \frac{d \cot(a + bx)}{2b^2} - \frac{(c + dx) \cot^2(a + bx)}{2b} - \frac{(c + dx) \log(1 - e^{2i(a+bx)})}{b} + \dots \\ &= -\frac{dx}{2b} + \frac{i(c + dx)^2}{2d} - \frac{d \cot(a + bx)}{2b^2} - \frac{(c + dx) \cot^2(a + bx)}{2b} - \frac{(c + dx) \log(1 - e^{2i(a+bx)})}{b} - \dots \\ &= -\frac{dx}{2b} + \frac{i(c + dx)^2}{2d} - \frac{d \cot(a + bx)}{2b^2} - \frac{(c + dx) \cot^2(a + bx)}{2b} - \frac{(c + dx) \log(1 - e^{2i(a+bx)})}{b} + \dots \end{aligned}$$

Mathematica [B] time = 6.14359, size = 240, normalized size = 2.2

$$d \csc(a) \sec(a) \left(\frac{\tan(a) \left(i \text{PolyLog} \left(2, e^{2i(\tan^{-1}(\tan(a))+bx)} \right) + ibx(2 \tan^{-1}(\tan(a)) - \pi) - 2(\tan^{-1}(\tan(a))+bx) \log \left(1 - e^{2i(\tan^{-1}(\tan(a))+bx)} \right) + 2 \tan^{-1}(\tan(a)) \log \left(1 - e^{2i(\tan^{-1}(\tan(a))+bx)} \right) \right)}{\sqrt{\tan^2(a)+1}} \right)}{2b^2 \sqrt{\sec^2(a) (\sin^2(a) + \cos^2(a))}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)*Cot[a + b*x]^3,x]

[Out] -(d*x^2*Cot[a])/2 - (d*x*Csc[a + b*x]^2)/(2*b) - (c*(Cot[a + b*x]^2 + 2*Log[Cos[a + b*x]] + 2*Log[Tan[a + b*x]]))/(2*b) + (d*Csc[a]*Csc[a + b*x]*Sin[b*x])/(2*b^2) + (d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]])]) + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]]) + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]])])]*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2))]

Maple [B] time = 0.161, size = 281, normalized size = 2.6

$$-icx + \frac{2 idax}{b} + \frac{2 bdx e^{2i(bx+a)} + 2 bce^{2i(bx+a)} - ide^{2i(bx+a)} + id}{b^2 (e^{2i(bx+a)} - 1)^2} - \frac{c \ln(e^{i(bx+a)} - 1)}{b} + 2 \frac{c \ln(e^{i(bx+a)})}{b} - \frac{c \ln(e^{i(bx+a)} + 1)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)*cot(b*x+a)^3,x)`

[Out]
$$-I*c*x+2*I/b*d*a*x+(2*b*d*x*\exp(2*I*(b*x+a))+2*b*c*\exp(2*I*(b*x+a))-I*d*\exp(2*I*(b*x+a))+I*d)/b^2/(\exp(2*I*(b*x+a))-1)^2-1/b*c*\ln(\exp(I*(b*x+a))-1)+2/b*c*\ln(\exp(I*(b*x+a)))-1/b*c*\ln(\exp(I*(b*x+a))+1)+I/b^2*d*a^2+I*d*\text{polylog}(2,-\exp(I*(b*x+a)))/b^2+1/2*I*d*x^2-1/b*d*\ln(1-\exp(I*(b*x+a)))*x-1/b^2*d*\ln(1-\exp(I*(b*x+a)))*a+I/b^2*d*\text{polylog}(2,\exp(I*(b*x+a)))-1/b*d*\ln(\exp(I*(b*x+a))+1)*x+1/b^2*d*a*\ln(\exp(I*(b*x+a))-1)-2/b^2*d*a*\ln(\exp(I*(b*x+a)))$$

Maxima [B] time = 1.6957, size = 1133, normalized size = 10.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*cot(b*x+a)^3,x, algorithm="maxima")`

[Out]
$$(b^2*d*x^2 + 2*b^2*c*x - (2*b*d*x + 2*b*c + 2*(b*d*x + b*c))*\cos(4*b*x + 4*a) - 4*(b*d*x + b*c)*\cos(2*b*x + 2*a) + (2*I*b*d*x + 2*I*b*c)*\sin(4*b*x + 4*a) + (-4*I*b*d*x - 4*I*b*c)*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) - (2*b*c*\cos(4*b*x + 4*a) - 4*b*c*\cos(2*b*x + 2*a) + 2*I*b*c*\sin(4*b*x + 4*a) - 4*I*b*c*\sin(2*b*x + 2*a) + 2*b*c)*\arctan2(\sin(b*x + a), \cos(b*x + a) - 1) + (2*b*d*x*\cos(4*b*x + 4*a) - 4*b*d*x*\cos(2*b*x + 2*a) + 2*I*b*d*x*\sin(4*b*x + 4*a) - 4*I*b*d*x*\sin(2*b*x + 2*a) + 2*b*d*x)*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) + (b^2*d*x^2 + 2*b^2*c*x)*\cos(4*b*x + 4*a) - (2*b^2*d*x^2 + 4*I*b*c + (4*b^2*c + 4*I*b*d)*x + 2*d)*\cos(2*b*x + 2*a) + (2*d*\cos(4*b*x + 4*a) - 4*d*\cos(2*b*x + 2*a) + 2*I*d*\sin(4*b*x + 4*a) - 4*I*d*\sin(2*b*x + 2*a) + 2*d)*\text{dilog}(-e^{(I*b*x + I*a)}) + (2*d*\cos(4*b*x + 4*a) - 4*d*\cos(2*b*x + 2*a) + 2*I*d*\sin(4*b*x + 4*a) - 4*I*d*\sin(2*b*x + 2*a) + 2*d)*\text{dilog}(e^{(I*b*x + I*a)}) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c))*\cos(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*\cos(2*b*x + 2*a) + (b*d*x + b*c)*\sin(4*b*x + 4*a) - 2*(b*d*x + b*c)*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c))*\cos(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*\cos(2*b*x + 2*a) + (b*d*x + b*c)*\sin(4*b*x + 4*a) - 2*(b*d*x + b*c)*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) - (-I*b^2*d*x^2 - 2*I*b^2*c*x)*\sin(4*b*x + 4*a) - (2*I*b^2*d*x^2 - 4*b*c - 4*(-I*b^2*c + b*d)*x + 2*I*d)*\sin(2*b*x + 2*a) + 2*d)/(-2*I*b^2*\cos(4*b*x + 4*a) + 4*I*b^2*\cos(2*b*x + 2*a) + 2*b^2*\sin(4*b*x + 4*a) - 4*b^2*\sin(2*b*x + 2*a) - 2*I*b^2)$$

Fricas [B] time = 0.537357, size = 859, normalized size = 7.88

$$4 b d x + 4 b c + (i d \cos (2 b x + 2 a) - i d) \text{Li}_2(\cos (2 b x + 2 a) + i \sin (2 b x + 2 a)) + (-i d \cos (2 b x + 2 a) + i d) \text{Li}_2(\cos$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*cot(b*x+a)^3,x, algorithm="fricas")`

[Out]
$$1/4*(4*b*d*x + 4*b*c + (I*d*\cos(2*b*x + 2*a) - I*d)*\text{dilog}(\cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a)) + (-I*d*\cos(2*b*x + 2*a) + I*d)*\text{dilog}(\cos(2*b*x + 2*a)$$

) - I*sin(2*b*x + 2*a)) + 2*(b*c - a*d - (b*c - a*d)*cos(2*b*x + 2*a))*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b*c - a*d - (b*c - a*d)*cos(2*b*x + 2*a))*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b*d*x + a*d - (b*d*x + a*d)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1) + 2*(b*d*x + a*d - (b*d*x + a*d)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) + 2*d*sin(2*b*x + 2*a))/(b^2*cos(2*b*x + 2*a) - b^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \cot^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cot(b*x+a)**3,x)

[Out] Integral((c + d*x)*cot(a + b*x)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \cot (bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cot(b*x+a)^3,x, algorithm="giac")

[Out] integrate((d*x + c)*cot(b*x + a)^3, x)

$$3.182 \quad \int \frac{\cot^3(a+bx)}{c+dx} dx$$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\cot^3(a+bx)}{c+dx}, x\right)$$

[Out] Unintegrable[Cot[a + b*x]^3/(c + d*x), x]

Rubi [A] time = 0.039856, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cot^3(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[Cot[a + b*x]^3/(c + d*x), x]

[Out] Defer[Int][Cot[a + b*x]^3/(c + d*x), x]

Rubi steps

$$\int \frac{\cot^3(a+bx)}{c+dx} dx = \int \frac{\cot^3(a+bx)}{c+dx} dx$$

Mathematica [A] time = 8.05091, size = 0, normalized size = 0.

$$\int \frac{\cot^3(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[Cot[a + b*x]^3/(c + d*x), x]

[Out] Integrate[Cot[a + b*x]^3/(c + d*x), x]

Maple [A] time = 2.391, size = 0, normalized size = 0.

$$\int \frac{(\cot(bx+a))^3}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(b*x+a)^3/(d*x+c), x)

[Out] int(cot(b*x+a)^3/(d*x+c), x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)^3/(d*x+c),x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cot(bx+a)^3}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)^3/(d*x+c),x, algorithm="fricas")

[Out] integral(cot(b*x + a)^3/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3(a+bx)}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)**3/(d*x+c),x)

[Out] Integral(cot(a + b*x)**3/(c + d*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(bx+a)^3}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)^3/(d*x+c),x, algorithm="giac")

[Out] integrate(cot(b*x + a)^3/(d*x + c), x)

$$3.183 \quad \int \frac{\cot^3(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\cot^3(a+bx)}{(c+dx)^2}, x\right)$$

[Out] Unintegrable[Cot[a + b*x]^3/(c + d*x)^2, x]

Rubi [A] time = 0.0373215, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[Cot[a + b*x]^3/(c + d*x)^2, x]

[Out] Defer[Int][Cot[a + b*x]^3/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2} dx = \int \frac{\cot^3(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 9.16524, size = 0, normalized size = 0.

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Cot[a + b*x]^3/(c + d*x)^2, x]

[Out] Integrate[Cot[a + b*x]^3/(c + d*x)^2, x]

Maple [A] time = 3.544, size = 0, normalized size = 0.

$$\int \frac{(\cot(bx+a))^3}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(b*x+a)^3/(d*x+c)^2, x)

[Out] int(cot(b*x+a)^3/(d*x+c)^2, x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cot(bx+a)^3}{d^2x^2+2cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(cot(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)**3/(d*x+c)**2,x)

[Out] Integral(cot(a + b*x)**3/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(bx+a)^3}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(cot(b*x + a)^3/(d*x + c)^2, x)

3.184 $\int (c + dx)^{5/2} \cos^3(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=407

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} - \frac{15\sqrt{\pi}d^{5/2}\cos\left(2a - \frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}} + \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(2a - \frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}$$

[Out] $(15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[2*a + 2*b*x])/(8*b) + (15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[4*a + 4*b*x])/(2048*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[4*a + 4*b*x])/(32*b) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[4*a - (4*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(4096*b^{(7/2)}) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]))/(256*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[4*a - (4*b*c)/d])/(4096*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}])]*\text{Sin}[2*a - (2*b*c)/d])/(256*b^{(7/2)}) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[2*a + 2*b*x])/(32*b^2) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[4*a + 4*b*x])/(256*b^2)$

Rubi [A] time = 0.796576, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} - \frac{15\sqrt{\pi}d^{5/2}\cos\left(2a - \frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}} + \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(2a - \frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^{(5/2)}*\text{Cos}[a + b*x]^3*\text{Sin}[a + b*x], x]$

[Out] $(15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[2*a + 2*b*x])/(8*b) + (15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[4*a + 4*b*x])/(2048*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[4*a + 4*b*x])/(32*b) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[4*a - (4*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(4096*b^{(7/2)}) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]))/(256*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[4*a - (4*b*c)/d])/(4096*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}])]*\text{Sin}[2*a - (2*b*c)/d])/(256*b^{(7/2)}) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[2*a + 2*b*x])/(32*b^2) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[4*a + 4*b*x])/(256*b^2)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^{5/2} \cos^3(a + bx) \sin(a + bx) dx &= \int \left(\frac{1}{4}(c + dx)^{5/2} \sin(2a + 2bx) + \frac{1}{8}(c + dx)^{5/2} \sin(4a + 4bx) \right) dx \\
&= \frac{1}{8} \int (c + dx)^{5/2} \sin(4a + 4bx) dx + \frac{1}{4} \int (c + dx)^{5/2} \sin(2a + 2bx) dx \\
&= -\frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} - \frac{(c + dx)^{5/2} \cos(4a + 4bx)}{32b} + \frac{(5d) \int (c + dx)^{3/2} \cos(2a + 2bx) dx}{64b} \\
&= -\frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} - \frac{(c + dx)^{5/2} \cos(4a + 4bx)}{32b} + \frac{5d(c + dx)^{3/2} \sin(2a + 2bx)}{32b^2} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{128b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} + \frac{15d^2 \sqrt{c + dx} \cos(4a + 4bx)}{2048b^3} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{128b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} + \frac{15d^2 \sqrt{c + dx} \cos(4a + 4bx)}{2048b^3} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{128b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} + \frac{15d^2 \sqrt{c + dx} \cos(4a + 4bx)}{2048b^3} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{128b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} + \frac{15d^2 \sqrt{c + dx} \cos(4a + 4bx)}{2048b^3}
\end{aligned}$$

Mathematica [A] time = 13.6521, size = 550, normalized size = 1.35

$$-1024b^3 c^2 \sqrt{c + dx} \cos(2(a + bx)) - 256b^3 c^2 \sqrt{c + dx} \cos(4(a + bx)) - 1024b^3 d^2 x^2 \sqrt{c + dx} \cos(2(a + bx)) - 256b^3 d^2 x^2 \sqrt{c + dx} \cos(4(a + bx))$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]

[Out] (-1024*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 960*b*d^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 2048*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 1024*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 256*b^3*c^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 60*b*d^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 512*b^3*c*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 256*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 480*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 480*Sqrt[b/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] + 1280*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 1280*b^2*d^2*x*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 160*b^2*c*d*Sqrt[c + d*x]*Sin[4*(a + b*x)] + 160*b^2*d^2*x*Sqrt[c + d*x]*Sin[4*(a + b*x)]/(8192*b^4)

Maple [A] time = 0.035, size = 470, normalized size = 1.2

$$2 \frac{1}{d} \left(-\frac{1}{16} \frac{d(dx+c)^{5/2}}{b} \cos \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) + \frac{5d}{16b} \left(\frac{1}{4} \frac{d(dx+c)^{3/2}}{b} \sin \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) - 3/4 \frac{d}{b} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x)

[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/16/b*d*(1/4/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))-1/64/b*d*(d*x+c)^(5/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+5/64/b*d*(1/8/b*d*(d*x+c)^(3/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))

Maxima [C] time = 2.31125, size = 1879, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")

[Out] 1/65536*sqrt(2)*(640*sqrt(2)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(4*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 5120*sqrt(2)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 16*(64*sqrt(2)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 15*sqrt(2)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(4*((d*x + c)*b - b*c + a*d)/d) - 256*(16*sqrt(2)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 15*sqrt(2)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(2*((d*x + c)*b - b*c + a*d)/d) - ((480*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqr

```
t(d^2))) + 480*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))) - 480*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) + 480*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) *d^3*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) - (480*
I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
480*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) + 480*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) - 480*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt
(d^2))) *d^3*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(2*I*b/d)) - (sqrt(2)*(15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2)))) *d^3*sqrt(abs(b)/abs(d))*cos(-4*(b*c - a*
d)/d) - sqrt(2)*(15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) *d^3*sqrt(abs(b)/abs(d))*sin(-4*(b*c - a*d)/d))*er
f(2*sqrt(d*x + c)*sqrt(I*b/d)) - (sqrt(2)*(15*sqrt(pi)*cos(1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*a
rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(-1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) *d^3*sqrt(abs(b)/abs(d)
)*cos(-4*(b*c - a*d)/d) - sqrt(2)*(-15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arct
an2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^3*sqrt(abs(b)/abs(d))*sin(-4*
(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - ((480*sqrt(pi)*cos(1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*sqrt(pi)*cos(-1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*I*sqrt(pi)*s
in(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 480*I*sqrt(p
i)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) *d^3*sqrt
(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) - (-480*I*sqrt(pi)*cos(1/4*pi + 1/2*a
rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 480*I*sqrt(pi)*cos(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*sqrt(pi)*sin(1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 480*sqrt(pi)*sin(-1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) *d^3*sqrt(abs(b)/abs(
d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d))*abs(d)/(b^3*d
*abs(b))
```

Fricas [A] time = 0.698365, size = 944, normalized size = 2.32

$$15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right) + 480\pi d^3\sqrt{\frac{b}{\pi d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")

[Out] -1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(192*b^3*d^2*x^2 + 384*b^3*c*d*x

$$+ 192*b^3*c^2 + 360*b*d^2*\cos(b*x + a)^2 - 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*x + 64*b^3*c^2 - 15*b*d^2)*\cos(b*x + a)^4 - 225*b*d^2 + 160*(2*(b^2*d^2*x + b^2*c*d)*\cos(b*x + a)^3 + 3*(b^2*d^2*x + b^2*c*d)*\cos(b*x + a))*\sin(b*x + a))*\sqrt{d*x + c})/b^4$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a), x)

[Out] Timed out

Giac [C] time = 1.77497, size = 2680, normalized size = 6.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a), x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/16384*(64*(\sqrt{2}*\sqrt{\pi})d^2*\operatorname{erf}(-\sqrt{2}*\sqrt{bd})*\sqrt{dx+c}*(I* \\ & b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((4*I*b*c - 4*I*a*d)/d)/(\sqrt{bd}*(I*b*d/\sqrt{ \\ & b^2*d^2} + 1)*b) + \sqrt{2}*\sqrt{\pi})d^2*\operatorname{erf}(-\sqrt{2}*\sqrt{bd})*\sqrt{dx+c} \\ &)*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-4*I*b*c + 4*I*a*d)/d)/(\sqrt{bd}*(-I*b \\ & *d/\sqrt{b^2*d^2} + 1)*b) + 8*\sqrt{\pi})d^2*\operatorname{erf}(-\sqrt{bd})*\sqrt{dx+c}*(I*b \\ & *d/\sqrt{b^2*d^2} + 1)/d)*e^{((2*I*b*c - 2*I*a*d)/d)/(\sqrt{bd}*(I*b*d/\sqrt{b \\ & ^2*d^2} + 1)*b) + 8*\sqrt{\pi})d^2*\operatorname{erf}(-\sqrt{bd})*\sqrt{dx+c}*(-I*b*d/\sqrt{ \\ & b^2*d^2} + 1)/d)*e^{((-2*I*b*c + 2*I*a*d)/d)/(\sqrt{bd}*(-I*b*d/\sqrt{b^2*d^2} \\ &) + 1)*b) + 4*\sqrt{dx+c}*d*e^{((4*I*(dx+c)*b - 4*I*b*c + 4*I*a*d)/d)/b \\ & + 16*\sqrt{dx+c}*d*e^{((2*I*(dx+c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*\sqrt{ \\ & rt(dx+c)*d*e^{((-2*I*(dx+c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*\sqrt{dx+c} \\ & *d*e^{((-4*I*(dx+c)*b + 4*I*b*c - 4*I*a*d)/d)/b}*c^2 + d^2*((I*\sqrt{2} \\ &)*\sqrt{\pi})*(-64*I*b^2*c^2*d + 48*b*c*d^2 + 15*I*d^3)*d*\operatorname{erf}(-\sqrt{2}*\sqrt{bd} \\ &)*\sqrt{dx+c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((4*I*b*c - 4*I*a*d)/d)/(\sqrt{ \\ & t(bd}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^3) - 4*I*(64*I*(dx+c)^{(5/2)*b^2*d - 1 \\ & 28*I*(dx+c)^{(3/2)*b^2*c*d + 64*I*\sqrt{dx+c}*b^2*c^2*d + 40*(dx+c)^{(\\ & 3/2)*b*d^2 - 48*\sqrt{dx+c}*b*c*d^2 - 15*I*\sqrt{dx+c}*d^3)*e^{((-4*I*(\\ & dx+c)*b + 4*I*b*c - 4*I*a*d)/d)/b^3)/d^2 + (I*\sqrt{2})*\sqrt{\pi})*(-64*I*b^ \\ & 2*c^2*d - 48*b*c*d^2 + 15*I*d^3)*d*\operatorname{erf}(-\sqrt{2}*\sqrt{bd})*\sqrt{dx+c}*(-I \\ & *b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-4*I*b*c + 4*I*a*d)/d)/(\sqrt{bd}*(-I*b*d/\sqrt{ \\ & rt(b^2*d^2} + 1)*b^3) - 4*I*(64*I*(dx+c)^{(5/2)*b^2*d - 128*I*(dx+c)^{(\\ & 3/2)*b^2*c*d + 64*I*\sqrt{dx+c}*b^2*c^2*d - 40*(dx+c)^{(3/2)*b*d^2 + 48 \\ & *\sqrt{dx+c}*b*c*d^2 - 15*I*\sqrt{dx+c}*d^3)*e^{((4*I*(dx+c)*b - 4*I* \\ & b*c + 4*I*a*d)/d)/b^3)/d^2 + 32*(I*\sqrt{\pi})*(-16*I*b^2*c^2*d + 24*b*c*d^2 + \\ & 15*I*d^3)*d*\operatorname{erf}(-\sqrt{bd})*\sqrt{dx+c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((\\ & 2*I*b*c - 2*I*a*d)/d)/(\sqrt{bd}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^3) - 2*I*(16*I \\ & *(dx+c)^{(5/2)*b^2*d - 32*I*(dx+c)^{(3/2)*b^2*c*d + 16*I*\sqrt{dx+c}* \\ & b^2*c^2*d + 20*(dx+c)^{(3/2)*b*d^2 - 24*\sqrt{dx+c}*b*c*d^2 - 15*I*\sqrt{ \\ & (dx+c)*d^3)*e^{((-2*I*(dx+c)*b + 2*I*b*c - 2*I*a*d)/d)/b^3)/d^2 + 32*(\\ & I*\sqrt{\pi})*(-16*I*b^2*c^2*d - 24*b*c*d^2 + 15*I*d^3)*d*\operatorname{erf}(-\sqrt{bd})*\sqrt{ \\ & dx+c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-2*I*b*c + 2*I*a*d)/d)/(\sqrt{bd} \end{aligned}$$

$$\begin{aligned}
&)*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^3) - 2*I*(16*I*(d*x + c)^{(5/2)}*b^2*d - 32*I*(d*x + c)^{(3/2)}*b^2*c*d + 16*I*\sqrt{d*x + c}*b^2*c^2*d - 20*(d*x + c)^{(3/2)}*b*d^2 + 24*\sqrt{d*x + c}*b*c*d^2 - 15*I*\sqrt{d*x + c}*d^3)*e^{((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^2} + 16*(I*\sqrt{2}*\sqrt{\pi}*(8*I*b*c*d - 3*d^2)*d*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((4*I*b*c - 4*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2)} + I*\sqrt{2}*\sqrt{\pi}*(8*I*b*c*d + 3*d^2)*d*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-4*I*b*c + 4*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2)} + 16*I*\sqrt{\pi}*(4*I*b*c*d - 3*d^2)*d*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((2*I*b*c - 2*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2)} + 16*I*\sqrt{\pi}*(4*I*b*c*d + 3*d^2)*d*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-2*I*b*c + 2*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2)} - 4*I*(8*I*(d*x + c)^{(3/2)}*b*d - 8*I*\sqrt{d*x + c}*b*c*d - 3*\sqrt{d*x + c}*d^2)*e^{((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2} - 32*I*(4*I*(d*x + c)^{(3/2)}*b*d - 4*I*\sqrt{d*x + c}*b*c*d - 3*\sqrt{d*x + c}*d^2)*e^{((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^2} - 32*I*(4*I*(d*x + c)^{(3/2)}*b*d - 4*I*\sqrt{d*x + c}*b*c*d + 3*\sqrt{d*x + c}*d^2)*e^{((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2} - 4*I*(8*I*(d*x + c)^{(3/2)}*b*d - 8*I*\sqrt{d*x + c}*b*c*d + 3*\sqrt{d*x + c}*d^2)*e^{((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2}*c)/d
\end{aligned}$$

3.185 $\int (c + dx)^{3/2} \cos^3(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=351

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3\sqrt{\pi}d^{3/2}\sin\left(2a - \frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}} - \frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a\right)}{512b^{5/2}}$$

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(8*b) - ((c + d*x)^(3/2)*Cos[4*a + 4*b*x])/(32*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]*Sqrt[Pi]])/(64*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]*Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) + (3*d*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rubi [A] time = 0.555808, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3\sqrt{\pi}d^{3/2}\sin\left(2a - \frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}} - \frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a\right)}{512b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x], x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(8*b) - ((c + d*x)^(3/2)*Cos[4*a + 4*b*x])/(32*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]*Sqrt[Pi]])/(64*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]*Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) + (3*d*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n * Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m * Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1) * Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
```

$*e - c*f)/d]$, $\text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /;$ $\text{FreeQ}\{c, d, e, f\}, x]$ && $\text{ComplexFreeQ}[f]$ && $\text{NeQ}[d*e - c*f, 0]$

Rule 3305

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] := \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Sin}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /;$ $\text{FreeQ}\{c, d, e, f\}, x]$ && $\text{ComplexFreeQ}[f]$ && $\text{EqQ}[d*e - c*f, 0]$

Rule 3351

$\text{Int}[\text{Sin}[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] := \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /;$ $\text{FreeQ}\{d, e, f\}, x]$

Rule 3304

$\text{Int}[\sin[\text{Pi}/2 + (e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] := \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Cos}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /;$ $\text{FreeQ}\{c, d, e, f\}, x]$ && $\text{ComplexFreeQ}[f]$ && $\text{EqQ}[d*e - c*f, 0]$

Rule 3352

$\text{Int}[\text{Cos}[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] := \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /;$ $\text{FreeQ}\{d, e, f\}, x]$

Rubi steps

$$\begin{aligned} \int (c + dx)^{3/2} \cos^3(a + bx) \sin(a + bx) dx &= \int \left(\frac{1}{4}(c + dx)^{3/2} \sin(2a + 2bx) + \frac{1}{8}(c + dx)^{3/2} \sin(4a + 4bx) \right) dx \\ &= \frac{1}{8} \int (c + dx)^{3/2} \sin(4a + 4bx) dx + \frac{1}{4} \int (c + dx)^{3/2} \sin(2a + 2bx) dx \\ &= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{8b} - \frac{(c + dx)^{3/2} \cos(4a + 4bx)}{32b} + \frac{(3d) \int \sqrt{c + dx} \cos(2a + 2bx) dx}{64b} \\ &= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{8b} - \frac{(c + dx)^{3/2} \cos(4a + 4bx)}{32b} + \frac{3d\sqrt{c + dx} \sin(2a + 2bx)}{32b^2} \\ &= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{8b} - \frac{(c + dx)^{3/2} \cos(4a + 4bx)}{32b} + \frac{3d\sqrt{c + dx} \sin(2a + 2bx)}{32b^2} \\ &= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{8b} - \frac{(c + dx)^{3/2} \cos(4a + 4bx)}{32b} + \frac{3d\sqrt{c + dx} \sin(2a + 2bx)}{32b^2} \\ &= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{8b} - \frac{(c + dx)^{3/2} \cos(4a + 4bx)}{32b} - \frac{3d^{3/2} \sqrt{\frac{\pi}{2}} \cos\left(4a - \frac{4bc}{d}\right)}{512b^2} \end{aligned}$$

Mathematica [A] time = 3.29738, size = 393, normalized size = 1.12

$$-3\sqrt{2\pi}d \sin\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c + dx}\right) - 48\sqrt{\pi}d \sin\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c + dx}}{\sqrt{\pi}}\right) - 3\sqrt{2\pi}d \cos\left(4a - \frac{4bc}{d}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x], x]

```
[Out] (-128*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 128*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] - 32*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*
x)] - 32*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 3*d*Sqrt[2*Pi]*Co
s[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 48*d*Sq
rt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
- 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (
4*b*c)/d] - 48*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Si
n[2*a - (2*b*c)/d] + 96*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 12*Sqr
t[b/d]*d*Sqrt[c + d*x]*Sin[4*(a + b*x)]/(1024*b^2*Sqrt[b/d])
```

Maple [A] time = 0.035, size = 376, normalized size = 1.1

$$2 \frac{1}{d} \left(-1/16 \frac{d(dx+c)^{3/2}}{b} \cos \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) + 3/16 \frac{d}{b} \left(1/4 \frac{d\sqrt{dx+c}}{b} \sin \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) - 1/8 \frac{d\sqrt{dx+c}}{b} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a), x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/16/b*d*(1/4
/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^(
1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d
)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))-1
/64/b*d*(d*x+c)^(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+3/64/b*d*(1/8/b*d*(d
*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)
^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d))
```

Maxima [C] time = 2.25609, size = 1806, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a), x, algorithm="maxima")
```

```
[Out] -1/8192*sqrt(2)*(128*sqrt(2)*(d*x + c)^(3/2)*b*d*abs(b)*cos(4*((d*x + c)*b
- b*c + a*d)/d)/abs(d) + 512*sqrt(2)*(d*x + c)^(3/2)*b*d*abs(b)*cos(2*((d*x
+ c)*b - b*c + a*d)/d)/abs(d) - 48*sqrt(2)*sqrt(d*x + c)*d^2*abs(b)*sin(4*
((d*x + c)*b - b*c + a*d)/d)/abs(d) - 384*sqrt(2)*sqrt(d*x + c)*d^2*abs(b)*
sin(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - ((-48*I*sqrt(pi)*cos(1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*I*sqrt(pi)*cos(-1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*sqrt(pi)*sin(1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi)*sin(-1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs
(d))*cos(-2*(b*c - a*d)/d) - (48*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) - 48*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-2*(b*c
- a*d)/d)*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - (sqrt(2)*(-3*I*sqrt(pi)*cos(1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos
```

```
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) * d^2 * sqrt(abs(b)/abs(d)) * cos(-4*(b*c - a*d)/d) - sqrt(2) * (3*sqrt(pi) * cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi) * cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi) * sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi) * sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * sin(-4*(b*c - a*d)/d) * erf(2*sqrt(d*x + c) * sqrt(I*b/d)) - (sqrt(2) * (3*I*sqrt(pi) * cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi) * cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi) * sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi) * sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * cos(-4*(b*c - a*d)/d) - sqrt(2) * (3*sqrt(pi) * cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi) * cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi) * sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi) * sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * sin(-4*(b*c - a*d)/d) * erf(2*sqrt(d*x + c) * sqrt(-I*b/d)) - ((48*I*sqrt(pi) * cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*I*sqrt(pi) * cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*sqrt(pi) * sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi) * sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * cos(-2*(b*c - a*d)/d) - (48*sqrt(pi) * cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi) * cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*I*sqrt(pi) * sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*I*sqrt(pi) * sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * sin(-2*(b*c - a*d)/d) * erf(sqrt(d*x + c) * sqrt(-2*I*b/d)) * abs(d) / (b^2 * d * abs(b))
```

Fricas [A] time = 0.675102, size = 749, normalized size = 2.13

$$3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+48\pi d^2\sqrt{\frac{b}{\pi d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")

[Out] -1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 48*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d))) + 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x + a)^4 - 6*b^2*d*x - 6*b^2*c - 3*(2*b*d*cos(b*x + a)^3 + 3*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c)/b^3

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.53265, size = 1485, normalized size = 4.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2048*(8*(sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c))*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c))*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c))*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b*c + I*sqrt(2)*sqrt(pi)*(8*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c))*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + I*sqrt(2)*sqrt(pi)*(8*I*b*c*d + 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c))*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 16*I*sqrt(pi)*(4*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c))*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 16*I*sqrt(pi)*(4*I*b*c*d + 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c))*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(8*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2 - 32*I*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^2 - 32*I*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2 - 4*I*(8*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d
```

3.186 $\int \sqrt{c + dx} \cos^3(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=299

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} + \frac{\sqrt{\pi} \sqrt{d} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b} \sqrt{c+dx}}{\sqrt{\pi} \sqrt{d}}\right)}{16b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \sin\left(4a - \frac{4bc}{d}\right) S\left(\frac{2\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}$$

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(8*b) - (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(
(32*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rubi [A] time = 0.446811, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} + \frac{\sqrt{\pi} \sqrt{d} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b} \sqrt{c+dx}}{\sqrt{\pi} \sqrt{d}}\right)}{16b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \sin\left(4a - \frac{4bc}{d}\right) S\left(\frac{2\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x], x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(8*b) - (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(
(32*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```


Rule 3305

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3304

Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}
 \int \sqrt{c+dx} \cos^3(a+bx) \sin(a+bx) dx &= \int \left(\frac{1}{4} \sqrt{c+dx} \sin(2a+2bx) + \frac{1}{8} \sqrt{c+dx} \sin(4a+4bx) \right) dx \\
 &= \frac{1}{8} \int \sqrt{c+dx} \sin(4a+4bx) dx + \frac{1}{4} \int \sqrt{c+dx} \sin(2a+2bx) dx \\
 &= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{8b} - \frac{\sqrt{c+dx} \cos(4a+4bx)}{32b} + \frac{d \int \frac{\cos(4a+4bx)}{\sqrt{c+dx}} dx}{64b} + \frac{d \int \frac{\cos(2a+2bx)}{\sqrt{c+dx}} dx}{64b} \\
 &= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{8b} - \frac{\sqrt{c+dx} \cos(4a+4bx)}{32b} + \frac{\left(d \cos\left(4a - \frac{4bc}{d}\right) \right) \int \frac{\cos(4a+4bx)}{\sqrt{c+dx}} dx}{64b} + \frac{\left(d \cos\left(2a - \frac{2bc}{d}\right) \right) \int \frac{\cos(2a+2bx)}{\sqrt{c+dx}} dx}{64b} \\
 &= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{8b} - \frac{\sqrt{c+dx} \cos(4a+4bx)}{32b} + \frac{\cos\left(4a - \frac{4bc}{d}\right) \text{Subst}\left(\int \frac{\cos(4a+4bx)}{\sqrt{c+dx}} dx, \sqrt{c+dx}, \frac{d}{2}\right)}{64b} + \frac{\cos\left(2a - \frac{2bc}{d}\right) \text{Subst}\left(\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}} dx, \sqrt{c+dx}, \frac{d}{2}\right)}{64b} \\
 &= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{8b} - \frac{\sqrt{c+dx} \cos(4a+4bx)}{32b} + \frac{\sqrt{d} \sqrt{\frac{\pi}{2}} \cos\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c+dx}\right)}{64b^{3/2}} + \frac{\sqrt{d} \sqrt{\frac{\pi}{2}} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c+dx}\right)}{64b^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.83486, size = 264, normalized size = 0.88

$$\sqrt{2\pi} \cos\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c+dx}\right) + 8\sqrt{\pi} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}} \sqrt{c+dx}}{\sqrt{\pi}}\right) - \sqrt{2\pi} \sin\left(4a - \frac{4bc}{d}\right) \text{FresnelS}\left(2\sqrt{\frac{2}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c+dx}\right) - 8\sqrt{\pi} \sin\left(2a - \frac{2bc}{d}\right) \text{FresnelS}\left(\frac{2\sqrt{\frac{b}{d}} \sqrt{c+dx}}{\sqrt{\pi}}\right)$$

128b

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x], x]

[Out] (-16*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 4*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*x)] + Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] + 8*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 8*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]

```
rt[c + d*x]]*Sin[4*a - (4*b*c)/d] - 8*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c
+ d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(128*b*Sqrt[b/d])
```

Maple [A] time = 0.032, size = 286, normalized size = 1.

$$2 \frac{1}{d} \left(-1/16 \frac{d\sqrt{dx+c}}{b} \cos\left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d}\right) + 1/32 \frac{d\sqrt{\pi}}{b} \left(\cos\left(2 \frac{ad-bc}{d}\right) \operatorname{FresnelC}\left(2 \frac{\sqrt{dx+cb}}{d\sqrt{\pi}} \frac{1}{\sqrt{\frac{b}{d}}}\right) - \sin\left(2 \frac{ad-bc}{d}\right) \operatorname{FresnelS}\left(2 \frac{\sqrt{dx+cb}}{d\sqrt{\pi}} \frac{1}{\sqrt{\frac{b}{d}}}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/32/b*d*Pi^(
1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c
)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d))-1/64/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+1/256/b*d*
2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2
)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))
```

Maxima [C] time = 2.1839, size = 1661, normalized size = 5.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/1024*sqrt(2)*(16*sqrt(2)*sqrt(d*x + c)*d*abs(b)*cos(4*((d*x + c)*b - b*c
+ a*d)/d)/abs(d) + 64*sqrt(2)*sqrt(d*x + c)*d*abs(b)*cos(2*((d*x + c)*b -
b*c + a*d)/d)/abs(d) - ((8*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))) + 8*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) - 8*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2
*arctan2(0, d/sqrt(d^2))) + 8*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) -
(8*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) + 8*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) + 8*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) - 8*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))))*d*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(
2*I*b/d)) - (sqrt(2)*(sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(I
*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I
*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sq
rt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sq
rt(abs(b)/abs(d))*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) -
(sqrt(2)*(sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
```

```

^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) *d*sqrt(abs(b)/abs(d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(-I*sqrt(pi)*
cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*
cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*s
in(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d*sqrt(abs(b)/
abs(d))*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - ((8*sqrt
(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*sqrt
(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*I*s
qrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 8*I
*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d
*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) - (-8*I*sqrt(pi)*cos(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 8*I*sqrt(pi)*cos(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*sqrt(pi)*sin(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 8*sqrt(pi)*sin(-1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d*sqrt(abs(b)/abs(d))*s
in(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d))) *abs(d)/(b*d*abs(b)
)

```

Fricas [A] time = 0.61023, size = 599, normalized size = 2.

$$\frac{\sqrt{2}\pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{4(bc-ad)}{d}\right) C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - \sqrt{2}\pi d \sqrt{\frac{b}{\pi d}} S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{4(bc-ad)}{d}\right) + 8\pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{4(bc-ad)}{d}\right)}{128b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a), x, algorithm="fricas")
```

```
[Out] 1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 8*pi*d*sq
rt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)
)) - 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(
-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 3*b)*sqrt(d*x + c)/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a), x)
```

```
[Out] Timed out
```

Giac [C] time = 1.3384, size = 643, normalized size = 2.15

$$\frac{\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right) e^{\left(\frac{4ibc-4iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} + \frac{\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right) e^{\left(\frac{-4ibc+4iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} + \frac{8\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right) e^{\left(\frac{2ibc-d}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/256*(\sqrt{2}*\sqrt{\pi})*d^2*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d}*\sqrt{d*x+c})*(I*b*d/\sqrt{b^2*d^2+1})/d * e^{((4*I*b*c-4*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2+1}) * b)} \\ & + \sqrt{2}*\sqrt{\pi}*d^2*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d}*\sqrt{d*x+c})*(-I*b*d/\sqrt{b^2*d^2+1})/d * e^{((-4*I*b*c+4*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2+1}) * b)} \\ & + 8*\sqrt{\pi}*d^2*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x+c})*(I*b*d/\sqrt{b^2*d^2+1})/d * e^{((2*I*b*c-2*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2+1}) * b)} \\ & + 8*\sqrt{\pi}*d^2*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x+c})*(-I*b*d/\sqrt{b^2*d^2+1})/d * e^{((-2*I*b*c+2*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2+1}) * b)} \\ & + 4*\sqrt{d*x+c}*d * e^{((4*I*(d*x+c)*b-4*I*b*c+4*I*a*d)/d)/b} + 16*\sqrt{d*x+c}*d * e^{((2*I*(d*x+c)*b-2*I*b*c+2*I*a*d)/d)/b} \\ & + 16*\sqrt{d*x+c}*d * e^{((-2*I*(d*x+c)*b+2*I*b*c-2*I*a*d)/d)/b} + 4*\sqrt{d*x+c}*d * e^{((-4*I*(d*x+c)*b+4*I*b*c-4*I*a*d)/d)/b}/d \end{aligned}$$

3.187 $\int \sqrt{c + dx} \cos^3(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=299

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} + \frac{\sqrt{\pi} \sqrt{d} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b} \sqrt{c+dx}}{\sqrt{\pi} \sqrt{d}}\right)}{16b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \sin\left(4a - \frac{4bc}{d}\right)}{64b^{3/2}}$$

[Out] $-(\text{Sqrt}[c + d*x]*\text{Cos}[2*a + 2*b*x])/(8*b) - (\text{Sqrt}[c + d*x]*\text{Cos}[4*a + 4*b*x])/(32*b) + (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[4*a - (4*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(64*b^{(3/2)}) + (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}])])/(16*b^{(3/2)}) - (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d])* \text{Sin}[4*a - (4*b*c)/d])/(64*b^{(3/2)}) - (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}])]* \text{Sin}[2*a - (2*b*c)/d])/(16*b^{(3/2)})$

Rubi [A] time = 0.454946, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \cos\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} + \frac{\sqrt{\pi} \sqrt{d} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b} \sqrt{c+dx}}{\sqrt{\pi} \sqrt{d}}\right)}{16b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \sin\left(4a - \frac{4bc}{d}\right)}{64b^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[c + d*x]*\text{Cos}[a + b*x]^3*\text{Sin}[a + b*x], x]$

[Out] $-(\text{Sqrt}[c + d*x]*\text{Cos}[2*a + 2*b*x])/(8*b) - (\text{Sqrt}[c + d*x]*\text{Cos}[4*a + 4*b*x])/(32*b) + (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[4*a - (4*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(64*b^{(3/2)}) + (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}])])/(16*b^{(3/2)}) - (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d])* \text{Sin}[4*a - (4*b*c)/d])/(64*b^{(3/2)}) - (\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}])]* \text{Sin}[2*a - (2*b*c)/d])/(16*b^{(3/2)})$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] := \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] := -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{GtQ}[m, 0]$

Rule 3306

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] := \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{ComplexFreeQ}[f] \&\& \text{NeQ}[d*e - c*f, 0]$

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned} \int \sqrt{c+dx} \cos^3(a+bx) \sin(a+bx) dx &= \int \left(\frac{1}{4} \sqrt{c+dx} \sin(2a+2bx) + \frac{1}{8} \sqrt{c+dx} \sin(4a+4bx) \right) dx \\ &= \frac{1}{8} \int \sqrt{c+dx} \sin(4a+4bx) dx + \frac{1}{4} \int \sqrt{c+dx} \sin(2a+2bx) dx \\ &= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{8b} - \frac{\sqrt{c+dx} \cos(4a+4bx)}{32b} + \frac{d \int \frac{\cos(4a+4bx)}{\sqrt{c+dx}} dx}{64b} + \frac{d \int \frac{\cos(2a+2bx)}{\sqrt{c+dx}} dx}{32b} \\ &= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{8b} - \frac{\sqrt{c+dx} \cos(4a+4bx)}{32b} + \frac{\left(d \cos\left(4a - \frac{4bc}{d}\right) \right) \int \frac{\cos\left(\frac{4b}{d}\sqrt{c+dx}\right)}{\sqrt{c+dx}} dx}{64b} \\ &= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{8b} - \frac{\sqrt{c+dx} \cos(4a+4bx)}{32b} + \frac{\cos\left(4a - \frac{4bc}{d}\right) \text{Subst}\left(\int \frac{\cos\left(\frac{4b}{d}\sqrt{c+dx}\right)}{\sqrt{c+dx}} dx\right)}{32b} \\ &= -\frac{\sqrt{c+dx} \cos(2a+2bx)}{8b} - \frac{\sqrt{c+dx} \cos(4a+4bx)}{32b} + \frac{\sqrt{d} \sqrt{\frac{\pi}{2}} \cos\left(4a - \frac{4bc}{d}\right) C\left(\frac{2\sqrt{\frac{b}{d}\sqrt{c+dx}}}{\sqrt{\pi}}\right)}{64b^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.391485, size = 264, normalized size = 0.88

$$\frac{\sqrt{2\pi} \cos\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c+dx}\right) + 8\sqrt{\pi} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}\sqrt{c+dx}}}{\sqrt{\pi}}\right) - \sqrt{2\pi} \sin\left(4a - \frac{4bc}{d}\right) S\left(2\sqrt{\frac{b}{d}\sqrt{c+dx}}\right)}{128b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x], x]
```

```
[Out] (-16*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 4*Sqrt[b/d]*Sqrt[c + d*x]*C
os[4*(a + b*x)] + Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[2/Pi]*Sqrt[c + d*x]] + 8*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/
d]*Sqrt[c + d*x])/Sqrt[Pi]] - Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*S
```



```

^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) * d*sqrt(abs(b)/abs(d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(-I*sqrt(pi)*
cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*
cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*s
in(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) * d*sqrt(abs(b)/
abs(d))*sin(-4*(b*c - a*d)/d)*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - ((8*sqrt
(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*sqrt
(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*I*s
qrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 8*I
*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) * d
*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) - (-8*I*sqrt(pi)*cos(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 8*I*sqrt(pi)*cos(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*sqrt(pi)*sin(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 8*sqrt(pi)*sin(-1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) * d*sqrt(abs(b)/abs(d))*s
in(-2*(b*c - a*d)/d)*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*abs(d)/(b*d*abs(b)
)

```

Fricas [A] time = 0.613858, size = 599, normalized size = 2.

$$\frac{\sqrt{2}\pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{4(bc-ad)}{d}\right) C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - \sqrt{2}\pi d \sqrt{\frac{b}{\pi d}} S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{4(bc-ad)}{d}\right) + 8\pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{2(bc-ad)}{d}\right)}{128b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 8*pi*d*sqrt
(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)
)) - 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(
-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 3*b)*sqrt(d*x + c))/b^2

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.32414, size = 643, normalized size = 2.15

$$\frac{\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right) e^{\left(\frac{4ibc-4iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} + \frac{\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right) e^{\left(\frac{-4ibc+4iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} + \frac{8\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right) e^{\left(\frac{2ibc-2iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)/d
```

3.188 $\int (c + dx)^{3/2} \cos^3(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=351

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3\sqrt{\pi}d^{3/2}\sin\left(2a - \frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}} - \frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a - \frac{4bc}{d}\right)}{512b^{5/2}}$$

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(8*b) - ((c + d*x)^(3/2)*Cos[4*a + 4*b*x])/(32*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(64*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) + (3*d*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rubi [A] time = 0.572442, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3\sqrt{\pi}d^{3/2}\sin\left(2a - \frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}} - \frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a - \frac{4bc}{d}\right)}{512b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x], x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(8*b) - ((c + d*x)^(3/2)*Cos[4*a + 4*b*x])/(32*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(64*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) + (3*d*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n * Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[(c + d*x)^m * Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1) * Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
```

$*e - c*f)/d]$, $\text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /;$ $\text{FreeQ}\{c, d, e, f\}, x]$ && $\text{ComplexFreeQ}[f]$ && $\text{NeQ}[d*e - c*f, 0]$

Rule 3305

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] := \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Sin}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /;$ $\text{FreeQ}\{c, d, e, f\}, x]$ && $\text{ComplexFreeQ}[f]$ && $\text{EqQ}[d*e - c*f, 0]$

Rule 3351

$\text{Int}[\text{Sin}[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] := \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /;$ $\text{FreeQ}\{d, e, f\}, x]$

Rule 3304

$\text{Int}[\text{sin}[\text{Pi}/2 + (e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] := \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Cos}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /;$ $\text{FreeQ}\{c, d, e, f\}, x]$ && $\text{ComplexFreeQ}[f]$ && $\text{EqQ}[d*e - c*f, 0]$

Rule 3352

$\text{Int}[\text{Cos}[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] := \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /;$ $\text{FreeQ}\{d, e, f\}, x]$

Rubi steps

$$\begin{aligned} \int (c + dx)^{3/2} \cos^3(a + bx) \sin(a + bx) dx &= \int \left(\frac{1}{4}(c + dx)^{3/2} \sin(2a + 2bx) + \frac{1}{8}(c + dx)^{3/2} \sin(4a + 4bx) \right) dx \\ &= \frac{1}{8} \int (c + dx)^{3/2} \sin(4a + 4bx) dx + \frac{1}{4} \int (c + dx)^{3/2} \sin(2a + 2bx) dx \\ &= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{8b} - \frac{(c + dx)^{3/2} \cos(4a + 4bx)}{32b} + \frac{(3d) \int \sqrt{c + dx} \cos(2a + 2bx) dx}{64b^2} \\ &= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{8b} - \frac{(c + dx)^{3/2} \cos(4a + 4bx)}{32b} + \frac{3d\sqrt{c + dx} \sin(2a + 2bx)}{32b^2} \\ &= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{8b} - \frac{(c + dx)^{3/2} \cos(4a + 4bx)}{32b} + \frac{3d\sqrt{c + dx} \sin(2a + 2bx)}{32b^2} \\ &= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{8b} - \frac{(c + dx)^{3/2} \cos(4a + 4bx)}{32b} + \frac{3d\sqrt{c + dx} \sin(2a + 2bx)}{32b^2} \\ &= -\frac{(c + dx)^{3/2} \cos(2a + 2bx)}{8b} - \frac{(c + dx)^{3/2} \cos(4a + 4bx)}{32b} - \frac{3d^{3/2} \sqrt{\frac{\pi}{2}} \cos\left(4a + \frac{2\sqrt{2}}{\sqrt{\pi}} \sqrt{b} \sqrt{c + dx}\right)}{32b} \end{aligned}$$

Mathematica [A] time = 1.61926, size = 393, normalized size = 1.12

$$-3\sqrt{2\pi}d \sin\left(4a - \frac{4bc}{d}\right) \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c + dx}\right) - 48\sqrt{\pi}d \sin\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c + dx}}{\sqrt{\pi}}\right) - 3\sqrt{2\pi}d \cos\left(4a - \frac{4bc}{d}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x], x]

```
[Out] (-128*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 128*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 32*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 32*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 3*d*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 48*d*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] - 48*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] + 96*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 12*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[4*(a + b*x)]/(1024*b^2*Sqrt[b/d])
```

Maple [A] time = 0.033, size = 376, normalized size = 1.1

$$2 \frac{1}{d} \left(-1/16 \frac{d(dx+c)^{3/2}}{b} \cos \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) + 3/16 \frac{d}{b} \left(1/4 \frac{d\sqrt{dx+c}}{b} \sin \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) - 1/8 \frac{d\sqrt{\pi}}{b} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/16/b*d*(1/4/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))-1/64/b*d*(d*x+c)^(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+3/64/b*d*(1/8/b*d*(d*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))
```

Maxima [C] time = 2.2755, size = 1806, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8192*sqrt(2)*(128*sqrt(2)*(d*x + c)^(3/2)*b*d*abs(b)*cos(4*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 512*sqrt(2)*(d*x + c)^(3/2)*b*d*abs(b)*cos(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 48*sqrt(2)*sqrt(d*x + c)*d^2*abs(b)*sin(4*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 384*sqrt(2)*sqrt(d*x + c)*d^2*abs(b)*sin(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - ((-48*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) - (48*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - (sqrt(2)*(-3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos
```

```
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) * d^2 * sqrt(abs(b)/abs(d)) * cos(-4*(b*c - a*d)/d) - sqrt(2) * (3*sqrt(pi) * cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi) * cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi) * sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi) * sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * sin(-4*(b*c - a*d)/d) * erf(2*sqrt(d*x + c) * sqrt(I*b/d)) - (sqrt(2) * (3*I*sqrt(pi) * cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi) * cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi) * sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi) * sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * cos(-4*(b*c - a*d)/d) - sqrt(2) * (3*sqrt(pi) * cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi) * cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi) * sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi) * sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * sin(-4*(b*c - a*d)/d) * erf(2*sqrt(d*x + c) * sqrt(-I*b/d)) - ((48*I*sqrt(pi) * cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*I*sqrt(pi) * cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*sqrt(pi) * sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi) * sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * cos(-2*(b*c - a*d)/d) - (48*sqrt(pi) * cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi) * cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*I*sqrt(pi) * sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*I*sqrt(pi) * sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) * d^2 * sqrt(abs(b)/abs(d)) * sin(-2*(b*c - a*d)/d) * erf(sqrt(d*x + c) * sqrt(-2*I*b/d))) * abs(d) / (b^2 * d * abs(b))
```

Fricas [A] time = 0.63998, size = 749, normalized size = 2.13

$$3\sqrt{2}\pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{4(bc-ad)}{d}\right) S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) + 3\sqrt{2}\pi d^2 \sqrt{\frac{b}{\pi d}} C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{4(bc-ad)}{d}\right) + 48\pi d^2 \sqrt{\frac{b}{\pi d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 48*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d))) + 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x + a)^4 - 6*b^2*d*x - 6*b^2*c - 3*(2*b*d*cos(b*x + a)^3 + 3*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a),x)

[Out] Timed out

Giac [C] time = 1.53742, size = 1485, normalized size = 4.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/2048*(8*(\sqrt{2}*\sqrt{\pi})*d^2*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d})*\sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((4*I*b*c-4*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2}+1)*b)} \\ & + \sqrt{2}*\sqrt{\pi}*d^2*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d})*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((-4*I*b*c+4*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2}+1)*b)} \\ & + 8*\sqrt{\pi}*d^2*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d})*\sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((2*I*b*c-2*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2}+1)*b)} \\ & + 8*\sqrt{\pi}*d^2*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d})*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((-2*I*b*c+2*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2}+1)*b)} \\ & + 4*\sqrt{d*x+c}*d*e^{((4*I*(d*x+c)*b-4*I*b*c+4*I*a*d)/d)/b} \\ & + 16*\sqrt{d*x+c}*d*e^{((2*I*(d*x+c)*b-2*I*b*c+2*I*a*d)/d)/b} \\ & + 16*\sqrt{d*x+c}*d*e^{((-2*I*(d*x+c)*b+2*I*b*c-2*I*a*d)/d)/b} \\ & + 4*\sqrt{d*x+c}*d*e^{((-4*I*(d*x+c)*b+4*I*b*c-4*I*a*d)/d)/b} \\ & *c + I*\sqrt{2}*\sqrt{\pi}*(8*I*b*c*d-3*d^2)*d*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d})*\sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((4*I*b*c-4*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2}+1)*b^2)} \\ & + I*\sqrt{2}*\sqrt{\pi}*(8*I*b*c*d+3*d^2)*d*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d})*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((-4*I*b*c+4*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2}+1)*b^2)} \\ & + 16*I*\sqrt{\pi}*(4*I*b*c*d-3*d^2)*d*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d})*\sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((2*I*b*c-2*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2}+1)*b^2)} \\ & + 16*I*\sqrt{\pi}*(4*I*b*c*d+3*d^2)*d*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d})*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((-2*I*b*c+2*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2}+1)*b^2)} \\ & - 4*I*(8*I*(d*x+c)^{(3/2)}*b*d-8*I*\sqrt{d*x+c}*b*c*d-3*\sqrt{d*x+c}*d^2)*e^{((4*I*(d*x+c)*b-4*I*b*c+4*I*a*d)/d)/b^2} \\ & - 32*I*(4*I*(d*x+c)^{(3/2)}*b*d-4*I*\sqrt{d*x+c}*b*c*d+3*\sqrt{d*x+c}*d^2)*e^{((2*I*(d*x+c)*b-2*I*b*c+2*I*a*d)/d)/b^2} \\ & - 32*I*(4*I*(d*x+c)^{(3/2)}*b*d-4*I*\sqrt{d*x+c}*b*c*d+3*\sqrt{d*x+c}*d^2)*e^{((-2*I*(d*x+c)*b+2*I*b*c-2*I*a*d)/d)/b^2} \\ & - 4*I*(8*I*(d*x+c)^{(3/2)}*b*d-8*I*\sqrt{d*x+c}*b*c*d+3*\sqrt{d*x+c}*d^2)*e^{((-4*I*(d*x+c)*b+4*I*b*c-4*I*a*d)/d)/b^2}/d \end{aligned}$$

3.189 $\int (c + dx)^{5/2} \cos^3(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=407

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} - \frac{15\sqrt{\pi}d^{5/2}\cos\left(2a - \frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}} + \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(2a - \frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} - \frac{15\sqrt{\pi}d^{5/2}\sin\left(2a - \frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}}$$

[Out] $(15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[2*a + 2*b*x])/(8*b) + (15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[4*a + 4*b*x])/(2048*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[4*a + 4*b*x])/(32*b) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[4*a - (4*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(4096*b^{(7/2)}) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]))/(256*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[4*a - (4*b*c)/d])/(4096*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}])]*\text{Sin}[2*a - (2*b*c)/d])/(256*b^{(7/2)}) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[2*a + 2*b*x])/(32*b^2) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[4*a + 4*b*x])/(256*b^2)$

Rubi [A] time = 0.672366, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a - \frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} - \frac{15\sqrt{\pi}d^{5/2}\cos\left(2a - \frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}} + \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(2a - \frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} - \frac{15\sqrt{\pi}d^{5/2}\sin\left(2a - \frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^{(5/2)}*\text{Cos}[a + b*x]^3*\text{Sin}[a + b*x], x]$

[Out] $(15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[2*a + 2*b*x])/(8*b) + (15*d^2*\text{Sqrt}[c + d*x]*\text{Cos}[4*a + 4*b*x])/(2048*b^3) - ((c + d*x)^{(5/2)}*\text{Cos}[4*a + 4*b*x])/(32*b) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[4*a - (4*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(4096*b^{(7/2)}) - (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]))/(256*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[4*a - (4*b*c)/d])/(4096*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}])]*\text{Sin}[2*a - (2*b*c)/d])/(256*b^{(7/2)}) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[2*a + 2*b*x])/(32*b^2) + (5*d*(c + d*x)^{(3/2)}*\text{Sin}[4*a + 4*b*x])/(256*b^2)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^{5/2} \cos^3(a + bx) \sin(a + bx) dx &= \int \left(\frac{1}{4}(c + dx)^{5/2} \sin(2a + 2bx) + \frac{1}{8}(c + dx)^{5/2} \sin(4a + 4bx) \right) dx \\
&= \frac{1}{8} \int (c + dx)^{5/2} \sin(4a + 4bx) dx + \frac{1}{4} \int (c + dx)^{5/2} \sin(2a + 2bx) dx \\
&= -\frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} - \frac{(c + dx)^{5/2} \cos(4a + 4bx)}{32b} + \frac{(5d) \int (c + dx)^{3/2} \cos(2a + 2bx) dx}{64b} \\
&= -\frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} - \frac{(c + dx)^{5/2} \cos(4a + 4bx)}{32b} + \frac{5d(c + dx)^{3/2} \sin(2a + 2bx)}{32b^2} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{128b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} + \frac{15d^2 \sqrt{c + dx} \cos(4a + 4bx)}{2048b^3} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{128b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} + \frac{15d^2 \sqrt{c + dx} \cos(4a + 4bx)}{2048b^3} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{128b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} + \frac{15d^2 \sqrt{c + dx} \cos(4a + 4bx)}{2048b^3} \\
&= \frac{15d^2 \sqrt{c + dx} \cos(2a + 2bx)}{128b^3} - \frac{(c + dx)^{5/2} \cos(2a + 2bx)}{8b} + \frac{15d^2 \sqrt{c + dx} \cos(4a + 4bx)}{2048b^3}
\end{aligned}$$

Mathematica [A] time = 10.9173, size = 550, normalized size = 1.35

$$-1024b^3 c^2 \sqrt{c + dx} \cos(2(a + bx)) - 256b^3 c^2 \sqrt{c + dx} \cos(4(a + bx)) - 1024b^3 d^2 x^2 \sqrt{c + dx} \cos(2(a + bx)) - 256b^3 d^2 x^2 \sqrt{c + dx} \cos(4(a + bx))$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]

[Out] (-1024*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 960*b*d^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 2048*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 1024*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 256*b^3*c^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 60*b*d^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 512*b^3*c*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 256*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 480*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 480*Sqrt[b/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] + 1280*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 1280*b^2*d^2*x*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 160*b^2*c*d*Sqrt[c + d*x]*Sin[4*(a + b*x)] + 160*b^2*d^2*x*Sqrt[c + d*x]*Sin[4*(a + b*x)]/(8192*b^4)

Maple [A] time = 0.034, size = 470, normalized size = 1.2

$$2 \frac{1}{d} \left(-\frac{1}{16} \frac{d(dx+c)^{5/2}}{b} \cos \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) + \frac{5d}{16b} \left(\frac{1}{4} \frac{d(dx+c)^{3/2}}{b} \sin \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) - 3/4 \frac{d}{b} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x)

[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/16/b*d*(1/4/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))-1/64/b*d*(d*x+c)^(5/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+5/64/b*d*(1/8/b*d*(d*x+c)^(3/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))

Maxima [C] time = 2.24238, size = 1879, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")

[Out] 1/65536*sqrt(2)*(640*sqrt(2)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(4*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 5120*sqrt(2)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 16*(64*sqrt(2)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 15*sqrt(2)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(4*((d*x + c)*b - b*c + a*d)/d) - 256*(16*sqrt(2)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 15*sqrt(2)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(2*((d*x + c)*b - b*c + a*d)/d) - ((480*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqr

```
t(d^2))) + 480*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))) - 480*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) + 480*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) *d^3*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) - (480*
I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
480*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) + 480*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) - 480*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt
(d^2))) *d^3*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(2*I*b/d)) - (sqrt(2)*(15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2)))) *d^3*sqrt(abs(b)/abs(d))*cos(-4*(b*c - a*
d)/d) - sqrt(2)*(15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) *d^3*sqrt(abs(b)/abs(d))*sin(-4*(b*c - a*d)/d))*er
f(2*sqrt(d*x + c)*sqrt(I*b/d)) - (sqrt(2)*(15*sqrt(pi)*cos(1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*a
rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(-1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) *d^3*sqrt(abs(b)/abs(d)
)*cos(-4*(b*c - a*d)/d) - sqrt(2)*(-15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arct
an2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^3*sqrt(abs(b)/abs(d))*sin(-4*
(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - ((480*sqrt(pi)*cos(1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*sqrt(pi)*cos(-1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*I*sqrt(pi)*s
in(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 480*I*sqrt(p
i)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) *d^3*sqrt
(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) - (-480*I*sqrt(pi)*cos(1/4*pi + 1/2*a
rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 480*I*sqrt(pi)*cos(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*sqrt(pi)*sin(1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 480*sqrt(pi)*sin(-1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))) *d^3*sqrt(abs(b)/abs(
d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d))*abs(d)/(b^3*d
*abs(b))
```

Fricas [A] time = 0.693544, size = 944, normalized size = 2.32

$$15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right) + 480\pi d^3\sqrt{\frac{b}{\pi d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")

[Out] -1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(192*b^3*d^2*x^2 + 384*b^3*c*d*x

$$+ 192*b^3*c^2 + 360*b*d^2*\cos(b*x + a)^2 - 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*x + 64*b^3*c^2 - 15*b*d^2)*\cos(b*x + a)^4 - 225*b*d^2 + 160*(2*(b^2*d^2*x + b^2*c*d)*\cos(b*x + a)^3 + 3*(b^2*d^2*x + b^2*c*d)*\cos(b*x + a))*\sin(b*x + a))*\sqrt{d*x + c})/b^4$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a), x)

[Out] Timed out

Giac [C] time = 1.74143, size = 2680, normalized size = 6.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a), x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/16384*(64*(\sqrt{2}*\sqrt{\pi})*d^2*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d})*\sqrt{d*x + c})*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((4*I*b*c - 4*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b) + \sqrt{2}*\sqrt{\pi})*d^2*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d})*\sqrt{d*x + c})*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-4*I*b*c + 4*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b) + 8*\sqrt{\pi})*d^2*\operatorname{erf}(-\sqrt{b*d})*\sqrt{d*x + c})*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((2*I*b*c - 2*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b) + 8*\sqrt{\pi})*d^2*\operatorname{erf}(-\sqrt{b*d})*\sqrt{d*x + c})*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-2*I*b*c + 2*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b) + 4*\sqrt{d*x + c})*d*e^{((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16*\sqrt{d*x + c})*d*e^{((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*\sqrt{d*x + c})*d*e^{((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*\sqrt{d*x + c})*d*e^{((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b}*c^2 + d^2*((I*\sqrt{2})*\sqrt{\pi})*(-64*I*b^2*c^2*d + 48*b*c*d^2 + 15*I*d^3)*d*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d})*\sqrt{d*x + c})*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((4*I*b*c - 4*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^3) - 4*I*(64*I*(d*x + c)^{(5/2)}*b^2*d - 128*I*(d*x + c)^{(3/2)}*b^2*c*d + 64*I*\sqrt{d*x + c})*b^2*c^2*d + 40*(d*x + c)^{(3/2)}*b*d^2 - 48*\sqrt{d*x + c})*b*c*d^2 - 15*I*\sqrt{d*x + c})*d^3)*e^{((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^3)/d^2 + (I*\sqrt{2})*\sqrt{\pi})*(-64*I*b^2*c^2*d - 48*b*c*d^2 + 15*I*d^3)*d*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d})*\sqrt{d*x + c})*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-4*I*b*c + 4*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^3) - 4*I*(64*I*(d*x + c)^{(5/2)}*b^2*d - 128*I*(d*x + c)^{(3/2)}*b^2*c*d + 64*I*\sqrt{d*x + c})*b^2*c^2*d - 40*(d*x + c)^{(3/2)}*b*d^2 + 48*\sqrt{d*x + c})*b*c*d^2 - 15*I*\sqrt{d*x + c})*d^3)*e^{((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^3)/d^2 + 32*(I*\sqrt{\pi})*(-16*I*b^2*c^2*d + 24*b*c*d^2 + 15*I*d^3)*d*\operatorname{erf}(-\sqrt{b*d})*\sqrt{d*x + c})*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((2*I*b*c - 2*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^3) - 2*I*(16*I*(d*x + c)^{(5/2)}*b^2*d - 32*I*(d*x + c)^{(3/2)}*b^2*c*d + 16*I*\sqrt{d*x + c})*b^2*c^2*d + 20*(d*x + c)^{(3/2)}*b*d^2 - 24*\sqrt{d*x + c})*b*c*d^2 - 15*I*\sqrt{d*x + c})*d^3)*e^{((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^3)/d^2 + 32*(I*\sqrt{\pi})*(-16*I*b^2*c^2*d - 24*b*c*d^2 + 15*I*d^3)*d*\operatorname{erf}(-\sqrt{b*d})*\sqrt{d*x + c})*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-2*I*b*c + 2*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-2*I*b*c + 2*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)} \end{aligned}$$

$$\begin{aligned}
&)*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^3) - 2*I*(16*I*(d*x + c)^{(5/2)}*b^2*d - 32*I*(d*x + c)^{(3/2)}*b^2*c*d + 16*I*\sqrt{d*x + c}*b^2*c^2*d - 20*(d*x + c)^{(3/2)}*b*d^2 + 24*\sqrt{d*x + c}*b*c*d^2 - 15*I*\sqrt{d*x + c}*d^3)*e^{((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3}/d^2) + 16*(I*\sqrt{2}*\sqrt{\pi}*(8*I*b*c*d - 3*d^2)*d*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((4*I*b*c - 4*I*a*d)/d)/(sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2)} + I*\sqrt{2}*\sqrt{\pi}*(8*I*b*c*d + 3*d^2)*d*\operatorname{erf}(-\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-4*I*b*c + 4*I*a*d)/d)/(sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2)} + 16*I*\sqrt{\pi}*(4*I*b*c*d - 3*d^2)*d*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((2*I*b*c - 2*I*a*d)/d)/(sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2)} + 16*I*\sqrt{\pi}*(4*I*b*c*d + 3*d^2)*d*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-2*I*b*c + 2*I*a*d)/d)/(sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2)} - 4*I*(8*I*(d*x + c)^{(3/2)}*b*d - 8*I*\sqrt{d*x + c}*b*c*d - 3*\sqrt{d*x + c}*d^2)*e^{((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2} - 32*I*(4*I*(d*x + c)^{(3/2)}*b*d - 4*I*\sqrt{d*x + c}*b*c*d - 3*\sqrt{d*x + c}*d^2)*e^{((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^2} - 32*I*(4*I*(d*x + c)^{(3/2)}*b*d - 4*I*\sqrt{d*x + c}*b*c*d + 3*\sqrt{d*x + c}*d^2)*e^{((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2} - 4*I*(8*I*(d*x + c)^{(3/2)}*b*d - 8*I*\sqrt{d*x + c}*b*c*d + 3*\sqrt{d*x + c}*d^2)*e^{((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2}*c)/d
\end{aligned}$$

3.190 $\int (c + dx)^{5/2} \cos^3(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=615

$$\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\sin\left(5a - \frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}} - \frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}} + \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}$$

[Out] $(5*d*(c + d*x)^{(3/2)}*\text{Cos}[a + b*x])/(16*b^2) - (5*d*(c + d*x)^{(3/2)}*\text{Cos}[3*a + 3*b*x])/(288*b^2) - (d*(c + d*x)^{(3/2)}*\text{Cos}[5*a + 5*b*x])/(160*b^2) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a - (b*c)/d]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(32*b^{(7/2)}) - (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(576*b^{(7/2)}) - (3*d^{(5/2)}*\text{Sqrt}[\text{Pi}/10]*\text{Cos}[5*a - (5*b*c)/d]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(1600*b^{(7/2)}) - (3*d^{(5/2)}*\text{Sqrt}[\text{Pi}/10]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[5*a - (5*b*c)/d])/(1600*b^{(7/2)}) - (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[3*a - (3*b*c)/d])/(576*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[a - (b*c)/d])/(32*b^{(7/2)}) - (15*d^2*\text{Sqrt}[c + d*x]*\text{Sin}[a + b*x])/(32*b^3) + ((c + d*x)^{(5/2)}*\text{Sin}[a + b*x])/(8*b) + (5*d^2*\text{Sqrt}[c + d*x]*\text{Sin}[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^{(5/2)}*\text{Sin}[3*a + 3*b*x])/(48*b) + (3*d^2*\text{Sqrt}[c + d*x]*\text{Sin}[5*a + 5*b*x])/(1600*b^3) - ((c + d*x)^{(5/2)}*\text{Sin}[5*a + 5*b*x])/(80*b)$

Rubi [A] time = 1.14485, antiderivative size = 615, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\sin\left(5a - \frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}} - \frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}} + \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^{(5/2)}*\text{Cos}[a + b*x]^3*\text{Sin}[a + b*x]^2, x]$

[Out] $(5*d*(c + d*x)^{(3/2)}*\text{Cos}[a + b*x])/(16*b^2) - (5*d*(c + d*x)^{(3/2)}*\text{Cos}[3*a + 3*b*x])/(288*b^2) - (d*(c + d*x)^{(3/2)}*\text{Cos}[5*a + 5*b*x])/(160*b^2) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a - (b*c)/d]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(32*b^{(7/2)}) - (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(576*b^{(7/2)}) - (3*d^{(5/2)}*\text{Sqrt}[\text{Pi}/10]*\text{Cos}[5*a - (5*b*c)/d]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(1600*b^{(7/2)}) - (3*d^{(5/2)}*\text{Sqrt}[\text{Pi}/10]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[5*a - (5*b*c)/d])/(1600*b^{(7/2)}) - (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[3*a - (3*b*c)/d])/(576*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[a - (b*c)/d])/(32*b^{(7/2)}) - (15*d^2*\text{Sqrt}[c + d*x]*\text{Sin}[a + b*x])/(32*b^3) + ((c + d*x)^{(5/2)}*\text{Sin}[a + b*x])/(8*b) + (5*d^2*\text{Sqrt}[c + d*x]*\text{Sin}[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^{(5/2)}*\text{Sin}[3*a + 3*b*x])/(48*b) + (3*d^2*\text{Sqrt}[c + d*x]*\text{Sin}[5*a + 5*b*x])/(1600*b^3) - ((c + d*x)^{(5/2)}*\text{Sin}[5*a + 5*b*x])/(80*b)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]$

$]^n \cos[a + b \cdot x]^p, x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3306

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]

Rule 3305

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)]]/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3304

Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)]]/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}
\int (c + dx)^{5/2} \cos^3(a + bx) \sin^2(a + bx) dx &= \int \left(\frac{1}{8} (c + dx)^{5/2} \cos(a + bx) - \frac{1}{16} (c + dx)^{5/2} \cos(3a + 3bx) - \frac{1}{16} (c + dx)^{5/2} \cos(5a + 5bx) \right) dx \\
&= - \left(\frac{1}{16} \int (c + dx)^{5/2} \cos(3a + 3bx) dx \right) - \frac{1}{16} \int (c + dx)^{5/2} \cos(5a + 5bx) dx \\
&= \frac{(c + dx)^{5/2} \sin(a + bx)}{8b} - \frac{(c + dx)^{5/2} \sin(3a + 3bx)}{48b} - \frac{(c + dx)^{5/2} \sin(5a + 5bx)}{80b} \\
&= \frac{5d(c + dx)^{3/2} \cos(a + bx)}{16b^2} - \frac{5d(c + dx)^{3/2} \cos(3a + 3bx)}{288b^2} - \frac{d(c + dx)^{3/2} \cos(5a + 5bx)}{160b^2} \\
&= \frac{5d(c + dx)^{3/2} \cos(a + bx)}{16b^2} - \frac{5d(c + dx)^{3/2} \cos(3a + 3bx)}{288b^2} - \frac{d(c + dx)^{3/2} \cos(5a + 5bx)}{160b^2} \\
&= \frac{5d(c + dx)^{3/2} \cos(a + bx)}{16b^2} - \frac{5d(c + dx)^{3/2} \cos(3a + 3bx)}{288b^2} - \frac{d(c + dx)^{3/2} \cos(5a + 5bx)}{160b^2} \\
&= \frac{5d(c + dx)^{3/2} \cos(a + bx)}{16b^2} - \frac{5d(c + dx)^{3/2} \cos(3a + 3bx)}{288b^2} - \frac{d(c + dx)^{3/2} \cos(5a + 5bx)}{160b^2} \\
&= \frac{5d(c + dx)^{3/2} \cos(a + bx)}{16b^2} - \frac{5d(c + dx)^{3/2} \cos(3a + 3bx)}{288b^2} - \frac{d(c + dx)^{3/2} \cos(5a + 5bx)}{160b^2}
\end{aligned}$$

Mathematica [C] time = 23.4923, size = 1795, normalized size = 2.92

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]

[Out]
$$\begin{aligned}
&((-I/16)*c^2*\text{Sqrt}[c + d*x]*((E^{((2*I)*a)}*\text{Gamma}[3/2, ((-I)*b*(c + d*x))/d])/ \\
&\text{Sqrt}[((-I)*b*(c + d*x))/d] - (E^{((2*I)*b*c)/d}*\text{Gamma}[3/2, (I*b*(c + d*x))/ \\
&d])/ \text{Sqrt}[(I*b*(c + d*x))/d]))/(b*E^{((I*(b*c + a*d))/d)} + (c*d*(\text{Sqrt}[b/d]*\text{S} \\
&\text{qrt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[2/Pi]*\text{Sqrt}[c + d*x]]*(-3*d*\text{Cos}[a - (b*c)/ \\
&d] + 2*b*c*\text{Sin}[a - (b*c)/d]) + \text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt} \\
&[2/Pi]*\text{Sqrt}[c + d*x]]*(2*b*c*\text{Cos}[a - (b*c)/d] + 3*d*\text{Sin}[a - (b*c)/d]) + 2*b \\
&* \text{Sqrt}[c + d*x]*(3*\text{Cos}[a + b*x] + 2*b*x*\text{Sin}[a + b*x])))/(16*b^3) + ((b/d)^(3 \\
&/2)*d^2*(-(\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[2/Pi]*\text{Sqrt}[c + d*x]]*((4*b^2*c \\
&c^2 - 15*d^2)*\text{Cos}[a - (b*c)/d] + 12*b*c*d*\text{Sin}[a - (b*c)/d])) - \text{Sqrt}[2*Pi]*\text{F} \\
&\text{resnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[2/Pi]*\text{Sqrt}[c + d*x]]*(-12*b*c*d*\text{Cos}[a - (b*c)/d] + (\\
&4*b^2*c^2 - 15*d^2)*\text{Sin}[a - (b*c)/d]) + 2*\text{Sqrt}[b/d]*d*\text{Sqrt}[c + d*x]*(-2*b*(\\
&c - 5*d*x)*\text{Cos}[a + b*x] + d*(-15 + 4*b^2*x^2)*\text{Sin}[a + b*x]))/(64*b^5) - (c \\
&^2*(-(\text{Sqrt}[2*Pi]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c \\
&+ d*x]]) - \text{Sqrt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*\text{Sin}[3*a \\
&- (3*b*c)/d] + 2*\text{Sqrt}[3]*\text{Sqrt}[b/d]*\text{Sqrt}[c + d*x]*\text{Sin}[3*(a + b*x)])))/(96*\text{Sqr} \\
&\text{t}[3]*b*\text{Sqrt}[b/d]) - (c*d*(\text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi \\
&]*\text{Sqrt}[c + d*x]]*(-(d*\text{Cos}[3*a - (3*b*c)/d]) + 2*b*c*\text{Sin}[3*a - (3*b*c)/d]) + \\
&\text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*(2*b*c*\text{C} \\
&\text{os}[3*a - (3*b*c)/d] + d*\text{Sin}[3*a - (3*b*c)/d]) + 2*\text{Sqrt}[3]*b*\text{Sqrt}[c + d*x]*(\\
&\text{Cos}[3*(a + b*x)] + 2*b*x*\text{Sin}[3*(a + b*x)])))/(96*\text{Sqrt}[3]*b^3) - ((b/d)^(3/2) \\
&)*d^2*(-(\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*((12*b^2*c \\
&^2 - 5*d^2)*\text{Cos}[3*a - (3*b*c)/d] + 12*b*c*d*\text{Sin}[3*a - (3*b*c)/d])) - \text{Sqrt}[2 \\
&*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*(-12*b*c*d*\text{Cos}[3*a - (3*b \\
&*c)/d] + (12*b^2*c^2 - 5*d^2)*\text{Sin}[3*a - (3*b*c)/d]) + 2*\text{Sqrt}[3]*\text{Sqrt}[b/d]*d \\
&*\text{Sqrt}[c + d*x]*(-2*b*(c - 5*d*x)*\text{Cos}[3*(a + b*x)] + d*(-5 + 12*b^2*x^2)*\text{Sin} \\
&[3*(a + b*x)])))/(1152*\text{Sqrt}[3]*b^5) - (c^2*(-(\text{Sqrt}[2*Pi]*\text{Cos}[5*a - (5*b*c)/
\end{aligned}$$

$$d] * \text{FresnelS}[\sqrt{b/d} * \sqrt{10/\pi} * \sqrt{c + dx}] - \sqrt{2\pi} * \text{FresnelC}[\sqrt{b/d} * \sqrt{10/\pi} * \sqrt{c + dx}] * \sin[5a - (5bc)/d] + 2\sqrt{5} * \sqrt{b/d} * \sqrt{c + dx} * \sin[5(a + bx)] / (160\sqrt{5} * b * \sqrt{b/d}) - (c * d * (\sqrt{b/d} * \sqrt{2\pi} * \text{FresnelC}[\sqrt{b/d} * \sqrt{10/\pi} * \sqrt{c + dx}] * (-3d * \cos[5a - (5bc)/d] + 10bc * \sin[5a - (5bc)/d]) + \sqrt{b/d} * \sqrt{2\pi} * \text{FresnelS}[\sqrt{b/d} * \sqrt{10/\pi} * \sqrt{c + dx}] * (10bc * \cos[5a - (5bc)/d] + 3d * \sin[5a - (5bc)/d]) + 2\sqrt{5} * b * \sqrt{c + dx} * (3\cos[5(a + bx)] + 10bx * \sin[5(a + bx)])) / (800\sqrt{5} * b^3) - ((b/d)^{(3/2)} * d^2 * (-\sqrt{2\pi} * \text{FresnelS}[\sqrt{b/d} * \sqrt{10/\pi} * \sqrt{c + dx}] * ((20b^2c^2 - 3d^2) * \cos[5a - (5bc)/d] + 12bc * d * \sin[5a - (5bc)/d])) - \sqrt{2\pi} * \text{FresnelC}[\sqrt{b/d} * \sqrt{10/\pi} * \sqrt{c + dx}] * (-12bc * d * \cos[5a - (5bc)/d] + (20b^2c^2 - 3d^2) * \sin[5a - (5bc)/d]) + 2\sqrt{5} * \sqrt{b/d} * d * \sqrt{c + dx} * (-2b * (c - 5dx) * \cos[5(a + bx)] + d * (-3 + 20b^2x^2) * \sin[5(a + bx)])) / (3200\sqrt{5} * b^5)$$

Maple [A] time = 0.051, size = 716, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((dx+c)^{(5/2)} * \cos(bx+a)^3 * \sin(bx+a)^2, x)$

[Out] $2/d * (1/16/b * d * (dx+c)^{(5/2)} * \sin(1/d * (dx+c) * b + (a*d-b*c)/d) - 5/16/b * d * (-1/2/b * d * (dx+c)^{(3/2)} * \cos(1/d * (dx+c) * b + (a*d-b*c)/d) + 3/2/b * d * (1/2/b * d * (dx+c)^{(1/2)} * \sin(1/d * (dx+c) * b + (a*d-b*c)/d) - 1/4/b * d * 2^{(1/2)} * \pi^{(1/2)} / (b/d)^{(1/2)} * (\cos((a*d-b*c)/d) * \text{FresnelS}(2^{(1/2)}/\pi^{(1/2)} / (b/d)^{(1/2)} * (dx+c)^{(1/2)} * b/d) + \sin((a*d-b*c)/d) * \text{FresnelC}(2^{(1/2)}/\pi^{(1/2)} / (b/d)^{(1/2)} * (dx+c)^{(1/2)} * b/d))) - 1/96/b * d * (dx+c)^{(5/2)} * \sin(3/d * (dx+c) * b + 3 * (a*d-b*c)/d) + 5/96/b * d * (-1/6/b * d * (dx+c)^{(3/2)} * \cos(3/d * (dx+c) * b + 3 * (a*d-b*c)/d) + 1/2/b * d * (1/6/b * d * (dx+c)^{(1/2)} * \sin(3/d * (dx+c) * b + 3 * (a*d-b*c)/d) - 1/36/b * d * 2^{(1/2)} * \pi^{(1/2)} * 3^{(1/2)} / (b/d)^{(1/2)} * (\cos(3 * (a*d-b*c)/d) * \text{FresnelS}(2^{(1/2)}/\pi^{(1/2)} * 3^{(1/2)} / (b/d)^{(1/2)} * (dx+c)^{(1/2)} * b/d) + \sin(3 * (a*d-b*c)/d) * \text{FresnelC}(2^{(1/2)}/\pi^{(1/2)} * 3^{(1/2)} / (b/d)^{(1/2)} * (dx+c)^{(1/2)} * b/d))) - 1/160/b * d * (dx+c)^{(5/2)} * \sin(5/d * (dx+c) * b + 5 * (a*d-b*c)/d) + 1/32/b * d * (-1/10/b * d * (dx+c)^{(3/2)} * \cos(5/d * (dx+c) * b + 5 * (a*d-b*c)/d) + 3/10/b * d * (1/10/b * d * (dx+c)^{(1/2)} * \sin(5/d * (dx+c) * b + 5 * (a*d-b*c)/d) - 1/100/b * d * 2^{(1/2)} * \pi^{(1/2)} * 5^{(1/2)} / (b/d)^{(1/2)} * (\cos(5 * (a*d-b*c)/d) * \text{FresnelS}(2^{(1/2)}/\pi^{(1/2)} * 5^{(1/2)} / (b/d)^{(1/2)} * (dx+c)^{(1/2)} * b/d) + \sin(5 * (a*d-b*c)/d) * \text{FresnelC}(2^{(1/2)}/\pi^{(1/2)} * 5^{(1/2)} / (b/d)^{(1/2)} * (dx+c)^{(1/2)} * b/d)))$

Maxima [C] time = 2.64826, size = 2940, normalized size = 4.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((dx+c)^{(5/2)} * \cos(bx+a)^3 * \sin(bx+a)^2, x, \text{algorithm}="maxima")$

[Out] $-1/1728000 * \sqrt{5} * \sqrt{3} * (720 * \sqrt{5} * \sqrt{3} * (dx + c)^{(3/2)} * b * d^2 * \sqrt{(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) * \cos(5 * ((dx + c) * b - b * c + a * d) / d) / \text{abs}(d)} + 2000 * \sqrt{5} * \sqrt{3} * (dx + c)^{(3/2)} * b * d^2 * \sqrt{(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) * \cos(3 * ((dx + c) * b - b * c + a * d) / d) / \text{abs}(d)} - 36000 * \sqrt{5} * \sqrt{3} * (dx + c)^{(3/2)} * b * d^2 * \sqrt{(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) * \cos(((dx + c) * b - b * c + a * d) / d) / \text{abs}(d)} + (\sqrt{3}) * (27 * I * \sqrt{\pi}) * \cos(1/4 * \pi + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\sqrt{d^2}))$

$$2(0, d/\sqrt{d^2})) * d^3 \text{abs}(b) \sin(-5*(b*c - a*d)/d) / \text{abs}(d) * \text{erf}(\sqrt{d*x + c} * \sqrt{-5*I*b/d}) + 72*(20*\sqrt{5}*\sqrt{3}*(d*x + c)^{(5/2)} * b^2 * d * \sqrt{\text{abs}(b)/\text{abs}(d)} * \text{abs}(b)/\text{abs}(d) - 3*\sqrt{5}*\sqrt{3}*\sqrt{d*x + c} * d^3 * \sqrt{\text{abs}(b)/\text{abs}(d)} * \text{abs}(b)/\text{abs}(d)) * \sin(5*((d*x + c)*b - b*c + a*d)/d) + 200*(12*\sqrt{5}*\sqrt{3}*(d*x + c)^{(5/2)} * b^2 * d * \sqrt{\text{abs}(b)/\text{abs}(d)} * \text{abs}(b)/\text{abs}(d) - 5*\sqrt{5}*\sqrt{3}*\sqrt{d*x + c} * d^3 * \sqrt{\text{abs}(b)/\text{abs}(d)} * \text{abs}(b)/\text{abs}(d)) * \sin(3*((d*x + c)*b - b*c + a*d)/d) - 3600*(4*\sqrt{5}*\sqrt{3}*(d*x + c)^{(5/2)} * b^2 * d * \sqrt{\text{abs}(b)/\text{abs}(d)} * \text{abs}(b)/\text{abs}(d) - 15*\sqrt{5}*\sqrt{3}*\sqrt{d*x + c} * d^3 * \sqrt{\text{abs}(b)/\text{abs}(d)} * \text{abs}(b)/\text{abs}(d)) * \sin(((d*x + c)*b - b*c + a*d)/d) * \text{abs}(d) / (b^3 * d * \sqrt{\text{abs}(b)/\text{abs}(d)} * \text{abs}(b))$$

Fricas [A] time = 0.839235, size = 1397, normalized size = 2.27

$$81 \sqrt{10} \pi d^3 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{5(bc-ad)}{d}\right) S\left(\sqrt{10} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) + 625 \sqrt{6} \pi d^3 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{3(bc-ad)}{d}\right) S\left(\sqrt{6} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) - 101250 \sqrt{2} \pi d^3 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{bc-ad}{d}\right) S\left(\sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) - 101250 \sqrt{2} \pi d^3 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{bc-ad}{d}\right) S\left(\sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) + 625 \sqrt{6} \pi d^3 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{3(bc-ad)}{d}\right) S\left(\sqrt{6} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) + 81 \sqrt{10} \pi d^3 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{5(bc-ad)}{d}\right) S\left(\sqrt{10} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) + 480(90(b^2 d^2 x + b^2 c d) \cos(bx+a)^5 - 50(b^2 d^2 x + b^2 c d) \cos(bx+a)^3 - 300(b^2 d^2 x + b^2 c d) \cos(bx+a) - (120 b^3 d^2 x^2 + 240 b^3 c d x + 120 b^3 c^2 - 9(20 b^3 d^2 x^2 + 40 b^3 c d x + 20 b^3 c^2 - 3 b d^2) \cos(bx+a)^4 - 428 b d^2 + (60 b^3 d^2 x^2 + 120 b^3 c d x + 60 b^3 c^2 + 11 b d^2) \cos(bx+a)^2) \sin(bx+a) \sqrt{d*x+c}) / b^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")

[Out] -1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_sin(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 625*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 625*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(90*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^5 - 50*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3 - 300*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) - (120*b^3*d^2*x^2 + 240*b^3*c*d*x + 120*b^3*c^2 - 9*(20*b^3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*cos(b*x + a)^4 - 428*b*d^2 + (60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 + 11*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^4

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)

[Out] Timed out

Giac [C] time = 2.16739, size = 4077, normalized size = 6.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")

```

[Out] -1/864000*(60*(9*I*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d
*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(
I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sq
rt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d
)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*I*sqrt(2)*sqrt(pi)*d^2*erf(
-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c
- I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*sqrt(p
i)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/
d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*s
qrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)*b) - 9*I*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I
*b*d/sqrt(b^2*d^2) + 1)*b) - 90*I*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I
*b*c + 5*I*a*d)/d)/b - 150*I*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c
+ 3*I*a*d)/d)/b + 900*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/
d)/b - 900*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 150
*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*I*sq
rt(d*x + c)*d*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)*c^2 - d^2*(27
*(sqrt(10)*sqrt(pi))*(-20*I*b^2*c^2*d + 12*b*c*d^2 + 3*I*d^3)*d*erf(-1/2*sq
rt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*
I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 10*(20*I*(d*x + c)^(5
/2)*b^2*d - 40*I*(d*x + c)^(3/2)*b^2*c*d + 20*I*sqrt(d*x + c)*b^2*c^2*d + 1
0*(d*x + c)^(3/2)*b*d^2 - 12*sqrt(d*x + c)*b*c*d^2 - 3*I*sqrt(d*x + c)*d^3)
*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^3)/d^2 + 125*(sqrt(6)*sqrt(
pi))*(-12*I*b^2*c^2*d + 12*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*s
qrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b
*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*(12*I*(d*x + c)^(5/2)*b^2*d - 24*I*(
d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*
b*d^2 - 12*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((-3*I*(d*x + c
)*b + 3*I*b*c - 3*I*a*d)/d)/b^3)/d^2 + 6750*(sqrt(2)*sqrt(pi))*(4*I*b^2*c^2*
d - 12*b*c*d^2 - 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b^3) - 2*(-4*I*(d*x + c)^(5/2)*b^2*d + 8*I*(d*x + c)^(3/2)*b^2*c*d
- 4*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 12*sqrt(d*x + c)
*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b
^3)/d^2 + 6750*(sqrt(2)*sqrt(pi))*(-4*I*b^2*c^2*d - 12*b*c*d^2 + 15*I*d^3)*d
*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(4*I*(d*
x + c)^(5/2)*b^2*d - 8*I*(d*x + c)^(3/2)*b^2*c*d + 4*I*sqrt(d*x + c)*b^2*c^
2*d - 10*(d*x + c)^(3/2)*b*d^2 + 12*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x +
c)*d^3)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^2 + 125*(sqrt(6)*sqrt
(pi))*(12*I*b^2*c^2*d + 12*b*c*d^2 - 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*s
qrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*(-12*I*(d*x + c)^(5/2)*b^2*d + 24
*I*(d*x + c)^(3/2)*b^2*c*d - 12*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3
/2)*b*d^2 - 12*sqrt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((3*I*(d*x
+ c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/d^2 + 27*(sqrt(10)*sqrt(pi))*(20*I*b^2*c
^2*d + 12*b*c*d^2 - 3*I*d^3)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/s
qrt(b^2*d^2) + 1)*b^3) - 10*(-20*I*(d*x + c)^(5/2)*b^2*d + 40*I*(d*x + c)^(
3/2)*b^2*c*d - 20*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 12
*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*x + c)*d^3)*e^((5*I*(d*x + c)*b - 5*I*b
*c + 5*I*a*d)/d)/b^3)/d^2 - 12*(9*sqrt(10)*sqrt(pi))*(10*I*b*c*d - 3*d^2)*d
*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
(5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 125*sqrt
(6)*sqrt(pi)*(2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b^2) + 2250*sqrt(2)*sqrt(pi))*(-2*I*b*c*d + 3*d^2)*d*erf(-1/

```

$$\begin{aligned}
& 2\sqrt{2}\sqrt{b*d}\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d*e^{((I*b*c - I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2)} + 2250\sqrt{2}\sqrt{\pi} \\
& *(2*I*b*c*d + 3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-I*b*c + I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2)} + 125\sqrt{6}\sqrt{\pi}*(-2*I*b*c*d - d^2)*d*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-3*I*b*c + 3*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2)} + 9\sqrt{10}\sqrt{\pi}*(-10*I*b*c*d - 3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{10}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-5*I*b*c + 5*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2)} - 90*(-10*I*(d*x + c)^{(3/2)}*b*d + 10*I*\sqrt{d*x + c}*b*c*d + 3*\sqrt{d*x + c}*d^2)*e^{((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^2} - 750*(-2*I*(d*x + c)^{(3/2)}*b*d + 2*I*\sqrt{d*x + c}*b*c*d + \sqrt{d*x + c}*d^2)*e^{((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2} - 4500*(2*I*(d*x + c)^{(3/2)}*b*d - 2*I*\sqrt{d*x + c}*b*c*d - 3*\sqrt{d*x + c}*d^2)*e^{((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2} - 4500*(-2*I*(d*x + c)^{(3/2)}*b*d + 2*I*\sqrt{d*x + c}*b*c*d - 3*\sqrt{d*x + c}*d^2)*e^{((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2} - 750*(2*I*(d*x + c)^{(3/2)}*b*d - 2*I*\sqrt{d*x + c}*b*c*d + \sqrt{d*x + c}*d^2)*e^{((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2} - 90*(10*I*(d*x + c)^{(3/2)}*b*d - 10*I*\sqrt{d*x + c}*b*c*d + 3*\sqrt{d*x + c}*d^2)*e^{((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^2}*c)/d
\end{aligned}$$

3.191 $\int (c + dx)^{3/2} \cos^3(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=534

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}} + \frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} + \frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\cos\left(5a - \frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}$$

[Out] $(3*d*\text{Sqrt}[c + d*x]*\text{Cos}[a + b*x])/(16*b^2) - (d*\text{Sqrt}[c + d*x]*\text{Cos}[3*a + 3*b*x])/(96*b^2) - (3*d*\text{Sqrt}[c + d*x]*\text{Cos}[5*a + 5*b*x])/(800*b^2) - (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a - (b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(16*b^{(5/2)}) + (d^{(3/2)}*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(96*b^{(5/2)}) + (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/10]*\text{Cos}[5*a - (5*b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(800*b^{(5/2)}) - (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/10]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]]*\text{Sin}[5*a - (5*b*c)/d])/(800*b^{(5/2)}) - (d^{(3/2)}*\text{Sqrt}[\text{Pi}/6]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]]*\text{Sin}[3*a - (3*b*c)/d])/(96*b^{(5/2)}) + (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]]*\text{Sin}[a - (b*c)/d])/(16*b^{(5/2)}) + ((c + d*x)^{(3/2)}*\text{Sin}[a + b*x])/(8*b) - ((c + d*x)^{(3/2)}*\text{Sin}[3*a + 3*b*x])/(48*b) - ((c + d*x)^{(3/2)}*\text{Sin}[5*a + 5*b*x])/(80*b)$

Rubi [A] time = 0.837616, antiderivative size = 534, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}} + \frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} + \frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\cos\left(5a - \frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^{(3/2)}*\text{Cos}[a + b*x]^3*\text{Sin}[a + b*x]^2, x]$

[Out] $(3*d*\text{Sqrt}[c + d*x]*\text{Cos}[a + b*x])/(16*b^2) - (d*\text{Sqrt}[c + d*x]*\text{Cos}[3*a + 3*b*x])/(96*b^2) - (3*d*\text{Sqrt}[c + d*x]*\text{Cos}[5*a + 5*b*x])/(800*b^2) - (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a - (b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(16*b^{(5/2)}) + (d^{(3/2)}*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(96*b^{(5/2)}) + (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/10]*\text{Cos}[5*a - (5*b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(800*b^{(5/2)}) - (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/10]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]]*\text{Sin}[5*a - (5*b*c)/d])/(800*b^{(5/2)}) - (d^{(3/2)}*\text{Sqrt}[\text{Pi}/6]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]]*\text{Sin}[3*a - (3*b*c)/d])/(96*b^{(5/2)}) + (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]]*\text{Sin}[a - (b*c)/d])/(16*b^{(5/2)}) + ((c + d*x)^{(3/2)}*\text{Sin}[a + b*x])/(8*b) - ((c + d*x)^{(3/2)}*\text{Sin}[3*a + 3*b*x])/(48*b) - ((c + d*x)^{(3/2)}*\text{Sin}[5*a + 5*b*x])/(80*b)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] := \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^{3/2} \cos^3(a + bx) \sin^2(a + bx) dx &= \int \left(\frac{1}{8}(c + dx)^{3/2} \cos(a + bx) - \frac{1}{16}(c + dx)^{3/2} \cos(3a + 3bx) - \frac{1}{16}(c + dx)^{3/2} \cos(5a + 5bx) \right) dx \\
&= -\left(\frac{1}{16} \int (c + dx)^{3/2} \cos(3a + 3bx) dx \right) - \frac{1}{16} \int (c + dx)^{3/2} \cos(5a + 5bx) dx + \frac{(c + dx)^{3/2} \sin(a + bx)}{8b} \\
&= \frac{(c + dx)^{3/2} \sin(a + bx)}{8b} - \frac{(c + dx)^{3/2} \sin(3a + 3bx)}{48b} - \frac{(c + dx)^{3/2} \sin(5a + 5bx)}{80b} \\
&= \frac{3d\sqrt{c + dx} \cos(a + bx)}{16b^2} - \frac{d\sqrt{c + dx} \cos(3a + 3bx)}{96b^2} - \frac{3d\sqrt{c + dx} \cos(5a + 5bx)}{800b^2} \\
&= \frac{3d\sqrt{c + dx} \cos(a + bx)}{16b^2} - \frac{d\sqrt{c + dx} \cos(3a + 3bx)}{96b^2} - \frac{3d\sqrt{c + dx} \cos(5a + 5bx)}{800b^2} \\
&= \frac{3d\sqrt{c + dx} \cos(a + bx)}{16b^2} - \frac{d\sqrt{c + dx} \cos(3a + 3bx)}{96b^2} - \frac{3d\sqrt{c + dx} \cos(5a + 5bx)}{800b^2} \\
&= \frac{3d\sqrt{c + dx} \cos(a + bx)}{16b^2} - \frac{d\sqrt{c + dx} \cos(3a + 3bx)}{96b^2} - \frac{3d\sqrt{c + dx} \cos(5a + 5bx)}{800b^2}
\end{aligned}$$

Mathematica [C] time = 12.5847, size = 1043, normalized size = 1.95

$$\frac{ice^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\Gamma\left(\frac{3}{2}-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{-\frac{2ibc}{d}}\Gamma\left(\frac{3}{2}+\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b}+d\left(\sqrt{\frac{b}{d}}\sqrt{2\pi}\text{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(2bc\sin\left(\frac{b(c+dx)}{d}\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]

[Out]
$$\frac{(-I/16)*c*\text{Sqrt}[c + d*x]*((E^{((2*I)*a)}*\Gamma[3/2, ((-I)*b*(c + d*x))/d])/ \text{Sqrt}[((-I)*b*(c + d*x))/d] - (E^{((2*I)*b*c)/d}*\Gamma[3/2, (I*b*(c + d*x))/d])/ \text{Sqrt}[(I*b*(c + d*x))/d])/(b*E^{((I*(b*c + a*d))/d)} + (d*(\text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[2/Pi]*\text{Sqrt}[c + d*x]]*(-3*d*\text{Cos}[a - (b*c)/d] + 2*b*c*\text{Sin}[a - (b*c)/d]) + \text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[2/Pi]*\text{Sqrt}[c + d*x]]*(2*b*c*\text{Cos}[a - (b*c)/d] + 3*d*\text{Sin}[a - (b*c)/d]) + 2*b*\text{Sqrt}[c + d*x]*(3*\text{Cos}[a + b*x] + 2*b*x*\text{Sin}[a + b*x])))/(32*b^3) - (c*(-\text{Sqrt}[2*Pi]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]) - \text{Sqrt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*\text{Sin}[3*a - (3*b*c)/d] + 2*\text{Sqrt}[3]*\text{Sqrt}[b/d]*\text{Sqrt}[c + d*x]*\text{Sin}[3*(a + b*x)]))/(96*\text{Sqrt}[3]*b*\text{Sqrt}[b/d]) - (d*(\text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*(-d*\text{Cos}[3*a - (3*b*c)/d]) + 2*b*c*\text{Sin}[3*a - (3*b*c)/d]) + \text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*(2*b*c*\text{Cos}[3*a - (3*b*c)/d] + d*\text{Sin}[3*a - (3*b*c)/d]) + 2*\text{Sqrt}[3]*b*\text{Sqrt}[c + d*x]*(\text{Cos}[3*(a + b*x)] + 2*b*x*\text{Sin}[3*(a + b*x)])))/(192*\text{Sqrt}[3]*b^3) - (c*(-\text{Sqrt}[2*Pi]*\text{Cos}[5*a - (5*b*c)/d]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[10/Pi]*\text{Sqrt}[c + d*x]]) - \text{Sqrt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[10/Pi]*\text{Sqrt}[c + d*x]]*\text{Sin}[5*a - (5*b*c)/d] + 2*\text{Sqrt}[5]*\text{Sqrt}[b/d]*\text{Sqrt}[c + d*x]*\text{Sin}[5*(a + b*x)]))/(160*\text{Sqrt}[5]*b*\text{Sqrt}[b/d]) - (d*(\text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[10/Pi]*\text{Sqrt}[c + d*x]]*(-3*d*\text{Cos}[5*a - (5*b*c)/d] + 10*b*c*\text{Sin}[5*a - (5*b*c)/d]) + \text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[10/Pi]*\text{Sqrt}[c + d*x]]*(10*b*c*\text{Cos}[5*a - (5*b*c)/d] + 3*d*\text{Sin}[5*a - (5*b*c)/d]) + 2*\text{Sqrt}[5]*b*\text{Sqrt}[c + d*x]*(3*\text{Cos}[5*(a + b*x)] + 10*b*x*\text{Sin}[5*(a + b*x)])))/(1600*\text{Sqrt}[5]*b^3)$$

Maple [A] time = 0.051, size = 583, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)

[Out]
$$\frac{2}{d}\left(\frac{1}{16b}\frac{d*(d*x+c)^{3/2}*\sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/16/b*d*(-1/2/b*d*(d*x+c)^{1/2}*\cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^{1/2}*Pi^{1/2}/(b/d)^{1/2}*(\cos((a*d-b*c)/d)*\text{FresnelC}(2^{1/2}/Pi^{1/2}/(b/d)^{1/2}*(d*x+c)^{1/2}*b/d)-\sin((a*d-b*c)/d)*\text{FresnelS}(2^{1/2}/Pi^{1/2}/(b/d)^{1/2}*(d*x+c)^{1/2}*b/d))}{196/b*d*(d*x+c)^{3/2}*\sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/32/b*d*(-1/6/b*d*(d*x+c)^{1/2}*\cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^{1/2}*Pi^{1/2}*3^{1/2}/(b/d)^{1/2}*(\cos(3*(a*d-b*c)/d)*\text{FresnelC}(2^{1/2}/Pi^{1/2}*3^{1/2}/(b/d)^{1/2}*(d*x+c)^{1/2}*b/d)-\sin(3*(a*d-b*c)/d)*\text{FresnelS}(2^{1/2}/Pi^{1/2}*3^{1/2}/(b/d)^{1/2}*(d*x+c)^{1/2}*b/d))}{160/b*d*(d*x+c)^{3/2}*\sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+3/160/b*d*(-1/10/b*d*(d*x+c)^{1/2}*\cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+1/100/b*d*2^{1/2}*Pi^{1/2}*5^{1/2}/(b/d)^{1/2}*(\cos(5*(a*d-b*c)/d)*\text{FresnelC}(2^{1/2}/Pi^{1/2}*5^{1/2}/(b/d)^{1/2}*(d*x+c)^{1/2}*b/d)-\sin(5*(a*d-b*c)/d)*\text{FresnelS}(2^{1/2}/Pi^{1/2}*5^{1/2}/(b/d)^{1/2}*(d*x+c)^{1/2}*b/d)}\right)$$

$$x+c)^{(1/2)*b/d))$$

Maxima [C] time = 2.61676, size = 2790, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")

[Out]
$$-1/288000*\sqrt{5}*\sqrt{3}*(240*\sqrt{5}*\sqrt{3}*(d*x + c)^{(3/2)}*b*d*\sqrt{\text{abs}(b)/\text{abs}(d)}*\text{abs}(b)*\sin(5*((d*x + c)*b - b*c + a*d)/d)/\text{abs}(d) + 400*\sqrt{5}*\sqrt{3}*(d*x + c)^{(3/2)}*b*d*\sqrt{\text{abs}(b)/\text{abs}(d)}*\text{abs}(b)*\sin(3*((d*x + c)*b - b*c + a*d)/d)/\text{abs}(d) - 2400*\sqrt{5}*\sqrt{3}*(d*x + c)^{(3/2)}*b*d*\sqrt{\text{abs}(b)/\text{abs}(d)}*\text{abs}(b)*\sin(((d*x + c)*b - b*c + a*d)/d)/\text{abs}(d) + 72*\sqrt{5}*\sqrt{3}*\sqrt{d*x + c}*d^2*\sqrt{\text{abs}(b)/\text{abs}(d)}*\text{abs}(b)*\cos(5*((d*x + c)*b - b*c + a*d)/d)/\text{abs}(d) + 200*\sqrt{5}*\sqrt{3}*\sqrt{d*x + c}*d^2*\sqrt{\text{abs}(b)/\text{abs}(d)}*\text{abs}(b)*\cos(3*((d*x + c)*b - b*c + a*d)/d)/\text{abs}(d) - 3600*\sqrt{5}*\sqrt{3}*\sqrt{d*x + c}*d^2*\sqrt{\text{abs}(b)/\text{abs}(d)}*\text{abs}(b)*\cos(((d*x + c)*b - b*c + a*d)/d)/\text{abs}(d) - (\sqrt{3}*(9*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 9*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 9*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 9*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) *d^2*\text{abs}(b)*\cos(-5*(b*c - a*d)/d)/\text{abs}(d) - \sqrt{3}*(9*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 9*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 9*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 9*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) *d^2*\text{abs}(b)*\sin(-5*(b*c - a*d)/d)/\text{abs}(d))*\text{erf}(\sqrt{d*x + c})*\sqrt{5*I*b/d) - (\sqrt{5}*(25*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 25*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 25*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 25*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) *d^2*\text{abs}(b)*\cos(-3*(b*c - a*d)/d)/\text{abs}(d) - \sqrt{5}*(25*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 25*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 25*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 25*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) *d^2*\text{abs}(b)*\sin(-3*(b*c - a*d)/d)/\text{abs}(d))*\text{erf}(\sqrt{d*x + c})*\sqrt{3*I*b/d) + (\sqrt{5}*\sqrt{3}*(450*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 450*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 450*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 450*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) *d^2*\text{abs}(b)*\cos(-(b*c - a*d)/d)/\text{abs}(d) + \sqrt{5}*\sqrt{3}*(-450*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 450*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 450*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 450*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) *d^2*\text{abs}(b)*\sin(-(b*c - a*d)/d)/\text{abs}(d))*\text{erf}(\sqrt{d*x + c})*\sqrt{I*b/d) + (\sqrt{5}*\sqrt{3}*(450*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 450*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 450*I*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 450*I*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) *d^2*\text{abs}(b)*\cos(-(b*c - a*d)/d)/\text{abs}(d) + \sqrt{5}*\sqrt{3}*(450*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 450*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 450*\sqrt{\pi}*\sin(1$$

$$\begin{aligned} & /4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 450*\sqrt{\pi}*\sin \\ & (-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^2 * \text{abs}(b) * \sin \\ & (-(b*c - a*d)/d) / \text{abs}(d) * \text{erf}(\sqrt{d*x + c} * \sqrt{-I*b/d}) - (\sqrt{5} * (25*\sqrt{\pi} * \cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 25*\sqrt{\pi} * \cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 25*I*\sqrt{\pi} * \sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 25*I*\sqrt{\pi} * \sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^2 * \text{abs}(b) * \cos(-3*(b*c - a*d)/d) / \text{abs}(d) - \sqrt{5} * (-25*I*\sqrt{\pi} * \cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 25*I*\sqrt{\pi} * \cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 25*\sqrt{\pi} * \sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 25*\sqrt{\pi} * \sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^2 * \text{abs}(b) * \sin(-3*(b*c - a*d)/d) / \text{abs}(d) * \text{erf}(\sqrt{d*x + c} * \sqrt{-3*I*b/d}) - (\sqrt{3} * (9*\sqrt{\pi} * \cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 9*\sqrt{\pi} * \cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 9*I*\sqrt{\pi} * \sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 9*I*\sqrt{\pi} * \sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^2 * \text{abs}(b) * \cos(-5*(b*c - a*d)/d) / \text{abs}(d) - \sqrt{3} * (-9*I*\sqrt{\pi} * \cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 9*I*\sqrt{\pi} * \cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 9*\sqrt{\pi} * \sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 9*\sqrt{\pi} * \sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^2 * \text{abs}(b) * \sin(-5*(b*c - a*d)/d) / \text{abs}(d) * \text{erf}(\sqrt{d*x + c} * \sqrt{-5*I*b/d}) * \text{abs}(d) / (b^2 * d * \sqrt{\text{abs}(b) / \text{abs}(d)}) * \text{abs}(b)) \end{aligned}$$

Fricas [A] time = 0.765724, size = 1162, normalized size = 2.18

$$27 \sqrt{10} \pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{5(bc-ad)}{d}\right) C\left(\sqrt{10} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) + 125 \sqrt{6} \pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{3(bc-ad)}{d}\right) C\left(\sqrt{6} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) - 6750 \sqrt{2} \pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{b*c - a*d}{d}\right) \text{fresnel_cos}\left(\sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) + 6750 \sqrt{2} \pi d^2 \sqrt{\frac{b}{\pi d}} \text{fresnel_sin}\left(\sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) * \sin\left(-\frac{b*c - a*d}{d}\right) - 125 \sqrt{6} \pi d^2 \sqrt{\frac{b}{\pi d}} \text{fresnel_sin}\left(\sqrt{6} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) * \sin\left(-\frac{3*(b*c - a*d)}{d}\right) - 27 \sqrt{10} \pi d^2 \sqrt{\frac{b}{\pi d}} \text{fresnel_sin}\left(\sqrt{10} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) * \sin\left(-\frac{5*(b*c - a*d)}{d}\right) - 480 * (9*b*d*\cos(b*x + a)^5 - 5*b*d*\cos(b*x + a)^3 - 30*b*d*\cos(b*x + a) + 10*(3*(b^2*d*x + b^2*c)*\cos(b*x + a)^4 - 2*b^2*d*x - 2*b^2*c - (b^2*d*x + b^2*c)*\cos(b*x + a)^2) * \sin(b*x + a)) * \sqrt{dx+c}) / b^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")

[Out] 1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_cos(sqrt(10)*sqrt(dx + c)*sqrt(b/(pi*d))) + 125*sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(dx + c)*sqrt(b/(pi*d))) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(dx + c)*sqrt(b/(pi*d))) + 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(dx + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(dx + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(10)*sqrt(dx + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(9*b*d*cos(b*x + a)^5 - 5*b*d*cos(b*x + a)^3 - 30*b*d*cos(b*x + a) + 10*(3*(b^2*d*x + b^2*c)*cos(b*x + a)^4 - 2*b^2*d*x - 2*b^2*c - (b^2*d*x + b^2*c)*cos(b*x + a)^2) * sin(b*x + a)) * sqrt(dx + c)) / b^3

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)

[Out] Timed out

Giac [C] time = 1.77981, size = 2271, normalized size = 4.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")

[Out]
$$-1/144000*(10*(9*I*\sqrt{10}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{10}*\sqrt{b*d}*\sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((5*I*b*c-5*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2}+1)*b)}+25*I*\sqrt{6}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((3*I*b*c-3*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2}+1)*b)}-450*I*\sqrt{2}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((I*b*c-I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2}+1)*b)}+450*I*\sqrt{2}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((-I*b*c+I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2}+1)*b)}-25*I*\sqrt{6}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((-3*I*b*c+3*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2}+1)*b)}-9*I*\sqrt{10}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{10}*\sqrt{b*d}*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((-5*I*b*c+5*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2}+1)*b)}-90*I*\sqrt{d*x+c})*d*e^{((5*I*(d*x+c)*b-5*I*b*c+5*I*a*d)/d)/b}-150*I*\sqrt{d*x+c})*d*e^{((3*I*(d*x+c)*b-3*I*b*c+3*I*a*d)/d)/b}+900*I*\sqrt{d*x+c})*d*e^{((I*(d*x+c)*b-I*b*c+I*a*d)/d)/b}-900*I*\sqrt{d*x+c})*d*e^{((-I*(d*x+c)*b+I*b*c-I*a*d)/d)/b}+150*I*\sqrt{d*x+c})*d*e^{((-3*I*(d*x+c)*b+3*I*b*c-3*I*a*d)/d)/b}+90*I*\sqrt{d*x+c})*d*e^{((-5*I*(d*x+c)*b+5*I*b*c-5*I*a*d)/d)/b}*c-9*\sqrt{10}*\sqrt{\pi})*(10*I*b*c*d-3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{10}*\sqrt{b*d}*\sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((5*I*b*c-5*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2}+1)*b^2)}-125*\sqrt{6}*\sqrt{\pi})*(2*I*b*c*d-d^2)*d*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((3*I*b*c-3*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2}+1)*b^2)}-2250*\sqrt{2}*\sqrt{\pi})*(-2*I*b*c*d+3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((I*b*c-I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2}+1)*b^2)}-2250*\sqrt{2}*\sqrt{\pi})*(2*I*b*c*d+3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((-I*b*c+I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2}+1)*b^2)}-125*\sqrt{6}*\sqrt{\pi})*(-2*I*b*c*d-d^2)*d*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((-3*I*b*c+3*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2}+1)*b^2)}-9*\sqrt{10}*\sqrt{\pi})*(-10*I*b*c*d-3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{10}*\sqrt{b*d}*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((-5*I*b*c+5*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2}+1)*b^2)}+90*(-10*I*(d*x+c)^(3/2)*b*d+10*I*\sqrt{d*x+c})*b*c*d+3*\sqrt{d*x+c})*d^2)*e^{((5*I*(d*x+c)*b-5*I*b*c+5*I*a*d)/d)/b^2}+750*(-2*I*(d*x+c)^(3/2)*b*d+2*I*\sqrt{d*x+c})*b*c*d+\sqrt{d*x+c})*d^2)*e^{((3*I*(d*x+c)*b-3*I*b*c+3*I*a*d)/d)/b^2}+4500*(2*I*(d*x+c)^(3/2)*b*d-2*I*\sqrt{d*x+c})*b*c*d-3*\sqrt{d*x+c})*d^2)*e^{((I*(d*x+c)*b-I*b*c+I*a*d)/d)/b^2}+4500*(-2*I*(d*x+c)^(3/2)*b*d+2*I*\sqrt{d*x+c})*b*c*d-3*\sqrt{d*x+c})*d^2)*e^{((-I*(d*x+c)*b+I*b*c-I*a*d)/d)/b^2}+750*(2*I*(d*x+c)^(3/2)*b*d-2*I*\sqrt{d*x+c})*b*c*d+\sqrt{d*x+c})*d^2)*e^{((-3*I*(d*x+c)*b+3*I*b*c-3*I*a*d)/d)/b^2}+90*(10*I*(d*x+c)^(3/2)*b*d-10*I*\sqrt{d*x+c})*b*c*d+3*\sqrt{d*x+c})*d^2)*e^{((-5*I*(d*x+c)*b+5*I*b*c-5*I*a*d)/d)/b^2)/d$$

3.192 $\int \sqrt{c + dx} \cos^3(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=459

$$\frac{\sqrt{\frac{\pi}{10}} \sqrt{d} \sin\left(5a - \frac{5bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}} + \frac{\sqrt{\frac{\pi}{6}} \sqrt{d} \sin\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \sin\left(a - \frac{bc}{d}\right)}{48b^{3/2}}$$

[Out] $-(\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/2] \text{Cos}[a - (b*c)/d] \text{FresnelS}[(\text{Sqrt}[b] \text{Sqrt}[2/\text{Pi}] \text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(8*b^{(3/2)}) + (\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/6] \text{Cos}[3*a - (3*b*c)/d] \text{FresnelS}[(\text{Sqrt}[b] \text{Sqrt}[6/\text{Pi}] \text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(48*b^{(3/2)}) + (\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/10] \text{Cos}[5*a - (5*b*c)/d] \text{FresnelS}[(\text{Sqrt}[b] \text{Sqrt}[10/\text{Pi}] \text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(80*b^{(3/2)}) + (\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/10] \text{FresnelC}[(\text{Sqrt}[b] \text{Sqrt}[10/\text{Pi}] \text{Sqrt}[c + d*x])/ \text{Sqrt}[d]] \text{Sin}[5*a - (5*b*c)/d])/(80*b^{(3/2)}) + (\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/6] \text{FresnelC}[(\text{Sqrt}[b] \text{Sqrt}[6/\text{Pi}] \text{Sqrt}[c + d*x])/ \text{Sqrt}[d]] \text{Sin}[3*a - (3*b*c)/d])/(48*b^{(3/2)}) - (\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/2] \text{FresnelC}[(\text{Sqrt}[b] \text{Sqrt}[2/\text{Pi}] \text{Sqrt}[c + d*x])/ \text{Sqrt}[d]] \text{Sin}[a - (b*c)/d])/(8*b^{(3/2)}) + (\text{Sqrt}[c + d*x] \text{Sin}[a + b*x])/(8*b) - (\text{Sqrt}[c + d*x] \text{Sin}[3*a + 3*b*x])/(48*b) - (\text{Sqrt}[c + d*x] \text{Sin}[5*a + 5*b*x])/(80*b)$

Rubi [A] time = 0.692595, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{10}} \sqrt{d} \sin\left(5a - \frac{5bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}} + \frac{\sqrt{\frac{\pi}{6}} \sqrt{d} \sin\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \sin\left(a - \frac{bc}{d}\right)}{48b^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[c + d*x] \text{Cos}[a + b*x]^3 \text{Sin}[a + b*x]^2, x]$

[Out] $-(\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/2] \text{Cos}[a - (b*c)/d] \text{FresnelS}[(\text{Sqrt}[b] \text{Sqrt}[2/\text{Pi}] \text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(8*b^{(3/2)}) + (\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/6] \text{Cos}[3*a - (3*b*c)/d] \text{FresnelS}[(\text{Sqrt}[b] \text{Sqrt}[6/\text{Pi}] \text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(48*b^{(3/2)}) + (\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/10] \text{Cos}[5*a - (5*b*c)/d] \text{FresnelS}[(\text{Sqrt}[b] \text{Sqrt}[10/\text{Pi}] \text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(80*b^{(3/2)}) + (\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/10] \text{FresnelC}[(\text{Sqrt}[b] \text{Sqrt}[10/\text{Pi}] \text{Sqrt}[c + d*x])/ \text{Sqrt}[d]] \text{Sin}[5*a - (5*b*c)/d])/(80*b^{(3/2)}) + (\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/6] \text{FresnelC}[(\text{Sqrt}[b] \text{Sqrt}[6/\text{Pi}] \text{Sqrt}[c + d*x])/ \text{Sqrt}[d]] \text{Sin}[3*a - (3*b*c)/d])/(48*b^{(3/2)}) - (\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/2] \text{FresnelC}[(\text{Sqrt}[b] \text{Sqrt}[2/\text{Pi}] \text{Sqrt}[c + d*x])/ \text{Sqrt}[d]] \text{Sin}[a - (b*c)/d])/(8*b^{(3/2)}) + (\text{Sqrt}[c + d*x] \text{Sin}[a + b*x])/(8*b) - (\text{Sqrt}[c + d*x] \text{Sin}[3*a + 3*b*x])/(48*b) - (\text{Sqrt}[c + d*x] \text{Sin}[5*a + 5*b*x])/(80*b)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)} * ((c_.) + (d_.)*(x_.))^{(m_.)} * \text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n \text{Cos}[a + b*x]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)} * \text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m \text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)} \text{Cos}[e + f*x], x], x]$

$e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] \&\& GtQ[m, 0]$

Rule 3306

$Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] \rightarrow Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d, e, f}, x] \&\& ComplexFreeQ[f] \&\& NeQ[d*e - c*f, 0]$

Rule 3305

$Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] \rightarrow Dist[2/d, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] \&\& ComplexFreeQ[f] \&\& EqQ[d*e - c*f, 0]$

Rule 3351

$Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] \rightarrow Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]$

Rule 3304

$Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] \rightarrow Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] \&\& ComplexFreeQ[f] \&\& EqQ[d*e - c*f, 0]$

Rule 3352

$Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] \rightarrow Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]$

Rubi steps

$$\begin{aligned} \int \sqrt{c + dx} \cos^3(a + bx) \sin^2(a + bx) dx &= \int \left(\frac{1}{8} \sqrt{c + dx} \cos(a + bx) - \frac{1}{16} \sqrt{c + dx} \cos(3a + 3bx) - \frac{1}{16} \sqrt{c + dx} \cos(5a + 5bx) \right) \sin^2(a + bx) dx \\ &= - \left(\frac{1}{16} \int \sqrt{c + dx} \cos(3a + 3bx) dx \right) - \frac{1}{16} \int \sqrt{c + dx} \cos(5a + 5bx) dx + \frac{1}{8} \int \sqrt{c + dx} \cos(a + bx) dx \\ &= \frac{\sqrt{c + dx} \sin(a + bx)}{8b} - \frac{\sqrt{c + dx} \sin(3a + 3bx)}{48b} - \frac{\sqrt{c + dx} \sin(5a + 5bx)}{80b} + \frac{d}{8} \int \frac{\sqrt{c + dx} \cos(a + bx)}{\sqrt{c + dx}} dx \\ &= \frac{\sqrt{c + dx} \sin(a + bx)}{8b} - \frac{\sqrt{c + dx} \sin(3a + 3bx)}{48b} - \frac{\sqrt{c + dx} \sin(5a + 5bx)}{80b} + \frac{(d \cos(a + bx))^{3/2}}{8b^{3/2}} \\ &= \frac{\sqrt{c + dx} \sin(a + bx)}{8b} - \frac{\sqrt{c + dx} \sin(3a + 3bx)}{48b} - \frac{\sqrt{c + dx} \sin(5a + 5bx)}{80b} + \frac{\cos(a + bx) \sqrt{c + dx}}{8b} \\ &= - \frac{\sqrt{d} \sqrt{\frac{\pi}{2}} \cos\left(a - \frac{bc}{d}\right) S\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c + dx}}{\sqrt{d}}\right)}{8b^{3/2}} + \frac{\sqrt{d} \sqrt{\frac{\pi}{6}} \cos\left(3a - \frac{3bc}{d}\right) S\left(\frac{\sqrt{b} \sqrt{\frac{6}{\pi}} \sqrt{c + dx}}{\sqrt{d}}\right)}{48b^{3/2}} \end{aligned}$$

Mathematica [C] time = 7.04336, size = 435, normalized size = 0.95

$$\frac{i\sqrt{c + dx} e^{-\frac{i(ad+bc)}{d}} \left(\frac{e^{2ia} \Gamma\left(\frac{3}{2}, -\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}} \Gamma\left(\frac{3}{2}, \frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}} \right) - \sqrt{2\pi} \sin\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\sqrt{\frac{6}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c + dx}\right) - \sqrt{2\pi} \cos\left(a - \frac{bc}{d}\right) \text{FresnelS}\left(\sqrt{\frac{2}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c + dx}\right)}{16b}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]

[Out]
$$\frac{(-I/16)\sqrt{c+dx}\left(E^{(2I)a}\Gamma\left(\frac{3}{2},\frac{(-I)b(c+dx)}{d}\right)\sqrt{\frac{(-I)b(c+dx)}{d}} - E^{\left(\frac{(2I)bc}{d}\right)}\Gamma\left(\frac{3}{2},\frac{Ib(c+dx)}{d}\right)\sqrt{\frac{Ib(c+dx)}{d}}\right)}{(bE^{\left(\frac{I(bc+ad)}{d}\right)} - (-\sqrt{2\pi}\cos\left[\frac{3a-(3bc)}{d}\right]\text{FresnelS}\left[\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right] - \sqrt{2\pi}\text{FresnelC}\left[\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right]\sin\left[\frac{3a-(3bc)}{d}\right] + 2\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin[3(a+bx)]})/(96\sqrt{3}b\sqrt{\frac{b}{d}}) - (-\sqrt{2\pi}\cos\left[\frac{5a-(5bc)}{d}\right]\text{FresnelS}\left[\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right] - \sqrt{2\pi}\text{FresnelC}\left[\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right]\sin\left[\frac{5a-(5bc)}{d}\right] + 2\sqrt{5}\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin[5(a+bx)]})/(160\sqrt{5}b\sqrt{\frac{b}{d}})}$$

Maple [A] time = 0.049, size = 444, normalized size = 1.

$$2\frac{1}{d}\left(1/16\frac{d\sqrt{dx+c}}{b}\sin\left(\frac{(dx+c)b}{d}+\frac{ad-bc}{d}\right)-1/32\frac{d\sqrt{2}\sqrt{\pi}}{b}\left(\cos\left(\frac{ad-bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\sqrt{dx+cb}}{\sqrt{\pi d}}\frac{1}{\sqrt{\frac{b}{d}}}\right)+\sin\left(\frac{ad}{d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)

[Out]
$$2/d*(1/16/b*d*(d*x+c)^{(1/2)}*\sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/32/b*d*2^{(1/2)}*\pi^{(1/2)}/(b/d)^{(1/2)}*(\cos((a*d-b*c)/d)*\text{FresnelS}(2^{(1/2)}/\pi^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d)+\sin((a*d-b*c)/d)*\text{FresnelC}(2^{(1/2)}/\pi^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d))-1/96/b*d*(d*x+c)^{(1/2)}*\sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/576/b*d*2^{(1/2)}*\pi^{(1/2)}*3^{(1/2)}/(b/d)^{(1/2)}*(\cos(3*(a*d-b*c)/d)*\text{FresnelS}(2^{(1/2)}/\pi^{(1/2)}*3^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d)+\sin(3*(a*d-b*c)/d)*\text{FresnelC}(2^{(1/2)}/\pi^{(1/2)}*3^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d))-1/160/b*d*(d*x+c)^{(1/2)}*\sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+1/1600/b*d*2^{(1/2)}*\pi^{(1/2)}*5^{(1/2)}/(b/d)^{(1/2)}*(\cos(5*(a*d-b*c)/d)*\text{FresnelS}(2^{(1/2)}/\pi^{(1/2)}*5^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d)+\sin(5*(a*d-b*c)/d)*\text{FresnelC}(2^{(1/2)}/\pi^{(1/2)}*5^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)}*b/d))$$

Maxima [C] time = 2.53509, size = 2531, normalized size = 5.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")

[Out]
$$-1/28800*\sqrt{5}*\sqrt{3}*(24*\sqrt{5}*\sqrt{3}*\sqrt{d*x+c}*d*\sqrt{\text{abs}(b)/\text{abs}(d)}*\text{abs}(b)*\sin(5*((d*x+c)*b-b*c+a*d)/d)/\text{abs}(d)+40*\sqrt{5}*\sqrt{3}*\sqrt{d*x+c}*d*\sqrt{\text{abs}(b)/\text{abs}(d)}*\text{abs}(b)*\sin(3*((d*x+c)*b-b*c+a*d)/d)/\text{abs}(d)-240*\sqrt{5}*\sqrt{3}*\sqrt{d*x+c}*d*\sqrt{\text{abs}(b)/\text{abs}(d)}*\text{abs}(b)*\sin(((d*x+c)*b-b*c+a*d)/d)/\text{abs}(d)+(\sqrt{3})*(-3*I*\sqrt{\pi}*\cos(1/4*\pi+1/2*\arctan2(0,b)+1/2*\arctan2(0,d/\sqrt{d^2})))-3*I*\sqrt{\pi}*\cos(-1/4*\pi+1/2*\arctan2(0,b)+1/2*\arctan2(0,d/\sqrt{d^2})))-3*\sqrt{\pi}*\sin(1/4*\pi+1/2*\arctan2(0,b)+1/2*\arctan2(0,d/\sqrt{d^2}))+3*\sqrt{\pi}*\sin(-1/4*\pi+1/2*\arctan2(0,b)+1/2*\arctan2(0,d/\sqrt{d^2}))*d*\text{abs}(b)*\cos(-5*$$

$$\begin{aligned}
& (b*c - a*d)/d)/\text{abs}(d) - \text{sqrt}(3)*(3*\text{sqrt}(\pi)*\cos(1/4*\pi + 1/2*\arctan2(0, b) \\
& + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 3*\text{sqrt}(\pi)*\cos(-1/4*\pi + 1/2*\arctan2(0, b) \\
& + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 3*I*\text{sqrt}(\pi)*\sin(1/4*\pi + 1/2*\arctan2(0, \\
& b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 3*I*\text{sqrt}(\pi)*\sin(-1/4*\pi + 1/2*\arctan2(\\
& 0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))))*d*\text{abs}(b)*\sin(-5*(b*c - a*d)/d)/\text{abs}(d) \\
&)*\text{erf}(\text{sqrt}(d*x + c)*\text{sqrt}(5*I*b/d)) + (\text{sqrt}(5)*(-5*I*\text{sqrt}(\pi)*\cos(1/4*\pi + 1 \\
& /2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 5*I*\text{sqrt}(\pi)*\cos(-1/4*\pi \\
& + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 5*\text{sqrt}(\pi)*\sin(1/4*\pi \\
& + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 5*\text{sqrt}(\pi)*\sin(-1/4*\pi \\
& + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))))*d*\text{abs}(b)*\cos(-3*(b*c - \\
& a*d)/d)/\text{abs}(d) - \text{sqrt}(5)*(5*\text{sqrt}(\pi)*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2* \\
& \arctan2(0, d/\text{sqrt}(d^2))) + 5*\text{sqrt}(\pi)*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2 \\
& *\arctan2(0, d/\text{sqrt}(d^2))) - 5*I*\text{sqrt}(\pi)*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1 \\
& /2*\arctan2(0, d/\text{sqrt}(d^2))) + 5*I*\text{sqrt}(\pi)*\sin(-1/4*\pi + 1/2*\arctan2(0, b) \\
& + 1/2*\arctan2(0, d/\text{sqrt}(d^2))))*d*\text{abs}(b)*\sin(-3*(b*c - a*d)/d)/\text{abs}(d))*\text{erf} \\
& (\text{sqrt}(d*x + c)*\text{sqrt}(3*I*b/d)) + (\text{sqrt}(5)*\text{sqrt}(3)*(30*I*\text{sqrt}(\pi)*\cos(1/4*\pi + \\
& 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 30*I*\text{sqrt}(\pi)*\cos(-1/4* \\
& \pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 30*\text{sqrt}(\pi)*\sin(1/4 \\
& *\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 30*\text{sqrt}(\pi)*\sin(-1 \\
& /4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))))*d*\text{abs}(b)*\cos(-(b* \\
& c - a*d)/d)/\text{abs}(d) + \text{sqrt}(5)*\text{sqrt}(3)*(30*\text{sqrt}(\pi)*\cos(1/4*\pi + 1/2*\arctan2(\\
& 0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 30*\text{sqrt}(\pi)*\cos(-1/4*\pi + 1/2*\arctan \\
& 2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 30*I*\text{sqrt}(\pi)*\sin(1/4*\pi + 1/2*arc \\
& tan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 30*I*\text{sqrt}(\pi)*\sin(-1/4*\pi + 1/2 \\
& *\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))))*d*\text{abs}(b)*\sin(-(b*c - a*d)/d) \\
&)/\text{abs}(d))*\text{erf}(\text{sqrt}(d*x + c)*\text{sqrt}(I*b/d)) + (\text{sqrt}(5)*\text{sqrt}(3)*(-30*I*\text{sqrt}(\pi)* \\
& \cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 30*I*\text{sqrt}(p \\
& i)*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 30*\text{sqrt} \\
& (\pi)*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 30*\text{sqrt} \\
& t(\pi)*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))))*d*\text{abs} \\
& (b)*\cos(-(b*c - a*d)/d)/\text{abs}(d) + \text{sqrt}(5)*\text{sqrt}(3)*(30*\text{sqrt}(\pi)*\cos(1/4*\pi + \\
& 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 30*\text{sqrt}(\pi)*\cos(-1/4*\pi \\
& + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 30*I*\text{sqrt}(\pi)*\sin(1/4* \\
& \pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 30*I*\text{sqrt}(\pi)*\sin(- \\
& 1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))))*d*\text{abs}(b)*\sin(-(b \\
& *c - a*d)/d)/\text{abs}(d))*\text{erf}(\text{sqrt}(d*x + c)*\text{sqrt}(-I*b/d)) + (\text{sqrt}(5)*(5*I*\text{sqrt}(\pi) \\
& i)*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 5*I*\text{sqrt} \\
& (\pi)*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 5*\text{sqrt} \\
& t(\pi)*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 5*\text{sqrt} \\
& t(\pi)*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))))*d*\text{abs} \\
& (b)*\cos(-3*(b*c - a*d)/d)/\text{abs}(d) - \text{sqrt}(5)*(5*\text{sqrt}(\pi)*\cos(1/4*\pi + 1/2*arc \\
& tan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 5*\text{sqrt}(\pi)*\cos(-1/4*\pi + 1/2*arc \\
& tan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 5*I*\text{sqrt}(\pi)*\sin(1/4*\pi + 1/2* \\
& arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 5*I*\text{sqrt}(\pi)*\sin(-1/4*\pi + 1 \\
& /2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))))*d*\text{abs}(b)*\sin(-3*(b*c - a*d) \\
&)/d)/\text{abs}(d))*\text{erf}(\text{sqrt}(d*x + c)*\text{sqrt}(-3*I*b/d)) + (\text{sqrt}(3)*(3*I*\text{sqrt}(\pi)*\cos \\
& (1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 3*I*\text{sqrt}(\pi)*c \\
& os(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 3*\text{sqrt}(\pi)* \\
& sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 3*\text{sqrt}(\pi)* \\
& sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))))*d*\text{abs}(b)*co \\
& s(-5*(b*c - a*d)/d)/\text{abs}(d) - \text{sqrt}(3)*(3*\text{sqrt}(\pi)*\cos(1/4*\pi + 1/2*\arctan2(0 \\
& , b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 3*\text{sqrt}(\pi)*\cos(-1/4*\pi + 1/2*\arctan2(\\
& 0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 3*I*\text{sqrt}(\pi)*\sin(1/4*\pi + 1/2*\arctan \\
& 2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 3*I*\text{sqrt}(\pi)*\sin(-1/4*\pi + 1/2*arc \\
& tan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))))*d*\text{abs}(b)*\sin(-5*(b*c - a*d)/d)/a \\
& bs(d))*\text{erf}(\text{sqrt}(d*x + c)*\text{sqrt}(-5*I*b/d))*\text{abs}(d)/(b*d*\text{sqrt}(\text{abs}(b)/\text{abs}(d))*a \\
& bs(b))
\end{aligned}$$

Fricas [A] time = 0.715721, size = 973, normalized size = 2.12

$$9\sqrt{10}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)S\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+25\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-450\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")

[Out] 1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_sin(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^4 - b*cos(b*x + a)^2 - 2*b)*sqrt(d*x + c)*sin(b*x + a)/b^2

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)

[Out] Timed out

Giac [C] time = 1.44228, size = 988, normalized size = 2.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")

[Out] -1/14400*(9*I*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 90*I*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b - 150*I*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I

$$\begin{aligned}
& *a*d)/d)/b + 900*I*\sqrt{d*x + c}*d*e^{(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b} \\
& - 900*I*\sqrt{d*x + c}*d*e^{(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b} + 150*I*\sqrt{d*x + c}*d*e^{((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b} + 90*I*\sqrt{d*x + c}*d*e^{(-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)/d}
\end{aligned}$$

3.193 $\int \sqrt{c + dx} \cos^3(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=459

$$\frac{\sqrt{\frac{\pi}{10}} \sqrt{d} \sin\left(5a - \frac{5bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}} + \frac{\sqrt{\frac{\pi}{6}} \sqrt{d} \sin\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \sin\left(a - \frac{bc}{d}\right)}{48b^{3/2}}$$

[Out] $-(\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/2] \text{Cos}[a - (b*c)/d] \text{FresnelS}[(\text{Sqrt}[b] \text{Sqrt}[2/\text{Pi}] \text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(8*b^{(3/2)}) + (\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/6] \text{Cos}[3*a - (3*b*c)/d] \text{FresnelS}[(\text{Sqrt}[b] \text{Sqrt}[6/\text{Pi}] \text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(48*b^{(3/2)}) + (\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/10] \text{Cos}[5*a - (5*b*c)/d] \text{FresnelS}[(\text{Sqrt}[b] \text{Sqrt}[10/\text{Pi}] \text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(80*b^{(3/2)}) + (\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/10] \text{FresnelC}[(\text{Sqrt}[b] \text{Sqrt}[10/\text{Pi}] \text{Sqrt}[c + d*x])/\text{Sqrt}[d]] \text{Sin}[5*a - (5*b*c)/d])/(80*b^{(3/2)}) + (\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/6] \text{FresnelC}[(\text{Sqrt}[b] \text{Sqrt}[6/\text{Pi}] \text{Sqrt}[c + d*x])/\text{Sqrt}[d]] \text{Sin}[3*a - (3*b*c)/d])/(48*b^{(3/2)}) - (\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/2] \text{FresnelC}[(\text{Sqrt}[b] \text{Sqrt}[2/\text{Pi}] \text{Sqrt}[c + d*x])/\text{Sqrt}[d]] \text{Sin}[a - (b*c)/d])/(8*b^{(3/2)}) + (\text{Sqrt}[c + d*x] \text{Sin}[a + b*x])/(8*b) - (\text{Sqrt}[c + d*x] \text{Sin}[3*a + 3*b*x])/(48*b) - (\text{Sqrt}[c + d*x] \text{Sin}[5*a + 5*b*x])/(80*b)$

Rubi [A] time = 0.671504, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{10}} \sqrt{d} \sin\left(5a - \frac{5bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}} + \frac{\sqrt{\frac{\pi}{6}} \sqrt{d} \sin\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} \sin\left(a - \frac{bc}{d}\right)}{48b^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[c + d*x] \text{Cos}[a + b*x]^3 \text{Sin}[a + b*x]^2, x]$

[Out] $-(\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/2] \text{Cos}[a - (b*c)/d] \text{FresnelS}[(\text{Sqrt}[b] \text{Sqrt}[2/\text{Pi}] \text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(8*b^{(3/2)}) + (\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/6] \text{Cos}[3*a - (3*b*c)/d] \text{FresnelS}[(\text{Sqrt}[b] \text{Sqrt}[6/\text{Pi}] \text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(48*b^{(3/2)}) + (\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/10] \text{Cos}[5*a - (5*b*c)/d] \text{FresnelS}[(\text{Sqrt}[b] \text{Sqrt}[10/\text{Pi}] \text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(80*b^{(3/2)}) + (\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/10] \text{FresnelC}[(\text{Sqrt}[b] \text{Sqrt}[10/\text{Pi}] \text{Sqrt}[c + d*x])/\text{Sqrt}[d]] \text{Sin}[5*a - (5*b*c)/d])/(80*b^{(3/2)}) + (\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/6] \text{FresnelC}[(\text{Sqrt}[b] \text{Sqrt}[6/\text{Pi}] \text{Sqrt}[c + d*x])/\text{Sqrt}[d]] \text{Sin}[3*a - (3*b*c)/d])/(48*b^{(3/2)}) - (\text{Sqrt}[d] \text{Sqrt}[\text{Pi}/2] \text{FresnelC}[(\text{Sqrt}[b] \text{Sqrt}[2/\text{Pi}] \text{Sqrt}[c + d*x])/\text{Sqrt}[d]] \text{Sin}[a - (b*c)/d])/(8*b^{(3/2)}) + (\text{Sqrt}[c + d*x] \text{Sin}[a + b*x])/(8*b) - (\text{Sqrt}[c + d*x] \text{Sin}[3*a + 3*b*x])/(48*b) - (\text{Sqrt}[c + d*x] \text{Sin}[5*a + 5*b*x])/(80*b)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)(x_.)]^{(p_.)} ((c_.) + (d_.)(x_.))^{(m_.)} \text{Sin}[(a_.) + (b_.)(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n \text{Cos}[a + b*x]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

$\text{Int}[(c_.) + (d_.)(x_.)]^{(m_.)} \text{sin}[(e_.) + (f_.)(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m \text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)} \text{Cos}[e + f*x], x], x]$

$e + f*x]$, $x]$, $x]$ /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3306

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]

Rule 3305

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3304

Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned} \int \sqrt{c + dx} \cos^3(a + bx) \sin^2(a + bx) dx &= \int \left(\frac{1}{8} \sqrt{c + dx} \cos(a + bx) - \frac{1}{16} \sqrt{c + dx} \cos(3a + 3bx) - \frac{1}{16} \sqrt{c + dx} \cos(5a + 5bx) \right) \sin^2(a + bx) dx \\ &= - \left(\frac{1}{16} \int \sqrt{c + dx} \cos(3a + 3bx) dx \right) - \frac{1}{16} \int \sqrt{c + dx} \cos(5a + 5bx) dx + \frac{1}{8} \int \sqrt{c + dx} \cos(a + bx) dx \\ &= \frac{\sqrt{c + dx} \sin(a + bx)}{8b} - \frac{\sqrt{c + dx} \sin(3a + 3bx)}{48b} - \frac{\sqrt{c + dx} \sin(5a + 5bx)}{80b} + \frac{d \int \sqrt{c + dx} \cos(a + bx) dx}{8b} \\ &= \frac{\sqrt{c + dx} \sin(a + bx)}{8b} - \frac{\sqrt{c + dx} \sin(3a + 3bx)}{48b} - \frac{\sqrt{c + dx} \sin(5a + 5bx)}{80b} + \frac{(d \int \sqrt{c + dx} \cos(a + bx) dx)}{8b} \\ &= \frac{\sqrt{c + dx} \sin(a + bx)}{8b} - \frac{\sqrt{c + dx} \sin(3a + 3bx)}{48b} - \frac{\sqrt{c + dx} \sin(5a + 5bx)}{80b} + \frac{\cos(a + bx) \sqrt{c + dx}}{8b} \\ &= - \frac{\sqrt{d} \sqrt{\frac{\pi}{2}} \cos\left(a - \frac{bc}{d}\right) S\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c + dx}}{\sqrt{d}}\right)}{8b^{3/2}} + \frac{\sqrt{d} \sqrt{\frac{\pi}{6}} \cos\left(3a - \frac{3bc}{d}\right) S\left(\frac{\sqrt{b} \sqrt{\frac{6}{\pi}} \sqrt{c + dx}}{\sqrt{d}}\right)}{48b^{3/2}} \end{aligned}$$

Mathematica [C] time = 7.04373, size = 435, normalized size = 0.95

$$\frac{i\sqrt{c + dx} e^{-\frac{i(ad+bc)}{d}} \left(\frac{e^{2ia} \Gamma\left(\frac{3}{2}, -\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}} \Gamma\left(\frac{3}{2}, \frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}} \right) - \sqrt{2\pi} \sin\left(3a - \frac{3bc}{d}\right) \text{FresnelC}\left(\sqrt{\frac{6}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c + dx}\right) - \sqrt{2\pi} \cos\left(a - \frac{bc}{d}\right) \text{FresnelS}\left(\sqrt{\frac{2}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c + dx}\right)}{16b}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]

[Out]
$$\frac{(-I/16)\sqrt{c+dx}\left(E^{(2I)a}\Gamma\left(\frac{3}{2},\frac{(-I)b(c+dx)}{d}\right)\sqrt{\frac{(-I)b(c+dx)}{d}} - E^{\left(\frac{(2I)bc}{d}\right)}\Gamma\left(\frac{3}{2},\frac{Ib(c+dx)}{d}\right)\sqrt{\frac{Ib(c+dx)}{d}}\right)}{(bE^{\left(\frac{I(bc+ad)}{d}\right)} - (-\sqrt{2\pi}\cos\left[\frac{3a-(3bc)}{d}\right]\text{FresnelS}\left[\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right] - \sqrt{2\pi}\text{FresnelC}\left[\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right]\sin\left[\frac{3a-(3bc)}{d}\right] + 2\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin[3(a+bx)]})/(96\sqrt{3}b\sqrt{\frac{b}{d}}) - (-\sqrt{2\pi}\cos\left[\frac{5a-(5bc)}{d}\right]\text{FresnelS}\left[\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right] - \sqrt{2\pi}\text{FresnelC}\left[\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right]\sin\left[\frac{5a-(5bc)}{d}\right] + 2\sqrt{5}\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin[5(a+bx)]})/(160\sqrt{5}b\sqrt{\frac{b}{d}})}$$

Maple [A] time = 0.05, size = 444, normalized size = 1.

$$2\frac{1}{d}\left(1/16\frac{d\sqrt{dx+c}}{b}\sin\left(\frac{(dx+c)b}{d} + \frac{ad-bc}{d}\right) - 1/32\frac{d\sqrt{2}\sqrt{\pi}}{b}\left(\cos\left(\frac{ad-bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\sqrt{dx+cb}}{\sqrt{\pi d}}\frac{1}{\sqrt{\frac{b}{d}}}\right) + \sin\left(\frac{ad}{d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)

[Out]
$$2/d*(1/16/b*d*(d*x+c)^{(1/2)}*\sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/32/b*d*2^{(1/2)}*\pi^{(1/2)}/(b/d)^{(1/2)}*(\cos((a*d-b*c)/d)*\text{FresnelS}(2^{(1/2)}/\pi^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)*b/d}+\sin((a*d-b*c)/d)*\text{FresnelC}(2^{(1/2)}/\pi^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)*b/d})-1/96/b*d*(d*x+c)^{(1/2)}*\sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/576/b*d*2^{(1/2)}*\pi^{(1/2)}*3^{(1/2)}/(b/d)^{(1/2)}*(\cos(3*(a*d-b*c)/d)*\text{FresnelS}(2^{(1/2)}/\pi^{(1/2)}*3^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)*b/d}+\sin(3*(a*d-b*c)/d)*\text{FresnelC}(2^{(1/2)}/\pi^{(1/2)}*3^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)*b/d})-1/160/b*d*(d*x+c)^{(1/2)}*\sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+1/1600/b*d*2^{(1/2)}*\pi^{(1/2)}*5^{(1/2)}/(b/d)^{(1/2)}*(\cos(5*(a*d-b*c)/d)*\text{FresnelS}(2^{(1/2)}/\pi^{(1/2)}*5^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)*b/d}+\sin(5*(a*d-b*c)/d)*\text{FresnelC}(2^{(1/2)}/\pi^{(1/2)}*5^{(1/2)}/(b/d)^{(1/2)}*(d*x+c)^{(1/2)*b/d}))$$

Maxima [C] time = 2.51465, size = 2531, normalized size = 5.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")

[Out]
$$-1/28800*\sqrt{5}*\sqrt{3}*(24*\sqrt{5}*\sqrt{3}*\sqrt{d*x+c}*d*\sqrt{\text{abs}(b)/\text{abs}(d)}*\text{abs}(b)*\sin(5*((d*x+c)*b-b*c+a*d)/d)/\text{abs}(d) + 40*\sqrt{5}*\sqrt{3}*\sqrt{d*x+c}*d*\sqrt{\text{abs}(b)/\text{abs}(d)}*\text{abs}(b)*\sin(3*((d*x+c)*b-b*c+a*d)/d)/\text{abs}(d) - 240*\sqrt{5}*\sqrt{3}*\sqrt{d*x+c}*d*\sqrt{\text{abs}(b)/\text{abs}(d)}*\text{abs}(b)*\sin(((d*x+c)*b-b*c+a*d)/d)/\text{abs}(d) + (\sqrt{3})*(-3*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 3*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 3*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 3*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d*\text{abs}(b)*\cos(-5*$$

$$\begin{aligned}
& (b*c - a*d)/d)/\text{abs}(d) - \text{sqrt}(3)*(3*\text{sqrt}(\pi)*\cos(1/4*\pi + 1/2*\arctan2(0, b) \\
& + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 3*\text{sqrt}(\pi)*\cos(-1/4*\pi + 1/2*\arctan2(0, b) \\
& + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 3*I*\text{sqrt}(\pi)*\sin(1/4*\pi + 1/2*\arctan2(0, \\
& b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 3*I*\text{sqrt}(\pi)*\sin(-1/4*\pi + 1/2*\arctan2(\\
& 0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))))*d*\text{abs}(b)*\sin(-5*(b*c - a*d)/d)/\text{abs}(d) \\
&)*\text{erf}(\text{sqrt}(d*x + c)*\text{sqrt}(5*I*b/d)) + (\text{sqrt}(5)*(-5*I*\text{sqrt}(\pi)*\cos(1/4*\pi + 1 \\
& /2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 5*I*\text{sqrt}(\pi)*\cos(-1/4*\pi \\
& + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 5*\text{sqrt}(\pi)*\sin(1/4*\pi \\
& + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 5*\text{sqrt}(\pi)*\sin(-1/4*\pi \\
& + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))))*d*\text{abs}(b)*\cos(-3*(b*c - \\
& a*d)/d)/\text{abs}(d) - \text{sqrt}(5)*(5*\text{sqrt}(\pi)*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2* \\
& \arctan2(0, d/\text{sqrt}(d^2))) + 5*\text{sqrt}(\pi)*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2 \\
& *\arctan2(0, d/\text{sqrt}(d^2))) - 5*I*\text{sqrt}(\pi)*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1 \\
& /2*\arctan2(0, d/\text{sqrt}(d^2))) + 5*I*\text{sqrt}(\pi)*\sin(-1/4*\pi + 1/2*\arctan2(0, b) \\
& + 1/2*\arctan2(0, d/\text{sqrt}(d^2))))*d*\text{abs}(b)*\sin(-3*(b*c - a*d)/d)/\text{abs}(d))*\text{erf} \\
& (\text{sqrt}(d*x + c)*\text{sqrt}(3*I*b/d)) + (\text{sqrt}(5)*\text{sqrt}(3)*(30*I*\text{sqrt}(\pi)*\cos(1/4*\pi + \\
& 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 30*I*\text{sqrt}(\pi)*\cos(-1/4* \\
& \pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 30*\text{sqrt}(\pi)*\sin(1/4 \\
& *\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 30*\text{sqrt}(\pi)*\sin(-1 \\
& /4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))))*d*\text{abs}(b)*\cos(-(b* \\
& c - a*d)/d)/\text{abs}(d) + \text{sqrt}(5)*\text{sqrt}(3)*(30*\text{sqrt}(\pi)*\cos(1/4*\pi + 1/2*\arctan2(\\
& 0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 30*\text{sqrt}(\pi)*\cos(-1/4*\pi + 1/2*\arctan \\
& 2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 30*I*\text{sqrt}(\pi)*\sin(1/4*\pi + 1/2*arc \\
& tan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 30*I*\text{sqrt}(\pi)*\sin(-1/4*\pi + 1/2 \\
& *\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))))*d*\text{abs}(b)*\sin(-(b*c - a*d)/d) \\
&)/\text{abs}(d))*\text{erf}(\text{sqrt}(d*x + c)*\text{sqrt}(I*b/d)) + (\text{sqrt}(5)*\text{sqrt}(3)*(-30*I*\text{sqrt}(\pi)* \\
& \cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 30*I*\text{sqrt}(p \\
& i)*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 30*\text{sqrt} \\
& (\pi)*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 30*\text{sqrt} \\
& t(\pi)*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))))*d*\text{abs} \\
& (b)*\cos(-(b*c - a*d)/d)/\text{abs}(d) + \text{sqrt}(5)*\text{sqrt}(3)*(30*\text{sqrt}(\pi)*\cos(1/4*\pi + \\
& 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 30*\text{sqrt}(\pi)*\cos(-1/4*\pi \\
& + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 30*I*\text{sqrt}(\pi)*\sin(1/4* \\
& \pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 30*I*\text{sqrt}(\pi)*\sin(- \\
& 1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))))*d*\text{abs}(b)*\sin(-(b \\
& *c - a*d)/d)/\text{abs}(d))*\text{erf}(\text{sqrt}(d*x + c)*\text{sqrt}(-I*b/d)) + (\text{sqrt}(5)*(5*I*\text{sqrt}(\pi) \\
& i)*\cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 5*I*\text{sqrt} \\
& (\pi)*\cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 5*\text{sqrt} \\
& t(\pi)*\sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 5*\text{sqrt} \\
& t(\pi)*\sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))))*d*\text{abs} \\
& (b)*\cos(-3*(b*c - a*d)/d)/\text{abs}(d) - \text{sqrt}(5)*(5*\text{sqrt}(\pi)*\cos(1/4*\pi + 1/2*arc \\
& tan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 5*\text{sqrt}(\pi)*\cos(-1/4*\pi + 1/2*arc \\
& tan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 5*I*\text{sqrt}(\pi)*\sin(1/4*\pi + 1/2* \\
& arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 5*I*\text{sqrt}(\pi)*\sin(-1/4*\pi + 1 \\
& /2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))))*d*\text{abs}(b)*\sin(-3*(b*c - a*d) \\
&)/d)/\text{abs}(d))*\text{erf}(\text{sqrt}(d*x + c)*\text{sqrt}(-3*I*b/d)) + (\text{sqrt}(3)*(3*I*\text{sqrt}(\pi)*\cos \\
& (1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 3*I*\text{sqrt}(\pi)*c \\
& os(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 3*\text{sqrt}(\pi)* \\
& sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 3*\text{sqrt}(\pi)* \\
& sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))))*d*\text{abs}(b)*co \\
& s(-5*(b*c - a*d)/d)/\text{abs}(d) - \text{sqrt}(3)*(3*\text{sqrt}(\pi)*\cos(1/4*\pi + 1/2*\arctan2(0 \\
& , b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 3*\text{sqrt}(\pi)*\cos(-1/4*\pi + 1/2*\arctan2(\\
& 0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) + 3*I*\text{sqrt}(\pi)*\sin(1/4*\pi + 1/2*\arctan \\
& 2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))) - 3*I*\text{sqrt}(\pi)*\sin(-1/4*\pi + 1/2*arc \\
& tan2(0, b) + 1/2*\arctan2(0, d/\text{sqrt}(d^2))))*d*\text{abs}(b)*\sin(-5*(b*c - a*d)/d)/a \\
& bs(d))*\text{erf}(\text{sqrt}(d*x + c)*\text{sqrt}(-5*I*b/d))*\text{abs}(d)/(b*d*\text{sqrt}(\text{abs}(b)/\text{abs}(d))*a \\
& bs(b))
\end{aligned}$$

Fricas [A] time = 0.71869, size = 973, normalized size = 2.12

$$9\sqrt{10}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)S\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) + 25\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 450\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")

[Out] 1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_sin(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^4 - b*cos(b*x + a)^2 - 2*b)*sqrt(d*x + c)*sin(b*x + a)/b^2

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)

[Out] Timed out

Giac [C] time = 1.42066, size = 988, normalized size = 2.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")

[Out] -1/14400*(9*I*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 90*I*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b - 150*I*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I

$$\begin{aligned}
& *a*d)/d)/b + 900*I*\sqrt{d*x + c}*d*e^{(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b} \\
& - 900*I*\sqrt{d*x + c}*d*e^{(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b} + 150*I*\sqrt{d*x + c}*d*e^{((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b} + 90*I*\sqrt{d*x + c}*d*e^{(-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)/d}
\end{aligned}$$

3.194 $\int (c + dx)^{3/2} \cos^3(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=534

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}} + \frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} + \frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\cos\left(5a - \frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}$$

[Out] $(3*d*\text{Sqrt}[c + d*x]*\text{Cos}[a + b*x])/(16*b^2) - (d*\text{Sqrt}[c + d*x]*\text{Cos}[3*a + 3*b*x])/(96*b^2) - (3*d*\text{Sqrt}[c + d*x]*\text{Cos}[5*a + 5*b*x])/(800*b^2) - (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a - (b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(16*b^{(5/2)}) + (d^{(3/2)}*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(96*b^{(5/2)}) + (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/10]*\text{Cos}[5*a - (5*b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(800*b^{(5/2)}) - (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/10]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]]*\text{Sin}[5*a - (5*b*c)/d])/(800*b^{(5/2)}) - (d^{(3/2)}*\text{Sqrt}[\text{Pi}/6]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]]*\text{Sin}[3*a - (3*b*c)/d])/(96*b^{(5/2)}) + (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]]*\text{Sin}[a - (b*c)/d])/(16*b^{(5/2)}) + ((c + d*x)^{(3/2)}*\text{Sin}[a + b*x])/(8*b) - ((c + d*x)^{(3/2)}*\text{Sin}[3*a + 3*b*x])/(48*b) - ((c + d*x)^{(3/2)}*\text{Sin}[5*a + 5*b*x])/(80*b)$

Rubi [A] time = 0.857925, antiderivative size = 534, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}} + \frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} + \frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\cos\left(5a - \frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^{(3/2)}*\text{Cos}[a + b*x]^3*\text{Sin}[a + b*x]^2, x]$

[Out] $(3*d*\text{Sqrt}[c + d*x]*\text{Cos}[a + b*x])/(16*b^2) - (d*\text{Sqrt}[c + d*x]*\text{Cos}[3*a + 3*b*x])/(96*b^2) - (3*d*\text{Sqrt}[c + d*x]*\text{Cos}[5*a + 5*b*x])/(800*b^2) - (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a - (b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(16*b^{(5/2)}) + (d^{(3/2)}*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(96*b^{(5/2)}) + (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/10]*\text{Cos}[5*a - (5*b*c)/d]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]])/(800*b^{(5/2)}) - (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/10]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]]*\text{Sin}[5*a - (5*b*c)/d])/(800*b^{(5/2)}) - (d^{(3/2)}*\text{Sqrt}[\text{Pi}/6]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]]*\text{Sin}[3*a - (3*b*c)/d])/(96*b^{(5/2)}) + (3*d^{(3/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]]*\text{Sin}[a - (b*c)/d])/(16*b^{(5/2)}) + ((c + d*x)^{(3/2)}*\text{Sin}[a + b*x])/(8*b) - ((c + d*x)^{(3/2)}*\text{Sin}[3*a + 3*b*x])/(48*b) - ((c + d*x)^{(3/2)}*\text{Sin}[5*a + 5*b*x])/(80*b)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] := \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^{3/2} \cos^3(a + bx) \sin^2(a + bx) dx &= \int \left(\frac{1}{8}(c + dx)^{3/2} \cos(a + bx) - \frac{1}{16}(c + dx)^{3/2} \cos(3a + 3bx) - \frac{1}{16}(c + dx)^{3/2} \cos(5a + 5bx) \right) dx \\
&= -\left(\frac{1}{16} \int (c + dx)^{3/2} \cos(3a + 3bx) dx \right) - \frac{1}{16} \int (c + dx)^{3/2} \cos(5a + 5bx) dx + \frac{(c + dx)^{3/2} \sin(a + bx)}{8b} \\
&= \frac{(c + dx)^{3/2} \sin(a + bx)}{8b} - \frac{(c + dx)^{3/2} \sin(3a + 3bx)}{48b} - \frac{(c + dx)^{3/2} \sin(5a + 5bx)}{80b} \\
&= \frac{3d\sqrt{c + dx} \cos(a + bx)}{16b^2} - \frac{d\sqrt{c + dx} \cos(3a + 3bx)}{96b^2} - \frac{3d\sqrt{c + dx} \cos(5a + 5bx)}{800b^2} \\
&= \frac{3d\sqrt{c + dx} \cos(a + bx)}{16b^2} - \frac{d\sqrt{c + dx} \cos(3a + 3bx)}{96b^2} - \frac{3d\sqrt{c + dx} \cos(5a + 5bx)}{800b^2} \\
&= \frac{3d\sqrt{c + dx} \cos(a + bx)}{16b^2} - \frac{d\sqrt{c + dx} \cos(3a + 3bx)}{96b^2} - \frac{3d\sqrt{c + dx} \cos(5a + 5bx)}{800b^2} \\
&= \frac{3d\sqrt{c + dx} \cos(a + bx)}{16b^2} - \frac{d\sqrt{c + dx} \cos(3a + 3bx)}{96b^2} - \frac{3d\sqrt{c + dx} \cos(5a + 5bx)}{800b^2}
\end{aligned}$$

Mathematica [C] time = 12.4057, size = 1043, normalized size = 1.95

$$\frac{ice^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\Gamma\left(\frac{3}{2}-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{-\frac{2ibc}{d}}\Gamma\left(\frac{3}{2}+\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b}+d\left(\sqrt{\frac{b}{d}}\sqrt{2\pi}\text{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(2bc\sin\left(\frac{2bc}{d}\sqrt{\frac{c+dx}{b}}\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]

[Out]
$$\frac{(-I/16)*c*\text{Sqrt}[c + d*x]*((E^{((2*I)*a)}*\Gamma[3/2, ((-I)*b*(c + d*x))/d])/ \text{Sqrt}[(I*b*(c + d*x))/d] - (E^{((2*I)*b*c)/d}*\Gamma[3/2, (I*b*(c + d*x))/d])/ \text{Sqrt}[(I*b*(c + d*x))/d])/(b*E^{((I*(b*c + a*d))/d)} + (d*(\text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[2/Pi]*\text{Sqrt}[c + d*x]]*(-3*d*\text{Cos}[a - (b*c)/d] + 2*b*c*\text{Sin}[a - (b*c)/d]) + \text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[2/Pi]*\text{Sqrt}[c + d*x]]*(2*b*c*\text{Cos}[a - (b*c)/d] + 3*d*\text{Sin}[a - (b*c)/d]) + 2*b*\text{Sqrt}[c + d*x]*(3*\text{Cos}[a + b*x] + 2*b*x*\text{Sin}[a + b*x])))/(32*b^3) - (c*(-\text{Sqrt}[2*Pi]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]) - \text{Sqrt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*\text{Sin}[3*a - (3*b*c)/d] + 2*\text{Sqrt}[3]*\text{Sqrt}[b/d]*\text{Sqrt}[c + d*x]*\text{Sin}[3*(a + b*x)]))/(96*\text{Sqrt}[3]*b*\text{Sqrt}[b/d]) - (d*(\text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*(-d*\text{Cos}[3*a - (3*b*c)/d]) + 2*b*c*\text{Sin}[3*a - (3*b*c)/d]) + \text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*(2*b*c*\text{Cos}[3*a - (3*b*c)/d] + d*\text{Sin}[3*a - (3*b*c)/d]) + 2*\text{Sqrt}[3]*b*\text{Sqrt}[c + d*x]*(\text{Cos}[3*(a + b*x)] + 2*b*x*\text{Sin}[3*(a + b*x)])))/(192*\text{Sqrt}[3]*b^3) - (c*(-\text{Sqrt}[2*Pi]*\text{Cos}[5*a - (5*b*c)/d]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[10/Pi]*\text{Sqrt}[c + d*x]]) - \text{Sqrt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[10/Pi]*\text{Sqrt}[c + d*x]]*\text{Sin}[5*a - (5*b*c)/d] + 2*\text{Sqrt}[5]*\text{Sqrt}[b/d]*\text{Sqrt}[c + d*x]*\text{Sin}[5*(a + b*x)]))/(160*\text{Sqrt}[5]*b*\text{Sqrt}[b/d]) - (d*(\text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[10/Pi]*\text{Sqrt}[c + d*x]]*(-3*d*\text{Cos}[5*a - (5*b*c)/d] + 10*b*c*\text{Sin}[5*a - (5*b*c)/d]) + \text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[10/Pi]*\text{Sqrt}[c + d*x]]*(10*b*c*\text{Cos}[5*a - (5*b*c)/d] + 3*d*\text{Sin}[5*a - (5*b*c)/d]) + 2*\text{Sqrt}[5]*b*\text{Sqrt}[c + d*x]*(3*\text{Cos}[5*(a + b*x)] + 10*b*x*\text{Sin}[5*(a + b*x)])))/(1600*\text{Sqrt}[5]*b^3)$$

Maple [A] time = 0.052, size = 583, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)

[Out]
$$\frac{2}{d}\left(\frac{1}{16b}\frac{d*(d*x+c)^{3/2}*\sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/16/b*d*(-1/2/b*d*(d*x+c)^{1/2}*\cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^{1/2}*Pi^{1/2}/(b/d)^{1/2}*(\cos((a*d-b*c)/d)*\text{FresnelC}(2^{1/2}/Pi^{1/2}/(b/d)^{1/2}*(d*x+c)^{1/2}*b/d)-\sin((a*d-b*c)/d)*\text{FresnelS}(2^{1/2}/Pi^{1/2}/(b/d)^{1/2}*(d*x+c)^{1/2}*b/d))}{196b}\frac{d*(d*x+c)^{3/2}*\sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/32/b*d*(-1/6/b*d*(d*x+c)^{1/2}*\cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^{1/2}*Pi^{1/2}*3^{1/2}/(b/d)^{1/2}*(\cos(3*(a*d-b*c)/d)*\text{FresnelC}(2^{1/2}/Pi^{1/2}*3^{1/2}/(b/d)^{1/2}*(d*x+c)^{1/2}*b/d)-\sin(3*(a*d-b*c)/d)*\text{FresnelS}(2^{1/2}/Pi^{1/2}*3^{1/2}/(b/d)^{1/2}*(d*x+c)^{1/2}*b/d))}{160b}\frac{d*(d*x+c)^{3/2}*\sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+3/160/b*d*(-1/10/b*d*(d*x+c)^{1/2}*\cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+1/100/b*d*2^{1/2}*Pi^{1/2}*5^{1/2}/(b/d)^{1/2}*(\cos(5*(a*d-b*c)/d)*\text{FresnelC}(2^{1/2}/Pi^{1/2}*5^{1/2}/(b/d)^{1/2}*(d*x+c)^{1/2}*b/d)-\sin(5*(a*d-b*c)/d)*\text{FresnelS}(2^{1/2}/Pi^{1/2}*5^{1/2}/(b/d)^{1/2}*(d*x+c)^{1/2}*b/d)}{1600b}$$

$$x+c)^{(1/2)*b/d))$$

Maxima [C] time = 2.62542, size = 2790, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/288000*sqrt(5)*sqrt(3)*(240*sqrt(5)*sqrt(3)*(d*x + c)^(3/2)*b*d*sqrt(abs(b)/abs(d))*abs(b)*sin(5*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 400*sqrt(5)*sqrt(3)*(d*x + c)^(3/2)*b*d*sqrt(abs(b)/abs(d))*abs(b)*sin(3*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 2400*sqrt(5)*sqrt(3)*(d*x + c)^(3/2)*b*d*sqrt(abs(b)/abs(d))*abs(b)*sin(((d*x + c)*b - b*c + a*d)/d)/abs(d) + 72*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d^2*sqrt(abs(b)/abs(d))*abs(b)*cos(5*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 200*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d^2*sqrt(abs(b)/abs(d))*abs(b)*cos(3*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 3600*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d^2*sqrt(abs(b)/abs(d))*abs(b)*cos(((d*x + c)*b - b*c + a*d)/d)/abs(d) - (sqrt(3)*(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d^2*abs(b)*cos(-5*(b*c - a*d)/d)/abs(d) - sqrt(3)*(9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d^2*abs(b)*sin(-5*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - (sqrt(5)*(25*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 25*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 25*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 25*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d^2*abs(b)*cos(-3*(b*c - a*d)/d)/abs(d) - sqrt(5)*(25*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 25*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 25*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 25*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d^2*abs(b)*sin(-3*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (sqrt(5)*sqrt(3)*(450*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 450*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d^2*abs(b)*cos(-(b*c - a*d)/d)/abs(d) + sqrt(5)*sqrt(3)*(-450*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 450*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 450*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d^2*abs(b)*sin(-(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (sqrt(5)*sqrt(3)*(450*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 450*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d^2*abs(b)*cos(-(b*c - a*d)/d)/abs(d) + sqrt(5)*sqrt(3)*(450*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 450*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 450*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d^2*abs(b)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 450*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 450*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
```

$$\begin{aligned} & /4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 450*\sqrt{\pi}*\sin \\ & (-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^2 * \text{abs}(b) * \sin \\ & (-(b*c - a*d)/d) / \text{abs}(d) * \text{erf}(\sqrt{d*x + c} * \sqrt{-I*b/d}) - (\sqrt{5} * (25*\sqrt{\pi} * \cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 25*\sqrt{\pi} * \cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 25*I*\sqrt{\pi} * \sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 25*I*\sqrt{\pi} * \sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^2 * \text{abs}(b) * \cos(-3*(b*c - a*d)/d) / \text{abs}(d) - \sqrt{5} * (-25*I*\sqrt{\pi} * \cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 25*I*\sqrt{\pi} * \cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 25*\sqrt{\pi} * \sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 25*\sqrt{\pi} * \sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^2 * \text{abs}(b) * \sin(-3*(b*c - a*d)/d) / \text{abs}(d) * \text{erf}(\sqrt{d*x + c} * \sqrt{-3*I*b/d}) - (\sqrt{3} * (9*\sqrt{\pi} * \cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 9*\sqrt{\pi} * \cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 9*I*\sqrt{\pi} * \sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 9*I*\sqrt{\pi} * \sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^2 * \text{abs}(b) * \cos(-5*(b*c - a*d)/d) / \text{abs}(d) - \sqrt{3} * (-9*I*\sqrt{\pi} * \cos(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 9*I*\sqrt{\pi} * \cos(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) + 9*\sqrt{\pi} * \sin(1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) - 9*\sqrt{\pi} * \sin(-1/4*\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))) * d^2 * \text{abs}(b) * \sin(-5*(b*c - a*d)/d) / \text{abs}(d) * \text{erf}(\sqrt{d*x + c} * \sqrt{-5*I*b/d}) * \text{abs}(d) / (b^2 * d * \sqrt{\text{abs}(b) / \text{abs}(d)}) * \text{abs}(b)) \end{aligned}$$

Fricas [A] time = 0.769284, size = 1162, normalized size = 2.18

$$27 \sqrt{10} \pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{5(bc-ad)}{d}\right) C\left(\sqrt{10} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) + 125 \sqrt{6} \pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{3(bc-ad)}{d}\right) C\left(\sqrt{6} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) - 6750 \sqrt{2} \pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{b*c - a*d}{d}\right) \text{fresnel_cos}\left(\sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) + 6750 \sqrt{2} \pi d^2 \sqrt{\frac{b}{\pi d}} \text{fresnel_sin}\left(\sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) * \sin\left(-\frac{b*c - a*d}{d}\right) - 125 \sqrt{6} \pi d^2 \sqrt{\frac{b}{\pi d}} \text{fresnel_sin}\left(\sqrt{6} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) * \sin\left(-\frac{3*(b*c - a*d)}{d}\right) - 27 \sqrt{10} \pi d^2 \sqrt{\frac{b}{\pi d}} \text{fresnel_sin}\left(\sqrt{10} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) * \sin\left(-\frac{5*(b*c - a*d)}{d}\right) - 480 * (9*b*d*\cos(b*x + a)^5 - 5*b*d*\cos(b*x + a)^3 - 30*b*d*\cos(b*x + a) + 10*(3*(b^2*d*x + b^2*c)*\cos(b*x + a)^4 - 2*b^2*d*x - 2*b^2*c - (b^2*d*x + b^2*c)*\cos(b*x + a)^2) * \sin(b*x + a)) * \sqrt{dx+c}) / b^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")

[Out] 1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_cos(sqrt(10)*sqrt(dx + c)*sqrt(b/(pi*d))) + 125*sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(dx + c)*sqrt(b/(pi*d))) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(dx + c)*sqrt(b/(pi*d))) + 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(dx + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(dx + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(10)*sqrt(dx + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(9*b*d*cos(b*x + a)^5 - 5*b*d*cos(b*x + a)^3 - 30*b*d*cos(b*x + a) + 10*(3*(b^2*d*x + b^2*c)*cos(b*x + a)^4 - 2*b^2*d*x - 2*b^2*c - (b^2*d*x + b^2*c)*cos(b*x + a)^2) * sin(b*x + a)) * sqrt(dx + c)) / b^3

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)

[Out] Timed out

Giac [C] time = 1.81203, size = 2271, normalized size = 4.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")

[Out]
$$-1/144000*(10*(9*I*\sqrt{10}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{10}*\sqrt{b*d}*\sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((5*I*b*c-5*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2}+1)*b)}+25*I*\sqrt{6}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((3*I*b*c-3*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2}+1)*b)}-450*I*\sqrt{2}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((I*b*c-I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2}+1)*b)}+450*I*\sqrt{2}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((-I*b*c+I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2}+1)*b)}-25*I*\sqrt{6}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((-3*I*b*c+3*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2}+1)*b)}-9*I*\sqrt{10}*\sqrt{\pi})*d^2*\operatorname{erf}(-1/2*\sqrt{10}*\sqrt{b*d}*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((-5*I*b*c+5*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2}+1)*b)}-90*I*\sqrt{d*x+c})*d*e^{((5*I*(d*x+c)*b-5*I*b*c+5*I*a*d)/d)/b}-150*I*\sqrt{d*x+c})*d*e^{((3*I*(d*x+c)*b-3*I*b*c+3*I*a*d)/d)/b}+900*I*\sqrt{d*x+c})*d*e^{((I*(d*x+c)*b-I*b*c+I*a*d)/d)/b}-900*I*\sqrt{d*x+c})*d*e^{((-I*(d*x+c)*b+I*b*c-I*a*d)/d)/b}+150*I*\sqrt{d*x+c})*d*e^{((-3*I*(d*x+c)*b+3*I*b*c-3*I*a*d)/d)/b}+90*I*\sqrt{d*x+c})*d*e^{((-5*I*(d*x+c)*b+5*I*b*c-5*I*a*d)/d)/b}*c-9*\sqrt{10}*\sqrt{\pi})*(10*I*b*c*d-3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{10}*\sqrt{b*d}*\sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((5*I*b*c-5*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2}+1)*b^2)}-125*\sqrt{6}*\sqrt{\pi})*(2*I*b*c*d-d^2)*d*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((3*I*b*c-3*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2}+1)*b^2)}-2250*\sqrt{2}*\sqrt{\pi})*(-2*I*b*c*d+3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((I*b*c-I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2}+1)*b^2)}-2250*\sqrt{2}*\sqrt{\pi})*(2*I*b*c*d+3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((-I*b*c+I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2}+1)*b^2)}-125*\sqrt{6}*\sqrt{\pi})*(-2*I*b*c*d-d^2)*d*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((-3*I*b*c+3*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2}+1)*b^2)}-9*\sqrt{10}*\sqrt{\pi})*(-10*I*b*c*d-3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{10}*\sqrt{b*d}*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2}+1)/d)*e^{((-5*I*b*c+5*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2}+1)*b^2)}+90*(-10*I*(d*x+c)^(3/2)*b*d+10*I*\sqrt{d*x+c})*b*c*d+3*\sqrt{d*x+c})*d^2)*e^{((5*I*(d*x+c)*b-5*I*b*c+5*I*a*d)/d)/b^2}+750*(-2*I*(d*x+c)^(3/2)*b*d+2*I*\sqrt{d*x+c})*b*c*d+\sqrt{d*x+c})*d^2)*e^{((3*I*(d*x+c)*b-3*I*b*c+3*I*a*d)/d)/b^2}+4500*(2*I*(d*x+c)^(3/2)*b*d-2*I*\sqrt{d*x+c})*b*c*d-3*\sqrt{d*x+c})*d^2)*e^{((I*(d*x+c)*b-I*b*c+I*a*d)/d)/b^2}+4500*(-2*I*(d*x+c)^(3/2)*b*d+2*I*\sqrt{d*x+c})*b*c*d-3*\sqrt{d*x+c})*d^2)*e^{((-I*(d*x+c)*b+I*b*c-I*a*d)/d)/b^2}+750*(2*I*(d*x+c)^(3/2)*b*d-2*I*\sqrt{d*x+c})*b*c*d+\sqrt{d*x+c})*d^2)*e^{((-3*I*(d*x+c)*b+3*I*b*c-3*I*a*d)/d)/b^2}+90*(10*I*(d*x+c)^(3/2)*b*d-10*I*\sqrt{d*x+c})*b*c*d+3*\sqrt{d*x+c})*d^2)*e^{((-5*I*(d*x+c)*b+5*I*b*c-5*I*a*d)/d)/b^2)/d$$

3.195 $\int (c + dx)^{5/2} \cos^3(a + bx) \sin^2(a + bx) dx$

Optimal. Leaf size=615

$$\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\sin\left(5a - \frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}} - \frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}} + \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}$$

[Out] $(5*d*(c + d*x)^{(3/2)}*\text{Cos}[a + b*x])/(16*b^2) - (5*d*(c + d*x)^{(3/2)}*\text{Cos}[3*a + 3*b*x])/(288*b^2) - (d*(c + d*x)^{(3/2)}*\text{Cos}[5*a + 5*b*x])/(160*b^2) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a - (b*c)/d]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(32*b^{(7/2)}) - (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(576*b^{(7/2)}) - (3*d^{(5/2)}*\text{Sqrt}[\text{Pi}/10]*\text{Cos}[5*a - (5*b*c)/d]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(1600*b^{(7/2)}) - (3*d^{(5/2)}*\text{Sqrt}[\text{Pi}/10]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[5*a - (5*b*c)/d])/(1600*b^{(7/2)}) - (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[3*a - (3*b*c)/d])/(576*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[a - (b*c)/d])/(32*b^{(7/2)}) - (15*d^2*\text{Sqrt}[c + d*x]*\text{Sin}[a + b*x])/(32*b^3) + ((c + d*x)^{(5/2)}*\text{Sin}[a + b*x])/(8*b) + (5*d^2*\text{Sqrt}[c + d*x]*\text{Sin}[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^{(5/2)}*\text{Sin}[3*a + 3*b*x])/(48*b) + (3*d^2*\text{Sqrt}[c + d*x]*\text{Sin}[5*a + 5*b*x])/(1600*b^3) - ((c + d*x)^{(5/2)}*\text{Sin}[5*a + 5*b*x])/(80*b)$

Rubi [A] time = 1.01548, antiderivative size = 615, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\sin\left(5a - \frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}} - \frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a - \frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}} + \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a - \frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^{(5/2)}*\text{Cos}[a + b*x]^3*\text{Sin}[a + b*x]^2, x]$

[Out] $(5*d*(c + d*x)^{(3/2)}*\text{Cos}[a + b*x])/(16*b^2) - (5*d*(c + d*x)^{(3/2)}*\text{Cos}[3*a + 3*b*x])/(288*b^2) - (d*(c + d*x)^{(3/2)}*\text{Cos}[5*a + 5*b*x])/(160*b^2) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a - (b*c)/d]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(32*b^{(7/2)}) - (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(576*b^{(7/2)}) - (3*d^{(5/2)}*\text{Sqrt}[\text{Pi}/10]*\text{Cos}[5*a - (5*b*c)/d]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(1600*b^{(7/2)}) - (3*d^{(5/2)}*\text{Sqrt}[\text{Pi}/10]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[10/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[5*a - (5*b*c)/d])/(1600*b^{(7/2)}) - (5*d^{(5/2)}*\text{Sqrt}[\text{Pi}/6]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[3*a - (3*b*c)/d])/(576*b^{(7/2)}) + (15*d^{(5/2)}*\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[a - (b*c)/d])/(32*b^{(7/2)}) - (15*d^2*\text{Sqrt}[c + d*x]*\text{Sin}[a + b*x])/(32*b^3) + ((c + d*x)^{(5/2)}*\text{Sin}[a + b*x])/(8*b) + (5*d^2*\text{Sqrt}[c + d*x]*\text{Sin}[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^{(5/2)}*\text{Sin}[3*a + 3*b*x])/(48*b) + (3*d^2*\text{Sqrt}[c + d*x]*\text{Sin}[5*a + 5*b*x])/(1600*b^3) - ((c + d*x)^{(5/2)}*\text{Sin}[5*a + 5*b*x])/(80*b)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]$

$]^n \cos[a + b*x]^p, x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3306

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]

Rule 3305

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3304

Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}
\int (c + dx)^{5/2} \cos^3(a + bx) \sin^2(a + bx) dx &= \int \left(\frac{1}{8} (c + dx)^{5/2} \cos(a + bx) - \frac{1}{16} (c + dx)^{5/2} \cos(3a + 3bx) - \frac{1}{16} (c + dx)^{5/2} \cos(5a + 5bx) \right) dx \\
&= - \left(\frac{1}{16} \int (c + dx)^{5/2} \cos(3a + 3bx) dx \right) - \frac{1}{16} \int (c + dx)^{5/2} \cos(5a + 5bx) dx \\
&= \frac{(c + dx)^{5/2} \sin(a + bx)}{8b} - \frac{(c + dx)^{5/2} \sin(3a + 3bx)}{48b} - \frac{(c + dx)^{5/2} \sin(5a + 5bx)}{80b} \\
&= \frac{5d(c + dx)^{3/2} \cos(a + bx)}{16b^2} - \frac{5d(c + dx)^{3/2} \cos(3a + 3bx)}{288b^2} - \frac{d(c + dx)^{3/2} \cos(5a + 5bx)}{160b^2} \\
&= \frac{5d(c + dx)^{3/2} \cos(a + bx)}{16b^2} - \frac{5d(c + dx)^{3/2} \cos(3a + 3bx)}{288b^2} - \frac{d(c + dx)^{3/2} \cos(5a + 5bx)}{160b^2} \\
&= \frac{5d(c + dx)^{3/2} \cos(a + bx)}{16b^2} - \frac{5d(c + dx)^{3/2} \cos(3a + 3bx)}{288b^2} - \frac{d(c + dx)^{3/2} \cos(5a + 5bx)}{160b^2} \\
&= \frac{5d(c + dx)^{3/2} \cos(a + bx)}{16b^2} - \frac{5d(c + dx)^{3/2} \cos(3a + 3bx)}{288b^2} - \frac{d(c + dx)^{3/2} \cos(5a + 5bx)}{160b^2} \\
&= \frac{5d(c + dx)^{3/2} \cos(a + bx)}{16b^2} - \frac{5d(c + dx)^{3/2} \cos(3a + 3bx)}{288b^2} - \frac{d(c + dx)^{3/2} \cos(5a + 5bx)}{160b^2}
\end{aligned}$$

Mathematica [C] time = 23.4094, size = 1795, normalized size = 2.92

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]

[Out]
$$\begin{aligned}
&((-I/16)*c^2*\text{Sqrt}[c + d*x]*((E^{((2*I)*a)}*\text{Gamma}[3/2, ((-I)*b*(c + d*x))/d])/ \\
&\text{Sqrt}[((-I)*b*(c + d*x))/d] - (E^{((2*I)*b*c)/d}*\text{Gamma}[3/2, (I*b*(c + d*x))/ \\
&d])/ \text{Sqrt}[(I*b*(c + d*x))/d]))/(b*E^{((I*(b*c + a*d))/d)} + (c*d*(\text{Sqrt}[b/d]*\text{S} \\
&\text{qrt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[2/Pi]*\text{Sqrt}[c + d*x]]*(-3*d*\text{Cos}[a - (b*c)/ \\
&d] + 2*b*c*\text{Sin}[a - (b*c)/d]) + \text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt} \\
&[2/Pi]*\text{Sqrt}[c + d*x]]*(2*b*c*\text{Cos}[a - (b*c)/d] + 3*d*\text{Sin}[a - (b*c)/d]) + 2*b \\
&* \text{Sqrt}[c + d*x]*(3*\text{Cos}[a + b*x] + 2*b*x*\text{Sin}[a + b*x])))/(16*b^3) + ((b/d)^(3 \\
&/2)*d^2*(-(\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[2/Pi]*\text{Sqrt}[c + d*x]]*((4*b^2*c \\
&c^2 - 15*d^2)*\text{Cos}[a - (b*c)/d] + 12*b*c*d*\text{Sin}[a - (b*c)/d])) - \text{Sqrt}[2*Pi]*\text{F} \\
&\text{resnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[2/Pi]*\text{Sqrt}[c + d*x]]*(-12*b*c*d*\text{Cos}[a - (b*c)/d] + (\\
&4*b^2*c^2 - 15*d^2)*\text{Sin}[a - (b*c)/d]) + 2*\text{Sqrt}[b/d]*d*\text{Sqrt}[c + d*x]*(-2*b*(\\
&c - 5*d*x)*\text{Cos}[a + b*x] + d*(-15 + 4*b^2*x^2)*\text{Sin}[a + b*x]))/(64*b^5) - (c \\
&^2*(-(\text{Sqrt}[2*Pi]*\text{Cos}[3*a - (3*b*c)/d]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c \\
&+ d*x]]) - \text{Sqrt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*\text{Sin}[3*a \\
&- (3*b*c)/d] + 2*\text{Sqrt}[3]*\text{Sqrt}[b/d]*\text{Sqrt}[c + d*x]*\text{Sin}[3*(a + b*x)])))/(96*\text{Sqr} \\
&\text{t}[3]*b*\text{Sqrt}[b/d]) - (c*d*(\text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi \\
&]*\text{Sqrt}[c + d*x]]*(-(d*\text{Cos}[3*a - (3*b*c)/d]) + 2*b*c*\text{Sin}[3*a - (3*b*c)/d]) + \\
&\text{Sqrt}[b/d]*\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*(2*b*c*\text{C} \\
&\text{os}[3*a - (3*b*c)/d] + d*\text{Sin}[3*a - (3*b*c)/d]) + 2*\text{Sqrt}[3]*b*\text{Sqrt}[c + d*x]*(\\
&\text{Cos}[3*(a + b*x)] + 2*b*x*\text{Sin}[3*(a + b*x)])))/(96*\text{Sqrt}[3]*b^3) - ((b/d)^(3/2) \\
&)*d^2*(-(\text{Sqrt}[2*Pi]*\text{FresnelS}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*((12*b^2*c \\
&^2 - 5*d^2)*\text{Cos}[3*a - (3*b*c)/d] + 12*b*c*d*\text{Sin}[3*a - (3*b*c)/d])) - \text{Sqrt}[2 \\
&*Pi]*\text{FresnelC}[\text{Sqrt}[b/d]*\text{Sqrt}[6/Pi]*\text{Sqrt}[c + d*x]]*(-12*b*c*d*\text{Cos}[3*a - (3*b \\
&*c)/d] + (12*b^2*c^2 - 5*d^2)*\text{Sin}[3*a - (3*b*c)/d]) + 2*\text{Sqrt}[3]*\text{Sqrt}[b/d]*d \\
&*\text{Sqrt}[c + d*x]*(-2*b*(c - 5*d*x)*\text{Cos}[3*(a + b*x)] + d*(-5 + 12*b^2*x^2)*\text{Sin} \\
&[3*(a + b*x)])))/(1152*\text{Sqrt}[3]*b^5) - (c^2*(-(\text{Sqrt}[2*Pi]*\text{Cos}[5*a - (5*b*c)/
\end{aligned}$$

$$d] * \text{FresnelS}[\sqrt{b/d} * \sqrt{10/\pi} * \sqrt{c + dx}] - \sqrt{2\pi} * \text{FresnelC}[\sqrt{b/d} * \sqrt{10/\pi} * \sqrt{c + dx}] * \sin[5a - (5bc)/d] + 2\sqrt{5} * \sqrt{b/d} * \sqrt{c + dx} * \sin[5(a + bx)] / (160\sqrt{5} * b * \sqrt{b/d}) - (c * d * (\sqrt{b/d} * \sqrt{2\pi} * \text{FresnelC}[\sqrt{b/d} * \sqrt{10/\pi} * \sqrt{c + dx}] * (-3d * \cos[5a - (5bc)/d] + 10bc * \sin[5a - (5bc)/d]) + \sqrt{b/d} * \sqrt{2\pi} * \text{FresnelS}[\sqrt{b/d} * \sqrt{10/\pi} * \sqrt{c + dx}] * (10bc * \cos[5a - (5bc)/d] + 3d * \sin[5a - (5bc)/d]) + 2\sqrt{5} * b * \sqrt{c + dx} * (3\cos[5(a + bx)] + 10bx * \sin[5(a + bx)])) / (800\sqrt{5} * b^3) - ((b/d)^{(3/2)} * d^2 * (-\sqrt{2\pi} * \text{FresnelS}[\sqrt{b/d} * \sqrt{10/\pi} * \sqrt{c + dx}] * ((20b^2c^2 - 3d^2) * \cos[5a - (5bc)/d] + 12bc * d * \sin[5a - (5bc)/d])) - \sqrt{2\pi} * \text{FresnelC}[\sqrt{b/d} * \sqrt{10/\pi} * \sqrt{c + dx}] * (-12bc * d * \cos[5a - (5bc)/d] + (20b^2c^2 - 3d^2) * \sin[5a - (5bc)/d]) + 2\sqrt{5} * \sqrt{b/d} * d * \sqrt{c + dx} * (-2b * (c - 5dx) * \cos[5(a + bx)] + d * (-3 + 20b^2x^2) * \sin[5(a + bx)])) / (3200\sqrt{5} * b^5)$$

Maple [A] time = 0.051, size = 716, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((dx+c)^{(5/2)} * \cos(bx+a)^3 * \sin(bx+a)^2, x)$

[Out] $2/d * (1/16/b * d * (dx+c)^{(5/2)} * \sin(1/d * (dx+c) * b + (a*d-b*c)/d) - 5/16/b * d * (-1/2/b * d * (dx+c)^{(3/2)} * \cos(1/d * (dx+c) * b + (a*d-b*c)/d) + 3/2/b * d * (1/2/b * d * (dx+c)^{(1/2)} * \sin(1/d * (dx+c) * b + (a*d-b*c)/d) - 1/4/b * d * 2^{(1/2)} * \pi^{(1/2)} / (b/d)^{(1/2)} * (\cos((a*d-b*c)/d) * \text{FresnelS}(2^{(1/2)}/\pi^{(1/2)} / (b/d)^{(1/2)} * (dx+c)^{(1/2)} * b/d) + \sin((a*d-b*c)/d) * \text{FresnelC}(2^{(1/2)}/\pi^{(1/2)} / (b/d)^{(1/2)} * (dx+c)^{(1/2)} * b/d))) - 1/96/b * d * (dx+c)^{(5/2)} * \sin(3/d * (dx+c) * b + 3 * (a*d-b*c)/d) + 5/96/b * d * (-1/6/b * d * (dx+c)^{(3/2)} * \cos(3/d * (dx+c) * b + 3 * (a*d-b*c)/d) + 1/2/b * d * (1/6/b * d * (dx+c)^{(1/2)} * \sin(3/d * (dx+c) * b + 3 * (a*d-b*c)/d) - 1/36/b * d * 2^{(1/2)} * \pi^{(1/2)} * 3^{(1/2)} / (b/d)^{(1/2)} * (\cos(3 * (a*d-b*c)/d) * \text{FresnelS}(2^{(1/2)}/\pi^{(1/2)} * 3^{(1/2)} / (b/d)^{(1/2)} * (dx+c)^{(1/2)} * b/d) + \sin(3 * (a*d-b*c)/d) * \text{FresnelC}(2^{(1/2)}/\pi^{(1/2)} * 3^{(1/2)} / (b/d)^{(1/2)} * (dx+c)^{(1/2)} * b/d))) - 1/160/b * d * (dx+c)^{(5/2)} * \sin(5/d * (dx+c) * b + 5 * (a*d-b*c)/d) + 1/32/b * d * (-1/10/b * d * (dx+c)^{(3/2)} * \cos(5/d * (dx+c) * b + 5 * (a*d-b*c)/d) + 3/10/b * d * (1/10/b * d * (dx+c)^{(1/2)} * \sin(5/d * (dx+c) * b + 5 * (a*d-b*c)/d) - 1/100/b * d * 2^{(1/2)} * \pi^{(1/2)} * 5^{(1/2)} / (b/d)^{(1/2)} * (\cos(5 * (a*d-b*c)/d) * \text{FresnelS}(2^{(1/2)}/\pi^{(1/2)} * 5^{(1/2)} / (b/d)^{(1/2)} * (dx+c)^{(1/2)} * b/d) + \sin(5 * (a*d-b*c)/d) * \text{FresnelC}(2^{(1/2)}/\pi^{(1/2)} * 5^{(1/2)} / (b/d)^{(1/2)} * (dx+c)^{(1/2)} * b/d)))$

Maxima [C] time = 2.69643, size = 2940, normalized size = 4.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((dx+c)^{(5/2)} * \cos(bx+a)^3 * \sin(bx+a)^2, x, \text{algorithm}="maxima")$

[Out] $-1/1728000 * \sqrt{5} * \sqrt{3} * (720 * \sqrt{5} * \sqrt{3} * (dx + c)^{(3/2)} * b * d^2 * \sqrt{(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) * \cos(5 * ((dx + c) * b - b * c + a * d) / d) / \text{abs}(d)} + 2000 * \sqrt{5} * \sqrt{3} * (dx + c)^{(3/2)} * b * d^2 * \sqrt{(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) * \cos(3 * ((dx + c) * b - b * c + a * d) / d) / \text{abs}(d)} - 36000 * \sqrt{5} * \sqrt{3} * (dx + c)^{(3/2)} * b * d^2 * \sqrt{(\text{abs}(b)/\text{abs}(d)) * \text{abs}(b) * \cos(((dx + c) * b - b * c + a * d) / d) / \text{abs}(d)} + (\sqrt{3} * (27 * I * \sqrt{\pi}) * \cos(1/4 * \pi + 1/2 * \arctan2(0, b) + 1/2 * \arctan2(0, d/\sqrt{d^2})))$

$$2(0, d/\sqrt{d^2})) * d^3 \text{abs}(b) \sin(-5*(b*c - a*d)/d) / \text{abs}(d) * \text{erf}(\sqrt{d*x + c} * \sqrt{-5*I*b/d}) + 72*(20*\sqrt{5}*\sqrt{3}*(d*x + c)^{(5/2)} * b^2*d*\sqrt{\text{abs}(b)/\text{abs}(d)} * \text{abs}(b)/\text{abs}(d) - 3*\sqrt{5}*\sqrt{3}*\sqrt{d*x + c} * d^3*\sqrt{\text{abs}(b)/\text{abs}(d)} * \text{abs}(b)/\text{abs}(d) * \sin(5*((d*x + c)*b - b*c + a*d)/d) + 200*(12*\sqrt{5})*\sqrt{3}*(d*x + c)^{(5/2)} * b^2*d*\sqrt{\text{abs}(b)/\text{abs}(d)} * \text{abs}(b)/\text{abs}(d) - 5*\sqrt{5}*\sqrt{3}*\sqrt{d*x + c} * d^3*\sqrt{\text{abs}(b)/\text{abs}(d)} * \text{abs}(b)/\text{abs}(d) * \sin(3*((d*x + c)*b - b*c + a*d)/d) - 3600*(4*\sqrt{5}*\sqrt{3}*(d*x + c)^{(5/2)} * b^2*d*\sqrt{\text{abs}(b)/\text{abs}(d)} * \text{abs}(b)/\text{abs}(d) - 15*\sqrt{5}*\sqrt{3}*\sqrt{d*x + c} * d^3*\sqrt{\text{abs}(b)/\text{abs}(d)} * \text{abs}(b)/\text{abs}(d) * \sin(((d*x + c)*b - b*c + a*d)/d)) * \text{abs}(d) / (b^3 * d*\sqrt{\text{abs}(b)/\text{abs}(d)} * \text{abs}(b))$$

Fricas [A] time = 0.830458, size = 1397, normalized size = 2.27

$$81 \sqrt{10} \pi d^3 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{5(bc-ad)}{d}\right) S\left(\sqrt{10} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) + 625 \sqrt{6} \pi d^3 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{3(bc-ad)}{d}\right) S\left(\sqrt{6} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right) - 101250 \sqrt{2} \pi d^3 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{bc-ad}{d}\right) S\left(\sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{\pi d}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")

[Out] $-1/432000*(81*\sqrt{10}*\pi*d^3*\sqrt{b/(pi*d)}*\cos(-5*(b*c - a*d)/d)*\text{fresnel_sin}(\sqrt{10}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) + 625*\sqrt{6}*\pi*d^3*\sqrt{b/(pi*d)}*\cos(-3*(b*c - a*d)/d)*\text{fresnel_sin}(\sqrt{6}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) - 101250*\sqrt{2}*\pi*d^3*\sqrt{b/(pi*d)}*\cos(-(b*c - a*d)/d)*\text{fresnel_sin}(\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/(pi*d)}) - 101250*\sqrt{2}*\pi*d^3*\sqrt{b/(pi*d)}*\text{fresnel_cos}(\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/(pi*d)})*\sin(-(b*c - a*d)/d) + 625*\sqrt{6}*\pi*d^3*\sqrt{b/(pi*d)}*\text{fresnel_cos}(\sqrt{6}*\sqrt{d*x + c}*\sqrt{b/(pi*d)})*\sin(-3*(b*c - a*d)/d) + 81*\sqrt{10}*\pi*d^3*\sqrt{b/(pi*d)}*\text{fresnel_cos}(\sqrt{10}*\sqrt{d*x + c}*\sqrt{b/(pi*d)})*\sin(-5*(b*c - a*d)/d) + 480*(90*(b^2*d^2*x + b^2*c*d)*\cos(b*x + a)^5 - 50*(b^2*d^2*x + b^2*c*d)*\cos(b*x + a)^3 - 300*(b^2*d^2*x + b^2*c*d)*\cos(b*x + a) - (120*b^3*d^2*x^2 + 240*b^3*c*d*x + 120*b^3*c^2 - 9*(20*b^3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*\cos(b*x + a)^4 - 428*b*d^2 + (60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 + 11*b*d^2)*\cos(b*x + a)^2)*\sin(b*x + a))*\sqrt{d*x + c})/b^4$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)

[Out] Timed out

Giac [C] time = 2.12865, size = 4077, normalized size = 6.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")

```

[Out] -1/864000*(60*(9*I*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d
*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(
I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sq
rt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d
)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*I*sqrt(2)*sqrt(pi)*d^2*erf(
-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c
- I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*sqrt(p
i)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/
d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*s
qrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)*b) - 9*I*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I
*b*d/sqrt(b^2*d^2) + 1)*b) - 90*I*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I
*b*c + 5*I*a*d)/d)/b - 150*I*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c
+ 3*I*a*d)/d)/b + 900*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/
d)/b - 900*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 150
*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*I*sq
rt(d*x + c)*d*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b*c^2 - d^2*(27
*(sqrt(10)*sqrt(pi))*(-20*I*b^2*c^2*d + 12*b*c*d^2 + 3*I*d^3)*d*erf(-1/2*sq
rt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*
I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 10*(20*I*(d*x + c)^(5
/2)*b^2*d - 40*I*(d*x + c)^(3/2)*b^2*c*d + 20*I*sqrt(d*x + c)*b^2*c^2*d + 1
0*(d*x + c)^(3/2)*b*d^2 - 12*sqrt(d*x + c)*b*c*d^2 - 3*I*sqrt(d*x + c)*d^3)
*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^3)/d^2 + 125*(sqrt(6)*sqrt(
pi))*(-12*I*b^2*c^2*d + 12*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*s
qrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b
*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*(12*I*(d*x + c)^(5/2)*b^2*d - 24*I*(
d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*
b*d^2 - 12*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((-3*I*(d*x + c
)*b + 3*I*b*c - 3*I*a*d)/d)/b^3)/d^2 + 6750*(sqrt(2)*sqrt(pi))*(4*I*b^2*c^2*
d - 12*b*c*d^2 - 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b^3) - 2*(-4*I*(d*x + c)^(5/2)*b^2*d + 8*I*(d*x + c)^(3/2)*b^2*c*d
- 4*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 12*sqrt(d*x + c)
*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b
^3)/d^2 + 6750*(sqrt(2)*sqrt(pi))*(-4*I*b^2*c^2*d - 12*b*c*d^2 + 15*I*d^3)*d
*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(4*I*(d*
x + c)^(5/2)*b^2*d - 8*I*(d*x + c)^(3/2)*b^2*c*d + 4*I*sqrt(d*x + c)*b^2*c^
2*d - 10*(d*x + c)^(3/2)*b*d^2 + 12*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x +
c)*d^3)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^2 + 125*(sqrt(6)*sqrt
(pi))*(12*I*b^2*c^2*d + 12*b*c*d^2 - 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*s
qrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*(-12*I*(d*x + c)^(5/2)*b^2*d + 24
*I*(d*x + c)^(3/2)*b^2*c*d - 12*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3
/2)*b*d^2 - 12*sqrt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((3*I*(d*x
+ c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/d^2 + 27*(sqrt(10)*sqrt(pi))*(20*I*b^2*c
^2*d + 12*b*c*d^2 - 3*I*d^3)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/s
qrt(b^2*d^2) + 1)*b^3) - 10*(-20*I*(d*x + c)^(5/2)*b^2*d + 40*I*(d*x + c)^(
3/2)*b^2*c*d - 20*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 12
*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*x + c)*d^3)*e^((5*I*(d*x + c)*b - 5*I*b
*c + 5*I*a*d)/d)/b^3)/d^2 - 12*(9*sqrt(10)*sqrt(pi))*(10*I*b*c*d - 3*d^2)*d
*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
(5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 125*sqrt
(6)*sqrt(pi)*(2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b^2) + 2250*sqrt(2)*sqrt(pi))*(-2*I*b*c*d + 3*d^2)*d*erf(-1/

```

$$\begin{aligned}
& 2\sqrt{2}\sqrt{b*d}\sqrt{d*x + c}*(I*b*d/\sqrt{b^2*d^2} + 1)/d*e^{((I*b*c - I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2)} + 2250\sqrt{2}\sqrt{\pi} \\
& *(2*I*b*c*d + 3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{2}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-I*b*c + I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2)} + 125\sqrt{6}\sqrt{\pi}*(-2*I*b*c*d - d^2)*d*\operatorname{erf}(-1/2*\sqrt{6}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-3*I*b*c + 3*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2)} + 9\sqrt{10}\sqrt{\pi}*(-10*I*b*c*d - 3*d^2)*d*\operatorname{erf}(-1/2*\sqrt{10}*\sqrt{b*d}*\sqrt{d*x + c}*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-5*I*b*c + 5*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2)} - 90*(-10*I*(d*x + c)^{(3/2)}*b*d + 10*I*\sqrt{d*x + c}*b*c*d + 3*\sqrt{d*x + c}*d^2)*e^{((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^2} - 750*(-2*I*(d*x + c)^{(3/2)}*b*d + 2*I*\sqrt{d*x + c}*b*c*d + \sqrt{d*x + c}*d^2)*e^{((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2} - 4500*(2*I*(d*x + c)^{(3/2)}*b*d - 2*I*\sqrt{d*x + c}*b*c*d - 3*\sqrt{d*x + c}*d^2)*e^{((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2} - 4500*(-2*I*(d*x + c)^{(3/2)}*b*d + 2*I*\sqrt{d*x + c}*b*c*d - 3*\sqrt{d*x + c}*d^2)*e^{((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2} - 750*(2*I*(d*x + c)^{(3/2)}*b*d - 2*I*\sqrt{d*x + c}*b*c*d + \sqrt{d*x + c}*d^2)*e^{((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2} - 90*(10*I*(d*x + c)^{(3/2)}*b*d - 10*I*\sqrt{d*x + c}*b*c*d + 3*\sqrt{d*x + c}*d^2)*e^{((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^2}*c)/d
\end{aligned}$$

3.196 $\int (c + dx)^{5/2} \cos^3(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=407

$$\frac{5\sqrt{\frac{\pi}{3}}d^{5/2} \cos\left(6a - \frac{6bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}} - \frac{45\sqrt{\pi}d^{5/2} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{2048b^{7/2}} - \frac{5\sqrt{\frac{\pi}{3}}d^{5/2} \sin\left(6a - \frac{6bc}{d}\right)}{18432b^{7/2}}$$

```
[Out] (45*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(1024*b^3) - (3*(c + d*x)^(5/2)*Cos[2*a + 2*b*x])/(64*b) - (5*d^2*Sqrt[c + d*x]*Cos[6*a + 6*b*x])/(9216*b^3) + ((c + d*x)^(5/2)*Cos[6*a + 6*b*x])/(192*b) + (5*d^(5/2)*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(18432*b^(7/2)) - (45*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2048*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/3]*FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(18432*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2048*b^(7/2)) + (15*d*(c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(256*b^2) - (5*d*(c + d*x)^(3/2)*Sin[6*a + 6*b*x])/(2304*b^2)
```

Rubi [A] time = 0.896867, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{5\sqrt{\frac{\pi}{3}}d^{5/2} \cos\left(6a - \frac{6bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}} - \frac{45\sqrt{\pi}d^{5/2} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{2048b^{7/2}} - \frac{5\sqrt{\frac{\pi}{3}}d^{5/2} \sin\left(6a - \frac{6bc}{d}\right)}{18432b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3, x]
```

```
[Out] (45*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(1024*b^3) - (3*(c + d*x)^(5/2)*Cos[2*a + 2*b*x])/(64*b) - (5*d^2*Sqrt[c + d*x]*Cos[6*a + 6*b*x])/(9216*b^3) + ((c + d*x)^(5/2)*Cos[6*a + 6*b*x])/(192*b) + (5*d^(5/2)*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(18432*b^(7/2)) - (45*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2048*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/3]*FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(18432*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2048*b^(7/2)) + (15*d*(c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(256*b^2) - (5*d*(c + d*x)^(3/2)*Sin[6*a + 6*b*x])/(2304*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^{5/2} \cos^3(a + bx) \sin^3(a + bx) dx &= \int \left(\frac{3}{32} (c + dx)^{5/2} \sin(2a + 2bx) - \frac{1}{32} (c + dx)^{5/2} \sin(6a + 6bx) \right) dx \\
&= -\left(\frac{1}{32} \int (c + dx)^{5/2} \sin(6a + 6bx) dx \right) + \frac{3}{32} \int (c + dx)^{5/2} \sin(2a + 2bx) dx \\
&= -\frac{3(c + dx)^{5/2} \cos(2a + 2bx)}{64b} + \frac{(c + dx)^{5/2} \cos(6a + 6bx)}{192b} - \frac{(5d) \int (c + dx)^{3/2} \sin(2a + 2bx) dx}{384b} \\
&= -\frac{3(c + dx)^{5/2} \cos(2a + 2bx)}{64b} + \frac{(c + dx)^{5/2} \cos(6a + 6bx)}{192b} + \frac{15d(c + dx)^{3/2} \sin(2a + 2bx)}{256b^2} \\
&= \frac{45d^2 \sqrt{c + dx} \cos(2a + 2bx)}{1024b^3} - \frac{3(c + dx)^{5/2} \cos(2a + 2bx)}{64b} - \frac{5d^2 \sqrt{c + dx} \cos(6a + 6bx)}{9216b^3} \\
&= \frac{45d^2 \sqrt{c + dx} \cos(2a + 2bx)}{1024b^3} - \frac{3(c + dx)^{5/2} \cos(2a + 2bx)}{64b} - \frac{5d^2 \sqrt{c + dx} \cos(6a + 6bx)}{9216b^3} \\
&= \frac{45d^2 \sqrt{c + dx} \cos(2a + 2bx)}{1024b^3} - \frac{3(c + dx)^{5/2} \cos(2a + 2bx)}{64b} - \frac{5d^2 \sqrt{c + dx} \cos(6a + 6bx)}{9216b^3} \\
&= \frac{45d^2 \sqrt{c + dx} \cos(2a + 2bx)}{1024b^3} - \frac{3(c + dx)^{5/2} \cos(2a + 2bx)}{64b} - \frac{5d^2 \sqrt{c + dx} \cos(6a + 6bx)}{9216b^3}
\end{aligned}$$

Mathematica [A] time = 5.34667, size = 550, normalized size = 1.35

$$-2592b^3c^2\sqrt{c + dx} \cos(2(a + bx)) + 288b^3c^2\sqrt{c + dx} \cos(6(a + bx)) - 2592b^3d^2x^2\sqrt{c + dx} \cos(2(a + bx)) + 288b^3d^2x^2\sqrt{c + dx} \cos(6(a + bx))$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]

[Out] (-2592*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 2430*b*d^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 5184*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 2592*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 288*b^3*c^2*Sqrt[c + d*x]*Cos[6*(a + b*x)] - 30*b*d^2*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 576*b^3*c*d*x*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 288*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 5*Sqrt[b/d]*d^3*Sqrt[3*Pi]*Cos[6*a - (6*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]] - 1215*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 5*Sqrt[b/d]*d^3*Sqrt[3*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]]*Sin[6*a - (6*b*c)/d] + 1215*Sqrt[b/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] + 3240*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 3240*b^2*d^2*x*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 120*b^2*c*d*Sqrt[c + d*x]*Sin[6*(a + b*x)] - 120*b^2*d^2*x*Sqrt[c + d*x]*Sin[6*(a + b*x)]/(55296*b^4)

Maple [A] time = 0.043, size = 477, normalized size = 1.2

$$2 \frac{1}{d} \left(-\frac{3d(dx+c)^{5/2}}{128b} \cos\left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d}\right) + \frac{15d}{128b} \left(\frac{1}{4} \frac{d(dx+c)^{3/2}}{b} \sin\left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d}\right) - 3/4 \frac{d}{b} \left(\right. \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)

[Out] 2/d*(-3/128/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+15/128/b*d*(1/4/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))+1/384/b*d*(d*x+c)^(5/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-5/384/b*d*(1/12/b*d*(d*x+c)^(3/2)*sin(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/4/b*d*(-1/12/b*d*(d*x+c)^(1/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))

Maxima [C] time = 2.24806, size = 1916, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")

[Out] -1/884736*sqrt(6)*sqrt(2)*(160*sqrt(6)*sqrt(2)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(6*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 4320*sqrt(6)*sqrt(2)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 8*(48*sqrt(6)*sqrt(2)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 5*sqrt(6)*sqrt(2)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(6*((d*x + c)*b - b*c + a*d)/d) + 216*(16*sqrt(6)*sqrt(2)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 15*sqrt(6)*sqrt(2)*sqrt(d*

```

x + c)*d^3*abs(b)/abs(d))*cos(2*((d*x + c)*b - b*c + a*d)/d) - (sqrt(2)*(5*
sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*
sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5
*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
5*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
))*d^3*sqrt(abs(b)/abs(d))*cos(-6*(b*c - a*d)/d) - sqrt(2)*(5*I*sqrt(pi)*co
s(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*
cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)
*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(a
bs(b)/abs(d))*sin(-6*(b*c - a*d)/d)*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) + (sq
rt(6)*(405*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) + 405*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) - 405*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) + 405*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + sqrt(6)
*(-405*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) - 405*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) - 405*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))) + 405*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d)*erf(sqrt(d*x
+ c)*sqrt(2*I*b/d)) + (sqrt(6)*(405*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) + 405*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 405*I*sqrt(pi)*sin(1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 405*I*sqrt(pi)*sin(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-
2*(b*c - a*d)/d) + sqrt(6)*(405*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) + 405*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) - 405*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 405*sqrt(pi)*sin(-1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-2*(b*
c - a*d)/d)*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)) - (sqrt(2)*(5*sqrt(pi)*cos(1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*cos(-
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*si
n(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*
sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(ab
s(b)/abs(d))*cos(-6*(b*c - a*d)/d) - sqrt(2)*(-5*I*sqrt(pi)*cos(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*cos(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*sin(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)*sin(-1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))
*sin(-6*(b*c - a*d)/d)*erf(sqrt(d*x + c)*sqrt(-6*I*b/d))*abs(d)/(b^3*d*ab
s(b))

```

Fricas [A] time = 0.767491, size = 1089, normalized size = 2.68

$$5\sqrt{3}\pi d^3 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{6(bc-ad)}{d}\right) C\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 5\sqrt{3}\pi d^3 \sqrt{\frac{b}{\pi d}} S\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{6(bc-ad)}{d}\right) - 1215\pi d^3 \sqrt{\frac{b}{\pi d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")

[Out] 1/55296*(5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 1215*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c


```
) * sqrt(b / (pi * d)) + 1215 * pi * d^3 * sqrt(b / (pi * d)) * fresnel_sin(2 * sqrt(d * x + c) *
sqrt(b / (pi * d))) * sin(-2 * (b * c - a * d) / d) + 96 * (24 * b^3 * d^2 * x^2 + 2 * (48 * b^3 * d^2 *
x^2 + 96 * b^3 * c * d * x + 48 * b^3 * c^2 - 5 * b * d^2) * cos(b * x + a)^6 + 48 * b^3 * c * d * x +
24 * b^3 * c^2 + 45 * b * d^2 * cos(b * x + a)^2 - 3 * (48 * b^3 * d^2 * x^2 + 96 * b^3 * c * d * x + 4
8 * b^3 * c^2 - 5 * b * d^2) * cos(b * x + a)^4 - 25 * b * d^2 - 20 * (2 * (b^2 * d^2 * x + b^2 * c * d
) * cos(b * x + a)^5 - 2 * (b^2 * d^2 * x + b^2 * c * d) * cos(b * x + a)^3 - 3 * (b^2 * d^2 * x +
b^2 * c * d) * cos(b * x + a)) * sin(b * x + a)) * sqrt(d * x + c) / b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 2.70005, size = 2677, normalized size = 6.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/110592*(48*(sqrt(3)*sqrt(pi)*d^2*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c))*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) + sqrt(3)*sqrt(pi)*d^2*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c
))*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)*b) - 27*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c))*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) - 27*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c))*(-I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d
^2) + 1)*b) + 6*sqrt(d*x + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)
/b - 54*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b - 54*
sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 6*sqrt(d*x
+ c)*d*e^((-6*I*(d*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b)*c^2 - d^2*((I*sqrt(
3)*sqrt(pi)*(48*I*b^2*c^2*d - 24*b*c*d^2 - 5*I*d^3)*d*erf(-sqrt(3)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(-48*I*(d*x + c)^(5/2)*b^2*d +
96*I*(d*x + c)^(3/2)*b^2*c*d - 48*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^(
3/2)*b*d^2 + 24*sqrt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((-6*I*(d
*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b^3)/d^2 + (I*sqrt(3)*sqrt(pi)*(48*I*b^2*
c^2*d + 24*b*c*d^2 - 5*I*d^3)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c))*(-I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
b^2*d^2) + 1)*b^3) - 6*I*(-48*I*(d*x + c)^(5/2)*b^2*d + 96*I*(d*x + c)^(3/2
)*b^2*c*d - 48*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 24*sq
rt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((6*I*(d*x + c)*b - 6*I*b*c
+ 6*I*a*d)/d)/b^3)/d^2 + 27*(I*sqrt(pi))*(-48*I*b^2*c^2*d + 72*b*c*d^2 + 45*
I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*
b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(48*I*(d*
x + c)^(5/2)*b^2*d - 96*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*
c^2*d + 60*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x + c)*b*c*d^2 - 45*I*sqrt(d*x
```

$$\begin{aligned}
& + c)*d^3)*e^{((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^3)/d^2 + 27*(I*\sqrt{\pi})*(-48*I*b^2*c^2*d - 72*b*c*d^2 + 45*I*d^3)*d*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x + c})*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-2*I*b*c + 2*I*a*d)/d)/(\sqrt{b*d})*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^3} - 2*I*(48*I*(d*x + c)^{(5/2)}*b^2*d - 96*I*(d*x + c)^{(3/2)}*b^2*c*d + 48*I*\sqrt{d*x + c}*b^2*c^2*d - 60*(d*x + c)^{(3/2)}*b*d^2 + 72*\sqrt{d*x + c}*b*c*d^2 - 45*I*\sqrt{d*x + c}*d^3)*e^{((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^2} - 24*(I*\sqrt{3})*\sqrt{\pi})*(-4*I*b*c*d + d^2)*d*\operatorname{erf}(-\sqrt{3}*\sqrt{b*d}*\sqrt{d*x + c})*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((6*I*b*c - 6*I*a*d)/d)/(\sqrt{b*d})*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2} + I*\sqrt{3})*\sqrt{\pi})*(-4*I*b*c*d - d^2)*d*\operatorname{erf}(-\sqrt{3}*\sqrt{b*d}*\sqrt{d*x + c})*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-6*I*b*c + 6*I*a*d)/d)/(\sqrt{b*d})*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2} + 9*I*\sqrt{\pi})*(12*I*b*c*d - 9*d^2)*d*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x + c})*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((2*I*b*c - 2*I*a*d)/d)/(\sqrt{b*d})*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2} + 9*I*\sqrt{\pi})*(12*I*b*c*d + 9*d^2)*d*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x + c})*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-2*I*b*c + 2*I*a*d)/d)/(\sqrt{b*d})*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2} - 6*I*(-4*I*(d*x + c)^{(3/2)}*b*d + 4*I*\sqrt{d*x + c}*b*c*d + \sqrt{d*x + c}*d^2)*e^{((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b^2} - 18*I*(12*I*(d*x + c)^{(3/2)}*b*d - 12*I*\sqrt{d*x + c}*b*c*d - 9*\sqrt{d*x + c}*d^2)*e^{((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^2} - 18*I*(12*I*(d*x + c)^{(3/2)}*b*d - 12*I*\sqrt{d*x + c}*b*c*d + 9*\sqrt{d*x + c}*d^2)*e^{((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2} - 6*I*(-4*I*(d*x + c)^{(3/2)}*b*d + 4*I*\sqrt{d*x + c}*b*c*d - \sqrt{d*x + c}*d^2)*e^{((-6*I*(d*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b^2}*c)/d
\end{aligned}$$

3.197 $\int (c + dx)^{3/2} \cos^3(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=351

$$\frac{\sqrt{\frac{\pi}{3}} d^{3/2} \sin\left(6a - \frac{6bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}} - \frac{9\sqrt{\pi} d^{3/2} \sin\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b} \sqrt{c+dx}}{\sqrt{\pi} \sqrt{d}}\right)}{512b^{5/2}} + \frac{\sqrt{\frac{\pi}{3}} d^{3/2} \cos\left(6a - \frac{6bc}{d}\right)}{1536}$$

```
[Out] (-3*(c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)^(3/2)*Cos[6*a + 6
*b*x])/(192*b) + (d^(3/2)*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelS[(2*Sqrt[
b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi]
)]/(512*b^(5/2)) + (d^(3/2)*Sqrt[Pi/3]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[
c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[P
i]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)
/d])/(512*b^(5/2)) + (9*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(256*b^2) - (d*Sq
rt[c + d*x]*Sin[6*a + 6*b*x])/(768*b^2)
```

Rubi [A] time = 0.628285, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{3}} d^{3/2} \sin\left(6a - \frac{6bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}} - \frac{9\sqrt{\pi} d^{3/2} \sin\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b} \sqrt{c+dx}}{\sqrt{\pi} \sqrt{d}}\right)}{512b^{5/2}} + \frac{\sqrt{\frac{\pi}{3}} d^{3/2} \cos\left(6a - \frac{6bc}{d}\right)}{1536}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3, x]
```

```
[Out] (-3*(c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)^(3/2)*Cos[6*a + 6
*b*x])/(192*b) + (d^(3/2)*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelS[(2*Sqrt[
b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi]
)]/(512*b^(5/2)) + (d^(3/2)*Sqrt[Pi/3]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[
c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[P
i]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)
/d])/(512*b^(5/2)) + (9*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(256*b^2) - (d*Sq
rt[c + d*x]*Sin[6*a + 6*b*x])/(768*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]
]^n*Cos[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
```

$*e - c*f)/d]$, $\text{Int}[\text{Cos}[(c*f)/d + f*x]/\text{Sqrt}[c + d*x], x], x] /;$ $\text{FreeQ}\{c, d, e, f\}, x]$ && $\text{ComplexFreeQ}[f]$ && $\text{NeQ}[d*e - c*f, 0]$

Rule 3305

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] :> \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Sin}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /;$ $\text{FreeQ}\{c, d, e, f\}, x]$ && $\text{ComplexFreeQ}[f]$ && $\text{EqQ}[d*e - c*f, 0]$

Rule 3351

$\text{Int}[\text{Sin}[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] :> \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /;$ $\text{FreeQ}\{d, e, f\}, x]$

Rule 3304

$\text{Int}[\sin[\text{Pi}/2 + (e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] :> \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Cos}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /;$ $\text{FreeQ}\{c, d, e, f\}, x]$ && $\text{ComplexFreeQ}[f]$ && $\text{EqQ}[d*e - c*f, 0]$

Rule 3352

$\text{Int}[\text{Cos}[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] :> \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /;$ $\text{FreeQ}\{d, e, f\}, x]$

Rubi steps

$$\begin{aligned} \int (c + dx)^{3/2} \cos^3(a + bx) \sin^3(a + bx) dx &= \int \left(\frac{3}{32}(c + dx)^{3/2} \sin(2a + 2bx) - \frac{1}{32}(c + dx)^{3/2} \sin(6a + 6bx) \right) dx \\ &= -\left(\frac{1}{32} \int (c + dx)^{3/2} \sin(6a + 6bx) dx \right) + \frac{3}{32} \int (c + dx)^{3/2} \sin(2a + 2bx) dx \\ &= -\frac{3(c + dx)^{3/2} \cos(2a + 2bx)}{64b} + \frac{(c + dx)^{3/2} \cos(6a + 6bx)}{192b} - \frac{d \int \sqrt{c + dx} \cos(2a + 2bx) dx}{128b} \\ &= -\frac{3(c + dx)^{3/2} \cos(2a + 2bx)}{64b} + \frac{(c + dx)^{3/2} \cos(6a + 6bx)}{192b} + \frac{9d\sqrt{c + dx} \sin(2a + 2bx)}{256b^2} \\ &= -\frac{3(c + dx)^{3/2} \cos(2a + 2bx)}{64b} + \frac{(c + dx)^{3/2} \cos(6a + 6bx)}{192b} + \frac{9d\sqrt{c + dx} \sin(2a + 2bx)}{256b^2} \\ &= -\frac{3(c + dx)^{3/2} \cos(2a + 2bx)}{64b} + \frac{(c + dx)^{3/2} \cos(6a + 6bx)}{192b} + \frac{9d\sqrt{c + dx} \sin(2a + 2bx)}{256b^2} \\ &= -\frac{3(c + dx)^{3/2} \cos(2a + 2bx)}{64b} + \frac{(c + dx)^{3/2} \cos(6a + 6bx)}{192b} + \frac{d^{3/2} \sqrt{\frac{\pi}{3}} \cos\left(6a - \frac{6bc}{d}\right)}{15} \end{aligned}$$

Mathematica [A] time = 3.08647, size = 391, normalized size = 1.11

$$\sqrt{3\pi}d \sin\left(6a - \frac{6bc}{d}\right) \text{FresnelC}\left(2\sqrt{\frac{3}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c + dx}\right) - 81\sqrt{\pi}d \sin\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c + dx}}{\sqrt{\pi}}\right) + \sqrt{3\pi}d \cos\left(6a - \frac{6bc}{d}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]

```
[Out] (-216*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 216*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 24*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 24*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[6*(a + b*x)] + d*Sqrt[3*Pi]*Cos[6*a - (6*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]] - 81*d*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + d*Sqrt[3*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]]*Sin[6*a - (6*b*c)/d] - 81*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] + 162*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 6*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[6*(a + b*x)])/(4608*b^2*Sqrt[b/d])
```

Maple [A] time = 0.041, size = 383, normalized size = 1.1

$$2 \frac{1}{d} \left(-\frac{3d(dx+c)^{3/2}}{128b} \cos\left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d}\right) + \frac{9d}{128b} \left(\frac{1}{4} \frac{d\sqrt{dx+c}}{b} \sin\left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d}\right) - \frac{1}{8} \frac{d\sqrt{\pi}}{b} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+9/128/b*d*(1/4/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))+1/384/b*d*(d*x+c)^(3/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/128/b*d*(1/12/b*d*(d*x+c)^(1/2)*sin(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/144/b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))
```

Maxima [C] time = 2.2209, size = 1829, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/73728*sqrt(6)*sqrt(2)*(32*sqrt(6)*sqrt(2)*(d*x + c)^(3/2)*b*d*abs(b)*cos(6*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 288*sqrt(6)*sqrt(2)*(d*x + c)^(3/2)*b*d*abs(b)*cos(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 8*sqrt(6)*sqrt(2)*sqrt(d*x + c)*d^2*abs(b)*sin(6*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 216*sqrt(6)*sqrt(2)*sqrt(d*x + c)*d^2*abs(b)*sin(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - (sqrt(2)*(-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-6*(b*c - a*d)/d) - sqrt(2)*(sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-6*(b*c - a*d)/d)*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) - (sqrt(6)*(27*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-6*(b*c - a*d)/d)
```

```

sqrt(d^2))) + 27*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) + 27*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) - 27*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) *d^2*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + sqrt(6)
*(27*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 27*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) - 27*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) + 27*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) *d^2*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)
*sqrt(2*I*b/d)) - (sqrt(6)*(-27*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) - 27*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) - 27*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^2*sqrt(abs(b)/abs(d))*cos(-2*(b*c -
a*d)/d) + sqrt(6)*(27*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))) + 27*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) + 27*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) - 27*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) *d^2*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d)
)*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)) - (sqrt(2)*(I*sqrt(pi)*cos(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*a
rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^2*sqrt(abs(b)/abs(d))*cos(-6*
(b*c - a*d)/d) - sqrt(2)*(sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) *d^2*sqrt(abs(b)/abs(d))*sin(-6*(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(-6*I*b/d))*abs(d)/(b^2*d*abs(b))

```

Fricas [A] time = 0.706436, size = 821, normalized size = 2.34

$$\sqrt{3}\pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{6(bc-ad)}{d}\right) S\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) + \sqrt{3}\pi d^2 \sqrt{\frac{b}{\pi d}} C\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{6(bc-ad)}{d}\right) - 81\pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{6(bc-ad)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4608*(sqrt(3)*pi*d^2*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_sin(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(3)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 81*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - 81*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 96*(8*(b^2*d*x + b^2*c)*cos(b*x + a)^6 - 12*(b^2*d*x + b^2*c)*cos(b*x + a)^4 + 2*b^2*d*x + 2*b^2*c - (2*b*d*cos(b*x + a))^5 - 2*b*d*cos(b*x + a)^3 - 3*b*d*cos(b*x + a))*sin(b*x + a)*sqrt(d*x + c))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 2.07558, size = 1481, normalized size = 4.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/9216*(4*(sqrt(3)*sqrt(pi)*d^2*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c))*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b) + sqrt(3)*sqrt(pi)*d^2*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c))*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)*b) - 27*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c))*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b) - 27*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c))*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)*b) + 6*sqrt(d*x + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b
- 54*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b - 54*sqrt
(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 6*sqrt(d*x +
c)*d*e^((-6*I*(d*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b*c - I*sqrt(3)*sqrt(pi)
*(-4*I*b*c*d + d^2)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c))*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b^2) - I*sqrt(3)*sqrt(pi)*(-4*I*b*c*d - d^2)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt
(d*x + c))*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 9*I*sqrt(pi)*(12*I*b*c*d - 9*d^2)*d*er
f(-sqrt(b*d)*sqrt(d*x + c))*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a
*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 9*I*sqrt(pi)*(12*I*b*c*d
+ 9*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c))*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 6*I*(-4
*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((6
*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b^2 + 18*I*(12*I*(d*x + c)^(3/2)*b*d
- 12*I*sqrt(d*x + c)*b*c*d - 9*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*
I*b*c + 2*I*a*d)/d)/b^2 + 18*I*(12*I*(d*x + c)^(3/2)*b*d - 12*I*sqrt(d*x +
c)*b*c*d + 9*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d
)/b^2 + 6*I*(-4*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*x
+ c)*d^2)*e^((-6*I*(d*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b^2)/d
```

3.198 $\int \sqrt{c + dx} \cos^3(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=299

$$\frac{\sqrt{\frac{\pi}{3}} \sqrt{d} \cos\left(6a - \frac{6bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}} + \frac{3\sqrt{\pi} \sqrt{d} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b} \sqrt{c+dx}}{\sqrt{\pi} \sqrt{d}}\right)}{128b^{3/2}} + \frac{\sqrt{\frac{\pi}{3}} \sqrt{d} \sin\left(6a - \frac{6bc}{d}\right)}{384b^{3/2}}$$

[Out] $(-3\sqrt{c + dx} \cos[2a + 2bx]) / (64b) + (\sqrt{c + dx} \cos[6a + 6bx]) / (192b) - (\sqrt{d} \sqrt{\pi/3} \cos[6a - (6bc)/d] \text{FresnelC}[(2\sqrt{b} \sqrt{c + dx}) / \sqrt{d}]) / (384b^{3/2}) + (3\sqrt{d} \sqrt{\pi} \cos[2a - (2bc)/d] \text{FresnelC}[(2\sqrt{b} \sqrt{c + dx}) / (\sqrt{d} \sqrt{\pi})]) / (128b^{3/2}) + (\sqrt{d} \sqrt{\pi/3} \text{FresnelS}[(2\sqrt{b} \sqrt{c + dx}) / \sqrt{d}] \sin[6a - (6bc)/d]) / (384b^{3/2}) - (3\sqrt{d} \sqrt{\pi} \text{FresnelS}[(2\sqrt{b} \sqrt{c + dx}) / (\sqrt{d} \sqrt{\pi})] \sin[2a - (2bc)/d]) / (128b^{3/2})$

Rubi [A] time = 0.457678, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{3}} \sqrt{d} \cos\left(6a - \frac{6bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}} + \frac{3\sqrt{\pi} \sqrt{d} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b} \sqrt{c+dx}}{\sqrt{\pi} \sqrt{d}}\right)}{128b^{3/2}} + \frac{\sqrt{\frac{\pi}{3}} \sqrt{d} \sin\left(6a - \frac{6bc}{d}\right)}{384b^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\sqrt{c + dx} \cos[a + bx]^3 \sin[a + bx]^3, x]$

[Out] $(-3\sqrt{c + dx} \cos[2a + 2bx]) / (64b) + (\sqrt{c + dx} \cos[6a + 6bx]) / (192b) - (\sqrt{d} \sqrt{\pi/3} \cos[6a - (6bc)/d] \text{FresnelC}[(2\sqrt{b} \sqrt{c + dx}) / \sqrt{d}]) / (384b^{3/2}) + (3\sqrt{d} \sqrt{\pi} \cos[2a - (2bc)/d] \text{FresnelC}[(2\sqrt{b} \sqrt{c + dx}) / (\sqrt{d} \sqrt{\pi})]) / (128b^{3/2}) + (\sqrt{d} \sqrt{\pi/3} \text{FresnelS}[(2\sqrt{b} \sqrt{c + dx}) / \sqrt{d}] \sin[6a - (6bc)/d]) / (384b^{3/2}) - (3\sqrt{d} \sqrt{\pi} \text{FresnelS}[(2\sqrt{b} \sqrt{c + dx}) / (\sqrt{d} \sqrt{\pi})] \sin[2a - (2bc)/d]) / (128b^{3/2})$

Rule 4406

$\text{Int}[\cos[(a_.) + (b_.)(x_)]^{(p_.)} ((c_.) + (d_.)(x_))^{(m_.)} \sin[(a_.) + (b_.)(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + dx)^m, \sin[a + bx]]^n \cos[a + bx]^p, x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

$\text{Int}[(c_.) + (d_.)(x_)]^{(m_.)} \sin[(e_.) + (f_.)(x_)], x_Symbol] \rightarrow -\text{Simp}[(c + dx)^m \cos[e + fx] / f, x] + \text{Dist}[(dm) / f, \text{Int}[(c + dx)^{(m-1)} \cos[e + fx], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3306

$\text{Int}[\sin[(e_.) + (f_.)(x_)] / \sqrt{(c_.) + (d_.)(x_)}, x_Symbol] \rightarrow \text{Dist}[\cos[(de - cf) / d], \text{Int}[\sin[(cf) / d + fx] / \sqrt{c + dx}, x], x] + \text{Dist}[\sin[(de - cf) / d], \text{Int}[\cos[(cf) / d + fx] / \sqrt{c + dx}, x], x] /;$ FreeQ[{c, d,

$e, f\}, x] \&\& \text{ComplexFreeQ}[f] \&\& \text{NeQ}[d*e - c*f, 0]$

Rule 3305

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \text{ :> } \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Sin}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{ComplexFreeQ}[f] \&\& \text{EqQ}[d*e - c*f, 0]$

Rule 3351

$\text{Int}[\text{Sin}[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] \text{ :> } \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /; \text{FreeQ}[\{d, e, f\}, x]$

Rule 3304

$\text{Int}[\sin[\text{Pi}/2 + (e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \text{ :> } \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Cos}[(f*x^2)/d], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{ComplexFreeQ}[f] \&\& \text{EqQ}[d*e - c*f, 0]$

Rule 3352

$\text{Int}[\text{Cos}[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] \text{ :> } \text{Simp}[(\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)])/(f*\text{Rt}[d, 2]), x] /; \text{FreeQ}[\{d, e, f\}, x]$

Rubi steps

$$\begin{aligned} \int \sqrt{c+dx} \cos^3(a+bx) \sin^3(a+bx) dx &= \int \left(\frac{3}{32} \sqrt{c+dx} \sin(2a+2bx) - \frac{1}{32} \sqrt{c+dx} \sin(6a+6bx) \right) dx \\ &= -\left(\frac{1}{32} \int \sqrt{c+dx} \sin(6a+6bx) dx \right) + \frac{3}{32} \int \sqrt{c+dx} \sin(2a+2bx) dx \\ &= -\frac{3\sqrt{c+dx} \cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx} \cos(6a+6bx)}{192b} - \frac{d \int \frac{\cos(6a+6bx)}{\sqrt{c+dx}} dx}{384b} + \\ &= -\frac{3\sqrt{c+dx} \cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx} \cos(6a+6bx)}{192b} - \frac{\left(d \cos\left(6a - \frac{6bc}{d}\right) \right) \int}{384b} \\ &= -\frac{3\sqrt{c+dx} \cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx} \cos(6a+6bx)}{192b} - \frac{\cos\left(6a - \frac{6bc}{d}\right) \text{Subst}}{384b} \\ &= -\frac{3\sqrt{c+dx} \cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx} \cos(6a+6bx)}{192b} - \frac{\sqrt{d} \sqrt{\frac{\pi}{3}} \cos\left(6a - \frac{6bc}{d}\right)}{384b} \end{aligned}$$

Mathematica [A] time = 1.38502, size = 264, normalized size = 0.88

$$-\sqrt{3\pi} \cos\left(6a - \frac{6bc}{d}\right) \text{FresnelC}\left(2\sqrt{\frac{3}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c+dx}\right) + 27\sqrt{\pi} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}} \sqrt{c+dx}}{\sqrt{\pi}}\right) + \sqrt{3\pi} \sin\left(6a - \frac{6bc}{d}\right)$$

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Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]

[Out] (-54*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 6*Sqrt[b/d]*Sqrt[c + d*x]*Cos[6*(a + b*x)] - Sqrt[3*Pi]*Cos[6*a - (6*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt

$$\left[\frac{3}{\pi} \sqrt{c + dx} \right] + 27 \sqrt{\pi} \cos\left[\frac{2a - (2bc)}{d} \right] \text{FresnelC}\left[\frac{2\sqrt{b/d} \sqrt{c + dx}}{\sqrt{\pi}} \right] + \sqrt{3\pi} \text{FresnelS}\left[\frac{2\sqrt{b/d} \sqrt{c + dx}}{\sqrt{\pi}} \right] \sin\left[\frac{6a - (6bc)}{d} \right] - 27 \sqrt{\pi} \text{FresnelS}\left[\frac{2\sqrt{b/d} \sqrt{c + dx}}{\sqrt{\pi}} \right] \sin\left[\frac{2a - (2bc)}{d} \right] \bigg/ (1152 b \sqrt{b/d})$$

Maple [A] time = 0.041, size = 293, normalized size = 1.

$$2 \frac{1}{d} \left(-\frac{3d\sqrt{dx+c}}{128b} \cos\left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d}\right) + \frac{3d\sqrt{\pi}}{256b} \left(\cos\left(2 \frac{ad-bc}{d}\right) \text{FresnelC}\left(2 \frac{\sqrt{dx+cb}}{d\sqrt{\pi}} \frac{1}{\sqrt{\frac{b}{d}}}\right) - \sin\left(2 \frac{ad-bc}{d}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)

[Out] 2/d*(-3/128/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/256/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))+1/384/b*d*(d*x+c)^(1/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/4608/b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))

Maxima [C] time = 2.19434, size = 1681, normalized size = 5.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")

[Out] 1/18432*sqrt(6)*sqrt(2)*(8*sqrt(6)*sqrt(2)*sqrt(d*x + c)*d*abs(b)*cos(6*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 72*sqrt(6)*sqrt(2)*sqrt(d*x + c)*d*abs(b)*cos(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - (sqrt(2)*(sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d*sqrt(abs(b)/abs(d))*cos(-6*(b*c - a*d)/d) - sqrt(2)*(I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d*sqrt(abs(b)/abs(d))*sin(-6*(b*c - a*d)/d)*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) + (sqrt(6)*(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + sqrt(6)*(-9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d)*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + (sqrt(6)*(9*sqrt(pi)*cos(1/4*pi +

$$\begin{aligned} & 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}) + 9*\sqrt{\pi}*\cos(-1/4*\pi + \\ & 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 9*I*\sqrt{\pi}*\sin(1/4*\pi + \\ & 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 9*I*\sqrt{\pi}*\sin(-1/4 \\ & *\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))*d*\sqrt{\text{abs}(b)/\text{abs}(d)} \\ &)*\cos(-2*(b*c - a*d)/d) + \sqrt{6}*(9*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0 \\ & , b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 9*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan \\ & 2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 9*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan \\ & 2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 9*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arcta \\ & n2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))*d*\sqrt{\text{abs}(b)/\text{abs}(d)}*\sin(-2*(b*c \\ & - a*d)/d))*\text{erf}(\sqrt{d*x + c}*\sqrt{-2*I*b/d}) - (\sqrt{2}*(\sqrt{\pi}*\cos(1/4*\pi \\ & + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + \sqrt{\pi}*\cos(-1/4*\pi \\ & + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + I*\sqrt{\pi}*\sin(1/4*\pi \\ & + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - I*\sqrt{\pi}*\sin(-1/4*\pi \\ & + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}))*d*\sqrt{\text{abs}(b)/\text{abs}(d)} \\ &)*\cos(-6*(b*c - a*d)/d) - \sqrt{2}*(-I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b \\ &) + 1/2*\arctan2(0, d/\sqrt{d^2})) - I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, \\ & b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + \sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b \\ & + 1/2*\arctan2(0, d/\sqrt{d^2})) - \sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b \\ & + 1/2*\arctan2(0, d/\sqrt{d^2}))*d*\sqrt{\text{abs}(b)/\text{abs}(d)}*\sin(-6*(b*c - a*d)/d \\ &)*\text{erf}(\sqrt{d*x + c}*\sqrt{-6*I*b/d}))*\text{abs}(d)/(b*d*\text{abs}(b)) \end{aligned}$$

Fricas [A] time = 0.64598, size = 632, normalized size = 2.11

$$\sqrt{3}\pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{6(bc-ad)}{d}\right) C\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - \sqrt{3}\pi d \sqrt{\frac{b}{\pi d}} S\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{6(bc-ad)}{d}\right) - 27\pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{6(bc-ad)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/1152*(\sqrt{3}*\pi*d*\sqrt{b/(\pi*d)}*\cos(-6*(b*c - a*d)/d)*\text{fresnel_cos}(2*\sqrt{3} \\ & *\sqrt{d*x + c}*\sqrt{b/(\pi*d)}) - \sqrt{3}*\pi*d*\sqrt{b/(\pi*d)}*\text{fresnel_sin}(2*\sqrt{3} \\ & *\sqrt{d*x + c}*\sqrt{b/(\pi*d)})*\sin(-6*(b*c - a*d)/d) - 27*\pi*d*\sqrt{b/(\pi*d)} \\ & *\cos(-2*(b*c - a*d)/d)*\text{fresnel_cos}(2*\sqrt{d*x + c}*\sqrt{b/(\pi*d)}) + 27*\pi*d*\sqrt{b/(\pi*d)} \\ & *\text{fresnel_sin}(2*\sqrt{d*x + c}*\sqrt{b/(\pi*d)})*\sin(-2*(b*c - a*d)/d) - 48*(4*b*\cos(b*x + a)^6 - 6*b*\cos(b*x + a)^4 + b)*\sqrt{d*x + c})/b^2 \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)

[Out] Timed out

Giac [C] time = 1.55546, size = 643, normalized size = 2.15

$$\frac{\sqrt{3}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{3}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\left(\frac{6ibc-6iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} + \frac{\sqrt{3}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{3}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\left(\frac{-6ibc+6iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} - \frac{27\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\left(\frac{2ibc-2iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")

[Out] 1/2304*(sqrt(3)*sqrt(pi)*d^2*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(3)*sqrt(pi)*d^2*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 27*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 27*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 6*sqrt(d*x + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b - 54*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b - 54*sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^((-6*I*(d*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b/d

3.199 $\int \sqrt{c + dx} \cos^3(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=299

$$\frac{\sqrt{\frac{\pi}{3}} \sqrt{d} \cos\left(6a - \frac{6bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}} + \frac{3\sqrt{\pi} \sqrt{d} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b} \sqrt{c+dx}}{\sqrt{\pi} \sqrt{d}}\right)}{128b^{3/2}} + \frac{\sqrt{\frac{\pi}{3}} \sqrt{d} \sin\left(6a - \frac{6bc}{d}\right)}{384b^{3/2}}$$

[Out] $(-3\sqrt{c + dx} \cos[2a + 2bx]) / (64b) + (\sqrt{c + dx} \cos[6a + 6bx]) / (192b) - (\sqrt{d} \sqrt{\pi/3} \cos[6a - (6bc)/d] \text{FresnelC}[(2\sqrt{b} \sqrt{c+dx}) / \sqrt{d}]) / (384b^{3/2}) + (3\sqrt{d} \sqrt{\pi} \cos[2a - (2bc)/d] \text{FresnelC}[(2\sqrt{b} \sqrt{c+dx}) / (\sqrt{d} \sqrt{\pi})]) / (128b^{3/2}) + (\sqrt{d} \sqrt{\pi/3} \text{FresnelS}[(2\sqrt{b} \sqrt{c+dx}) / \sqrt{d}]) \sin[6a - (6bc)/d] / (384b^{3/2}) - (3\sqrt{d} \sqrt{\pi} \text{FresnelS}[(2\sqrt{b} \sqrt{c+dx}) / (\sqrt{d} \sqrt{\pi})]) \sin[2a - (2bc)/d] / (128b^{3/2})$

Rubi [A] time = 0.448135, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{3}} \sqrt{d} \cos\left(6a - \frac{6bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}} + \frac{3\sqrt{\pi} \sqrt{d} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b} \sqrt{c+dx}}{\sqrt{\pi} \sqrt{d}}\right)}{128b^{3/2}} + \frac{\sqrt{\frac{\pi}{3}} \sqrt{d} \sin\left(6a - \frac{6bc}{d}\right)}{384b^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\sqrt{c + dx} \cos[a + bx]^3 \sin[a + bx]^3, x]$

[Out] $(-3\sqrt{c + dx} \cos[2a + 2bx]) / (64b) + (\sqrt{c + dx} \cos[6a + 6bx]) / (192b) - (\sqrt{d} \sqrt{\pi/3} \cos[6a - (6bc)/d] \text{FresnelC}[(2\sqrt{b} \sqrt{c+dx}) / \sqrt{d}]) / (384b^{3/2}) + (3\sqrt{d} \sqrt{\pi} \cos[2a - (2bc)/d] \text{FresnelC}[(2\sqrt{b} \sqrt{c+dx}) / (\sqrt{d} \sqrt{\pi})]) / (128b^{3/2}) + (\sqrt{d} \sqrt{\pi/3} \text{FresnelS}[(2\sqrt{b} \sqrt{c+dx}) / \sqrt{d}]) \sin[6a - (6bc)/d] / (384b^{3/2}) - (3\sqrt{d} \sqrt{\pi} \text{FresnelS}[(2\sqrt{b} \sqrt{c+dx}) / (\sqrt{d} \sqrt{\pi})]) \sin[2a - (2bc)/d] / (128b^{3/2})$

Rule 4406

$\text{Int}[\cos[(a_.) + (b_.)(x_.)]^{(p_.)} ((c_.) + (d_.)(x_.))^{(m_.)} \sin[(a_.) + (b_.)(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + dx)^m, \sin[a + bx]^n \cos[a + bx]^p, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

$\text{Int}[(c_.) + (d_.)(x_.)]^{(m_.)} \sin[(e_.) + (f_.)(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + dx)^m \cos[e + fx] / f, x] + \text{Dist}[(d \cdot m) / f, \text{Int}[(c + dx)^{(m-1)} \cos[e + fx], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3306

$\text{Int}[\sin[(e_.) + (f_.)(x_.)] / \sqrt{(c_.) + (d_.)(x_.)}, x_Symbol] \rightarrow \text{Dist}[\cos[(d \cdot e - c \cdot f) / d], \text{Int}[\sin[(c \cdot f) / d + f \cdot x] / \sqrt{c + dx}, x], x] + \text{Dist}[\sin[(d \cdot e - c \cdot f) / d], \text{Int}[\cos[(c \cdot f) / d + f \cdot x] / \sqrt{c + dx}, x], x] /;$ FreeQ[{c, d,

e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]

Rule 3305

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3304

Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned} \int \sqrt{c + dx} \cos^3(a + bx) \sin^3(a + bx) dx &= \int \left(\frac{3}{32} \sqrt{c + dx} \sin(2a + 2bx) - \frac{1}{32} \sqrt{c + dx} \sin(6a + 6bx) \right) dx \\ &= -\left(\frac{1}{32} \int \sqrt{c + dx} \sin(6a + 6bx) dx \right) + \frac{3}{32} \int \sqrt{c + dx} \sin(2a + 2bx) dx \\ &= -\frac{3\sqrt{c + dx} \cos(2a + 2bx)}{64b} + \frac{\sqrt{c + dx} \cos(6a + 6bx)}{192b} - \frac{d \int \frac{\cos(6a + 6bx)}{\sqrt{c + dx}} dx}{384b} + \dots \\ &= -\frac{3\sqrt{c + dx} \cos(2a + 2bx)}{64b} + \frac{\sqrt{c + dx} \cos(6a + 6bx)}{192b} - \frac{\left(d \cos\left(6a - \frac{6bc}{d}\right) \right) \int \frac{\cos}{384b}}{384b} \\ &= -\frac{3\sqrt{c + dx} \cos(2a + 2bx)}{64b} + \frac{\sqrt{c + dx} \cos(6a + 6bx)}{192b} - \frac{\cos\left(6a - \frac{6bc}{d}\right) \text{Subst}\left(\int \frac{\cos}{1}\right)}{384b} \\ &= -\frac{3\sqrt{c + dx} \cos(2a + 2bx)}{64b} + \frac{\sqrt{c + dx} \cos(6a + 6bx)}{192b} - \frac{\sqrt{d} \sqrt{\frac{\pi}{3}} \cos\left(6a - \frac{6bc}{d}\right) C}{384b^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.612968, size = 264, normalized size = 0.88

$$-\sqrt{3\pi} \cos\left(6a - \frac{6bc}{d}\right) \text{FresnelC}\left(2\sqrt{\frac{3}{\pi}} \sqrt{\frac{b}{d}} \sqrt{c + dx}\right) + 27\sqrt{\pi} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}} \sqrt{c + dx}}{\sqrt{\pi}}\right) + \sqrt{3\pi} \sin\left(6a - \frac{6bc}{d}\right) S$$

1152b

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]

[Out] (-54*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 6*Sqrt[b/d]*Sqrt[c + d*x]*Cos[6*(a + b*x)] - Sqrt[3*Pi]*Cos[6*a - (6*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt

$$\frac{[3/\text{Pi}]*\text{Sqrt}[c + d*x]}{d} + 27*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelC}[(2*\text{Sqrt}[b/d]*\text{Sqrt}[c + d*x])/\text{Sqrt}[\text{Pi}]] + \text{Sqrt}[3*\text{Pi}]*\text{FresnelS}[2*\text{Sqrt}[b/d]*\text{Sqrt}[3/\text{Pi}]*\text{Sqrt}[c + d*x]]*\text{Sin}[6*a - (6*b*c)/d] - 27*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[b/d]*\text{Sqrt}[c + d*x])/\text{Sqrt}[\text{Pi}]]*\text{Sin}[2*a - (2*b*c)/d]/(1152*b*\text{Sqrt}[b/d])$$

Maple [A] time = 0.037, size = 293, normalized size = 1.

$$2\frac{1}{d}\left(-\frac{3d\sqrt{dx+c}}{128b}\cos\left(2\frac{(dx+c)b}{d}+2\frac{ad-bc}{d}\right)+\frac{3d\sqrt{\pi}}{256b}\left(\cos\left(2\frac{ad-bc}{d}\right)\text{FresnelC}\left(2\frac{\sqrt{dx+cb}}{d\sqrt{\pi}}\frac{1}{\sqrt{\frac{b}{d}}}\right)-\sin\left(2\frac{ad-bc}{d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)

[Out] 2/d*(-3/128/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/256/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))+1/384/b*d*(d*x+c)^(1/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/4608/b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))

Maxima [C] time = 2.16878, size = 1681, normalized size = 5.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")

[Out] 1/18432*sqrt(6)*sqrt(2)*(8*sqrt(6)*sqrt(2)*sqrt(d*x + c)*d*abs(b)*cos(6*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 72*sqrt(6)*sqrt(2)*sqrt(d*x + c)*d*abs(b)*cos(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - (sqrt(2)*(sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d*sqrt(abs(b)/abs(d))*cos(-6*(b*c - a*d)/d) - sqrt(2)*(I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d*sqrt(abs(b)/abs(d))*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) + (sqrt(6)*(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + sqrt(6)*(-9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + (sqrt(6)*(9*sqrt(pi)*cos(1/4*pi +

$$\begin{aligned} & 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2}) + 9*\sqrt{\pi}*\cos(-1/4*\pi + \\ & 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 9*I*\sqrt{\pi}*\sin(1/4*\pi + \\ & 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 9*I*\sqrt{\pi}*\sin(-1/4 \\ & *\pi + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d*\sqrt{\text{abs}(b)/\text{abs}(d)} \\ &)*\cos(-2*(b*c - a*d)/d) + \sqrt{6}*(9*I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0 \\ & , b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 9*I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan \\ & 2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - 9*\sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan \\ & 2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + 9*\sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arcta \\ & n2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d*\sqrt{\text{abs}(b)/\text{abs}(d)}*\sin(-2*(b*c \\ & - a*d)/d))*\text{erf}(\sqrt{d*x + c}*\sqrt{-2*I*b/d}) - (\sqrt{2}*(\sqrt{\pi}*\cos(1/4*\pi \\ & i + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + \sqrt{\pi}*\cos(-1/4*\pi \\ & + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + I*\sqrt{\pi}*\sin(1/4*\pi \\ & + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})) - I*\sqrt{\pi}*\sin(-1/4*\pi \\ & i + 1/2*\arctan2(0, b) + 1/2*\arctan2(0, d/\sqrt{d^2})))*d*\sqrt{\text{abs}(b)/\text{abs}(d)} \\ & *\cos(-6*(b*c - a*d)/d) - \sqrt{2}*(-I*\sqrt{\pi}*\cos(1/4*\pi + 1/2*\arctan2(0, b \\ &) + 1/2*\arctan2(0, d/\sqrt{d^2})) - I*\sqrt{\pi}*\cos(-1/4*\pi + 1/2*\arctan2(0, \\ & b) + 1/2*\arctan2(0, d/\sqrt{d^2})) + \sqrt{\pi}*\sin(1/4*\pi + 1/2*\arctan2(0, b \\ & + 1/2*\arctan2(0, d/\sqrt{d^2})) - \sqrt{\pi}*\sin(-1/4*\pi + 1/2*\arctan2(0, b \\ & + 1/2*\arctan2(0, d/\sqrt{d^2})))*d*\sqrt{\text{abs}(b)/\text{abs}(d)}*\sin(-6*(b*c - a*d)/d) \\ &)*\text{erf}(\sqrt{d*x + c}*\sqrt{-6*I*b/d}))*\text{abs}(d)/(b*d*\text{abs}(b)) \end{aligned}$$

Fricas [A] time = 0.651762, size = 632, normalized size = 2.11

$$\sqrt{3}\pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{6(bc-ad)}{d}\right) C\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - \sqrt{3}\pi d \sqrt{\frac{b}{\pi d}} S\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{6(bc-ad)}{d}\right) - 27\pi d \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{6(bc-ad)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/1152*(\sqrt{3}*\pi*d*\sqrt{b}/(\pi*d))*\cos(-6*(b*c - a*d)/d)*\text{fresnel_cos}(2*\sqrt{3} \\ & *\sqrt{d*x + c}*\sqrt{b}/(\pi*d)) - \sqrt{3}*\pi*d*\sqrt{b}/(\pi*d)*\text{fresnel_s} \\ & \text{in}(2*\sqrt{3}*\sqrt{d*x + c}*\sqrt{b}/(\pi*d))*\sin(-6*(b*c - a*d)/d) - 27*\pi*d* \\ & \sqrt{b}/(\pi*d)*\cos(-2*(b*c - a*d)/d)*\text{fresnel_cos}(2*\sqrt{d*x + c}*\sqrt{b}/(\pi \\ & *d)) + 27*\pi*d*\sqrt{b}/(\pi*d)*\text{fresnel_sin}(2*\sqrt{d*x + c}*\sqrt{b}/(\pi*d))* \\ & \sin(-2*(b*c - a*d)/d) - 48*(4*b*\cos(b*x + a)^6 - 6*b*\cos(b*x + a)^4 + b)*\sqrt{d*x + c})/b^2 \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)

[Out] Timed out

Giac [C] time = 1.56079, size = 643, normalized size = 2.15

$$\frac{\sqrt{3}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{3}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\left(\frac{6ibc-6iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} + \frac{\sqrt{3}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{3}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\left(\frac{-6ibc+6iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}+1}\right)b} - \frac{27\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)}{d}\right)e^{\left(\frac{2ibc-2iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}+1}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")

[Out] 1/2304*(sqrt(3)*sqrt(pi)*d^2*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(3)*sqrt(pi)*d^2*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 27*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 27*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 6*sqrt(d*x + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b - 54*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b - 54*sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^((-6*I*(d*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b)/d

3.200 $\int (c + dx)^{3/2} \cos^3(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=351

$$\frac{\sqrt{\frac{\pi}{3}} d^{3/2} \sin\left(6a - \frac{6bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}} - \frac{9\sqrt{\pi} d^{3/2} \sin\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b} \sqrt{c+dx}}{\sqrt{\pi} \sqrt{d}}\right)}{512b^{5/2}} + \frac{\sqrt{\frac{\pi}{3}} d^{3/2} \cos\left(6a - \frac{6bc}{d}\right)}{1536b^{5/2}}$$

[Out] $(-3*(c + d*x)^{(3/2)}*\text{Cos}[2*a + 2*b*x])/(64*b) + ((c + d*x)^{(3/2)}*\text{Cos}[6*a + 6*b*x])/(192*b) + (d^{(3/2)}*\text{Sqrt}[\text{Pi}/3]*\text{Cos}[6*a - (6*b*c)/d]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[3/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(1536*b^{(5/2)}) - (9*d^{(3/2)}*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]))/(512*b^{(5/2)}) + (d^{(3/2)}*\text{Sqrt}[\text{Pi}/3]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[3/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[6*a - (6*b*c)/d])/(1536*b^{(5/2)}) - (9*d^{(3/2)}*\text{Sqrt}[\text{Pi}]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]))*\text{Sin}[2*a - (2*b*c)/d])/(512*b^{(5/2)}) + (9*d*\text{Sqrt}[c + d*x]*\text{Sin}[2*a + 2*b*x])/(256*b^2) - (d*\text{Sqrt}[c + d*x]*\text{Sin}[6*a + 6*b*x])/(768*b^2)$

Rubi [A] time = 0.55918, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{3}} d^{3/2} \sin\left(6a - \frac{6bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}} - \frac{9\sqrt{\pi} d^{3/2} \sin\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b} \sqrt{c+dx}}{\sqrt{\pi} \sqrt{d}}\right)}{512b^{5/2}} + \frac{\sqrt{\frac{\pi}{3}} d^{3/2} \cos\left(6a - \frac{6bc}{d}\right)}{1536b^{5/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^{(3/2)}*\text{Cos}[a + b*x]^3*\text{Sin}[a + b*x]^3, x]$

[Out] $(-3*(c + d*x)^{(3/2)}*\text{Cos}[2*a + 2*b*x])/(64*b) + ((c + d*x)^{(3/2)}*\text{Cos}[6*a + 6*b*x])/(192*b) + (d^{(3/2)}*\text{Sqrt}[\text{Pi}/3]*\text{Cos}[6*a - (6*b*c)/d]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[3/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])/(1536*b^{(5/2)}) - (9*d^{(3/2)}*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a - (2*b*c)/d]*\text{FresnelS}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]))/(512*b^{(5/2)}) + (d^{(3/2)}*\text{Sqrt}[\text{Pi}/3]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[3/\text{Pi}]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]*\text{Sin}[6*a - (6*b*c)/d])/(1536*b^{(5/2)}) - (9*d^{(3/2)}*\text{Sqrt}[\text{Pi}]*\text{FresnelC}[(2*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/(\text{Sqrt}[d]*\text{Sqrt}[\text{Pi}]))*\text{Sin}[2*a - (2*b*c)/d])/(512*b^{(5/2)}) + (9*d*\text{Sqrt}[c + d*x]*\text{Sin}[2*a + 2*b*x])/(256*b^2) - (d*\text{Sqrt}[c + d*x]*\text{Sin}[6*a + 6*b*x])/(768*b^2)$

Rule 4406

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] /;$ $\text{FreeQ}\{a, b, c, d, m\}, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[p, 0]$

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\text{sin}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /;$ $\text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{GtQ}[m, 0]$

Rule 3306

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/\text{Sqrt}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[c*f/d + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d$

*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]

Rule 3305

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^(2)], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)]]/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3304

Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^(2)], x_Symbol] := Simp[(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)]]/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\begin{aligned}
 \int (c + dx)^{3/2} \cos^3(a + bx) \sin^3(a + bx) dx &= \int \left(\frac{3}{32} (c + dx)^{3/2} \sin(2a + 2bx) - \frac{1}{32} (c + dx)^{3/2} \sin(6a + 6bx) \right) dx \\
 &= - \left(\frac{1}{32} \int (c + dx)^{3/2} \sin(6a + 6bx) dx \right) + \frac{3}{32} \int (c + dx)^{3/2} \sin(2a + 2bx) dx \\
 &= - \frac{3(c + dx)^{3/2} \cos(2a + 2bx)}{64b} + \frac{(c + dx)^{3/2} \cos(6a + 6bx)}{192b} - \frac{d \int \sqrt{c + dx} \cos(6a + 6bx) dx}{12b} \\
 &= - \frac{3(c + dx)^{3/2} \cos(2a + 2bx)}{64b} + \frac{(c + dx)^{3/2} \cos(6a + 6bx)}{192b} + \frac{9d\sqrt{c + dx} \sin(6a + 6bx)}{256b} \\
 &= - \frac{3(c + dx)^{3/2} \cos(2a + 2bx)}{64b} + \frac{(c + dx)^{3/2} \cos(6a + 6bx)}{192b} + \frac{9d\sqrt{c + dx} \sin(6a + 6bx)}{256b} \\
 &= - \frac{3(c + dx)^{3/2} \cos(2a + 2bx)}{64b} + \frac{(c + dx)^{3/2} \cos(6a + 6bx)}{192b} + \frac{9d\sqrt{c + dx} \sin(6a + 6bx)}{256b} \\
 &= - \frac{3(c + dx)^{3/2} \cos(2a + 2bx)}{64b} + \frac{(c + dx)^{3/2} \cos(6a + 6bx)}{192b} + \frac{d^{3/2} \sqrt{\frac{\pi}{3}} \cos(6a + 6bx)}{12b}
 \end{aligned}$$

Mathematica [A] time = 0.216276, size = 391, normalized size = 1.11

$$\sqrt{3\pi}d \sin\left(6a - \frac{6bc}{d}\right) \text{FresnelC}\left(2\sqrt{\frac{3}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c + dx}\right) - 81\sqrt{\pi}d \sin\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c + dx}}{\sqrt{\pi}}\right) + \sqrt{3\pi}d \cos\left(6a - \frac{6bc}{d}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]

```
[Out] (-216*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 216*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] + 24*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[6*(a + b*
x)] + 24*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[6*(a + b*x)] + d*Sqrt[3*Pi]*Cos[
6*a - (6*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]] - 81*d*Sqrt
[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] +
d*Sqrt[3*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]]*Sin[6*a - (6*b*
c)/d] - 81*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*
a - (2*b*c)/d] + 162*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 6*Sqrt[b/
d]*d*Sqrt[c + d*x]*Sin[6*(a + b*x)]/(4608*b^2*Sqrt[b/d])
```

Maple [A] time = 0.039, size = 383, normalized size = 1.1

$$2 \frac{1}{d} \left(-\frac{3d(dx+c)^{3/2}}{128b} \cos \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) + \frac{9d}{128b} \left(\frac{1}{4} \frac{d\sqrt{dx+c}}{b} \sin \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) - \frac{1}{8} \frac{d\sqrt{\pi}}{b} \cos \left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+9/128/b*d*(1
/4/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d
)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b
/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))
+1/384/b*d*(d*x+c)^(3/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/128/b*d*(1/12/b
*d*(d*x+c)^(1/2)*sin(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/144/b*d*2^(1/2)*Pi^(1/2
)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)
/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2
)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))
```

Maxima [C] time = 2.21766, size = 1829, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/73728*sqrt(6)*sqrt(2)*(32*sqrt(6)*sqrt(2)*(d*x + c)^(3/2)*b*d*abs(b)*cos(
6*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 288*sqrt(6)*sqrt(2)*(d*x + c)^(3/2)
*b*d*abs(b)*cos(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 8*sqrt(6)*sqrt(2)*s
qrt(d*x + c)*d^2*abs(b)*sin(6*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 216*sq
rt(6)*sqrt(2)*sqrt(d*x + c)*d^2*abs(b)*sin(2*((d*x + c)*b - b*c + a*d)/d)/ab
s(d) - (sqrt(2)*(-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-6*(b*c - a*d)/d) - sqrt(2)*(sq
rt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(
pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt
(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt
(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sq
rt(abs(b)/abs(d))*sin(-6*(b*c - a*d)/d)*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) -
(sqrt(6)*(27*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
```

```

sqrt(d^2))) + 27*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) + 27*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) - 27*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) *d^2*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + sqrt(6)
*(27*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 27*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) - 27*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) + 27*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) *d^2*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d)*erf(sqrt(d*x + c)
*sqrt(2*I*b/d)) - (sqrt(6)*(-27*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) - 27*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) - 27*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^2*sqrt(abs(b)/abs(d))*cos(-2*(b*c -
a*d)/d) + sqrt(6)*(27*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))) + 27*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) + 27*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) - 27*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) *d^2*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d)
)*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)) - (sqrt(2)*(I*sqrt(pi)*cos(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*a
rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) *d^2*sqrt(abs(b)/abs(d))*cos(-6*
(b*c - a*d)/d) - sqrt(2)*(sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) *d^2*sqrt(abs(b)/abs(d))*sin(-6*(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(-6*I*b/d))*abs(d)/(b^2*d*abs(b))

```

Fricas [A] time = 0.719563, size = 821, normalized size = 2.34

$$\sqrt{3}\pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{6(bc-ad)}{d}\right) S\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) + \sqrt{3}\pi d^2 \sqrt{\frac{b}{\pi d}} C\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{6(bc-ad)}{d}\right) - 81\pi d^2 \sqrt{\frac{b}{\pi d}} c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")

[Out] 1/4608*(sqrt(3)*pi*d^2*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_sin(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(3)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 81*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - 81*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 96*(8*(b^2*d*x + b^2*c)*cos(b*x + a)^6 - 12*(b^2*d*x + b^2*c)*cos(b*x + a)^4 + 2*b^2*d*x + 2*b^2*c - (2*b*d*cos(b*x + a))^5 - 2*b*d*cos(b*x + a)^3 - 3*b*d*cos(b*x + a))*sin(b*x + a)*sqrt(d*x + c))/b^3

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)

[Out] Timed out

Giac [C] time = 2.07807, size = 1481, normalized size = 4.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/9216*(4*(\sqrt{3}*\sqrt{\pi})*d^2*\operatorname{erf}(-\sqrt{3}*\sqrt{b*d})*\sqrt{d*x+c}*(I*b*d \\ & / \sqrt{b^2*d^2+1})/d)*e^{((6*I*b*c-6*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2+1})*b)} + \sqrt{3}*\sqrt{\pi}*d^2*\operatorname{erf}(-\sqrt{3}*\sqrt{b*d})*\sqrt{d*x+c}*(\\ & -I*b*d/\sqrt{b^2*d^2+1})/d)*e^{((-6*I*b*c+6*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2+1})*b)} - 27*\sqrt{\pi}*d^2*\operatorname{erf}(-\sqrt{b*d})*\sqrt{d*x+c}*(I*b*d \\ & / \sqrt{b^2*d^2+1})/d)*e^{((2*I*b*c-2*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2+1})*b)} - 27*\sqrt{\pi}*d^2*\operatorname{erf}(-\sqrt{b*d})*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2+1})/d)*e^{((-2*I*b*c+2*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2+1})*b)} + 6*\sqrt{d*x+c}*d*e^{((6*I*(d*x+c)*b-6*I*b*c+6*I*a*d)/d)/b} \\ & - 54*\sqrt{d*x+c}*d*e^{((2*I*(d*x+c)*b-2*I*b*c+2*I*a*d)/d)/b} - 54*\sqrt{d*x+c}*d*e^{((-2*I*(d*x+c)*b+2*I*b*c-2*I*a*d)/d)/b} + 6*\sqrt{d*x+c}*d*e^{((-6*I*(d*x+c)*b+6*I*b*c-6*I*a*d)/d)/b}*c - I*\sqrt{3}*\sqrt{\pi} \\ & *(-4*I*b*c*d+d^2)*d*\operatorname{erf}(-\sqrt{3}*\sqrt{b*d})*\sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2+1})/d)*e^{((6*I*b*c-6*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2+1})*b^2)} - I*\sqrt{3}*\sqrt{\pi}*(-4*I*b*c*d-d^2)*d*\operatorname{erf}(-\sqrt{3}*\sqrt{b*d})*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2+1})/d)*e^{((-6*I*b*c+6*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2+1})*b^2)} - 9*I*\sqrt{\pi}*(12*I*b*c*d-9*d^2)*d*\operatorname{erf}(-\sqrt{b*d})*\sqrt{d*x+c}*(I*b*d/\sqrt{b^2*d^2+1})/d)*e^{((2*I*b*c-2*I*a*d)/d)/(\sqrt{b*d}*(I*b*d/\sqrt{b^2*d^2+1})*b^2)} - 9*I*\sqrt{\pi}*(12*I*b*c*d+9*d^2)*d*\operatorname{erf}(-\sqrt{b*d})*\sqrt{d*x+c}*(-I*b*d/\sqrt{b^2*d^2+1})/d)*e^{((-2*I*b*c+2*I*a*d)/d)/(\sqrt{b*d}*(-I*b*d/\sqrt{b^2*d^2+1})*b^2)} + 6*I*(-4*I*(d*x+c)^(3/2)*b*d+4*I*\sqrt{d*x+c}*b*c*d+\sqrt{d*x+c}*d^2)*e^{((6*I*(d*x+c)*b-6*I*b*c+6*I*a*d)/d)/b^2} + 18*I*(12*I*(d*x+c)^(3/2)*b*d-12*I*\sqrt{d*x+c}*b*c*d-9*\sqrt{d*x+c}*d^2)*e^{((2*I*(d*x+c)*b-2*I*b*c+2*I*a*d)/d)/b^2} + 18*I*(12*I*(d*x+c)^(3/2)*b*d-12*I*\sqrt{d*x+c}*b*c*d+9*\sqrt{d*x+c}*d^2)*e^{((-2*I*(d*x+c)*b+2*I*b*c-2*I*a*d)/d)/b^2} + 6*I*(-4*I*(d*x+c)^(3/2)*b*d+4*I*\sqrt{d*x+c}*b*c*d-\sqrt{d*x+c}*d^2)*e^{((-6*I*(d*x+c)*b+6*I*b*c-6*I*a*d)/d)/b^2}/d \end{aligned}$$

3.201 $\int (c + dx)^{5/2} \cos^3(a + bx) \sin^3(a + bx) dx$

Optimal. Leaf size=407

$$\frac{5\sqrt{\frac{\pi}{3}}d^{5/2} \cos\left(6a - \frac{6bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}} - \frac{45\sqrt{\pi}d^{5/2} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{2048b^{7/2}} - \frac{5\sqrt{\frac{\pi}{3}}d^{5/2} \sin\left(6a - \frac{6bc}{d}\right)}{18432b^{7/2}}$$

```
[Out] (45*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(1024*b^3) - (3*(c + d*x)^(5/2)*Cos[2*a + 2*b*x])/(64*b) - (5*d^2*Sqrt[c + d*x]*Cos[6*a + 6*b*x])/(9216*b^3) + ((c + d*x)^(5/2)*Cos[6*a + 6*b*x])/(192*b) + (5*d^(5/2)*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(18432*b^(7/2)) - (45*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2048*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/3]*FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(18432*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2048*b^(7/2)) + (15*d*(c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(256*b^2) - (5*d*(c + d*x)^(3/2)*Sin[6*a + 6*b*x])/(2304*b^2)
```

Rubi [A] time = 0.669301, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{5\sqrt{\frac{\pi}{3}}d^{5/2} \cos\left(6a - \frac{6bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}} - \frac{45\sqrt{\pi}d^{5/2} \cos\left(2a - \frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{2048b^{7/2}} - \frac{5\sqrt{\frac{\pi}{3}}d^{5/2} \sin\left(6a - \frac{6bc}{d}\right)}{18432b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3, x]
```

```
[Out] (45*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(1024*b^3) - (3*(c + d*x)^(5/2)*Cos[2*a + 2*b*x])/(64*b) - (5*d^2*Sqrt[c + d*x]*Cos[6*a + 6*b*x])/(9216*b^3) + ((c + d*x)^(5/2)*Cos[6*a + 6*b*x])/(192*b) + (5*d^(5/2)*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(18432*b^(7/2)) - (45*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2048*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/3]*FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(18432*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2048*b^(7/2)) + (15*d*(c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(256*b^2) - (5*d*(c + d*x)^(3/2)*Sin[6*a + 6*b*x])/(2304*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^{5/2} \cos^3(a + bx) \sin^3(a + bx) dx &= \int \left(\frac{3}{32} (c + dx)^{5/2} \sin(2a + 2bx) - \frac{1}{32} (c + dx)^{5/2} \sin(6a + 6bx) \right) dx \\
&= -\left(\frac{1}{32} \int (c + dx)^{5/2} \sin(6a + 6bx) dx \right) + \frac{3}{32} \int (c + dx)^{5/2} \sin(2a + 2bx) dx \\
&= -\frac{3(c + dx)^{5/2} \cos(2a + 2bx)}{64b} + \frac{(c + dx)^{5/2} \cos(6a + 6bx)}{192b} - \frac{(5d) \int (c + dx)^{3/2} \sin(6a + 6bx) dx}{384b} \\
&= -\frac{3(c + dx)^{5/2} \cos(2a + 2bx)}{64b} + \frac{(c + dx)^{5/2} \cos(6a + 6bx)}{192b} + \frac{15d(c + dx)^{3/2} \sin(6a + 6bx)}{256b^2} \\
&= \frac{45d^2 \sqrt{c + dx} \cos(2a + 2bx)}{1024b^3} - \frac{3(c + dx)^{5/2} \cos(2a + 2bx)}{64b} - \frac{5d^2 \sqrt{c + dx} \cos(6a + 6bx)}{9216b^3} \\
&= \frac{45d^2 \sqrt{c + dx} \cos(2a + 2bx)}{1024b^3} - \frac{3(c + dx)^{5/2} \cos(2a + 2bx)}{64b} - \frac{5d^2 \sqrt{c + dx} \cos(6a + 6bx)}{9216b^3} \\
&= \frac{45d^2 \sqrt{c + dx} \cos(2a + 2bx)}{1024b^3} - \frac{3(c + dx)^{5/2} \cos(2a + 2bx)}{64b} - \frac{5d^2 \sqrt{c + dx} \cos(6a + 6bx)}{9216b^3} \\
&= \frac{45d^2 \sqrt{c + dx} \cos(2a + 2bx)}{1024b^3} - \frac{3(c + dx)^{5/2} \cos(2a + 2bx)}{64b} - \frac{5d^2 \sqrt{c + dx} \cos(6a + 6bx)}{9216b^3}
\end{aligned}$$

Mathematica [A] time = 3.00165, size = 550, normalized size = 1.35

$$-2592b^3c^2\sqrt{c + dx} \cos(2(a + bx)) + 288b^3c^2\sqrt{c + dx} \cos(6(a + bx)) - 2592b^3d^2x^2\sqrt{c + dx} \cos(2(a + bx)) + 288b^3d^2x^2\sqrt{c + dx} \cos(6(a + bx))$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]

[Out] (-2592*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 2430*b*d^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 5184*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 2592*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 288*b^3*c^2*Sqrt[c + d*x]*Cos[6*(a + b*x)] - 30*b*d^2*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 576*b^3*c*d*x*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 288*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 5*Sqrt[b/d]*d^3*Sqrt[3*Pi]*Cos[6*a - (6*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]] - 1215*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 5*Sqrt[b/d]*d^3*Sqrt[3*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]]*Sin[6*a - (6*b*c)/d] + 1215*Sqrt[b/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d] + 3240*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 3240*b^2*d^2*x*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 120*b^2*c*d*Sqrt[c + d*x]*Sin[6*(a + b*x)] - 120*b^2*d^2*x*Sqrt[c + d*x]*Sin[6*(a + b*x)]/(55296*b^4)

Maple [A] time = 0.04, size = 477, normalized size = 1.2

$$2 \frac{1}{d} \left(-\frac{3d(dx+c)^{5/2}}{128b} \cos\left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d}\right) + \frac{15d}{128b} \left(\frac{1}{4} \frac{d(dx+c)^{3/2}}{b} \sin\left(2 \frac{(dx+c)b}{d} + 2 \frac{ad-bc}{d}\right) - 3/4 \frac{d}{b} \left(\right. \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)

[Out] 2/d*(-3/128/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+15/128/b*d*(1/4/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))+1/384/b*d*(d*x+c)^(5/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-5/384/b*d*(1/12/b*d*(d*x+c)^(3/2)*sin(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/4/b*d*(-1/12/b*d*(d*x+c)^(1/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))

Maxima [C] time = 2.20164, size = 1916, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")

[Out] -1/884736*sqrt(6)*sqrt(2)*(160*sqrt(6)*sqrt(2)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(6*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 4320*sqrt(6)*sqrt(2)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 8*(48*sqrt(6)*sqrt(2)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 5*sqrt(6)*sqrt(2)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(6*((d*x + c)*b - b*c + a*d)/d) + 216*(16*sqrt(6)*sqrt(2)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 15*sqrt(6)*sqrt(2)*sqrt(d*

```

x + c)*d^3*abs(b)/abs(d))*cos(2*((d*x + c)*b - b*c + a*d)/d) - (sqrt(2)*(5*
sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*
sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5
*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
5*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
))*d^3*sqrt(abs(b)/abs(d))*cos(-6*(b*c - a*d)/d) - sqrt(2)*(5*I*sqrt(pi)*co
s(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*
cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)
*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(a
bs(b)/abs(d))*sin(-6*(b*c - a*d)/d)*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) + (sq
rt(6)*(405*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) + 405*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) - 405*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) + 405*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + sqrt(6)
*(-405*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) - 405*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) - 405*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))) + 405*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d)*erf(sqrt(d*x
+ c)*sqrt(2*I*b/d)) + (sqrt(6)*(405*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) + 405*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 405*I*sqrt(pi)*sin(1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 405*I*sqrt(pi)*sin(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-
2*(b*c - a*d)/d) + sqrt(6)*(405*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) + 405*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) - 405*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 405*sqrt(pi)*sin(-1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-2*(b*
c - a*d)/d)*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)) - (sqrt(2)*(5*sqrt(pi)*cos(1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*cos(-
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*si
n(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*
sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(ab
s(b)/abs(d))*cos(-6*(b*c - a*d)/d) - sqrt(2)*(-5*I*sqrt(pi)*cos(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*cos(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*sin(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)*sin(-1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d)
)*sin(-6*(b*c - a*d)/d)*erf(sqrt(d*x + c)*sqrt(-6*I*b/d))*abs(d)/(b^3*d*ab
s(b))

```

Fricas [A] time = 0.764722, size = 1089, normalized size = 2.68

$$5\sqrt{3}\pi d^3 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{6(bc-ad)}{d}\right) C\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 5\sqrt{3}\pi d^3 \sqrt{\frac{b}{\pi d}} S\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) \sin\left(-\frac{6(bc-ad)}{d}\right) - 1215\pi d^3 \sqrt{\frac{b}{\pi d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")

[Out] 1/55296*(5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 1215*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c

```
) * sqrt(b / (pi * d)) + 1215 * pi * d^3 * sqrt(b / (pi * d)) * fresnel_sin(2 * sqrt(d * x + c) *
sqrt(b / (pi * d))) * sin(-2 * (b * c - a * d) / d) + 96 * (24 * b^3 * d^2 * x^2 + 2 * (48 * b^3 * d^2 *
x^2 + 96 * b^3 * c * d * x + 48 * b^3 * c^2 - 5 * b * d^2) * cos(b * x + a)^6 + 48 * b^3 * c * d * x +
24 * b^3 * c^2 + 45 * b * d^2 * cos(b * x + a)^2 - 3 * (48 * b^3 * d^2 * x^2 + 96 * b^3 * c * d * x + 4
8 * b^3 * c^2 - 5 * b * d^2) * cos(b * x + a)^4 - 25 * b * d^2 - 20 * (2 * (b^2 * d^2 * x + b^2 * c * d)
) * cos(b * x + a)^5 - 2 * (b^2 * d^2 * x + b^2 * c * d) * cos(b * x + a)^3 - 3 * (b^2 * d^2 * x +
b^2 * c * d) * cos(b * x + a)) * sin(b * x + a) * sqrt(d * x + c) / b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 2.74327, size = 2677, normalized size = 6.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/110592*(48*(sqrt(3)*sqrt(pi)*d^2*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c))*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) + sqrt(3)*sqrt(pi)*d^2*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)*b) - 27*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c))*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) - 27*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c))*(-I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d
^2) + 1)*b) + 6*sqrt(d*x + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)
/b - 54*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b - 54*
sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 6*sqrt(d*x
+ c)*d*e^((-6*I*(d*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b)*c^2 - d^2*((I*sqrt(
3)*sqrt(pi)*(48*I*b^2*c^2*d - 24*b*c*d^2 - 5*I*d^3)*d*erf(-sqrt(3)*sqrt(b*d)
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(-48*I*(d*x + c)^(5/2)*b^2*d +
96*I*(d*x + c)^(3/2)*b^2*c*d - 48*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^(
3/2)*b*d^2 + 24*sqrt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((-6*I*(d
*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b^3)/d^2 + (I*sqrt(3)*sqrt(pi)*(48*I*b^2*
c^2*d + 24*b*c*d^2 - 5*I*d^3)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c))*(-I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
b^2*d^2) + 1)*b^3) - 6*I*(-48*I*(d*x + c)^(5/2)*b^2*d + 96*I*(d*x + c)^(3/2)
)*b^2*c*d - 48*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 24*sq
rt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((6*I*(d*x + c)*b - 6*I*b*c
+ 6*I*a*d)/d)/b^3)/d^2 + 27*(I*sqrt(pi))*(-48*I*b^2*c^2*d + 72*b*c*d^2 + 45*
I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c))*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*
b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(48*I*(d*
x + c)^(5/2)*b^2*d - 96*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*
c^2*d + 60*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x + c)*b*c*d^2 - 45*I*sqrt(d*x
```

$$\begin{aligned}
& + c)*d^3)*e^{((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^3)/d^2 + 27*(I*\sqrt{\pi})*(-48*I*b^2*c^2*d - 72*b*c*d^2 + 45*I*d^3)*d*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x + c})*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-2*I*b*c + 2*I*a*d)/d)/(\sqrt{b*d})*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^3} - 2*I*(48*I*(d*x + c)^{(5/2)}*b^2*d - 96*I*(d*x + c)^{(3/2)}*b^2*c*d + 48*I*\sqrt{d*x + c}*b^2*c^2*d - 60*(d*x + c)^{(3/2)}*b*d^2 + 72*\sqrt{d*x + c}*b*c*d^2 - 45*I*\sqrt{d*x + c}*d^3)*e^{((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^2} - 24*(I*\sqrt{3})*\sqrt{\pi})*(-4*I*b*c*d + d^2)*d*\operatorname{erf}(-\sqrt{3}*\sqrt{b*d}*\sqrt{d*x + c})*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((6*I*b*c - 6*I*a*d)/d)/(\sqrt{b*d})*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2} + I*\sqrt{3})*\sqrt{\pi})*(-4*I*b*c*d - d^2)*d*\operatorname{erf}(-\sqrt{3}*\sqrt{b*d}*\sqrt{d*x + c})*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-6*I*b*c + 6*I*a*d)/d)/(\sqrt{b*d})*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2} + 9*I*\sqrt{\pi})*(12*I*b*c*d - 9*d^2)*d*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x + c})*(I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((2*I*b*c - 2*I*a*d)/d)/(\sqrt{b*d})*(I*b*d/\sqrt{b^2*d^2} + 1)*b^2} + 9*I*\sqrt{\pi})*(12*I*b*c*d + 9*d^2)*d*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x + c})*(-I*b*d/\sqrt{b^2*d^2} + 1)/d)*e^{((-2*I*b*c + 2*I*a*d)/d)/(\sqrt{b*d})*(-I*b*d/\sqrt{b^2*d^2} + 1)*b^2} - 6*I*(-4*I*(d*x + c)^{(3/2)}*b*d + 4*I*\sqrt{d*x + c}*b*c*d + \sqrt{d*x + c}*d^2)*e^{((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b^2} - 18*I*(12*I*(d*x + c)^{(3/2)}*b*d - 12*I*\sqrt{d*x + c}*b*c*d - 9*\sqrt{d*x + c}*d^2)*e^{((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^2} - 18*I*(12*I*(d*x + c)^{(3/2)}*b*d - 12*I*\sqrt{d*x + c}*b*c*d + 9*\sqrt{d*x + c}*d^2)*e^{((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2} - 6*I*(-4*I*(d*x + c)^{(3/2)}*b*d + 4*I*\sqrt{d*x + c}*b*c*d - \sqrt{d*x + c}*d^2)*e^{((-6*I*(d*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b^2}*c)/d
\end{aligned}$$

3.202 $\int x^3 \cos^2(x) \cot^2(x) dx$

Optimal. Leaf size=112

$$-3ix \operatorname{PolyLog}(2, e^{2ix}) + \frac{3}{2} \operatorname{PolyLog}(3, e^{2ix}) - \frac{3x^4}{8} - ix^3 + \frac{3x^2}{8} + 3x^2 \log(1 - e^{2ix}) - \frac{3}{4}x^2 \cos^2(x) - x^3 \cot(x) - \frac{1}{2}x^3 \operatorname{si}$$

```
[Out] (3*x^2)/8 - I*x^3 - (3*x^4)/8 + (3*Cos[x]^2)/8 - (3*x^2*Cos[x]^2)/4 - x^3*C
ot[x] + 3*x^2*Log[1 - E^((2*I)*x)] - (3*I)*x*PolyLog[2, E^((2*I)*x)] + (3*P
olyLog[3, E^((2*I)*x)])/2 + (3*x*Cos[x]*Sin[x])/4 - (x^3*Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.185145, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {4408, 3311, 30, 3310, 3720, 3717, 2190, 2531, 2282, 6589}

$$-3ix \operatorname{PolyLog}(2, e^{2ix}) + \frac{3}{2} \operatorname{PolyLog}(3, e^{2ix}) - \frac{3x^4}{8} - ix^3 + \frac{3x^2}{8} + 3x^2 \log(1 - e^{2ix}) - \frac{3}{4}x^2 \cos^2(x) - x^3 \cot(x) - \frac{1}{2}x^3 \operatorname{si}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] (3*x^2)/8 - I*x^3 - (3*x^4)/8 + (3*Cos[x]^2)/8 - (3*x^2*Cos[x]^2)/4 - x^3*C
ot[x] + 3*x^2*Log[1 - E^((2*I)*x)] - (3*I)*x*PolyLog[2, E^((2*I)*x)] + (3*P
olyLog[3, E^((2*I)*x)])/2 + (3*x*Cos[x]*Sin[x])/4 - (x^3*Cos[x]*Sin[x])/2
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] := -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbo
l] := Simp[(d*m*(c + d*x)^(m - 1)*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sine[e + f*x])^(n - 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sine[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sine[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :=
Simp[(d*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sine[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sine[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*(b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
:= Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[
(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol]
:= Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^3 \cos^2(x) \cot^2(x) dx &= -\int x^3 \cos^2(x) dx + \int x^3 \cot^2(x) dx \\
&= -\frac{3}{4}x^2 \cos^2(x) - x^3 \cot(x) - \frac{1}{2}x^3 \cos(x) \sin(x) - \frac{\int x^3 dx}{2} + \frac{3}{2} \int x \cos^2(x) dx + 3 \int x^2 \cot(x) dx \\
&= -ix^3 - \frac{3x^4}{8} + \frac{3 \cos^2(x)}{8} - \frac{3}{4}x^2 \cos^2(x) - x^3 \cot(x) + \frac{3}{4}x \cos(x) \sin(x) - \frac{1}{2}x^3 \cos(x) \sin(x) - \frac{3x^2}{4} \log(1 - e^{2ix}) + \frac{3}{4}x \cos(x) \sin(x) \\
&= \frac{3x^2}{8} - ix^3 - \frac{3x^4}{8} + \frac{3 \cos^2(x)}{8} - \frac{3}{4}x^2 \cos^2(x) - x^3 \cot(x) + 3x^2 \log(1 - e^{2ix}) + \frac{3}{4}x \cos(x) \sin(x) \\
&= \frac{3x^2}{8} - ix^3 - \frac{3x^4}{8} + \frac{3 \cos^2(x)}{8} - \frac{3}{4}x^2 \cos^2(x) - x^3 \cot(x) + 3x^2 \log(1 - e^{2ix}) - 3ix \operatorname{Li}_2(e^{2ix}) + \frac{3}{4}x \cos(x) \sin(x) \\
&= \frac{3x^2}{8} - ix^3 - \frac{3x^4}{8} + \frac{3 \cos^2(x)}{8} - \frac{3}{4}x^2 \cos^2(x) - x^3 \cot(x) + 3x^2 \log(1 - e^{2ix}) - 3ix \operatorname{Li}_2(e^{2ix}) + \frac{3}{4}x \cos(x) \sin(x) \\
&= \frac{3x^2}{8} - ix^3 - \frac{3x^4}{8} + \frac{3 \cos^2(x)}{8} - \frac{3}{4}x^2 \cos^2(x) - x^3 \cot(x) + 3x^2 \log(1 - e^{2ix}) - 3ix \operatorname{Li}_2(e^{2ix}) + \frac{3}{4}x \cos(x) \sin(x)
\end{aligned}$$

Mathematica [A] time = 0.175528, size = 104, normalized size = 0.93

$$\frac{1}{16} (48ix \operatorname{PolyLog}(2, e^{-2ix}) + 24 \operatorname{PolyLog}(3, e^{-2ix}) - 6x^4 + 16ix^3 + 48x^2 \log(1 - e^{-2ix}) - 4x^3 \sin(2x) - 6x^2 \cos(2x) - \dots)$$

Antiderivative was successfully verified.

[In] Integrate[x^3*Cos[x]^2*Cot[x]^2,x]

[Out] ((-2*I)*Pi^3 + (16*I)*x^3 - 6*x^4 + 3*Cos[2*x] - 6*x^2*Cos[2*x] - 16*x^3*Cot[x] + 48*x^2*Log[1 - E^((-2*I)*x)] + (48*I)*x*PolyLog[2, E^((-2*I)*x)] + 24*PolyLog[3, E^((-2*I)*x)] + 6*x*Sin[2*x] - 4*x^3*Sin[2*x])/16

Maple [A] time = 0.106, size = 150, normalized size = 1.3

$$-\frac{3x^4}{8} + \frac{i}{32} (6ix^2 + 4x^3 - 3i - 6x) e^{2ix} - \frac{i}{32} (-6ix^2 + 4x^3 + 3i - 6x) e^{-2ix} - \frac{2ix^3}{e^{2ix} - 1} - 2ix^3 + 3x^2 \ln(1 - e^{ix}) - 6 \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*cos(x)^2*cot(x)^2,x)

[Out] -3/8*x^4+1/32*I*(6*I*x^2+4*x^3-3*I-6*x)*exp(2*I*x)-1/32*I*(-6*I*x^2+4*x^3+3*I-6*x)*exp(-2*I*x)-2*I*x^3/(exp(2*I*x)-1)-2*I*x^3+3*x^2*ln(1-exp(I*x))-6*I*x*polylog(2,exp(I*x))+6*polylog(3,exp(I*x))+3*x^2*ln(1+exp(I*x))-6*I*x*polylog(2,-exp(I*x))+6*polylog(3,-exp(I*x))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cos(x)^2*cot(x)^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError

Fricas [C] time = 0.594582, size = 851, normalized size = 7.6

$$4(2x^3 - 3x)\cos(x)^3 + 24x^2\log(\cos(x) + i\sin(x) + 1)\sin(x) + 24x^2\log(\cos(x) - i\sin(x) + 1)\sin(x) + 24x^2\log(-$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cos(x)^2*cot(x)^2,x, algorithm="fricas")

[Out] 1/16*(4*(2*x^3 - 3*x)*cos(x)^3 + 24*x^2*log(cos(x) + I*sin(x) + 1)*sin(x) + 24*x^2*log(cos(x) - I*sin(x) + 1)*sin(x) + 24*x^2*log(-cos(x) + I*sin(x) + 1)*sin(x) + 24*x^2*log(-cos(x) - I*sin(x) + 1)*sin(x) - 48*I*x*dilog(cos(x) + I*sin(x))*sin(x) + 48*I*x*dilog(cos(x) - I*sin(x))*sin(x) + 48*I*x*dilog(-cos(x) + I*sin(x))*sin(x) - 48*I*x*dilog(-cos(x) - I*sin(x))*sin(x) - 12*(2*x^3 - x)*cos(x) - 3*(2*x^4 + 2*(2*x^2 - 1)*cos(x)^2 - 2*x^2 + 1)*sin(x) + 48*polylog(3, cos(x) + I*sin(x))*sin(x) + 48*polylog(3, cos(x) - I*sin(x))*sin(x) + 48*polylog(3, -cos(x) + I*sin(x))*sin(x) + 48*polylog(3, -cos(x) - I*sin(x))*sin(x))/sin(x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cos^2(x) \cot^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*cos(x)**2*cot(x)**2,x)

[Out] Integral(x**3*cos(x)**2*cot(x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cos(x)^2 \cot(x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cos(x)^2*cot(x)^2,x, algorithm="giac")

[Out] integrate(x^3*cos(x)^2*cot(x)^2, x)

3.203 $\int x^2 \cos^2(x) \cot^2(x) dx$

Optimal. Leaf size=83

$$-i\text{PolyLog}\left(2, e^{2ix}\right) - \frac{x^3}{2} - ix^2 - x^2 \cot(x) - \frac{1}{2}x^2 \sin(x) \cos(x) + \frac{x}{4} + 2x \log\left(1 - e^{2ix}\right) - \frac{1}{2}x \cos^2(x) + \frac{1}{4} \sin(x) \cos(x)$$

[Out] x/4 - I*x^2 - x^3/2 - (x*Cos[x]^2)/2 - x^2*Cot[x] + 2*x*Log[1 - E^((2*I)*x)] - I*PolyLog[2, E^((2*I)*x)] + (Cos[x]*Sin[x])/4 - (x^2*Cos[x]*Sin[x])/2

Rubi [A] time = 0.169514, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {4408, 3311, 30, 2635, 8, 3720, 3717, 2190, 2279, 2391}

$$-i\text{PolyLog}\left(2, e^{2ix}\right) - \frac{x^3}{2} - ix^2 - x^2 \cot(x) - \frac{1}{2}x^2 \sin(x) \cos(x) + \frac{x}{4} + 2x \log\left(1 - e^{2ix}\right) - \frac{1}{2}x \cos^2(x) + \frac{1}{4} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[x^2*Cos[x]^2*Cot[x]^2,x]

[Out] x/4 - I*x^2 - x^3/2 - (x*Cos[x]^2)/2 - x^2*Cot[x] + 2*x*Log[1 - E^((2*I)*x)] - I*PolyLog[2, E^((2*I)*x)] + (Cos[x]*Sin[x])/4 - (x^2*Cos[x]*Sin[x])/2

Rule 4408

Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3311

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Simp[(d*m*(c + d*x)^(m - 1)*(b*SIN[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*SIN[e + f*x])^(n - 2), x], x] - Dist[(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*SIN[e + f*x])^n, x], x] - Simp[(b*(c + d*x)^m*COS[e + f*x]*(b*SIN[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_.), x_Symbol] := -Simp[(b*COS[c + d*x]*(b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
:= Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol]
:= Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[(c + d*x)^m*E^(2*I*k*Pi)*E^(2*I*(e + f*x))]/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol]
:= Simp[(c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol]
:= -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int x^2 \cos^2(x) \cot^2(x) dx &= -\int x^2 \cos^2(x) dx + \int x^2 \cot^2(x) dx \\ &= -\frac{1}{2}x \cos^2(x) - x^2 \cot(x) - \frac{1}{2}x^2 \cos(x) \sin(x) - \frac{\int x^2 dx}{2} + \frac{1}{2} \int \cos^2(x) dx + 2 \int x \cot(x) dx - \\ &= -ix^2 - \frac{x^3}{2} - \frac{1}{2}x \cos^2(x) - x^2 \cot(x) + \frac{1}{4} \cos(x) \sin(x) - \frac{1}{2}x^2 \cos(x) \sin(x) - 4i \int \frac{e^{2ix} x}{1 - e^{2ix}} dx + \\ &= \frac{x}{4} - ix^2 - \frac{x^3}{2} - \frac{1}{2}x \cos^2(x) - x^2 \cot(x) + 2x \log(1 - e^{2ix}) + \frac{1}{4} \cos(x) \sin(x) - \frac{1}{2}x^2 \cos(x) \sin(x) \\ &= \frac{x}{4} - ix^2 - \frac{x^3}{2} - \frac{1}{2}x \cos^2(x) - x^2 \cot(x) + 2x \log(1 - e^{2ix}) + \frac{1}{4} \cos(x) \sin(x) - \frac{1}{2}x^2 \cos(x) \sin(x) \\ &= \frac{x}{4} - ix^2 - \frac{x^3}{2} - \frac{1}{2}x \cos^2(x) - x^2 \cot(x) + 2x \log(1 - e^{2ix}) - i\text{Li}_2(e^{2ix}) + \frac{1}{4} \cos(x) \sin(x) - \frac{1}{2}x^2 \end{aligned}$$

Mathematica [A] time = 0.10048, size = 72, normalized size = 0.87

$$\frac{1}{8} \left(-8i \text{PolyLog}\left(2, e^{2ix}\right) - 4x^3 - 8ix^2 - 2x^2 \sin(2x) - 8x^2 \cot(x) + 16x \log(1 - e^{2ix}) + \sin(2x) - 2x \cos(2x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[x]^2*Cot[x]^2,x]
```

[Out] $((-8I)x^2 - 4x^3 - 2x\cos[2x] - 8x^2\cot[x] + 16x\log[1 - E^{((2I)x)}]) - (8I)\text{PolyLog}[2, E^{((2I)x)}] + \sin[2x] - 2x^2\sin[2x])/8$

Maple [A] time = 0.106, size = 112, normalized size = 1.4

$$-\frac{x^3}{2} + \frac{i}{16}(2ix + 2x^2 - 1)e^{2ix} - \frac{i}{16}(-2ix + 2x^2 - 1)e^{-2ix} - \frac{2ix^2}{e^{2ix} - 1} + 2x \ln(1 - e^{ix}) + 2x \ln(1 + e^{ix}) - 2ix^2 - 2ix$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cos(x)^2*cot(x)^2,x)`

[Out] $-1/2x^3 + 1/16I(2Ix + 2x^2 - 1)\exp(2Ix) - 1/16I(-2Ix + 2x^2 - 1)\exp(-2Ix) - 2Ix^2 / (\exp(2Ix) - 1) + 2x \ln(1 - \exp(Ix)) + 2x \ln(1 + \exp(Ix)) - 2Ix^2 - 2Ix \text{polylog}(2, \exp(Ix)) - 2Ix \text{polylog}(2, -\exp(Ix))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(x)^2*cot(x)^2,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError

Fricas [B] time = 0.558489, size = 551, normalized size = 6.64

$$(2x^2 - 1)\cos(x)^3 + 4x \log(\cos(x) + i \sin(x) + 1) \sin(x) + 4x \log(\cos(x) - i \sin(x) + 1) \sin(x) + 4x \log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(x)^2*cot(x)^2,x, algorithm="fricas")`

[Out] $1/4*((2x^2 - 1)\cos(x)^3 + 4x \log(\cos(x) + I\sin(x) + 1)\sin(x) + 4x \log(\cos(x) - I\sin(x) + 1)\sin(x) + 4x \log(-\cos(x) + I\sin(x) + 1)\sin(x) + 4x \log(-\cos(x) - I\sin(x) + 1)\sin(x) - (6x^2 - 1)\cos(x) - (2x^3 + 2x\cos(x)^2 - x)\sin(x) - 4I\text{dilog}(\cos(x) + I\sin(x))\sin(x) + 4I\text{dilog}(\cos(x) - I\sin(x))\sin(x) + 4I\text{dilog}(-\cos(x) + I\sin(x))\sin(x) - 4I\text{dilog}(-\cos(x) - I\sin(x))\sin(x))/\sin(x)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cos^2(x) \cot^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*cos(x)**2*cot(x)**2,x)`

[Out] Integral(x**2*cos(x)**2*cot(x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cos(x)^2 \cot(x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cos(x)^2*cot(x)^2,x, algorithm="giac")

[Out] integrate(x^2*cos(x)^2*cot(x)^2, x)

3.204 $\int x \cos^2(x) \cot^2(x) dx$

Optimal. Leaf size=33

$$-\frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x \cot(x) + \log(\sin(x)) - \frac{1}{2}x \sin(x) \cos(x)$$

[Out] $(-3*x^2)/4 - \text{Cos}[x]^2/4 - x*\text{Cot}[x] + \text{Log}[\text{Sin}[x]] - (x*\text{Cos}[x]*\text{Sin}[x])/2$

Rubi [A] time = 0.0546779, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {4408, 3310, 30, 3720, 3475}

$$-\frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x \cot(x) + \log(\sin(x)) - \frac{1}{2}x \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{Cos}[x]^2*\text{Cot}[x]^2, x]$

[Out] $(-3*x^2)/4 - \text{Cos}[x]^2/4 - x*\text{Cot}[x] + \text{Log}[\text{Sin}[x]] - (x*\text{Cos}[x]*\text{Sin}[x])/2$

Rule 4408

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(n_.)}*\text{Cot}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \text{ :> } -\text{Int}[(c + d*x)^m*\text{Cos}[a + b*x]^n*\text{Cot}[a + b*x]^{(p - 2)}, x] + \text{Int}[(c + d*x)^m*\text{Cos}[a + b*x]^{(n - 2)}*\text{Cot}[a + b*x]^p, x] \text{ /; FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[p, 0]$

Rule 3310

$\text{Int}[(c_.) + (d_.)*(x_.)]*((b_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^{(n_.)}, x_Symbol] \text{ :> } \text{Simp}[(d*(b*\text{Sin}[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n - 1))/n, \text{Int}[(c + d*x)*(b*\text{Sin}[e + f*x])^{(n - 2)}, x], x] - \text{Simp}[(b*(c + d*x)*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(n - 1)})/(f*n), x]) \text{ /; FreeQ}\{b, c, d, e, f\}, x \ \&\& \ \text{GtQ}[n, 1]$

Rule 30

$\text{Int}[(x_.)^{(m_.)}, x_Symbol] \text{ :> } \text{Simp}[x^{(m + 1)}/(m + 1), x] \text{ /; FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1]$

Rule 3720

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*((b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^{(n_.)}, x_Symbol] \text{ :> } \text{Simp}[(b*(c + d*x)^m*(b*\text{Tan}[e + f*x])^{(n - 1)})/(f*(n - 1)), x] + (-\text{Dist}[(b*d*m)/(f*(n - 1)), \text{Int}[(c + d*x)^{(m - 1)}*(b*\text{Tan}[e + f*x])^{(n - 1)}, x], x] - \text{Dist}[b^2, \text{Int}[(c + d*x)^m*(b*\text{Tan}[e + f*x])^{(n - 2)}, x], x]) \text{ /; FreeQ}\{b, c, d, e, f\}, x \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{GtQ}[m, 0]$

Rule 3475

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x_Symbol] \text{ :> } -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] \text{ /; FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \int x \cos^2(x) \cot^2(x) dx &= - \int x \cos^2(x) dx + \int x \cot^2(x) dx \\ &= -\frac{1}{4} \cos^2(x) - x \cot(x) - \frac{1}{2} x \cos(x) \sin(x) - \frac{\int x dx}{2} - \int x dx + \int \cot(x) dx \\ &= -\frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x \cot(x) + \log(\sin(x)) - \frac{1}{2} x \cos(x) \sin(x) \end{aligned}$$

Mathematica [A] time = 0.0250926, size = 33, normalized size = 1.

$$-\frac{3x^2}{4} - \frac{1}{4}x \sin(2x) - \frac{1}{8} \cos(2x) - x \cot(x) + \log(\sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[x]^2*Cot[x]^2,x]

[Out] (-3*x^2)/4 - Cos[2*x]/8 - x*Cot[x] + Log[Sin[x]] - (x*Sin[2*x])/4

Maple [B] time = 0.093, size = 76, normalized size = 2.3

$$\frac{1}{2 \tan(x)} \left(-x - \frac{x^2 \tan(x)}{2} \right) - \frac{\ln((\tan(x))^2 + 1)}{2} + \ln(\tan(x)) + \frac{1}{2 \tan(x) ((\tan(x))^2 + 1)} \left(-\frac{\tan(x)}{2} - x - 2x(\tan(x)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(x)^2*cot(x)^2,x)

[Out] 1/2*(-x-1/2*x^2*tan(x))/tan(x)-1/2*ln(tan(x)^2+1)+ln(tan(x))+1/2*(-1/2*tan(x)-x-2*x*tan(x)^2-x^2*tan(x)-x^2*tan(x)^3)/tan(x)/(tan(x)^2+1)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x)^2*cot(x)^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError

Fricas [A] time = 0.5048, size = 138, normalized size = 4.18

$$\frac{4x \cos(x)^3 - 12x \cos(x) - (6x^2 + 2 \cos(x)^2 - 1) \sin(x) + 8 \log\left(\frac{1}{2} \sin(x)\right) \sin(x)}{8 \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x)^2*cot(x)^2,x, algorithm="fricas")

[Out] $\frac{1}{8}(4x^2\cos(x)^3 - 12x\cos(x) - (6x^2 + 2\cos(x)^2 - 1)\sin(x) + 8\log(\frac{1}{2}\sin(x))\sin(x))/\sin(x)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos^2(x) \cot^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x)**2*cot(x)**2,x)`

[Out] `Integral(x*cos(x)**2*cot(x)**2, x)`

Giac [B] time = 1.18014, size = 278, normalized size = 8.42

$$6x^2 \tan\left(\frac{1}{2}x\right)^5 - 4x \tan\left(\frac{1}{2}x\right)^6 - 4 \log\left(\frac{16 \tan\left(\frac{1}{2}x\right)^2}{\tan\left(\frac{1}{2}x\right)^4 + 2 \tan\left(\frac{1}{2}x\right)^2 + 1}\right) \tan\left(\frac{1}{2}x\right)^5 + 12x^2 \tan\left(\frac{1}{2}x\right)^3 - 12x \tan\left(\frac{1}{2}x\right)^4 + \tan\left(\frac{1}{2}x\right)^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x)^2*cot(x)^2,x, algorithm="giac")`

[Out] $-\frac{1}{8}(6x^2\tan(1/2x)^5 - 4x\tan(1/2x)^6 - 4\log(16\tan(1/2x)^2/(\tan(1/2x)^4 + 2\tan(1/2x)^2 + 1))\tan(1/2x)^5 + 12x^2\tan(1/2x)^3 - 12x\tan(1/2x)^4 + \tan(1/2x)^5 - 8\log(16\tan(1/2x)^2/(\tan(1/2x)^4 + 2\tan(1/2x)^2 + 1))\tan(1/2x)^3 + 6x^2\tan(1/2x) + 12x\tan(1/2x)^2 - 6\tan(1/2x)^3 - 4\log(16\tan(1/2x)^2/(\tan(1/2x)^4 + 2\tan(1/2x)^2 + 1))\tan(1/2x) + 4x + \tan(1/2x))/(\tan(1/2x)^5 + 2\tan(1/2x)^3 + \tan(1/2x))$

3.205 $\int x^3 \cos^2(x) \cot^3(x) dx$

Optimal. Leaf size=180

$$3ix^2\text{PolyLog}(2, e^{2ix}) - 3x\text{PolyLog}(3, e^{2ix}) - \frac{3}{2}i\text{PolyLog}(2, e^{2ix}) - \frac{3}{2}i\text{PolyLog}(4, e^{2ix}) + \frac{ix^4}{2} - \frac{3x^3}{4} - \frac{3ix^2}{2} - 2x^3 \log(\dots)$$

```
[Out] (3*x)/8 - ((3*I)/2)*x^2 - (3*x^3)/4 + (I/2)*x^4 - (3*x^2*Cot[x])/2 - (x^3*Cot[x]^2)/2 + 3*x*Log[1 - E^((2*I)*x)] - 2*x^3*Log[1 - E^((2*I)*x)] - ((3*I)/2)*PolyLog[2, E^((2*I)*x)] + (3*I)*x^2*PolyLog[2, E^((2*I)*x)] - 3*x*PolyLog[3, E^((2*I)*x)] - ((3*I)/2)*PolyLog[4, E^((2*I)*x)] - (3*Cos[x]*Sin[x])/8 + (3*x^2*Cos[x]*Sin[x])/4 - (3*x*Sin[x]^2)/4 + (x^3*Sin[x]^2)/2
```

Rubi [A] time = 0.400615, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 15, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.25$, Rules used = {4408, 3443, 3311, 30, 2635, 8, 3717, 2190, 2531, 6609, 2282, 6589, 3720, 2279, 2391}

$$3ix^2\text{PolyLog}(2, e^{2ix}) - 3x\text{PolyLog}(3, e^{2ix}) - \frac{3}{2}i\text{PolyLog}(2, e^{2ix}) - \frac{3}{2}i\text{PolyLog}(4, e^{2ix}) + \frac{ix^4}{2} - \frac{3x^3}{4} - \frac{3ix^2}{2} - 2x^3 \log(\dots)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[x]^2*Cot[x]^3, x]
```

```
[Out] (3*x)/8 - ((3*I)/2)*x^2 - (3*x^3)/4 + (I/2)*x^4 - (3*x^2*Cot[x])/2 - (x^3*Cot[x]^2)/2 + 3*x*Log[1 - E^((2*I)*x)] - 2*x^3*Log[1 - E^((2*I)*x)] - ((3*I)/2)*PolyLog[2, E^((2*I)*x)] + (3*I)*x^2*PolyLog[2, E^((2*I)*x)] - 3*x*PolyLog[3, E^((2*I)*x)] - ((3*I)/2)*PolyLog[4, E^((2*I)*x)] - (3*Cos[x]*Sin[x])/8 + (3*x^2*Cos[x]*Sin[x])/4 - (3*x*Sin[x]^2)/4 + (x^3*Sin[x]^2)/2
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Ssin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Ssin[e + f*x])^(n - 2), x], x] - Dist[(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Ssin[e + f*x])^n, x], x] - Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Ssin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2635

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] * (b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3717

Int[((c_) + (d_)*(x_))^(m_)*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m * E^(2*I*k*Pi) * E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi) * E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 2190

Int[((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_)/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[((c + d*x)^m * Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1) * Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m * PolyLog[2, -(e*(F^(c*(a + b*x)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1) * PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 6609

Int[((e_) + (f_)*(x_))^(m_)*PolyLog[n_, (d_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(p_)], x_Symbol] := Simp[((e + f*x)^m * PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1) * PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
:> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int x^3 \cos^2(x) \cot^3(x) dx &= -\int x^3 \cos^2(x) \cot(x) dx + \int x^3 \cot^3(x) dx \\
&= -\frac{1}{2}x^3 \cot^2(x) + \frac{3}{2} \int x^2 \cot^2(x) dx - 2 \int x^3 \cot(x) dx + \int x^3 \cos(x) \sin(x) dx \\
&= -\frac{3}{2}x^2 \cot(x) - \frac{1}{2}x^3 \cot^2(x) + \frac{1}{2}x^3 \sin^2(x) - 2 \left(-\frac{ix^4}{4} - 2i \int \frac{e^{2ix}x^3}{1 - e^{2ix}} dx \right) - \frac{3}{2} \int x^2 dx - \frac{3}{2} \int x^2 dx \\
&= -\frac{3ix^2}{2} - \frac{x^3}{2} - \frac{3}{2}x^2 \cot(x) - \frac{1}{2}x^3 \cot^2(x) + \frac{3}{4}x^2 \cos(x) \sin(x) - \frac{3}{4}x \sin^2(x) + \frac{1}{2}x^3 \sin^2(x) - 6i \int x^2 dx \\
&= -\frac{3ix^2}{2} - \frac{3x^3}{4} - \frac{3}{2}x^2 \cot(x) - \frac{1}{2}x^3 \cot^2(x) + 3x \log(1 - e^{2ix}) - \frac{3}{8} \cos(x) \sin(x) + \frac{3}{4}x^2 \cos(x) \sin(x) \\
&= \frac{3x}{8} - \frac{3ix^2}{2} - \frac{3x^3}{4} - \frac{3}{2}x^2 \cot(x) - \frac{1}{2}x^3 \cot^2(x) + 3x \log(1 - e^{2ix}) - \frac{3}{8} \cos(x) \sin(x) + \frac{3}{4}x^2 \cos(x) \sin(x) \\
&= \frac{3x}{8} - \frac{3ix^2}{2} - \frac{3x^3}{4} - \frac{3}{2}x^2 \cot(x) - \frac{1}{2}x^3 \cot^2(x) + 3x \log(1 - e^{2ix}) - \frac{3}{2}i\text{Li}_2(e^{2ix}) - \frac{3}{8} \cos(x) \sin(x) \\
&= \frac{3x}{8} - \frac{3ix^2}{2} - \frac{3x^3}{4} - \frac{3}{2}x^2 \cot(x) - \frac{1}{2}x^3 \cot^2(x) + 3x \log(1 - e^{2ix}) - \frac{3}{2}i\text{Li}_2(e^{2ix}) - 2 \left(-\frac{ix^4}{4} + x^3 \right)
\end{aligned}$$

Mathematica [A] time = 0.416099, size = 159, normalized size = 0.88

$$\frac{1}{32} \left(-96ix^2 \text{PolyLog}(2, e^{-2ix}) - 96x \text{PolyLog}(3, e^{-2ix}) - 48i \text{PolyLog}(2, e^{2ix}) + 48i \text{PolyLog}(4, e^{-2ix}) - 16ix^4 - 48ix^2 - \dots \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3*Cos[x]^2*Cot[x]^3,x]

[Out] (I*Pi^4 - (48*I)*x^2 - (16*I)*x^4 + 12*x*Cos[2*x] - 8*x^3*Cos[2*x] - 48*x^2*Cot[x] - 16*x^3*Csc[x]^2 - 64*x^3*Log[1 - E^((-2*I)*x)] + 96*x*Log[1 - E^((2*I)*x)] - (96*I)*x^2*PolyLog[2, E^((-2*I)*x)] - (48*I)*PolyLog[2, E^((2*I)*x)] - 96*x*PolyLog[3, E^((-2*I)*x)] + (48*I)*PolyLog[4, E^((-2*I)*x)] - 6*Sin[2*x] + 12*x^2*Sin[2*x])/32

Maple [A] time = 0.157, size = 240, normalized size = 1.3

$$-3ix^2 - \frac{(6ix^2 + 4x^3 - 3i - 6x)e^{2ix}}{32} - \frac{(-6ix^2 + 4x^3 + 3i - 6x)e^{-2ix}}{32} + \frac{x^2(2xe^{2ix} - 3ie^{2ix} + 3i)}{(e^{2ix} - 1)^2} - 2x^3 \ln(1 - e^{ix})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*cos(x)^2*cot(x)^3,x)

[Out] $-3Ix^2 - 1/32*(6Ix^2 + 4x^3 - 3I - 6x)*\exp(2Ix) - 1/32*(-6Ix^2 + 4x^3 + 3I - 6x)*\exp(-2Ix) + x^2*(2x*\exp(2Ix) - 3I*\exp(2Ix) + 3I)/(\exp(2Ix) - 1)^2 - 2x^3*\ln(1 - \exp(Ix)) - 2x^3*\ln(1 + \exp(Ix)) + 6Ix^2*\text{polylog}(2, \exp(Ix)) + 3x*\ln(1 - \exp(Ix)) - 12x*\text{polylog}(3, \exp(Ix)) - 12x*\text{polylog}(3, -\exp(Ix)) + 3x*\ln(1 + \exp(Ix)) + 1/2Ix^4 + 6Ix^2*\text{polylog}(2, -\exp(Ix)) - 3I*\text{polylog}(2, \exp(Ix)) - 12I*\text{polylog}(4, \exp(Ix)) - 12I*\text{polylog}(4, -\exp(Ix)) - 3I*\text{polylog}(2, -\exp(Ix))$

Maxima [B] time = 3.47439, size = 5022, normalized size = 27.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cos(x)^2*cot(x)^3,x, algorithm="maxima")

[Out] $-(4x^3 + (4x^3 + 6Ix^2 - 6x - 3I)*\cos(6x))^2 - (-32Ix^4 - 16x^3 + 168Ix^2 + 24x + 12I)*\cos(4x)^2 - (-32Ix^4 + 56x^3 + 96Ix^2 + 12x)*\cos(2x)^2 - (4x^3 + 6Ix^2 - 6x - 3I)*\sin(6x)^2 - (32Ix^4 + 16x^3 - 168Ix^2 - 24x - 12I)*\sin(4x)^2 - (32Ix^4 - 56x^3 - 96Ix^2 - 12x)*\sin(2x)^2 - 6Ix^2 - ((128Ix^3 - 192Ix)*\cos(4x))^2 + (128Ix^3 - 192Ix)*\cos(2x)^2 + (-128Ix^3 + 192Ix)*\sin(4x)^2 + (-128Ix^3 + 192Ix)*\sin(2x)^2 + (-64Ix^3 + (-64Ix^3 + 96Ix)*\cos(4x) + (128Ix^3 - 192Ix)*\cos(2x) + 32*(2x^3 - 3x)*\sin(4x) - 64*(2x^3 - 3x)*\sin(2x) + 96Ix*\cos(6x) + (128Ix^3 + (-320Ix^3 + 480Ix)*\cos(2x) + 160*(2x^3 - 3x)*\sin(2x) - 192Ix)*\cos(4x) + (-64Ix^3 + 96Ix)*\cos(2x) + (64x^3 + 32*(2x^3 - 3x)*\cos(4x) - 64*(2x^3 - 3x)*\cos(2x) + (64Ix^3 - 96Ix)*\sin(4x) + (-128Ix^3 + 192Ix)*\sin(2x) - 96x)*\sin(6x) - (128x^3 + 128*(2x^3 - 3x)*\cos(4x) - 160*(2x^3 - 3x)*\cos(2x) - (320Ix^3 - 480Ix)*\sin(2x) - 192x)*\sin(4x) + 32*(2x^3 - 4*(2x^3 - 3x)*\cos(2x) - 3x)*\sin(2x))*\arctan2(\sin(x), \cos(x) + 1) - ((-128Ix^3 + 192Ix)*\cos(4x))^2 + (-128Ix^3 + 192Ix)*\cos(2x)^2 + (128Ix^3 - 192Ix)*\sin(4x)^2 + (128Ix^3 - 192Ix)*\sin(2x)^2 + (64Ix^3 + (64Ix^3 - 96Ix)*\cos(4x) + (-128Ix^3 + 192Ix)*\cos(2x) - 32*(2x^3 - 3x)*\sin(4x) + 64*(2x^3 - 3x)*\sin(2x) - 96Ix)*\cos(6x) + (-128Ix^3 + (320Ix^3 - 480Ix)*\cos(2x) - 160*(2x^3 - 3x)*\sin(2x) + 192Ix)*\cos(4x) + (64Ix^3 - 96Ix)*\cos(2x) - (64x^3 + 32*(2x^3 - 3x)*\cos(4x) - 64*(2x^3 - 3x)*\cos(2x) - (-64Ix^3 + 96Ix)*\sin(4x) - (128Ix^3 - 192Ix)*\sin(2x) - 96x)*\sin(6x) + (128x^3 + 128*(2x^3 - 3x)*\cos(4x) - 160*(2x^3 - 3x)*\cos(2x) + (-320Ix^3 + 480Ix)*\sin(2x) - 192x)*\sin(4x) - 32*(2x^3 - 4*(2x^3 - 3x)*\cos(2x) - 3x)*\sin(2x))*\arctan2(\sin(x), -\cos(x) + 1) - (16Ix^4 + 8x^3 - 12Ix^2 + (16Ix^4 + 16x^3 - 72Ix^2 - 24x - 12I)*\cos(4x) + (-32Ix^4 + 52x^3 + 90Ix^2 + 18x + 3I)*\cos(2x) - (16x^4 - 16Ix^3 - 72x^2 + 24Ix - 12)*\sin(4x) + (32x^4 + 52Ix^3 - 90x^2 + 18Ix - 3)*\sin(2x) - 12x + 6I)*\cos(6x) - (-32Ix^4 - 20x^3 + 30Ix^2 + (80Ix^4 - 104x^3 - 276Ix^2 - 36x - 6I)*\cos(2x) - (80x^4 + 104Ix^3 - 276x^2 + 36Ix - 6)*\sin(2x) + 30x - 15I)*\cos(4x) - (16Ix^4 + 16x^3 - 24Ix^2 - 24x + 12I)*\cos(2x) - ((-384Ix^2 + 192I)*$

$$\begin{aligned}
& \cos(4x)^2 + (-384Ix^2 + 192I)\cos(2x)^2 + (384Ix^2 - 192I)\sin(4x) \\
& ^2 + (384Ix^2 - 192I)\sin(2x)^2 + (192Ix^2 + (192Ix^2 - 96I)\cos(4 \\
& *x) + (-384Ix^2 + 192I)\cos(2x) - 96*(2x^2 - 1)\sin(4x) + 192*(2x^2 \\
& - 1)\sin(2x) - 96I)\cos(6x) + (-384Ix^2 + (960Ix^2 - 480I)\cos(2x) \\
& - 480*(2x^2 - 1)\sin(2x) + 192I)\cos(4x) + (192Ix^2 - 96I)\cos(2x) \\
& - (192x^2 + 96*(2x^2 - 1)\cos(4x) - 192*(2x^2 - 1)\cos(2x) - (-192Ix \\
& x^2 + 96I)\sin(4x) - (384Ix^2 - 192I)\sin(2x) - 96)\sin(6x) + (384x \\
& ^2 + 384*(2x^2 - 1)\cos(4x) - 480*(2x^2 - 1)\cos(2x) + (-960Ix^2 + 48 \\
& 0I)\sin(2x) - 192)\sin(4x) - 96*(2x^2 - 4*(2x^2 - 1)\cos(2x) - 1)\sin \\
& (2x))*\operatorname{dilog}(-e^{Ix}) - ((-384Ix^2 + 192I)\cos(4x)^2 + (-384Ix^2 + 1 \\
& 92I)\cos(2x)^2 + (384Ix^2 - 192I)\sin(4x)^2 + (384Ix^2 - 192I)\sin \\
& (2x)^2 + (192Ix^2 + (192Ix^2 - 96I)\cos(4x) + (-384Ix^2 + 192I)*c \\
& os(2x) - 96*(2x^2 - 1)\sin(4x) + 192*(2x^2 - 1)\sin(2x) - 96I)\cos(6 \\
& x) + (-384Ix^2 + (960Ix^2 - 480I)\cos(2x) - 480*(2x^2 - 1)\sin(2x) \\
& + 192I)\cos(4x) + (192Ix^2 - 96I)\cos(2x) - (192x^2 + 96*(2x^2 - 1) \\
& *\cos(4x) - 192*(2x^2 - 1)\cos(2x) - (-192Ix^2 + 96I)\sin(4x) - (384 \\
& Ix^2 - 192I)\sin(2x) - 96)\sin(6x) + (384x^2 + 384*(2x^2 - 1)\cos(4x \\
&) - 480*(2x^2 - 1)\cos(2x) + (-960Ix^2 + 480I)\sin(2x) - 192)\sin(4x \\
&) - 96*(2x^2 - 4*(2x^2 - 1)\cos(2x) - 1)\sin(2x))*\operatorname{dilog}(e^{Ix}) - (32* \\
& (2x^3 - 3x)\cos(4x)^2 + 32*(2x^3 - 3x)\cos(2x)^2 - 32*(2x^3 - 3x)*s \\
& in(4x)^2 - 32*(2x^3 - 3x)\sin(2x)^2 - (32x^3 + 16*(2x^3 - 3x)\cos(4 \\
& x) - 32*(2x^3 - 3x)\cos(2x) - (-32Ix^3 + 48Ix)\sin(4x) - (64Ix^3 \\
& - 96Ix)\sin(2x) - 48x)\cos(6x) + (64x^3 - 80*(2x^3 - 3x)\cos(2x) + \\
& (-160Ix^3 + 240Ix)\sin(2x) - 96x)\cos(4x) - 16*(2x^3 - 3x)\cos(2 \\
& x) + (-32Ix^3 + (-32Ix^3 + 48Ix)\cos(4x) + (64Ix^3 - 96Ix)\cos(2 \\
& *x) + 16*(2x^3 - 3x)\sin(4x) - 32*(2x^3 - 3x)\sin(2x) + 48Ix)\sin(6 \\
& *x) + (64Ix^3 + (128Ix^3 - 192Ix)\cos(4x) + (-160Ix^3 + 240Ix)*c \\
& os(2x) + 80*(2x^3 - 3x)\sin(2x) - 96Ix)\sin(4x) + (-32Ix^3 + (128 \\
& Ix^3 - 192Ix)\cos(2x) + 48Ix)\sin(2x))*\log(\cos(x)^2 + \sin(x)^2 + 2*c \\
& os(x) + 1) - (32*(2x^3 - 3x)\cos(4x)^2 + 32*(2x^3 - 3x)\cos(2x)^2 - 3 \\
& 2*(2x^3 - 3x)\sin(4x)^2 - 32*(2x^3 - 3x)\sin(2x)^2 - (32x^3 + 16*(2 \\
& x^3 - 3x)\cos(4x) - 32*(2x^3 - 3x)\cos(2x) - (-32Ix^3 + 48Ix)\sin(\\
& 4x) - (64Ix^3 - 96Ix)\sin(2x) - 48x)\cos(6x) + (64x^3 - 80*(2x^3 \\
& - 3x)\cos(2x) + (-160Ix^3 + 240Ix)\sin(2x) - 96x)\cos(4x) - 16*(2 \\
& x^3 - 3x)\cos(2x) + (-32Ix^3 + (-32Ix^3 + 48Ix)\cos(4x) + (64Ix^3 \\
& - 96Ix)\cos(2x) + 16*(2x^3 - 3x)\sin(4x) - 32*(2x^3 - 3x)\sin(2x \\
&) + 48Ix)\sin(6x) + (64Ix^3 + (128Ix^3 - 192Ix)\cos(4x) + (-160I \\
& x^3 + 240Ix)\cos(2x) + 80*(2x^3 - 3x)\sin(2x) - 96Ix)\sin(4x) + (\\
& -32Ix^3 + (128Ix^3 - 192Ix)\cos(2x) + 48Ix)\sin(2x))*\log(\cos(x)^2 \\
& + \sin(x)^2 - 2*\cos(x) + 1) - ((-384I*\cos(4x) + 768I*\cos(2x) + 384*\sin(\\
& 4x) - 768*\sin(2x) - 384I)\cos(6x) + (-1920I*\cos(2x) + 1920*\sin(2x) + \\
& 768I)\cos(4x) + 768I*\cos(4x)^2 + 768I*\cos(2x)^2 + (384*\cos(4x) - 76 \\
& 8*\cos(2x) + 384I*\sin(4x) - 768I*\sin(2x) + 384)\sin(6x) - 384*(4*\cos(4 \\
& *x) - 5*\cos(2x) - 5I*\sin(2x) + 2)*\sin(4x) - 768I*\sin(4x)^2 - 384*(4*c \\
& os(2x) - 1)\sin(2x) - 768I*\sin(2x)^2 - 384I*\cos(2x))*\operatorname{polylog}(4, -e^{I \\
& *x}) - ((-384I*\cos(4x) + 768I*\cos(2x) + 384*\sin(4x) - 768*\sin(2x) - 3 \\
& 84I)\cos(6x) + (-1920I*\cos(2x) + 1920*\sin(2x) + 768I)\cos(4x) + 768 \\
& I*\cos(4x)^2 + 768I*\cos(2x)^2 + (384*\cos(4x) - 768*\cos(2x) + 384I*\sin(\\
& 4x) - 768I*\sin(2x) + 384)\sin(6x) - 384*(4*\cos(4x) - 5*\cos(2x) - 5I* \\
& sin(2x) + 2)*\sin(4x) - 768I*\sin(4x)^2 - 384*(4*\cos(2x) - 1)\sin(2x) - \\
& 768I*\sin(2x)^2 - 384I*\cos(2x))*\operatorname{polylog}(4, e^{Ix}) - (768*x*\cos(4x)^2 \\
& + 768*x*\cos(2x)^2 - 768*x*\sin(4x)^2 - 768*x*\sin(2x)^2 - (384*x*\cos(4x) \\
& - 768*x*\cos(2x) + 384Ix*\sin(4x) - 768Ix*\sin(2x) + 384x)\cos(6x) - \\
& (1920*x*\cos(2x) + 1920Ix*\sin(2x) - 768x)\cos(4x) - 384x*\cos(2x) + \\
& (-384Ix*\cos(4x) + 768Ix*\cos(2x) + 384x*\sin(4x) - 768x*\sin(2x) - 3 \\
& 84Ix)\sin(6x) + (1536Ix*\cos(4x) - 1920Ix*\cos(2x) + 1920x*\sin(2x) \\
& + 768Ix)\sin(4x) + (1536Ix*\cos(2x) - 384Ix)\sin(2x))*\operatorname{polylog}(3, - \\
& e^{Ix}) - (768*x*\cos(4x)^2 + 768*x*\cos(2x)^2 - 768*x*\sin(4x)^2 - 768*x* \\
& \sin(2x)^2 - (384*x*\cos(4x) - 768*x*\cos(2x) + 384Ix*\sin(4x) - 768Ix*
\end{aligned}$$

```

sin(2*x) + 384*x*cos(6*x) - (1920*x*cos(2*x) + 1920*I*x*sin(2*x) - 768*x)*
cos(4*x) - 384*x*cos(2*x) + (-384*I*x*cos(4*x) + 768*I*x*cos(2*x) + 384*x*s
in(4*x) - 768*x*sin(2*x) - 384*I*x)*sin(6*x) + (1536*I*x*cos(4*x) - 1920*I*
x*cos(2*x) + 1920*x*sin(2*x) + 768*I*x)*sin(4*x) + (1536*I*x*cos(2*x) - 384
*I*x)*sin(2*x))*polylog(3, e^(I*x)) + (16*x^4 - 8*I*x^3 - 12*x^2 - (-8*I*x^
3 + 12*x^2 + 12*I*x - 6)*cos(6*x) + (16*x^4 - 16*I*x^3 - 72*x^2 + 24*I*x -
12)*cos(4*x) - (32*x^4 + 52*I*x^3 - 90*x^2 + 18*I*x - 3)*cos(2*x) - (-16*I*
x^4 - 16*x^3 + 72*I*x^2 + 24*x + 12*I)*sin(4*x) - (32*I*x^4 - 52*x^3 - 90*I
*x^2 - 18*x - 3*I)*sin(2*x) + 12*I*x + 6)*sin(6*x) - (32*x^4 - 20*I*x^3 - 3
0*x^2 + (64*x^4 - 32*I*x^3 - 336*x^2 + 48*I*x - 24)*cos(4*x) - (80*x^4 + 10
4*I*x^3 - 276*x^2 + 36*I*x - 6)*cos(2*x) + (-80*I*x^4 + 104*x^3 + 276*I*x^2
+ 36*x + 6*I)*sin(2*x) + 30*I*x + 15)*sin(4*x) + (16*x^4 - 16*I*x^3 - 24*x
^2 - (64*x^4 + 112*I*x^3 - 192*x^2 + 24*I*x)*cos(2*x) + 24*I*x + 12)*sin(2*
x) - 6*x + 3*I)/((32*cos(4*x) - 64*cos(2*x) + 32*I*sin(4*x) - 64*I*sin(2*x)
+ 32)*cos(6*x) + 32*(5*cos(2*x) + 5*I*sin(2*x) - 2)*cos(4*x) - 64*cos(4*x)
^2 - 64*cos(2*x)^2 - (-32*I*cos(4*x) + 64*I*cos(2*x) + 32*sin(4*x) - 64*sin
(2*x) - 32*I)*sin(6*x) - (128*I*cos(4*x) - 160*I*cos(2*x) + 160*sin(2*x) +
64*I)*sin(4*x) + 64*sin(4*x)^2 - (128*I*cos(2*x) - 32*I)*sin(2*x) + 64*sin(
2*x)^2 + 32*cos(2*x))

```

Fricas [C] time = 0.643887, size = 1566, normalized size = 8.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cos(x)^2*cot(x)^3,x, algorithm="fricas")

```

[Out] -1/8*(2*(2*x^3 - 3*x)*cos(x)^4 - 2*x^3 - 3*(2*x^3 - 3*x)*cos(x)^2 - ((24*I*
x^2 - 12*I)*cos(x)^2 - 24*I*x^2 + 12*I)*dilog(cos(x) + I*sin(x)) - ((-24*I*
x^2 + 12*I)*cos(x)^2 + 24*I*x^2 - 12*I)*dilog(cos(x) - I*sin(x)) - ((-24*I*
x^2 + 12*I)*cos(x)^2 + 24*I*x^2 - 12*I)*dilog(-cos(x) + I*sin(x)) - ((24*I*
x^2 - 12*I)*cos(x)^2 - 24*I*x^2 + 12*I)*dilog(-cos(x) - I*sin(x)) - 4*(2*x^
3 - (2*x^3 - 3*x)*cos(x)^2 - 3*x)*log(cos(x) + I*sin(x) + 1) - 4*(2*x^3 - (
2*x^3 - 3*x)*cos(x)^2 - 3*x)*log(cos(x) - I*sin(x) + 1) - 4*(2*x^3 - (2*x^3
- 3*x)*cos(x)^2 - 3*x)*log(-cos(x) + I*sin(x) + 1) - 4*(2*x^3 - (2*x^3 - 3
*x)*cos(x)^2 - 3*x)*log(-cos(x) - I*sin(x) + 1) - (-48*I*cos(x)^2 + 48*I)*p
olylog(4, cos(x) + I*sin(x)) - (48*I*cos(x)^2 - 48*I)*polylog(4, cos(x) - I
*sin(x)) - (48*I*cos(x)^2 - 48*I)*polylog(4, -cos(x) + I*sin(x)) - (-48*I*c
os(x)^2 + 48*I)*polylog(4, -cos(x) - I*sin(x)) + 48*(x*cos(x)^2 - x)*polylo
g(3, cos(x) + I*sin(x)) + 48*(x*cos(x)^2 - x)*polylog(3, cos(x) - I*sin(x))
+ 48*(x*cos(x)^2 - x)*polylog(3, -cos(x) + I*sin(x)) + 48*(x*cos(x)^2 - x)
*polylog(3, -cos(x) - I*sin(x)) - 3*((2*x^2 - 1)*cos(x)^3 + (2*x^2 + 1)*cos
(x))*sin(x) - 3*x)/(cos(x)^2 - 1)

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*cos(x)**2*cot(x)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cos(x)^2 \cot(x)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*cos(x)^2*cot(x)^3, x)
```

3.206 $\int x^2 \cos^2(x) \cot^3(x) dx$

Optimal. Leaf size=106

$$2ix \operatorname{PolyLog}(2, e^{2ix}) - \operatorname{PolyLog}(3, e^{2ix}) + \frac{2ix^3}{3} - \frac{3x^2}{4} - 2x^2 \log(1 - e^{2ix}) + \frac{1}{2}x^2 \sin^2(x) - \frac{1}{2}x^2 \cot^2(x) - \frac{\sin^2(x)}{4} - x$$

```
[Out] (-3*x^2)/4 + ((2*I)/3)*x^3 - x*Cot[x] - (x^2*Cot[x]^2)/2 - 2*x^2*Log[1 - E^
((2*I)*x)] + Log[Sin[x]] + (2*I)*x*PolyLog[2, E^((2*I)*x)] - PolyLog[3, E^
(2*I)*x] + (x*Cos[x]*Sin[x])/2 - Sin[x]^2/4 + (x^2*Sin[x]^2)/2
```

Rubi [A] time = 0.278309, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {4408, 3443, 3310, 30, 3717, 2190, 2531, 2282, 6589, 3720, 3475}

$$2ix \operatorname{PolyLog}(2, e^{2ix}) - \operatorname{PolyLog}(3, e^{2ix}) + \frac{2ix^3}{3} - \frac{3x^2}{4} - 2x^2 \log(1 - e^{2ix}) + \frac{1}{2}x^2 \sin^2(x) - \frac{1}{2}x^2 \cot^2(x) - \frac{\sin^2(x)}{4} - x$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (-3*x^2)/4 + ((2*I)/3)*x^3 - x*Cot[x] - (x^2*Cot[x]^2)/2 - 2*x^2*Log[1 - E^
((2*I)*x)] + Log[Sin[x]] + (2*I)*x*PolyLog[2, E^((2*I)*x)] - PolyLog[3, E^
(2*I)*x] + (x*Cos[x]*Sin[x])/2 - Sin[x]^2/4 + (x^2*Sin[x]^2)/2
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] := -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] := Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=>
Simp[(d*(b*Sine + f*x))^n/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sine + f*x)^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sine + f*x)^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :=> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :=> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[(c + d*x)^
```

```
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*(a_) + (b_)*(x_)))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3720

```
Int[(((c_) + (d_)*(x_))^(m_))*((b_)*tan[(e_) + (f_)*(x_)]^(n_)), x_Symb
ol] := Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_) + (d_)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^2 \cos^2(x) \cot^3(x) dx &= - \int x^2 \cos^2(x) \cot(x) dx + \int x^2 \cot^3(x) dx \\
&= -\frac{1}{2}x^2 \cot^2(x) - 2 \int x^2 \cot(x) dx + \int x \cot^2(x) dx + \int x^2 \cos(x) \sin(x) dx \\
&= -x \cot(x) - \frac{1}{2}x^2 \cot^2(x) + \frac{1}{2}x^2 \sin^2(x) - 2 \left(-\frac{ix^3}{3} - 2i \int \frac{e^{2ix}x^2}{1-e^{2ix}} dx \right) - \int x dx + \int \cot(x) dx \\
&= -\frac{x^2}{2} - x \cot(x) - \frac{1}{2}x^2 \cot^2(x) + \log(\sin(x)) + \frac{1}{2}x \cos(x) \sin(x) - \frac{\sin^2(x)}{4} + \frac{1}{2}x^2 \sin^2(x) - \frac{1}{2}x^2 \\
&= -\frac{3x^2}{4} - x \cot(x) - \frac{1}{2}x^2 \cot^2(x) + \log(\sin(x)) + \frac{1}{2}x \cos(x) \sin(x) - \frac{\sin^2(x)}{4} + \frac{1}{2}x^2 \sin^2(x) - \frac{1}{2}x^2 \\
&= -\frac{3x^2}{4} - x \cot(x) - \frac{1}{2}x^2 \cot^2(x) + \log(\sin(x)) + \frac{1}{2}x \cos(x) \sin(x) - \frac{\sin^2(x)}{4} + \frac{1}{2}x^2 \sin^2(x) - \frac{1}{2}x^2 \\
&= -\frac{3x^2}{4} - x \cot(x) - \frac{1}{2}x^2 \cot^2(x) + \log(\sin(x)) - 2 \left(-\frac{ix^3}{3} + x^2 \log(1-e^{2ix}) - ix \text{Li}_2(e^{2ix}) \right) + \frac{1}{2}x^2
\end{aligned}$$

Mathematica [A] time = 0.323761, size = 108, normalized size = 1.02

$$-2ix \text{PolyLog}(2, e^{-2ix}) - \text{PolyLog}(3, e^{-2ix}) - \frac{2ix^3}{3} - 2x^2 \log(1 - e^{-2ix}) - \frac{1}{4}x^2 \cos(2x) - \frac{1}{2}x^2 \csc^2(x) + \frac{1}{4}x \sin(2x) +$$

Antiderivative was successfully verified.

[In] Integrate[x^2*cos[x]^2*cot[x]^3,x]

[Out] (I/12)*Pi^3 - ((2*I)/3)*x^3 + Cos[2*x]/8 - (x^2*cos[2*x])/4 - x*cot[x] - (x^2*csc[x]^2)/2 - 2*x^2*Log[1 - E^((-2*I)*x)] + Log[Sin[x]] - (2*I)*x*PolyLog[2, E^((-2*I)*x)] - PolyLog[3, E^((-2*I)*x)] + (x*Sin[2*x])/4

Maple [A] time = 0.224, size = 170, normalized size = 1.6

$$\frac{2i}{3}x^3 - \frac{(2ix + 2x^2 - 1)e^{2ix}}{16} - \frac{(-2ix + 2x^2 - 1)e^{-2ix}}{16} + 2 \frac{x(xe^{2ix} - ie^{2ix} + i)}{(e^{2ix} - 1)^2} + \ln(e^{ix} - 1) - 2 \ln(e^{ix}) + \ln(1 + e^{ix})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cos(x)^2*cot(x)^3,x)

[Out] 2/3*I*x^3-1/16*(2*I*x+2*x^2-1)*exp(2*I*x)-1/16*(-2*I*x+2*x^2-1)*exp(-2*I*x)+2*x*(x*exp(2*I*x)-I*exp(2*I*x)+I)/(exp(2*I*x)-1)^2+ln(exp(I*x)-1)-2*ln(exp(I*x))+ln(1+exp(I*x))-2*x^2*ln(1-exp(I*x))+4*I*x*polylog(2,exp(I*x))-4*polylog(3,exp(I*x))-2*x^2*ln(1+exp(I*x))+4*I*x*polylog(2,-exp(I*x))-4*polylog(3,-exp(I*x))

Maxima [B] time = 2.63055, size = 3856, normalized size = 36.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cos(x)^2*cot(x)^3,x, algorithm="maxima")

```

[Out] -(3*(2*x^2 + 2*I*x - 1)*cos(6*x)^2 - (-64*I*x^3 - 24*x^2 + 168*I*x + 12)*co
s(4*x)^2 - (-64*I*x^3 + 84*x^2 + 96*I*x + 6)*cos(2*x)^2 - 3*(2*x^2 + 2*I*x
- 1)*sin(6*x)^2 - (64*I*x^3 + 24*x^2 - 168*I*x - 12)*sin(4*x)^2 - (64*I*x^3
- 84*x^2 - 96*I*x - 6)*sin(2*x)^2 + 6*x^2 - ((192*I*x^2 - 96*I)*cos(4*x)^2
+ (192*I*x^2 - 96*I)*cos(2*x)^2 + (-192*I*x^2 + 96*I)*sin(4*x)^2 + (-192*I
*x^2 + 96*I)*sin(2*x)^2 + (-96*I*x^2 + (-96*I*x^2 + 48*I)*cos(4*x) + (192*I
*x^2 - 96*I)*cos(2*x) + 48*(2*x^2 - 1)*sin(4*x) - 96*(2*x^2 - 1)*sin(2*x) +
48*I)*cos(6*x) + (192*I*x^2 + (-480*I*x^2 + 240*I)*cos(2*x) + 240*(2*x^2 -
1)*sin(2*x) - 96*I)*cos(4*x) + (-96*I*x^2 + 48*I)*cos(2*x) + (96*x^2 + 48*
(2*x^2 - 1)*cos(4*x) - 96*(2*x^2 - 1)*cos(2*x) + (96*I*x^2 - 48*I)*sin(4*x)
+ (-192*I*x^2 + 96*I)*sin(2*x) - 48)*sin(6*x) - (192*x^2 + 192*(2*x^2 - 1)
*cos(4*x) - 240*(2*x^2 - 1)*cos(2*x) - (480*I*x^2 - 240*I)*sin(2*x) - 96)*s
in(4*x) + 48*(2*x^2 - 4*(2*x^2 - 1)*cos(2*x) - 1)*sin(2*x))*arctan2(sin(x),
cos(x) + 1) - ((48*I*cos(4*x) - 96*I*cos(2*x) - 48*sin(4*x) + 96*sin(2*x)
+ 48*I)*cos(6*x) + (240*I*cos(2*x) - 240*sin(2*x) - 96*I)*cos(4*x) - 96*I*c
os(4*x)^2 - 96*I*cos(2*x)^2 - (48*cos(4*x) - 96*cos(2*x) + 48*I*sin(4*x) -
96*I*sin(2*x) + 48)*sin(6*x) + 48*(4*cos(4*x) - 5*cos(2*x) - 5*I*sin(2*x) +
2)*sin(4*x) + 96*I*sin(4*x)^2 + 48*(4*cos(2*x) - 1)*sin(2*x) + 96*I*sin(2*
x)^2 + 48*I*cos(2*x))*arctan2(sin(x), cos(x) - 1) - (-192*I*x^2*cos(4*x)^2
- 192*I*x^2*cos(2*x)^2 + 192*I*x^2*sin(4*x)^2 + 192*I*x^2*sin(2*x)^2 + 96*I
*x^2*cos(2*x) + (96*I*x^2*cos(4*x) - 192*I*x^2*cos(2*x) - 96*x^2*sin(4*x) +
192*x^2*sin(2*x) + 96*I*x^2)*cos(6*x) + (480*I*x^2*cos(2*x) - 480*x^2*sin(
2*x) - 192*I*x^2)*cos(4*x) - (96*x^2*cos(4*x) - 192*x^2*cos(2*x) + 96*I*x^2
*sin(4*x) - 192*I*x^2*sin(2*x) + 96*x^2)*sin(6*x) + (384*x^2*cos(4*x) - 480
*x^2*cos(2*x) - 480*I*x^2*sin(2*x) + 192*x^2)*sin(4*x) + 96*(4*x^2*cos(2*x)
- x^2)*sin(2*x))*arctan2(sin(x), -cos(x) + 1) - (32*I*x^3 + 12*x^2 + (32*I
*x^3 + 24*x^2 - 72*I*x - 12)*cos(4*x) + (-64*I*x^3 + 78*x^2 + 90*I*x + 9)*c
os(2*x) - (32*x^3 - 24*I*x^2 - 72*x + 12*I)*sin(4*x) + (64*x^3 + 78*I*x^2 -
90*x + 9*I)*sin(2*x) - 12*I*x - 6)*cos(6*x) - (-64*I*x^3 - 30*x^2 + (160*I
*x^3 - 156*x^2 - 276*I*x - 18)*cos(2*x) - (160*x^3 + 156*I*x^2 - 276*x + 18
*I)*sin(2*x) + 30*I*x + 15)*cos(4*x) - (32*I*x^3 + 24*x^2 - 24*I*x - 12)*co
s(2*x) - (-384*I*x*cos(4*x)^2 - 384*I*x*cos(2*x)^2 + 384*I*x*sin(4*x)^2 + 3
84*I*x*sin(2*x)^2 + (192*I*x*cos(4*x) - 384*I*x*cos(2*x) - 192*x*sin(4*x) +
384*x*sin(2*x) + 192*I*x)*cos(6*x) + (960*I*x*cos(2*x) - 960*x*sin(2*x) -
384*I*x)*cos(4*x) + 192*I*x*cos(2*x) - (192*x*cos(4*x) - 384*x*cos(2*x) + 1
92*I*x*sin(4*x) - 384*I*x*sin(2*x) + 192*x)*sin(6*x) + (768*x*cos(4*x) - 96
0*x*cos(2*x) - 960*I*x*sin(2*x) + 384*x)*sin(4*x) + 192*(4*x*cos(2*x) - x)*
sin(2*x))*dilog(-e^(I*x)) - (-384*I*x*cos(4*x)^2 - 384*I*x*cos(2*x)^2 + 384
*I*x*sin(4*x)^2 + 384*I*x*sin(2*x)^2 + (192*I*x*cos(4*x) - 384*I*x*cos(2*x)
- 192*x*sin(4*x) + 384*x*sin(2*x) + 192*I*x)*cos(6*x) + (960*I*x*cos(2*x)
- 960*x*sin(2*x) - 384*I*x)*cos(4*x) + 192*I*x*cos(2*x) - (192*x*cos(4*x) -
384*x*cos(2*x) + 192*I*x*sin(4*x) - 384*I*x*sin(2*x) + 192*x)*sin(6*x) + (
768*x*cos(4*x) - 960*x*cos(2*x) - 960*I*x*sin(2*x) + 384*x)*sin(4*x) + 192*
(4*x*cos(2*x) - x)*sin(2*x))*dilog(e^(I*x)) - (48*(2*x^2 - 1)*cos(4*x)^2 +
48*(2*x^2 - 1)*cos(2*x)^2 - 48*(2*x^2 - 1)*sin(4*x)^2 - 48*(2*x^2 - 1)*sin(
2*x)^2 - (48*x^2 + 24*(2*x^2 - 1)*cos(4*x) - 48*(2*x^2 - 1)*cos(2*x) - (-48
*I*x^2 + 24*I)*sin(4*x) - (96*I*x^2 - 48*I)*sin(2*x) - 24)*cos(6*x) + (96*x
^2 - 120*(2*x^2 - 1)*cos(2*x) + (-240*I*x^2 + 120*I)*sin(2*x) - 48)*cos(4*x
) - 24*(2*x^2 - 1)*cos(2*x) + (-48*I*x^2 + (-48*I*x^2 + 24*I)*cos(4*x) + (9
6*I*x^2 - 48*I)*cos(2*x) + 24*(2*x^2 - 1)*sin(4*x) - 48*(2*x^2 - 1)*sin(2*x
) + 24*I)*sin(6*x) + (96*I*x^2 + (192*I*x^2 - 96*I)*cos(4*x) + (-240*I*x^2
+ 120*I)*cos(2*x) + 120*(2*x^2 - 1)*sin(2*x) - 48*I)*sin(4*x) + (-48*I*x^2
+ (192*I*x^2 - 96*I)*cos(2*x) + 24*I)*sin(2*x))*log(cos(x)^2 + sin(x)^2 + 2
*cos(x) + 1) - (48*(2*x^2 - 1)*cos(4*x)^2 + 48*(2*x^2 - 1)*cos(2*x)^2 - 48*
(2*x^2 - 1)*sin(4*x)^2 - 48*(2*x^2 - 1)*sin(2*x)^2 - (48*x^2 + 24*(2*x^2 -
1)*cos(4*x) - 48*(2*x^2 - 1)*cos(2*x) - (-48*I*x^2 + 24*I)*sin(4*x) - (96*I
*x^2 - 48*I)*sin(2*x) - 24)*cos(6*x) + (96*x^2 - 120*(2*x^2 - 1)*cos(2*x) +
(-240*I*x^2 + 120*I)*sin(2*x) - 48)*cos(4*x) - 24*(2*x^2 - 1)*cos(2*x) + (
-48*I*x^2 + (-48*I*x^2 + 24*I)*cos(4*x) + (96*I*x^2 - 48*I)*cos(2*x) + 24*(

```

$$\begin{aligned}
& 2x^2 - 1) \sin(4x) - 48(2x^2 - 1) \sin(2x) + 24I \sin(6x) + (96Ix^2 \\
& + (192Ix^2 - 96I) \cos(4x) + (-240Ix^2 + 120I) \cos(2x) + 120(2x^2 \\
& - 1) \sin(2x) - 48I \sin(4x) + (-48Ix^2 + (192Ix^2 - 96I) \cos(2x) + \\
& 24I) \sin(2x)) \log(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1) + ((192\cos(4x) - \\
& 384\cos(2x) + 192I \sin(4x) - 384I \sin(2x) + 192) \cos(6x) + 192(5\cos \\
& (2x) + 5I \sin(2x) - 2) \cos(4x) - 384\cos(4x)^2 - 384\cos(2x)^2 - (-1 \\
& 92I \cos(4x) + 384I \cos(2x) + 192\sin(4x) - 384\sin(2x) - 192I) \sin(6 \\
& x) - (768I \cos(4x) - 960I \cos(2x) + 960\sin(2x) + 384I) \sin(4x) + 3 \\
& 84\sin(4x)^2 - (768I \cos(2x) - 192I) \sin(2x) + 384\sin(2x)^2 + 192\cos \\
& (2x)) \operatorname{polylog}(3, -e^{Ix}) + ((192\cos(4x) - 384\cos(2x) + 192I \sin(4x) \\
& - 384I \sin(2x) + 192) \cos(6x) + 192(5\cos(2x) + 5I \sin(2x) - 2) \cos \\
& (4x) - 384\cos(4x)^2 - 384\cos(2x)^2 - (-192I \cos(4x) + 384I \cos(2x) \\
& + 192\sin(4x) - 384\sin(2x) - 192I) \sin(6x) - (768I \cos(4x) - 960I \\
& I \cos(2x) + 960\sin(2x) + 384I) \sin(4x) + 384\sin(4x)^2 - (768I \cos(2x) \\
& - 192I) \sin(2x) + 384\sin(2x)^2 + 192\cos(2x)) \operatorname{polylog}(3, e^{Ix}) \\
& + (32x^3 - 12Ix^2 - (-12Ix^2 + 12x + 6I) \cos(6x) + (32x^3 - 24Ix^2 \\
& ^2 - 72x + 12I) \cos(4x) - (64x^3 + 78Ix^2 - 90x + 9I) \cos(2x) - (- \\
& 32Ix^3 - 24x^2 + 72Ix + 12) \sin(4x) - (64Ix^3 - 78x^2 - 90Ix - 9 \\
&) \sin(2x) - 12x + 6I) \sin(6x) - (64x^3 - 30Ix^2 + (128x^3 - 48Ix^2 \\
& 2 - 336x + 24I) \cos(4x) - (160x^3 + 156Ix^2 - 276x + 18I) \cos(2x) \\
& + (-160Ix^3 + 156x^2 + 276Ix + 18) \sin(2x) - 30x + 15I) \sin(4x) + \\
& (32x^3 - 24Ix^2 - (128x^3 + 168Ix^2 - 192x + 12I) \cos(2x) - 24x + \\
& 12I) \sin(2x) - 6Ix - 3) / ((48\cos(4x) - 96\cos(2x) + 48I \sin(4x) - \\
& 96I \sin(2x) + 48) \cos(6x) + 48(5\cos(2x) + 5I \sin(2x) - 2) \cos(4x) \\
& - 96\cos(4x)^2 - 96\cos(2x)^2 - (-48I \cos(4x) + 96I \cos(2x) + 48\sin(4x) \\
& - 96\sin(2x) - 48I) \sin(6x) - (192I \cos(4x) - 240I \cos(2x) + 24 \\
& 0\sin(2x) + 96I) \sin(4x) + 96\sin(4x)^2 - (192I \cos(2x) - 48I) \sin(2x) \\
& + 96\sin(2x)^2 + 48\cos(2x))
\end{aligned}$$

Fricas [C] time = 0.613128, size = 1204, normalized size = 11.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cos(x)^2*cot(x)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned}
& -1/8*(2*(2x^2 - 1)\cos(x)^4 - 3*(2x^2 - 1)\cos(x)^2 - 2x^2 - (16Ix*\cos \\
& (x)^2 - 16Ix)*\operatorname{dilog}(\cos(x) + I\sin(x)) - (-16Ix*\cos(x)^2 + 16Ix)*\operatorname{dilog} \\
& (\cos(x) - I\sin(x)) - (-16Ix*\cos(x)^2 + 16Ix)*\operatorname{dilog}(-\cos(x) + I\sin(x) \\
&) - (16Ix*\cos(x)^2 - 16Ix)*\operatorname{dilog}(-\cos(x) - I\sin(x)) + 4*((2x^2 - 1)*\cos \\
& (x)^2 - 2x^2 + 1)\log(\cos(x) + I\sin(x) + 1) + 4*((2x^2 - 1)\cos(x)^2 - \\
& 2x^2 + 1)\log(\cos(x) - I\sin(x) + 1) - 4*(\cos(x)^2 - 1)\log(-1/2\cos(x) + \\
& 1/2I\sin(x) + 1/2) - 4*(\cos(x)^2 - 1)\log(-1/2\cos(x) - 1/2I\sin(x) + 1/ \\
& 2) + 8*(x^2\cos(x)^2 - x^2)\log(-\cos(x) + I\sin(x) + 1) + 8*(x^2\cos(x)^2 - \\
& x^2)\log(-\cos(x) - I\sin(x) + 1) + 16*(\cos(x)^2 - 1)\operatorname{polylog}(3, \cos(x) + I \\
& * \sin(x)) + 16*(\cos(x)^2 - 1)\operatorname{polylog}(3, \cos(x) - I\sin(x)) + 16*(\cos(x)^2 - \\
& 1)\operatorname{polylog}(3, -\cos(x) + I\sin(x)) + 16*(\cos(x)^2 - 1)\operatorname{polylog}(3, -\cos(x) - \\
& I\sin(x)) - 4*(x*\cos(x)^3 + x*\cos(x))*\sin(x) - 1)/(\cos(x)^2 - 1)
\end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cos^2(x) \cot^3(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(x)**2*cot(x)**3,x)
```

```
[Out] Integral(x**2*cos(x)**2*cot(x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cos(x)^2 \cot(x)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*cos(x)^2*cot(x)^3, x)
```

3.207 $\int x \cos^2(x) \cot^3(x) dx$

Optimal. Leaf size=73

$$i\text{PolyLog}(2, e^{2ix}) + ix^2 - \frac{3x}{4} - 2x \log(1 - e^{2ix}) + \frac{1}{2}x \sin^2(x) - \frac{1}{2}x \cot^2(x) - \frac{\cot(x)}{2} + \frac{1}{4} \sin(x) \cos(x)$$

[Out] $(-3*x)/4 + I*x^2 - \text{Cot}[x]/2 - (x*\text{Cot}[x]^2)/2 - 2*x*\text{Log}[1 - E^{((2*I)*x)}] + I$
 $*\text{PolyLog}[2, E^{((2*I)*x)}] + (\text{Cos}[x]*\text{Sin}[x])/4 + (x*\text{Sin}[x]^2)/2$

Rubi [A] time = 0.163592, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.$, Rules used = {4408, 3443, 2635, 8, 3717, 2190, 2279, 2391, 3720, 3473}

$$i\text{PolyLog}(2, e^{2ix}) + ix^2 - \frac{3x}{4} - 2x \log(1 - e^{2ix}) + \frac{1}{2}x \sin^2(x) - \frac{1}{2}x \cot^2(x) - \frac{\cot(x)}{2} + \frac{1}{4} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{Cos}[x]^2*\text{Cot}[x]^3, x]$

[Out] $(-3*x)/4 + I*x^2 - \text{Cot}[x]/2 - (x*\text{Cot}[x]^2)/2 - 2*x*\text{Log}[1 - E^{((2*I)*x)}] + I$
 $*\text{PolyLog}[2, E^{((2*I)*x)}] + (\text{Cos}[x]*\text{Sin}[x])/4 + (x*\text{Sin}[x]^2)/2$

Rule 4408

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(n_.)}*\text{Cot}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] := -\text{Int}[(c + d*x)^m*\text{Cos}[a + b*x]^n*\text{Cot}[a + b*x]^{(p - 2)}, x] + \text{Int}[(c + d*x)^m*\text{Cos}[a + b*x]^{(n - 2)}*\text{Cot}[a + b*x]^p, x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3443

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(n_.)}*(x_.)^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)]^{(p_.)}, x_Symbol] := \text{Simp}[(x^{(m - n + 1)}*\text{Sin}[a + b*x]^n)^{(p + 1)} / (b*n*(p + 1)), x] - \text{Dist}[(m - n + 1) / (b*n*(p + 1)), \text{Int}[x^{(m - n)}*\text{Sin}[a + b*x]^n]^{(p + 1)}, x] /;$ FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]

Rule 2635

$\text{Int}[(b_.)*\text{sin}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] := -\text{Simp}[(b*\text{Cos}[c + d*x] * (b*\text{Sin}[c + d*x])^{(n - 1)}) / (d*n), x] + \text{Dist}[(b^2*(n - 1)) / n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n - 2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

$\text{Int}[a_, x_Symbol] := \text{Simp}[a*x, x] /;$ FreeQ[a, x]

Rule 3717

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\text{tan}[(e_.) + \text{Pi}*(k_.) + (f_.)*(x_.)], x_Symbol] := \text{Simp}[(I*(c + d*x)^{(m + 1)}) / (d*(m + 1)), x] - \text{Dist}[2*I, \text{Int}[(c + d*x)^m * E^{(2*I*k*Pi)} * E^{(2*I*(e + f*x))} / (1 + E^{(2*I*k*Pi)} * E^{(2*I*(e + f*x))}), x], x] /;$ FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3720

```
Int[((c_) + (d_)*(x_))^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symb
ol] := Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
x] - Dist[b^2, Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3473

```
Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int x \cos^2(x) \cot^3(x) dx &= -\int x \cos^2(x) \cot(x) dx + \int x \cot^3(x) dx \\
&= -\frac{1}{2}x \cot^2(x) + \frac{1}{2} \int \cot^2(x) dx - 2 \int x \cot(x) dx + \int x \cos(x) \sin(x) dx \\
&= -\frac{\cot(x)}{2} - \frac{1}{2}x \cot^2(x) + \frac{1}{2}x \sin^2(x) - 2 \left(-\frac{ix^2}{2} - 2i \int \frac{e^{2ix}}{1 - e^{2ix}} dx \right) - \frac{\int 1 dx}{2} - \frac{1}{2} \int \sin^2(x) dx \\
&= -\frac{x}{2} - \frac{\cot(x)}{2} - \frac{1}{2}x \cot^2(x) + \frac{1}{4} \cos(x) \sin(x) + \frac{1}{2}x \sin^2(x) - \frac{\int 1 dx}{4} - 2 \left(-\frac{ix^2}{2} + x \log(1 - e^{2ix}) \right) \\
&= -\frac{3x}{4} - \frac{\cot(x)}{2} - \frac{1}{2}x \cot^2(x) + \frac{1}{4} \cos(x) \sin(x) + \frac{1}{2}x \sin^2(x) - 2 \left(-\frac{ix^2}{2} + x \log(1 - e^{2ix}) + \frac{1}{2}i \operatorname{Su} \right) \\
&= -\frac{3x}{4} - \frac{\cot(x)}{2} - \frac{1}{2}x \cot^2(x) - 2 \left(-\frac{ix^2}{2} + x \log(1 - e^{2ix}) - \frac{1}{2}i \operatorname{Li}_2(e^{2ix}) \right) + \frac{1}{4} \cos(x) \sin(x) + \frac{1}{2}x \sin^2(x)
\end{aligned}$$

Mathematica [A] time = 0.107574, size = 62, normalized size = 0.85

$$\frac{1}{8} \left(8i \operatorname{PolyLog}(2, e^{2ix}) + 8ix^2 - 16x \log(1 - e^{2ix}) + \sin(2x) - 2x \cos(2x) - 4 \cot(x) - 4x \csc^2(x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x]^2*Cot[x]^3,x]
```

[Out] $((8*I)*x^2 - 2*x*\cos[2*x] - 4*\cot[x] - 4*x*\csc[x]^2 - 16*x*\log[1 - E^{((2*I)*x)}] + (8*I)*\text{PolyLog}[2, E^{((2*I)*x)}] + \sin[2*x])/8$

Maple [A] time = 0.126, size = 109, normalized size = 1.5

$$ix^2 - \frac{(i+2x)e^{2ix}}{16} - \frac{(2x-i)e^{-2ix}}{16} + \frac{2xe^{2ix} - ie^{2ix} + i}{(e^{2ix} - 1)^2} - 2x \ln(1 - e^{ix}) - 2x \ln(1 + e^{ix}) + 2i \text{polylog}(2, e^{ix}) + 2i \text{polylog}(2, -e^{ix})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cos(x)^2*cot(x)^3,x)`

[Out] $I*x^2 - 1/16*(I+2*x)*\exp(2*I*x) - 1/16*(2*x-I)*\exp(-2*I*x) + (2*x*\exp(2*I*x) - I*\exp(2*I*x) + I)/(\exp(2*I*x) - 1)^2 - 2*x*\ln(1 - \exp(I*x)) - 2*x*\ln(1 + \exp(I*x)) + 2*I*\text{polylog}(2, \exp(I*x)) + 2*I*\text{polylog}(2, -\exp(I*x))$

Maxima [B] time = 1.88111, size = 2349, normalized size = 32.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x)^2*cot(x)^3,x, algorithm="maxima")`

[Out] $-\left((2*x + I)*\cos(6*x)^2 - (-32*I*x^2 - 8*x - 4*I)*\cos(4*x)^2 - (-32*I*x^2 + 28*x - 16*I)*\cos(2*x)^2 - (2*x + I)*\sin(6*x)^2 - (32*I*x^2 + 8*x + 4*I)*\sin(4*x)^2 - (32*I*x^2 - 28*x + 16*I)*\sin(2*x)^2 - (64*I*x*\cos(4*x)^2 + 64*I*x*\cos(2*x)^2 - 64*I*x*\sin(4*x)^2 - 64*I*x*\sin(2*x)^2 + (-32*I*x*\cos(4*x) + 64*I*x*\cos(2*x) + 32*x*\sin(4*x) - 64*x*\sin(2*x) - 32*I*x)*\cos(6*x) + (-160*I*x*\cos(2*x) + 160*x*\sin(2*x) + 64*I*x)*\cos(4*x) - 32*I*x*\cos(2*x) + (32*x*\cos(4*x) - 64*x*\cos(2*x) + 32*I*x*\sin(4*x) - 64*I*x*\sin(2*x) + 32*x)*\sin(6*x) - (128*x*\cos(4*x) - 160*x*\cos(2*x) - 160*I*x*\sin(2*x) + 64*x)*\sin(4*x) - 32*(4*x*\cos(2*x) - x)*\sin(2*x)\right)*\arctan2(\sin(x), \cos(x) + 1) - (-64*I*x*\cos(4*x)^2 - 64*I*x*\cos(2*x)^2 + 64*I*x*\sin(4*x)^2 + 64*I*x*\sin(2*x)^2 + (32*I*x*\cos(4*x) - 64*I*x*\cos(2*x) - 32*x*\sin(4*x) + 64*x*\sin(2*x) + 32*I*x)*\cos(6*x) + (160*I*x*\cos(2*x) - 160*x*\sin(2*x) - 64*I*x)*\cos(4*x) + 32*I*x*\cos(2*x) - (32*x*\cos(4*x) - 64*x*\cos(2*x) + 32*I*x*\sin(4*x) - 64*I*x*\sin(2*x) + 32*x)*\sin(6*x) + (128*x*\cos(4*x) - 160*x*\cos(2*x) - 160*I*x*\sin(2*x) + 64*x)*\sin(4*x) + 32*(4*x*\cos(2*x) - x)*\sin(2*x)\right)*\arctan2(\sin(x), -\cos(x) + 1) - (16*I*x^2 + (16*I*x^2 + 8*x + 4*I)*\cos(4*x) + (-32*I*x^2 + 26*x - 17*I)*\cos(2*x) - 4*(4*x^2 - 2*I*x + 1)*\sin(4*x) + (32*x^2 + 26*I*x + 17)*\sin(2*x) + 4*x + 14*I)*\cos(6*x) - (-32*I*x^2 + (80*I*x^2 - 52*x + 34*I)*\cos(2*x) - 2*(40*x^2 + 26*I*x + 17)*\sin(2*x) - 10*x - 27*I)*\cos(4*x) - (16*I*x^2 + 8*x + 12*I)*\cos(2*x) - ((32*I*\cos(4*x) - 64*I*\cos(2*x) - 32*\sin(4*x) + 64*\sin(2*x) + 32*I)*\cos(6*x) + (160*I*\cos(2*x) - 160*\sin(2*x) - 64*I)*\cos(4*x) - 64*I*\cos(4*x)^2 - 64*I*\cos(2*x)^2 - (32*\cos(4*x) - 64*\cos(2*x) + 32*I*\sin(4*x) - 64*I*\sin(2*x) + 32)*\sin(6*x) + 32*(4*\cos(4*x) - 5*\cos(2*x) - 5*I*\sin(2*x) + 2)*\sin(4*x) + 64*I*\sin(4*x)^2 + 32*(4*\cos(2*x) - 1)*\sin(2*x) + 64*I*\sin(2*x)^2 + 32*I*\cos(2*x))*\text{dilog}(-e^{I*x}) - ((32*I*\cos(4*x) - 64*I*\cos(2*x) - 32*\sin(4*x) + 64*\sin(2*x) + 32*I)*\cos(6*x) + (160*I*\cos(2*x) - 160*\sin(2*x) - 64*I)*\cos(4*x) - 64*I*\cos(4*x)^2 - 64*I*\cos(2*x)^2 - (32*\cos(4*x) - 64*\cos(2*x) + 32*I*\sin(4*x) - 64*I*\sin(2*x) + 32)*\sin(6*x) + 32*(4*\cos(4*x) - 5*\cos(2*x) - 5*I*\sin(2*x) + 2)*\sin(4*x) + 64*I*\sin(4*x)^2 + 32*(4*\cos(2*x) - 1)*\sin(2*x) + 64*I*\sin(2*x)^2 + 32*I*\cos(2*x))*\text{dilog}(e^{I*x}) - (32*x*\cos(2*x) + 64*I*\sin(2*x)^2 + 32*I*\cos(2*x))*\text{dilog}(e^{I*x}) - (32*x*\cos(2*x) + 64*I*\sin(2*x)^2 + 32*I*\cos(2*x))*\text{dilog}(e^{I*x})$

```
s(4*x)^2 + 32*x*cos(2*x)^2 - 32*x*sin(4*x)^2 - 32*x*sin(2*x)^2 - (16*x*cos(
4*x) - 32*x*cos(2*x) + 16*I*x*sin(4*x) - 32*I*x*sin(2*x) + 16*x)*cos(6*x) -
(80*x*cos(2*x) + 80*I*x*sin(2*x) - 32*x)*cos(4*x) - 16*x*cos(2*x) + (-16*I
*x*cos(4*x) + 32*I*x*cos(2*x) + 16*x*sin(4*x) - 32*x*sin(2*x) - 16*I*x)*sin
(6*x) + (64*I*x*cos(4*x) - 80*I*x*cos(2*x) + 80*x*sin(2*x) + 32*I*x)*sin(4*
x) + (64*I*x*cos(2*x) - 16*I*x)*sin(2*x))*log(cos(x)^2 + sin(x)^2 + 2*cos(x
) + 1) - (32*x*cos(4*x)^2 + 32*x*cos(2*x)^2 - 32*x*sin(4*x)^2 - 32*x*sin(2*
x)^2 - (16*x*cos(4*x) - 32*x*cos(2*x) + 16*I*x*sin(4*x) - 32*I*x*sin(2*x) +
16*x)*cos(6*x) - (80*x*cos(2*x) + 80*I*x*sin(2*x) - 32*x)*cos(4*x) - 16*x*
cos(2*x) + (-16*I*x*cos(4*x) + 32*I*x*cos(2*x) + 16*x*sin(4*x) - 32*x*sin(2
*x) - 16*I*x)*sin(6*x) + (64*I*x*cos(4*x) - 80*I*x*cos(2*x) + 80*x*sin(2*x)
+ 32*I*x)*sin(4*x) + (64*I*x*cos(2*x) - 16*I*x)*sin(2*x))*log(cos(x)^2 + s
in(x)^2 - 2*cos(x) + 1) + (16*x^2 + 2*(2*I*x - 1)*cos(6*x) + 4*(4*x^2 - 2*I
*x + 1)*cos(4*x) - (32*x^2 + 26*I*x + 17)*cos(2*x) - (-16*I*x^2 - 8*x - 4*I
)*sin(4*x) - (32*I*x^2 - 26*x + 17*I)*sin(2*x) - 4*I*x + 14)*sin(6*x) - (32
*x^2 + 8*(8*x^2 - 2*I*x + 1)*cos(4*x) - 2*(40*x^2 + 26*I*x + 17)*cos(2*x) +
(-80*I*x^2 + 52*x - 34*I)*sin(2*x) - 10*I*x + 27)*sin(4*x) + (16*x^2 - 8*(
8*x^2 + 7*I*x + 4)*cos(2*x) - 8*I*x + 12)*sin(2*x) + 2*x - I)/((16*cos(4*x)
- 32*cos(2*x) + 16*I*sin(4*x) - 32*I*sin(2*x) + 16)*cos(6*x) + 16*(5*cos(2
*x) + 5*I*sin(2*x) - 2)*cos(4*x) - 32*cos(4*x)^2 - 32*cos(2*x)^2 - (-16*I*c
os(4*x) + 32*I*cos(2*x) + 16*sin(4*x) - 32*sin(2*x) - 16*I)*sin(6*x) - (64*
I*cos(4*x) - 80*I*cos(2*x) + 80*sin(2*x) + 32*I)*sin(4*x) + 32*sin(4*x)^2 -
(64*I*cos(2*x) - 16*I)*sin(2*x) + 32*sin(2*x)^2 + 16*cos(2*x))
```

Fricas [B] time = 0.581824, size = 640, normalized size = 8.77

$$2x \cos(x)^4 - 3x \cos(x)^2 - (4i \cos(x)^2 - 4i) \operatorname{Li}_2(\cos(x) + i \sin(x)) - (-4i \cos(x)^2 + 4i) \operatorname{Li}_2(\cos(x) - i \sin(x)) - (-4i \cos(x)^2 + 4i) \operatorname{Li}_2(\cos(x) + i \sin(x)) - (-4i \cos(x)^2 + 4i) \operatorname{Li}_2(\cos(x) - i \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*x*cos(x)^4 - 3*x*cos(x)^2 - (4*I*cos(x)^2 - 4*I)*dilog(cos(x) + I*s
in(x)) - (-4*I*cos(x)^2 + 4*I)*dilog(cos(x) - I*sin(x)) - (-4*I*cos(x)^2 +
4*I)*dilog(-cos(x) + I*sin(x)) - (4*I*cos(x)^2 - 4*I)*dilog(-cos(x) - I*sin
(x)) + 4*(x*cos(x)^2 - x)*log(cos(x) + I*sin(x) + 1) + 4*(x*cos(x)^2 - x)*l
og(cos(x) - I*sin(x) + 1) + 4*(x*cos(x)^2 - x)*log(-cos(x) + I*sin(x) + 1)
+ 4*(x*cos(x)^2 - x)*log(-cos(x) - I*sin(x) + 1) - (cos(x)^3 + cos(x))*sin(
x) - x)/(cos(x)^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos^2(x) \cot^3(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)**2*cot(x)**3,x)
```

```
[Out] Integral(x*cos(x)**2*cot(x)**3, x)
```


Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(x)^2 \cot(x)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cos(x)^2*cot(x)^3, x)
```

3.208 $\int (c + dx)^m \tan(a + bx) dx$

Optimal. Leaf size=16

Unintegrable(tan(a + bx)(c + dx)^m, x)

[Out] Unintegrable[(c + d*x)^m*Tan[a + b*x], x]

Rubi [A] time = 0.0177912, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \tan(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Tan[a + b*x], x]

[Out] Defer[Int] [(c + d*x)^m*Tan[a + b*x], x]

Rubi steps

$$\int (c + dx)^m \tan(a + bx) dx = \int (c + dx)^m \tan(a + bx) dx$$

Mathematica [A] time = 2.51709, size = 0, normalized size = 0.

$$\int (c + dx)^m \tan(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Tan[a + b*x], x]

[Out] Integrate[(c + d*x)^m*Tan[a + b*x], x]

Maple [A] time = 0.2, size = 0, normalized size = 0.

$$\int (dx + c)^m \sec(bx + a) \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a), x)

[Out] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \sec(bx + a) \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx + c)^m \sec(bx + a) \sin(bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \sec(bx + a) \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

3.209 $\int (c + dx)^4 \tan(a + bx) dx$

Optimal. Leaf size=158

$$\frac{3d^2(c + dx)^2 \text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{b^3} - \frac{3id^3(c + dx) \text{PolyLog}\left(4, -e^{2i(a+bx)}\right)}{b^4} + \frac{2id(c + dx)^3 \text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^2} + \frac{3d^4}{b^5}$$

[Out] ((I/5)*(c + d*x)^5)/d - ((c + d*x)^4*Log[1 + E^((2*I)*(a + b*x))])/b + ((2*I)*d*(c + d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(c + d*x)*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/(2*b^5)

Rubi [A] time = 0.209578, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3719, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c + dx)^2 \text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{b^3} - \frac{3id^3(c + dx) \text{PolyLog}\left(4, -e^{2i(a+bx)}\right)}{b^4} + \frac{2id(c + dx)^3 \text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^2} + \frac{3d^4}{b^5}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^4*Tan[a + b*x], x]

[Out] ((I/5)*(c + d*x)^5)/d - ((c + d*x)^4*Log[1 + E^((2*I)*(a + b*x))])/b + ((2*I)*d*(c + d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(c + d*x)*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/(2*b^5)

Rule 3719

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*(f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> -Simp[(f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 6609

Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,

d, e, f, n, p}, x] && GtQ[m, 0]

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^4 \tan(a + bx) dx &= \frac{i(c + dx)^5}{5d} - 2i \int \frac{e^{2i(a+bx)}(c + dx)^4}{1 + e^{2i(a+bx)}} dx \\ &= \frac{i(c + dx)^5}{5d} - \frac{(c + dx)^4 \log(1 + e^{2i(a+bx)})}{b} + \frac{(4d) \int (c + dx)^3 \log(1 + e^{2i(a+bx)}) dx}{b} \\ &= \frac{i(c + dx)^5}{5d} - \frac{(c + dx)^4 \log(1 + e^{2i(a+bx)})}{b} + \frac{2id(c + dx)^3 \text{Li}_2(-e^{2i(a+bx)})}{b^2} - \frac{(6id^2) \int (c + dx)^2 \log(1 + e^{2i(a+bx)}) dx}{b^2} \\ &= \frac{i(c + dx)^5}{5d} - \frac{(c + dx)^4 \log(1 + e^{2i(a+bx)})}{b} + \frac{2id(c + dx)^3 \text{Li}_2(-e^{2i(a+bx)})}{b^2} - \frac{3d^2(c + dx)^2 \text{Li}_2(-e^{2i(a+bx)})}{b^3} \\ &= \frac{i(c + dx)^5}{5d} - \frac{(c + dx)^4 \log(1 + e^{2i(a+bx)})}{b} + \frac{2id(c + dx)^3 \text{Li}_2(-e^{2i(a+bx)})}{b^2} - \frac{3d^2(c + dx)^2 \text{Li}_2(-e^{2i(a+bx)})}{b^3} \\ &= \frac{i(c + dx)^5}{5d} - \frac{(c + dx)^4 \log(1 + e^{2i(a+bx)})}{b} + \frac{2id(c + dx)^3 \text{Li}_2(-e^{2i(a+bx)})}{b^2} - \frac{3d^2(c + dx)^2 \text{Li}_2(-e^{2i(a+bx)})}{b^3} \\ &= \frac{i(c + dx)^5}{5d} - \frac{(c + dx)^4 \log(1 + e^{2i(a+bx)})}{b} + \frac{2id(c + dx)^3 \text{Li}_2(-e^{2i(a+bx)})}{b^2} - \frac{3d^2(c + dx)^2 \text{Li}_2(-e^{2i(a+bx)})}{b^3} \end{aligned}$$

Mathematica [A] time = 0.0687408, size = 157, normalized size = 0.99

$$\frac{3d^2 \left(2b^2(c + dx)^2 \text{PolyLog}\left(3, -e^{2i(a+bx)}\right) + d \left(2ib(c + dx) \text{PolyLog}\left(4, -e^{2i(a+bx)}\right) - d \text{PolyLog}\left(5, -e^{2i(a+bx)}\right) \right) \right)}{2b^5} + \frac{2id(c + dx)^4 \log(1 + e^{2i(a+bx)})}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Tan[a + b*x], x]
```

```
[Out] ((I/5)*(c + d*x)^5)/d - ((c + d*x)^4*Log[1 + E^((2*I)*(a + b*x))])/b + ((2*I)*d*(c + d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(2*b^2*(c + d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))] + d*((2*I)*b*(c + d*x)*PolyLog[4, -E^((2*I)*(a + b*x))] - d*PolyLog[5, -E^((2*I)*(a + b*x))]))/(2*b^5)
```

Maple [B] time = 0.315, size = 616, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x)`

[Out] $\frac{3}{2}d^4 \text{polylog}(5, -\exp(2I*(b*x+a))) / b^5 + \frac{1}{5}I*d^4*x^5 + I*c*d^3*x^4 - 3I/b^4*c*d^3 \text{polylog}(4, -\exp(2I*(b*x+a))) - 3/b^3*c^2*d^2 \text{polylog}(3, -\exp(2I*(b*x+a))) - 3/b^3*d^4 \text{polylog}(3, -\exp(2I*(b*x+a))) * x^2 - 4/b*c*d^3 \ln(\exp(2I*(b*x+a)) + 1) * x^3 - 1/b*c^4 \ln(\exp(2I*(b*x+a)) + 1) - I*c^4*x - 6/b*c^2*d^2 \ln(\exp(2I*(b*x+a)) + 1) * x^2 - 12I/b^2*a^2*c^2*d^2*x + 8I/b^3*c*d^3*a^3*x + 6I/b^2*c*d^3 \text{polylog}(2, -\exp(2I*(b*x+a))) * x^2 + 6I/b^2*c^2*d^2 \text{polylog}(2, -\exp(2I*(b*x+a))) * x - 6/b^3*c*d^3 \text{polylog}(3, -\exp(2I*(b*x+a))) * x - 1/b*d^4 \ln(\exp(2I*(b*x+a)) + 1) * x^4 - 8/5I/b^5*d^4*a^5 + 8I/b*a*c^3*d*x - 4/b*c^3*d \ln(\exp(2I*(b*x+a)) + 1) * x - 2I/b^4*d^4*a^4*x - 8I/b^3*a^3*c^2*d^2 + 4I/b^2*a^2*c^3*d + 6I/b^4*c*d^3*a^4 + 2I*c^2*d^2*x^3 + 2I*c^3*d*x^2 + 2/b^5*d^4*a^4 \ln(\exp(I*(b*x+a))) - 8/b^2*c^3*d*a \ln(\exp(I*(b*x+a))) + 12/b^3*c^2*d^2*a^2 \ln(\exp(I*(b*x+a))) - 8/b^4*c*d^3*a^3 \ln(\exp(I*(b*x+a))) + 2/b*c^4 \ln(\exp(I*(b*x+a))) + 2I/b^2*c^3*d \text{polylog}(2, -\exp(2I*(b*x+a))) - 3I/b^4*d^4 \text{polylog}(4, -\exp(2I*(b*x+a))) * x + 2I/b^2*d^4 \text{polylog}(2, -\exp(2I*(b*x+a))) * x^3$

Maxima [B] time = 2.05808, size = 1069, normalized size = 6.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")`

[Out] $-\frac{1}{30}(15c^4 \log(-\sin(bx+a)^2+1) - 60a^3c^3d \log(-\sin(bx+a)^2+1)) / b + 90a^2c^2d^2 \log(-\sin(bx+a)^2+1) / b^2 - 60a^3c^3d^3 \log(-\sin(bx+a)^2+1) / b^3 + 15a^4d^4 \log(-\sin(bx+a)^2+1) / b^4 + 2(-3I(bx+a)^5d^4 + (-15I*b*c*d^3 + 15I*a*d^4)*(bx+a)^4 - 45d^4 \text{polylog}(5, -e^{(2I*bx+2Ia)}) + (-30I*b^2*c^2*d^2 + 60I*a*b*c*d^3 - 30I*a^2*d^4)*(bx+a)^3 + (-30I*b^3*c^3*d + 90I*a*b^2*c^2*d^2 - 90I*a^2*b*c*d^3 + 30I*a^3*d^4)*(bx+a)^2 + (30I*(bx+a)^4*d^4 + (80I*b*c*d^3 - 80I*a*d^4)*(bx+a)^3 + (90I*b^2*c^2*d^2 - 180I*a*b*c*d^3 + 90I*a^2*d^4)*(bx+a)^2 + (60I*b^3*c^3*d - 180I*a*b^2*c^2*d^2 + 180I*a^2*b*c*d^3 - 60I*a^3*d^4)*(bx+a)) \arctan2(\sin(2*bx+2*a), \cos(2*bx+2*a)+1) + (-30I*b^3*c^3*d + 90I*a*b^2*c^2*d^2 - 90I*a^2*b*c*d^3 - 60I*(bx+a)^3*d^4 + 30I*a^3*d^4 + (-120I*b*c*d^3 + 120I*a*d^4)*(bx+a)^2 + (-90I*b^2*c^2*d^2 + 180I*a*b*c*d^3 - 90I*a^2*d^4)*(bx+a)) \text{dilog}(-e^{(2I*bx+2Ia)}) + 5(3(bx+a)^4*d^4 + 8(b*c*d^3 - a*d^4)*(bx+a)^3 + 9(b^2*c^2*d^2 - 2a*b*c*d^3 + a^2*d^4)*(bx+a)^2 + 6(b^3*c^3*d - 3a*b^2*c^2*d^2 + 3a^2*b*c*d^3 - a^3*d^4)*(bx+a)) \log(\cos(2*bx+2*a)^2 + \sin(2*bx+2*a)^2 + 2\cos(2*bx+2*a)+1) + (60I*b*c*d^3 + 90I*(bx+a)*d^4 - 60I*a*d^4) \text{polylog}(4, -e^{(2I*bx+2Ia)}) + 15(3b^2*c^2*d^2 - 6a*b*c*d^3 + 6(bx+a)^2*d^4 + 3a^2*d^4 + 8(b*c*d^3 - a*d^4)*(bx+a)) \text{polylog}(3, -e^{(2I*bx+2Ia)}) / b^4) / b$

Fricas [C] time = 0.741923, size = 3355, normalized size = 21.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")`

```
[Out] 1/2*(24*d^4*polylog(5, I*cos(b*x + a) + sin(b*x + a)) + 24*d^4*polylog(5, I
*cos(b*x + a) - sin(b*x + a)) + 24*d^4*polylog(5, -I*cos(b*x + a) + sin(b*x
+ a)) + 24*d^4*polylog(5, -I*cos(b*x + a) - sin(b*x + a)) + (-4*I*b^3*d^4*
x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(I*cos(
b*x + a) + sin(b*x + a)) + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3
*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (4*I*b^3
*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(-
I*cos(b*x + a) + sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 1
2*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) -
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log
(cos(b*x + a) + I*sin(b*x + a) + I) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*
c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(cos(b*x + a) - I*sin(b*x + a) + I) -
(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b
^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(I*cos(b*x + a)
+ sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 +
4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4
)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 +
6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*
a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^4*d^4*x
^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d -
6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) - sin(b*x
+ a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + I) - (b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-cos(b*x + a) - I*sin(b
*x + a) + I) + (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, I*cos(b*x + a) + si
n(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, I*cos(b*x + a) - si
n(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, -I*cos(b*x + a) + s
in(b*x + a)) + (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, -I*cos(b*x + a) - s
in(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, I*
cos(b*x + a) + sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^
2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^
3*x + b^2*c^2*d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 12*(b^2*d^4
*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -I*cos(b*x + a) - sin(b*x +
a)))/b^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^4 \sec(bx + a) \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*sec(b*x + a)*sin(b*x + a), x)
```

3.210 $\int (c + dx)^3 \tan(a + bx) dx$

Optimal. Leaf size=132

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} + \frac{3id(c+dx)^2\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^2} - \frac{3id^3\text{PolyLog}\left(4, -e^{2i(a+bx)}\right)}{4b^4} - \frac{(c+dx)^3 \log}{b^4}$$

[Out] $((I/4)*(c + d*x)^4)/d - ((c + d*x)^3*\text{Log}[1 + E^{((2*I)*(a + b*x))}])/b + (((3*I)/2)*d*(c + d*x)^2*\text{PolyLog}[2, -E^{((2*I)*(a + b*x))}])/b^2 - (3*d^2*(c + d*x)*\text{PolyLog}[3, -E^{((2*I)*(a + b*x))}])/(2*b^3) - (((3*I)/4)*d^3*\text{PolyLog}[4, -E^{((2*I)*(a + b*x))}])/b^4$

Rubi [A] time = 0.182795, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3719, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} + \frac{3id(c+dx)^2\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^2} - \frac{3id^3\text{PolyLog}\left(4, -e^{2i(a+bx)}\right)}{4b^4} - \frac{(c+dx)^3 \log}{b^4}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^3*Tan[a + b*x], x]

[Out] $((I/4)*(c + d*x)^4)/d - ((c + d*x)^3*\text{Log}[1 + E^{((2*I)*(a + b*x))}])/b + (((3*I)/2)*d*(c + d*x)^2*\text{PolyLog}[2, -E^{((2*I)*(a + b*x))}])/b^2 - (3*d^2*(c + d*x)*\text{PolyLog}[3, -E^{((2*I)*(a + b*x))}])/(2*b^3) - (((3*I)/4)*d^3*\text{PolyLog}[4, -E^{((2*I)*(a + b*x))}])/b^4$

Rule 3719

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*(f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[(f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 6609

Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,

d, e, f, n, p}, x] && GtQ[m, 0]

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^3 \tan(a + bx) dx &= \frac{i(c + dx)^4}{4d} - 2i \int \frac{e^{2i(a+bx)}(c + dx)^3}{1 + e^{2i(a+bx)}} dx \\ &= \frac{i(c + dx)^4}{4d} - \frac{(c + dx)^3 \log(1 + e^{2i(a+bx)})}{b} + \frac{(3d) \int (c + dx)^2 \log(1 + e^{2i(a+bx)}) dx}{b} \\ &= \frac{i(c + dx)^4}{4d} - \frac{(c + dx)^3 \log(1 + e^{2i(a+bx)})}{b} + \frac{3id(c + dx)^2 \text{Li}_2(-e^{2i(a+bx)})}{2b^2} - \frac{(3id^2) \int (c + dx) \log(1 + e^{2i(a+bx)}) dx}{b} \\ &= \frac{i(c + dx)^4}{4d} - \frac{(c + dx)^3 \log(1 + e^{2i(a+bx)})}{b} + \frac{3id(c + dx)^2 \text{Li}_2(-e^{2i(a+bx)})}{2b^2} - \frac{3d^2(c + dx) \text{Li}_3(-e^{2i(a+bx)})}{2b^3} \\ &= \frac{i(c + dx)^4}{4d} - \frac{(c + dx)^3 \log(1 + e^{2i(a+bx)})}{b} + \frac{3id(c + dx)^2 \text{Li}_2(-e^{2i(a+bx)})}{2b^2} - \frac{3d^2(c + dx) \text{Li}_3(-e^{2i(a+bx)})}{2b^3} \\ &= \frac{i(c + dx)^4}{4d} - \frac{(c + dx)^3 \log(1 + e^{2i(a+bx)})}{b} + \frac{3id(c + dx)^2 \text{Li}_2(-e^{2i(a+bx)})}{2b^2} - \frac{3d^2(c + dx) \text{Li}_3(-e^{2i(a+bx)})}{2b^3} \end{aligned}$$

Mathematica [A] time = 0.0874103, size = 126, normalized size = 0.95

$$\frac{1}{4} \left(\frac{3d \left(2b^2(c + dx)^2 \text{PolyLog}\left(2, -e^{2i(a+bx)}\right) + d \left(2ib(c + dx) \text{PolyLog}\left(3, -e^{2i(a+bx)}\right) - d \text{PolyLog}\left(4, -e^{2i(a+bx)}\right) \right) \right)}{b^4} + \frac{4i(c + dx)^3 \log(1 + e^{2i(a+bx)})}{b} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Tan[a + b*x], x]
```

```
[Out] (I/4)*((c + d*x)^4/d + ((4*I)*(c + d*x)^3*Log[1 + E^((2*I)*(a + b*x))])/b + (3*d*(2*b^2*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))] + d*((2*I)*b*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))] - d*PolyLog[4, -E^((2*I)*(a + b*x))]))/b^4)
```

Maple [B] time = 0.289, size = 423, normalized size = 3.2

$$-\frac{c^3 \ln(e^{2i(bx+a)} + 1)}{b} - \frac{3d^2 c \text{polylog}(3, -e^{2i(bx+a)})}{2b^3} - \frac{3d^3 \text{polylog}(3, -e^{2i(bx+a)})x}{2b^3} + \frac{\frac{3i}{2}d^3 a^4}{b^4} + \frac{i}{4}d^3 x^4 + icd^2 x^3 - ic^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x)`

[Out]
$$-1/b*c^3*\ln(\exp(2*I*(b*x+a))+1)-3/2/b^3*c*d^2*\text{polylog}(3,-\exp(2*I*(b*x+a)))-3/2/b^3*d^3*\text{polylog}(3,-\exp(2*I*(b*x+a)))*x+3/2*I/b^4*d^3*a^4+1/4*I*d^3*x^4+I*c*d^2*x^3-I*c^3*x+3/2*I*c^2*d*x^2+3/2*I/b^2*c^2*d*\text{polylog}(2,-\exp(2*I*(b*x+a)))-3/b*c^2*d*\ln(\exp(2*I*(b*x+a))+1)*x-3/b*c*d^2*\ln(\exp(2*I*(b*x+a))+1)*x^2-1/b*d^3*\ln(\exp(2*I*(b*x+a))+1)*x^3+3/2*I/b^2*d^3*\text{polylog}(2,-\exp(2*I*(b*x+a)))*x^2+6/b^3*c*d^2*a^2*\ln(\exp(I*(b*x+a)))-6/b^2*c^2*d*a*\ln(\exp(I*(b*x+a)))-4*I/b^3*a^3*c*d^2+2*I/b^3*d^3*a^3*x+3*I/b^2*a^2*c^2*d-3/4*I*d^3*\text{polylog}(4,-\exp(2*I*(b*x+a)))/b^4+3*I/b^2*c*d^2*\text{polylog}(2,-\exp(2*I*(b*x+a)))*x-2/b^4*d^3*a^3*\ln(\exp(I*(b*x+a)))+6*I/b*a*c^2*d*x-6*I/b^2*a^2*c*d^2*x+2/b*c^3*\ln(\exp(I*(b*x+a)))$$

Maxima [B] time = 1.857, size = 662, normalized size = 5.02

$$\frac{6c^3 \log(-\sin(bx+a)^2+1) - \frac{18ac^2d \log(-\sin(bx+a)^2+1)}{b} + \frac{18a^2cd^2 \log(-\sin(bx+a)^2+1)}{b^2} - \frac{6a^3d^3 \log(-\sin(bx+a)^2+1)}{b^3} + \frac{-3i(bx+a)^4d^3+(-$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")`

[Out]
$$-1/12*(6*c^3*\log(-\sin(b*x+a)^2+1) - 18*a*c^2*d*\log(-\sin(b*x+a)^2+1)/b + 18*a^2*c*d^2*\log(-\sin(b*x+a)^2+1)/b^2 - 6*a^3*d^3*\log(-\sin(b*x+a)^2+1)/b^3 + (-3*I*(b*x+a)^4*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x+a)^3 + 12*I*d^3*\text{polylog}(4, -e^{(2*I*b*x+2*I*a)}) + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*a^2*d^3)*(b*x+a)^2 + (16*I*(b*x+a)^3*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x+a)^2 + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 36*I*a^2*d^3)*(b*x+a))*\arctan2(\sin(2*b*x+2*a), \cos(2*b*x+2*a)+1) + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 24*I*(b*x+a)^2*d^3 - 18*I*a^2*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x+a))*\text{dilog}(-e^{(2*I*b*x+2*I*a)}) + 2*(4*(b*x+a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x+a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x+a))*\log(\cos(2*b*x+2*a)^2 + \sin(2*b*x+2*a)^2 + 2*\cos(2*b*x+2*a)+1) + 6*(3*b*c*d^2 + 4*(b*x+a)*d^3 - 3*a*d^3)*\text{polylog}(3, -e^{(2*I*b*x+2*I*a)})/b^3)/b$$

Fricas [C] time = 0.664148, size = 2367, normalized size = 17.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")`

[Out]
$$1/2*(6*I*d^3*\text{polylog}(4, I*\cos(b*x+a) + \sin(b*x+a)) - 6*I*d^3*\text{polylog}(4, I*\cos(b*x+a) - \sin(b*x+a)) - 6*I*d^3*\text{polylog}(4, -I*\cos(b*x+a) + \sin(b*x+a)) + 6*I*d^3*\text{polylog}(4, -I*\cos(b*x+a) - \sin(b*x+a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*\text{dilog}(I*\cos(b*x+a) + \sin(b*x+a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*\text{dilog}(I*\cos(b*x+a) - \sin(b*x+a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*\text{dilog}(-I*\cos(b*x+a) + \sin(b*x+a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*\text{dilog}(-I*\cos(b*x+a) - \sin(b*x+a)) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\log(\cos(b*x+a) + I*\sin(b*x+a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\log(\cos(b*x+a) - I*s$$

```

in(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1) -
(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*
d^2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(
-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) -
sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log
(-cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b*d^3*x + b*c*
d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*poly
log(3, I*cos(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I
*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x
+ a) - sin(b*x + a))/b^4

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*sec(b*x+a)*sin(b*x+a),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \sec(bx + a) \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^3*sec(b*x + a)*sin(b*x + a), x)

3.211 $\int (c + dx)^2 \tan(a + bx) dx$

Optimal. Leaf size=96

$$\frac{id(c + dx)\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^2} - \frac{d^2\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} - \frac{(c + dx)^2 \log\left(1 + e^{2i(a+bx)}\right)}{b} + \frac{i(c + dx)^3}{3d}$$

[Out] $((I/3)*(c + d*x)^3)/d - ((c + d*x)^2*\text{Log}[1 + E^((2*I)*(a + b*x))])/b + (I*d*(c + d*x)*\text{PolyLog}[2, -E^((2*I)*(a + b*x))])/b^2 - (d^2*\text{PolyLog}[3, -E^((2*I)*(a + b*x))])/(2*b^3)$

Rubi [A] time = 0.152983, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3719, 2190, 2531, 2282, 6589}

$$\frac{id(c + dx)\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^2} - \frac{d^2\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} - \frac{(c + dx)^2 \log\left(1 + e^{2i(a+bx)}\right)}{b} + \frac{i(c + dx)^3}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^2*\text{Tan}[a + b*x], x]$

[Out] $((I/3)*(c + d*x)^3)/d - ((c + d*x)^2*\text{Log}[1 + E^((2*I)*(a + b*x))])/b + (I*d*(c + d*x)*\text{PolyLog}[2, -E^((2*I)*(a + b*x))])/b^2 - (d^2*\text{PolyLog}[3, -E^((2*I)*(a + b*x))])/(2*b^3)$

Rule 3719

$\text{Int}[(c + d*x)^m*\text{Tan}[e + f*x], x] \text{ :> } \text{Simp}[(I*(c + d*x)^{m+1})/(d*(m+1)), x] - \text{Dist}[2*I, \text{Int}[(c + d*x)^m*E^{2*I*(e + f*x)}]/(1 + E^{2*I*(e + f*x)}), x] /; \text{FreeQ}\{c, d, e, f, x\} \ \&\& \ \text{IGtQ}[m, 0]$

Rule 2190

$\text{Int}[(c + d*x)^m*\text{Log}[1 + (b*(F^{g*(e + f*x)})^n)/a]]/(b*f*g^n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g^n*\text{Log}[F]), \text{Int}[(c + d*x)^{m-1}*\text{Log}[1 + (b*(F^{g*(e + f*x)})^n)/a]], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n, x\} \ \&\& \ \text{IGtQ}[m, 0]$

Rule 2531

$\text{Int}[\text{Log}[1 + (e + f*x)^n]*((c + d*x)^m), x] \text{ :> } -\text{Simp}[(f + g*x)^m*\text{PolyLog}[2, -(e*(F^{c*(a + b*x)})^n)]/(b*c^n*\text{Log}[F]), x] + \text{Dist}[(g*m)/(b*c^n*\text{Log}[F]), \text{Int}[(f + g*x)^{m-1}*\text{PolyLog}[2, -(e*(F^{c*(a + b*x)})^n)], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n, x\} \ \&\& \ \text{GtQ}[m, 0]$

Rule 2282

$\text{Int}[u, x] \text{ :> } \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (w_1)*(a_1)*(v_1)^{n_1}] /; \text{FreeQ}\{a, m, n, x\} \ \&\& \ \text{IntegerQ}[m*n] \ \&\& \ !\text{MatchQ}[u, E^{(c + d*x)*F[x]}] /; \text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{InverseFunctionQ}[F[x]]$

Rule 6589

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \tan(a + bx) dx &= \frac{i(c + dx)^3}{3d} - 2i \int \frac{e^{2i(a+bx)}(c + dx)^2}{1 + e^{2i(a+bx)}} dx \\ &= \frac{i(c + dx)^3}{3d} - \frac{(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b} + \frac{(2d) \int (c + dx) \log(1 + e^{2i(a+bx)}) dx}{b} \\ &= \frac{i(c + dx)^3}{3d} - \frac{(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b} + \frac{id(c + dx) \text{Li}_2(-e^{2i(a+bx)})}{b^2} - \frac{(id^2) \int \text{Li}_2(-e^{2i(a+bx)})}{b^2} \\ &= \frac{i(c + dx)^3}{3d} - \frac{(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b} + \frac{id(c + dx) \text{Li}_2(-e^{2i(a+bx)})}{b^2} - \frac{d^2 \text{Subst}\left(\int \frac{\text{Li}_2(-e^{2i(a+bx)})}{x}\right)}{2b^2} \\ &= \frac{i(c + dx)^3}{3d} - \frac{(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b} + \frac{id(c + dx) \text{Li}_2(-e^{2i(a+bx)})}{b^2} - \frac{d^2 \text{Li}_3(-e^{2i(a+bx)})}{2b^3} \end{aligned}$$

Mathematica [A] time = 0.0409372, size = 100, normalized size = 1.04

$$\frac{6ibd^2(c + dx)\text{PolyLog}\left(2, -e^{2i(a+bx)}\right) - 3d^3\text{PolyLog}\left(3, -e^{2i(a+bx)}\right) + 2ib^2(c + dx)^2\left(b(c + dx) + 3id \log\left(1 + e^{2i(a+bx)}\right)\right)}{6b^3d}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2*Tan[a + b*x], x]

[Out] ((2*I)*b^2*(c + d*x)^2*(b*(c + d*x) + (3*I)*d*Log[1 + E^((2*I)*(a + b*x))]) + (6*I)*b*d^2*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))] - 3*d^3*PolyLog[3, -E^((2*I)*(a + b*x))])/(6*b^3*d)

Maple [B] time = 0.277, size = 257, normalized size = 2.7

$$icdx^2 + \frac{2ia^2cd}{b^2} - ic^2x - \frac{c^2 \ln(e^{2i(bx+a)} + 1)}{b} + 2 \frac{c^2 \ln(e^{i(bx+a)})}{b} + 2 \frac{a^2 d^2 \ln(e^{i(bx+a)})}{b^3} - \frac{2ia^2 d^2 x}{b^2} + \frac{i}{3} d^2 x^3 - 2 \frac{cd \ln(e^{2i(bx+a)})}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*sec(b*x+a)*sin(b*x+a), x)

[Out] I*c*d*x^2+2*I/b^2*a^2*c*d-I*c^2*x-1/b*c^2*ln(exp(2*I*(b*x+a))+1)+2/b*c^2*ln(exp(I*(b*x+a)))+2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))-2*I/b^2*a^2*d^2*x+1/3*I*d^2*x^3-2/b*c*d*ln(exp(2*I*(b*x+a))+1)*x+I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-1/b*d^2*ln(exp(2*I*(b*x+a))+1)*x^2+I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-4/b^2*c*d*a*ln(exp(I*(b*x+a)))-4/3*I/b^3*a^3*d^2+4*I/b*a*c*d*x

Maxima [B] time = 1.85091, size = 378, normalized size = 3.94

$$\frac{3c^2 \log(-\sin(bx + a)^2 + 1) - \frac{6acd \log(-\sin(bx + a)^2 + 1)}{b} + \frac{3a^2 d^2 \log(-\sin(bx + a)^2 + 1)}{b^2} + \frac{-2i(bx + a)^3 d^2 + (-6ibcd + 6i ad^2)(bx + a)^2 + 3d^2 \text{Li}_3(-e^{2i(bx + a)})}{b^3}}{6b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")

[Out]
$$-1/6*(3*c^2*\log(-\sin(b*x + a)^2 + 1) - 6*a*c*d*\log(-\sin(b*x + a)^2 + 1)/b + 3*a^2*d^2*\log(-\sin(b*x + a)^2 + 1)/b^2 + (-2*I*(b*x + a)^3*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a)^2 + 3*d^2*\text{polylog}(3, -e^{(2*I*b*x + 2*I*a)}) + (6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a))*\text{arctan2}(\sin(2*b*x + 2*a), \cos(2*b*x + 2*a) + 1) + (-6*I*b*c*d - 6*I*(b*x + a)*d^2 + 6*I*a*d^2)*\text{dilog}(-e^{(2*I*b*x + 2*I*a)}) + 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1))/b^2)/b$$

Fricas [C] time = 0.597114, size = 1534, normalized size = 15.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")

[Out]
$$-1/2*(2*d^2*\text{polylog}(3, I*\cos(b*x + a) + \sin(b*x + a)) + 2*d^2*\text{polylog}(3, I*\cos(b*x + a) - \sin(b*x + a)) + 2*d^2*\text{polylog}(3, -I*\cos(b*x + a) + \sin(b*x + a)) + 2*d^2*\text{polylog}(3, -I*\cos(b*x + a) - \sin(b*x + a)) - (-2*I*b*d^2*x - 2*I*b*c*d)*\text{dilog}(I*\cos(b*x + a) + \sin(b*x + a)) - (2*I*b*d^2*x + 2*I*b*c*d)*\text{dilog}(I*\cos(b*x + a) - \sin(b*x + a)) - (2*I*b*d^2*x + 2*I*b*c*d)*\text{dilog}(-I*\cos(b*x + a) + \sin(b*x + a)) - (-2*I*b*d^2*x - 2*I*b*c*d)*\text{dilog}(-I*\cos(b*x + a) - \sin(b*x + a)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\log(\cos(b*x + a) + I*\sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\log(\cos(b*x + a) - I*\sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\log(I*\cos(b*x + a) + \sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\log(I*\cos(b*x + a) - \sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\log(-I*\cos(b*x + a) + \sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\log(-I*\cos(b*x + a) - \sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\log(-\cos(b*x + a) - I*\sin(b*x + a) + I))/b^3$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \sin(a + bx) \sec(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*sec(b*x+a)*sin(b*x+a),x)

[Out] Integral((c + d*x)**2*sin(a + b*x)*sec(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \sec(bx + a) \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)*sin(b*x + a), x)
```

3.212 $\int (c + dx) \tan(a + bx) dx$

Optimal. Leaf size=66

$$\frac{id\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^2} - \frac{(c + dx) \log\left(1 + e^{2i(a+bx)}\right)}{b} + \frac{i(c + dx)^2}{2d}$$

[Out] $((I/2)*(c + d*x)^2)/d - ((c + d*x)*\text{Log}[1 + E^((2*I)*(a + b*x))])/b + ((I/2)*d*\text{PolyLog}[2, -E^((2*I)*(a + b*x))])/b^2$

Rubi [A] time = 0.0925894, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3719, 2190, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^2} - \frac{(c + dx) \log\left(1 + e^{2i(a+bx)}\right)}{b} + \frac{i(c + dx)^2}{2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)*\text{Tan}[a + b*x], x]$

[Out] $((I/2)*(c + d*x)^2)/d - ((c + d*x)*\text{Log}[1 + E^((2*I)*(a + b*x))])/b + ((I/2)*d*\text{PolyLog}[2, -E^((2*I)*(a + b*x))])/b^2$

Rule 3719

$\text{Int}[(c + d*x)^m \tan(a + b*x), x] \rightarrow \text{Simp}[(I*(c + d*x)^{m+1})/(d*(m+1)), x] - \text{Dist}[2*I, \text{Int}[(c + d*x)^m * E^{2*I*(a + b*x)}]/(1 + E^{2*I*(a + b*x)}), x] /;$ $\text{FreeQ}\{c, d, e, f, x\} \ \&\& \ \text{IGtQ}[m, 0]$

Rule 2190

$\text{Int}[(F + (g*(e + f*x))^n)^m * (c + d*x)^m, x] \rightarrow \text{Simp}[(c + d*x)^m * \text{Log}[1 + (b*(F + (g*(e + f*x))^n)/a)]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{m-1} * \text{Log}[1 + (b*(F + (g*(e + f*x))^n)/a)], x] /;$ $\text{FreeQ}\{F, a, b, c, d, e, f, g, n, x\} \ \&\& \ \text{IGtQ}[m, 0]$

Rule 2279

$\text{Int}[\text{Log}[a + b*x] * (F + (g*(e + f*x))^n)^m, x] \rightarrow \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F + (g*(e + f*x))^n)^m], x] /;$ $\text{FreeQ}\{F, a, b, c, d, e, n, x\} \ \&\& \ \text{GtQ}[a, 0]$

Rule 2391

$\text{Int}[\text{Log}[c + d*x] * (e + f*x)^n, x] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /;$ $\text{FreeQ}\{c, d, e, n, x\} \ \&\& \ \text{EqQ}[c*d, 1]$

Rubi steps

$$\begin{aligned}
\int (c + dx) \tan(a + bx) dx &= \frac{i(c + dx)^2}{2d} - 2i \int \frac{e^{2i(a+bx)}(c + dx)}{1 + e^{2i(a+bx)}} dx \\
&= \frac{i(c + dx)^2}{2d} - \frac{(c + dx) \log(1 + e^{2i(a+bx)})}{b} + \frac{d \int \log(1 + e^{2i(a+bx)}) dx}{b} \\
&= \frac{i(c + dx)^2}{2d} - \frac{(c + dx) \log(1 + e^{2i(a+bx)})}{b} - \frac{(id) \text{Subst} \left(\int \frac{\log(1+x)}{x} dx, x, e^{2i(a+bx)} \right)}{2b^2} \\
&= \frac{i(c + dx)^2}{2d} - \frac{(c + dx) \log(1 + e^{2i(a+bx)})}{b} + \frac{id \text{Li}_2(-e^{2i(a+bx)})}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.0147743, size = 70, normalized size = 1.06

$$\frac{id \text{PolyLog}(2, -e^{2i(a+bx)})}{2b^2} - \frac{c \log(\cos(a + bx))}{b} - \frac{dx \log(1 + e^{2i(a+bx)})}{b} + \frac{1}{2} id x^2$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Tan[a + b*x], x]

[Out] (I/2)*d*x^2 - (d*x*Log[1 + E^((2*I)*(a + b*x))])/b - (c*Log[Cos[a + b*x]])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2

Maple [B] time = 0.252, size = 123, normalized size = 1.9

$$\frac{i}{2} dx^2 - icx + 2 \frac{c \ln(e^{i(bx+a)})}{b} - \frac{c \ln(e^{2i(bx+a)} + 1)}{b} + \frac{2 idax}{b} + \frac{ida^2}{b^2} - \frac{d \ln(e^{2i(bx+a)} + 1)x}{b} + \frac{i}{2} d \text{polylog}(2, -e^{2i(bx+a)})/b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*sec(b*x+a)*sin(b*x+a), x)

[Out] 1/2*I*d*x^2 - I*c*x + 2/b*c*ln(exp(I*(b*x+a))) - 1/b*c*ln(exp(2*I*(b*x+a))+1) + 2*I/b*d*a*x + I/b^2*d*a^2 - 1/b*d*ln(exp(2*I*(b*x+a))+1)*x + 1/2*I*d*polylog(2, -exp(2*I*(b*x+a)))/b^2 - 2/b^2*d*a*ln(exp(I*(b*x+a)))

Maxima [B] time = 1.72612, size = 154, normalized size = 2.33

$$\frac{-i b^2 dx^2 - 2i b^2 cx + (2i b dx + 2i bc) \arctan(\sin(2bx + 2a), \cos(2bx + 2a) + 1) - id \text{Li}_2(-e^{(2i bx + 2i a)}) + (bdx + bc)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a), x, algorithm="maxima")

[Out] -1/2*(-I*b^2*d*x^2 - 2*I*b^2*c*x + (2*I*b*d*x + 2*I*b*c)*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - I*d*dilog(-e^(2*I*b*x + 2*I*a)) + (b*d*x + b*c)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1))/b^2

Fricas [B] time = 0.571796, size = 837, normalized size = 12.68

$$-i d\text{Li}_2(i \cos(bx + a) + \sin(bx + a)) + i d\text{Li}_2(i \cos(bx + a) - \sin(bx + a)) + i d\text{Li}_2(-i \cos(bx + a) + \sin(bx + a)) - i d\text{Li}_2(-i \cos(bx + a) - \sin(bx + a))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")

[Out] $\frac{1}{2} * (-I * d * \text{dilog}(I * \cos(b * x + a) + \sin(b * x + a)) + I * d * \text{dilog}(I * \cos(b * x + a) - \sin(b * x + a)) + I * d * \text{dilog}(-I * \cos(b * x + a) + \sin(b * x + a)) - I * d * \text{dilog}(-I * \cos(b * x + a) - \sin(b * x + a)) - (b * c - a * d) * \log(\cos(b * x + a) + I * \sin(b * x + a) + I) - (b * c - a * d) * \log(\cos(b * x + a) - I * \sin(b * x + a) + I) - (b * d * x + a * d) * \log(I * \cos(b * x + a) + \sin(b * x + a) + 1) - (b * d * x + a * d) * \log(I * \cos(b * x + a) - \sin(b * x + a) + 1) - (b * d * x + a * d) * \log(-I * \cos(b * x + a) + \sin(b * x + a) + 1) - (b * d * x + a * d) * \log(-I * \cos(b * x + a) - \sin(b * x + a) + 1) - (b * c - a * d) * \log(-\cos(b * x + a) + I * \sin(b * x + a) + I) - (b * c - a * d) * \log(-\cos(b * x + a) - I * \sin(b * x + a) + I)) / b^2$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \sin(a + bx) \sec(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x)

[Out] Integral((c + d*x)*sin(a + b*x)*sec(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \sec(bx + a) \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)*sec(b*x + a)*sin(b*x + a), x)

$$3.213 \quad \int \frac{\tan(a+bx)}{c+dx} dx$$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\tan(a+bx)}{c+dx}, x\right)$$

[Out] Unintegrable[Tan[a + b*x]/(c + d*x), x]

Rubi [A] time = 0.0199929, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\tan(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[Tan[a + b*x]/(c + d*x), x]

[Out] Defer[Int][Tan[a + b*x]/(c + d*x), x]

Rubi steps

$$\int \frac{\tan(a+bx)}{c+dx} dx = \int \frac{\tan(a+bx)}{c+dx} dx$$

Mathematica [A] time = 3.56978, size = 0, normalized size = 0.

$$\int \frac{\tan(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[Tan[a + b*x]/(c + d*x), x]

[Out] Integrate[Tan[a + b*x]/(c + d*x), x]

Maple [A] time = 0.222, size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)\sin(bx+a)}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)*sin(b*x+a)/(d*x+c), x)

[Out] int(sec(b*x+a)*sin(b*x+a)/(d*x+c), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (bx+a) \sin (bx+a)}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="maxima")

[Out] integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec (bx+a) \sin (bx+a)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="fricas")

[Out] integral(sec(b*x + a)*sin(b*x + a)/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin (a+bx) \sec (a+bx)}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x)

[Out] Integral(sin(a + b*x)*sec(a + b*x)/(c + d*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (bx+a) \sin (bx+a)}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="giac")

[Out] integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c), x)

$$3.214 \quad \int \frac{\tan(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\tan(a+bx)}{(c+dx)^2}, x\right)$$

[Out] Unintegrable[Tan[a + b*x]/(c + d*x)^2, x]

Rubi [A] time = 0.0196721, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\tan(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[Tan[a + b*x]/(c + d*x)^2, x]

[Out] Defer[Int][Tan[a + b*x]/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\tan(a+bx)}{(c+dx)^2} dx = \int \frac{\tan(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 5.36351, size = 0, normalized size = 0.

$$\int \frac{\tan(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Tan[a + b*x]/(c + d*x)^2, x]

[Out] Integrate[Tan[a + b*x]/(c + d*x)^2, x]

Maple [A] time = 0.235, size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)\sin(bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2, x)

[Out] int(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (bx+a) \sin (bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")

[Out] integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c)^2, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec (bx+a) \sin (bx+a)}{d^2 x^2+2 c d x+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(sec(b*x + a)*sin(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin (a+bx) \sec (a+bx)}{(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)**2,x)

[Out] Integral(sin(a + b*x)*sec(a + b*x)/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (bx+a) \sin (bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c)^2, x)

3.215 $\int (c + dx)^m \sin(a + bx) \tan(a + bx) dx$

Optimal. Leaf size=147

$$\text{Unintegrable}(\sec(a + bx)(c + dx)^m, x) + \frac{ie^{i\left(a - \frac{bc}{d}\right)}(c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m + 1, -\frac{ib(c+dx)}{d}\right)}{2b} - \frac{ie^{-i\left(a - \frac{bc}{d}\right)}(c + dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m + 1, \frac{ib(c+dx)}{d}\right)}{2b}$$

[Out] $((I/2)*E^{(I*(a - (b*c)/d))}*(c + d*x)^m*\text{Gamma}[1 + m, ((-I)*b*(c + d*x))/d]) / (b*((-I)*b*(c + d*x))/d)^m - ((I/2)*(c + d*x)^m*\text{Gamma}[1 + m, (I*b*(c + d*x))/d]) / (b*E^{(I*(a - (b*c)/d))}*((I*b*(c + d*x))/d)^m) + \text{Unintegrable}[(c + d*x)^m*\text{Sec}[a + b*x], x]$

Rubi [A] time = 0.135594, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \sin(a + bx) \tan(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x], x]

[Out] $((I/2)*E^{(I*(a - (b*c)/d))}*(c + d*x)^m*\text{Gamma}[1 + m, ((-I)*b*(c + d*x))/d]) / (b*((-I)*b*(c + d*x))/d)^m - ((I/2)*(c + d*x)^m*\text{Gamma}[1 + m, (I*b*(c + d*x))/d]) / (b*E^{(I*(a - (b*c)/d))}*((I*b*(c + d*x))/d)^m) + \text{Defer}[\text{Int}[(c + d*x)^m*\text{Sec}[a + b*x], x]]$

Rubi steps

$$\begin{aligned} \int (c + dx)^m \sin(a + bx) \tan(a + bx) dx &= - \int (c + dx)^m \cos(a + bx) dx + \int (c + dx)^m \sec(a + bx) dx \\ &= - \left(\frac{1}{2} \int e^{-i(a+bx)} (c + dx)^m dx \right) - \frac{1}{2} \int e^{i(a+bx)} (c + dx)^m dx + \int (c + dx)^m \sec(a + bx) dx \\ &= \frac{ie^{i\left(a - \frac{bc}{d}\right)}(c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, -\frac{ib(c+dx)}{d}\right)}{2b} - \frac{ie^{-i\left(a - \frac{bc}{d}\right)}(c + dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, \frac{ib(c+dx)}{d}\right)}{2b} \end{aligned}$$

Mathematica [A] time = 6.58576, size = 0, normalized size = 0.

$$\int (c + dx)^m \sin(a + bx) \tan(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x], x]

[Out] Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x], x]

Maple [A] time = 0.323, size = 0, normalized size = 0.

$$\int (dx + c)^m \sec(bx + a) (\sin(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x)`

[Out] `int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x)`

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \sec (bx + a) \sin (bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")`

[Out] `integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^2, x)`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos (bx + a)^2 - 1\right)(dx + c)^m \sec (bx + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")`

[Out] `integral(-(cos(b*x + a)^2 - 1)*(d*x + c)^m*sec(b*x + a), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)**m*sec(b*x+a)*sin(b*x+a)**2,x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \sec (bx + a) \sin (bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")`

[Out] `integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^2, x)`

3.216 $\int (c + dx)^3 \sin(a + bx) \tan(a + bx) dx$

Optimal. Leaf size=275

$$\frac{6d^2(c + dx)\text{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^3} + \frac{6d^2(c + dx)\text{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^3} + \frac{3id(c + dx)^2\text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^2} - \frac{3id(c + dx)^2\text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^2}$$

```
[Out] ((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + (6*d^3*Cos[a + b*x])/b^4 -
(3*d*(c + d*x)^2*Cos[a + b*x])/b^2 + ((3*I)*d*(c + d*x)^2*PolyLog[2, (-I)*
E^(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/
b^2 - (6*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 + (6*d^2*(c +
d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4, (-I)*E^(I*
(a + b*x))])/b^4 + ((6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4 + (6*d^2*(c
+ d*x)*Sin[a + b*x])/b^3 - ((c + d*x)^3*Sin[a + b*x])/b
```

Rubi [A] time = 0.214907, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {4407, 3296, 2638, 4181, 2531, 6609, 2282, 6589}

$$\frac{6d^2(c + dx)\text{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^3} + \frac{6d^2(c + dx)\text{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^3} + \frac{3id(c + dx)^2\text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^2} - \frac{3id(c + dx)^2\text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x], x]
```

```
[Out] ((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + (6*d^3*Cos[a + b*x])/b^4 -
(3*d*(c + d*x)^2*Cos[a + b*x])/b^2 + ((3*I)*d*(c + d*x)^2*PolyLog[2, (-I)*
E^(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/
b^2 - (6*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 + (6*d^2*(c +
d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4, (-I)*E^(I*
(a + b*x))])/b^4 + ((6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4 + (6*d^2*(c
+ d*x)*Sin[a + b*x])/b^3 - ((c + d*x)^3*Sin[a + b*x])/b
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*Sin[(a_.) + (b_.)*(x_.)]^(n_.)*Tan[(a_.) + (b
_.)*(x_.)]^(p_.), x_Symbol] := -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol
] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
```

```
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_)))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_.))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 \sin(a + bx) \tan(a + bx) dx &= - \int (c + dx)^3 \cos(a + bx) dx + \int (c + dx)^3 \sec(a + bx) dx \\
&= - \frac{2i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{(c + dx)^3 \sin(a + bx)}{b} - \frac{(3d) \int (c + dx)^2 \log(1 - ie^{i(a+bx)})}{b} \\
&= - \frac{2i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{3d(c + dx)^2 \cos(a + bx)}{b^2} + \frac{3id(c + dx)^2 \text{Li}_2(-ie^{i(a+bx)})}{b^2} \\
&= - \frac{2i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{3d(c + dx)^2 \cos(a + bx)}{b^2} + \frac{3id(c + dx)^2 \text{Li}_2(-ie^{i(a+bx)})}{b^2} \\
&= - \frac{2i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} + \frac{6d^3 \cos(a + bx)}{b^4} - \frac{3d(c + dx)^2 \cos(a + bx)}{b^2} + \frac{3id(c + dx)^2 \text{Li}_2(-ie^{i(a+bx)})}{b^2} \\
&= - \frac{2i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} + \frac{6d^3 \cos(a + bx)}{b^4} - \frac{3d(c + dx)^2 \cos(a + bx)}{b^2} + \frac{3id(c + dx)^2 \text{Li}_2(-ie^{i(a+bx)})}{b^2}
\end{aligned}$$

Mathematica [B] time = 1.5463, size = 557, normalized size = 2.03

$$-3ib^2d(c + dx)^2 \text{PolyLog}(2, -ie^{i(a+bx)}) + 3ib^2d(c + dx)^2 \text{PolyLog}(2, ie^{i(a+bx)}) + 6bcd^2 \text{PolyLog}(3, -ie^{i(a+bx)}) - 6bcd^2 \text{PolyLog}(3, ie^{i(a+bx)})$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x],x]

[Out] -(((2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^2*c^2*d*Cos[a + b*x] - 6*d^3*Cos[a + b*x] + 6*b^2*c*d^2*x*Cos[a + b*x] + 3*b^2*d^3*x^2*Cos[a + b*x] - 3*b^3*c^2*d*x*Log[1 - I*E^(I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 - I*E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1 + I*E^(I*(a + b*x))] - (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] + 6*b*c*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, I*E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))] + b^3*c^3*Sin[a + b*x] - 6*b*c*d^2*Sin[a + b*x] + 3*b^3*c^2*d*x*Sin[a + b*x] - 6*b*d^3*x*Sin[a + b*x] + 3*b^3*c*d^2*x^2*Sin[a + b*x] + b^3*d^3*x^3*Sin[a + b*x])/b^4)

Maple [B] time = 0.418, size = 901, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x)

[Out] 6*I/b^2*c^2*d*a*arctan(exp(I*(b*x+a)))+6*I/b^2*c*d^2*polylog(2,-I*exp(I*(b*x+a)))*x-6*I/b^2*c*d^2*polylog(2,I*exp(I*(b*x+a)))*x-6*I/b^3*c*d^2*a^2*arctan(exp(I*(b*x+a)))-1/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))-6*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4-3*I/b^2*c^2*d*polylog(2,I*exp(I*(b*x+a)))+3*I/b^2*d^3*polylog(2,-I*exp(I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog(2,I*exp(I*(b*x+a)))*x^2+2*I/b^4*d^3*a^3*arctan(exp(I*(b*x+a)))+3*I/b^2*c^2*d*polylog(2,-I*exp(I*(b*x+a)))+1/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))-1/b*d^3*ln(1+I*exp(I*(b*x+a)))*x^3+1/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3+6/b^3*d^3*polylog(3,I*exp(I*(b*x+a)))*x-6/b^3*d^3*polylog(3,-I*exp(I*(b*x+a)))*x+1/b^4*a^3*d^3*ln(1-I*exp(I*(b*x+a)))-6/b^3*c*d^2*polylog(3,-I*exp(I*(b*x+a)))-1/b^4*a^3*d^3*ln(1+I*exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,I*exp(I*(b*x+a)))-2*I/b*c^3*arctan(exp(I*(b*x+a)))+6*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-3/b^3*a^2*c*d^2*ln(1-I*exp(I*(b*x+a)))+3/b*c*d^2*ln(1-I*exp(I*(b*x+a)))*x^2-3/b*c*d^2*ln(1+I*exp(I*(b*x+a)))*x^2+3/b^3*a^2*c*d^2*ln(1+I*exp(I*(b*x+a)))+3/b*c^2*d*ln(1-I*exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-I*exp(I*(b*x+a)))*a-3/b*c^2*d*ln(1+I*exp(I*(b*x+a)))*a

Maxima [B] time = 2.24289, size = 1247, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")

[Out] 1/2*(c^3*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a)) - 3*a*c^2*d*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))

$$\begin{aligned} & /b + 3*a^2*c*d^2*(\log(\sin(b*x + a) + 1) - \log(\sin(b*x + a) - 1) - 2*\sin(b*x \\ & + a))/b^2 - a^3*d^3*(\log(\sin(b*x + a) + 1) - \log(\sin(b*x + a) - 1) - 2*\sin \\ & (b*x + a))/b^3 + (12*I*d^3*polylog(4, I*e^(I*b*x + I*a)) - 12*I*d^3*polylog \\ & (4, -I*e^(I*b*x + I*a)) + (-2*I*(b*x + a)^3*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3 \\ &)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*a^2*d^3)*(b*x + a))* \\ & arctan2(\cos(b*x + a), \sin(b*x + a) + 1) + (-2*I*(b*x + a)^3*d^3 + (-6*I*b*c \\ & *d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*a^2* \\ & d^3)*(b*x + a))*arctan2(\cos(b*x + a), -\sin(b*x + a) + 1) - 6*(b^2*c^2*d - 2 \\ & *a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a \\ &))*\cos(b*x + a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - \\ & 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*dilog(I*e^(I*b*x + I* \\ & a)) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + \\ & (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*dilog(-I*e^(I*b*x + I*a)) + ((b*x + \\ & a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + \\ & a^2*d^3)*(b*x + a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\sin(b*x + a) + \\ & 1) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2* \\ & a*b*c*d^2 + a^2*d^3)*(b*x + a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\sin \\ & (b*x + a) + 1) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, I*e^(I*b*x \\ & + I*a)) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, -I*e^(I*b*x + I* \\ & a)) - 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + \\ & a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*\sin(b*x + a) \\ &)/b^3)/b \end{aligned}$$

Fricas [C] time = 0.77632, size = 2591, normalized size = 9.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")

[Out] $\frac{1}{2}*(6*I*d^3*polylog(4, I*\cos(b*x + a) + \sin(b*x + a)) + 6*I*d^3*polylog(4, I*\cos(b*x + a) - \sin(b*x + a)) - 6*I*d^3*polylog(4, -I*\cos(b*x + a) + \sin(b*x + a)) - 6*I*d^3*polylog(4, -I*\cos(b*x + a) - \sin(b*x + a)) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*\cos(b*x + a) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(I*\cos(b*x + a) + \sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(I*\cos(b*x + a) - \sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-I*\cos(b*x + a) + \sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-I*\cos(b*x + a) - \sin(b*x + a)) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\log(\cos(b*x + a) + I*\sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\log(\cos(b*x + a) - I*\sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*\log(I*\cos(b*x + a) + \sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*\log(I*\cos(b*x + a) - \sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*\log(-I*\cos(b*x + a) + \sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*\log(-I*\cos(b*x + a) - \sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\log(-\cos(b*x + a) - I*\sin(b*x + a) + I) - 6*(b*d^3*x + b*c*d^2)*polylog(3, I*\cos(b*x + a) + \sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, I*\cos(b*x + a) - \sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*\cos(b*x + a) + \sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*\cos(b*x + a) - \sin(b*x + a)) - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*\sin(b*x + a))/b^4$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*sec(b*x+a)*sin(b*x+a)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \sec(bx + a) \sin(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^3*sec(b*x + a)*sin(b*x + a)^2, x)

3.217 $\int (c + dx)^2 \sin(a + bx) \tan(a + bx) dx$

Optimal. Leaf size=186

$$\frac{2id(c + dx)\text{PolyLog}(2, -ie^{i(a+bx)})}{b^2} - \frac{2id(c + dx)\text{PolyLog}(2, ie^{i(a+bx)})}{b^2} - \frac{2d^2\text{PolyLog}(3, -ie^{i(a+bx)})}{b^3} + \frac{2d^2\text{PolyLog}(3, ie^{i(a+bx)})}{b^3}$$

[Out] $((-2*I)*(c + d*x)^2*\text{ArcTan}[E^{(I*(a + b*x))}])/b - (2*d*(c + d*x)*\text{Cos}[a + b*x])/b^2 + ((2*I)*d*(c + d*x)*\text{PolyLog}[2, (-I)*E^{(I*(a + b*x))}])/b^2 - ((2*I)*d*(c + d*x)*\text{PolyLog}[2, I*E^{(I*(a + b*x))}])/b^2 - (2*d^2*\text{PolyLog}[3, (-I)*E^{(I*(a + b*x))}])/b^3 + (2*d^2*\text{PolyLog}[3, I*E^{(I*(a + b*x))}])/b^3 + (2*d^2*\text{Sin}[a + b*x])/b^3 - ((c + d*x)^2*\text{Sin}[a + b*x])/b$

Rubi [A] time = 0.146066, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$, Rules used = {4407, 3296, 2637, 4181, 2531, 2282, 6589}

$$\frac{2id(c + dx)\text{PolyLog}(2, -ie^{i(a+bx)})}{b^2} - \frac{2id(c + dx)\text{PolyLog}(2, ie^{i(a+bx)})}{b^2} - \frac{2d^2\text{PolyLog}(3, -ie^{i(a+bx)})}{b^3} + \frac{2d^2\text{PolyLog}(3, ie^{i(a+bx)})}{b^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x], x]

[Out] $((-2*I)*(c + d*x)^2*\text{ArcTan}[E^{(I*(a + b*x))}])/b - (2*d*(c + d*x)*\text{Cos}[a + b*x])/b^2 + ((2*I)*d*(c + d*x)*\text{PolyLog}[2, (-I)*E^{(I*(a + b*x))}])/b^2 - ((2*I)*d*(c + d*x)*\text{PolyLog}[2, I*E^{(I*(a + b*x))}])/b^2 - (2*d^2*\text{PolyLog}[3, (-I)*E^{(I*(a + b*x))}])/b^3 + (2*d^2*\text{PolyLog}[3, I*E^{(I*(a + b*x))}])/b^3 + (2*d^2*\text{Sin}[a + b*x])/b^3 - ((c + d*x)^2*\text{Sin}[a + b*x])/b$

Rule 4407

Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4181

Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \sin(a + bx) \tan(a + bx) dx &= - \int (c + dx)^2 \cos(a + bx) dx + \int (c + dx)^2 \sec(a + bx) dx \\ &= - \frac{2i(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{(c + dx)^2 \sin(a + bx)}{b} - \frac{(2d) \int (c + dx) \log(1 - ie^{i(a+bx)})}{b} \\ &= - \frac{2i(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{2d(c + dx) \cos(a + bx)}{b^2} + \frac{2id(c + dx) \text{Li}_2(-ie^{i(a+bx)})}{b^2} \\ &= - \frac{2i(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{2d(c + dx) \cos(a + bx)}{b^2} + \frac{2id(c + dx) \text{Li}_2(-ie^{i(a+bx)})}{b^2} \\ &= - \frac{2i(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{2d(c + dx) \cos(a + bx)}{b^2} + \frac{2id(c + dx) \text{Li}_2(-ie^{i(a+bx)})}{b^2} \end{aligned}$$

Mathematica [A] time = 0.883213, size = 315, normalized size = 1.69

$$\frac{-2ibd(c + dx)\text{PolyLog}(2, -ie^{i(a+bx)}) + 2ibd(c + dx)\text{PolyLog}(2, ie^{i(a+bx)}) + 2d^2\text{PolyLog}(3, -ie^{i(a+bx)}) - 2d^2\text{PolyLog}(3, ie^{i(a+bx)})}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x], x]
```

```
[Out] -(((2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b*c*d*Cos[a + b*x] + 2*b*d^2*x
*Cos[a + b*x] - 2*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1
- I*E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x))] + b^2*d^2*x^2
*Log[1 + I*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a +
b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] + 2*d^2*PolyLog
[3, (-I)*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, I*E^(I*(a + b*x))] + b^2*c^2*S
in[a + b*x] - 2*d^2*Sin[a + b*x] + 2*b^2*c*d*x*Sin[a + b*x] + b^2*d^2*x^2*S
in[a + b*x])/b^3)
```

Maple [B] time = 0.361, size = 512, normalized size = 2.8

$$\frac{2id^2 \operatorname{polylog}\left(2, -ie^{i(bx+a)}\right)x}{b^2} - \frac{2id^2a^2 \arctan\left(e^{i(bx+a)}\right)}{b^3} - 2 \frac{cd \ln\left(1 + ie^{i(bx+a)}\right)a}{b^2} - \frac{d^2 \ln\left(1 + ie^{i(bx+a)}\right)x^2}{b} + 2 \frac{d^2 \operatorname{polylog}\left(2, -ie^{i(bx+a)}\right)x^2}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x)

[Out] $2*I/b^2*d^2*\operatorname{polylog}(2, -I*\exp(I*(b*x+a)))*x - 2*I/b^3*d^2*a^2*\arctan(\exp(I*(b*x+a))) - 2/b^2*c*d*\ln(1+I*\exp(I*(b*x+a)))*a - 1/b*d^2*\ln(1+I*\exp(I*(b*x+a)))*x^2 + 2*d^2*\operatorname{polylog}(3, I*\exp(I*(b*x+a)))/b^3 + 1/b*d^2*\ln(1-I*\exp(I*(b*x+a)))*x^2 + 1/b^3*a^2*d^2*\ln(1+I*\exp(I*(b*x+a))) - 2*I/b^2*c*d*\operatorname{polylog}(2, I*\exp(I*(b*x+a))) - 1/2*I*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)/b^3*\exp(-I*(b*x+a)) - 2*I/b^2*d^2*\operatorname{polylog}(2, I*\exp(I*(b*x+a)))*x + 2*I/b^2*c*d*\operatorname{polylog}(2, -I*\exp(I*(b*x+a))) + 4*I/b^2*c*d*a*\arctan(\exp(I*(b*x+a))) + 2/b*c*d*\ln(1-I*\exp(I*(b*x+a)))*x + 2/b^2*c*d*\ln(1-I*\exp(I*(b*x+a)))*a - 2*d^2*\operatorname{polylog}(3, -I*\exp(I*(b*x+a)))/b^3 - 2*I/b*c^2*\arctan(\exp(I*(b*x+a))) - 2/b*c*d*\ln(1+I*\exp(I*(b*x+a)))*x - 1/b^3*a^2*d^2*\ln(1-I*\exp(I*(b*x+a))) + 1/2*I*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*\exp(I*(b*x+a))$

Maxima [B] time = 1.98873, size = 689, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")

[Out] $1/2*(c^2*(\log(\sin(b*x + a) + 1) - \log(\sin(b*x + a) - 1) - 2*\sin(b*x + a)) - 2*a*c*d*(\log(\sin(b*x + a) + 1) - \log(\sin(b*x + a) - 1) - 2*\sin(b*x + a))/b + a^2*d^2*(\log(\sin(b*x + a) + 1) - \log(\sin(b*x + a) - 1) - 2*\sin(b*x + a))/b^2 + (4*d^2*\operatorname{polylog}(3, I*e^{I*b*x + I*a}) - 4*d^2*\operatorname{polylog}(3, -I*e^{I*b*x + I*a})) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a))*\arctan^2(\cos(b*x + a), \sin(b*x + a) + 1) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a))*\arctan^2(\cos(b*x + a), -\sin(b*x + a) + 1) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*\cos(b*x + a) + (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*\operatorname{dilog}(I*e^{I*b*x + I*a}) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*\operatorname{dilog}(-I*e^{I*b*x + I*a}) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\sin(b*x + a) + 1) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\sin(b*x + a) + 1) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*\sin(b*x + a))/b^2)/b$

Fricas [C] time = 0.676838, size = 1666, normalized size = 8.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")

[Out] $-1/2*(2*d^2*\operatorname{polylog}(3, I*\cos(b*x + a) + \sin(b*x + a)) - 2*d^2*\operatorname{polylog}(3, I*\cos(b*x + a) - \sin(b*x + a)) + 2*d^2*\operatorname{polylog}(3, -I*\cos(b*x + a) + \sin(b*x + a))$

a)) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 4*(b*d^2*x + b*c*d)*cos(b*x + a) - (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*x + a)) - (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) - (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b*x + a))/b^3

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*sec(b*x+a)*sin(b*x+a)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \sec(bx + a) \sin(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^2*sec(b*x + a)*sin(b*x + a)^2, x)

3.218 $\int (c + dx) \sin(a + bx) \tan(a + bx) dx$

Optimal. Leaf size=103

$$\frac{idPolyLog\left(2, -ie^{i(a+bx)}\right)}{b^2} - \frac{idPolyLog\left(2, ie^{i(a+bx)}\right)}{b^2} - \frac{d \cos(a + bx)}{b^2} - \frac{(c + dx) \sin(a + bx)}{b} - \frac{2i(c + dx) \tan^{-1}\left(e^{i(a+bx)}\right)}{b}$$

[Out] $((-2*I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b - (d*Cos[a + b*x])/b^2 + (I*d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - ((c + d*x)*Sin[a + b*x])/b$

Rubi [A] time = 0.0675388, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4407, 3296, 2638, 4181, 2279, 2391}

$$\frac{idPolyLog\left(2, -ie^{i(a+bx)}\right)}{b^2} - \frac{idPolyLog\left(2, ie^{i(a+bx)}\right)}{b^2} - \frac{d \cos(a + bx)}{b^2} - \frac{(c + dx) \sin(a + bx)}{b} - \frac{2i(c + dx) \tan^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Sin[a + b*x]*Tan[a + b*x], x]

[Out] $((-2*I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b - (d*Cos[a + b*x])/b^2 + (I*d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - ((c + d*x)*Sin[a + b*x])/b$

Rule 4407

Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4181

Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_.) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))]

)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned} \int (c + dx) \sin(a + bx) \tan(a + bx) dx &= - \int (c + dx) \cos(a + bx) dx + \int (c + dx) \sec(a + bx) dx \\ &= - \frac{2i(c + dx) \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{(c + dx) \sin(a + bx)}{b} - \frac{d \int \log\left(1 - ie^{i(a+bx)}\right) dx}{b} \\ &= - \frac{2i(c + dx) \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d \cos(a + bx)}{b^2} - \frac{(c + dx) \sin(a + bx)}{b} + \frac{(id) \operatorname{Subst}\left(\int \log\left(1 - ie^{i(a+bx)}\right) dx\right)}{b} \\ &= - \frac{2i(c + dx) \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d \cos(a + bx)}{b^2} + \frac{id \operatorname{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2} - \frac{id \operatorname{Li}_2\left(ie^{i(a+bx)}\right)}{b^2} \end{aligned}$$

Mathematica [B] time = 0.423804, size = 213, normalized size = 2.07

$$\frac{d \left(i \left(\operatorname{PolyLog}\left(2, -e^{i(-a-bx+\frac{\pi}{2})}\right) - \operatorname{PolyLog}\left(2, e^{i(-a-bx+\frac{\pi}{2})}\right) \right) + (-a - bx + \frac{\pi}{2}) \left(\log\left(1 - e^{i(-a-bx+\frac{\pi}{2})}\right) - \log\left(1 + e^{i(-a-bx+\frac{\pi}{2})}\right) \right) \right)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Sin[a + b*x]*Tan[a + b*x], x]

[Out] (c*ArcTanh[Sin[a + b*x]])/b + (d*((-a + Pi/2 - b*x)*(Log[1 - E^(I*(-a + Pi/2 - b*x))] - Log[1 + E^(I*(-a + Pi/2 - b*x))]) - (-a + Pi/2)*Log[Tan[(-a + Pi/2 - b*x)/2]] + I*(PolyLog[2, -E^(I*(-a + Pi/2 - b*x))] - PolyLog[2, E^(I*(-a + Pi/2 - b*x))]))/b^2 - (d*Cos[b*x]*(Cos[a] + b*x*Sin[a]))/b^2 - (d*(b*x*Cos[a] - Sin[a])*Sin[b*x])/b^2 - (c*Sin[a + b*x])/b

Maple [B] time = 0.227, size = 221, normalized size = 2.2

$$\frac{\frac{i}{2}(dxb + bc + id)e^{i(bx+a)}}{b^2} - \frac{\frac{i}{2}(dxb + bc - id)e^{-i(bx+a)}}{b^2} - \frac{2ic \arctan(e^{i(bx+a)})}{b} - \frac{d \ln(1 + ie^{i(bx+a)})x}{b} - \frac{d \ln(1 + ie^{i(bx+a)})}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x)

[Out] 1/2*I*(d*x*b+b*c+I*d)/b^2*exp(I*(b*x+a))-1/2*I*(d*x*b+b*c-I*d)/b^2*exp(-I*(b*x+a))-2*I/b*c*arctan(exp(I*(b*x+a)))-1/b*d*ln(1+I*exp(I*(b*x+a)))*x-1/b^2*d*ln(1+I*exp(I*(b*x+a)))*a+1/b*d*ln(1-I*exp(I*(b*x+a)))*x+1/b^2*d*ln(1-I*exp(I*(b*x+a)))*a+I/b^2*d*dilog(1+I*exp(I*(b*x+a)))-I/b^2*d*dilog(1-I*exp(I*(b*x+a)))+2*I/b^2*d*a*arctan(exp(I*(b*x+a)))

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 0.594791, size = 905, normalized size = 8.79

$$\frac{2d \cos(bx + a) + i d \operatorname{Li}_2(i \cos(bx + a) + \sin(bx + a)) + i d \operatorname{Li}_2(i \cos(bx + a) - \sin(bx + a)) - i d \operatorname{Li}_2(-i \cos(bx + a) + \sin(bx + a)) - i d \operatorname{Li}_2(-i \cos(bx + a) - \sin(bx + a))}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*cos(b*x + a) + I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(I*cos(b*x + a) - sin(b*x + a)) - I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b*d*x + a*d)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x + a*d)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b*c - a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 2*(b*d*x + b*c)*sin(b*x + a))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \sin^2(a + bx) \sec(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*sin(a + b*x)**2*sec(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \sec(bx + a) \sin(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a)*sin(b*x + a)^2, x)
```

$$3.219 \quad \int \frac{\sin(a+bx) \tan(a+bx)}{c+dx} dx$$

Optimal. Leaf size=68

$$\text{Unintegrable}\left(\frac{\sec(a+bx)}{c+dx}, x\right) - \frac{\cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{d} + \frac{\sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d}$$

[Out] -((Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d) + (Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d + Unintegrable[Sec[a + b*x]/(c + d*x), x]

Rubi [A] time = 0.110089, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sin(a+bx) \tan(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x), x]

[Out] -((Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d) + (Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d + Defer[Int][Sec[a + b*x]/(c + d*x), x]

Rubi steps

$$\begin{aligned} \int \frac{\sin(a+bx) \tan(a+bx)}{c+dx} dx &= - \int \frac{\cos(a+bx)}{c+dx} dx + \int \frac{\sec(a+bx)}{c+dx} dx \\ &= - \left(\cos\left(a - \frac{bc}{d}\right) \int \frac{\cos\left(\frac{bc}{d} + bx\right)}{c+dx} dx \right) + \sin\left(a - \frac{bc}{d}\right) \int \frac{\sin\left(\frac{bc}{d} + bx\right)}{c+dx} dx + \int \frac{\sec(a+bx)}{c+dx} dx \\ &= - \frac{\cos\left(a - \frac{bc}{d}\right) \text{Ci}\left(\frac{bc}{d} + bx\right)}{d} + \frac{\sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d} + \int \frac{\sec(a+bx)}{c+dx} dx \end{aligned}$$

Mathematica [A] time = 6.07811, size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx) \tan(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x), x]

[Out] Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x), x]

Maple [A] time = 0.34, size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a) (\sin(bx+a))^2}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(\cos(bx+a)^2-1)\sec(bx+a)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a+bx)\sec(a+bx)}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**2*sec(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)\sin(bx+a)^2}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(b*x + a)^2/(d*x + c), x)
```

$$3.220 \quad \int \frac{\sin(a+bx) \tan(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=86

$$\text{Unintegrable}\left(\frac{\sec(a+bx)}{(c+dx)^2}, x\right) + \frac{b \sin\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{d^2} + \frac{b \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d^2} + \frac{\cos(a+bx)}{d(c+dx)}$$

[Out] Cos[a + b*x]/(d*(c + d*x)) + (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d^2 + (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Unintegrable[Sec[a + b*x]/(c + d*x)^2, x]

Rubi [A] time = 0.138904, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sin(a+bx) \tan(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]

[Out] Cos[a + b*x]/(d*(c + d*x)) + (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d^2 + (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Defer[Int][Sec[a + b*x]/(c + d*x)^2, x]

Rubi steps

$$\begin{aligned} \int \frac{\sin(a+bx) \tan(a+bx)}{(c+dx)^2} dx &= - \int \frac{\cos(a+bx)}{(c+dx)^2} dx + \int \frac{\sec(a+bx)}{(c+dx)^2} dx \\ &= \frac{\cos(a+bx)}{d(c+dx)} + \frac{b \int \frac{\sin(a+bx)}{c+dx} dx}{d} + \int \frac{\sec(a+bx)}{(c+dx)^2} dx \\ &= \frac{\cos(a+bx)}{d(c+dx)} + \frac{\left(b \cos\left(a - \frac{bc}{d}\right)\right) \int \frac{\sin\left(\frac{bc}{d} + bx\right)}{c+dx} dx}{d} + \frac{\left(b \sin\left(a - \frac{bc}{d}\right)\right) \int \frac{\cos\left(\frac{bc}{d} + bx\right)}{c+dx} dx}{d} + \int \frac{\sec(a+bx)}{(c+dx)^2} dx \\ &= \frac{\cos(a+bx)}{d(c+dx)} + \frac{b \text{Ci}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{d^2} + \frac{b \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d^2} + \int \frac{\sec(a+bx)}{(c+dx)^2} dx \end{aligned}$$

Mathematica [A] time = 7.23992, size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx) \tan(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]

[Out] Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]

Maple [A] time = 0.607, size = 0, normalized size = 0.

$$\int \frac{\sec(bx + a) (\sin(bx + a))^2}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x)

[Out] int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(\cos(bx + a)^2 - 1) \sec(bx + a)}{d^2x^2 + 2cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a + bx) \sec(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(b*x+a)**2/(d*x+c)**2,x)

[Out] Integral(sin(a + b*x)**2*sec(a + b*x)/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx + a) \sin(bx + a)^2}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(b*x + a)^2/(d*x + c)^2, x)
```

3.221 $\int (c + dx)^m \sin^2(a + bx) \tan(a + bx) dx$

Optimal. Leaf size=151

$$\text{Unintegrable}(\tan(a + bx)(c + dx)^m, x) + \frac{2^{-m-3} e^{2i\left(a - \frac{bc}{d}\right)} (c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m + 1, -\frac{2ib(c+dx)}{d}\right)}{b} + \frac{2^{-m-3} e^{-2i\left(a - \frac{bc}{d}\right)} (c + dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m + 1, \frac{2ib(c+dx)}{d}\right)}{b}$$

[Out] $(2^{(-3 - m)} E^{((2I)*(a - (b*c)/d)}) * (c + d*x)^m * \text{Gamma}[1 + m, ((-2I)*b*(c + d*x))/d]) / (b * (((-I)*b*(c + d*x))/d)^m) + (2^{(-3 - m)} * (c + d*x)^m * \text{Gamma}[1 + m, ((2I)*b*(c + d*x))/d]) / (b * E^{((2I)*(a - (b*c)/d)}) * ((I*b*(c + d*x))/d)^m) + \text{Unintegrable}[(c + d*x)^m * \text{Tan}[a + b*x], x]$

Rubi [A] time = 0.170835, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \sin^2(a + bx) \tan(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m * Sin[a + b*x]^2 * Tan[a + b*x], x]

[Out] $(2^{(-3 - m)} E^{((2I)*(a - (b*c)/d)}) * (c + d*x)^m * \text{Gamma}[1 + m, ((-2I)*b*(c + d*x))/d]) / (b * (((-I)*b*(c + d*x))/d)^m) + (2^{(-3 - m)} * (c + d*x)^m * \text{Gamma}[1 + m, ((2I)*b*(c + d*x))/d]) / (b * E^{((2I)*(a - (b*c)/d)}) * ((I*b*(c + d*x))/d)^m) + \text{Defer}[\text{Int}][(c + d*x)^m * \text{Tan}[a + b*x], x]$

Rubi steps

$$\begin{aligned} \int (c + dx)^m \sin^2(a + bx) \tan(a + bx) dx &= - \int (c + dx)^m \cos(a + bx) \sin(a + bx) dx + \int (c + dx)^m \tan(a + bx) dx \\ &= - \int \frac{1}{2} (c + dx)^m \sin(2a + 2bx) dx + \int (c + dx)^m \tan(a + bx) dx \\ &= - \left(\frac{1}{2} \int (c + dx)^m \sin(2a + 2bx) dx \right) + \int (c + dx)^m \tan(a + bx) dx \\ &= - \left(\frac{1}{4} i \int e^{-i(2a+2bx)} (c + dx)^m dx \right) + \frac{1}{4} i \int e^{i(2a+2bx)} (c + dx)^m dx + \int (c + dx)^m \tan(a + bx) dx \\ &= \frac{2^{-3-m} e^{2i\left(a - \frac{bc}{d}\right)} (c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, -\frac{2ib(c+dx)}{d}\right)}{b} + \frac{2^{-3-m} e^{-2i\left(a - \frac{bc}{d}\right)} (c + dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, \frac{2ib(c+dx)}{d}\right)}{b} + \int (c + dx)^m \tan(a + bx) dx \end{aligned}$$

Mathematica [A] time = 7.77205, size = 0, normalized size = 0.

$$\int (c + dx)^m \sin^2(a + bx) \tan(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m * Sin[a + b*x]^2 * Tan[a + b*x], x]

[Out] Integrate[(c + d*x)^m * Sin[a + b*x]^2 * Tan[a + b*x], x]

Maple [A] time = 0.274, size = 0, normalized size = 0.

$$\int (dx + c)^m \sec(bx + a) (\sin(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x)

[Out] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \sec(bx + a) \sin(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")

[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^3, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-(\cos(bx + a))^2 - 1\right)(dx + c)^m \sec(bx + a) \sin(bx + a), x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")

[Out] integral(-(cos(b*x + a)^2 - 1)*(d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**m*sec(b*x+a)*sin(b*x+a)**3,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \sec(bx + a) \sin(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")

[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^3, x)

3.222 $\int (c + dx)^3 \sin^2(a + bx) \tan(a + bx) dx$

Optimal. Leaf size=251

$$-\frac{3d^2(c + dx)\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} + \frac{3id(c + dx)^2\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^2} - \frac{3id^3\text{PolyLog}\left(4, -e^{2i(a+bx)}\right)}{4b^4} + \frac{3d^2(c + dx)}{4b}$$

```
[Out] (-3*d^3*x)/(8*b^3) + (c + d*x)^3/(4*b) + ((I/4)*(c + d*x)^4)/d - ((c + d*x)^3*Log[1 + E^((2*I)*(a + b*x))])/b + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (3*d^3*Cos[a + b*x]*Sin[a + b*x])/(8*b^4) - (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(4*b^2) + (3*d^2*(c + d*x)*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)^3*Sin[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.297539, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {4407, 4404, 3311, 32, 2635, 8, 3719, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c + dx)\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} + \frac{3id(c + dx)^2\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^2} - \frac{3id^3\text{PolyLog}\left(4, -e^{2i(a+bx)}\right)}{4b^4} + \frac{3d^2(c + dx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sin[a + b*x]^2*Tan[a + b*x], x]
```

```
[Out] (-3*d^3*x)/(8*b^3) + (c + d*x)^3/(4*b) + ((I/4)*(c + d*x)^4)/d - ((c + d*x)^3*Log[1 + E^((2*I)*(a + b*x))])/b + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (3*d^3*Cos[a + b*x]*Sin[a + b*x])/(8*b^4) - (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(4*b^2) + (3*d^2*(c + d*x)*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)^3*Sin[a + b*x]^2)/(2*b)
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3719

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n]/a)/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 6609

Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[(e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d}

, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned}
 \int (c + dx)^3 \sin^2(a + bx) \tan(a + bx) dx &= - \int (c + dx)^3 \cos(a + bx) \sin(a + bx) dx + \int (c + dx)^3 \tan(a + bx) dx \\
 &= \frac{i(c + dx)^4}{4d} - \frac{(c + dx)^3 \sin^2(a + bx)}{2b} - 2i \int \frac{e^{2i(a+bx)}(c + dx)^3}{1 + e^{2i(a+bx)}} dx + \frac{(3d) \int (c + dx)^3 \tan(a + bx) dx}{4b^2} \\
 &= \frac{i(c + dx)^4}{4d} - \frac{(c + dx)^3 \log(1 + e^{2i(a+bx)})}{b} - \frac{3d(c + dx)^2 \cos(a + bx) \sin(a + bx)}{4b^2} \\
 &= \frac{(c + dx)^3}{4b} + \frac{i(c + dx)^4}{4d} - \frac{(c + dx)^3 \log(1 + e^{2i(a+bx)})}{b} + \frac{3id(c + dx)^2 \text{Li}_2(-e^{2i(a+bx)})}{2b^2} \\
 &= -\frac{3d^3x}{8b^3} + \frac{(c + dx)^3}{4b} + \frac{i(c + dx)^4}{4d} - \frac{(c + dx)^3 \log(1 + e^{2i(a+bx)})}{b} + \frac{3id(c + dx)^2 \text{Li}_2(-e^{2i(a+bx)})}{2b^2} \\
 &= -\frac{3d^3x}{8b^3} + \frac{(c + dx)^3}{4b} + \frac{i(c + dx)^4}{4d} - \frac{(c + dx)^3 \log(1 + e^{2i(a+bx)})}{b} + \frac{3id(c + dx)^2 \text{Li}_2(-e^{2i(a+bx)})}{2b^2} \\
 &= -\frac{3d^3x}{8b^3} + \frac{(c + dx)^3}{4b} + \frac{i(c + dx)^4}{4d} - \frac{(c + dx)^3 \log(1 + e^{2i(a+bx)})}{b} + \frac{3id(c + dx)^2 \text{Li}_2(-e^{2i(a+bx)})}{2b^2}
 \end{aligned}$$

Mathematica [B] time = 7.14703, size = 1720, normalized size = 6.85

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^3*Sin[a + b*x]^2*Tan[a + b*x], x]

[Out] ((-I/4)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))]*Sec[a]/(b^3*E^(I*a)) - (I/8)*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a) - ((4*I)*(1 + E^((-2*I)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1 + E^((2*I)*a))*(2*b^2*x^2*PolyLog[2, -E^((-2*I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-2*I)*(a + b*x))] - PolyLog[4, -E^((-2*I)*(a + b*x))]))/(b^4*E^((2*I)*a))*Sec[a] - (c^3*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x]) - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]])]) + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]) + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]])])))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + Sec[a]*(Cos[2*a + 2*b*x]/(64*b^4) - ((I/64)*Sin[2*a + 2*b*x])/b^4)*(8*b^3*c^3*Cos[a] - (12*I)*b^2*c^2*d*Cos[a] - 12*b*c*d^2*Cos[a] + (6*I)*d^3*Cos[a] + 24*b^3*c^2*d*x*Cos[a] - (24*I)*b^2*c*d^2*x*Cos[a] - 12*b*d^3*x*Cos[a] + 24*b^3*c*d^2*x^2*Cos[a] - (12*I)*b^2*d^3*x^2*Cos[a] + 8*b^3*d^3*x^3*Cos[a] + (32*I)*b^4*c^3*x*Cos[a + 2*b*x] + (48*I)*b^4*c^2*d*x^2*Cos[a + 2*b*x] + (32*I)*b^4*c*d^2*x^3*Cos[a + 2*b*x] + (8*I)*b^4*d^3*x^4*Cos[a + 2*b*x] - (32*I)*b^4*c^3*x*Cos[3*a + 2*b*x] - (48*I)*b^4*c^2*d*x^2*Cos[3*a + 2*b*x] - (32*I)*b^4*c*d^2*x^3*Cos[3*a + 2*b*x] - (8*I)*b^4*d^3*x^4*Cos[3*a + 2*b*x] + 4*b^3*c^3*Cos[3*a + 4*b*x] + (6*I)*b^2*c^2*d*Cos[3*a + 4*b*x] - 6*b*c*d^2*Cos[3*a + 4*b*x] - (3*I)*d^3*Cos[3*a + 4*b*x] + 12*b^3*c^2*d*x*Cos[3*a + 4*b*x] + (12*I)*b^2*c*d^2*x*Cos[3*a + 4*b*x] - 6*b*d^3*x*Cos[3*a + 4*b*x] + 12*b^3*c*d^2*x^2*Cos[3*a + 4*b*x] + (6*I)*b^2*d^3*x^2*Cos[3*a + 4*b*x] + 4*b^3*d^3*x^3*Cos[3*a + 4*b*x] + 4*b^3*c^3*Cos[5*a + 4*b*x] + (6*I)*b^2*c^2*d*Cos[5*a + 4*b*x] - 6*b*c*d^2*Cos[5*a + 4*b*x] - (3*I)*d^3*Cos[5*a + 4*b*x] + 12*b^3*c^2*d*x*Cos[5*a + 4*b*x] + (12*I)*b^2*c*

$$d^2*x*\cos[5*a + 4*b*x] - 6*b*d^3*x*\cos[5*a + 4*b*x] + 12*b^3*c*d^2*x^2*\cos[5*a + 4*b*x] + (6*I)*b^2*d^3*x^2*\cos[5*a + 4*b*x] + 4*b^3*d^3*x^3*\cos[5*a + 4*b*x] - 32*b^4*c^3*x*\sin[a + 2*b*x] - 48*b^4*c^2*d*x^2*\sin[a + 2*b*x] - 32*b^4*c*d^2*x^3*\sin[a + 2*b*x] - 8*b^4*d^3*x^4*\sin[a + 2*b*x] + 32*b^4*c^3*x*\sin[3*a + 2*b*x] + 48*b^4*c^2*d*x^2*\sin[3*a + 2*b*x] + 32*b^4*c*d^2*x^3*\sin[3*a + 2*b*x] + 8*b^4*d^3*x^4*\sin[3*a + 2*b*x] + (4*I)*b^3*c^3*\sin[3*a + 4*b*x] - 6*b^2*c^2*d*\sin[3*a + 4*b*x] - (6*I)*b*c*d^2*\sin[3*a + 4*b*x] + 3*d^3*\sin[3*a + 4*b*x] + (12*I)*b^3*c^2*d*x*\sin[3*a + 4*b*x] - 12*b^2*c*d^2*x*\sin[3*a + 4*b*x] - (6*I)*b*d^3*x*\sin[3*a + 4*b*x] + (12*I)*b^3*c*d^2*x^2*\sin[3*a + 4*b*x] - 6*b^2*d^3*x^2*\sin[3*a + 4*b*x] + (4*I)*b^3*d^3*x^3*\sin[3*a + 4*b*x] + (4*I)*b^3*c^3*\sin[5*a + 4*b*x] - 6*b^2*c^2*d*\sin[5*a + 4*b*x] - (6*I)*b*c*d^2*\sin[5*a + 4*b*x] + 3*d^3*\sin[5*a + 4*b*x] + (12*I)*b^3*c^2*d*x*\sin[5*a + 4*b*x] - 12*b^2*c*d^2*x*\sin[5*a + 4*b*x] - (6*I)*b*d^3*x*\sin[5*a + 4*b*x] + (12*I)*b^3*c*d^2*x^2*\sin[5*a + 4*b*x] - 6*b^2*d^3*x^2*\sin[5*a + 4*b*x] + (4*I)*b^3*d^3*x^3*\sin[5*a + 4*b*x]$$

Maple [B] time = 0.285, size = 641, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x)

[Out] $-1/b*c^3*\ln(\exp(2*I*(b*x+a))+1)-3/2/b^3*c*d^2*\text{polylog}(3,-\exp(2*I*(b*x+a)))-3/2/b^3*d^3*\text{polylog}(3,-\exp(2*I*(b*x+a)))*x+3/2*I/b^4*d^3*a^4+1/4*I*d^3*x^4+I*c*d^2*x^3-I*c^3*x+3/2*I*c^2*d*x^2+3/2*I/b^2*c^2*d*\text{polylog}(2,-\exp(2*I*(b*x+a)))-3/b*c^2*d*\ln(\exp(2*I*(b*x+a))+1)*x-3/b*c*d^2*\ln(\exp(2*I*(b*x+a))+1)*x^2-1/b*d^3*\ln(\exp(2*I*(b*x+a))+1)*x^3+3/2*I/b^2*d^3*\text{polylog}(2,-\exp(2*I*(b*x+a)))*x^2+6/b^3*c*d^2*a^2*\ln(\exp(I*(b*x+a)))-6/b^2*c^2*d*a*\ln(\exp(I*(b*x+a)))-4*I/b^3*a^3*c*d^2+2*I/b^3*d^3*a^3*x+3*I/b^2*a^2*c^2*d-3/4*I*d^3*\text{polylog}(4,-\exp(2*I*(b*x+a)))/b^4+3*I/b^2*c*d^2*\text{polylog}(2,-\exp(2*I*(b*x+a)))*x-2/b^4*d^3*a^3*\ln(\exp(I*(b*x+a)))+6*I/b*a*c^2*d*x-6*I/b^2*a^2*c*d^2*x+1/32*(4*d^3*x^3*b^3+6*I*b^2*d^3*x^2+12*b^3*c*d^2*x^2+12*I*b^2*c*d^2*x+12*b^3*c^2*d*x+6*I*b^2*c^2*d+4*b^3*c^3-6*b*d^3*x-3*I*d^3-6*c*d^2*b)/b^4*\exp(2*I*(b*x+a))+2/b*c^3*\ln(\exp(I*(b*x+a)))+1/32*(4*d^3*x^3*b^3-6*I*b^2*d^3*x^2+12*b^3*c*d^2*x^2-12*I*b^2*c*d^2*x+12*b^3*c^2*d*x-6*I*b^2*c^2*d+4*b^3*c^3-6*b*d^3*x+3*I*d^3-6*c*d^2*b)/b^4*\exp(-2*I*(b*x+a))$

Maxima [B] time = 2.09739, size = 925, normalized size = 3.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")

[Out] $-1/48*(24*(\sin(b*x + a)^2 + \log(\sin(b*x + a)^2 - 1))*c^3 - 72*(\sin(b*x + a)^2 + \log(\sin(b*x + a)^2 - 1))*a*c^2*d/b + 72*(\sin(b*x + a)^2 + \log(\sin(b*x + a)^2 - 1))*a^2*c*d^2/b^2 - 24*(\sin(b*x + a)^2 + \log(\sin(b*x + a)^2 - 1))*a^3*d^3/b^3 + (-12*I*(b*x + a)^4*d^3 + (-48*I*b*c*d^2 + 48*I*a*d^3)*(b*x + a)^3 + 48*I*d^3*\text{polylog}(4, -e^{(2*I*b*x + 2*I*a)}) + (-72*I*b^2*c^2*d + 144*I*a*b*c*d^2 - 72*I*a^2*d^3)*(b*x + a)^2 + (64*I*(b*x + a)^3*d^3 + (144*I*b*c*d^2 - 144*I*a*d^3)*(b*x + a)^2 + (144*I*b^2*c^2*d - 288*I*a*b*c*d^2 + 144*$

$$\begin{aligned} & I*a^2*d^3*(b*x + a)*\arctan2(\sin(2*b*x + 2*a), \cos(2*b*x + 2*a) + 1) - 6*(\\ & 2*(b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + \\ & 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 - 1)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) \\ &) + (-72*I*b^2*c^2*d + 144*I*a*b*c*d^2 - 96*I*(b*x + a)^2*d^3 - 72*I*a^2*d^ \\ & 3 + (-144*I*b*c*d^2 + 144*I*a*d^3)*(b*x + a))*\operatorname{dilog}(-e^{(2*I*b*x + 2*I*a)}) + \\ & 8*(4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2* \\ & a*b*c*d^2 + a^2*d^3)*(b*x + a))*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 \\ & + 2*\cos(2*b*x + 2*a) + 1) + 24*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*\operatorname{poly} \\ & \operatorname{log}(3, -e^{(2*I*b*x + 2*I*a)}) + 9*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^ \\ & 2*d^3 + (2*a^2 - 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))/ \\ & b^3)/b \end{aligned}$$

Fricas [C] time = 0.843416, size = 2753, normalized size = 10.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 24*I*d^3*\operatorname{polylog}(4, I*\cos(b*x + a) \\ & + \sin(b*x + a)) + 24*I*d^3*\operatorname{polylog}(4, I*\cos(b*x + a) - \sin(b*x + a)) + 24*I \\ & *d^3*\operatorname{polylog}(4, -I*\cos(b*x + a) + \sin(b*x + a)) - 24*I*d^3*\operatorname{polylog}(4, -I*\cos \\ & (b*x + a) - \sin(b*x + a)) - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 \\ & - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*\cos(b*x + a)^2 + 3*(2*b^2*d^3*x^2 \\ & + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*\cos(b*x + a)*\sin(b*x + a) + 3*(2*b^3* \\ & c^2*d - b*d^3)*x - (-12*I*b^2*d^3*x^2 - 24*I*b^2*c*d^2*x - 12*I*b^2*c^2*d)* \\ & \operatorname{dilog}(I*\cos(b*x + a) + \sin(b*x + a)) - (12*I*b^2*d^3*x^2 + 24*I*b^2*c*d^2*x \\ & + 12*I*b^2*c^2*d)*\operatorname{dilog}(I*\cos(b*x + a) - \sin(b*x + a)) - (12*I*b^2*d^3*x^2 \\ & + 24*I*b^2*c*d^2*x + 12*I*b^2*c^2*d)*\operatorname{dilog}(-I*\cos(b*x + a) + \sin(b*x + a)) \\ & - (-12*I*b^2*d^3*x^2 - 24*I*b^2*c*d^2*x - 12*I*b^2*c^2*d)*\operatorname{dilog}(-I*\cos(b*x \\ & + a) - \sin(b*x + a)) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^ \\ & 3)*\log(\cos(b*x + a) + I*\sin(b*x + a) + I) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3* \\ & a^2*b*c*d^2 - a^3*d^3)*\log(\cos(b*x + a) - I*\sin(b*x + a) + I) + 4*(b^3*d^3* \\ & x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3 \\ & *d^3)*\log(I*\cos(b*x + a) + \sin(b*x + a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2 \\ & *x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*\log(I*\cos(b \\ & *x + a) - \sin(b*x + a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2* \\ & d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*\log(-I*\cos(b*x + a) + \sin(b* \\ & x + a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^ \\ & 2*d - 3*a^2*b*c*d^2 + a^3*d^3)*\log(-I*\cos(b*x + a) - \sin(b*x + a) + 1) + 4* \\ & (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\log(-\cos(b*x + a) + I*s \\ & \sin(b*x + a) + I) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\log \\ & (-\cos(b*x + a) - I*\sin(b*x + a) + I) + 24*(b*d^3*x + b*c*d^2)*\operatorname{polylog}(3, I \\ & *\cos(b*x + a) + \sin(b*x + a)) + 24*(b*d^3*x + b*c*d^2)*\operatorname{polylog}(3, I*\cos(b*x \\ & + a) - \sin(b*x + a)) + 24*(b*d^3*x + b*c*d^2)*\operatorname{polylog}(3, -I*\cos(b*x + a) + \\ & \sin(b*x + a)) + 24*(b*d^3*x + b*c*d^2)*\operatorname{polylog}(3, -I*\cos(b*x + a) - \sin(b* \\ & x + a)))/b^4 \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((d*x+c)**3*sec(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \sec (bx + a) \sin (bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)*sin(b*x + a)^3, x)
```

3.223 $\int (c + dx)^2 \sin^2(a + bx) \tan(a + bx) dx$

Optimal. Leaf size=184

$$\frac{id(c + dx)\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^2} - \frac{d^2\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} - \frac{d(c + dx)\sin(a + bx)\cos(a + bx)}{2b^2} + \frac{d^2\sin^2(a + bx)}{4b^3} - (c$$

```
[Out] (c*d*x)/(2*b) + (d^2*x^2)/(4*b) + ((I/3)*(c + d*x)^3)/d - ((c + d*x)^2*Log[
1 + E^((2*I)*(a + b*x))])/b + (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x)
)])/b^2 - (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) - (d*(c + d*x)*Cos
[a + b*x]*Sin[a + b*x])/(2*b^2) + (d^2*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)
^2*Sin[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.228075, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4407, 4404, 3310, 3719, 2190, 2531, 2282, 6589}

$$\frac{id(c + dx)\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^2} - \frac{d^2\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} - \frac{d(c + dx)\sin(a + bx)\cos(a + bx)}{2b^2} + \frac{d^2\sin^2(a + bx)}{4b^3} - (c$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sin[a + b*x]^2*Tan[a + b*x], x]
```

```
[Out] (c*d*x)/(2*b) + (d^2*x^2)/(4*b) + ((I/3)*(c + d*x)^3)/d - ((c + d*x)^2*Log[
1 + E^((2*I)*(a + b*x))])/b + (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x)
)])/b^2 - (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) - (d*(c + d*x)*Cos
[a + b*x]*Sin[a + b*x])/(2*b^2) + (d^2*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)
^2*Sin[a + b*x]^2)/(2*b)
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :>
Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
+ f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
```

[m, 0]

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))] * ((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \sin^2(a + bx) \tan(a + bx) dx &= - \int (c + dx)^2 \cos(a + bx) \sin(a + bx) dx + \int (c + dx)^2 \tan(a + bx) dx \\ &= \frac{i(c + dx)^3}{3d} - \frac{(c + dx)^2 \sin^2(a + bx)}{2b} - 2i \int \frac{e^{2i(a+bx)}(c + dx)^2}{1 + e^{2i(a+bx)}} dx + \frac{d \int (c + dx) \tan(a + bx) dx}{b} \\ &= \frac{i(c + dx)^3}{3d} - \frac{(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b} - \frac{d(c + dx) \cos(a + bx) \sin(a + bx)}{2b^2} \\ &= \frac{cdx}{2b} + \frac{d^2x^2}{4b} + \frac{i(c + dx)^3}{3d} - \frac{(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b} + \frac{id(c + dx) \text{Li}_2(-e^{2i(a+bx)})}{b^2} \\ &= \frac{cdx}{2b} + \frac{d^2x^2}{4b} + \frac{i(c + dx)^3}{3d} - \frac{(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b} + \frac{id(c + dx) \text{Li}_2(-e^{2i(a+bx)})}{b^2} \\ &= \frac{cdx}{2b} + \frac{d^2x^2}{4b} + \frac{i(c + dx)^3}{3d} - \frac{(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b} + \frac{id(c + dx) \text{Li}_2(-e^{2i(a+bx)})}{b^2} \end{aligned}$$

Mathematica [B] time = 6.4593, size = 518, normalized size = 2.82

$$cd \csc(a) \sec(a) \left(b^2 x^2 e^{-i \tan^{-1}(\cot(a))} - \frac{\cot(a) \left(i \text{PolyLog} \left(2, e^{2i(bx - \tan^{-1}(\cot(a)))} \right) \right) + ibx(-2 \tan^{-1}(\cot(a)) - \pi) - 2(bx - \tan^{-1}(\cot(a))) \log \left(1 - e^{2i(bx - \tan^{-1}(\cot(a)))} \right)}{\sqrt{\cot^2(a) - 1}} \right)}{b^2 \sqrt{\csc^2(a) (\sin^2(a) + \cos^2(a))}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^2*Sin[a + b*x]^2*Tan[a + b*x], x]

[Out] ((-I/12)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))]*Sec[a])/(b^3*E^(I*a)) - (c^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x]) - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]])])]) + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]]) + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]])])]/Sqrt[1 + Cot[a]^2])*Sec[a]/(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Cos[2*b*x]*(2*b^2*c^2*Cos[2*a] - d^2*Cos[2*a] + 4*b^2*c*d*x*Cos[2*a] + 2*b^2*d^2*x^2*Cos[2*a] - 2*b*c*d*Sin[2*a] - 2*b*d^2*x*Sin[2*a]))/(8*b^3) - ((2*b*c*d*Cos[2*a] + 2*b*d^2*x*Cos[2*a] + 2*b^2*c^2*Sin[2*a] - d^2*Sin[2*a] + 4*b^2*c*d*x*Sin[2*a] + 2*b^2*d^2*x^2*Sin[2*a])*Sin[2*b*x])/(8*b^3) + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Tan[a])/3

Maple [B] time = 0.353, size = 379, normalized size = 2.1

$$\frac{2ia^2cd}{b^2} + icdx^2 - \frac{4i}{3}a^3d^2 + \frac{(2d^2x^2b^2 + 2ibd^2x + 4b^2cdx + 2ibcd + 2b^2c^2 - d^2)e^{2i(bx+a)}}{16b^3} + \frac{(2d^2x^2b^2 - 2ibd^2x + 4b^2cdx + 2ibcd + 2b^2c^2 - d^2)e^{2i(bx+a)}}{16b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^3,x)

[Out] 2*I/b^2*a^2*c*d+I*c*d*x^2-4/3*I/b^3*a^3*d^2+1/16*(2*d^2*x^2*b^2+2*I*b*d^2*x+4*b^2*c*d*x+2*I*b*c*d+2*b^2*c^2-d^2)/b^3*exp(2*I*(b*x+a))+1/16*(2*d^2*x^2*b^2-2*I*b*d^2*x+4*b^2*c*d*x-2*I*b*c*d+2*b^2*c^2-d^2)/b^3*exp(-2*I*(b*x+a))-1/b*c^2*ln(exp(2*I*(b*x+a))+1)+2/b*c^2*ln(exp(I*(b*x+a)))+2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))+4*I/b*a*c*d*x+I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))-I*c^2*x+I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-1/b*d^2*ln(exp(2*I*(b*x+a))+1)*x^2-2*I/b^2*a^2*d^2*x-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-4/b^2*c*d*a*ln(exp(I*(b*x+a)))+1/3*I*d^2*x^3-2/b*c*d*ln(exp(2*I*(b*x+a))+1)*x

Maxima [B] time = 1.83483, size = 512, normalized size = 2.78

$$\frac{12(\sin(bx+a)^2 + \log(\sin(bx+a)^2 - 1))c^2 - \frac{24(\sin(bx+a)^2 + \log(\sin(bx+a)^2 - 1))acd}{b} + \frac{12(\sin(bx+a)^2 + \log(\sin(bx+a)^2 - 1))a^2d^2}{b^2} + \frac{-8i(bx+a)^3d^2 + (-24I*b*c*d + 24I*a*d^2)(bx+a)^2 + 12d^2*polylog(3, -e^{(2I*bx + 2I*a)}) + (24I*(bx+a)^2*d^2 + (48I*b*c*d - 48I*a*d^2)*(bx+a))*arctan2(\sin(2*b*x + 2*a), \cos(2*b*x + 2*a) + 1) - 3*(2*(bx+a)^2*d^2 + 4*(b*c*d - a*d^2)*(bx+a) - d^2)*\cos(2*b*x + 2*a) + (-24I*b*c*d - 24I*(bx+a)*d^2 + 24I*a*d^2)*dilog(-$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")

[Out] -1/24*(12*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))*c^2 - 24*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))*a*c*d/b + 12*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))*a^2*d^2/b^2 + (-8*I*(b*x + a)^3*d^2 + (-24*I*b*c*d + 24*I*a*d^2)*(b*x + a)^2 + 12*d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) + (24*I*(b*x + a)^2*d^2 + (48*I*b*c*d - 48*I*a*d^2)*(b*x + a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 3*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(2*b*x + 2*a) + (-24*I*b*c*d - 24*I*(b*x + a)*d^2 + 24*I*a*d^2)*dilog(-

$$e^{(2Ibx + 2Ia)} + 12*((bx + a)^2d^2 + 2*(b*c*d - a*d^2)*(bx + a))*\log(\cos(2bx + 2a)^2 + \sin(2bx + 2a)^2 + 2*\cos(2bx + 2a) + 1) + 6*(b*c*d + (bx + a)*d^2 - a*d^2)*\sin(2bx + 2a))/b^2/b$$

Fricas [C] time = 0.706088, size = 1746, normalized size = 9.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((dx+c)^2*sec(bx+a)*sin(bx+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 \\ & - d^2)*\cos(bx + a)^2 + 4*d^2*\text{polylog}(3, I*\cos(bx + a) + \sin(bx + a)) + 4 \\ & *d^2*\text{polylog}(3, I*\cos(bx + a) - \sin(bx + a)) + 4*d^2*\text{polylog}(3, -I*\cos(bx \\ & x + a) + \sin(bx + a)) + 4*d^2*\text{polylog}(3, -I*\cos(bx + a) - \sin(bx + a)) + \\ & 2*(b*d^2*x + b*c*d)*\cos(bx + a)*\sin(bx + a) - (-4*I*b*d^2*x - 4*I*b*c*d) \\ & *dilog(I*\cos(bx + a) + \sin(bx + a)) - (4*I*b*d^2*x + 4*I*b*c*d)*dilog(I*\cos \\ & os(bx + a) - \sin(bx + a)) - (4*I*b*d^2*x + 4*I*b*c*d)*dilog(-I*\cos(bx + \\ & a) + \sin(bx + a)) - (-4*I*b*d^2*x - 4*I*b*c*d)*dilog(-I*\cos(bx + a) - \sin \\ & (bx + a)) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\log(\cos(bx + a) + I*\sin(bx \\ & + a) + I) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\log(\cos(bx + a) - I*\sin(bx \\ & + a) + I) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\log(I*\cos(\\ & bx + a) + \sin(bx + a) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a \\ & ^2*d^2)*\log(I*\cos(bx + a) - \sin(bx + a) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d \\ & *x + 2*a*b*c*d - a^2*d^2)*\log(-I*\cos(bx + a) + \sin(bx + a) + 1) + 2*(b^2* \\ & d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\log(-I*\cos(bx + a) - \sin(bx \\ & + a) + 1) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\log(-\cos(bx + a) + I*\sin(bx \\ & + a) + I) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\log(-\cos(bx + a) - I*\sin(bx \\ & x + a) + I))/b^3 \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((dx+c)**2*sec(bx+a)*sin(bx+a)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \sec(bx + a) \sin(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((dx+c)^2*sec(bx+a)*sin(bx+a)^3,x, algorithm="giac")

[Out] integrate((dx + c)^2*sec(bx + a)*sin(bx + a)^3, x)

3.224 $\int (c + dx) \sin^2(a + bx) \tan(a + bx) dx$

Optimal. Leaf size=115

$$\frac{id\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^2} - \frac{d \sin(a + bx) \cos(a + bx)}{4b^2} - \frac{(c + dx) \log\left(1 + e^{2i(a+bx)}\right)}{b} - \frac{(c + dx) \sin^2(a + bx)}{2b} + \frac{dx}{4b} + \frac{i(c + dx)}{2d}$$

[Out] (d*x)/(4*b) + ((I/2)*(c + d*x)^2)/d - ((c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (d*Cos[a + b*x]*Sin[a + b*x])/(4*b^2) - ((c + d*x)*Sin[a + b*x]^2)/(2*b)

Rubi [A] time = 0.127852, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {4407, 4404, 2635, 8, 3719, 2190, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^2} - \frac{d \sin(a + bx) \cos(a + bx)}{4b^2} - \frac{(c + dx) \log\left(1 + e^{2i(a+bx)}\right)}{b} - \frac{(c + dx) \sin^2(a + bx)}{2b} + \frac{dx}{4b} + \frac{i(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Sin[a + b*x]^2*Tan[a + b*x], x]

[Out] (d*x)/(4*b) + ((I/2)*(c + d*x)^2)/d - ((c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (d*Cos[a + b*x]*Sin[a + b*x])/(4*b^2) - ((c + d*x)*Sin[a + b*x]^2)/(2*b)

Rule 4407

Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 4404

Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_.), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 3719

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ

[m, 0]

Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int (c + dx) \sin^2(a + bx) \tan(a + bx) dx &= - \int (c + dx) \cos(a + bx) \sin(a + bx) dx + \int (c + dx) \tan(a + bx) dx \\
&= \frac{i(c + dx)^2}{2d} - \frac{(c + dx) \sin^2(a + bx)}{2b} - 2i \int \frac{e^{2i(a+bx)}(c + dx)}{1 + e^{2i(a+bx)}} dx + \frac{d \int \sin^2(a + bx) dx}{2b} \\
&= \frac{i(c + dx)^2}{2d} - \frac{(c + dx) \log(1 + e^{2i(a+bx)})}{b} - \frac{d \cos(a + bx) \sin(a + bx)}{4b^2} - \frac{(c + dx) \sin^2(a + bx)}{2b} \\
&= \frac{dx}{4b} + \frac{i(c + dx)^2}{2d} - \frac{(c + dx) \log(1 + e^{2i(a+bx)})}{b} - \frac{d \cos(a + bx) \sin(a + bx)}{4b^2} - \frac{(c + dx) \sin^2(a + bx)}{2b} \\
&= \frac{dx}{4b} + \frac{i(c + dx)^2}{2d} - \frac{(c + dx) \log(1 + e^{2i(a+bx)})}{b} + \frac{id \operatorname{Li}_2(-e^{2i(a+bx)})}{2b^2} - \frac{d \cos(a + bx) \sin(a + bx)}{4b^2} - \frac{(c + dx) \sin^2(a + bx)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.303667, size = 134, normalized size = 1.17

$$\frac{d \left(\frac{1}{2} i \operatorname{PolyLog} \left(2, -e^{2i(a+bx)} \right) + \frac{1}{2} i (a + bx)^2 - (a + bx) \log \left(1 + e^{2i(a+bx)} \right) \right)}{b^2} - \frac{d \sin(2(a + bx))}{8b^2} + \frac{ad \log(\cos(a + bx))}{b^2} - \frac{c}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Sin[a + b*x]^2*Tan[a + b*x], x]

```
[Out] (d*x*Cos[2*(a + b*x)])/(4*b) + (a*d*Log[Cos[a + b*x]])/b^2 - (c*(-Cos[a + b
*x]^2/2 + Log[Cos[a + b*x]]))/b + (d*((I/2)*(a + b*x)^2 - (a + b*x)*Log[1 +
E^((2*I)*(a + b*x))] + (I/2)*PolyLog[2, -E^((2*I)*(a + b*x))]))/b^2 - (d*S
in[2*(a + b*x)])/(8*b^2)
```

Maple [A] time = 0.355, size = 179, normalized size = 1.6

$$\frac{i}{2} dx^2 - icx + \frac{(2dxb + id + 2bc) e^{2i(bx+a)}}{16b^2} + \frac{(2dxb - id + 2bc) e^{-2i(bx+a)}}{16b^2} + 2 \frac{c \ln(e^{i(bx+a)})}{b} - \frac{c \ln(e^{2i(bx+a)} + 1)}{b} + \frac{2id}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x)`

[Out] $\frac{1}{2}I^2d^2x^2 - I^2cx + \frac{1}{16}(2d^2xb + Id + 2b^2c)/b^2 \exp(2I(b*x+a)) + \frac{1}{16}(2d^2x^2b - Id + 2b^2c)/b^2 \exp(-2I(b*x+a)) + \frac{2}{b^2c} \ln(\exp(I(b*x+a))) - \frac{1}{b^2c} \ln(\exp(2I(b*x+a)) + 1) + \frac{2I}{b^2da} x + \frac{I}{b^2d^2a^2} - \frac{1}{b^2d} \ln(\exp(2I(b*x+a)) + 1) * x + \frac{1}{2}I^2d^2 \text{polylog}(2, -\exp(2I(b*x+a))) / b^2 - \frac{2}{b^2d^2a} \ln(\exp(I(b*x+a)))$

Maxima [A] time = 1.77019, size = 196, normalized size = 1.7

$$\frac{-4ib^2dx^2 - 8ib^2cx + (8ibdx + 8ibc) \arctan(\sin(2bx + 2a), \cos(2bx + 2a) + 1) - 2(bdx + bc) \cos(2bx + 2a) - 4id \operatorname{dilog}(\exp(2I(b*x+a)))}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")`

[Out] $-\frac{1}{8}(-4I^2b^2d^2x^2 - 8I^2b^2c^2x + (8I^2b^2d^2x + 8I^2b^2c^2) \arctan_2(\sin(2bx + 2a), \cos(2bx + 2a) + 1) - 2(b^2dx + b^2c) \cos(2bx + 2a) - 4I^2d^2 \operatorname{dilog}(\exp(2I(b*x+a))) + 4(b^2d^2x + b^2c) \log(\cos(2bx + 2a)^2 + \sin(2bx + 2a)^2 + 2\cos(2bx + 2a) + 1) + d^2 \sin(2bx + 2a)) / b^2$

Fricas [B] time = 0.61366, size = 965, normalized size = 8.39

$$\frac{bdx - 2(bdx + bc) \cos(bx + a)^2 + d \cos(bx + a) \sin(bx + a) + 2i d \operatorname{Li}_2(i \cos(bx + a) + \sin(bx + a)) - 2i d \operatorname{Li}_2(i \cos(bx + a) - \sin(bx + a))}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")`

[Out] $-\frac{1}{4}(b^2d^2x - 2(b^2d^2x + b^2c) \cos(bx + a)^2 + d^2 \cos(bx + a) \sin(bx + a) + 2I^2d^2 \operatorname{dilog}(I \cos(bx + a) + \sin(bx + a)) - 2I^2d^2 \operatorname{dilog}(I \cos(bx + a) - \sin(bx + a)) - 2I^2d^2 \operatorname{dilog}(-I \cos(bx + a) + \sin(bx + a)) + 2I^2d^2 \operatorname{dilog}(-I \cos(bx + a) - \sin(bx + a)) + 2(b^2c - a^2d) \log(\cos(bx + a) + I \sin(bx + a) + I) + 2(b^2c - a^2d) \log(\cos(bx + a) - I \sin(bx + a) + I) + 2(b^2d^2x + a^2d) \log(I \cos(bx + a) + \sin(bx + a) + 1) + 2(b^2d^2x + a^2d) \log(I \cos(bx + a) - \sin(bx + a) + 1) + 2(b^2d^2x + a^2d) \log(-I \cos(bx + a) + \sin(bx + a) + 1) + 2(b^2d^2x + a^2d) \log(-I \cos(bx + a) - \sin(bx + a) + 1) + 2(b^2c - a^2d) \log(-\cos(bx + a) + I \sin(bx + a) + I) + 2(b^2c - a^2d) \log(-\cos(bx + a) - I \sin(bx + a) + I)) / b^2$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)**3,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \sec (bx + a) \sin (bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")

[Out] integrate((d*x + c)*sec(b*x + a)*sin(b*x + a)^3, x)

$$3.225 \quad \int \frac{\sin^2(a+bx) \tan(a+bx)}{c+dx} dx$$

Optimal. Leaf size=81

$$\text{Unintegrable}\left(\frac{\tan(a+bx)}{c+dx}, x\right) - \frac{\sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{2d} - \frac{\cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{2d}$$

[Out] -(CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d) - (Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Unintegrable[Tan[a + b*x]/(c + d*x), x]

Rubi [A] time = 0.151151, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sin^2(a+bx) \tan(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]

[Out] -(CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d) - (Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Defer[Int][Tan[a + b*x]/(c + d*x), x]

Rubi steps

$$\begin{aligned} \int \frac{\sin^2(a+bx) \tan(a+bx)}{c+dx} dx &= - \int \frac{\cos(a+bx) \sin(a+bx)}{c+dx} dx + \int \frac{\tan(a+bx)}{c+dx} dx \\ &= - \int \frac{\sin(2a+2bx)}{2(c+dx)} dx + \int \frac{\tan(a+bx)}{c+dx} dx \\ &= - \left(\frac{1}{2} \int \frac{\sin(2a+2bx)}{c+dx} dx \right) + \int \frac{\tan(a+bx)}{c+dx} dx \\ &= - \left(\frac{1}{2} \cos\left(2a - \frac{2bc}{d}\right) \int \frac{\sin\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx \right) - \frac{1}{2} \sin\left(2a - \frac{2bc}{d}\right) \int \frac{\cos\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx + \\ &= - \frac{\text{Ci}\left(\frac{2bc}{d} + 2bx\right) \sin\left(2a - \frac{2bc}{d}\right)}{2d} - \frac{\cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{2d} + \int \frac{\tan(a+bx)}{c+dx} dx \end{aligned}$$

Mathematica [A] time = 0.779359, size = 0, normalized size = 0.

$$\int \frac{\sin^2(a+bx) \tan(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]

[Out] Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]

Maple [A] time = 0.488, size = 0, normalized size = 0.

$$\int \frac{\sec (bx+a) (\sin (bx+a))^3}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x)

[Out] int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(\cos (bx+a)^2-1) \sec (bx+a) \sin (bx+a)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")

[Out] integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)*sin(b*x + a)/(d*x + c), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(b*x+a)**3/(d*x+c),x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (bx+a) \sin (bx+a)^3}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(b*x + a)^3/(d*x + c), x)
```

$$3.226 \quad \int \frac{\sin^2(a+bx) \tan(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=101

$$\text{Unintegrable}\left(\frac{\tan(a+bx)}{(c+dx)^2}, x\right) - \frac{b \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{d^2} + \frac{b \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^2} + \frac{\sin(2a + 2bx)}{2d(c+dx)}$$

[Out] -((b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2) + Sin[2*a + 2*b*x]/(2*d*(c + d*x)) + (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^2 + Unintegrable[Tan[a + b*x]/(c + d*x)^2, x]

Rubi [A] time = 0.176326, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sin^2(a+bx) \tan(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]

[Out] -((b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2) + Sin[2*a + 2*b*x]/(2*d*(c + d*x)) + (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^2 + Defer[Int][Tan[a + b*x]/(c + d*x)^2, x]

Rubi steps

$$\begin{aligned} \int \frac{\sin^2(a+bx) \tan(a+bx)}{(c+dx)^2} dx &= - \int \frac{\cos(a+bx) \sin(a+bx)}{(c+dx)^2} dx + \int \frac{\tan(a+bx)}{(c+dx)^2} dx \\ &= - \int \frac{\sin(2a+2bx)}{2(c+dx)^2} dx + \int \frac{\tan(a+bx)}{(c+dx)^2} dx \\ &= - \left(\frac{1}{2} \int \frac{\sin(2a+2bx)}{(c+dx)^2} dx \right) + \int \frac{\tan(a+bx)}{(c+dx)^2} dx \\ &= \frac{\sin(2a+2bx)}{2d(c+dx)} - \frac{b \int \frac{\cos(2a+2bx)}{c+dx} dx}{d} + \int \frac{\tan(a+bx)}{(c+dx)^2} dx \\ &= \frac{\sin(2a+2bx)}{2d(c+dx)} - \frac{\left(b \cos\left(2a - \frac{2bc}{d}\right)\right) \int \frac{\cos\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx}{d} + \frac{\left(b \sin\left(2a - \frac{2bc}{d}\right)\right) \int \frac{\sin\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx}{d} \\ &= - \frac{b \cos\left(2a - \frac{2bc}{d}\right) \text{Ci}\left(\frac{2bc}{d} + 2bx\right)}{d^2} + \frac{\sin(2a+2bx)}{2d(c+dx)} + \frac{b \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^2} \end{aligned}$$

Mathematica [A] time = 2.50299, size = 0, normalized size = 0.

$$\int \frac{\sin^2(a+bx) \tan(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]

[Out] Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]

Maple [A] time = 0.812, size = 0, normalized size = 0.

$$\int \frac{\sec(bx + a) (\sin(bx + a))^3}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x)

[Out] int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(\cos(bx + a)^2 - 1) \sec(bx + a) \sin(bx + a)}{d^2x^2 + 2cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)*sin(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(b*x+a)**3/(d*x+c)**2,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (bx + a) \sin (bx + a)^3}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(b*x + a)^3/(d*x + c)^2, x)
```

3.227 $\int (c + dx)^m \csc(a + bx) \sec(a + bx) dx$

Optimal. Leaf size=22

CannotIntegrate(csc(a + bx) sec(a + bx)(c + dx)^m, x)

[Out] CannotIntegrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x], x]

Rubi [A] time = 0.183179, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \csc(a + bx) \sec(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x], x]

[Out] Defer[Int] [(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x], x]

Rubi steps

$$\int (c + dx)^m \csc(a + bx) \sec(a + bx) dx = \int (c + dx)^m \csc(a + bx) \sec(a + bx) dx$$

Mathematica [A] time = 5.94163, size = 0, normalized size = 0.

$$\int (c + dx)^m \csc(a + bx) \sec(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x], x]

[Out] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x], x]

Maple [A] time = 0.141, size = 0, normalized size = 0.

$$\int (dx + c)^m \csc(bx + a) \sec(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a), x)

[Out] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \csc(bx + a) \sec(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx + c)^m \csc(bx + a) \sec(bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a)*sec(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \csc(bx + a) \sec(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a), x)
```

3.228 $\int (c + dx)^4 \csc(a + bx) \sec(a + bx) dx$

Optimal. Leaf size=247

$$-\frac{3id^3(c+dx)\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4} + \frac{3id^3(c+dx)\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4} - \frac{3d^2(c+dx)^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3} + \frac{3d^2(c+dx)^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}$$

[Out] $(-2*(c + d*x)^4*\text{ArcTanh}[E^{((2*I)*(a + b*x))}])/b + ((2*I)*d*(c + d*x)^3*\text{PolyLog}[2, -E^{((2*I)*(a + b*x))}])/b^2 - ((2*I)*d*(c + d*x)^3*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^2 - (3*d^2*(c + d*x)^2*\text{PolyLog}[3, -E^{((2*I)*(a + b*x))}])/b^3 + (3*d^2*(c + d*x)^2*\text{PolyLog}[3, E^{((2*I)*(a + b*x))}])/b^3 - ((3*I)*d^3*(c + d*x)*\text{PolyLog}[4, -E^{((2*I)*(a + b*x))}])/b^4 + ((3*I)*d^3*(c + d*x)*\text{PolyLog}[4, E^{((2*I)*(a + b*x))}])/b^4 + (3*d^4*\text{PolyLog}[5, -E^{((2*I)*(a + b*x))}])/(2*b^5) - (3*d^4*\text{PolyLog}[5, E^{((2*I)*(a + b*x))}])/(2*b^5)$

Rubi [A] time = 0.229067, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {4419, 4183, 2531, 6609, 2282, 6589}

$$-\frac{3id^3(c+dx)\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4} + \frac{3id^3(c+dx)\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4} - \frac{3d^2(c+dx)^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3} + \frac{3d^2(c+dx)^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x],x]

[Out] $(-2*(c + d*x)^4*\text{ArcTanh}[E^{((2*I)*(a + b*x))}])/b + ((2*I)*d*(c + d*x)^3*\text{PolyLog}[2, -E^{((2*I)*(a + b*x))}])/b^2 - ((2*I)*d*(c + d*x)^3*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^2 - (3*d^2*(c + d*x)^2*\text{PolyLog}[3, -E^{((2*I)*(a + b*x))}])/b^3 + (3*d^2*(c + d*x)^2*\text{PolyLog}[3, E^{((2*I)*(a + b*x))}])/b^3 - ((3*I)*d^3*(c + d*x)*\text{PolyLog}[4, -E^{((2*I)*(a + b*x))}])/b^4 + ((3*I)*d^3*(c + d*x)*\text{PolyLog}[4, E^{((2*I)*(a + b*x))}])/b^4 + (3*d^4*\text{PolyLog}[5, -E^{((2*I)*(a + b*x))}])/(2*b^5) - (3*d^4*\text{PolyLog}[5, E^{((2*I)*(a + b*x))}])/(2*b^5)$

Rule 4419

Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]

Rule 4183

Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/ (b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 6609

```
Int[((e_.) + (f_.)*(x_.))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_.)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_ /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^4 \csc(a + bx) \sec(a + bx) dx &= 2 \int (c + dx)^4 \csc(2a + 2bx) dx \\ &= -\frac{2(c + dx)^4 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{(4d) \int (c + dx)^3 \log(1 - e^{i(2a+2bx)}) dx}{b} + \frac{(4d) \int (c + dx)^3 \log(1 + e^{i(2a+2bx)}) dx}{b} \\ &= -\frac{2(c + dx)^4 \tanh^{-1}(e^{2i(a+bx)})}{b} + \frac{2id(c + dx)^3 \text{Li}_2(-e^{2i(a+bx)})}{b^2} - \frac{2id(c + dx)^3 \text{Li}_2(e^{2i(a+bx)})}{b^2} \\ &= -\frac{2(c + dx)^4 \tanh^{-1}(e^{2i(a+bx)})}{b} + \frac{2id(c + dx)^3 \text{Li}_2(-e^{2i(a+bx)})}{b^2} - \frac{2id(c + dx)^3 \text{Li}_2(e^{2i(a+bx)})}{b^2} \\ &= -\frac{2(c + dx)^4 \tanh^{-1}(e^{2i(a+bx)})}{b} + \frac{2id(c + dx)^3 \text{Li}_2(-e^{2i(a+bx)})}{b^2} - \frac{2id(c + dx)^3 \text{Li}_2(e^{2i(a+bx)})}{b^2} \\ &= -\frac{2(c + dx)^4 \tanh^{-1}(e^{2i(a+bx)})}{b} + \frac{2id(c + dx)^3 \text{Li}_2(-e^{2i(a+bx)})}{b^2} - \frac{2id(c + dx)^3 \text{Li}_2(e^{2i(a+bx)})}{b^2} \\ &= -\frac{2(c + dx)^4 \tanh^{-1}(e^{2i(a+bx)})}{b} + \frac{2id(c + dx)^3 \text{Li}_2(-e^{2i(a+bx)})}{b^2} - \frac{2id(c + dx)^3 \text{Li}_2(e^{2i(a+bx)})}{b^2} \end{aligned}$$

Mathematica [B] time = 1.37853, size = 578, normalized size = 2.34

$$-6b^2c^2d^2\text{PolyLog}\left(3, -e^{2i(a+bx)}\right) + 6b^2c^2d^2\text{PolyLog}\left(3, e^{2i(a+bx)}\right) - 12b^2cd^3x\text{PolyLog}\left(3, -e^{2i(a+bx)}\right) + 12b^2cd^3x\text{PolyLog}\left(3, e^{2i(a+bx)}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x], x]

[Out] (-4*b^4*c^4*ArcTanh[E^((2*I)*(a + b*x))] + 8*b^4*c^3*d*x*Log[1 - E^((2*I)*(a + b*x))] + 12*b^4*c^2*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] + 8*b^4*c*d^3*x^3*Log[1 - E^((2*I)*(a + b*x))] + 2*b^4*d^4*x^4*Log[1 - E^((2*I)*(a + b*x))] - 8*b^4*c^3*d*x*Log[1 + E^((2*I)*(a + b*x))] - 12*b^4*c^2*d^2*x^2*Log[1 + E^((2*I)*(a + b*x))] - 8*b^4*c*d^3*x^3*Log[1 + E^((2*I)*(a + b*x))] - 2*b^4*d^4*x^4*Log[1 + E^((2*I)*(a + b*x))] + (4*I)*b^3*d*(c + d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))] - (4*I)*b^3*d*(c + d*x)^3*PolyLog[2, E^((2*I)*(a + b*x))])

$$\begin{aligned}
& b*x)) - 6*b^2*c^2*d^2*PolyLog[3, -E^((2*I)*(a + b*x))] - 12*b^2*c*d^3*x*PolyLog[3, -E^((2*I)*(a + b*x))] - 6*b^2*d^4*x^2*PolyLog[3, -E^((2*I)*(a + b*x))] + 6*b^2*c^2*d^2*PolyLog[3, E^((2*I)*(a + b*x))] + 12*b^2*c*d^3*x*PolyLog[3, E^((2*I)*(a + b*x))] + 6*b^2*d^4*x^2*PolyLog[3, E^((2*I)*(a + b*x))] - (6*I)*b*c*d^3*PolyLog[4, -E^((2*I)*(a + b*x))] - (6*I)*b*d^4*x*PolyLog[4, -E^((2*I)*(a + b*x))] + (6*I)*b*c*d^3*PolyLog[4, E^((2*I)*(a + b*x))] + (6*I)*b*d^4*x*PolyLog[4, E^((2*I)*(a + b*x))] + 3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))] - 3*d^4*PolyLog[5, E^((2*I)*(a + b*x))]/(2*b^5)
\end{aligned}$$

Maple [B] time = 0.329, size = 1242, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x)

[Out]
$$\begin{aligned}
& 3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5+1/b*d^4*\ln(1-exp(I*(b*x+a)))*x^4-1/b^5*d^4*\ln(1-exp(I*(b*x+a)))*a^4-3*I/b^4*c*d^3*polylog(4,-exp(2*I*(b*x+a)))-3/b^3*c^2*d^2*polylog(3,-exp(2*I*(b*x+a)))-3/b^3*d^4*polylog(3,-exp(2*I*(b*x+a)))*x^2-4/b*c*d^3*\ln(exp(2*I*(b*x+a))+1)*x^3+4/b*c*d^3*\ln(1-exp(I*(b*x+a)))*x^3-1/b*c^4*\ln(exp(2*I*(b*x+a))+1)+4/b^4*c*d^3*\ln(1-exp(I*(b*x+a)))*a^3-6/b*c^2*d^2*\ln(exp(2*I*(b*x+a))+1)*x^2+4/b*c*d^3*\ln(exp(I*(b*x+a))+1)*x^3+6*I/b^2*c*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2+6*I/b^2*c^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x+12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x^2+12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))+12/b^3*c^2*d^2*polylog(3,-exp(I*(b*x+a)))+12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2-6/b^3*c*d^3*polylog(3,-exp(2*I*(b*x+a)))*x-1/b*d^4*\ln(exp(2*I*(b*x+a))+1)*x^4+6/b*c^2*d^2*\ln(1-exp(I*(b*x+a)))*x^2+1/b*d^4*\ln(exp(I*(b*x+a))+1)*x^4-6/b^3*c^2*d^2*a^2*\ln(1-exp(I*(b*x+a)))+4/b*c^3*d*\ln(1-exp(I*(b*x+a)))*x+4/b^2*c^3*d*\ln(1-exp(I*(b*x+a)))*a+4/b*c^3*d*\ln(exp(I*(b*x+a))+1)*x+24/b^3*c*d^3*polylog(3,-exp(I*(b*x+a)))*x+6/b*c^2*d^2*\ln(exp(I*(b*x+a))+1)*x^2+24/b^3*c*d^3*polylog(3,exp(I*(b*x+a)))*x-4/b*c^3*d*\ln(exp(2*I*(b*x+a))+1)*x+1/b^5*d^4*a^4*\ln(exp(I*(b*x+a))-1)+6/b^3*c^2*d^2*a^2*\ln(exp(I*(b*x+a))-1)-4/b^4*c*d^3*a^3*\ln(exp(I*(b*x+a))-1)-4/b^2*c^3*d*a*\ln(exp(I*(b*x+a))-1)-4*I/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))-4*I/b^2*c^3*d*polylog(2,-exp(I*(b*x+a)))+24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-4*I/b^2*d^4*polylog(2,exp(I*(b*x+a)))*x^3-4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)))*x+24*I/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))+24*I/b^4*c*d^3*polylog(4,-exp(I*(b*x+a)))-24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^5-12*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-12*I/b^2*c^2*d^2*polylog(2,exp(I*(b*x+a)))*x-12*I/b^2*c^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+1/b*c^4*\ln(exp(I*(b*x+a))+1)+1/b*c^4*\ln(exp(I*(b*x+a))-1)-12*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2+2*I/b^2*c^3*d*polylog(2,-exp(2*I*(b*x+a)))-3*I/b^4*d^4*polylog(4,-exp(2*I*(b*x+a)))*x+2*I/b^2*d^4*polylog(2,-exp(2*I*(b*x+a)))*x^3
\end{aligned}$$

Maxima [B] time = 2.50605, size = 2402, normalized size = 9.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")

```
[Out] -1/6*(3*c^4*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)) - 12*a*c^3*d*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b + 18*a^2*c^2*d^2*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - 12*a^3*c*d^3*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^3 + 3*a^4*d^4*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^4 - (18*d^4*polylog(5, -e^(2*I*b*x + 2*I*a)) - 144*d^4*polylog(5, -e^(I*b*x + I*a)) - 144*d^4*polylog(5, e^(I*b*x + I*a)) - (12*I*(b*x + a)^4*d^4 + (32*I*b*c*d^3 - 32*I*a*d^4)*(b*x + a)^3 + (36*I*b^2*c^2*d^2 - 72*I*a*b*c*d^3 + 36*I*a^2*d^4)*(b*x + a)^2 + (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 - 24*I*a^3*d^4)*(b*x + a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - (-6*I*(b*x + a)^4*d^4 + (-24*I*b*c*d^3 + 24*I*a*d^4)*(b*x + a)^3 + (-36*I*b^2*c^2*d^2 + 72*I*a*b*c*d^3 - 36*I*a^2*d^4)*(b*x + a)^2 + (-24*I*b^3*c^3*d + 72*I*a*b^2*c^2*d^2 - 72*I*a^2*b*c*d^3 + 24*I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (6*I*(b*x + a)^4*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a)^3 + (36*I*b^2*c^2*d^2 - 72*I*a*b*c*d^3 + 36*I*a^2*d^4)*(b*x + a)^2 + (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 - 24*I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (-12*I*b^3*c^3*d + 36*I*a*b^2*c^2*d^2 - 36*I*a^2*b*c*d^3 - 24*I*(b*x + a)^3*d^4 + 12*I*a^3*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*x + a)^2 + (-36*I*b^2*c^2*d^2 + 72*I*a*b*c*d^3 - 36*I*a^2*d^4)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a)) - (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 + 24*I*(b*x + a)^3*d^4 - 24*I*a^3*d^4 + (72*I*b*c*d^3 - 72*I*a*d^4)*(b*x + a)^2 + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*a^2*d^4)*(b*x + a))*dilog(-e^(I*b*x + I*a)) - (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 + 24*I*(b*x + a)^3*d^4 - 24*I*a^3*d^4 + (72*I*b*c*d^3 - 72*I*a*d^4)*(b*x + a)^2 + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*a^2*d^4)*(b*x + a))*dilog(e^(I*b*x + I*a)) - 2*(3*(b*x + a)^4*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 6*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 3*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + 3*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (24*I*b*c*d^3 + 36*I*(b*x + a)*d^4 - 24*I*a*d^4)*polylog(4, -e^(2*I*b*x + 2*I*a)) - (-144*I*b*c*d^3 - 144*I*(b*x + a)*d^4 + 144*I*a*d^4)*polylog(4, -e^(I*b*x + I*a)) - (-144*I*b*c*d^3 - 144*I*(b*x + a)*d^4 + 144*I*a*d^4)*polylog(4, e^(I*b*x + I*a)) - 6*(3*b^2*c^2*d^2 - 6*a*b*c*d^3 + 6*(b*x + a)^2*d^4 + 3*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, -e^(2*I*b*x + 2*I*a)) + 72*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, -e^(I*b*x + I*a)) + 72*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, e^(I*b*x + I*a)))/b^4)/b
```

Fricas [C] time = 1.03994, size = 6280, normalized size = 25.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(24*d^4*polylog(5, cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*polylog(5, cos(b*x + a) - I*sin(b*x + a)) - 24*d^4*polylog(5, I*cos(b*x + a) + sin(b*x + a)) - 24*d^4*polylog(5, I*cos(b*x + a) - sin(b*x + a)) - 24*d^4*polylog(5, -I*cos(b*x + a) + sin(b*x + a)) - 24*d^4*polylog(5, -I*cos(b*x + a) - si
```

$$\begin{aligned}
& n(b*x + a)) + 24*d^4*\text{polylog}(5, -\cos(b*x + a) + I*\sin(b*x + a)) + 24*d^4*\text{polylog}(5, -\cos(b*x + a) - I*\sin(b*x + a)) - (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*\text{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*\text{dilog}(\cos(b*x + a) - I*\sin(b*x + a)) - (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*\text{dilog}(I*\cos(b*x + a) + \sin(b*x + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*\text{dilog}(I*\cos(b*x + a) - \sin(b*x + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*\text{dilog}(-I*\cos(b*x + a) + \sin(b*x + a)) - (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*\text{dilog}(-I*\cos(b*x + a) - \sin(b*x + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*\text{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) - (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*\text{dilog}(-\cos(b*x + a) - I*\sin(b*x + a)) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*\log(\cos(b*x + a) + I*\sin(b*x + a) + I) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*\log(\cos(b*x + a) - I*\sin(b*x + a) + I) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*\log(I*\cos(b*x + a) + \sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*\log(I*\cos(b*x + a) - \sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*\log(-I*\cos(b*x + a) + \sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*\log(-I*\cos(b*x + a) - \sin(b*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*\log(-1/2*\cos(b*x + a) + 1/2*I*\sin(b*x + a) + 1/2) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*\log(-1/2*\cos(b*x + a) - 1/2*I*\sin(b*x + a) + 1/2) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + I) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*\log(-\cos(b*x + a) - I*\sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*\log(-\cos(b*x + a) - I*\sin(b*x + a) + I) - (24*I*b*d^4*x + 24*I*b*c*d^3)*\text{polylog}(4, \cos(b*x + a) + I*\sin(b*x + a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*\text{polylog}(4, \cos(b*x + a) - I*\sin(b*x + a)) - (24*I*b*d^4*x + 24*I*b*c*d^3)*\text{polylog}(4, I*\cos(b*x + a) + \sin(b*x + a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*\text{polylog}(4, I*\cos(b*x + a) - \sin(b*x + a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*\text{polylog}(4, -I*\cos(b*x + a) + \sin(b*x + a)) - (24*I*b*d^4*x + 24*I*b*c*d^3)*\text{polylog}(4, -I*\cos(b*x + a) - \sin(b*x + a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*\text{polylog}(4, -\cos(b*x + a) + I*\sin(b*x + a)) - (24*I*b*d^4*x + 24*I*b*c*d^3)*\text{polylog}(4, -\cos(b*x + a) - I*\sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*\text{polylog}(3, \cos(b*x + a) + I*\sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*\text{polylog}(3, \cos(b*x + a) - I*\sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*\text{polylog}(3, I*\cos(b*x + a) + \sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*\text{polylog}(3, I*\cos(b*x + a) - \sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*\text{polylog}(3, -I*\cos(b*x + a) + \sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*\text{polylog}(3, -I*\cos(b*x + a) - \sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*\text{polylog}(3, -\cos(b*x + a) + I*\sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*\text{polylog}(3, -\cos(b*x + a) - I*\sin(b*x + a))/b^5
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**4*csc(b*x+a)*sec(b*x+a),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^4 \csc(bx + a) \sec(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^4*csc(b*x + a)*sec(b*x + a), x)

3.229 $\int (c + dx)^3 \csc(a + bx) \sec(a + bx) dx$

Optimal. Leaf size=197

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} + \frac{3d^2(c+dx)\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3} + \frac{3id(c+dx)^2\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^2} - \frac{3id(c+dx)^2\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2}$$

[Out] $(-2*(c + d*x)^3*\text{ArcTanh}[E^{((2*I)*(a + b*x))}])/b + (((3*I)/2)*d*(c + d*x)^2*\text{PolyLog}[2, -E^{((2*I)*(a + b*x))}])/b^2 - (((3*I)/2)*d*(c + d*x)^2*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^2 - (3*d^2*(c + d*x)*\text{PolyLog}[3, -E^{((2*I)*(a + b*x))}])/(2*b^3) + (3*d^2*(c + d*x)*\text{PolyLog}[3, E^{((2*I)*(a + b*x))}])/(2*b^3) - (((3*I)/4)*d^3*\text{PolyLog}[4, -E^{((2*I)*(a + b*x))}])/b^4 + (((3*I)/4)*d^3*\text{PolyLog}[4, E^{((2*I)*(a + b*x))}])/b^4$

Rubi [A] time = 0.166281, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {4419, 4183, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} + \frac{3d^2(c+dx)\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3} + \frac{3id(c+dx)^2\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^2} - \frac{3id(c+dx)^2\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^3*\text{Csc}[a + b*x]*\text{Sec}[a + b*x], x]$

[Out] $(-2*(c + d*x)^3*\text{ArcTanh}[E^{((2*I)*(a + b*x))}])/b + (((3*I)/2)*d*(c + d*x)^2*\text{PolyLog}[2, -E^{((2*I)*(a + b*x))}])/b^2 - (((3*I)/2)*d*(c + d*x)^2*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^2 - (3*d^2*(c + d*x)*\text{PolyLog}[3, -E^{((2*I)*(a + b*x))}])/(2*b^3) + (3*d^2*(c + d*x)*\text{PolyLog}[3, E^{((2*I)*(a + b*x))}])/(2*b^3) - (((3*I)/4)*d^3*\text{PolyLog}[4, -E^{((2*I)*(a + b*x))}])/b^4 + (((3*I)/4)*d^3*\text{PolyLog}[4, E^{((2*I)*(a + b*x))}])/b^4$

Rule 4419

$\text{Int}[\text{Csc}[(a_.) + (b_.)*(x_.)]^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sec}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[2^n, \text{Int}[(c + d*x)^m*\text{Csc}[2*a + 2*b*x]^n, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]

Rule 4183

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(-2*(c + d*x)^m*\text{ArcTanh}[E^{(I*(e + f*x))}])/f, x] + (-\text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 - E^{(I*(e + f*x))}], x], x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + E^{(I*(e + f*x))}], x], x]) /;$ FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2531

$\text{Int}[\text{Log}[1 + (e_.)*((F_)^{((c_.)*((a_.) + (b_.)*(x_.)))})^{(n_.)}]*((f_.) + (g_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow -\text{Simp}[(f + g*x)^m*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x))))^n])/((b*c*n*\text{Log}[F]), x] + \text{Dist}[(g*m)/(b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^{(m-1)}*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x))))^n}], x], x] /;$ FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 6609


```
Int[((e_.) + (f_.)*(x_.))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_.)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*(a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^3 \csc(a + bx) \sec(a + bx) dx &= 2 \int (c + dx)^3 \csc(2a + 2bx) dx \\ &= -\frac{2(c + dx)^3 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{(3d) \int (c + dx)^2 \log(1 - e^{i(2a+2bx)}) dx}{b} + \frac{(3d) \int (c + dx)^2 \log(1 + e^{i(2a+2bx)}) dx}{b} \\ &= -\frac{2(c + dx)^3 \tanh^{-1}(e^{2i(a+bx)})}{b} + \frac{3id(c + dx)^2 \text{Li}_2(-e^{2i(a+bx)})}{2b^2} - \frac{3id(c + dx)^2 \text{Li}_2(e^{2i(a+bx)})}{2b^2} \\ &= -\frac{2(c + dx)^3 \tanh^{-1}(e^{2i(a+bx)})}{b} + \frac{3id(c + dx)^2 \text{Li}_2(-e^{2i(a+bx)})}{2b^2} - \frac{3id(c + dx)^2 \text{Li}_2(e^{2i(a+bx)})}{2b^2} \\ &= -\frac{2(c + dx)^3 \tanh^{-1}(e^{2i(a+bx)})}{b} + \frac{3id(c + dx)^2 \text{Li}_2(-e^{2i(a+bx)})}{2b^2} - \frac{3id(c + dx)^2 \text{Li}_2(e^{2i(a+bx)})}{2b^2} \\ &= -\frac{2(c + dx)^3 \tanh^{-1}(e^{2i(a+bx)})}{b} + \frac{3id(c + dx)^2 \text{Li}_2(-e^{2i(a+bx)})}{2b^2} - \frac{3id(c + dx)^2 \text{Li}_2(e^{2i(a+bx)})}{2b^2} \end{aligned}$$

Mathematica [A] time = 1.0162, size = 350, normalized size = 1.78

$$6ib^2d(c + dx)^2 \text{PolyLog}(2, -e^{2i(a+bx)}) - 6ib^2d(c + dx)^2 \text{PolyLog}(2, e^{2i(a+bx)}) - 6bd^2(c + dx) \text{PolyLog}(3, -e^{2i(a+bx)}) + \dots$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-8*b^3*c^3*ArcTanh[E^((2*I)*(a + b*x))]) + 12*b^3*c^2*d*x*Log[1 - E^((2*I)*(a + b*x))] + 12*b^3*c*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] + 4*b^3*d^3*x^3*Log[1 - E^((2*I)*(a + b*x))] - 12*b^3*c^2*d*x*Log[1 + E^((2*I)*(a + b*x))] - 12*b^3*c*d^2*x^2*Log[1 + E^((2*I)*(a + b*x))] - 4*b^3*d^3*x^3*Log[1 + E^((2*I)*(a + b*x))] + (6*I)*b^2*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))] - (6*I)*b^2*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))] - 6*b*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))] + 6*b*c*d^2*PolyLog[3, E^((2*I)*(a + b*x))] + 6*b*d^3*x*PolyLog[3, E^((2*I)*(a + b*x))] - (3*I)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))] + (3*I)*d^3*PolyLog[4, E^((2*I)*(a + b*x))]/(4*b^4)
```

Maple [B] time = 0.313, size = 816, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x)`

[Out]
$$\begin{aligned} & 3/b*c*d^2*\ln(\exp(I*(b*x+a))+1)*x^2+3/b*c*d^2*\ln(1-\exp(I*(b*x+a)))*x^2-3*I/b \\ & ^2*d^3*polylog(2,\exp(I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog(2,-\exp(I*(b*x+a))) \\ & *x^2-3*I/b^2*c^2*d*polylog(2,\exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,-\exp(I \\ & *(b*x+a)))-1/b*c^3*\ln(\exp(2*I*(b*x+a))+1)-3/2/b^3*c*d^2*polylog(3,-\exp(2*I \\ & *(b*x+a)))-3/2/b^3*d^3*polylog(3,-\exp(2*I*(b*x+a)))*x+6*I*d^3*polylog(4,\exp(\\ & I*(b*x+a)))/b^4+3/2*I/b^2*c^2*d*polylog(2,-\exp(2*I*(b*x+a)))-3/b*c^2*d*\ln(e \\ & xp(2*I*(b*x+a))+1)*x-3/b*c*d^2*\ln(\exp(2*I*(b*x+a))+1)*x^2-1/b*d^3*\ln(\exp(2* \\ & I*(b*x+a))+1)*x^3+3/2*I/b^2*d^3*polylog(2,-\exp(2*I*(b*x+a)))*x^2+1/b*d^3*\ln \\ & (1-\exp(I*(b*x+a)))*x^3+1/b^4*d^3*\ln(1-\exp(I*(b*x+a)))*a^3+1/b*d^3*\ln(\exp(I \\ & *(b*x+a))+1)*x^3+3/b^3*c*d^2*a^2*\ln(\exp(I*(b*x+a))-1)-3/b^2*c^2*d*a*\ln(\exp(I \\ & *(b*x+a))-1)-3/b^3*c*d^2*a^2*\ln(1-\exp(I*(b*x+a)))+3/b*c^2*d*\ln(\exp(I*(b*x+a \\ &))+1)*x+3/b*c^2*d*\ln(1-\exp(I*(b*x+a)))*x+3/b^2*c^2*d*\ln(1-\exp(I*(b*x+a)))*a \\ & -3/4*I*d^3*polylog(4,-\exp(2*I*(b*x+a)))/b^4+3*I/b^2*c*d^2*polylog(2,-\exp(2* \\ & I*(b*x+a)))*x+6/b^3*d^3*polylog(3,\exp(I*(b*x+a)))*x+6/b^3*d^3*polylog(3,-\exp \\ & (I*(b*x+a)))*x+6/b^3*c*d^2*polylog(3,\exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3 \\ & ,-\exp(I*(b*x+a)))-1/b^4*d^3*a^3*\ln(\exp(I*(b*x+a))-1)+6*I/b^4*d^3*polylog(4, \\ & -\exp(I*(b*x+a)))-6*I/b^2*c*d^2*polylog(2,\exp(I*(b*x+a)))*x-6*I/b^2*c*d^2*po \\ & lylog(2,-\exp(I*(b*x+a)))*x+1/b*c^3*\ln(\exp(I*(b*x+a))-1)+1/b*c^3*\ln(\exp(I*(b \\ & *x+a))+1) \end{aligned}$$

Maxima [B] time = 2.20054, size = 1435, normalized size = 7.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")`

[Out]
$$\begin{aligned} & -1/6*(3*c^3*(\log(\sin(b*x + a)^2 - 1) - \log(\sin(b*x + a)^2)) - 9*a*c^2*d*(\log \\ & (\sin(b*x + a)^2 - 1) - \log(\sin(b*x + a)^2))/b + 9*a^2*c*d^2*(\log(\sin(b*x + \\ & a)^2 - 1) - \log(\sin(b*x + a)^2))/b^2 - 3*a^3*d^3*(\log(\sin(b*x + a)^2 - 1) \\ & - \log(\sin(b*x + a)^2))/b^3 + (6*I*d^3*polylog(4, -e^(2*I*b*x + 2*I*a)) - 36 \\ & *I*d^3*polylog(4, -e^(I*b*x + I*a)) - 36*I*d^3*polylog(4, e^(I*b*x + I*a)) \\ & + (8*I*(b*x + a)^3*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^ \\ & 2*c^2*d - 36*I*a*b*c*d^2 + 18*I*a^2*d^3)*(b*x + a))*arctan2(\sin(2*b*x + 2*a \\ &), \cos(2*b*x + 2*a) + 1) + (-6*I*(b*x + a)^3*d^3 + (-18*I*b*c*d^2 + 18*I*a* \\ & d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*a^2*d^3)*(b*x + \\ & a))*arctan2(\sin(b*x + a), \cos(b*x + a) + 1) + (6*I*(b*x + a)^3*d^3 + (18*I \\ & *b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18* \\ & I*a^2*d^3)*(b*x + a))*arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) + (-9*I*b^2* \\ & c^2*d + 18*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 9*I*a^2*d^3 + (-18*I*b*c*d^ \\ & 2 + 18*I*a*d^3)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a)) + (18*I*b^2*c^2*d - \\ & 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + 18*I*a^2*d^3 + (36*I*b*c*d^2 - 36*I \\ & *a*d^3)*(b*x + a))*dilog(-e^(I*b*x + I*a)) + (18*I*b^2*c^2*d - 36*I*a*b*c*d \\ & ^2 + 18*I*(b*x + a)^2*d^3 + 18*I*a^2*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x \\ & + a))*dilog(e^(I*b*x + I*a)) + (4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b \\ & *x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*\log(\cos(2*b*x \end{aligned}$$

$$\begin{aligned}
& + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) - 3*((b*x + a)^3*d^3 \\
& + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) \\
& *(b*x + a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - 3*(\\
& (b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c* \\
& d^2 + a^2*d^3)*(b*x + a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + \\
& a) + 1) + 3*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*\text{polylog}(3, -e^{(2*I*b*x \\
& + 2*I*a)}) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\text{polylog}(3, -e^{(I*b*x + I* \\
& a)}) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\text{polylog}(3, e^{(I*b*x + I*a)})))/b^3 \\
&)/b
\end{aligned}$$

Fricas [C] time = 0.864937, size = 4410, normalized size = 22.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")

[Out] $1/2*(6*I*d^3*\text{polylog}(4, \cos(b*x + a) + I*\sin(b*x + a)) - 6*I*d^3*\text{polylog}(4, \cos(b*x + a) - I*\sin(b*x + a)) + 6*I*d^3*\text{polylog}(4, I*\cos(b*x + a) + \sin(b*x + a)) - 6*I*d^3*\text{polylog}(4, I*\cos(b*x + a) - \sin(b*x + a)) - 6*I*d^3*\text{polylog}(4, -I*\cos(b*x + a) + \sin(b*x + a)) + 6*I*d^3*\text{polylog}(4, -I*\cos(b*x + a) - \sin(b*x + a)) - 6*I*d^3*\text{polylog}(4, -\cos(b*x + a) + I*\sin(b*x + a)) + 6*I*d^3*\text{polylog}(4, -\cos(b*x + a) - I*\sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*\text{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*\text{dilog}(\cos(b*x + a) - I*\sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*\text{dilog}(I*\cos(b*x + a) + \sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*\text{dilog}(I*\cos(b*x + a) - \sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*\text{dilog}(-I*\cos(b*x + a) + \sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*\text{dilog}(-I*\cos(b*x + a) - \sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*\text{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*\text{dilog}(-\cos(b*x + a) - I*\sin(b*x + a)) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\log(\cos(b*x + a) + I*\sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\log(\cos(b*x + a) - I*\sin(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*\log(I*\cos(b*x + a) - \sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*\log(-I*\cos(b*x + a) + \sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*\log(-I*\cos(b*x + a) - \sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\log(-1/2*\cos(b*x + a) + 1/2*I*\sin(b*x + a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*\log(-\cos(b*x + a) - I*\sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\log(-\cos(b*x + a) - I*\sin(b*x + a) + I) + 6*(b*d^3*x + b*c*d^2)*\text{polylog}(3, \cos(b*x + a) - I*\sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*\text{polylog}(3, I*\cos(b*x + a) + \sin(b*x + a))$

```
) - 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \csc(bx + a) \sec(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)*sec(b*x + a), x)
```

3.230 $\int (c + dx)^2 \csc(a + bx) \sec(a + bx) dx$

Optimal. Leaf size=127

$$\frac{id(c + dx)\text{PolyLog}(2, -e^{2i(a+bx)})}{b^2} - \frac{id(c + dx)\text{PolyLog}(2, e^{2i(a+bx)})}{b^2} - \frac{d^2\text{PolyLog}(3, -e^{2i(a+bx)})}{2b^3} + \frac{d^2\text{PolyLog}(3, e^{2i(a+bx)})}{2b^3}$$

```
[Out] (-2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/b + (I*d*(c + d*x)*PolyLog[2,
-E^((2*I)*(a + b*x))])/b^2 - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
])/b^2 - (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (d^2*PolyLog[3, E
^((2*I)*(a + b*x))])/(2*b^3)
```

Rubi [A] time = 0.116645, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {4419, 4183, 2531, 2282, 6589}

$$\frac{id(c + dx)\text{PolyLog}(2, -e^{2i(a+bx)})}{b^2} - \frac{id(c + dx)\text{PolyLog}(2, e^{2i(a+bx)})}{b^2} - \frac{d^2\text{PolyLog}(3, -e^{2i(a+bx)})}{2b^3} + \frac{d^2\text{PolyLog}(3, e^{2i(a+bx)})}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x], x]
```

```
[Out] (-2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/b + (I*d*(c + d*x)*PolyLog[2,
-E^((2*I)*(a + b*x))])/b^2 - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
])/b^2 - (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (d^2*PolyLog[3, E
^((2*I)*(a + b*x))])/(2*b^3)
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] := Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-
2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(
m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*(a_.) + (b_.)*x)]*
```

(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \csc(a + bx) \sec(a + bx) dx &= 2 \int (c + dx)^2 \csc(2a + 2bx) dx \\ &= -\frac{2(c + dx)^2 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{(2d) \int (c + dx) \log\left(1 - e^{i(2a+2bx)}\right) dx}{b} + \frac{(2d) \int (c + dx) \log\left(1 + e^{i(2a+2bx)}\right) dx}{b} \\ &= -\frac{2(c + dx)^2 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{id(c + dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{id(c + dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} \\ &= -\frac{2(c + dx)^2 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{id(c + dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{id(c + dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} \\ &= -\frac{2(c + dx)^2 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{id(c + dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{id(c + dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} \end{aligned}$$

Mathematica [A] time = 0.628039, size = 213, normalized size = 1.68

$$\frac{2ibd(c + dx)\text{PolyLog}\left(2, -e^{2i(a+bx)}\right) - 2ibd(c + dx)\text{PolyLog}\left(2, e^{2i(a+bx)}\right) - d^2\text{PolyLog}\left(3, -e^{2i(a+bx)}\right) + d^2\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{(2b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x], x]

[Out] (-4*b^2*c^2*ArcTanh[E^((2*I)*(a + b*x))]) + 4*b^2*c*d*x*Log[1 - E^((2*I)*(a + b*x))] + 2*b^2*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] - 4*b^2*c*d*x*Log[1 + E^((2*I)*(a + b*x))] - 2*b^2*d^2*x^2*Log[1 + E^((2*I)*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))] - d^2*PolyLog[3, -E^((2*I)*(a + b*x))] + d^2*PolyLog[3, E^((2*I)*(a + b*x))]/(2*b^3)

Maple [B] time = 0.297, size = 469, normalized size = 3.7

$$-\frac{d^2 \text{polylog}\left(3, -e^{2i(bx+a)}\right)}{2b^3} - \frac{d^2 \ln\left(e^{2i(bx+a)} + 1\right) x^2}{b} - \frac{d^2 \ln\left(1 - e^{i(bx+a)}\right) a^2}{b^3} + \frac{d^2 \ln\left(e^{i(bx+a)} + 1\right) x^2}{b} + \frac{d^2 \ln\left(1 - e^{i(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*csc(b*x+a)*sec(b*x+a), x)

[Out] -1/2*d^2*polylog(3, -exp(2*I*(b*x+a)))/b^3 - 1/b*d^2*ln(exp(2*I*(b*x+a))+1)*x^2 - 1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2 + 1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2 + 1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2 - 1/b*c^2*ln(exp(2*I*(b*x+a))+1) - 2*I/b^2*c*d*polylog(2, -exp(I*(b*x+a))) - 2*I/b^2*c*d*polylog(2, exp(I*(b*x+a))) + I/b^2*d^2*polylog(2, -exp(2*I*(b*x+a)))*x - 2*I/b^2*d^2*polylog(2, -exp(I*(b*x+a)))*x + I/b^2*c*d*polylog(2, -exp(2*I*(b*x+a))) + 2/b*c*d*ln(exp(I*(b*x+a))+1)*x + 1/b*c^2*ln(e

$$\begin{aligned} & \text{xp}(I*(b*x+a))+1)+1/b*c^2*\ln(\exp(I*(b*x+a))-1)+2*d^2*\text{polylog}(3, -\exp(I*(b*x+a))) \\ &)/b^3+2*d^2*\text{polylog}(3, \exp(I*(b*x+a)))/b^3-2*I/b^2*d^2*\text{polylog}(2, \exp(I*(b*x+a))) \\ &)*x-2/b*c*d*\ln(\exp(2*I*(b*x+a))+1)*x+2/b*c*d*\ln(1-\exp(I*(b*x+a)))*x+2/ \\ & b^2*c*d*\ln(1-\exp(I*(b*x+a)))*a+1/b^3*d^2*a^2*\ln(\exp(I*(b*x+a))-1)-2/b^2*c*d \\ & *a*\ln(\exp(I*(b*x+a))-1) \end{aligned}$$

Maxima [B] time = 1.97525, size = 797, normalized size = 6.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/2*(c^2*(\log(\sin(b*x + a)^2 - 1) - \log(\sin(b*x + a)^2)) - 2*a*c*d*(\log(\sin(b*x + a)^2 - 1) - \log(\sin(b*x + a)^2))/b + a^2*d^2*(\log(\sin(b*x + a)^2 - 1) - \log(\sin(b*x + a)^2))/b^2 + (d^2*\text{polylog}(3, -e^{(2*I*b*x + 2*I*a)}) - 4*d^2*\text{polylog}(3, -e^{(I*b*x + I*a)}) - 4*d^2*\text{polylog}(3, e^{(I*b*x + I*a)}) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*\arctan2(\sin(2*b*x + 2*a), \cos(2*b*x + 2*a) + 1) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a))*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) + (-2*I*b*c*d - 2*I*(b*x + a)*d^2 + 2*I*a*d^2)*\text{dilog}(-e^{(2*I*b*x + 2*I*a)}) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*\text{dilog}(-e^{(I*b*x + I*a)}) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*\text{dilog}(e^{(I*b*x + I*a)}) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1))/b^2)/b \end{aligned}$$

Fricas [C] time = 0.737504, size = 2850, normalized size = 22.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & 1/2*(2*d^2*\text{polylog}(3, \cos(b*x + a) + I*\sin(b*x + a)) + 2*d^2*\text{polylog}(3, \cos(b*x + a) - I*\sin(b*x + a)) - 2*d^2*\text{polylog}(3, I*\cos(b*x + a) + \sin(b*x + a)) - 2*d^2*\text{polylog}(3, I*\cos(b*x + a) - \sin(b*x + a)) - 2*d^2*\text{polylog}(3, -I*\cos(b*x + a) + \sin(b*x + a)) - 2*d^2*\text{polylog}(3, -I*\cos(b*x + a) - \sin(b*x + a)) + 2*d^2*\text{polylog}(3, -\cos(b*x + a) + I*\sin(b*x + a)) + 2*d^2*\text{polylog}(3, -\cos(b*x + a) - I*\sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*\text{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*\text{dilog}(\cos(b*x + a) - I*\sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*\text{dilog}(I*\cos(b*x + a) + \sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*\text{dilog}(I*\cos(b*x + a) - \sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*\text{dilog}(-I*\cos(b*x + a) + \sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*\text{dilog}(-I*\cos(b*x + a) - \sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*\text{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*\text{dilog}(-\cos(b*x + a) - I*\sin(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\log(\cos(b*x + a) + I*\sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)* \end{aligned}$$

```
log(cos(b*x + a) - I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \csc(a + bx) \sec(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*csc(a + b*x)*sec(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \csc(bx + a) \sec(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)*sec(b*x + a), x)
```


3.231 $\int (c + dx) \csc(a + bx) \sec(a + bx) dx$

Optimal. Leaf size=71

$$\frac{idPolyLog\left(2, -e^{2i(a+bx)}\right)}{2b^2} - \frac{idPolyLog\left(2, e^{2i(a+bx)}\right)}{2b^2} - \frac{2(c + dx) \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}$$

[Out] $(-2*(c + d*x)*ArcTanh[E^((2*I)*(a + b*x))])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2$

Rubi [A] time = 0.0555391, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4419, 4183, 2279, 2391}

$$\frac{idPolyLog\left(2, -e^{2i(a+bx)}\right)}{2b^2} - \frac{idPolyLog\left(2, e^{2i(a+bx)}\right)}{2b^2} - \frac{2(c + dx) \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Csc[a + b*x]*Sec[a + b*x], x]

[Out] $(-2*(c + d*x)*ArcTanh[E^((2*I)*(a + b*x))])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2$

Rule 4419

Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]

Rule 4183

Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m-1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_.) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned}
\int (c + dx) \csc(a + bx) \sec(a + bx) dx &= 2 \int (c + dx) \csc(2a + 2bx) dx \\
&= -\frac{2(c + dx) \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{d \int \log(1 - e^{i(2a+2bx)}) dx}{b} + \frac{d \int \log(1 + e^{i(2a+2bx)})}{b} \\
&= -\frac{2(c + dx) \tanh^{-1}(e^{2i(a+bx)})}{b} + \frac{(id) \operatorname{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^{i(2a+2bx)}\right)}{2b^2} - \frac{(id) \operatorname{Subst}\left(\int \frac{\log(1+x)}{x} dx, x, e^{i(2a+2bx)}\right)}{2b^2} \\
&= -\frac{2(c + dx) \tanh^{-1}(e^{2i(a+bx)})}{b} + \frac{id \operatorname{Li}_2(-e^{2i(a+bx)})}{2b^2} - \frac{id \operatorname{Li}_2(e^{2i(a+bx)})}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.11855, size = 141, normalized size = 1.99

$$\frac{d \left(i \left(\operatorname{PolyLog} \left(2, -e^{i(2a+2bx)} \right) - \operatorname{PolyLog} \left(2, e^{i(2a+2bx)} \right) \right) + (2a + 2bx) \left(\log \left(1 - e^{i(2a+2bx)} \right) - \log \left(1 + e^{i(2a+2bx)} \right) \right) - 2a \log \left(\tan \left(\frac{2a + 2bx}{2} \right) \right) \right)}{2b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Csc[a + b*x]*Sec[a + b*x], x]

[Out] -((c*Log[Cos[a + b*x]])/b) + (c*Log[Sin[a + b*x]])/b + (d*((2*a + 2*b*x)*(Log[1 - E^(I*(2*a + 2*b*x))] - Log[1 + E^(I*(2*a + 2*b*x))]) - 2*a*Log[Tan[(2*a + 2*b*x)/2]] + I*(PolyLog[2, -E^(I*(2*a + 2*b*x))] - PolyLog[2, E^(I*(2*a + 2*b*x))])))/(2*b^2)

Maple [B] time = 0.234, size = 208, normalized size = 2.9

$$-\frac{c \ln(e^{2i(bx+a)} + 1)}{b} + \frac{c \ln(e^{i(bx+a)} - 1)}{b} + \frac{c \ln(e^{i(bx+a)} + 1)}{b} + \frac{d \ln(1 - e^{i(bx+a)})x}{b} + \frac{d \ln(1 - e^{i(bx+a)})a}{b^2} - \frac{id \operatorname{polylog}(2, e^{i(bx+a)})}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*csc(b*x+a)*sec(b*x+a), x)

[Out] -1/b*c*ln(exp(2*I*(b*x+a))+1)+1/b*c*ln(exp(I*(b*x+a))-1)+1/b*c*ln(exp(I*(b*x+a))+1)+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(b*x+a)))*a-I*d*polylog(2, exp(I*(b*x+a)))/b^2-1/b*d*ln(exp(2*I*(b*x+a))+1)*x+1/2*I*d*polylog(2, -exp(2*I*(b*x+a)))/b^2+1/b*d*ln(exp(I*(b*x+a))+1)*x-I*d*polylog(2, -exp(I*(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1)

Maxima [B] time = 1.94968, size = 360, normalized size = 5.07

$$\frac{2i b dx \arctan(\sin(bx + a), -\cos(bx + a) + 1) - 2i bc \arctan(\sin(bx + a), \cos(bx + a) - 1) + (2i b dx + 2i bc) \arctan(\sin(bx + a), \cos(bx + a) - 1)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a), x, algorithm="maxima")

[Out] -1/2*(2*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*I*b*c*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*I*b*d*x + 2*I*b*c)*arctan2(sin(2*b*x + a), cos(2*b*x + a) - 1))/b^2

$2*a), \cos(2*b*x + 2*a) + 1) + (-2*I*b*d*x - 2*I*b*c)*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) - I*d*dilog(-e^{(2*I*b*x + 2*I*a)}) + 2*I*d*dilog(-e^{(I*b*x + I*a)}) + 2*I*d*dilog(e^{(I*b*x + I*a)}) + (b*d*x + b*c)*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) - (b*d*x + b*c)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - (b*d*x + b*c)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1))/b^2$

Fricas [B] time = 0.630837, size = 1544, normalized size = 21.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")

[Out] $1/2*(-I*d*dilog(\cos(b*x + a) + I*\sin(b*x + a)) + I*d*dilog(\cos(b*x + a) - I*\sin(b*x + a)) - I*d*dilog(I*\cos(b*x + a) + \sin(b*x + a)) + I*d*dilog(I*\cos(b*x + a) - \sin(b*x + a)) + I*d*dilog(-I*\cos(b*x + a) + \sin(b*x + a)) - I*d*dilog(-I*\cos(b*x + a) - \sin(b*x + a)) + I*d*dilog(-\cos(b*x + a) + I*\sin(b*x + a)) - I*d*dilog(-\cos(b*x + a) - I*\sin(b*x + a)) + (b*d*x + b*c)*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) - (b*c - a*d)*\log(\cos(b*x + a) + I*\sin(b*x + a) + I) + (b*d*x + b*c)*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1) - (b*c - a*d)*\log(\cos(b*x + a) - I*\sin(b*x + a) + I) - (b*d*x + a*d)*\log(I*\cos(b*x + a) + \sin(b*x + a) + 1) - (b*d*x + a*d)*\log(I*\cos(b*x + a) - \sin(b*x + a) + 1) - (b*d*x + a*d)*\log(-I*\cos(b*x + a) + \sin(b*x + a) + 1) - (b*d*x + a*d)*\log(-I*\cos(b*x + a) - \sin(b*x + a) + 1) + (b*c - a*d)*\log(-1/2*\cos(b*x + a) + 1/2*I*\sin(b*x + a) + 1/2) + (b*c - a*d)*\log(-1/2*\cos(b*x + a) - 1/2*I*\sin(b*x + a) + 1/2) + (b*d*x + a*d)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1) - (b*c - a*d)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + I) + (b*d*x + a*d)*\log(-\cos(b*x + a) - I*\sin(b*x + a) + 1) - (b*c - a*d)*\log(-\cos(b*x + a) - I*\sin(b*x + a) + I))/b^2$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \csc(a + bx) \sec(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x)

[Out] Integral((c + d*x)*csc(a + b*x)*sec(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \csc(bx + a) \sec(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)*csc(b*x + a)*sec(b*x + a), x)

$$3.232 \quad \int \frac{\csc(a+bx) \sec(a+bx)}{c+dx} dx$$

Optimal. Leaf size=21

$$2\text{Unintegrable}\left(\frac{\csc(2a+2bx)}{c+dx}, x\right)$$

[Out] 2*Unintegrable[Csc[2*a + 2*b*x]/(c + d*x), x]

Rubi [A] time = 0.04482, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc(a+bx) \sec(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x), x]

[Out] 2*Defer[Int][Csc[2*a + 2*b*x]/(c + d*x), x]

Rubi steps

$$\int \frac{\csc(a+bx) \sec(a+bx)}{c+dx} dx = 2 \int \frac{\csc(2a+2bx)}{c+dx} dx$$

Mathematica [A] time = 4.52966, size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx) \sec(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x), x]

[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x), x]

Maple [A] time = 0.2, size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a) \sec(bx+a)}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)*sec(b*x+a)/(d*x+c), x)

[Out] int(csc(b*x+a)*sec(b*x+a)/(d*x+c), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (bx + a) \sec (bx + a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x, algorithm="maxima")

[Out] integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc (bx + a) \sec (bx + a)}{dx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x, algorithm="fricas")

[Out] integral(csc(b*x + a)*sec(b*x + a)/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (a + bx) \sec (a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x)

[Out] Integral(csc(a + b*x)*sec(a + b*x)/(c + d*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (bx + a) \sec (bx + a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x, algorithm="giac")

[Out] integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c), x)

$$3.233 \quad \int \frac{\csc(a+bx) \sec(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=21

$$2\text{Unintegrable}\left(\frac{\csc(2a+2bx)}{(c+dx)^2}, x\right)$$

[Out] 2*Unintegrable[Csc[2*a + 2*b*x]/(c + d*x)^2, x]

Rubi [A] time = 0.0410168, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc(a+bx) \sec(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x)^2, x]

[Out] 2*Defer[Int][Csc[2*a + 2*b*x]/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\csc(a+bx) \sec(a+bx)}{(c+dx)^2} dx = 2 \int \frac{\csc(2a+2bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 5.88274, size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx) \sec(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x)^2, x]

[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x)^2, x]

Maple [A] time = 0.197, size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a) \sec(bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2, x)

[Out] int(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx + a) \sec(bx + a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x, algorithm="maxima")

[Out] integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c)^2, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx + a) \sec(bx + a)}{d^2x^2 + 2cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(csc(b*x + a)*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(a + bx) \sec(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)**2,x)

[Out] Integral(csc(a + b*x)*sec(a + b*x)/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx + a) \sec(bx + a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c)^2, x)

3.234 $\int (c + dx)^m \csc^2(a + bx) \sec(a + bx) dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\csc^2(a + bx) \sec(a + bx)(c + dx)^m, x\right)$$

[Out] CannotIntegrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x], x]

Rubi [A] time = 0.196717, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \csc^2(a + bx) \sec(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x], x]

[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x], x]

Rubi steps

$$\int (c + dx)^m \csc^2(a + bx) \sec(a + bx) dx = \int (c + dx)^m \csc^2(a + bx) \sec(a + bx) dx$$

Mathematica [A] time = 9.15385, size = 0, normalized size = 0.

$$\int (c + dx)^m \csc^2(a + bx) \sec(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x], x]

[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x], x]

Maple [A] time = 0.163, size = 0, normalized size = 0.

$$\int (dx + c)^m (\csc(bx + a))^2 \sec(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a), x)

[Out] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \csc(bx + a)^2 \sec(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left((dx + c)^m \csc(bx + a)^2 \sec(bx + a), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**2*sec(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \csc(bx + a)^2 \sec(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a), x)
```

3.235 $\int (c + dx)^3 \csc^2(a + bx) \sec(a + bx) dx$

Optimal. Leaf size=350

$$\frac{6id^2(c + dx)\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} - \frac{6id^2(c + dx)\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3} - \frac{6d^2(c + dx)\text{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^3} + \frac{6d^2(c + dx)\text{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^3}$$

```
[Out] ((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b - (6*d*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^2 - ((c + d*x)^3*Csc[a + b*x])/b + ((6*I)*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^3 + ((3*I)*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^3 - (6*d^3*PolyLog[3, -E^(I*(a + b*x))])/b^4 - (6*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 + (6*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + (6*d^3*PolyLog[3, E^(I*(a + b*x))])/b^4 - ((6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^4 + ((6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4
```

Rubi [A] time = 0.640135, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {2621, 321, 207, 4420, 6741, 12, 6742, 6273, 4181, 2531, 6609, 2282, 6589, 4183}

$$\frac{6id^2(c + dx)\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} - \frac{6id^2(c + dx)\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3} - \frac{6d^2(c + dx)\text{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^3} + \frac{6d^2(c + dx)\text{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x], x]
```

```
[Out] ((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b - (6*d*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^2 - ((c + d*x)^3*Csc[a + b*x])/b + ((6*I)*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^3 + ((3*I)*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^3 - (6*d^3*PolyLog[3, -E^(I*(a + b*x))])/b^4 - (6*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 + (6*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + (6*d^3*PolyLog[3, E^(I*(a + b*x))])/b^4 - ((6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^4 + ((6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 4420

Int[Csc[(a_) + (b_)*(x_)]^(n_)*((c_) + (d_)*(x_))^(m_)*Sec[(a_) + (b_)*(x_)]^(p_), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]

Rule 6741

Int[u_, x_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rule 6273

Int[((a_) + ArcTanh[u_]*(b_))*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m + 1, x]]

Rule 4181

Int[csc[(e_) + Pi*(k_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_)*((F_)^(c_)*((a_) + (b_)*(x_)))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 6609

Int[((e_) + (f_)*(x_))^(m_)*PolyLog[n_, (d_)*((F_)^(c_)*((a_) + (b_)*(x_)))^(p_)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 \csc^2(a + bx) \sec(a + bx) dx &= \frac{(c + dx)^3 \tanh^{-1}(\sin(a + bx))}{b} - \frac{(c + dx)^3 \csc(a + bx)}{b} - (3d) \int (c + dx)^2 \left(\frac{\tanh^{-1}(\sin(a + bx))}{b} - \frac{\csc(a + bx)}{b} \right) dx \\
&= \frac{(c + dx)^3 \tanh^{-1}(\sin(a + bx))}{b} - \frac{(c + dx)^3 \csc(a + bx)}{b} - (3d) \int \frac{(c + dx)^2 (\tanh^{-1}(\sin(a + bx)) - \csc(a + bx))}{b} dx \\
&= \frac{(c + dx)^3 \tanh^{-1}(\sin(a + bx))}{b} - \frac{(c + dx)^3 \csc(a + bx)}{b} - \frac{(3d) \int (c + dx)^2 (\tanh^{-1}(\sin(a + bx)) - \csc(a + bx)) dx}{b} \\
&= \frac{(c + dx)^3 \tanh^{-1}(\sin(a + bx))}{b} - \frac{(c + dx)^3 \csc(a + bx)}{b} - \frac{(3d) \int ((c + dx)^2 \tanh^{-1}(\sin(a + bx)) - (c + dx)^2 \csc(a + bx)) dx}{b} \\
&= \frac{(c + dx)^3 \tanh^{-1}(\sin(a + bx))}{b} - \frac{(c + dx)^3 \csc(a + bx)}{b} - \frac{(3d) \int (c + dx)^2 \tanh^{-1}(\sin(a + bx)) dx}{b} + \frac{(3d) \int (c + dx)^2 \csc(a + bx) dx}{b} \\
&= -\frac{6d(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^3 \csc(a + bx)}{b} + \frac{\int b(c + dx)^3 \sec(a + bx) dx}{b} \\
&= -\frac{6d(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^3 \csc(a + bx)}{b} + \frac{6id^2(c + dx)\text{Li}_2(-e^{i(a+bx)})}{b^3} \\
&= -\frac{2i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{6d(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^3 \csc(a + bx)}{b} \\
&= -\frac{2i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{6d(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^3 \csc(a + bx)}{b} \\
&= -\frac{2i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{6d(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^3 \csc(a + bx)}{b} \\
&= -\frac{2i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{6d(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^3 \csc(a + bx)}{b} \\
&= -\frac{2i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{6d(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^3 \csc(a + bx)}{b}
\end{aligned}$$

Mathematica [B] time = 6.41717, size = 739, normalized size = 2.11

$$3ib^2d(c + dx)^2 \text{PolyLog}(2, -ie^{i(a+bx)}) - 3ib^2d(c + dx)^2 \text{PolyLog}(2, ie^{i(a+bx)}) - 6bcd^2 \text{PolyLog}(3, -ie^{i(a+bx)}) + 6bcd^2 \text{PolyLog}(3, ie^{i(a+bx)})$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] -(((c + d*x)^3*Csc[a])/b) + (3*d*((c + d*x)^2*Log[1 - E^(I*(a + b*x))] - (c + d*x)^2*Log[1 + E^(I*(a + b*x))] + ((2*I)*d*(b*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] + I*d*PolyLog[3, -E^(I*(a + b*x))]))/b^2 + (2*d*((-I)*b*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] + d*PolyLog[3, E^(I*(a + b*x))])/b^2 + (((-2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 - I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1 - I*E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + I*E^(I*(a + b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))] - (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*c*d^2*PolyLog[3, I*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4 + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-c^3*Sin[(b*x)/2]) - 3*c^2*d*x*Sin[(b*x)/2] - 3*c*d^2*x^2*Sin[(b*x)/2] - d^3*x^3*Sin[(b*x)/2]))/(2*b) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*(c^3*Sin[(b*x)/2] + 3*c^2*d*x*Sin[(b*x)/2] + 3*c*d^2*x^2*Sin[(b*x)/2] + d^3*x^3*Sin[(b*x)/2]))/(2*b)
```

Maple [B] time = 0.679, size = 1158, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x)
```

```
[Out] -6*d^3*polylog(3,-exp(I*(b*x+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4+3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a^2-3*d^3/b^2*ln(exp(I*(b*x+a))+1)*x^2+3*d^3/b^2*ln(1-exp(I*(b*x+a)))*x^2+6/b^3*d^3*ln(1-exp(I*(b*x+a)))*a*x-6/b^3*c*d^2*a*ln(exp(I*(b*x+a))-1)-2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)+6*I/b^2*c^2*d*a*arctan(exp(I*(b*x+a)))+6*I/b^2*c*d^2*polylog(2,-I*exp(I*(b*x+a)))*x-6*I/b^2*c*d^2*polylog(2,I*exp(I*(b*x+a)))*x-6*I/b^3*c*d^2*a^2*arctan(exp(I*(b*x+a)))-6*I/b^4*d^3*a*dilog(exp(I*(b*x+a))+1)-6*d^2/b^2*c*ln(exp(I*(b*x+a))+1)*x-6*I*d^3/b^3*polylog(2,exp(I*(b*x+a)))*x-6*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4-3*I/b^2*c^2*d*polylog(2,I*exp(I*(b*x+a)))+3*I/b^2*d^3*polylog(2,-I*exp(I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog(2,I*exp(I*(b*x+a)))*x^2+2*I/b^4*d^3*a^3*arctan(exp(I*(b*x+a)))+3*I/b^2*c^2*d*polylog(2,-I*exp(I*(b*x+a)))-3/b^2*c^2*d*ln(exp(I*(b*x+a))+1)+3/b^2*c^2*d*ln(exp(I*(b*x+a))-1)+3/b^4*d^3*a^2*ln(exp(I*(b*x+a))-1)-1/b*d^3*ln(1+I*exp(I*(b*x+a)))*x^3+1/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3+6/b^3*d^3*polylog(3,I*exp(I*(b*x+a)))*x-6/b^3*d^3*polylog(3,-I*exp(I*(b*x+a)))*x+1/b^4*a^3*d^3*ln(1-I*exp(I*(b*x+a)))-6/b^3*c*d^2*polylog(3,-I*exp(I*(b*x+a)))-1/b^4*a^3*d^3*ln(1+I*exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,I*exp(I*(b*x+a)))-2*I/b*c^3*arctan(exp(I*(b*x+a)))+6*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-3/b^3*a^2*c*d^2*ln(1-I*exp(I*(b*x+a)))+3/b*c*d^2*ln(1-I*exp(I*(b*x+a)))*x^2-3/b*c*d^2*ln(1+I*exp(I*(b*x+a)))*x^2+3/b^3*a^2*c*d^2*ln(1+I*exp(I*(b*x+a)))+3/b*c^2*d*ln(1-I*exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-I*exp(I*(b*x+a)))*a-3/b*c^2*d*ln(1+I*exp(I*(b*x+a)))*x-3/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))*a+6*I*d^3/b^3*polylog(2,-exp(I*(b*x+a)))*x-6*I/b^4*d^3*a*dilog(exp(I*(b*x+a)))+6*I/b^3*d^2*c*dilog(exp(I*(b*x+a)))+6*I/b^4*d^3*polylog(2,-exp(I*(b*x+a)))*a+6*I/b^3*d^2*c*dilog(exp(I*(b*x+a))+1)-6*I/b^4*d^3*polylog(2,exp(I*(b*x+a)))*a
```

Maxima [B] time = 4.38934, size = 4374, normalized size = 12.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")

[Out]
$$-1/2*(c^3*(2/\sin(b*x + a) - \log(\sin(b*x + a) + 1) + \log(\sin(b*x + a) - 1)) - 3*a*c^2*d*(2/\sin(b*x + a) - \log(\sin(b*x + a) + 1) + \log(\sin(b*x + a) - 1))/b + 3*a^2*c*d^2*(2/\sin(b*x + a) - \log(\sin(b*x + a) + 1) + \log(\sin(b*x + a) - 1))/b^2 - a^3*d^3*(2/\sin(b*x + a) - \log(\sin(b*x + a) + 1) + \log(\sin(b*x + a) - 1))/b^3 - 2*((2*(b*x + a)^3*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a) - 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (-2*I*(b*x + a)^3*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*a^2*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\cos(b*x + a), \sin(b*x + a) + 1) + (2*(b*x + a)^3*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a) - 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (-2*I*(b*x + a)^3*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*a^2*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\cos(b*x + a), -\sin(b*x + a) + 1) + (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*a^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) - (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*a^2*d^3 - 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3))*\cos(2*b*x + 2*a) - (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*a^2*d^3))*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) - 1) + (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) - 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*\cos(b*x + a) + (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*a^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{dilog}(I*e^(I*b*x + I*a)) - (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*a^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{dilog}(-I*e^(I*b*x + I*a)) - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3))*\cos(2*b*x + 2*a) - (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3))*\sin(2*b*x + 2*a))*\operatorname{dilog}(-e^(I*b*x + I*a)) + (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3))*\cos(2*b*x + 2*a) + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3))*\sin(2*b*x + 2*a))*\operatorname{dilog}(e^(I*b*x + I*a)) + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3*I*(b*x + a)^2*d^3 - 3*I*a^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a) + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*(b*x + a)^2*d^3 + 3*I*a^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*(b*x + a)^2*d^3 + 3*I*a^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a) + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3*I*(b*x + a)^$$

$$\begin{aligned}
& 2*d^3 - 3*I*a^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)*\cos(2*b*x + 2*a) \\
& + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) + (I*(b*x + a)^3*d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*a^2*d^3)*(b*x + a) + (-I*(b*x + a)^3*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3*I*a^2*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\sin(b*x + a) + 1) + (-I*(b*x + a)^3*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3*I*a^2*d^3)*(b*x + a) + (I*(b*x + a)^3*d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*a^2*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\sin(b*x + a) + 1) + 12*(d^3*\cos(2*b*x + 2*a) + I*d^3*\sin(2*b*x + 2*a) - d^3)*\text{polylog}(4, I*e^(I*b*x + I*a)) - 12*(d^3*\cos(2*b*x + 2*a) + I*d^3*\sin(2*b*x + 2*a) - d^3)*\text{polylog}(4, -I*e^(I*b*x + I*a)) + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3 + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*\cos(2*b*x + 2*a) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\sin(2*b*x + 2*a))*\text{polylog}(3, I*e^(I*b*x + I*a)) + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3 + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*\cos(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\sin(2*b*x + 2*a))*\text{polylog}(3, -I*e^(I*b*x + I*a)) + (12*I*d^3*\cos(2*b*x + 2*a) - 12*d^3*\sin(2*b*x + 2*a) - 12*I*d^3)*\text{polylog}(3, -e^(I*b*x + I*a)) + (-12*I*d^3*\cos(2*b*x + 2*a) + 12*d^3*\sin(2*b*x + 2*a) + 12*I*d^3)*\text{polylog}(3, e^(I*b*x + I*a)) + (-4*I*(b*x + a)^3*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*a^2*d^3)*(b*x + a))*\sin(b*x + a))/(-2*I*b^3*\cos(2*b*x + 2*a) + 2*b^3*\sin(2*b*x + 2*a) + 2*I*b^3))/b
\end{aligned}$$

Fricas [C] time = 0.951221, size = 4446, normalized size = 12.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")

[Out] $-1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 6*I*d^3*\text{polylog}(4, I*\cos(b*x + a) + \sin(b*x + a))*\sin(b*x + a) - 6*I*d^3*\text{polylog}(4, I*\cos(b*x + a) - \sin(b*x + a))*\sin(b*x + a) + 6*I*d^3*\text{polylog}(4, -I*\cos(b*x + a) + \sin(b*x + a))*\sin(b*x + a) + 6*I*d^3*\text{polylog}(4, -I*\cos(b*x + a) - \sin(b*x + a))*\sin(b*x + a) - 6*d^3*\text{polylog}(3, \cos(b*x + a) + I*\sin(b*x + a))*\sin(b*x + a) - 6*d^3*\text{polylog}(3, \cos(b*x + a) - I*\sin(b*x + a))*\sin(b*x + a) + 6*d^3*\text{polylog}(3, -\cos(b*x + a) + I*\sin(b*x + a))*\sin(b*x + a) + 6*d^3*\text{polylog}(3, -\cos(b*x + a) - I*\sin(b*x + a))*\sin(b*x + a) - (-6*I*b*d^3*x - 6*I*b*c*d^2)*\text{dilog}(\cos(b*x + a) + I*\sin(b*x + a))*\sin(b*x + a) - (6*I*b*d^3*x + 6*I*b*c*d^2)*\text{dilog}(\cos(b*x + a) - I*\sin(b*x + a))*\sin(b*x + a) - (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*\text{dilog}(I*\cos(b*x + a) + \sin(b*x + a))*\sin(b*x + a) - (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*\text{dilog}(I*\cos(b*x + a) - \sin(b*x + a))*\sin(b*x + a) - (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*\text{dilog}(-I*\cos(b*x + a) + \sin(b*x + a))*\sin(b*x + a) - (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*\text{dilog}(-I*\cos(b*x + a) - \sin(b*x + a))*\sin(b*x + a) - (-6*I*b*d^3*x - 6*I*b*c*d^2)*\text{dilog}(-\cos(b*x + a) + I*\sin(b*x + a))*\sin(b*x + a) - (6*I*b*d^3*x + 6*I*b*c*d^2)*\text{dilog}(-\cos(b*x + a) - I*\sin(b*x + a))*\sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1)*\sin(b*x + a) -$

```
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a))/(b^4*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*csc(b*x+a)**2*sec(b*x+a),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \csc(bx + a)^2 \sec(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^3*csc(b*x + a)^2*sec(b*x + a), x)

3.236 $\int (c + dx)^2 \csc^2(a + bx) \sec(a + bx) dx$

Optimal. Leaf size=226

$$\frac{2id(c + dx)\text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^2} - \frac{2id(c + dx)\text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^2} + \frac{2id^2\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} - \frac{2id^2\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3}$$

```
[Out] ((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b - (4*d*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^2 - ((c + d*x)^2*Csc[a + b*x])/b + ((2*I)*d^2*PolyLog[2, -E^(I*(a + b*x))])/b^3 + ((2*I)*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - ((2*I)*d^2*PolyLog[2, E^(I*(a + b*x))])/b^3 - (2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 + (2*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3
```

Rubi [A] time = 0.383878, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {2621, 321, 207, 4420, 6741, 12, 6742, 6273, 4181, 2531, 2282, 6589, 4183, 2279, 2391}

$$\frac{2id(c + dx)\text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^2} - \frac{2id(c + dx)\text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^2} + \frac{2id^2\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} - \frac{2id^2\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x], x]
```

```
[Out] ((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b - (4*d*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^2 - ((c + d*x)^2*Csc[a + b*x])/b + ((2*I)*d^2*PolyLog[2, -E^(I*(a + b*x))])/b^3 + ((2*I)*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - ((2*I)*d^2*PolyLog[2, E^(I*(a + b*x))])/b^3 - (2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 + (2*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_.)]^(n_), x_Symbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_.))^(m_)*((a_.) + (b_.)*(x_.)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6741

```
Int[u_, x_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x]] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m + 1, x]]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x)) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \csc^2(a + bx) \sec(a + bx) dx &= \frac{(c + dx)^2 \tanh^{-1}(\sin(a + bx))}{b} - \frac{(c + dx)^2 \csc(a + bx)}{b} - (2d) \int (c + dx) \left(\frac{\tan(a + bx)}{b} \right) dx \\ &= \frac{(c + dx)^2 \tanh^{-1}(\sin(a + bx))}{b} - \frac{(c + dx)^2 \csc(a + bx)}{b} - (2d) \int \frac{(c + dx) (\tan(a + bx))}{b} dx \\ &= \frac{(c + dx)^2 \tanh^{-1}(\sin(a + bx))}{b} - \frac{(c + dx)^2 \csc(a + bx)}{b} - \frac{(2d) \int (c + dx) (\tan(a + bx)) dx}{b} \\ &= \frac{(c + dx)^2 \tanh^{-1}(\sin(a + bx))}{b} - \frac{(c + dx)^2 \csc(a + bx)}{b} - \frac{(2d) \int ((c + dx) \tan(a + bx)) dx}{b} \\ &= \frac{(c + dx)^2 \tanh^{-1}(\sin(a + bx))}{b} - \frac{(c + dx)^2 \csc(a + bx)}{b} - \frac{(2d) \int (c + dx) \tan(a + bx) dx}{b} \\ &= -\frac{4d(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^2 \csc(a + bx)}{b} + \frac{\int b(c + dx)^2 \sec(a + bx) dx}{b} \\ &= -\frac{4d(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^2 \csc(a + bx)}{b} + \frac{(2id^2) \text{Subst}\left(\int \frac{\log(x)}{b^2} dx\right)}{b^2} \\ &= -\frac{2i(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{4d(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^2 \csc(a + bx)}{b} \\ &= -\frac{2i(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{4d(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^2 \csc(a + bx)}{b} \\ &= -\frac{2i(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{4d(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^2 \csc(a + bx)}{b} \\ &= -\frac{2i(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{4d(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{(c + dx)^2 \csc(a + bx)}{b} \end{aligned}$$

Mathematica [B] time = 6.29071, size = 593, normalized size = 2.62

$$\frac{2ibd(c + dx)\text{PolyLog}\left(2, -ie^{i(a+bx)}\right) - 2ibd(c + dx)\text{PolyLog}\left(2, ie^{i(a+bx)}\right) - 2d^2\text{PolyLog}\left(3, -ie^{i(a+bx)}\right) + 2d^2\text{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x],x]

[Out] -(((c + d*x)^2*Csc[a])/b) + ((-2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] + b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - 2*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + I*E^(I*(a + b*x))]) + (2*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 2*d^2*PolyLog[3, I*E^(I*(a + b*x))]/b^3 + ((4*I)*c*d*ArcTan[(I*Cos[a] - I*Sin[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]]/(b^2*Sqrt[Cos[a]^2 + Sin[a]^2]) + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-c^2*Sin[(b*x)/2]) - 2*c*d*x*Sin[(b*x)/2] - d^2*x^2*Sin[(b*x)/2]))/(2*b) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*(c^2*Sin[(b*x)/2] + 2*c*d*x*Sin[(b*x)/2] + d^2*x^2*Sin[(b*x)/2]))/(2*b) + (2*d^2*(-2*ArcTan[Tan[a]]*ArcTanh[(-Cos[a] + Sin[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]]/Sqrt[Cos[a]^2 + Sin[a]^2] + ((b*x + ArcTan[Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[Tan[a]])]) - Log[1 + E^(I*(b*x + ArcTan[Tan[a]])])]) + I*(PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]])]) - PolyLog[2, E^(I*(b*x + ArcTan[Tan[a]])])]))*Sec[a])/Sqrt[1 + Tan[a]^2])/b^3

Maple [B] time = 0.449, size = 556, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x)

[Out] 2*I/b^2*d^2*polylog(2,-I*exp(I*(b*x+a)))*x-2*d^2/b^3*a*ln(exp(I*(b*x+a))-1)+1/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2+2*d/b^2*c*ln(exp(I*(b*x+a))-1)-1/b^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)))-1/b*d^2*ln(1+I*exp(I*(b*x+a)))*x^2+1/b^3*a^2*d^2*ln(1+I*exp(I*(b*x+a)))-2/b*c*d*ln(1+I*exp(I*(b*x+a)))*x+2*I/b^2*c*d*polylog(2,-I*exp(I*(b*x+a)))-2*I/b^2*d^2*polylog(2,I*exp(I*(b*x+a)))*x+2*I/b^3*d^2*dilog(exp(I*(b*x+a))+1)+4*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))-2*d/b^2*c*ln(exp(I*(b*x+a))+1)+2/b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a+2*d^2*polylog(3,I*exp(I*(b*x+a)))/b^3+2*I/b^3*d^2*dilog(exp(I*(b*x+a)))-2*I/b^2*c*d*polylog(2,I*exp(I*(b*x+a)))-2*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3-2*I*(d^2*x^2+2*c*d*x+c^2)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)-2*d^2/b^2*ln(exp(I*(b*x+a))+1)*x-2/b^2*c*d*ln(1+I*exp(I*(b*x+a)))*a+2/b*c*d*ln(1-I*exp(I*(b*x+a)))*x-2*I/b^3*d^2*a^2*arctan(exp(I*(b*x+a)))-2*I/b*c^2*arctan(exp(I*(b*x+a)))

Maxima [B] time = 2.35497, size = 2202, normalized size = 9.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")

[Out] -1/2*(c^2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1)) - 2*a*c*d*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b + a^2*d^2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b^2 - 2*((2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a)

$$\begin{aligned}
& - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\cos(2*b*x + 2*a) + (-2* \\
& I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a))*\sin(2*b*x + 2*a))* \\
& \operatorname{rctan2}(\cos(b*x + a), -\sin(b*x + a) + 1) + (4*b*c*d + 4*(b*x + a)*d^2 - 4*a* \\
& d^2 - 4*(b*c*d + (b*x + a)*d^2 - a*d^2))*\cos(2*b*x + 2*a) + (-4*I*b*c*d - 4* \\
& I*(b*x + a)*d^2 + 4*I*a*d^2))*\sin(2*b*x + 2*a))*\operatorname{arctan2}(\sin(b*x + a), \cos(b* \\
& x + a) + 1) - (4*b*c*d - 4*a*d^2 - 4*(b*c*d - a*d^2))*\cos(2*b*x + 2*a) - (4* \\
& I*b*c*d - 4*I*a*d^2))*\sin(2*b*x + 2*a))*\operatorname{arctan2}(\sin(b*x + a), \cos(b*x + a) - \\
& 1) - 4*((b*x + a)*d^2*\cos(2*b*x + 2*a) + I*(b*x + a)*d^2*\sin(2*b*x + 2*a) \\
& - (b*x + a)*d^2)*\operatorname{arctan2}(\sin(b*x + a), -\cos(b*x + a) + 1) - 4*((b*x + a)^2*d^2 \\
& + 2*(b*c*d - a*d^2)*(b*x + a))*\cos(b*x + a) + (4*b*c*d + 4*(b*x + a)*d^2 \\
& - 4*a*d^2 - 4*(b*c*d + (b*x + a)*d^2 - a*d^2))*\cos(2*b*x + 2*a) + (-4*I*b* \\
& c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2))*\sin(2*b*x + 2*a))*\operatorname{dilog}(I*e^(I*b*x + I \\
& *a)) - (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 - 4*(b*c*d + (b*x + a)*d^2 - a* \\
& d^2))*\cos(2*b*x + 2*a) - (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2))*\sin(2*b \\
& *x + 2*a))*\operatorname{dilog}(-I*e^(I*b*x + I*a)) + 4*(d^2*\cos(2*b*x + 2*a) + I*d^2*\sin(\\
& 2*b*x + 2*a) - d^2)*\operatorname{dilog}(-e^(I*b*x + I*a)) - 4*(d^2*\cos(2*b*x + 2*a) + I*d \\
& ^2*\sin(2*b*x + 2*a) - d^2)*\operatorname{dilog}(e^(I*b*x + I*a)) + (-2*I*b*c*d - 2*I*(b*x \\
& + a)*d^2 + 2*I*a*d^2 + (2*I*b*c*d + 2*I*(b*x + a)*d^2 - 2*I*a*d^2))*\cos(2*b* \\
& x + 2*a) - 2*(b*c*d + (b*x + a)*d^2 - a*d^2))*\sin(2*b*x + 2*a))*\log(\cos(b*x \\
& + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) + (2*I*b*c*d + 2*I*(b*x + a)* \\
& d^2 - 2*I*a*d^2 + (-2*I*b*c*d - 2*I*(b*x + a)*d^2 + 2*I*a*d^2))*\cos(2*b*x + \\
& 2*a) + 2*(b*c*d + (b*x + a)*d^2 - a*d^2))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a) \\
& ^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) + (I*(b*x + a)^2*d^2 + (2*I*b*c*d \\
& - 2*I*a*d^2)*(b*x + a) + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b \\
& *x + a))*\cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a)) \\
& *\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\sin(b*x + a) + 1 \\
&) + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) + (I*(b*x + a) \\
& ^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a))*\cos(2*b*x + 2*a) - ((b*x + a)^2 \\
& *d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \\
& \sin(b*x + a)^2 - 2*\sin(b*x + a) + 1) + (-4*I*d^2*\cos(2*b*x + 2*a) + 4*d^2*s \\
& \sin(2*b*x + 2*a) + 4*I*d^2)*\operatorname{polylog}(3, I*e^(I*b*x + I*a)) + (4*I*d^2*\cos(2*b \\
& *x + 2*a) - 4*d^2*\sin(2*b*x + 2*a) - 4*I*d^2)*\operatorname{polylog}(3, -I*e^(I*b*x + I*a) \\
&) + (-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a))*\sin(b*x + a \\
&))/((-2*I*b^2*\cos(2*b*x + 2*a) + 2*b^2*\sin(2*b*x + 2*a) + 2*I*b^2))/b
\end{aligned}$$

Fricas [C] time = 0.764123, size = 2844, normalized size = 12.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")

[Out] $-1/2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 2*I*d^2*\operatorname{dilog}(\cos(b*x + a) + I*\sin(b*x + a))*\sin(b*x + a) - 2*I*d^2*\operatorname{dilog}(\cos(b*x + a) - I*\sin(b*x + a))*\sin(b*x + a) + 2*I*d^2*\operatorname{dilog}(-\cos(b*x + a) + I*\sin(b*x + a))*\sin(b*x + a) - 2*I*d^2*\operatorname{dilog}(-\cos(b*x + a) - I*\sin(b*x + a))*\sin(b*x + a) + 2*d^2*\operatorname{polylog}(3, I*\cos(b*x + a) + \sin(b*x + a))*\sin(b*x + a) - 2*d^2*\operatorname{polylog}(3, I*\cos(b*x + a) - \sin(b*x + a))*\sin(b*x + a) + 2*d^2*\operatorname{polylog}(3, -I*\cos(b*x + a) + \sin(b*x + a))*\sin(b*x + a) - 2*d^2*\operatorname{polylog}(3, -I*\cos(b*x + a) - \sin(b*x + a))*\sin(b*x + a) - (-2*I*b*d^2*x - 2*I*b*c*d)*\operatorname{dilog}(I*\cos(b*x + a) + \sin(b*x + a))*\sin(b*x + a) - (-2*I*b*d^2*x - 2*I*b*c*d)*\operatorname{dilog}(I*\cos(b*x + a) - \sin(b*x + a))*\sin(b*x + a) - (2*I*b*d^2*x + 2*I*b*c*d)*\operatorname{dilog}(-I*\cos(b*x + a) + \sin(b*x + a))*\sin(b*x + a) - (2*I*b*d^2*x + 2*I*b*c*d)*\operatorname{dilog}(-I*\cos(b*x + a) - \sin(b*x + a))*\sin(b*x + a) + 2*(b*d^2*x + b*c*d)*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1)*\sin(b*x + a) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\log(\cos(b*x + a) + I*\sin(b*x + a) + I)*\sin(b*x + a) + 2*(b*d^2*x + b*c*d)*\log(\cos(b*$

$$\begin{aligned}
& x + a) - I \sin(bx + a) + 1) \sin(bx + a) + (b^2 c^2 - 2ab^2 c d + a^2 d^2) \\
& * \log(\cos(bx + a) - I \sin(bx + a) + I) \sin(bx + a) - (b^2 d^2 x^2 + 2b^2 \\
& * c d x + 2a^2 b^2 c d - a^2 d^2) * \log(I \cos(bx + a) + \sin(bx + a) + 1) \sin(bx \\
& x + a) + (b^2 d^2 x^2 + 2b^2 c d x + 2a^2 b^2 c d - a^2 d^2) * \log(I \cos(bx + \\
& a) - \sin(bx + a) + 1) \sin(bx + a) - (b^2 d^2 x^2 + 2b^2 c d x + 2a^2 b^2 c \\
& d - a^2 d^2) * \log(-I \cos(bx + a) + \sin(bx + a) + 1) \sin(bx + a) + (b^2 d^ \\
& 2 x^2 + 2b^2 c d x + 2a^2 b^2 c d - a^2 d^2) * \log(-I \cos(bx + a) - \sin(bx + \\
& a) + 1) \sin(bx + a) - 2(b^2 c d - a^2 d^2) * \log(-1/2 \cos(bx + a) + 1/2 I \sin \\
& (bx + a) + 1/2) \sin(bx + a) - 2(b^2 c d - a^2 d^2) * \log(-1/2 \cos(bx + a) - 1/ \\
& 2 I \sin(bx + a) + 1/2) \sin(bx + a) - 2(b^2 d^2 x + a^2 d^2) * \log(-\cos(bx + a \\
&) + I \sin(bx + a) + 1) \sin(bx + a) - (b^2 c^2 - 2ab^2 c d + a^2 d^2) * \log \\
& (-\cos(bx + a) + I \sin(bx + a) + I) \sin(bx + a) - 2(b^2 d^2 x + a^2 d^2) * \log \\
& (-\cos(bx + a) - I \sin(bx + a) + 1) \sin(bx + a) + (b^2 c^2 - 2ab^2 c d + a \\
& ^2 d^2) * \log(-\cos(bx + a) - I \sin(bx + a) + I) \sin(bx + a)) / (b^3 \sin(bx \\
& + a))
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*csc(b*x+a)**2*sec(b*x+a),x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")

[Out] Timed out

3.237 $\int (c + dx) \csc^2(a + bx) \sec(a + bx) dx$

Optimal. Leaf size=131

$$\frac{idPolyLog(2, -ie^{i(a+bx)})}{b^2} - \frac{idPolyLog(2, ie^{i(a+bx)})}{b^2} - \frac{d \tanh^{-1}(\cos(a + bx))}{b^2} - \frac{(c + dx) \csc(a + bx)}{b} + \frac{(c + dx) \tanh^{-1}(\cos(a + bx))}{b}$$

```
[Out] ((-2*I)*d*x*ArcTan[E^(I*(a + b*x))])/b - (d*ArcTanh[Cos[a + b*x]])/b^2 - (d*x*ArcTanh[Sin[a + b*x]])/b + ((c + d*x)*ArcTanh[Sin[a + b*x]])/b - ((c + d*x)*Csc[a + b*x])/b + (I*d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2
```

Rubi [A] time = 0.134538, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {2621, 321, 207, 4420, 6271, 12, 4181, 2279, 2391, 3770}

$$\frac{idPolyLog(2, -ie^{i(a+bx)})}{b^2} - \frac{idPolyLog(2, ie^{i(a+bx)})}{b^2} - \frac{d \tanh^{-1}(\cos(a + bx))}{b^2} - \frac{(c + dx) \csc(a + bx)}{b} + \frac{(c + dx) \tanh^{-1}(\cos(a + bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*d*x*ArcTan[E^(I*(a + b*x))])/b - (d*ArcTanh[Cos[a + b*x]])/b^2 - (d*x*ArcTanh[Sin[a + b*x]])/b + ((c + d*x)*ArcTanh[Sin[a + b*x]])/b - ((c + d*x)*Csc[a + b*x])/b + (I*d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_.))^(m_)*((a_.) + (b_.)*(x_.)^(n_.))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_.)]^(n_.)*((c_.) + (d_.)*(x_.))^(m_.)*Sec[(a_.) + (b_.)*(x_.)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] := Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c + dx) \csc^2(a + bx) \sec(a + bx) dx &= \frac{(c + dx) \tanh^{-1}(\sin(a + bx))}{b} - \frac{(c + dx) \csc(a + bx)}{b} - d \int \left(\frac{\tanh^{-1}(\sin(a + bx))}{b} \right. \\
&= \frac{(c + dx) \tanh^{-1}(\sin(a + bx))}{b} - \frac{(c + dx) \csc(a + bx)}{b} - \frac{d \int \tanh^{-1}(\sin(a + bx)) dx}{b} \\
&= -\frac{d \tanh^{-1}(\cos(a + bx))}{b^2} - \frac{dx \tanh^{-1}(\sin(a + bx))}{b} + \frac{(c + dx) \tanh^{-1}(\sin(a + bx))}{b} \\
&= -\frac{d \tanh^{-1}(\cos(a + bx))}{b^2} - \frac{dx \tanh^{-1}(\sin(a + bx))}{b} + \frac{(c + dx) \tanh^{-1}(\sin(a + bx))}{b} \\
&= -\frac{2idx \tan^{-1}(e^{i(a+bx)})}{b} - \frac{d \tanh^{-1}(\cos(a + bx))}{b^2} - \frac{dx \tanh^{-1}(\sin(a + bx))}{b} + \frac{(c + dx) \tanh^{-1}(\sin(a + bx))}{b} \\
&= -\frac{2idx \tan^{-1}(e^{i(a+bx)})}{b} - \frac{d \tanh^{-1}(\cos(a + bx))}{b^2} - \frac{dx \tanh^{-1}(\sin(a + bx))}{b} + \frac{(c + dx) \tanh^{-1}(\sin(a + bx))}{b} \\
&= -\frac{2idx \tan^{-1}(e^{i(a+bx)})}{b} - \frac{d \tanh^{-1}(\cos(a + bx))}{b^2} - \frac{dx \tanh^{-1}(\sin(a + bx))}{b} + \frac{(c + dx) \tanh^{-1}(\sin(a + bx))}{b}
\end{aligned}$$

Mathematica [C] time = 2.94677, size = 517, normalized size = 3.95

$$\frac{c \csc(a + bx) \operatorname{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, \sin^2(a + bx)\right)}{b} - dx \left(-i \left(\operatorname{PolyLog}\left(2, \frac{1}{2} \left((1 + i) - (1 - i) \tan\left(\frac{1}{2}(a + bx)\right)\right)\right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x],x]

[Out] $(d*(a*\cos[(a + b*x)/2] - (a + b*x)*\cos[(a + b*x)/2])*Csc[(a + b*x)/2])/(2*b^2) - (c*Csc[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, \sin[a + b*x]^2])/b - (d*\log[\cos[(a + b*x)/2]])/b^2 + (d*\log[\sin[(a + b*x)/2]])/b^2 - (d*x*(a*\log[1 - \tan[(a + b*x)/2]] - a*\log[1 + \tan[(a + b*x)/2]] - I*(\log[1 + I*\tan[(a + b*x)/2]]*\log[(1/2 - I/2)*(1 + \tan[(a + b*x)/2]]) + \text{PolyLog}[2, ((1 + I) - (1 - I)*\tan[(a + b*x)/2])/2]) + I*(\log[1 - I*\tan[(a + b*x)/2]]*\log[(1/2 + I/2)*(1 + \tan[(a + b*x)/2]]) + \text{PolyLog}[2, (-1/2 - I/2)*(I + \tan[(a + b*x)/2])) - I*(\log[1 - I*\tan[(a + b*x)/2]]*\log[(-1/2 + I/2)*(-1 + \tan[(a + b*x)/2]]) + \text{PolyLog}[2, ((1 + I) + (1 - I)*\tan[(a + b*x)/2])/2]) + I*(\log[1 + I*\tan[(a + b*x)/2]]*\log[(-1/2 - I/2)*(-1 + \tan[(a + b*x)/2]]) + \text{PolyLog}[2, ((1 - I) + (1 + I)*\tan[(a + b*x)/2])/2]))/(b*(a - I*\log[1 - I*\tan[(a + b*x)/2]] + I*\log[1 + I*\tan[(a + b*x)/2]]) + (d*\sec[(a + b*x)/2]*(a*\sin[(a + b*x)/2] - (a + b*x)*\sin[(a + b*x)/2]))/(2*b^2)$

Maple [A] time = 0.299, size = 235, normalized size = 1.8

$$\frac{-2ie^{i(bx+a)}(dx+c)}{b(e^{2i(bx+a)}-1)} + \frac{2iad \arctan(e^{i(bx+a)})}{b^2} - \frac{d \ln(e^{i(bx+a)}+1)}{b^2} + \frac{d \ln(e^{i(bx+a)}-1)}{b^2} - \frac{2ic \arctan(e^{i(bx+a)})}{b} - \frac{d \ln(1 - e^{i(bx+a)})}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x)

[Out] $-2*I*\exp(I*(b*x+a))*(d*x+c)/b/(\exp(2*I*(b*x+a))-1)+2*I/b^2*d*a*\arctan(\exp(I*(b*x+a)))-d/b^2*\ln(\exp(I*(b*x+a))+1)+d/b^2*\ln(\exp(I*(b*x+a))-1)-2*I/b*c*\arctan(\exp(I*(b*x+a)))-1/b*d*\ln(1+I*\exp(I*(b*x+a)))*x-1/b^2*d*\ln(1+I*\exp(I*(b*x+a)))*a-I/b^2*d*dilog(1-I*\exp(I*(b*x+a)))+I/b^2*d*dilog(1+I*\exp(I*(b*x+a)))+1/b*d*\ln(1-I*\exp(I*(b*x+a)))*x+1/b^2*d*\ln(1-I*\exp(I*(b*x+a)))*a$

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")

[Out] Timed out

Fricas [B] time = 0.607381, size = 1215, normalized size = 9.27

$$2bdx + i dLi_2(i \cos(bx + a) + \sin(bx + a)) \sin(bx + a) + i dLi_2(i \cos(bx + a) - \sin(bx + a)) \sin(bx + a) - i dLi_2(i \cos(bx + a) + \sin(bx + a)) \sin(bx + a) - i dLi_2(i \cos(bx + a) - \sin(bx + a)) \sin(bx + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")

```
[Out] -1/2*(2*b*d*x + I*d*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + I*d
*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - I*d*dilog(-I*cos(b*x +
a) + sin(b*x + a))*sin(b*x + a) - I*d*dilog(-I*cos(b*x + a) - sin(b*x + a)
)*sin(b*x + a) - (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x
+ a) + (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + d
*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - (b*d*x + a*d)*log(I*cos(b*x + a
) + sin(b*x + a) + 1)*sin(b*x + a) + (b*d*x + a*d)*log(I*cos(b*x + a) - sin
(b*x + a) + 1)*sin(b*x + a) - (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x +
a) + 1)*sin(b*x + a) + (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) +
1)*sin(b*x + a) - d*log(-1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - (b*c - a*d)
*log(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + (b*c - a*d)*log(-co
s(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + 2*b*c)/(b^2*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \csc^2(a + bx) \sec(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**2*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)**2*sec(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \csc(bx + a)^2 \sec(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^2*sec(b*x + a), x)
```

$$3.238 \quad \int \frac{\csc^2(a+bx) \sec(a+bx)}{c+dx} dx$$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\csc^2(a+bx) \sec(a+bx)}{c+dx}, x\right)$$

[Out] CannotIntegrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x), x]

Rubi [A] time = 0.159846, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^2(a+bx) \sec(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x), x]

[Out] Defer[Int] [(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x), x]

Rubi steps

$$\int \frac{\csc^2(a+bx) \sec(a+bx)}{c+dx} dx = \int \frac{\csc^2(a+bx) \sec(a+bx)}{c+dx} dx$$

Mathematica [A] time = 11.6999, size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx) \sec(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x), x]

[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x), x]

Maple [A] time = 2.356, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx+a))^2 \sec(bx+a)}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c), x)

[Out] int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c), x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)^2 \sec(bx+a)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x, algorithm="fricas")

[Out] integral(csc(b*x + a)^2*sec(b*x + a)/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx) \sec(a+bx)}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)**2*sec(b*x+a)/(d*x+c),x)

[Out] Integral(csc(a + b*x)**2*sec(a + b*x)/(c + d*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^2 \sec(bx+a)}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x, algorithm="giac")

[Out] integrate(csc(b*x + a)^2*sec(b*x + a)/(d*x + c), x)

$$3.239 \quad \int \frac{\csc^2(a+bx) \sec(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\csc^2(a+bx) \sec(a+bx)}{(c+dx)^2}, x\right)$$

[Out] CannotIntegrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2, x]

Rubi [A] time = 0.189883, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^2(a+bx) \sec(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2, x]

[Out] Defer[Int] [(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\csc^2(a+bx) \sec(a+bx)}{(c+dx)^2} dx = \int \frac{\csc^2(a+bx) \sec(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 11.9084, size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx) \sec(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2, x]

[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2, x]

Maple [A] time = 2.649, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx+a))^2 \sec(bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2, x)

[Out] int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2, x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)^2 \sec(bx+a)}{d^2x^2 + 2cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(csc(b*x + a)^2*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(a + bx) \sec(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)**2*sec(b*x+a)/(d*x+c)**2,x)

[Out] Integral(csc(a + b*x)**2*sec(a + b*x)/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

sage0*x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x, algorithm="giac")

[Out] sage0*x

3.240 $\int (c + dx)^m \csc^3(a + bx) \sec(a + bx) dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}(\csc^3(a + bx) \sec(a + bx)(c + dx)^m, x)$$

[Out] CannotIntegrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x], x]

Rubi [A] time = 0.236741, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \csc^3(a + bx) \sec(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x], x]

[Out] Defer[Int] [(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x], x]

Rubi steps

$$\int (c + dx)^m \csc^3(a + bx) \sec(a + bx) dx = \int (c + dx)^m \csc^3(a + bx) \sec(a + bx) dx$$

Mathematica [A] time = 11.0166, size = 0, normalized size = 0.

$$\int (c + dx)^m \csc^3(a + bx) \sec(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x], x]

[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x], x]

Maple [A] time = 0.178, size = 0, normalized size = 0.

$$\int (dx + c)^m (\csc(bx + a))^3 \sec(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a), x)

[Out] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \csc(bx + a)^3 \sec(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")

[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx + c)^m \csc(bx + a)^3 \sec(bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")

[Out] integral((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**m*csc(b*x+a)**3*sec(b*x+a),x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \csc(bx + a)^3 \sec(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a), x)

3.241 $\int (c + dx)^3 \csc^3(a + bx) \sec(a + bx) dx$

Optimal. Leaf size=325

$$-\frac{3d^2(c + dx)\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} + \frac{3d^2(c + dx)\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3} + \frac{3id(c + dx)^2\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^2} - \frac{3id(c + dx)^2\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2}$$

```
[Out] (((-3*I)/2)*d*(c + d*x)^2)/b^2 - (c + d*x)^3/(2*b) - (2*(c + d*x)^3*ArcTanh[E^((2*I)*(a + b*x))])/b - (3*d*(c + d*x)^2*Cot[a + b*x])/(2*b^2) - ((c + d*x)^3*Cot[a + b*x]^2)/(2*b) + (3*d^2*(c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b^3 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (((3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^4 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/b^3 + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/b^3 - (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/4)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4
```

Rubi [A] time = 0.82345, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 18, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {2620, 14, 4420, 6741, 12, 6742, 3720, 3717, 2190, 2279, 2391, 32, 2551, 4183, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c + dx)\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} + \frac{3d^2(c + dx)\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3} + \frac{3id(c + dx)^2\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^2} - \frac{3id(c + dx)^2\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x], x]
```

```
[Out] (((-3*I)/2)*d*(c + d*x)^2)/b^2 - (c + d*x)^3/(2*b) - (2*(c + d*x)^3*ArcTanh[E^((2*I)*(a + b*x))])/b - (3*d*(c + d*x)^2*Cot[a + b*x])/(2*b^2) - ((c + d*x)^3*Cot[a + b*x]^2)/(2*b) + (3*d^2*(c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b^3 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (((3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^4 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/b^3 + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/b^3 - (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/4)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x],
x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)
*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*x]^p, x]},
Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
```

x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]

Rule 6741

Int[u_, x_Symbol] :=> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]

Rule 12

Int[(a_)*(u_), x_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 6742

Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rule 3720

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 3717

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] :=> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m * E^(2*I*k*Pi) * E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi) * E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :=> Simp[((c + d*x)^m * Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n * Log[F]), x] - Dist[(d*m)/(b*f*g*n * Log[F]), Int[(c + d*x)^(m - 1) * Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] :=> Dist[1/(d*e*n * Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :=> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :=> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[((a + b*x)^(m + 1)
)*Log[u]/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(
m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/ (b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/ (b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
 \int (c + dx)^3 \csc^3(a + bx) \sec(a + bx) dx &= -\frac{(c + dx)^3 \cot^2(a + bx)}{2b} + \frac{(c + dx)^3 \log(\tan(a + bx))}{b} - (3d) \int (c + dx)^2 \left(-\frac{\cot^2(a + bx)}{2b}\right) dx \\
 &= -\frac{(c + dx)^3 \cot^2(a + bx)}{2b} + \frac{(c + dx)^3 \log(\tan(a + bx))}{b} - (3d) \int \frac{(c + dx)^2 (-\cot^2(a + bx))}{2b} dx \\
 &= -\frac{(c + dx)^3 \cot^2(a + bx)}{2b} + \frac{(c + dx)^3 \log(\tan(a + bx))}{b} - \frac{(3d) \int (c + dx)^2 (-\cot^2(a + bx)) dx}{2b} \\
 &= -\frac{(c + dx)^3 \cot^2(a + bx)}{2b} + \frac{(c + dx)^3 \log(\tan(a + bx))}{b} - \frac{(3d) \int (-c + dx)^2 \cot^2(a + bx) dx}{2b} \\
 &= -\frac{(c + dx)^3 \cot^2(a + bx)}{2b} + \frac{(c + dx)^3 \log(\tan(a + bx))}{b} + \frac{(3d) \int (c + dx)^2 \cot^2(a + bx) dx}{2b} \\
 &= -\frac{3d(c + dx)^2 \cot(a + bx)}{2b^2} - \frac{(c + dx)^3 \cot^2(a + bx)}{2b} + \frac{\int 2b(c + dx)^3 \csc(2a + 2bx) dx}{b} \\
 &= -\frac{3id(c + dx)^2}{2b^2} - \frac{(c + dx)^3}{2b} - \frac{3d(c + dx)^2 \cot(a + bx)}{2b^2} - \frac{(c + dx)^3 \cot^2(a + bx)}{2b} \\
 &= -\frac{3id(c + dx)^2}{2b^2} - \frac{(c + dx)^3}{2b} - \frac{2(c + dx)^3 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{3d(c + dx)^2 \cot(a + bx)}{2b^2} \\
 &= -\frac{3id(c + dx)^2}{2b^2} - \frac{(c + dx)^3}{2b} - \frac{2(c + dx)^3 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{3d(c + dx)^2 \cot(a + bx)}{2b^2} \\
 &= -\frac{3id(c + dx)^2}{2b^2} - \frac{(c + dx)^3}{2b} - \frac{2(c + dx)^3 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{3d(c + dx)^2 \cot(a + bx)}{2b^2} \\
 &= -\frac{3id(c + dx)^2}{2b^2} - \frac{(c + dx)^3}{2b} - \frac{2(c + dx)^3 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{3d(c + dx)^2 \cot(a + bx)}{2b^2} \\
 &= -\frac{3id(c + dx)^2}{2b^2} - \frac{(c + dx)^3}{2b} - \frac{2(c + dx)^3 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{3d(c + dx)^2 \cot(a + bx)}{2b^2}
 \end{aligned}$$

Mathematica [B] time = 6.99256, size = 1477, normalized size = 4.54

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x], x]
```

```
[Out] -((c + d*x)^3*Csc[a + b*x]^2)/(2*b) - (c*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^
((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] +
(3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^
(2*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a +
b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a +
b*x))] - I*PolyLog[3, E^((-I)*(a + b*x))]))/E^((2*I)*a))/(2*b^3) - (d^3*E^
(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[
1 - E^((-I)*(a + b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*
(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))
] - (2*I)*b*x*PolyLog[3, -E^((-I)*(a + b*x))] - 2*PolyLog[4, -E^((-I)*(a +
b*x))])))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b^2*x^2*PolyLog[2, E^((-I)*(a
+ b*x))] - (2*I)*b*x*PolyLog[3, E^((-I)*(a + b*x))] - 2*PolyLog[4, E^((-I)
*(a + b*x))])))/E^((2*I)*a))/(4*b^4) + (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2
+ d^3*x^3)*Csc[a]*Sec[a])/4 - ((I/4)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E
^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLo
g[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)
*(a + b*x))])*Sec[a])/(b^3*E^(I*a)) - (I/8)*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a
) - ((4*I)*(1 + E^((-2*I)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1
+ E^((2*I)*a))*(2*b^2*x^2*PolyLog[2, -E^((-2*I)*(a + b*x))] - (2*I)*b*x*Pol
```

$$\begin{aligned} & y \log[3, -E^{((-2*I)*(a + b*x))}] - \text{PolyLog}[4, -E^{((-2*I)*(a + b*x))}]/(b^4 * E^{((2*I)*a)}) * \text{Sec}[a] - (c^3 * \text{Sec}[a] * (\text{Cos}[a] * \text{Log}[\text{Cos}[a] * \text{Cos}[b*x] - \text{Sin}[a] * \text{Sin}[b*x]] + b*x * \text{Sin}[a])) / (b * (\text{Cos}[a]^2 + \text{Sin}[a]^2)) + (c^3 * \text{Csc}[a] * (-b*x * \text{Cos}[a]) \\ & + \text{Log}[\text{Cos}[b*x] * \text{Sin}[a] + \text{Cos}[a] * \text{Sin}[b*x]] * \text{Sin}[a])) / (b * (\text{Cos}[a]^2 + \text{Sin}[a]^2)) + (3 * c * d^2 * \text{Csc}[a] * (-b*x * \text{Cos}[a]) + \text{Log}[\text{Cos}[b*x] * \text{Sin}[a] + \text{Cos}[a] * \text{Sin}[b*x]] * \text{Sin}[a])) / (b^3 * (\text{Cos}[a]^2 + \text{Sin}[a]^2)) - (3 * c^2 * d * \text{Csc}[a] * ((b^2 * x^2) / E^{(I * \text{ArcTan}[\text{Cot}[a]])}) - (\text{Cot}[a] * (I * b * x * (-\text{Pi} - 2 * \text{ArcTan}[\text{Cot}[a])) - \text{Pi} * \text{Log}[1 + E^{((-2 * I) * b * x)}] - 2 * (b * x - \text{ArcTan}[\text{Cot}[a]]) * \text{Log}[1 - E^{((2 * I) * (b * x - \text{ArcTan}[\text{Cot}[a]])})}] + \text{Pi} * \text{Log}[\text{Cos}[b*x]] - 2 * \text{ArcTan}[\text{Cot}[a]] * \text{Log}[\text{Sin}[b*x - \text{ArcTan}[\text{Cot}[a]]]]) + I * \text{PolyLog}[2, E^{((2 * I) * (b * x - \text{ArcTan}[\text{Cot}[a]])})}]) / \text{Sqrt}[1 + \text{Cot}[a]^2] * \text{Sec}[a]) / (2 * b^2 * \text{Sqrt}[\text{Csc}[a]^2 * (\text{Cos}[a]^2 + \text{Sin}[a]^2)]) + (3 * \text{Csc}[a] * \text{Csc}[a + b*x] * (c^2 * d * \text{Sin}[b*x] + 2 * c * d^2 * x * \text{Sin}[b*x] + d^3 * x^2 * \text{Sin}[b*x])) / (2 * b^2) - (3 * c^2 * d * \text{Csc}[a] * \text{Sec}[a] * (b^2 * E^{(I * \text{ArcTan}[\text{Tan}[a]])} * x^2 + ((I * b * x * (-\text{Pi} + 2 * \text{ArcTan}[\text{Tan}[a]]) - \text{Pi} * \text{Log}[1 + E^{((-2 * I) * b * x)}] - 2 * (b * x + \text{ArcTan}[\text{Tan}[a]]) * \text{Log}[1 - E^{((2 * I) * (b * x + \text{ArcTan}[\text{Tan}[a]])})}] + \text{Pi} * \text{Log}[\text{Cos}[b*x]] + 2 * \text{ArcTan}[\text{Tan}[a]] * \text{Log}[\text{Sin}[b*x + \text{ArcTan}[\text{Tan}[a]]]]) + I * \text{PolyLog}[2, E^{((2 * I) * (b * x + \text{ArcTan}[\text{Tan}[a]])})}]) * \text{Tan}[a]) / \text{Sqrt}[1 + \text{Tan}[a]^2]) / (2 * b^2 * \text{Sqrt}[\text{Sec}[a]^2 * (\text{Cos}[a]^2 + \text{Sin}[a]^2)]) - (3 * d^3 * \text{Csc}[a] * \text{Sec}[a] * (b^2 * E^{(I * \text{ArcTan}[\text{Tan}[a]])} * x^2 + ((I * b * x * (-\text{Pi} + 2 * \text{ArcTan}[\text{Tan}[a]]) - \text{Pi} * \text{Log}[1 + E^{((-2 * I) * b * x)}] - 2 * (b * x + \text{ArcTan}[\text{Tan}[a]]) * \text{Log}[1 - E^{((2 * I) * (b * x + \text{ArcTan}[\text{Tan}[a]])})}] + \text{Pi} * \text{Log}[\text{Cos}[b*x]] + 2 * \text{ArcTan}[\text{Tan}[a]] * \text{Log}[\text{Sin}[b*x + \text{ArcTan}[\text{Tan}[a]]]]) + I * \text{PolyLog}[2, E^{((2 * I) * (b * x + \text{ArcTan}[\text{Tan}[a]])})}]) * \text{Tan}[a]) / \text{Sqrt}[1 + \text{Tan}[a]^2]) / (2 * b^4 * \text{Sqrt}[\text{Sec}[a]^2 * (\text{Cos}[a]^2 + \text{Sin}[a]^2)]) \end{aligned}$$

Maple [B] time = 0.421, size = 1223, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d*x+c)^3 * \text{csc}(b*x+a)^3 * \text{sec}(b*x+a), x)$

[Out] $\begin{aligned} & 3/b * c * d^2 * \ln(\exp(I * (b*x+a)) + 1) * x^2 + 3/b * c * d^2 * \ln(1 - \exp(I * (b*x+a))) * x^2 - 3 * I / b \\ & ^2 * d^3 * \text{polylog}(2, \exp(I * (b*x+a))) * x^2 - 3 * I / b^2 * d^3 * \text{polylog}(2, -\exp(I * (b*x+a))) \\ & * x^2 - 3 * I / b^2 * c^2 * d * \text{polylog}(2, \exp(I * (b*x+a))) - 3 * I / b^2 * c^2 * d * \text{polylog}(2, -\exp(I \\ & * (b*x+a))) - 1 / b * c^3 * \ln(\exp(2 * I * (b*x+a)) + 1) - 3 / 2 / b^3 * c * d^2 * \text{polylog}(3, -\exp(2 * I * \\ & (b*x+a))) - 3 / 2 / b^3 * d^3 * \text{polylog}(3, -\exp(2 * I * (b*x+a))) * x + 3 / b^3 * d^3 * \ln(\exp(I * (b * \\ & x+a)) + 1) * x + 3 / b^3 * d^3 * \ln(1 - \exp(I * (b*x+a))) * x + 3 / b^4 * d^3 * \ln(1 - \exp(I * (b*x+a))) * \\ & a - 3 * I / b^2 * d^3 * x^2 - 3 * I / b^4 * d^3 * \text{polylog}(2, -\exp(I * (b*x+a))) - 3 * I / b^4 * d^3 * a^2 + 6 * \\ & I * d^3 * \text{polylog}(4, \exp(I * (b*x+a))) / b^4 + 3 / 2 * I / b^2 * c^2 * d * \text{polylog}(2, -\exp(2 * I * (b*x \\ & +a))) - 3 / b * c^2 * d * \ln(\exp(2 * I * (b*x+a)) + 1) * x - 3 / b * c * d^2 * \ln(\exp(2 * I * (b*x+a)) + 1) * x \\ & ^2 - 1 / b * d^3 * \ln(\exp(2 * I * (b*x+a)) + 1) * x^3 + 3 / 2 * I / b^2 * d^3 * \text{polylog}(2, -\exp(2 * I * (b*x \\ & +a))) * x^2 - 3 / b^4 * d^3 * a * \ln(\exp(I * (b*x+a)) - 1) + 6 / b^4 * d^3 * a * \ln(\exp(I * (b*x+a))) + 3 \\ & / b^3 * d^2 * c * \ln(\exp(I * (b*x+a)) + 1) - 6 / b^3 * d^2 * c * \ln(\exp(I * (b*x+a))) + 3 / b^3 * d^2 * c * \\ & \ln(\exp(I * (b*x+a)) - 1) + 1 / b * d^3 * \ln(1 - \exp(I * (b*x+a))) * x^3 + 1 / b^4 * d^3 * \ln(1 - \exp(I * \\ & (b*x+a))) * a^3 + 1 / b * d^3 * \ln(\exp(I * (b*x+a)) + 1) * x^3 + 3 / b^3 * c * d^2 * a^2 * \ln(\exp(I * (b * \\ & x+a)) - 1) - 3 / b^2 * c^2 * d * a * \ln(\exp(I * (b*x+a)) - 1) - 3 / b^3 * c * d^2 * a^2 * \ln(1 - \exp(I * (b*x \\ & +a))) + 3 / b * c^2 * d * \ln(\exp(I * (b*x+a)) + 1) * x + 3 / b * c^2 * d * \ln(1 - \exp(I * (b*x+a))) * x + 3 / b \\ & ^2 * c^2 * d * \ln(1 - \exp(I * (b*x+a))) * a - 6 * I / b^3 * d^3 * a * x + (2 * b * d^3 * x^3 * \exp(2 * I * (b*x+a)) \\ &) - 3 * I * d^3 * x^2 * \exp(2 * I * (b*x+a)) + 6 * b * c * d^2 * x^2 * \exp(2 * I * (b*x+a)) - 6 * I * c * d^2 * x * \\ & \exp(2 * I * (b*x+a)) + 6 * b * c^2 * d * x * \exp(2 * I * (b*x+a)) - 3 * I * c^2 * d * \exp(2 * I * (b*x+a)) + 3 * \\ & I * d^3 * x^2 + 2 * b * c^3 * \exp(2 * I * (b*x+a)) + 6 * I * c * d^2 * x + 3 * I * c^2 * d) / b^2 / (\exp(2 * I * (b*x \\ & +a)) - 1)^2 - 3 / 4 * I * d^3 * \text{polylog}(4, -\exp(2 * I * (b*x+a))) / b^4 + 3 * I / b^2 * c * d^2 * \text{polylog}(\\ & 2, -\exp(2 * I * (b*x+a))) * x + 6 / b^3 * d^3 * \text{polylog}(3, \exp(I * (b*x+a))) * x + 6 / b^3 * d^3 * \text{poly} \\ & \log(3, -\exp(I * (b*x+a))) * x + 6 / b^3 * c * d^2 * \text{polylog}(3, \exp(I * (b*x+a))) + 6 / b^3 * c * d^2 * \\ & \text{polylog}(3, -\exp(I * (b*x+a))) - 1 / b^4 * d^3 * a^3 * \ln(\exp(I * (b*x+a)) - 1) + 6 * I / b^4 * d^3 * \text{p} \\ & \text{olylog}(4, -\exp(I * (b*x+a))) - 6 * I / b^2 * c * d^2 * \text{polylog}(2, \exp(I * (b*x+a))) * x - 6 * I / b^2 \\ & * c * d^2 * \text{polylog}(2, -\exp(I * (b*x+a))) * x + 1 / b * c^3 * \ln(\exp(I * (b*x+a)) - 1) + 1 / b * c^3 * \ln \end{aligned}$

$(\exp(I*(b*x+a))+1)-3*I*d^3*\text{polylog}(2,\exp(I*(b*x+a)))/b^4$

Maxima [B] time = 7.78119, size = 6939, normalized size = 21.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")

[Out]
$$-1/2*(c^3*(1/\sin(b*x + a)^2 + \log(\sin(b*x + a)^2 - 1) - \log(\sin(b*x + a)^2)) - 3*a*c^2*d*(1/\sin(b*x + a)^2 + \log(\sin(b*x + a)^2 - 1) - \log(\sin(b*x + a)^2))/b + 3*a^2*c*d^2*(1/\sin(b*x + a)^2 + \log(\sin(b*x + a)^2 - 1) - \log(\sin(b*x + a)^2))/b^2 - a^3*d^3*(1/\sin(b*x + a)^2 + \log(\sin(b*x + a)^2 - 1) - \log(\sin(b*x + a)^2))/b^3 - 2*(18*b^2*c^2*d - 36*a*b*c*d^2 + 18*a^2*d^3 - (8*(b*x + a)^3*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a) + 2*(4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*\cos(4*b*x + 4*a) - 4*(4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (8*I*(b*x + a)^3*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*a^2*d^3)*(b*x + a))*\sin(4*b*x + 4*a) + (-16*I*(b*x + a)^3*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a)^2 + (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2 - 36*I*a^2*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(2*b*x + 2*a), \cos(2*b*x + 2*a) + 1) + (6*(b*x + a)^3*d^3 + 18*b*c*d^2 - 18*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a) + 6*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*\cos(4*b*x + 4*a) - 12*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - (-6*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2 + 18*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 18*I)*d^3)*(b*x + a))*\sin(4*b*x + 4*a) - (12*I*(b*x + a)^3*d^3 + 36*I*b*c*d^2 - 36*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + (36*I*a^2 + 36*I)*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) + (18*b*c*d^2 - 18*a*d^3 + 18*(b*c*d^2 - a*d^3))*\cos(4*b*x + 4*a) - 36*(b*c*d^2 - a*d^3))*\cos(2*b*x + 2*a) - (-18*I*b*c*d^2 + 18*I*a*d^3))*\sin(4*b*x + 4*a) - (36*I*b*c*d^2 - 36*I*a*d^3))*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) - 1) - (6*(b*x + a)^3*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a) + 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*\cos(4*b*x + 4*a) - 12*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (6*I*(b*x + a)^3*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + (18*I*a^2 + 18*I)*d^3)*(b*x + a))*\sin(4*b*x + 4*a) + (-12*I*(b*x + a)^3*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a)^2 + (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2 + (-36*I*a^2 - 36*I)*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) - 18*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) - (12*I*(b*x + a)^3*d^3 + 18*b^2*c^2*d - 36*a*b*c*d^2 + 18*a^2*d^3 + (36*I*b*c*d^2 - 18*(2*I*a + 1)*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 36*(2*I*a + 1)*b*c*d^2 + (36*I*a^2 + 36*a)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (9*b^2*c^2*d - 18*a*b*c*d^2 + 12*(b*x + a)^2*d^3 + 9*a^2*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a) + 3*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^2*d^3 + 3*a^2*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^2*d^3 + 3*a^2*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 -$$

$$\begin{aligned}
& 12*I*(b*x + a)^2*d^3 - 9*I*a^2*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a) \\
&))*\sin(4*b*x + 4*a) - (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 24*I*(b*x + a)^2*d^3 \\
& + 18*I*a^2*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a) \\
&)*\operatorname{dilog}(-e^{(2*I*b*x + 2*I*a)}) - (18*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b*x + a)^2*d^3 \\
& + 18*(a^2 + 1)*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a) + 18*(b^2*c^2*d - 2*a*b*c*d^2 \\
& + (b*x + a)^2*d^3 + (a^2 + 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) \\
& - 36*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a)) \\
& *\cos(2*b*x + 2*a) + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I*a^2 + 18*I)*d^3 \\
& + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*\sin(4*b*x + 4*a) + (-36*I*b^2*c^2*d + 72 \\
& *I*a*b*c*d^2 - 36*I*(b*x + a)^2*d^3 + (-36*I*a^2 - 36*I)*d^3 + (-72*I*b*c*d^2 + 72*I*a*d^3) \\
& *(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{dilog}(e^{(I*b*x + I*a)}) - (18*b^2*c^2*d - 36*a*b*c*d^2 \\
& + 18*(b*x + a)^2*d^3 + 18*(a^2 + 1)*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a) + 18*(b^2*c^2*d - 2*a*b*c*d^2 \\
& + (b*x + a)^2*d^3 + (a^2 + 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) \\
& - 36*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a)) \\
& *\cos(2*b*x + 2*a) + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I*a^2 + 18*I)*d^3 \\
& + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*\sin(4*b*x + 4*a) + (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2 \\
& - 36*I*(b*x + a)^2*d^3 + (-36*I*a^2 - 36*I)*d^3 + (-72*I*b*c*d^2 + 72*I*a*d^3)*(b*x + a))*\sin(2*b \\
& *x + 2*a))*\operatorname{dilog}(e^{(I*b*x + I*a)}) - (-4*I*(b*x + a)^3*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3) \\
& *(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 - 9*I*a^2*d^3)*(b*x + a) + (-4*I*(b*x + a)^3*d^3 \\
& + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 - 9*I*a^2*d^3) \\
& *(b*x + a))*\cos(4*b*x + 4*a) + (8*I*(b*x + a)^3*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 \\
& + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*a^2*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (4 \\
& *(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c \\
& *d^2 + a^2*d^3)*(b*x + a))*\sin(4*b*x + 4*a) - 2*(4*(b*x + a)^3*d^3 + 9*(b*c \\
& *d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a) \\
&)*\sin(2*b*x + 2*a))*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b \\
& *x + 2*a) + 1) - (3*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3) \\
& *(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 9*I)*d^3)*(b*x + a) + (3*I*(b*x + a)^3*d^3 \\
& + 9*I*b*c*d^2 - 9*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I \\
& *a^2 + 9*I)*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + (-6*I*(b*x + a)^3*d^3 - 18*I \\
& *b*c*d^2 + 18*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 \\
& + (-18*I*a^2 - 18*I)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - 3*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 \\
& + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*\sin(4*b*x + 4*a) \\
& + 6*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 \\
& + (a^2 + 1)*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) \\
& - (3*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 \\
& + (9*I*a^2 + 9*I)*d^3)*(b*x + a) + (3*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3) \\
& *(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 9*I)*d^3)*(b*x + a) + (3*I*(b*x + a)^3*d^3 \\
& + 9*I*b*c*d^2 - 9*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 9*I) \\
& *d^3)*(b*x + a))*\cos(4*b*x + 4*a) + (-6*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2 + 18*I*a*d^3 + (-18*I*b*c*d^2 \\
& + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 18*I)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) \\
& - 3*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 \\
& + (a^2 + 1)*d^3)*(b*x + a))*\sin(4*b*x + 4*a) + 6*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3) \\
& *(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 \\
& + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) - (6*d^3*\cos(4*b*x + 4*a) - 12*d^3*\cos(2*b*x + 2*a) + 6*I*d^3*\sin(4*b*x + 4*a) \\
& - 12*I*d^3*\sin(2*b*x + 2*a) + 6*d^3)*\operatorname{polylog}(4, -e^{(2*I*b*x + 2*I*a)}) + (36*d^3*\cos(4*b*x + 4*a) - 72*d^3*\cos(2*b*x + 2*a) \\
& + 36*I*d^3*\sin(4*b*x + 4*a) - 72*I*d^3*\sin(2*b*x + 2*a) + 36*d^3)*\operatorname{polylog}(4, -e^{(I*b*x + I*a)}) + (36*d^3*\cos(4*b*x + 4*a) - 72*d^3*\cos(2*b*x + 2*a) \\
& + 36*I*d^3*\sin(4*b*x + 4*a) - 72*I*d^3*\sin(2*b*x + 2*a) +
\end{aligned}$$

$$\begin{aligned}
& 36*d^3)*polylog(4, e^{(I*b*x + I*a)}) - (-9*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + \\
& 9*I*a*d^3 + (-9*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 9*I*a*d^3)*cos(4*b*x + 4*a) \\
& + (18*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 18*I*a*d^3)*cos(2*b*x + 2*a) + 3*(\\
& 3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*sin(4*b*x + 4*a) - 6*(3*b*c*d^2 + 4* \\
& (b*x + a)*d^3 - 3*a*d^3)*sin(2*b*x + 2*a))*polylog(3, -e^{(2*I*b*x + 2*I*a)}) \\
& - (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3 + (36*I*b*c*d^2 + 36*I*(\\
& b*x + a)*d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a) + (-72*I*b*c*d^2 - 72*I*(b*x + \\
& a)*d^3 + 72*I*a*d^3)*cos(2*b*x + 2*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3) \\
&)*sin(4*b*x + 4*a) + 72*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a)) \\
& *polylog(3, -e^{(I*b*x + I*a)}) - (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a \\
& *d^3 + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a) + \\
& (-72*I*b*c*d^2 - 72*I*(b*x + a)*d^3 + 72*I*a*d^3)*cos(2*b*x + 2*a) - 36*(b* \\
& c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) + 72*(b*c*d^2 + (b*x + a)*d \\
& ^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, e^{(I*b*x + I*a)}) - (18*I*(b*x + a) \\
& ^2*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (12*(b*x \\
& + a)^3*d^3 - 18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*a^2*d^3 + (36*b*c*d^2 \\
& - (36*a - 18*I)*d^3)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a - 36*I)*b*c*d^2 + \\
& 36*(a^2 - I*a)*d^3)*(b*x + a))*sin(2*b*x + 2*a))/(-6*I*b^3*cos(4*b*x + 4*a) \\
& + 12*I*b^3*cos(2*b*x + 2*a) + 6*b^3*sin(4*b*x + 4*a) - 12*b^3*sin(2*b*x + \\
& 2*a) - 6*I*b^3))/b
\end{aligned}$$

Fricas [C] time = 1.26092, size = 8092, normalized size = 24.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")

[Out] $1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 3*I*d^3 + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 3*I*d^3 + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 3*I*d^3 + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 3*I*d^3 + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 3*b*c*d^2 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 3*b*c*d^2 + 3*(b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d + b*d^3)*x)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 3*b*c*d^2 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 3*b*c*d^2 + 3*(b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d + b*d^3)*x)*log(cos(b*x + a) - I*sin(b*x + a) +$

$$\begin{aligned}
& 1) + (b^3c^3 - 3ab^2c^2d + 3a^2b^2cd^2 - a^3d^3 - (b^3c^3 - 3ab^2c^2d + 3a^2b^2cd^2 - a^3d^3) \cos(bx + a)^2) \log(\cos(bx + a) - I \sin(bx + a) + I) \\
& + (b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + 3ab^2c^2d - 3a^2b^2cd^2 + a^3d^3 - (b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + 3ab^2c^2d - 3a^2b^2cd^2 + a^3d^3) \cos(bx + a)^2) \log(I \cos(bx + a) + \sin(bx + a) + 1) \\
& + (b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + 3ab^2c^2d - 3a^2b^2cd^2 + a^3d^3 - (b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + 3ab^2c^2d - 3a^2b^2cd^2 + a^3d^3) \cos(bx + a)^2) \log(I \cos(bx + a) - \sin(bx + a) + 1) \\
& + (b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + 3ab^2c^2d - 3a^2b^2cd^2 + a^3d^3 - (b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + 3ab^2c^2d - 3a^2b^2cd^2 + a^3d^3) \cos(bx + a)^2) \log(-I \cos(bx + a) + \sin(bx + a) + 1) \\
& + (b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + 3ab^2c^2d - 3a^2b^2cd^2 + a^3d^3 - (b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + 3ab^2c^2d - 3a^2b^2cd^2 + a^3d^3) \cos(bx + a)^2) \log(-I \cos(bx + a) - \sin(bx + a) + 1) \\
& - (b^3c^3 - 3ab^2c^2d + 3(a^2 + 1)b^2cd^2 - (a^3 + 3a)d^3 - (b^3c^3 - 3ab^2c^2d + 3(a^2 + 1)b^2cd^2 - (a^3 + 3a)d^3) \cos(bx + a)^2) \log(-1/2 \cos(bx + a) + 1/2 I \sin(bx + a) + 1/2) \\
& - (b^3c^3 - 3ab^2c^2d + 3(a^2 + 1)b^2cd^2 - (a^3 + 3a)d^3 - (b^3c^3 - 3ab^2c^2d + 3(a^2 + 1)b^2cd^2 - (a^3 + 3a)d^3) \cos(bx + a)^2) \log(-1/2 \cos(bx + a) - 1/2 I \sin(bx + a) + 1/2) \\
& - (b^3d^3x^3 + 3b^3cd^2x^2 + 3ab^2c^2d - 3a^2b^2cd^2 + (a^3 + 3a)d^3 - (b^3d^3x^3 + 3b^3cd^2x^2 + 3ab^2c^2d - 3a^2b^2cd^2 + (a^3 + 3a)d^3) \cos(bx + a)^2) \log(-\cos(bx + a) + I \sin(bx + a) + 1) \\
& + (b^3c^3 - 3ab^2c^2d + 3a^2b^2cd^2 - a^3d^3 - (b^3c^3 - 3ab^2c^2d + 3a^2b^2cd^2 - a^3d^3) \cos(bx + a)^2) \log(-\cos(bx + a) + I \sin(bx + a) + I) \\
& - (b^3d^3x^3 + 3b^3cd^2x^2 + 3ab^2c^2d - 3a^2b^2cd^2 + (a^3 + 3a)d^3 - (b^3d^3x^3 + 3b^3cd^2x^2 + 3ab^2c^2d - 3a^2b^2cd^2 + (a^3 + 3a)d^3) \cos(bx + a)^2) \log(-\cos(bx + a) - I \sin(bx + a) + 1) \\
& + (b^3c^3 - 3ab^2c^2d + 3a^2b^2cd^2 - a^3d^3 - (b^3c^3 - 3ab^2c^2d + 3a^2b^2cd^2 - a^3d^3) \cos(bx + a)^2) \log(-\cos(bx + a) - I \sin(bx + a) + I) \\
& + (6I d^3 \cos(bx + a)^2 - 6I d^3) \text{polylog}(4, \cos(bx + a) + I \sin(bx + a)) \\
& + (-6I d^3 \cos(bx + a)^2 + 6I d^3) \text{polylog}(4, \cos(bx + a) - I \sin(bx + a)) \\
& + (6I d^3 \cos(bx + a)^2 - 6I d^3) \text{polylog}(4, I \cos(bx + a) + \sin(bx + a)) \\
& + (-6I d^3 \cos(bx + a)^2 + 6I d^3) \text{polylog}(4, I \cos(bx + a) - \sin(bx + a)) \\
& + (-6I d^3 \cos(bx + a)^2 + 6I d^3) \text{polylog}(4, -I \cos(bx + a) + \sin(bx + a)) \\
& + (6I d^3 \cos(bx + a)^2 - 6I d^3) \text{polylog}(4, -I \cos(bx + a) - \sin(bx + a)) \\
& + (-6I d^3 \cos(bx + a)^2 + 6I d^3) \text{polylog}(4, -\cos(bx + a) + I \sin(bx + a)) \\
& + (6I d^3 \cos(bx + a)^2 - 6I d^3) \text{polylog}(4, -\cos(bx + a) - I \sin(bx + a)) \\
& - 6(b^3d^3x + b^3cd^2 - (b^3d^3x + b^3cd^2) \cos(bx + a)^2) \text{polylog}(3, \cos(bx + a) + I \sin(bx + a)) \\
& - 6(b^3d^3x + b^3cd^2 - (b^3d^3x + b^3cd^2) \cos(bx + a)^2) \text{polylog}(3, \cos(bx + a) - I \sin(bx + a)) \\
& + 6(b^3d^3x + b^3cd^2 - (b^3d^3x + b^3cd^2) \cos(bx + a)^2) \text{polylog}(3, I \cos(bx + a) + \sin(bx + a)) \\
& + 6(b^3d^3x + b^3cd^2 - (b^3d^3x + b^3cd^2) \cos(bx + a)^2) \text{polylog}(3, I \cos(bx + a) - \sin(bx + a)) \\
& + 6(b^3d^3x + b^3cd^2 - (b^3d^3x + b^3cd^2) \cos(bx + a)^2) \text{polylog}(3, -I \cos(bx + a) + \sin(bx + a)) \\
& + 6(b^3d^3x + b^3cd^2 - (b^3d^3x + b^3cd^2) \cos(bx + a)^2) \text{polylog}(3, -I \cos(bx + a) - \sin(bx + a)) \\
& - 6(b^3d^3x + b^3cd^2 - (b^3d^3x + b^3cd^2) \cos(bx + a)^2) \text{polylog}(3, -\cos(bx + a) + I \sin(bx + a)) \\
& - 6(b^3d^3x + b^3cd^2 - (b^3d^3x + b^3cd^2) \cos(bx + a)^2) \text{polylog}(3, -\cos(bx + a) - I \sin(bx + a)) \\
& \Big/ (b^4 \cos(bx + a)^2 - b^4)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**3*sec(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \csc(bx + a)^3 \sec(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^3*sec(b*x + a), x)
```

3.242 $\int (c + dx)^2 \csc^3(a + bx) \sec(a + bx) dx$

Optimal. Leaf size=201

$$\frac{id(c + dx)\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^2} - \frac{id(c + dx)\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^2} - \frac{d^2\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} + \frac{d^2\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3}$$

```
[Out] -((c*d*x)/b) - (d^2*x^2)/(2*b) - (2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))
])/b - (d*(c + d*x)*Cot[a + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]^2)/(2*b)
+ (d^2*Log[Sin[a + b*x]])/b^3 + (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*
x))])/b^2 - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d^2*Poly
Log[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (d^2*PolyLog[3, E^((2*I)*(a + b*x)
)])/ (2*b^3)
```

Rubi [A] time = 0.442508, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {2620, 14, 4420, 6741, 12, 6742, 3720, 3475, 2551, 4183, 2531, 2282, 6589}

$$\frac{id(c + dx)\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^2} - \frac{id(c + dx)\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^2} - \frac{d^2\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} + \frac{d^2\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x], x]
```

```
[Out] -((c*d*x)/b) - (d^2*x^2)/(2*b) - (2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))
])/b - (d*(c + d*x)*Cot[a + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]^2)/(2*b)
+ (d^2*Log[Sin[a + b*x]])/b^3 + (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*
x))])/b^2 - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d^2*Poly
Log[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (d^2*PolyLog[3, E^((2*I)*(a + b*x)
)])/ (2*b^3)
```

Rule 2620

```
Int[Csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v !=
= u]
```

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rule 3720

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 2551

Int[Log[u_] * ((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[((a + b*x)^(m + 1) * Log[u]) / (b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a + b*x)^(m + 1) * D[u, x]) / u, x], x] /; FreeQ[{a, b, m}, x] && InverseFunctionFreeQ[u, x] && NeQ[m, -1]

Rule 4183

Int[csc[(e_.) + (f_.)*(x_)] * ((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m * ArcTanh[E^(I*(e + f*x))]) / f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1) * Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1) * Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)] * ((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m * PolyLog[2, -(e*(F^(c*(a + b*x))))^n]) / (b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1) * PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)] / ((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p] / (e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned}
 \int (c+dx)^2 \csc^3(a+bx) \sec(a+bx) dx &= -\frac{(c+dx)^2 \cot^2(a+bx)}{2b} + \frac{(c+dx)^2 \log(\tan(a+bx))}{b} - (2d) \int (c+dx) \left(-\frac{c}{b} + \frac{dx}{b}\right) \csc^2(a+bx) dx \\
 &= -\frac{(c+dx)^2 \cot^2(a+bx)}{2b} + \frac{(c+dx)^2 \log(\tan(a+bx))}{b} - (2d) \int \frac{(c+dx) \left(-\frac{c}{b} + \frac{dx}{b}\right)}{\csc^2(a+bx)} dx \\
 &= -\frac{(c+dx)^2 \cot^2(a+bx)}{2b} + \frac{(c+dx)^2 \log(\tan(a+bx))}{b} - \frac{d \int (c+dx) \left(-\cot^2(a+bx)\right) dx}{\csc^2(a+bx)} \\
 &= -\frac{(c+dx)^2 \cot^2(a+bx)}{2b} + \frac{(c+dx)^2 \log(\tan(a+bx))}{b} - \frac{d \int \left(-\frac{c+dx}{\csc^2(a+bx)}\right) dx}{\csc^2(a+bx)} \\
 &= -\frac{(c+dx)^2 \cot^2(a+bx)}{2b} + \frac{(c+dx)^2 \log(\tan(a+bx))}{b} + \frac{d \int (c+dx) \cot^2(a+bx) dx}{\csc^2(a+bx)} \\
 &= -\frac{d(c+dx) \cot(a+bx)}{b^2} - \frac{(c+dx)^2 \cot^2(a+bx)}{2b} + \frac{\int 2b(c+dx)^2 \csc(2a+2bx) dx}{b} \\
 &= -\frac{cdx}{b} - \frac{d^2x^2}{2b} - \frac{d(c+dx) \cot(a+bx)}{b^2} - \frac{(c+dx)^2 \cot^2(a+bx)}{2b} + \frac{d^2 \log(\sin(a+bx))}{b^2} \\
 &= -\frac{cdx}{b} - \frac{d^2x^2}{2b} - \frac{2(c+dx)^2 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx) \cot(a+bx)}{b^2} - \frac{(c+dx)^2 \cot^2(a+bx)}{b^2} \\
 &= -\frac{cdx}{b} - \frac{d^2x^2}{2b} - \frac{2(c+dx)^2 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx) \cot(a+bx)}{b^2} - \frac{(c+dx)^2 \cot^2(a+bx)}{b^2} \\
 &= -\frac{cdx}{b} - \frac{d^2x^2}{2b} - \frac{2(c+dx)^2 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx) \cot(a+bx)}{b^2} - \frac{(c+dx)^2 \cot^2(a+bx)}{b^2} \\
 &= -\frac{cdx}{b} - \frac{d^2x^2}{2b} - \frac{2(c+dx)^2 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx) \cot(a+bx)}{b^2} - \frac{(c+dx)^2 \cot^2(a+bx)}{b^2}
 \end{aligned}$$

Mathematica [B] time = 6.75669, size = 872, normalized size = 4.34

$$\frac{\sec(a)(\cos(a) \log(\cos(a) \cos(bx) - \sin(a) \sin(bx)) + bx \sin(a))c^2}{b(\cos^2(a) + \sin^2(a))} + \frac{\csc(a)(\log(\cos(bx) \sin(a) + \cos(a) \sin(bx)) \sin(a))}{b(\cos^2(a) + \sin^2(a))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x], x]
```

```
[Out] -((c + d*x)^2*Csc[a + b*x]^2)/(2*b) - (d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a + b*x))]) - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))]) - I*PolyLog[3, E^((-I)*(a + b*x))]))/E^((2*I)*a))/(6*b^3) + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Csc[a]*Sec[a])/3 - ((I/12)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))]) - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(I*a)) - (c^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (c^2*Csc[a]*(-b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a])/(b*(Cos[a]^2 + Sin[a]^2)) + (d^2*Csc[a]*(-b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) - (c*d*Csc[a]*(b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x]) - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x
```

$$- \text{ArcTan}[\text{Cot}[a]])) + \text{Pi} * \text{Log}[\text{Cos}[b*x]] - 2 * \text{ArcTan}[\text{Cot}[a]] * \text{Log}[\text{Sin}[b*x - \text{ArcTan}[\text{Cot}[a]]]] + \text{I} * \text{PolyLog}[2, \text{E}^{((2*I)*(b*x - \text{ArcTan}[\text{Cot}[a]]))}] / \text{Sqrt}[1 + \text{Cot}[a]^2] * \text{Sec}[a]] / (b^2 * \text{Sqrt}[\text{Csc}[a]^2 * (\text{Cos}[a]^2 + \text{Sin}[a]^2)]) + (\text{Csc}[a] * \text{Csc}[a + b*x] * (c*d * \text{Sin}[b*x] + d^2 * x * \text{Sin}[b*x])) / b^2 - (c*d * \text{Csc}[a] * \text{Sec}[a] * (b^2 * \text{E}^{(\text{I} * \text{ArcTan}[\text{Tan}[a]]) * x^2 + ((\text{I} * b * x * (-\text{Pi} + 2 * \text{ArcTan}[\text{Tan}[a]]) - \text{Pi} * \text{Log}[1 + \text{E}^{(-2 * \text{I} * b * x)] - 2 * (b * x + \text{ArcTan}[\text{Tan}[a]]) * \text{Log}[1 - \text{E}^{((2 * \text{I} * (b * x + \text{ArcTan}[\text{Tan}[a]])]))} + \text{Pi} * \text{Log}[\text{Cos}[b*x]] + 2 * \text{ArcTan}[\text{Tan}[a]] * \text{Log}[\text{Sin}[b*x + \text{ArcTan}[\text{Tan}[a]]]] + \text{I} * \text{PolyLog}[2, \text{E}^{((2 * \text{I} * (b * x + \text{ArcTan}[\text{Tan}[a]])]))} * \text{Tan}[a]) / \text{Sqrt}[1 + \text{Tan}[a]^2])]) / (b^2 * \text{Sqrt}[\text{Sec}[a]^2 * (\text{Cos}[a]^2 + \text{Sin}[a]^2)])$$

Maple [B] time = 0.369, size = 632, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x)`

[Out]
$$-1/2*d^2*\text{polylog}(3, -\exp(2*I*(b*x+a))) / b^3 - 1/b*d^2*\ln(\exp(2*I*(b*x+a))+1)*x^2 - 1/b^3*d^2*\ln(1-\exp(I*(b*x+a)))*a^2 + 1/b*d^2*\ln(\exp(I*(b*x+a))+1)*x^2 + 1/b*d^2*\ln(1-\exp(I*(b*x+a)))*x^2 - 1/b*c^2*\ln(\exp(2*I*(b*x+a))+1) + I/b^2*c*d*\text{polylog}(2, -\exp(2*I*(b*x+a))) - 2*I/b^2*d^2*\text{polylog}(2, \exp(I*(b*x+a)))*x - 2*I/b^2*d^2*\text{polylog}(2, -\exp(I*(b*x+a)))*x - 2*I/b^2*c*d*\text{polylog}(2, \exp(I*(b*x+a))) - 2*I/b^2*c*d*\text{polylog}(2, -\exp(I*(b*x+a))) - 2/b^3*d^2*\ln(\exp(I*(b*x+a)))+1/b^3*d^2*\ln(\exp(I*(b*x+a))+1)+1/b^3*d^2*\ln(\exp(I*(b*x+a))-1)+2*(b*d^2*x^2*\exp(2*I*(b*x+a))+2*b*c*d*x*\exp(2*I*(b*x+a))+b*c^2*\exp(2*I*(b*x+a))-I*d^2*x*\exp(2*I*(b*x+a))-I*c*d*\exp(2*I*(b*x+a))+I*d^2*x+I*d*c)/b^2 / (\exp(2*I*(b*x+a))-1)^2 + 2/b*c*d*\ln(\exp(I*(b*x+a))+1)*x + 1/b*c^2*\ln(\exp(I*(b*x+a))+1)+1/b*c^2*\ln(\exp(I*(b*x+a))-1)+2*d^2*\text{polylog}(3, -\exp(I*(b*x+a))) / b^3 + 2*d^2*\text{polylog}(3, \exp(I*(b*x+a))) / b^3 + I/b^2*d^2*\text{polylog}(2, -\exp(2*I*(b*x+a)))*x - 2/b*c*d*\ln(\exp(2*I*(b*x+a))+1)*x + 2/b*c*d*\ln(1-\exp(I*(b*x+a)))*x + 2/b^2*c*d*\ln(1-\exp(I*(b*x+a)))*a + 1/b^3*d^2*a^2*\ln(\exp(I*(b*x+a))-1) - 2/b^2*c*d*a*\ln(\exp(I*(b*x+a))-1)$$

Maxima [B] time = 2.94517, size = 3405, normalized size = 16.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")`

[Out]
$$-1/2*(c^2*(1/\sin(b*x + a)^2 + \log(\sin(b*x + a)^2 - 1) - \log(\sin(b*x + a)^2) - 2*a*c*d*(1/\sin(b*x + a)^2 + \log(\sin(b*x + a)^2 - 1) - \log(\sin(b*x + a)^2)) / b + a^2*d^2*(1/\sin(b*x + a)^2 + \log(\sin(b*x + a)^2 - 1) - \log(\sin(b*x + a)^2)) / b^2 + 2*(4*(b*x + a)*d^2*\cos(4*b*x + 4*a) + 4*I*(b*x + a)*d^2*\sin(4*b*x + 4*a) - 4*b*c*d + 4*a*d^2 + (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\cos(4*b*x + 4*a) - 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\cos(2*b*x + 2*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*\sin(4*b*x + 4*a) + (-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a))*\sin(2*b*x + 2*a)) * \arctan2(\sin(2*b*x + 2*a), \cos(2*b*x + 2*a) + 1) - (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2)*\cos(4*b*x + 4*a) - 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2)*\cos(2*b*x + 2*a) - (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d +$$

$$\begin{aligned}
& 4*I*a*d^2*(b*x + a) - 2*I*d^2*\sin(4*b*x + 4*a) - (4*I*(b*x + a)^2*d^2 + (8*I*b*c*d - 8*I*a*d^2)*(b*x + a) + 4*I*d^2)*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) - (2*d^2*\cos(4*b*x + 4*a) - 4*d^2*\cos(2*b*x + 2*a) + 2*I*d^2*\sin(4*b*x + 4*a) - 4*I*d^2*\sin(2*b*x + 2*a) + 2*d^2)*\arctan2(\sin(b*x + a), \cos(b*x + a) - 1) + (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\cos(4*b*x + 4*a) - 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\cos(2*b*x + 2*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*\sin(4*b*x + 4*a) + (-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) + (4*I*(b*x + a)^2*d^2 + 4*b*c*d - 4*a*d^2 + (8*I*b*c*d - 4*(2*I*a + 1)*d^2)*(b*x + a))*\cos(2*b*x + 2*a) - (2*b*c*d + 2*(b*x + a)*d^2 - 2*a*d^2 + 2*(b*c*d + (b*x + a)*d^2 - a*d^2))*\cos(4*b*x + 4*a) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*\cos(2*b*x + 2*a) - (-2*I*b*c*d - 2*I*(b*x + a)*d^2 + 2*I*a*d^2)*\sin(4*b*x + 4*a) - (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*\sin(2*b*x + 2*a))*\operatorname{dilog}(-e^{(2*I*b*x + 2*I*a)}) + (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2))*\cos(4*b*x + 4*a) - 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*\cos(2*b*x + 2*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*\sin(4*b*x + 4*a) + (-8*I*b*c*d - 8*I*(b*x + a)*d^2 + 8*I*a*d^2)*\sin(2*b*x + 2*a))*\operatorname{dilog}(-e^{(I*b*x + I*a)}) + (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2))*\cos(4*b*x + 4*a) - 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*\cos(2*b*x + 2*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*\sin(4*b*x + 4*a) + (-8*I*b*c*d - 8*I*(b*x + a)*d^2 + 8*I*a*d^2)*\sin(2*b*x + 2*a))*\operatorname{dilog}(e^{(I*b*x + I*a)}) + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a))*\cos(4*b*x + 4*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*\cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\sin(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a) + I*d^2)*\cos(4*b*x + 4*a) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a) - 2*I*d^2)*\cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2)*\sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2)*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a) + I*d^2 + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a) + I*d^2))*\cos(4*b*x + 4*a) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a) - 2*I*d^2)*\cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2)*\sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2)*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) + (-I*d^2*\cos(4*b*x + 4*a) + 2*I*d^2*\cos(2*b*x + 2*a) + d^2*\sin(4*b*x + 4*a) - 2*d^2*\sin(2*b*x + 2*a) - I*d^2)*\operatorname{polylog}(3, -e^{(2*I*b*x + 2*I*a)}) + (4*I*d^2*\cos(4*b*x + 4*a) - 8*I*d^2*\cos(2*b*x + 2*a) - 4*d^2*\sin(4*b*x + 4*a) + 8*d^2*\sin(2*b*x + 2*a) + 4*I*d^2)*\operatorname{polylog}(3, -e^{(I*b*x + I*a)}) + (4*I*d^2*\cos(4*b*x + 4*a) - 8*I*d^2*\cos(2*b*x + 2*a) - 4*d^2*\sin(4*b*x + 4*a) + 8*d^2*\sin(2*b*x + 2*a) + 4*I*d^2)*\operatorname{polylog}(3, e^{(I*b*x + I*a)}) - (4*(b*x + a)^2*d^2 - 4*I*b*c*d + 4*I*a*d^2 + (8*b*c*d - (8*a - 4*I)*d^2)*(b*x + a))*\sin(2*b*x + 2*a))/(-2*I*b^2*\cos(4*b*x + 4*a) + 4*I*b^2*\cos(2*b*x + 2*a) + 2*b^2*\sin(4*b*x + 4*a) - 4*b^2*\sin(2*b*x + 2*a) - 2*I*b^2))/b
\end{aligned}$$

Fricas [C] time = 0.915492, size = 4868, normalized size = 24.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")

```
[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a)
*sin(b*x + a) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b
*x + a)^2)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d
+ (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2)*dilog(cos(b*x + a) - I*sin(b*x
+ a)) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x + a)
^2)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d + (2*I
*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) - sin(b*x + a))
+ (-2*I*b*d^2*x - 2*I*b*c*d + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2)*dil
og(-I*cos(b*x + a) + sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2
*x - 2*I*b*c*d)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (-2
*I*b*d^2*x - 2*I*b*c*d + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2)*dilog(-c
os(b*x + a) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2*x -
2*I*b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + d^2)*
cos(b*x + a)^2 + d^2)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*c^2 - 2
*a*b*c*d + a^2*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2)*log(co
s(b*x + a) + I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + d^2)*cos(b*x + a)^2 + d^2)*log(cos(b*
x + a) - I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*c^2 -
2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) - I*sin(b*x + a) + I)
+ (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*
c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(I*cos(b*x + a) + sin(b*x +
a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2
+ 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(I*cos(b*x + a) - s
in(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*
d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-I*cos(b*x
+ a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^
2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-
I*cos(b*x + a) - sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2 -
(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a
) + 1/2*I*sin(b*x + a) + 1/2) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2 - (b^2
*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1
/2*I*sin(b*x + a) + 1/2) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2
- (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-c
os(b*x + a) + I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a
) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 +
2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*si
n(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 2*(d^2*co
s(b*x + a)^2 - d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*(d^2*cos(
b*x + a)^2 - d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 2*(d^2*cos(b*
x + a)^2 - d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*(d^2*cos(b*x
+ a)^2 - d^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 2*(d^2*cos(b*x +
a)^2 - d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 2*(d^2*cos(b*x + a
)^2 - d^2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 2*(d^2*cos(b*x + a)
^2 - d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 2*(d^2*cos(b*x + a)^
2 - d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^3*cos(b*x + a)^2 -
b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((d*x+c)**2*csc(b*x+a)**3*sec(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \csc(bx + a)^3 \sec(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^3*sec(b*x + a), x)
```

3.243 $\int (c + dx) \csc^3(a + bx) \sec(a + bx) dx$

Optimal. Leaf size=141

$$\frac{idPolyLog\left(2, -e^{2i(a+bx)}\right)}{2b^2} - \frac{idPolyLog\left(2, e^{2i(a+bx)}\right)}{2b^2} - \frac{d \cot(a + bx)}{2b^2} - \frac{(c + dx) \cot^2(a + bx)}{2b} + \frac{(c + dx) \log(\tan(a + bx))}{b}$$

[Out] $-(d*x)/(2*b) - (2*d*x*ArcTanh[E^((2*I)*(a + b*x))])/b - (d*Cot[a + b*x])/(2*b^2) - ((c + d*x)*Cot[a + b*x]^2)/(2*b) - (d*x*Log[Tan[a + b*x]])/b + ((c + d*x)*Log[Tan[a + b*x]])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2$

Rubi [A] time = 0.138571, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {2620, 14, 4420, 3473, 8, 2548, 12, 4183, 2279, 2391}

$$\frac{idPolyLog\left(2, -e^{2i(a+bx)}\right)}{2b^2} - \frac{idPolyLog\left(2, e^{2i(a+bx)}\right)}{2b^2} - \frac{d \cot(a + bx)}{2b^2} - \frac{(c + dx) \cot^2(a + bx)}{2b} + \frac{(c + dx) \log(\tan(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x], x]

[Out] $-(d*x)/(2*b) - (2*d*x*ArcTanh[E^((2*I)*(a + b*x))])/b - (d*Cot[a + b*x])/(2*b^2) - ((c + d*x)*Cot[a + b*x]^2)/(2*b) - (d*x*Log[Tan[a + b*x]])/b + ((c + d*x)*Log[Tan[a + b*x]])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2$

Rule 2620

Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Dist[1/f, Subst[Int[(1 + x^2)^(m + n)/2 - 1]/x^m, x], x, Tan[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 4420

Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)]^(n_.), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2548

```
Int[Log[u_], x_Symbol] := Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(
m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int (c + dx) \csc^3(a + bx) \sec(a + bx) dx &= -\frac{(c + dx) \cot^2(a + bx)}{2b} + \frac{(c + dx) \log(\tan(a + bx))}{b} - d \int \left(-\frac{\cot^2(a + bx)}{2b} + \right. \\
&= -\frac{(c + dx) \cot^2(a + bx)}{2b} + \frac{(c + dx) \log(\tan(a + bx))}{b} + \frac{d \int \cot^2(a + bx) dx}{2b} - \\
&= -\frac{d \cot(a + bx)}{2b^2} - \frac{(c + dx) \cot^2(a + bx)}{2b} - \frac{dx \log(\tan(a + bx))}{b} + \frac{(c + dx) \log}{b} \\
&= -\frac{dx}{2b} - \frac{d \cot(a + bx)}{2b^2} - \frac{(c + dx) \cot^2(a + bx)}{2b} - \frac{dx \log(\tan(a + bx))}{b} + \frac{(c + dx) \log}{b} \\
&= -\frac{dx}{2b} - \frac{2dx \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{d \cot(a + bx)}{2b^2} - \frac{(c + dx) \cot^2(a + bx)}{2b} - \frac{dx \log}{b} \\
&= -\frac{dx}{2b} - \frac{2dx \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{d \cot(a + bx)}{2b^2} - \frac{(c + dx) \cot^2(a + bx)}{2b} - \frac{dx \log}{b} \\
&= -\frac{dx}{2b} - \frac{2dx \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{d \cot(a + bx)}{2b^2} - \frac{(c + dx) \cot^2(a + bx)}{2b} - \frac{dx \log}{b}
\end{aligned}$$

Mathematica [A] time = 0.957523, size = 210, normalized size = 1.49

$$\frac{d \left(\frac{1}{2} i \text{PolyLog} \left(2, -e^{2i(a+bx)} \right) + \frac{1}{2} i (a + bx)^2 - (a + bx) \log \left(1 + e^{2i(a+bx)} \right) \right)}{b^2} + \frac{d \left((a + bx) \log \left(1 - e^{2i(a+bx)} \right) - \frac{1}{2} i (a + bx)^2 \right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x], x]
```

```
[Out] -(d*Cot[a + b*x])/(2*b^2) - (d*x*Csc[a + b*x]^2)/(2*b) + (a*d*Log[Cos[a + b*x]])/b^2 - (c*(Csc[a + b*x]^2 + 2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]]))/(2*b) - (a*d*Log[Sin[a + b*x]])/b^2 + (d*((I/2)*(a + b*x)^2 - (a + b*x)*Log[1 + E^((2*I)*(a + b*x))] + (I/2)*PolyLog[2, -E^((2*I)*(a + b*x))]))/b^2 + (d*((a + b*x)*Log[1 - E^((2*I)*(a + b*x))] - (I/2)*((a + b*x)^2 + PolyLog[2, E^((2*I)*(a + b*x))])))/b^2
```

Maple [B] time = 0.173, size = 270, normalized size = 1.9

$$\frac{2bdxe^{2i(bx+a)} + 2bce^{2i(bx+a)} - ide^{2i(bx+a)} + id}{b^2(e^{2i(bx+a)} - 1)^2} - \frac{c \ln(e^{2i(bx+a)} + 1)}{b} + \frac{c \ln(e^{i(bx+a)} - 1)}{b} + \frac{c \ln(e^{i(bx+a)} + 1)}{b} + \frac{d \ln(1 - e^{i(bx+a)})}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x)
```

```
[Out] (2*b*d*x*exp(2*I*(b*x+a))+2*b*c*exp(2*I*(b*x+a))-I*d*exp(2*I*(b*x+a))+I*d)/b^2/(exp(2*I*(b*x+a))-1)^2-1/b*c*ln(exp(2*I*(b*x+a))+1)+1/b*c*ln(exp(I*(b*x+a))-1)+1/b*c*ln(exp(I*(b*x+a))+1)+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(b*x+a)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2-1/b*d*ln(exp(2*I*(b*x+a))+1)*x+1/2*I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+1/b*d*ln(exp(I*(b*x+a))+1)*x-I*d*polylog(2,-exp(I*(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

Maxima [B] time = 2.21863, size = 1397, normalized size = 9.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -((2*b*d*x + 2*b*c + 2*(b*d*x + b*c))*cos(4*b*x + 4*a) - 4*(b*d*x + b*c)*cos(2*b*x + 2*a) + (2*I*b*d*x + 2*I*b*c)*sin(4*b*x + 4*a) + (-4*I*b*d*x - 4*I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - (2*b*d*x + 2*b*c + 2*(b*d*x + b*c))*cos(4*b*x + 4*a) - 4*(b*d*x + b*c)*cos(2*b*x + 2*a) - (-2*I*b*d*x - 2*I*b*c)*sin(4*b*x + 4*a) - (4*I*b*d*x + 4*I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*b*c*cos(4*b*x + 4*a) - 4*b*c*cos(2*b*x + 2*a) + 2*I*b*c*sin(4*b*x + 4*a) - 4*I*b*c*sin(2*b*x + 2*a) + 2*b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*b*d*x*cos(4*b*x + 4*a) - 4*b*d*x*cos(2*b*x + 2*a) + 2*I*b*d*x*sin(4*b*x + 4*a) - 4*I*b*d*x*sin(2*b*x + 2*a) + 2*b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (4*I*b*d*x + 4*I*b*c + 2*d)*cos(2*b*x + 2*a) - (d*cos(4*b*x + 4*a) - 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) - 2*I*d*sin(2*b*x + 2*a) + d)*dilog(-e^(2*I*b*x + 2*I*a)) + (2*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) + 2*I*d*sin(4*b*x + 4*a) - 4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(-e^(I*b*x + I*a)) + (2*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) + 2*I*d*sin(4*b*x + 4*a) - 4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c))*cos(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(4*b*x + 4*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (I*b*d*x + I*b*c + (I*b*d*x + I*b*c))*cos(4*b*x + 4*a) + (-2*I*b*d*x - 2*I*b*c)*cos(2*b*x + 2*a) - (b*d*x + b*c)*sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*b*d*x + I*b*c + (I*b*d*x + I*b*c))*cos(4*b*x + 4*a) + (-2*I*b*d*x - 2*I*b*c)*
```

$$\cos(2bx + 2a) - (bdx + bc)\sin(4bx + 4a) + 2(bdx + bc)\sin(2bx + 2a) \log(\cos(bx + a)^2 + \sin(bx + a)^2 - 2\cos(bx + a) + 1) - 2(2bdx + 2bc - Id)\sin(2bx + 2a) - 2d / (-2Ib^2\cos(4bx + 4a) + 4Ib^2\cos(2bx + 2a) + 2b^2\sin(4bx + 4a) - 4b^2\sin(2bx + 2a) - 2Ib^2)$$

Fricas [B] time = 0.722204, size = 2457, normalized size = 17.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((dx+c)*csc(bx+a)^3*sec(bx+a),x, algorithm="fricas")

[Out] $\frac{1}{2}(bdx + d\cos(bx + a)\sin(bx + a) + bc + (-Id\cos(bx + a)^2 + Id) \text{dilog}(\cos(bx + a) + I\sin(bx + a)) + (Id\cos(bx + a)^2 - Id) \text{dilog}(\cos(bx + a) - I\sin(bx + a)) + (-Id\cos(bx + a)^2 + Id) \text{dilog}(I\cos(bx + a) + \sin(bx + a)) + (Id\cos(bx + a)^2 - Id) \text{dilog}(I\cos(bx + a) - \sin(bx + a)) + (Id\cos(bx + a)^2 - Id) \text{dilog}(-I\cos(bx + a) + \sin(bx + a)) + (-Id\cos(bx + a)^2 + Id) \text{dilog}(-I\cos(bx + a) - \sin(bx + a)) + (Id\cos(bx + a)^2 - Id) \text{dilog}(-\cos(bx + a) + I\sin(bx + a)) + (-Id\cos(bx + a)^2 + Id) \text{dilog}(-\cos(bx + a) - I\sin(bx + a)) - (bdx - (bdx + bc)\cos(bx + a)^2 + bc) \log(\cos(bx + a) + I\sin(bx + a) + 1) - ((bc - ad)\cos(bx + a)^2 - bc + ad) \log(\cos(bx + a) + I\sin(bx + a) + I) - (bdx - (bdx + bc)\cos(bx + a)^2 + bc) \log(\cos(bx + a) - I\sin(bx + a) + 1) - ((bc - ad)\cos(bx + a)^2 - bc + ad) \log(\cos(bx + a) - I\sin(bx + a) + I) + (bdx - (bdx + ad)\cos(bx + a)^2 + ad) \log(I\cos(bx + a) + \sin(bx + a) + 1) + (bdx - (bdx + ad)\cos(bx + a)^2 + ad) \log(I\cos(bx + a) - \sin(bx + a) + 1) + (bdx - (bdx + ad)\cos(bx + a)^2 + ad) \log(-I\cos(bx + a) + \sin(bx + a) + 1) + (bdx - (bdx + ad)\cos(bx + a)^2 + ad) \log(-I\cos(bx + a) - \sin(bx + a) + 1) + ((bc - ad)\cos(bx + a)^2 - bc + ad) \log(-\frac{1}{2}\cos(bx + a) + \frac{1}{2}I\sin(bx + a) + \frac{1}{2}) + ((bc - ad)\cos(bx + a)^2 - bc + ad) \log(-\frac{1}{2}\cos(bx + a) - \frac{1}{2}I\sin(bx + a) + \frac{1}{2}) - (bdx - (bdx + ad)\cos(bx + a)^2 + ad) \log(-\cos(bx + a) + I\sin(bx + a) + 1) - ((bc - ad)\cos(bx + a)^2 - bc + ad) \log(-\cos(bx + a) + I\sin(bx + a) + I) - (bdx - (bdx + ad)\cos(bx + a)^2 + ad) \log(-\cos(bx + a) - I\sin(bx + a) + 1) - ((bc - ad)\cos(bx + a)^2 - bc + ad) \log(-\cos(bx + a) - I\sin(bx + a) + I)) / (b^2\cos(bx + a)^2 - b^2)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((dx+c)*csc(bx+a)**3*sec(bx+a),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \csc(bx + a)^3 \sec(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^3*sec(b*x + a), x)
```

$$3.244 \quad \int \frac{\csc^3(a+bx) \sec(a+bx)}{c+dx} dx$$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\csc^3(a+bx) \sec(a+bx)}{c+dx}, x\right)$$

[Out] CannotIntegrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x), x]

Rubi [A] time = 0.146745, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^3(a+bx) \sec(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x), x]

[Out] Defer[Int] [(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x), x]

Rubi steps

$$\int \frac{\csc^3(a+bx) \sec(a+bx)}{c+dx} dx = \int \frac{\csc^3(a+bx) \sec(a+bx)}{c+dx} dx$$

Mathematica [A] time = 14.3861, size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx) \sec(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x), x]

[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x), x]

Maple [A] time = 3.076, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx+a))^3 \sec(bx+a)}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c), x)

[Out] int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c), x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)^3 \sec(bx+a)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x, algorithm="fricas")

[Out] integral(csc(b*x + a)^3*sec(b*x + a)/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx) \sec(a+bx)}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)**3*sec(b*x+a)/(d*x+c),x)

[Out] Integral(csc(a + b*x)**3*sec(a + b*x)/(c + d*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^3 \sec(bx+a)}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x, algorithm="giac")

[Out] integrate(csc(b*x + a)^3*sec(b*x + a)/(d*x + c), x)

$$3.245 \quad \int \frac{\csc^3(a+bx) \sec(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\csc^3(a+bx) \sec(a+bx)}{(c+dx)^2}, x\right)$$

[Out] CannotIntegrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2, x]

Rubi [A] time = 0.191978, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^3(a+bx) \sec(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2, x]

[Out] Defer[Int] [(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\csc^3(a+bx) \sec(a+bx)}{(c+dx)^2} dx = \int \frac{\csc^3(a+bx) \sec(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 16.889, size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx) \sec(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2, x]

[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2, x]

Maple [A] time = 4.781, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx+a))^3 \sec(bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2, x)

[Out] int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2, x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)^3 \sec(bx+a)}{d^2x^2 + 2cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(csc(b*x + a)^3*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx) \sec(a+bx)}{(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)**3*sec(b*x+a)/(d*x+c)**2,x)

[Out] Integral(csc(a + b*x)**3*sec(a + b*x)/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^3 \sec(bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(csc(b*x + a)^3*sec(b*x + a)/(d*x + c)^2, x)

$$3.246 \quad \int (c + dx)^m \sec(a + bx) \tan(a + bx) dx$$

Optimal. Leaf size=22

CannotIntegrate (tan(a + bx) sec(a + bx)(c + dx)^m, x)

[Out] CannotIntegrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x], x]

Rubi [A] time = 0.142017, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \sec(a + bx) \tan(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x], x]

[Out] Defer[Int] [(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x], x]

Rubi steps

$$\int (c + dx)^m \sec(a + bx) \tan(a + bx) dx = \int (c + dx)^m \sec(a + bx) \tan(a + bx) dx$$

Mathematica [A] time = 0.930321, size = 0, normalized size = 0.

$$\int (c + dx)^m \sec(a + bx) \tan(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x], x]

[Out] Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x], x]

Maple [A] time = 0.162, size = 0, normalized size = 0.

$$\int (dx + c)^m \sec(bx + a) \tan(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*sec(b*x+a)*tan(b*x+a), x)

[Out] int((d*x+c)^m*sec(b*x+a)*tan(b*x+a), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \sec(bx + a) \tan(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")

[Out] integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left((dx + c)^m \sec (bx + a) \tan (bx + a), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")

[Out] integral((d*x + c)^m*sec(b*x + a)*tan(b*x + a), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^m \tan (a + bx) \sec (a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**m*sec(b*x+a)*tan(b*x+a),x)

[Out] Integral((c + d*x)**m*tan(a + b*x)*sec(a + b*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \sec (bx + a) \tan (bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a), x)

3.247 $\int (c + dx)^4 \sec(a + bx) \tan(a + bx) dx$

Optimal. Leaf size=227

$$\frac{24d^3(c + dx)\text{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^4} - \frac{24d^3(c + dx)\text{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^4} - \frac{12id^2(c + dx)^2\text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} + \dots$$

```
[Out] ((8*I)*d*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b^2 - ((12*I)*d^2*(c + d*x)^2
*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((12*I)*d^2*(c + d*x)^2*PolyLog[2,
I*E^(I*(a + b*x))])/b^3 + (24*d^3*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x)
)])/b^4 - (24*d^3*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^4 + ((24*I)*d^
4*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^5 - ((24*I)*d^4*PolyLog[4, I*E^(I*(a
+ b*x))])/b^5 + ((c + d*x)^4*Sec[a + b*x])/b
```

Rubi [A] time = 0.185714, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {4409, 4181, 2531, 6609, 2282, 6589}

$$\frac{24d^3(c + dx)\text{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^4} - \frac{24d^3(c + dx)\text{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^4} - \frac{12id^2(c + dx)^2\text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} + \dots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sec[a + b*x]*Tan[a + b*x], x]
```

```
[Out] ((8*I)*d*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b^2 - ((12*I)*d^2*(c + d*x)^2
*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((12*I)*d^2*(c + d*x)^2*PolyLog[2,
I*E^(I*(a + b*x))])/b^3 + (24*d^3*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x)
)])/b^4 - (24*d^3*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^4 + ((24*I)*d^
4*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^5 - ((24*I)*d^4*PolyLog[4, I*E^(I*(a
+ b*x))])/b^5 + ((c + d*x)^4*Sec[a + b*x])/b
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] := Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x)
)], x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^4 \sec(a + bx) \tan(a + bx) dx &= \frac{(c + dx)^4 \sec(a + bx)}{b} - \frac{(4d) \int (c + dx)^3 \sec(a + bx) dx}{b} \\ &= \frac{8id(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{(c + dx)^4 \sec(a + bx)}{b} + \frac{(12d^2) \int (c + dx)^2 \log(1 - ie^{i(a+bx)})}{b^2} \\ &= \frac{8id(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{12id^2(c + dx)^2 \text{Li}_2(-ie^{i(a+bx)})}{b^3} + \frac{12id^2(c + dx)^2 \text{Li}_2(ie^{i(a+bx)})}{b^3} \\ &= \frac{8id(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{12id^2(c + dx)^2 \text{Li}_2(-ie^{i(a+bx)})}{b^3} + \frac{12id^2(c + dx)^2 \text{Li}_2(ie^{i(a+bx)})}{b^3} \\ &= \frac{8id(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{12id^2(c + dx)^2 \text{Li}_2(-ie^{i(a+bx)})}{b^3} + \frac{12id^2(c + dx)^2 \text{Li}_2(ie^{i(a+bx)})}{b^3} \\ &= \frac{8id(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{12id^2(c + dx)^2 \text{Li}_2(-ie^{i(a+bx)})}{b^3} + \frac{12id^2(c + dx)^2 \text{Li}_2(ie^{i(a+bx)})}{b^3} \end{aligned}$$

Mathematica [A] time = 1.23976, size = 428, normalized size = 1.89

$$\frac{(c + dx)^4 \sec(a + bx)}{b} - \frac{4d(3ib^2d(c + dx)^2 \text{PolyLog}(2, -ie^{i(a+bx)}) - 3ib^2d(c + dx)^2 \text{PolyLog}(2, ie^{i(a+bx)}) - 6bcd^2 \text{PolyLog}(2, e^{i(a+bx)}) - 6bcd^2 \text{PolyLog}(2, -e^{i(a+bx)}) - 6bcd^2 \text{PolyLog}(2, -1) - 6bcd^2 \text{PolyLog}(2, 1))}{b^5} + ((c + d*x)^4 \text{Sec}[a + b*x])/b$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Sec[a + b*x]*Tan[a + b*x], x]
```

```
[Out] (-4*d*((-2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 - I*E^(
I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] + b^3*d^3*x^3*Lo
g[1 - I*E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] - 3*b^3
*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + I*E^(I*(a + b*x
))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))] - (3*I)*b^2*
d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, (-I)*E^(
I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*c*d^2*Poly
Log[3, I*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))] - (6*I)
*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, I*E^(I*(a + b
x)))]/b^5 + ((c + d*x)^4*Sec[a + b*x])/b
```

Maple [B] time = 0.315, size = 767, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^4*sec(b*x+a)*tan(b*x+a), x)`

[Out] $24*I*d^3/b^3*c*polylog(2, I*\exp(I*(b*x+a)))*x+24*I*d^3/b^4*c*a^2*\arctan(\exp(I*(b*x+a)))-24*I*d^2/b^3*c^2*a*\arctan(\exp(I*(b*x+a)))+24*I*d^4*polylog(4, -I*\exp(I*(b*x+a)))/b^5+2*\exp(I*(b*x+a))*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*x^2+4*c^3*d*x+c^4)/b/(\exp(2*I*(b*x+a))+1)-24*d^4/b^4*polylog(3, I*\exp(I*(b*x+a)))*x+24*d^4/b^4*polylog(3, -I*\exp(I*(b*x+a)))*x-4*d^4/b^5*a^3*\ln(1-I*\exp(I*(b*x+a)))+4*d^4/b^2*\ln(1+I*\exp(I*(b*x+a)))*x^3-4*d^4/b^2*\ln(1-I*\exp(I*(b*x+a)))*x^3-24*d^3/b^4*c*polylog(3, I*\exp(I*(b*x+a)))+4*d^4/b^5*a^3*\ln(1+I*\exp(I*(b*x+a)))+24*d^3/b^4*c*polylog(3, -I*\exp(I*(b*x+a)))-24*I*d^4*polylog(4, I*\exp(I*(b*x+a)))/b^5-24*I*d^3/b^3*c*polylog(2, -I*\exp(I*(b*x+a)))*x-12*d^2/b^3*c^2*\ln(1-I*\exp(I*(b*x+a)))*a+12*d^3/b^2*c*\ln(1+I*\exp(I*(b*x+a)))*x^2-12*d^3/b^2*c*\ln(1-I*\exp(I*(b*x+a)))*x^2+12*I*d^2/b^3*c^2*polylog(2, I*\exp(I*(b*x+a)))-12*I*d^4/b^3*polylog(2, -I*\exp(I*(b*x+a)))*x^2+12*I*d^4/b^3*polylog(2, I*\exp(I*(b*x+a)))*x^2-12*I*d^2/b^3*c^2*polylog(2, -I*\exp(I*(b*x+a)))-8*I*d^4/b^5*a^3*\arctan(\exp(I*(b*x+a)))+8*I*d/b^2*c^3*\arctan(\exp(I*(b*x+a)))+12*d^3/b^4*c*a^2*\ln(1-I*\exp(I*(b*x+a)))+12*d^2/b^2*c^2*\ln(1+I*\exp(I*(b*x+a)))*x+12*d^2/b^3*c^2*\ln(1+I*\exp(I*(b*x+a)))*a-12*d^3/b^4*c*a^2*\ln(1+I*\exp(I*(b*x+a)))-12*d^2/b^2*c^2*\ln(1-I*\exp(I*(b*x+a)))*x$

Maxima [B] time = 3.0425, size = 3974, normalized size = 17.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^4*sec(b*x+a)*tan(b*x+a), x, algorithm="maxima")`

[Out] $(2*(4*(b*x + a)*\cos(2*b*x + 2*a)*\cos(b*x + a) + 4*(b*x + a)*\sin(2*b*x + 2*a)*\sin(b*x + a) + 4*(b*x + a)*\cos(b*x + a) - (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\sin(b*x + a) + 1) + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\sin(b*x + a) + 1))*c^3*d/((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)*b) - 6*(4*(b*x + a)*\cos(2*b*x + 2*a)*\cos(b*x + a) + 4*(b*x + a)*\sin(2*b*x + 2*a)*\sin(b*x + a) + 4*(b*x + a)*\cos(b*x + a) - (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\sin(b*x + a) + 1) + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\sin(b*x + a) + 1))*a*c^2*d^2/((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)*b^2) + 6*(4*(b*x + a)*\cos(2*b*x + 2*a)*\cos(b*x + a) + 4*(b*x + a)*\sin(2*b*x + 2*a)*\sin(b*x + a) + 4*(b*x + a)*\cos(b*x + a) - (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\sin(b*x + a) + 1) + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\sin(b*x + a) + 1))*a^2*c*d^3/((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)*b^3) - 2*(4*(b*x + a)*\cos(2*b*x + 2*a)*\cos(b*x + a) + 4*(b*x + a)*\sin(2*b*x + 2*a)*\sin(b*x + a) + 4*(b*x + a)*\cos(b*x + a) - (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\sin(b*x + a) + 1) + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\sin(b*x + a) + 1))*a$

$$\begin{aligned}
& x + a)^2 + 2*\sin(b*x + a) + 1) + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + \\
& 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\sin(b*x + \\
& a) + 1))*a^3*d^4/((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + \\
& 2*a) + 1)*b^4) + c^4/\cos(b*x + a) - 4*a*c^3*d/(b*\cos(b*x + a)) + 6*a^2*c^2* \\
& d^2/(b^2*\cos(b*x + a)) - 4*a^3*c*d^3/(b^3*\cos(b*x + a)) + a^4*d^4/(b^4*\cos(\\
& b*x + a)) + ((4*(b*x + a)^3*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 12*(b^ \\
& 2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) + 4*((b*x + a)^3*d^4 + 3*(b*c* \\
& d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a \\
&))*\cos(2*b*x + 2*a) - (-4*I*(b*x + a)^3*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)* \\
& (b*x + a)^2 + (-12*I*b^2*c^2*d^2 + 24*I*a*b*c*d^3 - 12*I*a^2*d^4)*(b*x + a) \\
&)*\sin(2*b*x + 2*a))*\arctan2(\cos(b*x + a), \sin(b*x + a) + 1) + (4*(b*x + a)^ \\
& 3*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + \\
& a^2*d^4)*(b*x + a) + 4*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + \\
& 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*\cos(2*b*x + 2*a) - (-4* \\
& I*(b*x + a)^3*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2 + (-12*I*b^2*c \\
& ^2*d^2 + 24*I*a*b*c*d^3 - 12*I*a^2*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan \\
& 2(\cos(b*x + a), -\sin(b*x + a) + 1) - (2*I*(b*x + a)^4*d^4 + (8*I*b*c*d^3 - \\
& 8*I*a*d^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4) \\
& *(b*x + a)^2)*\cos(b*x + a) + (12*b^2*c^2*d^2 - 24*a*b*c*d^3 + 12*(b*x + a)^ \\
& 2*d^4 + 12*a^2*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a) + 12*(b^2*c^2*d^2 - 2*a \\
& *b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*\cos(2 \\
& *b*x + 2*a) - (-12*I*b^2*c^2*d^2 + 24*I*a*b*c*d^3 - 12*I*(b*x + a)^2*d^4 - \\
& 12*I*a^2*d^4 + (-24*I*b*c*d^3 + 24*I*a*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*di \\
& log(I*e^(I*b*x + I*a)) - (12*b^2*c^2*d^2 - 24*a*b*c*d^3 + 12*(b*x + a)^2*d^ \\
& 4 + 12*a^2*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a) + 12*(b^2*c^2*d^2 - 2*a*b*c \\
& *d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*\cos(2*b*x \\
& + 2*a) + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*(b*x + a)^2*d^4 + 12*I* \\
& a^2*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*dilog(-I \\
& *e^(I*b*x + I*a)) - (-2*I*(b*x + a)^3*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x \\
& + a)^2 + (-6*I*b^2*c^2*d^2 + 12*I*a*b*c*d^3 - 6*I*a^2*d^4)*(b*x + a) + (-2 \\
& *I*(b*x + a)^3*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x + a)^2 + (-6*I*b^2*c^2 \\
& *d^2 + 12*I*a*b*c*d^3 - 6*I*a^2*d^4)*(b*x + a))*\cos(2*b*x + 2*a) + 2*((b*x \\
& + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 \\
& + a^2*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^ \\
& 2 + 2*\sin(b*x + a) + 1) - (2*I*(b*x + a)^3*d^4 + (6*I*b*c*d^3 - 6*I*a*d^4)* \\
& (b*x + a)^2 + (6*I*b^2*c^2*d^2 - 12*I*a*b*c*d^3 + 6*I*a^2*d^4)*(b*x + a) + \\
& (2*I*(b*x + a)^3*d^4 + (6*I*b*c*d^3 - 6*I*a*d^4)*(b*x + a)^2 + (6*I*b^2*c^2 \\
& *d^2 - 12*I*a*b*c*d^3 + 6*I*a^2*d^4)*(b*x + a))*\cos(2*b*x + 2*a) - 2*((b*x \\
& + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 \\
& + a^2*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^ \\
& 2 - 2*\sin(b*x + a) + 1) - 24*(d^4*\cos(2*b*x + 2*a) + I*d^4*\sin(2*b*x + 2*a) \\
& + d^4)*polylog(4, I*e^(I*b*x + I*a)) + 24*(d^4*\cos(2*b*x + 2*a) + I*d^4*si \\
& n(2*b*x + 2*a) + d^4)*polylog(4, -I*e^(I*b*x + I*a)) - (-24*I*b*c*d^3 - 24* \\
& I*(b*x + a)*d^4 + 24*I*a*d^4 + (-24*I*b*c*d^3 - 24*I*(b*x + a)*d^4 + 24*I*a \\
& *d^4)*\cos(2*b*x + 2*a) + 24*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*\sin(2*b*x + 2 \\
& *a))*polylog(3, I*e^(I*b*x + I*a)) - (24*I*b*c*d^3 + 24*I*(b*x + a)*d^4 - 2 \\
& 4*I*a*d^4 + (24*I*b*c*d^3 + 24*I*(b*x + a)*d^4 - 24*I*a*d^4)*\cos(2*b*x + 2* \\
& a) - 24*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*\sin(2*b*x + 2*a))*polylog(3, -I*e \\
& ^-(I*b*x + I*a)) + 2*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6* \\
& (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2)*\sin(b*x + a))/(-I*b^4*co \\
& s(2*b*x + 2*a) + b^4*\sin(2*b*x + 2*a) - I*b^4))/b
\end{aligned}$$

Fricas [C] time = 0.784666, size = 2930, normalized size = 12.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")

[Out] $(b^4d^4x^4 + 4b^4cd^3x^3 + 6b^4c^2d^2x^2 + 4b^4c^3dx + b^4c^4 - 12I*d^4*\cos(b*x + a)*\text{polylog}(4, I*\cos(b*x + a) + \sin(b*x + a)) - 12I*d^4*\cos(b*x + a)*\text{polylog}(4, I*\cos(b*x + a) - \sin(b*x + a)) + 12I*d^4*\cos(b*x + a)*\text{polylog}(4, -I*\cos(b*x + a) + \sin(b*x + a)) + 12I*d^4*\cos(b*x + a)*\text{polylog}(4, -I*\cos(b*x + a) - \sin(b*x + a)) + (6I*b^2*d^4*x^2 + 12I*b^2*c*d^3*x + 6I*b^2*c^2*d^2)*\cos(b*x + a)*\text{dilog}(I*\cos(b*x + a) + \sin(b*x + a)) + (6I*b^2*d^4*x^2 + 12I*b^2*c*d^3*x + 6I*b^2*c^2*d^2)*\cos(b*x + a)*\text{dilog}(I*\cos(b*x + a) - \sin(b*x + a)) + (-6I*b^2*d^4*x^2 - 12I*b^2*c*d^3*x - 6I*b^2*c^2*d^2)*\cos(b*x + a)*\text{dilog}(-I*\cos(b*x + a) + \sin(b*x + a)) + (-6I*b^2*d^4*x^2 - 12I*b^2*c*d^3*x - 6I*b^2*c^2*d^2)*\cos(b*x + a)*\text{dilog}(-I*\cos(b*x + a) - \sin(b*x + a)) - 2*(b^3*c^3*d - 3a*b^2*c^2*d^2 + 3a^2*b*c*d^3 - a^3*d^4)*\cos(b*x + a)*\log(\cos(b*x + a) + I*\sin(b*x + a) + I) + 2*(b^3*c^3*d - 3a*b^2*c^2*d^2 + 3a^2*b*c*d^3 - a^3*d^4)*\cos(b*x + a)*\log(\cos(b*x + a) - I*\sin(b*x + a) + I) - 2*(b^3*d^4*x^3 + 3b^3*c*d^3*x^2 + 3b^3*c^2*d^2*x + 3a*b^2*c^2*d^2 - 3a^2*b*c*d^3 + a^3*d^4)*\cos(b*x + a)*\log(I*\cos(b*x + a) + \sin(b*x + a) + 1) + 2*(b^3*d^4*x^3 + 3b^3*c*d^3*x^2 + 3b^3*c^2*d^2*x + 3a*b^2*c^2*d^2 - 3a^2*b*c*d^3 + a^3*d^4)*\cos(b*x + a)*\log(I*\cos(b*x + a) - \sin(b*x + a) + 1) - 2*(b^3*d^4*x^3 + 3b^3*c*d^3*x^2 + 3b^3*c^2*d^2*x + 3a*b^2*c^2*d^2 - 3a^2*b*c*d^3 + a^3*d^4)*\cos(b*x + a)*\log(-I*\cos(b*x + a) + \sin(b*x + a) + 1) + 2*(b^3*d^4*x^3 + 3b^3*c*d^3*x^2 + 3b^3*c^2*d^2*x + 3a*b^2*c^2*d^2 - 3a^2*b*c*d^3 + a^3*d^4)*\cos(b*x + a)*\log(-I*\cos(b*x + a) - \sin(b*x + a) + 1) - 2*(b^3*c^3*d - 3a*b^2*c^2*d^2 + 3a^2*b*c*d^3 - a^3*d^4)*\cos(b*x + a)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + I) + 2*(b^3*c^3*d - 3a*b^2*c^2*d^2 + 3a^2*b*c*d^3 - a^3*d^4)*\cos(b*x + a)*\log(-\cos(b*x + a) - I*\sin(b*x + a) + I) + 12*(b*d^4*x + b*c*d^3)*\cos(b*x + a)*\text{polylog}(3, I*\cos(b*x + a) + \sin(b*x + a)) - 12*(b*d^4*x + b*c*d^3)*\cos(b*x + a)*\text{polylog}(3, I*\cos(b*x + a) - \sin(b*x + a)) + 12*(b*d^4*x + b*c*d^3)*\cos(b*x + a)*\text{polylog}(3, -I*\cos(b*x + a) + \sin(b*x + a)) - 12*(b*d^4*x + b*c*d^3)*\cos(b*x + a)*\text{polylog}(3, -I*\cos(b*x + a) - \sin(b*x + a)))/(b^5*\cos(b*x + a))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^4 \tan(a + bx) \sec(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**4*sec(b*x+a)*tan(b*x+a),x)

[Out] Integral((c + d*x)**4*tan(a + b*x)*sec(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^4 \sec(bx + a) \tan(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^4*sec(b*x + a)*tan(b*x + a), x)

3.248 $\int (c + dx)^3 \sec(a + bx) \tan(a + bx) dx$

Optimal. Leaf size=159

$$\frac{6id^2(c + dx)\text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} + \frac{6id^2(c + dx)\text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3} + \frac{6d^3\text{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^4} - \frac{6d^3\text{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^4}$$

[Out] $((6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))]/b^3 + ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))]/b^3 + (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))]/b^4 - (6*d^3*PolyLog[3, I*E^(I*(a + b*x))]/b^4 + ((c + d*x)^3*Sec[a + b*x])/b$

Rubi [A] time = 0.125766, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {4409, 4181, 2531, 2282, 6589}

$$\frac{6id^2(c + dx)\text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} + \frac{6id^2(c + dx)\text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3} + \frac{6d^3\text{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^4} - \frac{6d^3\text{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^4}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x], x]

[Out] $((6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))]/b^3 + ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))]/b^3 + (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))]/b^4 - (6*d^3*PolyLog[3, I*E^(I*(a + b*x))]/b^4 + ((c + d*x)^3*Sec[a + b*x])/b$

Rule 4409

Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] - Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

Rule 4181

Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^(n)])/b*c*n*Log[F], x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^(n)]]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*

(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]

Rule 6589

Int [PolyLog [n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)] / ((d_.) + (e_.)*(x_)), x_Symbol] :> Simp [PolyLog [n + 1, c*(a + b*x)^p] / (e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ [b*d, a*e]

Rubi steps

$$\begin{aligned} \int (c + dx)^3 \sec(a + bx) \tan(a + bx) dx &= \frac{(c + dx)^3 \sec(a + bx)}{b} - \frac{(3d) \int (c + dx)^2 \sec(a + bx) dx}{b} \\ &= \frac{6id(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{(c + dx)^3 \sec(a + bx)}{b} + \frac{(6d^2) \int (c + dx) \log(1 - I e^{i(a+bx)})}{b^2} \\ &= \frac{6id(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{6id^2(c + dx) \text{Li}_2(-ie^{i(a+bx)})}{b^3} + \frac{6id^2(c + dx) \text{Li}_2(Ie^{i(a+bx)})}{b^3} \\ &= \frac{6id(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{6id^2(c + dx) \text{Li}_2(-ie^{i(a+bx)})}{b^3} + \frac{6id^2(c + dx) \text{Li}_2(Ie^{i(a+bx)})}{b^3} \\ &= \frac{6id(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{6id^2(c + dx) \text{Li}_2(-ie^{i(a+bx)})}{b^3} + \frac{6id^2(c + dx) \text{Li}_2(Ie^{i(a+bx)})}{b^3} \end{aligned}$$

Mathematica [A] time = 0.854534, size = 256, normalized size = 1.61

$$\frac{(c + dx)^3 \sec(a + bx)}{b} - \frac{3d(2ibd(c + dx) \text{PolyLog}(2, -ie^{i(a+bx)}) - 2ibd(c + dx) \text{PolyLog}(2, ie^{i(a+bx)}) - 2d^2 \text{PolyLog}(3, Ie^{i(a+bx)}))}{b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x], x]

[Out] (-3*d*((-2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))]) + 2*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] + b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - 2*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 2*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^4 + ((c + d*x)^3*Sec[a + b*x])/b

Maple [B] time = 0.274, size = 463, normalized size = 2.9

$$2 \frac{(d^3 x^3 + 3cd^2 x^2 + 3c^2 dx + c^3) e^{i(bx+a)}}{b(e^{2i(bx+a)} + 1)} - 3 \frac{d^3 \ln(1 - ie^{i(bx+a)}) x^2}{b^2} + \frac{6id^3 a^2 \arctan(e^{i(bx+a)})}{b^4} + \frac{6id^3 x \text{polylog}(2, ie^{i(bx+a)})}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*sec(b*x+a)*tan(b*x+a), x)

[Out] 2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))+1) -3*d^3/b^2*ln(1-I*exp(I*(b*x+a)))*x^2+6*I*d^3/b^4*a^2*arctan(exp(I*(b*x+a)))+6*I*d^3*x*polylog(2,I*exp(I*(b*x+a)))/b^3-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4+6*I*d/b^2*c^2*arctan(exp(I*(b*x+a)))+6*d^2/b^3*c*ln(1+I*exp(I*(b*x+a)))*a-3*d^3/b^4*a^2*ln(1+I*exp(I*(b*x+a)))-6*I*c*d^2*polylog(2,-I*exp(I*(b*x+a)))/b^3

$$\begin{aligned} & x+a)))/b^3+6*I*c*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3-6*d^2/b^2*c*\ln(1-I*exp \\ & (I*(b*x+a)))*x+3*d^3/b^2*\ln(1+I*exp(I*(b*x+a)))*x^2-6*I*d^3*x*polylog(2,-I* \\ & exp(I*(b*x+a)))/b^3-6*d^2/b^3*c*\ln(1-I*exp(I*(b*x+a)))*a+3*d^3/b^4*a^2*\ln(1 \\ & -I*exp(I*(b*x+a)))+6*d^2/b^2*c*\ln(1+I*exp(I*(b*x+a)))*x-12*I*d^2/b^3*c*a*ar \\ & ctan(exp(I*(b*x+a)))+6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4 \end{aligned}$$

Maxima [B] time = 2.26116, size = 2395, normalized size = 15.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")

[Out]
$$\begin{aligned} & 1/2*(3*(4*(b*x + a)*\cos(2*b*x + 2*a)*\cos(b*x + a) + 4*(b*x + a)*\sin(2*b*x + \\ & 2*a)*\sin(b*x + a) + 4*(b*x + a)*\cos(b*x + a) - (\cos(2*b*x + 2*a)^2 + \sin(2 \\ & *b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 \\ & + 2*\sin(b*x + a) + 1) + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2 \\ & *b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\sin(b*x + a) + 1)) \\ & *c^2*d/((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)* \\ & b) - 6*(4*(b*x + a)*\cos(2*b*x + 2*a)*\cos(b*x + a) + 4*(b*x + a)*\sin(2*b*x + \\ & 2*a)*\sin(b*x + a) + 4*(b*x + a)*\cos(b*x + a) - (\cos(2*b*x + 2*a)^2 + \sin(2 \\ & *b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 \\ & + 2*\sin(b*x + a) + 1) + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2 \\ & *b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\sin(b*x + a) + 1)) \\ & *a*c*d^2/((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1 \\ &)*b^2) + 3*(4*(b*x + a)*\cos(2*b*x + 2*a)*\cos(b*x + a) + 4*(b*x + a)*\sin(2*b \\ & *x + 2*a)*\sin(b*x + a) + 4*(b*x + a)*\cos(b*x + a) - (\cos(2*b*x + 2*a)^2 + s \\ & in(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + \\ & a)^2 + 2*\sin(b*x + a) + 1) + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*c \\ & os(2*b*x + 2*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\sin(b*x + a) + \\ & 1))*a^2*d^3/((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) \\ & + 1)*b^3) + 2*c^3/\cos(b*x + a) - 6*a*c^2*d/(b*cos(b*x + a)) + 6*a^2*c*d^2/ \\ & (b^2*cos(b*x + a)) - 2*a^3*d^3/(b^3*cos(b*x + a)) + 2*((6*(b*x + a)^2*d^3 + \\ & 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)* \\ & (b*x + a))*\cos(2*b*x + 2*a) - (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I \\ & *a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\cos(b*x + a), \sin(b*x + a) + 1 \\ &) + (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*((b*x + a)^2*d^ \\ & 3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - (-6*I*(b*x + a)^2*d^3 \\ & + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\cos(b* \\ & x + a), -\sin(b*x + a) + 1) - (4*I*(b*x + a)^3*d^3 + (12*I*b*c*d^2 - 12*I*a* \\ & d^3)*(b*x + a)^2)*\cos(b*x + a) + (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 \\ & + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\cos(2*b*x + 2*a) - (-12*I*b*c*d^2 - \\ & 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*\sin(2*b*x + 2*a))*\operatorname{dilog}(I*e^{(I*b*x + I*a)}) \\ & - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 + 12*(b*c*d^2 + (b*x + a)*d^3 \\ & - a*d^3)*\cos(2*b*x + 2*a) + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3 \\ &)*\sin(2*b*x + 2*a))*\operatorname{dilog}(-I*e^{(I*b*x + I*a)}) - (-3*I*(b*x + a)^2*d^3 + (-6 \\ & *I*b*c*d^2 + 6*I*a*d^3)*(b*x + a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + \\ & 6*I*a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - \\ & a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + \\ & 2*\sin(b*x + a) + 1) - (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x \\ & + a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*\cos(2*b \\ & *x + 2*a) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\sin(2*b*x + \\ & 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\sin(b*x + a) + 1) - (-12*I*d \\ & ^3*\cos(2*b*x + 2*a) + 12*d^3*\sin(2*b*x + 2*a) - 12*I*d^3)*\operatorname{polylog}(3, I*e^{(I \\ & *b*x + I*a)}) - (12*I*d^3*\cos(2*b*x + 2*a) - 12*d^3*\sin(2*b*x + 2*a) + 12*I* \\ & d^3)*\operatorname{polylog}(3, -I*e^{(I*b*x + I*a)}) + 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d \end{aligned}$$

$$\frac{(b^3(x+a)^2 \sin(bx+a)) / (-2Ib^3 \cos(2bx+2a) + 2b^3 \sin(2bx+2a) - 2Ib^3)}{b}$$

Fricas [C] time = 0.669218, size = 1993, normalized size = 12.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")

[Out] $\frac{1}{2} (2b^3d^3x^3 + 6b^3c^2d^2x^2 + 6b^3c^2d^2x + 2b^3c^3 + 6d^3 \cos(bx+a) \operatorname{polylog}(3, I \cos(bx+a) + \sin(bx+a)) - 6d^3 \cos(bx+a) \operatorname{polylog}(3, I \cos(bx+a) - \sin(bx+a)) + 6d^3 \cos(bx+a) \operatorname{polylog}(3, -I \cos(bx+a) + \sin(bx+a)) - 6d^3 \cos(bx+a) \operatorname{polylog}(3, -I \cos(bx+a) - \sin(bx+a)) + (6Ib^3d^3x + 6Ib^3c^2d^2) \cos(bx+a) \operatorname{dilog}(I \cos(bx+a) + \sin(bx+a)) + (6Ib^3d^3x + 6Ib^3c^2d^2) \cos(bx+a) \operatorname{dilog}(I \cos(bx+a) - \sin(bx+a)) + (-6Ib^3d^3x - 6Ib^3c^2d^2) \cos(bx+a) \operatorname{dilog}(-I \cos(bx+a) + \sin(bx+a)) + (-6Ib^3d^3x - 6Ib^3c^2d^2) \cos(bx+a) \operatorname{dilog}(-I \cos(bx+a) - \sin(bx+a)) - 3(b^2c^2d - 2a^2d^3) \cos(bx+a) \log(\cos(bx+a) + I \sin(bx+a) + I) + 3(b^2c^2d - 2a^2d^3) \cos(bx+a) \log(\cos(bx+a) - I \sin(bx+a) + I) - 3(b^2d^3x^2 + 2b^2c^2d^2x + 2a^2d^3) \cos(bx+a) \log(I \cos(bx+a) + \sin(bx+a) + 1) + 3(b^2d^3x^2 + 2b^2c^2d^2x + 2a^2d^3) \cos(bx+a) \log(I \cos(bx+a) - \sin(bx+a) + 1) - 3(b^2d^3x^2 + 2b^2c^2d^2x + 2a^2d^3) \cos(bx+a) \log(-I \cos(bx+a) + \sin(bx+a) + 1) + 3(b^2d^3x^2 + 2b^2c^2d^2x + 2a^2d^3) \cos(bx+a) \log(-I \cos(bx+a) - \sin(bx+a) + 1) - 3(b^2c^2d - 2a^2d^3) \cos(bx+a) \log(-\cos(bx+a) + I \sin(bx+a) + I) + 3(b^2c^2d - 2a^2d^3) \cos(bx+a) \log(-\cos(bx+a) - I \sin(bx+a) + I)) / (b^4 \cos(bx+a))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \tan(a + bx) \sec(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*sec(b*x+a)*tan(b*x+a),x)

[Out] Integral((c + d*x)**3*tan(a + b*x)*sec(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \sec(bx + a) \tan(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^3*sec(b*x + a)*tan(b*x + a), x)

3.249 $\int (c + dx)^2 \sec(a + bx) \tan(a + bx) dx$

Optimal. Leaf size=97

$$-\frac{2id^2 \text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} + \frac{2id^2 \text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3} + \frac{4id(c+dx) \tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2 \sec(a+bx)}{b}$$

[Out] $((4*I)*d*(c + d*x)*\text{ArcTan}[E^{(I*(a + b*x))}])/b^2 - ((2*I)*d^2*\text{PolyLog}[2, (-I)*E^{(I*(a + b*x))}])/b^3 + ((2*I)*d^2*\text{PolyLog}[2, I*E^{(I*(a + b*x))}])/b^3 + (c + d*x)^2*\text{Sec}[a + b*x])/b$

Rubi [A] time = 0.0684212, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {4409, 4181, 2279, 2391}

$$-\frac{2id^2 \text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} + \frac{2id^2 \text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3} + \frac{4id(c+dx) \tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2 \sec(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^2*\text{Sec}[a + b*x]*\text{Tan}[a + b*x], x]$

[Out] $((4*I)*d*(c + d*x)*\text{ArcTan}[E^{(I*(a + b*x))}])/b^2 - ((2*I)*d^2*\text{PolyLog}[2, (-I)*E^{(I*(a + b*x))}])/b^3 + ((2*I)*d^2*\text{PolyLog}[2, I*E^{(I*(a + b*x))}])/b^3 + (c + d*x)^2*\text{Sec}[a + b*x])/b$

Rule 4409

$\text{Int}[(c + d*x)^m*\text{Sec}[a + b*x]^n*\text{Tan}[a + b*x]^p, x] \rightarrow \text{Simp}[(c + d*x)^m*\text{Sec}[a + b*x]^n/(b*n), x] - \text{Dist}[(d*m)/(b*n), \text{Int}[(c + d*x)^{m-1}*\text{Sec}[a + b*x]^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

Rule 4181

$\text{Int}[\text{csc}[e + \text{Pi}*k + f*x]^m, x] \rightarrow \text{Simp}[-2*(c + d*x)^m*\text{ArcTanh}[E^{(I*k*\text{Pi})}*E^{(I*(e + f*x))}]/f, x] + (-\text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{m-1}*\text{Log}[1 - E^{(I*k*\text{Pi})}*E^{(I*(e + f*x))}], x], x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{m-1}*\text{Log}[1 + E^{(I*k*\text{Pi})}*E^{(I*(e + f*x))}], x], x]) /;$ FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]

Rule 2279

$\text{Int}[\text{Log}[a + b*x]^n, x] \rightarrow \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /;$ FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

$\text{Int}[\text{Log}[c + d*x]^n, x] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /;$ FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\int (c + dx)^2 \sec(a + bx) \tan(a + bx) dx = \frac{(c + dx)^2 \sec(a + bx)}{b} - \frac{(2d) \int (c + dx) \sec(a + bx) dx}{b}$$

$$= \frac{4id(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{(c + dx)^2 \sec(a + bx)}{b} + \frac{(2d^2) \int \log(1 - ie^{i(a+bx)})}{b^2}$$

$$= \frac{4id(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{(c + dx)^2 \sec(a + bx)}{b} - \frac{(2id^2) \text{Subst}\left(\int \frac{\log(1-x)}{x}\right)}{b^3}$$

$$= \frac{4id(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{2id^2 \text{Li}_2(-ie^{i(a+bx)})}{b^3} + \frac{2id^2 \text{Li}_2(ie^{i(a+bx)})}{b^3} + \frac{(c + dx)^2 \sec(a + bx)}{b}$$

Mathematica [A] time = 1.71315, size = 174, normalized size = 1.79

$$\frac{2d^2 \csc(a) \left(i \text{PolyLog}\left(2, -e^{i(bx - \tan^{-1}(\cot(a)))}\right) - i \text{PolyLog}\left(2, e^{i(bx - \tan^{-1}(\cot(a)))}\right) + (bx - \tan^{-1}(\cot(a))) \left(\log\left(1 - e^{i(bx - \tan^{-1}(\cot(a)))}\right) - \log\left(1 + e^{i(bx - \tan^{-1}(\cot(a)))}\right) \right) \right)}{\sqrt{\csc^2(a)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x], x]
```

```
[Out] (-4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - 4*d^2*ArcTan[Cot[a]]*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + (2*d^2*Csc[a]*((b*x - ArcTan[Cot[a]])*(Log[1 - E^(I*(b*x - ArcTan[Cot[a]])]) - Log[1 + E^(I*(b*x - ArcTan[Cot[a]])])]) + I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]])]) - I*PolyLog[2, E^(I*(b*x - ArcTan[Cot[a]])])]))/Sqrt[Csc[a]^2] + b^2*(c + d*x)^2*Sec[a + b*x])/b^3
```

Maple [B] time = 0.135, size = 227, normalized size = 2.3

$$2 \frac{e^{i(bx+a)} (d^2 x^2 + 2cdx + c^2)}{b(e^{2i(bx+a)} + 1)} + \frac{4idc \arctan(e^{i(bx+a)})}{b^2} + 2 \frac{d^2 \ln(1 + ie^{i(bx+a)})x}{b^2} + 2 \frac{d^2 \ln(1 + ie^{i(bx+a)})a}{b^3} - 2 \frac{d^2 \ln(1 - ie^{i(bx+a)})x}{b^2} - 2 \frac{d^2 \ln(1 - ie^{i(bx+a)})a}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*tan(b*x+a), x)
```

```
[Out] 2*exp(I*(b*x+a))*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x+a))+1)+4*I*d/b^2*c*a*arctan(exp(I*(b*x+a)))+2*d^2/b^2*ln(1+I*exp(I*(b*x+a)))*x+2*d^2/b^3*ln(1+I*exp(I*(b*x+a)))*a-2*d^2/b^2*ln(1-I*exp(I*(b*x+a)))*x-2*d^2/b^3*ln(1-I*exp(I*(b*x+a)))*a-2*I*d^2/b^3*dilog(1+I*exp(I*(b*x+a)))+2*I*d^2/b^3*dilog(1-I*exp(I*(b*x+a)))-4*I*d^2/b^3*a*arctan(exp(I*(b*x+a)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a), x, algorithm="maxima")
```

[Out] Timed out

Fricas [B] time = 0.595668, size = 1166, normalized size = 12.02

$$b^2 d^2 x^2 + 2 b^2 c d x + b^2 c^2 + i d^2 \cos(bx + a) \operatorname{Li}_2(i \cos(bx + a) + \sin(bx + a)) + i d^2 \cos(bx + a) \operatorname{Li}_2(i \cos(bx + a) - \sin(bx + a))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")

[Out] $(b^2 d^2 x^2 + 2 b^2 c d x + b^2 c^2 + I d^2 \cos(bx + a) \operatorname{dilog}(I \cos(bx + a) + \sin(bx + a)) + I d^2 \cos(bx + a) \operatorname{dilog}(I \cos(bx + a) - \sin(bx + a)) - I d^2 \cos(bx + a) \operatorname{dilog}(-I \cos(bx + a) + \sin(bx + a)) - I d^2 \cos(bx + a) \operatorname{dilog}(-I \cos(bx + a) - \sin(bx + a)) - (b^2 c d - a d^2) \cos(bx + a) \log(\cos(bx + a) + I \sin(bx + a) + I) + (b^2 c d - a d^2) \cos(bx + a) \log(\cos(bx + a) - I \sin(bx + a) + I) - (b^2 d^2 x + a d^2) \cos(bx + a) \log(I \cos(bx + a) + \sin(bx + a) + 1) + (b^2 d^2 x + a d^2) \cos(bx + a) \log(I \cos(bx + a) - \sin(bx + a) + 1) - (b^2 d^2 x + a d^2) \cos(bx + a) \log(-I \cos(bx + a) + \sin(bx + a) + 1) + (b^2 d^2 x + a d^2) \cos(bx + a) \log(-I \cos(bx + a) - \sin(bx + a) + 1) - (b^2 c d - a d^2) \cos(bx + a) \log(-\cos(bx + a) + I \sin(bx + a) + I) + (b^2 c d - a d^2) \cos(bx + a) \log(-\cos(bx + a) - I \sin(bx + a) + I)) / (b^3 \cos(bx + a))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \tan(a + bx) \sec(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*sec(b*x+a)*tan(b*x+a),x)

[Out] Integral((c + d*x)**2*tan(a + b*x)*sec(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \sec(bx + a) \tan(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^2*sec(b*x + a)*tan(b*x + a), x)

3.250 $\int (c + dx) \sec(a + bx) \tan(a + bx) dx$

Optimal. Leaf size=29

$$\frac{(c + dx) \sec(a + bx)}{b} - \frac{d \tanh^{-1}(\sin(a + bx))}{b^2}$$

[Out] $-\left(\frac{d \operatorname{ArcTanh}[\sin[a + b x]]}{b^2}\right) + \left(\frac{(c + d x) \operatorname{Sec}[a + b x]}{b}\right)$

Rubi [A] time = 0.0193704, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4409, 3770}

$$\frac{(c + dx) \sec(a + bx)}{b} - \frac{d \tanh^{-1}(\sin(a + bx))}{b^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Sec[a + b*x]*Tan[a + b*x], x]

[Out] $-\left(\frac{d \operatorname{ArcTanh}[\sin[a + b x]]}{b^2}\right) + \left(\frac{(c + d x) \operatorname{Sec}[a + b x]}{b}\right)$

Rule 4409

Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] - Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (c + dx) \sec(a + bx) \tan(a + bx) dx &= \frac{(c + dx) \sec(a + bx)}{b} - \frac{d \int \sec(a + bx) dx}{b} \\ &= -\frac{d \tanh^{-1}(\sin(a + bx))}{b^2} + \frac{(c + dx) \sec(a + bx)}{b} \end{aligned}$$

Mathematica [B] time = 0.0471758, size = 93, normalized size = 3.21

$$\frac{d \log\left(\cos\left(\frac{a}{2} + \frac{bx}{2}\right) - \sin\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{b^2} - \frac{d \log\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right) + \cos\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{b^2} + \frac{c \sec(a + bx)}{b} + \frac{dx \sec(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Sec[a + b*x]*Tan[a + b*x], x]

[Out] $(d \operatorname{Log}[\cos[a/2 + (b x)/2] - \sin[a/2 + (b x)/2]])/b^2 - (d \operatorname{Log}[\cos[a/2 + (b x)/2] + \sin[a/2 + (b x)/2]])/b^2 + (c \operatorname{Sec}[a + b x])/b + (d x \operatorname{Sec}[a + b x])/b$

b

Maple [A] time = 0.022, size = 49, normalized size = 1.7

$$\frac{dx}{b \cos(bx + a)} - \frac{d \ln(\sec(bx + a) + \tan(bx + a))}{b^2} + \frac{c}{b \cos(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*sec(b*x+a)*tan(b*x+a),x)

[Out] 1/b*d/cos(b*x+a)*x-1/b^2*d*ln(sec(b*x+a)+tan(b*x+a))+1/b*c/cos(b*x+a)

Maxima [B] time = 1.49983, size = 350, normalized size = 12.07

$$\frac{4(bx+a)\cos(2bx+2a)\cos(bx+a)+4(bx+a)\sin(2bx+2a)\sin(bx+a)+4(bx+a)\cos(bx+a)-(\cos(2bx+2a)^2+\sin(2bx+2a)^2+2\cos(2bx+2a)+1)\log(\cos(bx+a)^2+(\cos(2bx+2a)^2+\sin(2bx+2a)^2+2\cos(2bx+2a)+1)\log(\cos(bx+a)^2+\sin(bx+a)^2-2\sin(bx+a)+1))}{(\cos(2bx+2a)^2+\sin(2bx+2a)^2+2\cos(2bx+2a)+1)*b} + 2*c/\cos(bx+a) - 2*a*d/(b*\cos(bx+a))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")

[Out] 1/2*((4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b) + 2*c/cos(b*x + a) - 2*a*d/(b*cos(b*x + a))/b

Fricas [B] time = 0.492755, size = 163, normalized size = 5.62

$$\frac{2 b d x - d \cos(b x + a) \log(\sin(b x + a) + 1) + d \cos(b x + a) \log(-\sin(b x + a) + 1) + 2 b c}{2 b^2 \cos(b x + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")

[Out] 1/2*(2*b*d*x - d*cos(b*x + a)*log(sin(b*x + a) + 1) + d*cos(b*x + a)*log(-sin(b*x + a) + 1) + 2*b*c)/(b^2*cos(b*x + a))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \tan(a + bx) \sec(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x)

$$3.251 \quad \int \frac{\sec(a+bx) \tan(a+bx)}{c+dx} dx$$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\tan(a+bx) \sec(a+bx)}{c+dx}, x\right)$$

[Out] CannotIntegrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x), x]

Rubi [A] time = 0.0940087, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sec(a+bx) \tan(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x), x]

[Out] Defer[Int] [(Sec[a + b*x]*Tan[a + b*x])/(c + d*x), x]

Rubi steps

$$\int \frac{\sec(a+bx) \tan(a+bx)}{c+dx} dx = \int \frac{\sec(a+bx) \tan(a+bx)}{c+dx} dx$$

Mathematica [A] time = 11.2622, size = 0, normalized size = 0.

$$\int \frac{\sec(a+bx) \tan(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x), x]

[Out] Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x), x]

Maple [A] time = 0.268, size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a) \tan(bx+a)}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)*tan(b*x+a)/(d*x+c), x)

[Out] int(sec(b*x+a)*tan(b*x+a)/(d*x+c), x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(bx+a)\tan(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="fricas")

[Out] integral(sec(b*x + a)*tan(b*x + a)/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(a+bx)\sec(a+bx)}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)

[Out] Integral(tan(a + b*x)*sec(a + b*x)/(c + d*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)\tan(bx+a)}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="giac")

[Out] integrate(sec(b*x + a)*tan(b*x + a)/(d*x + c), x)

$$3.252 \quad \int \frac{\sec(a+bx) \tan(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\tan(a+bx) \sec(a+bx)}{(c+dx)^2}, x\right)$$

[Out] CannotIntegrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]

Rubi [A] time = 0.114617, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sec(a+bx) \tan(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]

[Out] Defer[Int] [(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\sec(a+bx) \tan(a+bx)}{(c+dx)^2} dx = \int \frac{\sec(a+bx) \tan(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 19.3385, size = 0, normalized size = 0.

$$\int \frac{\sec(a+bx) \tan(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]

[Out] Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]

Maple [A] time = 0.32, size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a) \tan(bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2, x)

[Out] int(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2, x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(bx+a)\tan(bx+a)}{d^2x^2+2cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(sec(b*x + a)*tan(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(a+bx)\sec(a+bx)}{(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)**2,x)

[Out] Integral(tan(a + b*x)*sec(a + b*x)/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)\tan(bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(sec(b*x + a)*tan(b*x + a)/(d*x + c)^2, x)

3.253 $\int (c + dx)^m \tan^2(a + bx) dx$

Optimal. Leaf size=18

Unintegrable($\tan^2(a + bx)(c + dx)^m, x$)

[Out] Unintegrable[(c + d*x)^m*Tan[a + b*x]^2, x]

Rubi [A] time = 0.0357392, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \tan^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Tan[a + b*x]^2, x]

[Out] Defer[Int] [(c + d*x)^m*Tan[a + b*x]^2, x]

Rubi steps

$$\int (c + dx)^m \tan^2(a + bx) dx = \int (c + dx)^m \tan^2(a + bx) dx$$

Mathematica [A] time = 3.14041, size = 0, normalized size = 0.

$$\int (c + dx)^m \tan^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Tan[a + b*x]^2, x]

[Out] Integrate[(c + d*x)^m*Tan[a + b*x]^2, x]

Maple [A] time = 0.166, size = 0, normalized size = 0.

$$\int (dx + c)^m (\tan(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*tan(b*x+a)^2, x)

[Out] int((d*x+c)^m*tan(b*x+a)^2, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \tan(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*tan(b*x+a)^2,x, algorithm="maxima")

[Out] integrate((d*x + c)^m*tan(b*x + a)^2, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx + c)^m \tan(bx + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*tan(b*x+a)^2,x, algorithm="fricas")

[Out] integral((d*x + c)^m*tan(b*x + a)^2, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^m \tan^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**m*tan(b*x+a)**2,x)

[Out] Integral((c + d*x)**m*tan(a + b*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \tan(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*tan(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^m*tan(b*x + a)^2, x)

3.254 $\int (c + dx)^3 \tan^2(a + bx) dx$

Optimal. Leaf size=128

$$-\frac{3id^2(c + dx)\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^3} + \frac{3d^3\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^4} + \frac{3d(c + dx)^2 \log\left(1 + e^{2i(a+bx)}\right)}{b^2} + \frac{(c + dx)^3 \tan(a + bx)}{b}$$

[Out] $((-I)*(c + d*x)^3)/b - (c + d*x)^4/(4*d) + (3*d*(c + d*x)^2*\text{Log}[1 + E^((2*I)*(a + b*x))])/b^2 - ((3*I)*d^2*(c + d*x)*\text{PolyLog}[2, -E^((2*I)*(a + b*x))])/b^3 + (3*d^3*\text{PolyLog}[3, -E^((2*I)*(a + b*x))])/(2*b^4) + ((c + d*x)^3*\text{Tan}[a + b*x])/b$

Rubi [A] time = 0.210269, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3720, 3719, 2190, 2531, 2282, 6589, 32}

$$-\frac{3id^2(c + dx)\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^3} + \frac{3d^3\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^4} + \frac{3d(c + dx)^2 \log\left(1 + e^{2i(a+bx)}\right)}{b^2} + \frac{(c + dx)^3 \tan(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^3*\text{Tan}[a + b*x]^2, x]$

[Out] $((-I)*(c + d*x)^3)/b - (c + d*x)^4/(4*d) + (3*d*(c + d*x)^2*\text{Log}[1 + E^((2*I)*(a + b*x))])/b^2 - ((3*I)*d^2*(c + d*x)*\text{PolyLog}[2, -E^((2*I)*(a + b*x))])/b^3 + (3*d^3*\text{PolyLog}[3, -E^((2*I)*(a + b*x))])/(2*b^4) + ((c + d*x)^3*\text{Tan}[a + b*x])/b$

Rule 3720

$\text{Int}[(c + d*x)^m*(b*\text{Tan}[e + f*x])^n, x] \rightarrow \text{Simp}[(b*(c + d*x)^m*(b*\text{Tan}[e + f*x])^{n-1})/(f*(n-1)), x] + (-\text{Dist}[(b*d*m)/(f*(n-1)), \text{Int}[(c + d*x)^{m-1}*(b*\text{Tan}[e + f*x])^{n-1}, x], x] - \text{Dist}[b^2, \text{Int}[(c + d*x)^m*(b*\text{Tan}[e + f*x])^{n-2}, x], x]) /;$ FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 3719

$\text{Int}[(c + d*x)^m*\text{Tan}[e + f*x], x] \rightarrow \text{Simp}[(I*(c + d*x)^{m+1})/(d*(m+1)), x] - \text{Dist}[2*I, \text{Int}[(c + d*x)^m*\text{E}^{(2*I*(e + f*x))}/(1 + \text{E}^{(2*I*(e + f*x))}), x], x] /;$ FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2190

$\text{Int}[(c + d*x)^m*\text{Log}[1 + (b*(F^{g*(e + f*x)})^n)/a], x] \rightarrow \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^{g*(e + f*x)})^n)/a], x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{m-1}*\text{Log}[1 + (b*(F^{g*(e + f*x)})^n)/a], x], x] /;$ FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

$\text{Int}[\text{Log}[1 + (e*(F^{c*(a + b*x)})^n)], x] \rightarrow -\text{Simp}[(f + g*x)^m*\text{PolyLog}[2, -(e*(F^{c*(a + b*x)})^n)], x] + \text{Dist}[(g*m)/(b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^{m-1}*\text{PolyLog}[2, -(e*(F^{c*(a + b*x)})^n)], x], x] /;$ FreeQ[{F, a, b, c, e, f}

, g, n}, x] && GtQ[m, 0]

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^3 \tan^2(a + bx) dx &= \frac{(c + dx)^3 \tan(a + bx)}{b} - \frac{(3d) \int (c + dx)^2 \tan(a + bx) dx}{b} - \int (c + dx)^3 dx \\ &= -\frac{i(c + dx)^3}{b} - \frac{(c + dx)^4}{4d} + \frac{(c + dx)^3 \tan(a + bx)}{b} + \frac{(6id) \int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}} dx}{b} \\ &= -\frac{i(c + dx)^3}{b} - \frac{(c + dx)^4}{4d} + \frac{3d(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b^2} + \frac{(c + dx)^3 \tan(a + bx)}{b} - \frac{(6id^2) \int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}} dx}{b^2} \\ &= -\frac{i(c + dx)^3}{b} - \frac{(c + dx)^4}{4d} + \frac{3d(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b^2} - \frac{3id^2(c + dx) \text{Li}_2(-e^{2i(a+bx)})}{b^3} \\ &= -\frac{i(c + dx)^3}{b} - \frac{(c + dx)^4}{4d} + \frac{3d(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b^2} - \frac{3id^2(c + dx) \text{Li}_2(-e^{2i(a+bx)})}{b^3} \\ &= -\frac{i(c + dx)^3}{b} - \frac{(c + dx)^4}{4d} + \frac{3d(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b^2} - \frac{3id^2(c + dx) \text{Li}_2(-e^{2i(a+bx)})}{b^3} \end{aligned}$$

Mathematica [B] time = 6.56442, size = 424, normalized size = 3.31

$$3cd^2 \csc(a) \sec(a) \left(b^2 x^2 e^{-i \tan^{-1}(\cot(a))} - \frac{\cot(a) \left(i \text{PolyLog}\left(2, e^{2i(bx - \tan^{-1}(\cot(a)))} \right) + ibx(-2 \tan^{-1}(\cot(a)) - \pi) - 2(bx - \tan^{-1}(\cot(a))) \log\left(1 - e^{2i(bx - \tan^{-1}(\cot(a)))} \right) \right)}{\sqrt{\cot^2(a)}} \right) / b^3 \sqrt{\csc^2(a) (\sin^2(a) + \cos^2(a))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^3*Tan[a + b*x]^2,x]

```
[Out] -(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 + ((I/4)*d^3*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a]/(b^4*E^(I*a)) + (3*c^2*d*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^2*(Cos[a]^2 + Sin[a]^2)) + (3*c*d^2*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]])) - (Cot[a]*
```

$$\frac{(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]])])] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]])]))/Sqrt[1 + Cot[a]^2]*Sec[a]/(b^3*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)] + (Sec[a]*Sec[a + b*x]*(c^3*Sin[b*x] + 3*c^2*d*x*Sin[b*x] + 3*c*d^2*x^2*Sin[b*x] + d^3*x^3*Sin[b*x]))/b$$

Maple [B] time = 0.197, size = 348, normalized size = 2.7

$$-\frac{d^3x^4}{4} - cd^2x^3 - \frac{3c^2dx^2}{2} - c^3x - \frac{3id^3\text{polylog}(2, -e^{2i(bx+a)})x}{b^3} + 3\frac{c^2d\ln(e^{2i(bx+a)} + 1)}{b^2} - 6\frac{c^2d\ln(e^{i(bx+a)})}{b^2} - 6\frac{d^3a^2\ln(\dots)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*tan(b*x+a)^2,x)

[Out] $-1/4*d^3*x^4 - c*d^2*x^3 - 3/2*c^2*d*x^2 - c^3*x - 3*I*d^3/b^3*\text{polylog}(2, -\exp(2*I*(b*x+a))) * x + 3*d/b^2*c^2*\ln(\exp(2*I*(b*x+a))+1) - 6*d/b^2*c^2*\ln(\exp(I*(b*x+a))) - 6*d^3/b^4*a^2*\ln(\exp(I*(b*x+a))) - 3*I*d^2/b^3*c*\text{polylog}(2, -\exp(2*I*(b*x+a))) + 6*I*d^3/b^3*a^2*x - 6*I*d^2/b^3*c*a^2 + 2*I*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)/b / (\exp(2*I*(b*x+a))+1) + 3*d^3/b^2*\ln(\exp(2*I*(b*x+a))+1) * x^2 - 2*I*d^3/b*x^3 + 3/2*d^3*\text{polylog}(3, -\exp(2*I*(b*x+a))) / b^4 + 12*d^2/b^3*c*a*\ln(\exp(I*(b*x+a))) + 6*d^2/b^2*c*\ln(\exp(2*I*(b*x+a))+1) * x + 4*I*d^3/b^4*a^3 - 12*I*d^2/b^2*c*a*x - 6*I*d^2/b*c*x^2$

Maxima [B] time = 1.93025, size = 1840, normalized size = 14.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*tan(b*x+a)^2,x, algorithm="maxima")

[Out] $-1/2*(2*(b*x + a - \tan(b*x + a))*c^3 - 6*(b*x + a - \tan(b*x + a))*a*c^2*d/b + 6*(b*x + a - \tan(b*x + a))*a^2*c*d^2/b^2 - 2*(b*x + a - \tan(b*x + a))*a^3*d^3/b^3 + 3*((b*x + a)^2*\cos(2*b*x + 2*a)^2 + (b*x + a)^2*\sin(2*b*x + 2*a)^2 + 2*(b*x + a)^2*\cos(2*b*x + 2*a) + (b*x + a)^2 - (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*\sin(2*b*x + 2*a))*c^2*d / ((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)*b) - 6*((b*x + a)^2*\cos(2*b*x + 2*a)^2 + (b*x + a)^2*\sin(2*b*x + 2*a)^2 + 2*(b*x + a)^2*\cos(2*b*x + 2*a) + (b*x + a)^2 - (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*\sin(2*b*x + 2*a))*a*c*d^2 / ((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)*b^2) + 3*((b*x + a)^2*\cos(2*b*x + 2*a)^2 + (b*x + a)^2*\sin(2*b*x + 2*a)^2 + 2*(b*x + a)^2*\cos(2*b*x + 2*a) + (b*x + a)^2 - (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*\sin(2*b*x + 2*a))*a^2*d^3 / ((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)*b^3) - 2*(I*(b*x + a))^4*d^3 + (4*I*b*c*d^2 - 4*I*a*d^3)*(b*x + a)^3 + (12*(b*x + a)^2*d^3 + 24*(b*c*d^2 - a*d^3)*(b*x + a) + 12*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (12*I*(b*x + a)^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*arctan2(\sin(2*b*x + 2*a), \cos(2*b*x + 2*a))$

+ 1) + (I*(b*x + a)^4*d^3 + (4*I*b*c*d^2 - 4*(I*a + 2)*d^3)*(b*x + a)^3 - 24*(b*c*d^2 - a*d^3)*(b*x + a)^2)*cos(2*b*x + 2*a) - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*sin(2*b*x + 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (-6*I*d^3*cos(2*b*x + 2*a) + 6*d^3*sin(2*b*x + 2*a) - 6*I*d^3)*polylog(3, -e^(2*I*b*x + 2*I*a)) - ((b*x + a)^4*d^3 + (4*b*c*d^2 - (4*a - 8*I)*d^3)*(b*x + a)^3 - (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a)^2)*sin(2*b*x + 2*a))/(-4*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin(2*b*x + 2*a) - 4*I*b^3)) /b

Fricas [C] time = 0.502994, size = 913, normalized size = 7.13

$$b^4 d^3 x^4 + 4 b^4 c d^2 x^3 + 6 b^4 c^2 d x^2 + 4 b^4 c^3 x - 3 d^3 \operatorname{polylog}\left(3, \frac{\tan(bx+a)^2 + 2i \tan(bx+a) - 1}{\tan(bx+a)^2 + 1}\right) - 3 d^3 \operatorname{polylog}\left(3, \frac{\tan(bx+a)^2 - 2i \tan(bx+a) - 1}{\tan(bx+a)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*tan(b*x+a)^2,x, algorithm="fricas")

[Out] -1/4*(b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 6*b^4*c^2*d*x^2 + 4*b^4*c^3*x - 3*d^3*polylog(3, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 3*d^3*polylog(3, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - (6*I*b*d^3*x + 6*I*b*c*d^2)*dilog(2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) - (-6*I*b*d^3*x - 6*I*b*c*d^2)*dilog(2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*tan(b*x + a))/b^4

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \tan^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*tan(b*x+a)**2,x)

[Out] Integral((c + d*x)**3*tan(a + b*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \tan(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*tan(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^3*tan(b*x + a)^2, x)

3.255 $\int (c + dx)^2 \tan^2(a + bx) dx$

Optimal. Leaf size=96

$$-\frac{id^2 \text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^3} + \frac{2d(c+dx) \log\left(1 + e^{2i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2 \tan(a+bx)}{b} - \frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d}$$

[Out] $((-I)*(c + d*x)^2)/b - (c + d*x)^3/(3*d) + (2*d*(c + d*x)*\text{Log}[1 + E^{((2*I)*(a + b*x))}])/b^2 - (I*d^2*\text{PolyLog}[2, -E^{((2*I)*(a + b*x))}])/b^3 + ((c + d*x)^2*\text{Tan}[a + b*x])/b$

Rubi [A] time = 0.141087, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3720, 3719, 2190, 2279, 2391, 32}

$$-\frac{id^2 \text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^3} + \frac{2d(c+dx) \log\left(1 + e^{2i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2 \tan(a+bx)}{b} - \frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2*Tan[a + b*x]^2,x]

[Out] $((-I)*(c + d*x)^2)/b - (c + d*x)^3/(3*d) + (2*d*(c + d*x)*\text{Log}[1 + E^{((2*I)*(a + b*x))}])/b^2 - (I*d^2*\text{PolyLog}[2, -E^{((2*I)*(a + b*x))}])/b^3 + ((c + d*x)^2*\text{Tan}[a + b*x])/b$

Rule 3720

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 3719

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

```
Int[Log[(c_.)*(d_) + (e_.)*(x_)^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_)^(m_)), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \tan^2(a + bx) dx &= \frac{(c + dx)^2 \tan(a + bx)}{b} - \frac{(2d) \int (c + dx) \tan(a + bx) dx}{b} - \int (c + dx)^2 dx \\ &= -\frac{i(c + dx)^2}{b} - \frac{(c + dx)^3}{3d} + \frac{(c + dx)^2 \tan(a + bx)}{b} + \frac{(4id) \int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}} dx}{b} \\ &= -\frac{i(c + dx)^2}{b} - \frac{(c + dx)^3}{3d} + \frac{2d(c + dx) \log(1 + e^{2i(a+bx)})}{b^2} + \frac{(c + dx)^2 \tan(a + bx)}{b} - \frac{(2d^2 x^2)}{2} \\ &= -\frac{i(c + dx)^2}{b} - \frac{(c + dx)^3}{3d} + \frac{2d(c + dx) \log(1 + e^{2i(a+bx)})}{b^2} + \frac{(c + dx)^2 \tan(a + bx)}{b} + \frac{(id^2 x^2)}{2} \\ &= -\frac{i(c + dx)^2}{b} - \frac{(c + dx)^3}{3d} + \frac{2d(c + dx) \log(1 + e^{2i(a+bx)})}{b^2} - \frac{id^2 \text{Li}_2(-e^{2i(a+bx)})}{b^3} + \frac{(c + dx)^2}{2} \end{aligned}$$

Mathematica [B] time = 6.38096, size = 276, normalized size = 2.88

$$\frac{d^2 \csc(a) \sec(a) \left(b^2 x^2 e^{-i \tan^{-1}(\cot(a))} - \frac{\cot(a) \left(i \text{PolyLog} \left(2, e^{2i(bx - \tan^{-1}(\cot(a)))} \right) \right) + ibx(-2 \tan^{-1}(\cot(a)) - \pi) - 2(bx - \tan^{-1}(\cot(a))) \log \left(1 - e^{2i(bx - \tan^{-1}(\cot(a)))} \right)}{\sqrt{\cot^2(a) + 1}} \right)}{b^3 \sqrt{\csc^2(a) (\sin^2(a) + \cos^2(a))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Tan[a + b*x]^2,x]
```

```
[Out] -(x*(3*c^2 + 3*c*d*x + d^2*x^2))/3 + (2*c*d*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^2*(Cos[a]^2 + Sin[a]^2)) + (d^2*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]])]) + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]]) + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]])])))/Sqrt[1 + Cot[a]^2]*Sec[a])/(b^3*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Sec[a]*Sec[a + b*x]*(c^2*Sin[b*x] + 2*c*d*x*Sin[b*x] + d^2*x^2*Sin[b*x]))/b
```

Maple [B] time = 0.162, size = 191, normalized size = 2.

$$-\frac{d^2 x^3}{3} - cdx^2 - c^2x + \frac{2i(d^2x^2 + 2cdx + c^2)}{b(e^{2i(bx+a)} + 1)} - 4\frac{cd \ln(e^{i(bx+a)})}{b^2} + 2\frac{cd \ln(e^{2i(bx+a)} + 1)}{b^2} - \frac{2id^2x^2}{b} - \frac{4id^2ax}{b^2} - \frac{2id^2a^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*tan(b*x+a)^2,x)
```

```
[Out] -1/3*d^2*x^3-c*d*x^2-c^2*x+2*I*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x+a))+1)
-4*d/b^2*c*ln(exp(I*(b*x+a)))+2*d/b^2*c*ln(exp(2*I*(b*x+a))+1)-2*I*d^2/b*x^
2-4*I*d^2/b^2*a*x-2*I*d^2/b^3*a^2+2*d^2/b^2*ln(exp(2*I*(b*x+a))+1)*x-I*d^2*
polylog(2,-exp(2*I*(b*x+a)))/b^3+4*d^2/b^3*a*ln(exp(I*(b*x+a)))
```

Maxima [B] time = 1.84466, size = 563, normalized size = 5.86

$$ib^3d^2x^3 + 3ib^3cdx^2 + 3ib^3c^2x + 6b^2c^2 + (6bd^2x + 6bcd + 6(bd^2x + bcd) \cos(2bx + 2a) + (6ibd^2x + 6ibcd) \sin(2bx + 2a)) / (-3Ib^3 \cos(2bx + 2a) + 3b^3 \sin(2bx + 2a) - 3Ib^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (I*b^3*d^2*x^3 + 3*I*b^3*c*d*x^2 + 3*I*b^3*c^2*x + 6*b^2*c^2 + (6*b*d^2*x +
6*b*c*d + 6*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a) + (6*I*b*d^2*x + 6*I*b*c*d)
*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + (I*b^3
*d^2*x^3 + (3*I*b^3*c*d - 6*b^2*d^2)*x^2 - 3*(-I*b^3*c^2 + 4*b^2*c*d)*x)*co
s(2*b*x + 2*a) - 3*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) + d^2)*di
log(-e^(2*I*b*x + 2*I*a)) + (-3*I*b*d^2*x - 3*I*b*c*d + (-3*I*b*d^2*x - 3*I
*b*c*d)*cos(2*b*x + 2*a) + 3*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a))*log(cos(2*
b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (b^3*d^2*x^3
+ 3*(b^3*c*d + 2*I*b^2*d^2)*x^2 + (3*b^3*c^2 + 12*I*b^2*c*d)*x)*sin(2*b*x +
2*a))/(-3*I*b^3*cos(2*b*x + 2*a) + 3*b^3*sin(2*b*x + 2*a) - 3*I*b^3)
```

Fricas [B] time = 0.490464, size = 522, normalized size = 5.44

$$\frac{2b^3d^2x^3 + 6b^3cdx^2 + 6b^3c^2x - 3id^2\text{Li}_2\left(\frac{2(i \tan(bx+a)-1)}{\tan(bx+a)^2+1} + 1\right) + 3id^2\text{Li}_2\left(\frac{2(-i \tan(bx+a)-1)}{\tan(bx+a)^2+1} + 1\right) - 6(bd^2x + bcd) \log\left(-\frac{2(i \tan(bx+a)-1)}{\tan(bx+a)^2+1}\right)}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*d^2*x^3 + 6*b^3*c*d*x^2 + 6*b^3*c^2*x - 3*I*d^2*dilog(2*(I*tan(
b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + 3*I*d^2*dilog(2*(-I*tan(b*x + a)
- 1)/(tan(b*x + a)^2 + 1) + 1) - 6*(b*d^2*x + b*c*d)*log(-2*(I*tan(b*x + a)
- 1)/(tan(b*x + a)^2 + 1)) - 6*(b*d^2*x + b*c*d)*log(-2*(-I*tan(b*x + a) -
1)/(tan(b*x + a)^2 + 1)) - 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*tan(b*x
+ a))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \tan^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*tan(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \tan (bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*tan(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^2*tan(b*x + a)^2, x)

3.256 $\int (c + dx) \tan^2(a + bx) dx$

Optimal. Leaf size=40

$$\frac{d \log(\cos(a + bx))}{b^2} + \frac{(c + dx) \tan(a + bx)}{b} - cx - \frac{dx^2}{2}$$

[Out] $-(c*x) - (d*x^2)/2 + (d*\text{Log}[\text{Cos}[a + b*x]])/b^2 + ((c + d*x)*\text{Tan}[a + b*x])/b$

Rubi [A] time = 0.0283952, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3720, 3475}

$$\frac{d \log(\cos(a + bx))}{b^2} + \frac{(c + dx) \tan(a + bx)}{b} - cx - \frac{dx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Tan[a + b*x]^2, x]

[Out] $-(c*x) - (d*x^2)/2 + (d*\text{Log}[\text{Cos}[a + b*x]])/b^2 + ((c + d*x)*\text{Tan}[a + b*x])/b$

Rule 3720

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (c + dx) \tan^2(a + bx) dx &= \frac{(c + dx) \tan(a + bx)}{b} - \frac{d \int \tan(a + bx) dx}{b} - \int (c + dx) dx \\ &= -cx - \frac{dx^2}{2} + \frac{d \log(\cos(a + bx))}{b^2} + \frac{(c + dx) \tan(a + bx)}{b} \end{aligned}$$

Mathematica [A] time = 0.265031, size = 76, normalized size = 1.9

$$\frac{d \log(\cos(a + bx))}{b^2} - \frac{c \tan^{-1}(\tan(a + bx))}{b} + \frac{c \tan(a + bx)}{b} + \frac{dx \sec(a) \sin(bx) \sec(a + bx)}{b} - \frac{dx \sec(a)(bx \cos(a) - 2 \sin(a))}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Tan[a + b*x]^2, x]

[Out] $-((c*\text{ArcTan}[\text{Tan}[a + b*x]])/b) + (d*\text{Log}[\text{Cos}[a + b*x]])/b^2 - (d*x*\text{Sec}[a]*(b*x*\text{Cos}[a] - 2*\text{Sin}[a]))/(2*b) + (d*x*\text{Sec}[a]*\text{Sec}[a + b*x]*\text{Sin}[b*x])/b + (c*\text{Tan}$

$[a + b*x])/b$

Maple [A] time = 0.044, size = 47, normalized size = 1.2

$$-\frac{dx^2}{2} - cx + \frac{d \tan (bx + a) x}{b} + \frac{d \ln (\cos (bx + a))}{b^2} + \frac{c \tan (bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*tan(b*x+a)^2,x)

[Out] -1/2*d*x^2-c*x+1/b*d*tan(b*x+a)*x+d*ln(cos(b*x+a))/b^2+1/b*c*tan(b*x+a)

Maxima [B] time = 1.4674, size = 320, normalized size = 8.

$$\frac{2(bx+a-\tan(bx+a))c - \frac{2(bx+a-\tan(bx+a))ad}{b} + \frac{((bx+a)^2 \cos(2bx+2a)^2 + (bx+a)^2 \sin(2bx+2a)^2 + 2(bx+a)^2 \cos(2bx+2a) + (bx+a)^2 - (\cos(2bx+2a))^2 - (\sin(2bx+2a))^2)}{2b}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*tan(b*x+a)^2,x, algorithm="maxima")

[Out] -1/2*(2*(b*x + a - tan(b*x + a))*c - 2*(b*x + a - tan(b*x + a))*a*d/b + ((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 + 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b))/b

Fricas [A] time = 0.471681, size = 131, normalized size = 3.28

$$-\frac{b^2 dx^2 + 2 b^2 cx - d \log \left(\frac{1}{\tan (bx+a)^2 + 1} \right) - 2 (bdx + bc) \tan (bx + a)}{2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*tan(b*x+a)^2,x, algorithm="fricas")

[Out] -1/2*(b^2*d*x^2 + 2*b^2*c*x - d*log(1/(tan(b*x + a)^2 + 1)) - 2*(b*d*x + b*c)*tan(b*x + a))/b^2

Sympy [A] time = 0.372713, size = 65, normalized size = 1.62

$$\begin{cases} -cx - \frac{dx^2}{2} + \frac{c \tan (a+bx)}{b} + \frac{dx \tan (a+bx)}{b} - \frac{d \log (\tan ^2 (a+bx)+1)}{2b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2} \right) \tan ^2 (a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*tan(b*x+a)**2,x)

[Out] Piecewise((-c*x - d*x**2/2 + c*tan(a + b*x)/b + d*x*tan(a + b*x)/b - d*log(tan(a + b*x)**2 + 1)/(2*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*tan(a)**2, True))

Giac [B] time = 1.40319, size = 301, normalized size = 7.52

$$b^2 dx^2 \tan(bx) \tan(a) + 2 b^2 cx \tan(bx) \tan(a) - b^2 dx^2 - 2 b^2 cx + 2 b dx \tan(bx) + 2 b dx \tan(a) - d \log\left(\frac{1}{\tan(bx)^4 \tan(a)^2 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*tan(b*x+a)^2,x, algorithm="giac")

[Out] -1/2*(b^2*d*x^2*tan(b*x)*tan(a) + 2*b^2*c*x*tan(b*x)*tan(a) - b^2*d*x^2 - 2*b^2*c*x + 2*b*d*x*tan(b*x) + 2*b*d*x*tan(a) - d*log(4*(tan(a)^2 + 1)/(tan(b*x)^4*tan(a)^2 - 2*tan(b*x)^3*tan(a) + tan(b*x)^2*tan(a)^2 + tan(b*x)^2 - 2*tan(b*x)*tan(a) + 1))*tan(b*x)*tan(a) + 2*b*c*tan(b*x) + 2*b*c*tan(a) + d*log(4*(tan(a)^2 + 1)/(tan(b*x)^4*tan(a)^2 - 2*tan(b*x)^3*tan(a) + tan(b*x)^2*tan(a)^2 + tan(b*x)^2 - 2*tan(b*x)*tan(a) + 1)))/(b^2*tan(b*x)*tan(a) - b^2)

$$3.257 \quad \int \frac{\tan^2(a+bx)}{c+dx} dx$$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\tan^2(a+bx)}{c+dx}, x\right)$$

[Out] Unintegrable[Tan[a + b*x]^2/(c + d*x), x]

Rubi [A] time = 0.0398115, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\tan^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[Tan[a + b*x]^2/(c + d*x), x]

[Out] Defer[Int][Tan[a + b*x]^2/(c + d*x), x]

Rubi steps

$$\int \frac{\tan^2(a+bx)}{c+dx} dx = \int \frac{\tan^2(a+bx)}{c+dx} dx$$

Mathematica [A] time = 3.95696, size = 0, normalized size = 0.

$$\int \frac{\tan^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[Tan[a + b*x]^2/(c + d*x), x]

[Out] Integrate[Tan[a + b*x]^2/(c + d*x), x]

Maple [A] time = 0.264, size = 0, normalized size = 0.

$$\int \frac{(\tan(bx+a))^2}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(b*x+a)^2/(d*x+c), x)

[Out] int(tan(b*x+a)^2/(d*x+c), x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a)^2/(d*x+c),x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tan(bx+a)^2}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a)^2/(d*x+c),x, algorithm="fricas")

[Out] integral(tan(b*x + a)^2/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(a+bx)}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a)**2/(d*x+c),x)

[Out] Integral(tan(a + b*x)**2/(c + d*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(bx+a)^2}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a)^2/(d*x+c),x, algorithm="giac")

[Out] integrate(tan(b*x + a)^2/(d*x + c), x)

$$3.258 \quad \int \frac{\tan^2(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\tan^2(a+bx)}{(c+dx)^2}, x\right)$$

[Out] Unintegrable[Tan[a + b*x]^2/(c + d*x)^2, x]

Rubi [A] time = 0.0385174, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[Tan[a + b*x]^2/(c + d*x)^2, x]

[Out] Defer[Int][Tan[a + b*x]^2/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2} dx = \int \frac{\tan^2(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 5.26563, size = 0, normalized size = 0.

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Tan[a + b*x]^2/(c + d*x)^2, x]

[Out] Integrate[Tan[a + b*x]^2/(c + d*x)^2, x]

Maple [A] time = 0.324, size = 0, normalized size = 0.

$$\int \frac{(\tan(bx+a))^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(b*x+a)^2/(d*x+c)^2, x)

[Out] int(tan(b*x+a)^2/(d*x+c)^2, x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tan^2(bx + a)}{d^2x^2 + 2cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(tan(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a)**2/(d*x+c)**2,x)

[Out] Integral(tan(a + b*x)**2/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(bx + a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(tan(b*x + a)^2/(d*x + c)^2, x)

3.259 $\int (c + dx)^m \sin(a + bx) \tan^2(a + bx) dx$

Optimal. Leaf size=149

$$\text{CannotIntegrate}(\tan(a + bx) \sec(a + bx)(c + dx)^m, x) + \frac{e^{i\left(a - \frac{bc}{d}\right)}(c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m + 1, -\frac{ib(c+dx)}{d}\right)}{2b}$$

[Out] CannotIntegrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x], x] + (E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(2*b*((-I)*b*(c + d*x))/d)^m) + ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(2*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)

Rubi [A] time = 0.157146, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \sin(a + bx) \tan^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x]^2,x]

[Out] (E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(2*b*((-I)*b*(c + d*x))/d)^m) + ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(2*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + Defer[Int] [(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x], x]

Rubi steps

$$\begin{aligned} \int (c + dx)^m \sin(a + bx) \tan^2(a + bx) dx &= - \int (c + dx)^m \sin(a + bx) dx + \int (c + dx)^m \sec(a + bx) \tan(a + bx) dx \\ &= - \left(\frac{1}{2} i \int e^{-i(a+bx)} (c + dx)^m dx \right) + \frac{1}{2} i \int e^{i(a+bx)} (c + dx)^m dx + \int (c + dx)^m \sec(a + bx) \tan(a + bx) dx \\ &= \frac{e^{i\left(a - \frac{bc}{d}\right)}(c + dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, -\frac{ib(c+dx)}{d}\right)}{2b} + \frac{e^{-i\left(a - \frac{bc}{d}\right)}(c + dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, \frac{ib(c+dx)}{d}\right)}{2b} \end{aligned}$$

Mathematica [A] time = 5.09666, size = 0, normalized size = 0.

$$\int (c + dx)^m \sin(a + bx) \tan^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x]^2,x]

[Out] Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x]^2, x]

Maple [A] time = 0.17, size = 0, normalized size = 0.

$$\int (dx + c)^m \sin(bx + a) (\tan(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x)`

[Out] `int((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x)`

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \sin (bx + a) \tan (bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")`

[Out] `integrate((d*x + c)^m*sin(b*x + a)*tan(b*x + a)^2, x)`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx + c)^m \sin (bx + a) \tan (bx + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")`

[Out] `integral((d*x + c)^m*sin(b*x + a)*tan(b*x + a)^2, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)**m*sin(b*x+a)*tan(b*x+a)**2,x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \sin (bx + a) \tan (bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")`

[Out] `integrate((d*x + c)^m*sin(b*x + a)*tan(b*x + a)^2, x)`

3.260 $\int (c + dx)^3 \sin(a + bx) \tan^2(a + bx) dx$

Optimal. Leaf size=228

$$\frac{6id^2(c + dx)\text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} + \frac{6id^2(c + dx)\text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3} + \frac{6d^3\text{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^4} - \frac{6d^3\text{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^4}$$

[Out] $((6*I)*d*(c + d*x)^2*\text{ArcTan}[E^{(I*(a + b*x))}])/b^2 - (6*d^2*(c + d*x)*\text{Cos}[a + b*x])/b^3 + ((c + d*x)^3*\text{Cos}[a + b*x])/b - ((6*I)*d^2*(c + d*x)*\text{PolyLog}[2, (-I)*E^{(I*(a + b*x))}])/b^3 + ((6*I)*d^2*(c + d*x)*\text{PolyLog}[2, I*E^{(I*(a + b*x))}])/b^3 + (6*d^3*\text{PolyLog}[3, (-I)*E^{(I*(a + b*x))}])/b^4 - (6*d^3*\text{PolyLog}[3, I*E^{(I*(a + b*x))}])/b^4 + ((c + d*x)^3*\text{Sec}[a + b*x])/b + (6*d^3*\text{Sin}[a + b*x])/b^4 - (3*d*(c + d*x)^2*\text{Sin}[a + b*x])/b^2$

Rubi [A] time = 0.21102, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4407, 3296, 2637, 4409, 4181, 2531, 2282, 6589}

$$\frac{6id^2(c + dx)\text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} + \frac{6id^2(c + dx)\text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3} + \frac{6d^3\text{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^4} - \frac{6d^3\text{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^3*\text{Sin}[a + b*x]*\text{Tan}[a + b*x]^2, x]$

[Out] $((6*I)*d*(c + d*x)^2*\text{ArcTan}[E^{(I*(a + b*x))}])/b^2 - (6*d^2*(c + d*x)*\text{Cos}[a + b*x])/b^3 + ((c + d*x)^3*\text{Cos}[a + b*x])/b - ((6*I)*d^2*(c + d*x)*\text{PolyLog}[2, (-I)*E^{(I*(a + b*x))}])/b^3 + ((6*I)*d^2*(c + d*x)*\text{PolyLog}[2, I*E^{(I*(a + b*x))}])/b^3 + (6*d^3*\text{PolyLog}[3, (-I)*E^{(I*(a + b*x))}])/b^4 - (6*d^3*\text{PolyLog}[3, I*E^{(I*(a + b*x))}])/b^4 + ((c + d*x)^3*\text{Sec}[a + b*x])/b + (6*d^3*\text{Sin}[a + b*x])/b^4 - (3*d*(c + d*x)^2*\text{Sin}[a + b*x])/b^2$

Rule 4407

$\text{Int}[(c + d*x)^m*\text{Sin}[a + b*x]^n*\text{Tan}[a + b*x]^p, x] := -\text{Int}[(c + d*x)^m*\text{Sin}[a + b*x]^n*\text{Tan}[a + b*x]^{p-2}, x] + \text{Int}[(c + d*x)^m*\text{Sin}[a + b*x]^{n-2}*\text{Tan}[a + b*x]^p, x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

$\text{Int}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] := -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{m-1}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

$\text{Int}[\text{sin}[\text{Pi}/2 + (c + d*x)], x] := \text{Simp}[\text{Sin}[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 4409

$\text{Int}[(c + d*x)^m*\text{Sec}[a + b*x]^n, x] := \text{Simp}[(c + d*x)^m*\text{Sec}[a + b*x]^n/(b*n), x] - \text{Dist}[(d*m)/(b*n), \text{Int}[(c + d*x)^{m-1}*\text{Sec}[a + b*x]^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Dist
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_.)))^(n_.)]*((f_.) + (g_.)
*(x_.))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_.)*((a_.)*(v_)^(n_.))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 \sin(a + bx) \tan^2(a + bx) dx &= - \int (c + dx)^3 \sin(a + bx) dx + \int (c + dx)^3 \sec(a + bx) \tan(a + bx) dx \\
&= \frac{(c + dx)^3 \cos(a + bx)}{b} + \frac{(c + dx)^3 \sec(a + bx)}{b} - \frac{(3d) \int (c + dx)^2 \cos(a + bx) dx}{b} \\
&= \frac{6id(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{(c + dx)^3 \cos(a + bx)}{b} + \frac{(c + dx)^3 \sec(a + bx)}{b} - \\
&= \frac{6id(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{6d^2(c + dx) \cos(a + bx)}{b^3} + \frac{(c + dx)^3 \cos(a + bx)}{b} \\
&= \frac{6id(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{6d^2(c + dx) \cos(a + bx)}{b^3} + \frac{(c + dx)^3 \cos(a + bx)}{b} \\
&= \frac{6id(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{6d^2(c + dx) \cos(a + bx)}{b^3} + \frac{(c + dx)^3 \cos(a + bx)}{b}
\end{aligned}$$

Mathematica [B] time = 1.60966, size = 532, normalized size = 2.33

$$\sec(a + bx) \left(-12ibd^2(c + dx) \cos(a + bx) \text{PolyLog}\left(2, -ie^{i(a+bx)}\right) + 12ibd^2(c + dx) \cos(a + bx) \text{PolyLog}\left(2, ie^{i(a+bx)}\right) + 12d^2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x]^2,x]

[Out] (Sec[a + b*x]*(3*b^3*c^3 - 6*b*c*d^2 + 9*b^3*c^2*d*x - 6*b*d^3*x + 9*b^3*c*d^2*x^2 + 3*b^3*d^3*x^3 + (12*I)*b^2*c^2*d*ArcTan[E^(I*(a + b*x))])*Cos[a +

$$\begin{aligned}
& b*x] + b^3*c^3*\text{Cos}[2*(a + b*x)] - 6*b*c*d^2*\text{Cos}[2*(a + b*x)] + 3*b^3*c^2*d*x*\text{Cos}[2*(a + b*x)] - 6*b*d^3*x*\text{Cos}[2*(a + b*x)] + 3*b^3*c*d^2*x^2*\text{Cos}[2*(a + b*x)] + b^3*d^3*x^3*\text{Cos}[2*(a + b*x)] - 12*b^2*c*d^2*x*\text{Cos}[a + b*x]*\text{Log}[1 - I*E^(I*(a + b*x))] - 6*b^2*d^3*x^2*\text{Cos}[a + b*x]*\text{Log}[1 - I*E^(I*(a + b*x))] + 12*b^2*c*d^2*x*\text{Cos}[a + b*x]*\text{Log}[1 + I*E^(I*(a + b*x))] + 6*b^2*d^3*x^2*\text{Cos}[a + b*x]*\text{Log}[1 + I*E^(I*(a + b*x))] - (12*I)*b*d^2*(c + d*x)*\text{Cos}[a + b*x]*\text{PolyLog}[2, (-I)*E^(I*(a + b*x))] + (12*I)*b*d^2*(c + d*x)*\text{Cos}[a + b*x]*\text{PolyLog}[2, I*E^(I*(a + b*x))] + 12*d^3*\text{Cos}[a + b*x]*\text{PolyLog}[3, (-I)*E^(I*(a + b*x))] - 12*d^3*\text{Cos}[a + b*x]*\text{PolyLog}[3, I*E^(I*(a + b*x))] - 3*b^2*c^2*d*\text{Sin}[2*(a + b*x)] + 6*d^3*\text{Sin}[2*(a + b*x)] - 6*b^2*c*d^2*x*\text{Sin}[2*(a + b*x)] - 3*b^2*d^3*x^2*\text{Sin}[2*(a + b*x)])))/(2*b^4)
\end{aligned}$$

Maple [B] time = 0.296, size = 677, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x)

[Out] $\frac{1}{2}*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*\exp(I*(b*x+a))+1/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*\exp(-I*(b*x+a))+2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*\exp(I*(b*x+a))/b/(\exp(2*I*(b*x+a))+1)-3*d^3/b^4*a^2*\ln(1+I*\exp(I*(b*x+a)))-6*I*c*d^2*\text{polylog}(2,-I*\exp(I*(b*x+a)))/b^3+6*d^2/b^3*c*\ln(1+I*\exp(I*(b*x+a)))*a+6*d^2/b^2*c*\ln(1+I*\exp(I*(b*x+a)))*x-6*d^2/b^2*c*\ln(1-I*\exp(I*(b*x+a)))*x+3*d^3/b^2*\ln(1+I*\exp(I*(b*x+a)))*x^2-6*d^2/b^3*c*\ln(1-I*\exp(I*(b*x+a)))*a-6*I*d^3*x*\text{polylog}(2,-I*\exp(I*(b*x+a)))/b^3-3*d^3/b^2*\ln(1-I*\exp(I*(b*x+a)))*x^2+3*d^3/b^4*a^2*\ln(1-I*\exp(I*(b*x+a)))+6*I*d^3*x*\text{polylog}(2,I*\exp(I*(b*x+a)))/b^3-6*d^3*\text{polylog}(3,I*\exp(I*(b*x+a)))/b^4+6*d^3*\text{polylog}(3,-I*\exp(I*(b*x+a)))/b^4+6*I*d^3/b^4*a^2*\arctan(\exp(I*(b*x+a)))+6*I*c*d^2*\text{polylog}(2,I*\exp(I*(b*x+a)))/b^3+6*I*d/b^2*c^2*\arctan(\exp(I*(b*x+a)))-12*I*d^2/b^3*c*a*\arctan(\exp(I*(b*x+a)))$

Maxima [B] time = 6.93555, size = 14923, normalized size = 65.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")

[Out] $\frac{1}{2}*(2*c^3*(1/\cos(b*x + a) + \cos(b*x + a)) - 6*a*c^2*d*(1/\cos(b*x + a) + \cos(b*x + a))/b + 6*a^2*c*d^2*(1/\cos(b*x + a) + \cos(b*x + a))/b^2 - 2*a^3*d^3*(1/\cos(b*x + a) + \cos(b*x + a))/b^3 + 3*((b*x + (b*x + a)*\cos(2*b*x + 2*a) + a + \sin(2*b*x + 2*a))*\cos(3*b*x + 3*a)^3 + 6*(b*x + a)*\cos(b*x + a)^3 + ((b*x + a)*\sin(2*b*x + 2*a) - \cos(2*b*x + 2*a) - 1)*\sin(3*b*x + 3*a)^3 + 6*(b*x + a)*\cos(b*x + a)*\sin(b*x + a)^2 + 2*(4*(b*x + a)*\cos(2*b*x + 2*a)*\cos(b*x + a) + 4*(b*x + a)*\cos(b*x + a) + (3*(b*x + a)*\sin(b*x + a) + \cos(b*x + a))*\sin(2*b*x + 2*a))*\cos(3*b*x + 3*a)^2 + ((b*x + a)*\cos(b*x + a) - \sin(b*x + a))*\cos(2*b*x + 2*a)^2 + (8*(b*x + a)*\sin(2*b*x + 2*a)*\sin(b*x + a) + (b*x + (b*x + a)*\cos(2*b*x + 2*a) + a + \sin(2*b*x + 2*a))*\cos(3*b*x + 3*a) + 2*(3*(b*x + a)*\cos(b*x + a) - \sin(b*x + a))*\cos(2*b*x + 2*a) + 6*(b*x +$

$$\begin{aligned}
& a) \cos(bx + a) - 2 \sin(bx + a) \sin(3bx + 3a)^2 + ((bx + a) \cos(bx + a) - \sin(bx + a)) \sin(2bx + 2a)^2 + ((bx + a) \cos(2bx + 2a)^2 + 13 \\
& * (bx + a) \cos(bx + a)^2 + (bx + a) \sin(2bx + 2a)^2 + (bx + a) \sin(bx + a)^2 + bx + (13 * (bx + a) \cos(bx + a)^2 + (bx + a) \sin(bx + a)^2 + \\
& 2 * bx + 2a) \cos(2bx + 2a) + (12 * (bx + a) \cos(bx + a) \sin(bx + a) + \cos(bx + a)^2 + \sin(bx + a)^2) \sin(2bx + 2a) + a) \cos(3bx + 3a) + 2 * \\
& (3 * (bx + a) \cos(bx + a)^3 + 3 * (bx + a) \cos(bx + a) \sin(bx + a)^2 + (bx + a) \cos(bx + a) - \sin(bx + a)) \cos(2bx + 2a) + (bx + a) \cos(bx + \\
& a) - ((\cos(2bx + 2a)^2 + \sin(2bx + 2a)^2 + 2 * \cos(2bx + 2a) + 1) \cos(3bx + 3a)^2 + (\cos(bx + a)^2 + \sin(bx + a)^2) \cos(2bx + 2a)^2 + (\\
& \cos(2bx + 2a)^2 + \sin(2bx + 2a)^2 + 2 * \cos(2bx + 2a) + 1) \sin(3bx + 3a)^2 + (\cos(bx + a)^2 + \sin(bx + a)^2) \sin(2bx + 2a)^2 + 2 * (\cos(2 \\
& * bx + 2a)^2 \cos(bx + a) + \cos(bx + a) \sin(2bx + 2a)^2 + 2 * \cos(2bx + 2a) \cos(bx + a) + \cos(bx + a)) \cos(3bx + 3a) + 2 * (\cos(bx + a)^2 + \\
& \sin(bx + a)^2) \cos(2bx + 2a) + \cos(bx + a)^2 + 2 * (\cos(2bx + 2a)^2 \sin(bx + a) + \sin(2bx + 2a)^2 \sin(bx + a) + 2 * \cos(2bx + 2a) \sin(bx \\
& + a) + \sin(bx + a)) \sin(3bx + 3a) + \sin(bx + a)^2 \log(\cos(bx + a)^2 \\
& + \sin(bx + a)^2 + 2 * \sin(bx + a) + 1) + ((\cos(2bx + 2a)^2 + \sin(2bx + 2a)^2 + 2 * \cos(2bx + 2a) + 1) \cos(3bx + 3a)^2 + (\cos(bx + a)^2 + \sin \\
& (bx + a)^2) \cos(2bx + 2a)^2 + (\cos(2bx + 2a)^2 + \sin(2bx + 2a)^2 + 2 * \cos(2bx + 2a) + 1) \sin(3bx + 3a)^2 + (\cos(bx + a)^2 + \sin(bx + \\
& a)^2) \sin(2bx + 2a)^2 + 2 * (\cos(2bx + 2a)^2 \cos(bx + a) + \cos(bx + a) \sin(2bx + 2a)^2 + 2 * \cos(2bx + 2a) \cos(bx + a) + \cos(bx + a)) \cos \\
& (3bx + 3a) + 2 * (\cos(bx + a)^2 + \sin(bx + a)^2) \cos(2bx + 2a) + \cos(bx + a)^2 + 2 * (\cos(2bx + 2a)^2 \sin(bx + a) + \sin(2bx + 2a)^2 \sin(bx \\
& + a) + 2 * \cos(2bx + 2a) \sin(bx + a) + \sin(bx + a)) \sin(3bx + 3a) + \sin(bx + a)^2 \log(\cos(bx + a)^2 + \sin(bx + a)^2 - 2 * \sin(bx + a) + 1) \\
& + (((bx + a) \sin(2bx + 2a) - \cos(2bx + 2a) - 1) \cos(3bx + 3a)^2 + 12 * (bx + a) \cos(bx + a) \sin(bx + a) + 2 * (((bx + a) \sin(bx + a) - \cos(bx \\
& + a)) \cos(2bx + 2a) + ((bx + a) \cos(bx + a) + \sin(bx + a)) \sin(2bx + 2a) + (bx + a) \sin(bx + a) - \cos(bx + a)) \cos(3bx + 3a) + (12 * \\
& (bx + a) \cos(bx + a) \sin(bx + a) - \cos(bx + a)^2 - \sin(bx + a)^2 - 2) * \cos(2bx + 2a) - \cos(2bx + 2a)^2 - \cos(bx + a)^2 + ((bx + a) \cos(bx \\
& + a)^2 + 13 * (bx + a) \sin(bx + a)^2) \sin(2bx + 2a) - \sin(2bx + 2a)^2 - \sin(bx + a)^2 - 1) \sin(3bx + 3a) + 6 * ((bx + a) \cos(bx + a)^2 \sin(bx \\
& + a) + (bx + a) \sin(bx + a)^3) \sin(2bx + 2a) - \sin(bx + a) * c^2 * d \\
& / (((\cos(2bx + 2a)^2 + \sin(2bx + 2a)^2 + 2 * \cos(2bx + 2a) + 1) \cos(3bx + 3a)^2 + (\cos(bx + a)^2 + \sin(bx + a)^2) \cos(2bx + 2a)^2 + (\cos \\
& (2bx + 2a)^2 + \sin(2bx + 2a)^2 + 2 * \cos(2bx + 2a) + 1) \sin(3bx + 3a)^2 + (\cos(bx + a)^2 + \sin(bx + a)^2) \sin(2bx + 2a)^2 + 2 * (\cos(2bx \\
& + 2a)^2 \cos(bx + a) + \cos(bx + a) \sin(2bx + 2a)^2 + 2 * \cos(2bx + 2a) \cos(bx + a) + \cos(bx + a)) \cos(3bx + 3a) + 2 * (\cos(bx + a)^2 + \sin \\
& (bx + a)^2) \cos(2bx + 2a) + \cos(bx + a)^2 + 2 * (\cos(2bx + 2a)^2 \sin(bx + a) + \sin(2bx + 2a)^2 \sin(bx + a) + 2 * \cos(2bx + 2a) \sin(bx + a) \\
& + \sin(bx + a)) \sin(3bx + 3a) + \sin(bx + a)^2 * b) - 6 * ((bx + (bx + a) \cos(2bx + 2a) + a + \sin(2bx + 2a)) \cos(3bx + 3a)^3 + 6 * (bx + a) \\
&) \cos(bx + a)^3 + ((bx + a) \sin(2bx + 2a) - \cos(2bx + 2a) - 1) \sin(3bx + 3a)^3 + 6 * (bx + a) \cos(bx + a) \sin(bx + a)^2 + 2 * (4 * (bx + a) \cos \\
& (2bx + 2a) \cos(bx + a) + 4 * (bx + a) \cos(bx + a) + (3 * (bx + a) \sin(bx + a) + \cos(bx + a)) \sin(2bx + 2a)) \cos(3bx + 3a)^2 + ((bx + a) \cos \\
& (bx + a) - \sin(bx + a)) \cos(2bx + 2a)^2 + (8 * (bx + a) \sin(2bx + 2a) \sin(bx + a) + (bx + (bx + a) \cos(2bx + 2a) + a + \sin(2bx + 2a) \\
&)) \cos(3bx + 3a) + 2 * (3 * (bx + a) \cos(bx + a) - \sin(bx + a)) \cos(2bx + 2a) + 6 * (bx + a) \cos(bx + a) - 2 * \sin(bx + a)) \sin(3bx + 3a)^2 + (\\
& (bx + a) \cos(bx + a) - \sin(bx + a)) \sin(2bx + 2a)^2 + ((bx + a) \cos(2bx + 2a)^2 + 13 * (bx + a) \cos(bx + a)^2 + (bx + a) \sin(2bx + 2a)^2 \\
& + (bx + a) \sin(bx + a)^2 + bx + (13 * (bx + a) \cos(bx + a)^2 + (bx + a) \sin(bx + a)^2 + 2 * bx + 2a) \cos(2bx + 2a) + (12 * (bx + a) \cos(bx + a) \sin(bx + a) + \cos(bx + a)^2 + \sin(bx + a)^2) \sin(2bx + 2a) + a) \cos
\end{aligned}$$

$$\begin{aligned}
& s(3bx + 3a) + 2*(3*(bx + a)*\cos(bx + a)^3 + 3*(bx + a)*\cos(bx + a)*\sin(bx + a)^2 + (bx + a)*\cos(bx + a) - \sin(bx + a))*\cos(2bx + 2a) + (\\
& bx + a)*\cos(bx + a) - ((\cos(2bx + 2a)^2 + \sin(2bx + 2a)^2 + 2*\cos(2 \\
& *bx + 2a) + 1)*\cos(3bx + 3a)^2 + (\cos(bx + a)^2 + \sin(bx + a)^2)*\cos \\
& (2bx + 2a)^2 + (\cos(2bx + 2a)^2 + \sin(2bx + 2a)^2 + 2*\cos(2bx + \\
& 2a) + 1)*\sin(3bx + 3a)^2 + (\cos(bx + a)^2 + \sin(bx + a)^2)*\sin(2bx \\
& + 2a)^2 + 2*(\cos(2bx + 2a)^2*\cos(bx + a) + \cos(bx + a)*\sin(2bx + 2 \\
& a)^2 + 2*\cos(2bx + 2a)*\cos(bx + a) + \cos(bx + a))*\cos(3bx + 3a) + 2 \\
& *(\cos(bx + a)^2 + \sin(bx + a)^2)*\cos(2bx + 2a) + \cos(bx + a)^2 + 2*(\cos \\
& (2bx + 2a)^2*\sin(bx + a) + \sin(2bx + 2a)^2*\sin(bx + a) + 2*\cos(2* \\
& bx + 2a)*\sin(bx + a) + \sin(bx + a))*\sin(3bx + 3a) + \sin(bx + a)^2)* \\
& \log(\cos(bx + a)^2 + \sin(bx + a)^2 + 2*\sin(bx + a) + 1) + ((\cos(2bx + 2 \\
& *a)^2 + \sin(2bx + 2a)^2 + 2*\cos(2bx + 2a) + 1)*\cos(3bx + 3a)^2 + (\\
& \cos(bx + a)^2 + \sin(bx + a)^2)*\cos(2bx + 2a)^2 + (\cos(2bx + 2a)^2 + \\
& \sin(2bx + 2a)^2 + 2*\cos(2bx + 2a) + 1)*\sin(3bx + 3a)^2 + (\cos(bx \\
& + a)^2 + \sin(bx + a)^2)*\sin(2bx + 2a)^2 + 2*(\cos(2bx + 2a)^2*\cos(b \\
& x + a) + \cos(bx + a)*\sin(2bx + 2a)^2 + 2*\cos(2bx + 2a)*\cos(bx + a) \\
& + \cos(bx + a))*\cos(3bx + 3a) + 2*(\cos(bx + a)^2 + \sin(bx + a)^2)*\cos(\\
& 2bx + 2a) + \cos(bx + a)^2 + 2*(\cos(2bx + 2a)^2*\sin(bx + a) + \sin(2* \\
& bx + 2a)^2*\sin(bx + a) + 2*\cos(2bx + 2a)*\sin(bx + a) + \sin(bx + a)) \\
& *\sin(3bx + 3a) + \sin(bx + a)^2)*\log(\cos(bx + a)^2 + \sin(bx + a)^2 - 2 \\
& *\sin(bx + a) + 1) + (((bx + a)*\sin(2bx + 2a) - \cos(2bx + 2a) - 1)*\cos \\
& (3bx + 3a)^2 + 12*(bx + a)*\cos(bx + a)*\sin(bx + a) + 2*((bx + a)* \\
& \sin(bx + a) - \cos(bx + a))*\cos(2bx + 2a) + ((bx + a)*\cos(bx + a) + \sin \\
& (bx + a))*\sin(2bx + 2a) + (bx + a)*\sin(bx + a) - \cos(bx + a))*\cos(\\
& 3bx + 3a) + (12*(bx + a)*\cos(bx + a)*\sin(bx + a) - \cos(bx + a)^2 - \sin \\
& (bx + a)^2 - 2)*\cos(2bx + 2a) - \cos(2bx + 2a)^2 - \cos(bx + a)^2 + \\
& ((bx + a)*\cos(bx + a)^2 + 13*(bx + a)*\sin(bx + a)^2)*\sin(2bx + 2a) \\
& - \sin(2bx + 2a)^2 - \sin(bx + a)^2 - 1)*\sin(3bx + 3a) + 6*((bx + a)* \\
& \cos(bx + a)^2*\sin(bx + a) + (bx + a)*\sin(bx + a)^3)*\sin(2bx + 2a) - \\
& \sin(bx + a)*a*c*d^2/(((\cos(2bx + 2a)^2 + \sin(2bx + 2a)^2 + 2*\cos(2* \\
& bx + 2a) + 1)*\cos(3bx + 3a)^2 + (\cos(bx + a)^2 + \sin(bx + a)^2)*\cos(\\
& 2bx + 2a)^2 + (\cos(2bx + 2a)^2 + \sin(2bx + 2a)^2 + 2*\cos(2bx + 2 \\
& *a) + 1)*\sin(3bx + 3a)^2 + (\cos(bx + a)^2 + \sin(bx + a)^2)*\sin(2bx + \\
& 2a)^2 + 2*(\cos(2bx + 2a)^2*\cos(bx + a) + \cos(bx + a)*\sin(2bx + 2a) \\
&)^2 + 2*\cos(2bx + 2a)*\cos(bx + a) + \cos(bx + a))*\cos(3bx + 3a) + 2* \\
& (\cos(bx + a)^2 + \sin(bx + a)^2)*\cos(2bx + 2a) + \cos(bx + a)^2 + 2*(\cos \\
& (2bx + 2a)^2*\sin(bx + a) + \sin(2bx + 2a)^2*\sin(bx + a) + 2*\cos(2bx \\
& + 2a)*\sin(bx + a) + \sin(bx + a))*\sin(3bx + 3a) + \sin(bx + a)^2)*b \\
& ^2) + 3*((bx + (bx + a)*\cos(2bx + 2a) + a + \sin(2bx + 2a))*\cos(3bx \\
& + 3a)^3 + 6*(bx + a)*\cos(bx + a)^3 + ((bx + a)*\sin(2bx + 2a) - \cos \\
& (2bx + 2a) - 1)*\sin(3bx + 3a)^3 + 6*(bx + a)*\cos(bx + a)*\sin(bx + \\
& a)^2 + 2*(4*(bx + a)*\cos(2bx + 2a)*\cos(bx + a) + 4*(bx + a)*\cos(bx + \\
& a) + (3*(bx + a)*\sin(bx + a) + \cos(bx + a))*\sin(2bx + 2a))*\cos(3bx \\
& + 3a)^2 + ((bx + a)*\cos(bx + a) - \sin(bx + a))*\cos(2bx + 2a)^2 + (8 \\
& *(bx + a)*\sin(2bx + 2a)*\sin(bx + a) + (bx + (bx + a)*\cos(2bx + 2a) \\
&) + a + \sin(2bx + 2a))*\cos(3bx + 3a) + 2*(3*(bx + a)*\cos(bx + a) - \\
& \sin(bx + a))*\cos(2bx + 2a) + 6*(bx + a)*\cos(bx + a) - 2*\sin(bx + a)) \\
& *\sin(3bx + 3a)^2 + ((bx + a)*\cos(bx + a) - \sin(bx + a))*\sin(2bx + 2 \\
& *a)^2 + ((bx + a)*\cos(2bx + 2a)^2 + 13*(bx + a)*\cos(bx + a)^2 + (bx \\
& + a)*\sin(2bx + 2a)^2 + (bx + a)*\sin(bx + a)^2 + bx + (13*(bx + a)*\cos \\
& (bx + a)^2 + (bx + a)*\sin(bx + a)^2 + 2*bx + 2a)*\cos(2bx + 2a) + (\\
& 12*(bx + a)*\cos(bx + a)*\sin(bx + a) + \cos(bx + a)^2 + \sin(bx + a)^2)*\sin \\
& (2bx + 2a) + a)*\cos(3bx + 3a) + 2*(3*(bx + a)*\cos(bx + a)^3 + 3*(\\
& bx + a)*\cos(bx + a)*\sin(bx + a)^2 + (bx + a)*\cos(bx + a) - \sin(bx + a) \\
&))*\cos(2bx + 2a) + (bx + a)*\cos(bx + a) - ((\cos(2bx + 2a)^2 + \sin(2 \\
& *bx + 2a)^2 + 2*\cos(2bx + 2a) + 1)*\cos(3bx + 3a)^2 + (\cos(bx + a)^2 \\
& + \sin(bx + a)^2)*\cos(2bx + 2a)^2 + (\cos(2bx + 2a)^2 + \sin(2bx + \\
& 2a)^2 + 2*\cos(2bx + 2a) + 1)*\sin(3bx + 3a)^2 + (\cos(bx + a)^2 + \sin
\end{aligned}$$

$$\begin{aligned}
& (b*x + a)^2*\sin(2*b*x + 2*a)^2 + 2*(\cos(2*b*x + 2*a)^2*\cos(b*x + a) + \cos(\\
& b*x + a)*\sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a)*\cos(b*x + a) + \cos(b*x + a \\
&))*\cos(3*b*x + 3*a) + 2*(\cos(b*x + a)^2 + \sin(b*x + a)^2)*\cos(2*b*x + 2*a) \\
& + \cos(b*x + a)^2 + 2*(\cos(2*b*x + 2*a)^2*\sin(b*x + a) + \sin(2*b*x + 2*a)^2* \\
& \sin(b*x + a) + 2*\cos(2*b*x + 2*a)*\sin(b*x + a) + \sin(b*x + a))*\sin(3*b*x + \\
& 3*a) + \sin(b*x + a)^2*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\sin(b*x + a) \\
& + 1) + ((\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) \\
& *\cos(3*b*x + 3*a)^2 + (\cos(b*x + a)^2 + \sin(b*x + a)^2)*\cos(2*b*x + 2*a)^2 \\
& + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)*\sin(3* \\
& b*x + 3*a)^2 + (\cos(b*x + a)^2 + \sin(b*x + a)^2)*\sin(2*b*x + 2*a)^2 + 2*(\cos \\
& (2*b*x + 2*a)^2*\cos(b*x + a) + \cos(b*x + a)*\sin(2*b*x + 2*a)^2 + 2*\cos(2*b \\
& *x + 2*a)*\cos(b*x + a) + \cos(b*x + a))*\cos(3*b*x + 3*a) + 2*(\cos(b*x + a)^2 \\
& + \sin(b*x + a)^2)*\cos(2*b*x + 2*a) + \cos(b*x + a)^2 + 2*(\cos(2*b*x + 2*a)^ \\
& 2*\sin(b*x + a) + \sin(2*b*x + 2*a)^2*\sin(b*x + a) + 2*\cos(2*b*x + 2*a)*\sin(b \\
& *x + a) + \sin(b*x + a))*\sin(3*b*x + 3*a) + \sin(b*x + a)^2*\log(\cos(b*x + a) \\
& ^2 + \sin(b*x + a)^2 - 2*\sin(b*x + a) + 1) + (((b*x + a)*\sin(2*b*x + 2*a) - \\
& \cos(2*b*x + 2*a) - 1)*\cos(3*b*x + 3*a)^2 + 12*(b*x + a)*\cos(b*x + a)*\sin(b* \\
& x + a) + 2*((b*x + a)*\sin(b*x + a) - \cos(b*x + a))*\cos(2*b*x + 2*a) + ((b* \\
& x + a)*\cos(b*x + a) + \sin(b*x + a))*\sin(2*b*x + 2*a) + (b*x + a)*\sin(b*x + \\
& a) - \cos(b*x + a))*\cos(3*b*x + 3*a) + (12*(b*x + a)*\cos(b*x + a)*\sin(b*x + \\
& a) - \cos(b*x + a)^2 - \sin(b*x + a)^2 - 2)*\cos(2*b*x + 2*a) - \cos(2*b*x + 2* \\
& a)^2 - \cos(b*x + a)^2 + ((b*x + a)*\cos(b*x + a)^2 + 13*(b*x + a)*\sin(b*x + \\
& a)^2)*\sin(2*b*x + 2*a) - \sin(2*b*x + 2*a)^2 - \sin(b*x + a)^2 - 1)*\sin(3*b*x \\
& + 3*a) + 6*((b*x + a)*\cos(b*x + a)^2*\sin(b*x + a) + (b*x + a)*\sin(b*x + a) \\
& ^3)*\sin(2*b*x + 2*a) - \sin(b*x + a)*a^2*d^3/(((\cos(2*b*x + 2*a)^2 + \sin(2* \\
& b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1)*\cos(3*b*x + 3*a)^2 + (\cos(b*x + a)^2 \\
& + \sin(b*x + a)^2)*\cos(2*b*x + 2*a)^2 + (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2 \\
& *a)^2 + 2*\cos(2*b*x + 2*a) + 1)*\sin(3*b*x + 3*a)^2 + (\cos(b*x + a)^2 + \sin(\\
& b*x + a)^2)*\sin(2*b*x + 2*a)^2 + 2*(\cos(2*b*x + 2*a)^2*\cos(b*x + a) + \cos(b \\
& *x + a)*\sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a)*\cos(b*x + a) + \cos(b*x + a) \\
&)*\cos(3*b*x + 3*a) + 2*(\cos(b*x + a)^2 + \sin(b*x + a)^2)*\cos(2*b*x + 2*a) + \\
& \cos(b*x + a)^2 + 2*(\cos(2*b*x + 2*a)^2*\sin(b*x + a) + \sin(2*b*x + 2*a)^2*s \\
& in(b*x + a) + 2*\cos(2*b*x + 2*a)*\sin(b*x + a) + \sin(b*x + a))*\sin(3*b*x + 3 \\
& *a) + \sin(b*x + a)^2*b^3) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + (6*a + 6*I)*d \\
& ^3 + (3*b*c*d^2 - (3*a + 3*I)*d^3)*(b*x + a)^2 + ((b*x + a)^3*d^3 - 6*b*c*d \\
& ^2 + (6*a - 6*I)*d^3 + (3*b*c*d^2 - (3*a - 3*I)*d^3)*(b*x + a)^2 + (6*I*b*c \\
& *d^2 - 6*(I*a + 1)*d^3)*(b*x + a))*\cos(3*b*x + 3*a)^2 + 6*((b*x + a)^3*d^3 \\
& - 2*b*c*d^2 - 2*(b*x + a)*d^3 + 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2)* \\
& \cos(b*x + a)^2 - ((b*x + a)^3*d^3 - 6*b*c*d^2 + (6*a - 6*I)*d^3 + (3*b*c*d^ \\
& 2 - (3*a - 3*I)*d^3)*(b*x + a)^2 - (-6*I*b*c*d^2 - 6*(-I*a - 1)*d^3)*(b*x + \\
& a))*\sin(3*b*x + 3*a)^2 + (12*I*(b*x + a)^3*d^3 - 24*I*b*c*d^2 - 24*I*(b*x \\
& + a)*d^3 + 24*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a)^2)*\cos(b*x + \\
& a)*\sin(b*x + a) - 6*((b*x + a)^3*d^3 - 2*b*c*d^2 - 2*(b*x + a)*d^3 + 2*a*d^ \\
& 3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2)*\sin(b*x + a)^2 + (-6*I*b*c*d^2 - 6*(-I \\
& *a + 1)*d^3)*(b*x + a) + ((6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3 \\
&)*(b*x + a) + (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a) \\
&)*\cos(2*b*x + 2*a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\sin \\
& (2*b*x + 2*a))*\cos(3*b*x + 3*a) + (((6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 1 \\
& 2*I*a*d^3)*(b*x + a))*\cos(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^ \\
& 3)*(b*x + a))*\sin(b*x + a))*\cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^3 + (12*I \\
& *b*c*d^2 - 12*I*a*d^3)*(b*x + a))*\cos(b*x + a) - (6*(b*x + a)^2*d^3 + 12*(b \\
& *c*d^2 - a*d^3)*(b*x + a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + \\
& a))*\cos(2*b*x + 2*a) - (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3 \\
&)*(b*x + a))*\sin(2*b*x + 2*a))*\sin(3*b*x + 3*a) - (6*((b*x + a)^2*d^3 + 2*(\\
& b*c*d^2 - a*d^3)*(b*x + a))*\cos(b*x + a) - (-6*I*(b*x + a)^2*d^3 + (-12*I*b \\
& *c*d^2 + 12*I*a*d^3)*(b*x + a))*\sin(b*x + a))*\sin(2*b*x + 2*a) - 6*((b*x + \\
& a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\sin(b*x + a))*\arctan2(\cos(b*x + a \\
&), \sin(b*x + a) + 1) + ((6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)* \\
& (b*x + a) + (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*c
\end{aligned}$$

$$\begin{aligned}
& \cos(2bx + 2a) - 6*((bx + a)^2d^3 + 2*(b^2cd^2 - a^2d^3)*(bx + a))*\sin(2bx + 2a) \\
& * \cos(3bx + 3a) + ((6I*(bx + a)^2d^3 + (12I*b^2cd^2 - 12I*a^2d^3)*(bx + a))*\cos(bx + a) \\
& - 6*((bx + a)^2d^3 + 2*(b^2cd^2 - a^2d^3)*(bx + a))*\sin(bx + a))*\cos(2bx + 2a) + (6I*(bx + a)^2d^3 + (12I*b^2cd^2 - 12I*a^2d^3)*(bx + a))*\cos(bx + a) \\
& - (6*(bx + a)^2d^3 + 12*(b^2cd^2 - a^2d^3)*(bx + a) + 6*((bx + a)^2d^3 + 2*(b^2cd^2 - a^2d^3)*(bx + a)))*\cos(2bx + 2a) \\
& - (-6I*(bx + a)^2d^3 + (-12I*b^2cd^2 + 12I*a^2d^3)*(bx + a))*\sin(2bx + 2a))*\sin(3bx + 3a) - (6*((bx + a)^2d^3 + 2*(b^2cd^2 - a^2d^3)*(bx + a))*\cos(bx + a) \\
& - (-6I*(bx + a)^2d^3 + (-12I*b^2cd^2 + 12I*a^2d^3)*(bx + a))*\sin(bx + a))*\sin(2bx + 2a) - 6*((bx + a)^2d^3 + 2*(b^2cd^2 - a^2d^3)*(bx + a))*\sin(bx + a) \\
& * \arctan2(\cos(bx + a), -\sin(bx + a) + 1) + ((7*(bx + a)^3d^3 - 18*b^2cd^2 + (18*a - 6I)*d^3 + (21*b^2cd^2 - (21*a - 3I)*d^3)*(bx + a)^2 + (6I*b^2cd^2 - 6I*(a + 3)*d^3)*(bx + a))*\cos(bx + a) + (7I*(bx + a)^3d^3 - 18I*b^2cd^2 - 6I*(-3I*a - 1)*d^3 + (21I*b^2cd^2 - 3*(7I*a + 1)*d^3)*(bx + a)^2 - (6*b^2cd^2 - (6*a - 18I)*d^3)*(bx + a))*\sin(bx + a))*\cos(3bx + 3a) + ((bx + a)^3d^3 - 6*b^2cd^2 + (6*a + 6I)*d^3 + (3*b^2cd^2 - (3*a + 3I)*d^3)*(bx + a)^2 + (-6I*b^2cd^2 - 6*(-I*a + 1)*d^3)*(bx + a))*\cos(2bx + 2a) + ((12I*b^2cd^2 + 12I*(bx + a)*d^3 - 12I*a^2d^3 + (12I*b^2cd^2 + 12I*(bx + a)*d^3 - 12I*a^2d^3))*\cos(2bx + 2a) - 12*(b^2cd^2 + (bx + a)*d^3 - a^2d^3)*\sin(2bx + 2a))*\cos(3bx + 3a) + ((12I*b^2cd^2 + 12I*(bx + a)*d^3 - 12I*a^2d^3))*\cos(bx + a) - 12*(b^2cd^2 + (bx + a)*d^3 - a^2d^3)*\sin(bx + a))*\cos(2bx + 2a) + (12I*b^2cd^2 + 12I*(bx + a)*d^3 - 12I*a^2d^3))*\cos(bx + a) - (12*b^2cd^2 + 12*(bx + a)*d^3 - 12*a^2d^3 + 12*(b^2cd^2 + (bx + a)*d^3 - a^2d^3))*\cos(2bx + 2a) - (-12I*b^2cd^2 - 12I*(bx + a)*d^3 + 12I*a^2d^3)*\sin(2bx + 2a))*\sin(3bx + 3a) - (12*(b^2cd^2 + (bx + a)*d^3 - a^2d^3))*\cos(bx + a) - (-12I*b^2cd^2 - 12I*(bx + a)*d^3 + 12I*a^2d^3))*\sin(bx + a))*\sin(2bx + 2a) - 12*(b^2cd^2 + (bx + a)*d^3 - a^2d^3)*\sin(bx + a))*\operatorname{dilog}(Ie^{I*bx + I*a}) + ((-12I*b^2cd^2 - 12I*(bx + a)*d^3 + 12I*a^2d^3 + (-12I*b^2cd^2 - 12I*(bx + a)*d^3 + 12I*a^2d^3))*\cos(2bx + 2a) + 12*(b^2cd^2 + (bx + a)*d^3 - a^2d^3)*\sin(2bx + 2a))*\cos(3bx + 3a) + ((-12I*b^2cd^2 - 12I*(bx + a)*d^3 + 12I*a^2d^3))*\cos(bx + a) + 12*(b^2cd^2 + (bx + a)*d^3 - a^2d^3)*\sin(bx + a))*\cos(2bx + 2a) + (-12I*b^2cd^2 - 12I*(bx + a)*d^3 + 12I*a^2d^3))*\cos(bx + a) + (12*b^2cd^2 + 12*(bx + a)*d^3 - 12*a^2d^3 + 12*(b^2cd^2 + (bx + a)*d^3 - a^2d^3))*\cos(2bx + 2a) + (12I*b^2cd^2 + 12I*(bx + a)*d^3 - 12I*a^2d^3))*\sin(2bx + 2a))*\sin(3bx + 3a) + (12*(b^2cd^2 + (bx + a)*d^3 - a^2d^3))*\cos(bx + a) + (12I*b^2cd^2 + 12I*(bx + a)*d^3 - 12I*a^2d^3))*\sin(bx + a))*\sin(2bx + 2a) + 12*(b^2cd^2 + (bx + a)*d^3 - a^2d^3)*\sin(bx + a))*\operatorname{dilog}(-Ie^{I*bx + I*a}) - ((3*(bx + a)^2d^3 + 6*(b^2cd^2 - a^2d^3)*(bx + a) + 3*((bx + a)^2d^3 + 2*(b^2cd^2 - a^2d^3)*(bx + a))*\cos(2bx + 2a) - (-3I*(bx + a)^2d^3 + (-6I*b^2cd^2 + 6I*a^2d^3)*(bx + a))*\sin(2bx + 2a))*\cos(3bx + 3a) + 3*((bx + a)^2d^3 + 2*(b^2cd^2 - a^2d^3)*(bx + a))*\cos(bx + a) - (-3I*(bx + a)^2d^3 + (-6I*b^2cd^2 + 6I*a^2d^3)*(bx + a))*\sin(bx + a))*\cos(2bx + 2a) + 3*((bx + a)^2d^3 + 2*(b^2cd^2 - a^2d^3)*(bx + a))*\cos(bx + a) - (-3I*(bx + a)^2d^3 + (-6I*b^2cd^2 + 6I*a^2d^3)*(bx + a))*\sin(2bx + 2a) + 3*((bx + a)^2d^3 + 2*(b^2cd^2 - a^2d^3)*(bx + a))*\sin(2bx + 2a))*\sin(3bx + 3a) - ((-3I*(bx + a)^2d^3 + (-6I*b^2cd^2 + 6I*a^2d^3))*(bx + a))*\cos(bx + a) + 3*((bx + a)^2d^3 + 2*(b^2cd^2 - a^2d^3)*(bx + a))*\sin(bx + a))*\sin(2bx + 2a) - (-3I*(bx + a)^2d^3 + (-6I*b^2cd^2 + 6I*a^2d^3)*(bx + a))*\sin(bx + a))*\log(\cos(bx + a)^2 + \sin(bx + a)^2 + 2*\sin(bx + a) + 1) + ((3*(bx + a)^2d^3 + 6*(b^2cd^2 - a^2d^3)*(bx + a) + 3*((bx + a)^2d^3 + 2*(b^2cd^2 - a^2d^3)*(bx + a))*\cos(2bx + 2a) + (3I*(bx + a)^2d^3 + (6I*b^2cd^2 - 6I*a^2d^3)*(bx + a))*\sin(2bx + 2a))*\cos(3bx + 3a) + (3*((bx + a)^2d^3 + 2*(b^2cd^2 - a^2d^3)*(bx + a))*\cos(bx + a) + (3I*(bx + a)^2d^3 + (6I*b^2cd^2 - 6I*a^2d^3)*(bx + a))*\sin(bx + a))*\cos(2bx + 2a) + 3*((bx + a)^2d^3 + 2*(b^2cd^2 - a^2d^3)*(bx + a))*\cos(bx + a) + (3I*(bx + a)^2d^3 + (6I*b^2cd^2 - 6I*a^2d^3)*(bx + a))*\sin(bx + a))
\end{aligned}$$

$$\begin{aligned}
& a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*\cos(2*b*x \\
& + 2*a) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\sin(2*b*x + 2 \\
& *a))*\sin(3*b*x + 3*a) + ((3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(\\
& b*x + a))*\cos(b*x + a) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) \\
&)*\sin(b*x + a))*\sin(2*b*x + 2*a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6* \\
& I*a*d^3)*(b*x + a))*\sin(b*x + a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*s \\
& \sin(b*x + a) + 1) - (12*d^3*\cos(b*x + a) + 12*I*d^3*\sin(b*x + a) + 12*(d^3*c \\
& \cos(2*b*x + 2*a) + I*d^3*\sin(2*b*x + 2*a) + d^3))*\cos(3*b*x + 3*a) + 12*(d^3*c \\
& \cos(b*x + a) + I*d^3*\sin(b*x + a))*\cos(2*b*x + 2*a) - (-12*I*d^3*\cos(2*b*x \\
& + 2*a) + 12*d^3*\sin(2*b*x + 2*a) - 12*I*d^3)*\sin(3*b*x + 3*a) - (-12*I*d^3*c \\
& \cos(b*x + a) + 12*d^3*\sin(b*x + a))*\sin(2*b*x + 2*a))*\text{polylog}(3, I*e^(I*b*x \\
& + I*a)) + (12*d^3*\cos(b*x + a) + 12*I*d^3*\sin(b*x + a) + 12*(d^3*\cos(2*b*x \\
& + 2*a) + I*d^3*\sin(2*b*x + 2*a) + d^3))*\cos(3*b*x + 3*a) + 12*(d^3*\cos(b*x \\
& + a) + I*d^3*\sin(b*x + a))*\cos(2*b*x + 2*a) + (12*I*d^3*\cos(2*b*x + 2*a) - \\
& 12*d^3*\sin(2*b*x + 2*a) + 12*I*d^3)*\sin(3*b*x + 3*a) + (12*I*d^3*\cos(b*x + \\
& a) - 12*d^3*\sin(b*x + a))*\sin(2*b*x + 2*a))*\text{polylog}(3, -I*e^(I*b*x + I*a)) \\
& + ((2*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 - 12*(-I*a - 1)*d^3 + (6*I*b*c*d^2 - \\
& 6*(I*a + 1)*d^3)*(b*x + a)^2 - (12*b*c*d^2 - (12*a - 12*I)*d^3)*(b*x + a)) \\
& *\cos(3*b*x + 3*a) + (7*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2 - 6*(-3*I*a - 1)*d^ \\
& 3 + (21*I*b*c*d^2 - 3*(7*I*a + 1)*d^3)*(b*x + a)^2 - (6*b*c*d^2 - (6*a - 18 \\
& *I)*d^3)*(b*x + a))*\cos(b*x + a) - (7*(b*x + a)^3*d^3 - 18*b*c*d^2 + (18*a \\
& - 6*I)*d^3 + (21*b*c*d^2 - (21*a - 3*I)*d^3)*(b*x + a)^2 - (-6*I*b*c*d^2 - \\
& 6*(-I*a - 3)*d^3)*(b*x + a))*\sin(b*x + a))*\sin(3*b*x + 3*a) + (I*(b*x + a)^ \\
& 3*d^3 - 6*I*b*c*d^2 - 6*(-I*a + 1)*d^3 + (3*I*b*c*d^2 - 3*(I*a - 1)*d^3)*(b \\
& *x + a)^2 + (6*b*c*d^2 - (6*a + 6*I)*d^3)*(b*x + a))*\sin(2*b*x + 2*a))/(2*b \\
& ^3*\cos(b*x + a) + 2*I*b^3*\sin(b*x + a) + (2*b^3*\cos(2*b*x + 2*a) + 2*I*b^3* \\
& \sin(2*b*x + 2*a) + 2*b^3)*\cos(3*b*x + 3*a) + (2*b^3*\cos(b*x + a) + 2*I*b^3* \\
& \sin(b*x + a))*\cos(2*b*x + 2*a) - (-2*I*b^3*\cos(2*b*x + 2*a) + 2*b^3*\sin(2*b \\
& *x + 2*a) - 2*I*b^3)*\sin(3*b*x + 3*a) + 2*(I*b^3*\cos(b*x + a) - b^3*\sin(b*x \\
& + a))*\sin(2*b*x + 2*a))/b
\end{aligned}$$

Fricas [C] time = 0.745178, size = 2237, normalized size = 9.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")

[Out] $1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*\cos(b*x + a))*\text{polylog}(3, I*\cos(b*x + a) + \sin(b*x + a)) - 6*d^3*\cos(b*x + a)*\text{polylog}(3, I*\cos(b*x + a) - \sin(b*x + a)) + 6*d^3*\cos(b*x + a)*\text{polylog}(3, -I*\cos(b*x + a) + \sin(b*x + a)) - 6*d^3*\cos(b*x + a)*\text{polylog}(3, -I*\cos(b*x + a) - \sin(b*x + a)) + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*\cos(b*x + a)^2 + (6*I*b*d^3*x + 6*I*b*c*d^2)*\cos(b*x + a)*\text{dilog}(I*\cos(b*x + a) + \sin(b*x + a)) + (6*I*b*d^3*x + 6*I*b*c*d^2)*\cos(b*x + a)*\text{dilog}(I*\cos(b*x + a) - \sin(b*x + a)) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*\cos(b*x + a)*\text{dilog}(-I*\cos(b*x + a) + \sin(b*x + a)) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*\cos(b*x + a)*\text{dilog}(-I*\cos(b*x + a) - \sin(b*x + a)) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*\cos(b*x + a)*\log(\cos(b*x + a) + I*\sin(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*\cos(b*x + a)*\log(\cos(b*x + a) - I*\sin(b*x + a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*\cos(b*x + a)*\log(I*\cos(b*x + a) + \sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*\cos(b*x + a)*\log(-I*\cos(b*x + a) + \sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*\cos(b*x + a)*\log(-I*\cos(b*x + a)$

- sin(b*x + a) + 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a)*sin(b*x + a))/(b^4*cos(b*x + a))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \sin(a + bx) \tan^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*sin(b*x+a)*tan(b*x+a)**2,x)

[Out] Integral((c + d*x)**3*sin(a + b*x)*tan(a + b*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \sin(bx + a) \tan(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^3*sin(b*x + a)*tan(b*x + a)^2, x)

3.261 $\int (c + dx)^2 \sin(a + bx) \tan^2(a + bx) dx$

Optimal. Leaf size=145

$$-\frac{2id^2 \text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} + \frac{2id^2 \text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3} - \frac{2d(c+dx)\sin(a+bx)}{b^2} + \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d^2 c}{b^2}$$

[Out] $((4*I)*d*(c + d*x)*\text{ArcTan}[E^{(I*(a + b*x))}])/b^2 - (2*d^2*\text{Cos}[a + b*x])/b^3 + ((c + d*x)^2*\text{Cos}[a + b*x])/b - ((2*I)*d^2*\text{PolyLog}[2, (-I)*E^{(I*(a + b*x))}])/b^3 + ((2*I)*d^2*\text{PolyLog}[2, I*E^{(I*(a + b*x))}])/b^3 + ((c + d*x)^2*\text{Sec}[a + b*x])/b - (2*d*(c + d*x)*\text{Sin}[a + b*x])/b^2$

Rubi [A] time = 0.131128, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {4407, 3296, 2638, 4409, 4181, 2279, 2391}

$$-\frac{2id^2 \text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} + \frac{2id^2 \text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3} - \frac{2d(c+dx)\sin(a+bx)}{b^2} + \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d^2 c}{b^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^2*\text{Sin}[a + b*x]*\text{Tan}[a + b*x]^2, x]$

[Out] $((4*I)*d*(c + d*x)*\text{ArcTan}[E^{(I*(a + b*x))}])/b^2 - (2*d^2*\text{Cos}[a + b*x])/b^3 + ((c + d*x)^2*\text{Cos}[a + b*x])/b - ((2*I)*d^2*\text{PolyLog}[2, (-I)*E^{(I*(a + b*x))}])/b^3 + ((2*I)*d^2*\text{PolyLog}[2, I*E^{(I*(a + b*x))}])/b^3 + ((c + d*x)^2*\text{Sec}[a + b*x])/b - (2*d*(c + d*x)*\text{Sin}[a + b*x])/b^2$

Rule 4407

$\text{Int}[(c + d*x)^m*\text{Sin}[a + b*x]^n*\text{Tan}[a + b*x]^p, x] \rightarrow -\text{Int}[(c + d*x)^m*\text{Sin}[a + b*x]^n*\text{Tan}[a + b*x]^{p-2}, x] + \text{Int}[(c + d*x)^m*\text{Sin}[a + b*x]^{n-2}*\text{Tan}[a + b*x]^p, x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

$\text{Int}[(c + d*x)^m*\text{Cos}[e + f*x], x] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{m-1}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

$\text{Int}[\text{Sin}[c + d*x], x] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 4409

$\text{Int}[(c + d*x)^m*\text{Sec}[a + b*x]^n*\text{Tan}[a + b*x]^p, x] \rightarrow \text{Simp}[(c + d*x)^m*\text{Sec}[a + b*x]^n/(b*n), x] - \text{Dist}[(d*m)/(b*n), \text{Int}[(c + d*x)^{m-1}*\text{Sec}[a + b*x]^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

Rule 4181

$\text{Int}[\text{csc}[e + f*x]*\text{Cos}[k*x], x] \rightarrow \text{Simp}[(-2*(c + d*x)^m*\text{ArcTanh}[E^{(I*k*Pi)*E^{(I*(e + f*x))}}])/f, x] + (-\text{Di}$

```
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \sin(a + bx) \tan^2(a + bx) dx &= - \int (c + dx)^2 \sin(a + bx) dx + \int (c + dx)^2 \sec(a + bx) \tan(a + bx) dx \\ &= \frac{(c + dx)^2 \cos(a + bx)}{b} + \frac{(c + dx)^2 \sec(a + bx)}{b} - \frac{(2d) \int (c + dx) \cos(a + bx) dx}{b} \\ &= \frac{4id(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{(c + dx)^2 \cos(a + bx)}{b} + \frac{(c + dx)^2 \sec(a + bx)}{b} \\ &= \frac{4id(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{2d^2 \cos(a + bx)}{b^3} + \frac{(c + dx)^2 \cos(a + bx)}{b} + \frac{(c + dx)^2 \sec(a + bx)}{b} \\ &= \frac{4id(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{2d^2 \cos(a + bx)}{b^3} + \frac{(c + dx)^2 \cos(a + bx)}{b} - \frac{2id(c + dx) \tan^{-1}(e^{i(a+bx)})}{b} \end{aligned}$$

Mathematica [B] time = 3.11789, size = 362, normalized size = 2.5

$$\frac{2d^2 \csc(a) \left(i \operatorname{PolyLog} \left(2, -e^{i(bx - \tan^{-1}(\cot(a)))} \right) - i \operatorname{PolyLog} \left(2, e^{i(bx - \tan^{-1}(\cot(a)))} \right) + (bx - \tan^{-1}(\cot(a))) \left(\log \left(1 - e^{i(bx - \tan^{-1}(\cot(a)))} \right) - \log \left(1 + e^{i(bx - \tan^{-1}(\cot(a)))} \right) \right) \right)}{\sqrt{\csc^2(a)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (-4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - 4*d^2*ArcTan[Cot[a]]*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + (2*d^2*Csc[a]*((b*x - ArcTan[Cot[a]])*(Log[1 - E^(I*(b*x - ArcTan[Cot[a]])]) - Log[1 + E^(I*(b*x - ArcTan[Cot[a]])])]) + I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]])]) - I*PolyLog[2, E^(I*(b*x - ArcTan[Cot[a]])])]) / Sqrt[Csc[a]^2 + b^2*(c + d*x)^2*Sec[a] + Cos[b*x]*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] - 2*b*d*(c + d*x)*Sin[a]) - (2*b*d*(c + d*x)*Cos[a] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x] + (b^2*(c + d*x)^2*Sin[(b*x)/2]) / ((Cos[a/2] - Sin[a/2])*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2]) - (b^2*(c + d*x)^2*Sin[(b*x)/2]) / ((Cos[a/2] + Sin[a/2])*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])))) / b^3
```

Maple [B] time = 0.158, size = 345, normalized size = 2.4

$$\frac{(d^2x^2b^2 + 2b^2cdx + b^2c^2 + 2ibd^2x - 2d^2 + 2ibcd) e^{i(bx+a)}}{2b^3} + \frac{(d^2x^2b^2 + 2b^2cdx + b^2c^2 - 2ibd^2x - 2d^2 - 2ibcd) e^{-i(bx+a)}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x)`

[Out] $\frac{1}{2}(d^2x^2b^2+2b^2cdx+b^2c^2+2Ibd^2x-2d^2+2Ib^2cd)/b^3\exp(I(bx+a))+\frac{1}{2}(d^2x^2b^2+2b^2cdx+b^2c^2-2Ibd^2x-2d^2-2Ib^2cd)/b^3\exp(-I(bx+a))+2\exp(I(bx+a))(d^2x^2+2cdx+c^2)/(\exp(2I(bx+a))+1)+4Id/b^2c\arctan(\exp(I(bx+a)))+2d^2/b^2\ln(1+\exp(I(bx+a)))x+2d^2/b^3\ln(1+\exp(I(bx+a)))a-2d^2/b^2\ln(1-\exp(I(bx+a)))x-2d^2/b^3\ln(1-\exp(I(bx+a)))a-2Id^2/b^3\operatorname{dilog}(1+\exp(I(bx+a)))+2Id^2/b^3\operatorname{dilog}(1-\exp(I(bx+a)))-4Id^2/b^3a\arctan(\exp(I(bx+a)))$

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")`

[Out] Timed out

Fricas [B] time = 0.637048, size = 1316, normalized size = 9.08

$b^2d^2x^2 + 2b^2cdx + b^2c^2 + id^2 \cos(bx + a) \operatorname{Li}_2(i \cos(bx + a) + \sin(bx + a)) + id^2 \cos(bx + a) \operatorname{Li}_2(i \cos(bx + a) - \sin(bx + a))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")`

[Out] $(b^2d^2x^2 + 2b^2cdx + b^2c^2 + Id^2\cos(bx + a)\operatorname{dilog}(I\cos(bx + a) + \sin(bx + a)) + Id^2\cos(bx + a)\operatorname{dilog}(I\cos(bx + a) - \sin(bx + a)) - Id^2\cos(bx + a)\operatorname{dilog}(-I\cos(bx + a) + \sin(bx + a)) - Id^2\cos(bx + a)\operatorname{dilog}(-I\cos(bx + a) - \sin(bx + a)) + (b^2d^2x^2 + 2b^2cdx + b^2c^2 - 2d^2)\cos(bx + a)^2 - (b^2cd - ad^2)\cos(bx + a)\log(\cos(bx + a) + I\sin(bx + a) + I) + (b^2cd - ad^2)\cos(bx + a)\log(\cos(bx + a) - I\sin(bx + a) + I) - (bd^2x + ad^2)\cos(bx + a)\log(I\cos(bx + a) + \sin(bx + a) + 1) + (bd^2x + ad^2)\cos(bx + a)\log(I\cos(bx + a) - \sin(bx + a) + 1) - (bd^2x + ad^2)\cos(bx + a)\log(-I\cos(bx + a) + \sin(bx + a) + 1) + (bd^2x + ad^2)\cos(bx + a)\log(-I\cos(bx + a) - \sin(bx + a) + 1) - (b^2cd - ad^2)\cos(bx + a)\log(-\cos(bx + a) + I\sin(bx + a) + I) + (b^2cd - ad^2)\cos(bx + a)\log(-\cos(bx + a) - I\sin(bx + a) + I) - 2(bd^2x + b^2cd)\cos(bx + a)\sin(bx + a))/(b^3\cos(bx + a))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \sin(a + bx) \tan^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sin(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*sin(a + b*x)*tan(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \sin(bx + a) \tan(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sin(b*x + a)*tan(b*x + a)^2, x)
```

3.262 $\int (c + dx) \sin(a + bx) \tan^2(a + bx) dx$

Optimal. Leaf size=56

$$-\frac{d \sin(a + bx)}{b^2} - \frac{d \tanh^{-1}(\sin(a + bx))}{b^2} + \frac{(c + dx) \cos(a + bx)}{b} + \frac{(c + dx) \sec(a + bx)}{b}$$

[Out] $-\left(\frac{d \operatorname{ArcTanh}[\sin[a + b*x]]}{b^2}\right) + \left(\frac{(c + d*x)*\cos[a + b*x]}{b}\right) + \left(\frac{(c + d*x)*\sec[a + b*x]}{b}\right) - \left(\frac{d*\sin[a + b*x]}{b^2}\right)$

Rubi [A] time = 0.0537627, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {4407, 3296, 2637, 4409, 3770}

$$-\frac{d \sin(a + bx)}{b^2} - \frac{d \tanh^{-1}(\sin(a + bx))}{b^2} + \frac{(c + dx) \cos(a + bx)}{b} + \frac{(c + dx) \sec(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)*\sin[a + b*x]*\tan[a + b*x]^2, x]$

[Out] $-\left(\frac{d*\operatorname{ArcTanh}[\sin[a + b*x]]}{b^2}\right) + \left(\frac{(c + d*x)*\cos[a + b*x]}{b}\right) + \left(\frac{(c + d*x)*\sec[a + b*x]}{b}\right) - \left(\frac{d*\sin[a + b*x]}{b^2}\right)$

Rule 4407

$\text{Int}[(c + d*x)^m \sin(a + b*x)^n \tan(a + b*x)^p, x] \rightarrow -\text{Int}[(c + d*x)^m \sin(a + b*x)^{n-2} \tan(a + b*x)^p, x] + \text{Int}[(c + d*x)^m \sin(a + b*x)^n \tan(a + b*x)^{p-2}, x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

$\text{Int}[(c + d*x)^m \sin(e + f*x), x] \rightarrow -\text{Simp}[(c + d*x)^m \cos(e + f*x)/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{m-1} \cos(e + f*x), x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

$\text{Int}[\sin[\pi/2 + (c + d*x)], x] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 4409

$\text{Int}[(c + d*x)^m \sec(a + b*x)^n \tan(a + b*x)^p, x] \rightarrow \text{Simp}[(c + d*x)^m \sec(a + b*x)^n / (b*n), x] - \text{Dist}[(d*m)/(b*n), \text{Int}[(c + d*x)^{m-1} \sec(a + b*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

Rule 3770

$\text{Int}[\csc(c + d*x), x] \rightarrow -\text{Simp}[\operatorname{ArcTanh}[\cos[c + d*x]]/d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (c + dx) \sin(a + bx) \tan^2(a + bx) dx &= - \int (c + dx) \sin(a + bx) dx + \int (c + dx) \sec(a + bx) \tan(a + bx) dx \\ &= \frac{(c + dx) \cos(a + bx)}{b} + \frac{(c + dx) \sec(a + bx)}{b} - \frac{d \int \cos(a + bx) dx}{b} - \frac{d \int \sec(a + bx) dx}{b} \\ &= -\frac{d \tanh^{-1}(\sin(a + bx))}{b^2} + \frac{(c + dx) \cos(a + bx)}{b} + \frac{(c + dx) \sec(a + bx)}{b} - \frac{d \sin(a + bx)}{b} \end{aligned}$$

Mathematica [A] time = 0.316159, size = 107, normalized size = 1.91

$$\frac{\sec(a + bx) \left(b(c + dx) \cos(2(a + bx)) - d \sin(2(a + bx)) + 2d \cos(a + bx) \left(\log \left(\cos \left(\frac{1}{2}(a + bx) \right) - \sin \left(\frac{1}{2}(a + bx) \right) \right) - \log \left(\cos \left(\frac{1}{2}(a + bx) \right) + \sin \left(\frac{1}{2}(a + bx) \right) \right) \right) - d \sin(2(a + bx)) \right)}{2b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Sin[a + b*x]*Tan[a + b*x]^2,x]

[Out] (Sec[a + b*x]*(3*b*c + 3*b*d*x + b*(c + d*x)*Cos[2*(a + b*x)] + 2*d*Cos[a + b*x]*(Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]] - Log[Cos[(a + b*x)/2] + Sin[(a + b*x)/2]]) - d*Sin[2*(a + b*x)])/(2*b^2)

Maple [C] time = 0.129, size = 123, normalized size = 2.2

$$\frac{(dxb + bc + id) e^{i(bx+a)}}{2b^2} + \frac{(dxb + bc - id) e^{-i(bx+a)}}{2b^2} + 2 \frac{e^{i(bx+a)} (dx + c)}{b(e^{2i(bx+a)} + 1)} - \frac{d \ln(e^{i(bx+a)} + i)}{b^2} + \frac{d \ln(e^{i(bx+a)} - i)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x)

[Out] 1/2*(d*x*b+b*c+I*d)/b^2*exp(I*(b*x+a))+1/2*(d*x*b+b*c-I*d)/b^2*exp(-I*(b*x+a))+2*exp(I*(b*x+a))*(d*x+c)/b/(exp(2*I*(b*x+a))+1)-d/b^2*ln(exp(I*(b*x+a))+I)+d/b^2*ln(exp(I*(b*x+a))-I)

Maxima [B] time = 1.76264, size = 2866, normalized size = 51.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")

[Out] 1/2*(2*c*(1/cos(b*x + a) + cos(b*x + a)) - 2*a*d*(1/cos(b*x + a) + cos(b*x + a))/b + ((b*x + (b*x + a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)^3 + ((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + 2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*cos(b*x + a) + (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x + a)*sin(2*b*x + 2*a)*sin(b*x + a) + (b*x + (b*x + a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a)

```

- sin(b*x + a))*cos(2*b*x + 2*a) + 6*(b*x + a)*cos(b*x + a) - 2*sin(b*x + a
))*sin(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(b*x + a))*sin(2*b*x +
2*a)^2 + ((b*x + a)*cos(2*b*x + 2*a)^2 + 13*(b*x + a)*cos(b*x + a)^2 + (b*
x + a)*sin(2*b*x + 2*a)^2 + (b*x + a)*sin(b*x + a)^2 + b*x + (13*(b*x + a)*
cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) +
(12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x + a)^2 + sin(b*x + a)^2)
*sin(2*b*x + 2*a) + a)*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a)^3 + 3
*(b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + (b*x + a)*cos(b*x + a) - sin(b*x +
a))*cos(2*b*x + 2*a) + (b*x + a)*cos(b*x + a) - ((cos(2*b*x + 2*a)^2 + sin
(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a
)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + s
in(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + co
s(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x +
a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a
) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^
2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x
+ 3*a) + sin(b*x + a)^2*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x +
a) + 1) + ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) +
1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^
2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(
3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(
cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2
*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)
^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a
)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin
(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2*log(cos(b*x +
a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + (((b*x + a)*sin(2*b*x + 2*a)
- cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a)^2 + 12*(b*x + a)*cos(b*x + a)*sin(
b*x + a) + 2*((b*x + a)*sin(b*x + a) - cos(b*x + a))*cos(2*b*x + 2*a) + ((
b*x + a)*cos(b*x + a) + sin(b*x + a))*sin(2*b*x + 2*a) + (b*x + a)*sin(b*x
+ a) - cos(b*x + a))*cos(3*b*x + 3*a) + (12*(b*x + a)*cos(b*x + a)*sin(b*x
+ a) - cos(b*x + a)^2 - sin(b*x + a)^2 - 2)*cos(2*b*x + 2*a) - cos(2*b*x +
2*a)^2 - cos(b*x + a)^2 + ((b*x + a)*cos(b*x + a)^2 + 13*(b*x + a)*sin(b*x
+ a)^2)*sin(2*b*x + 2*a) - sin(2*b*x + 2*a)^2 - sin(b*x + a)^2 - 1)*sin(3*b
*x + 3*a) + 6*((b*x + a)*cos(b*x + a)^2*sin(b*x + a) + (b*x + a)*sin(b*x +
a)^3)*sin(2*b*x + 2*a) - sin(b*x + a)*d/(((cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + s
in(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^
2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x
+ a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x +
a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*co
s(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos
(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b
*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a)
+ sin(b*x + a)^2)*b))/b

```

Fricas [A] time = 0.511821, size = 251, normalized size = 4.48

$$\frac{2bdx + 2(bdx + bc) \cos(bx + a)^2 - d \cos(bx + a) \log(\sin(bx + a) + 1) + d \cos(bx + a) \log(-\sin(bx + a) + 1) - 2d \cos(bx + a)}{2b^2 \cos(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")

[Out] 1/2*(2*b*d*x + 2*(b*d*x + b*c)*cos(b*x + a)^2 - d*cos(b*x + a)*log(sin(b*x + a) + 1) + d*cos(b*x + a)*log(-sin(b*x + a) + 1) - 2*d*cos(b*x + a)*sin(b*x + a)^2)*b

$$\begin{aligned}
& \tan(1/2*b*x)^2 + \tan(1/2*a)^2 - 2*\tan(1/2*b*x) - 2*\tan(1/2*a) + 1))*\tan(1/2 \\
& *b*x)^3*\tan(1/2*a) + 4*d*\log(2*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4*\tan(1/2*a \\
&)^2 - 2*\tan(1/2*b*x)^4*\tan(1/2*a) - 2*\tan(1/2*b*x)^3*\tan(1/2*a)^2 + \tan(1/2 \\
& *b*x)^4 + 2*\tan(1/2*b*x)^2*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^3 - 2*\tan(1/2*b*x) \\
& *\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^2 + \tan(1/2*a)^2 + 2*\tan(1/2*b*x) + 2*\tan(1/ \\
& 2*a) + 1))*\tan(1/2*b*x)^3*\tan(1/2*a) - 4*d*\tan(1/2*b*x)^4*\tan(1/2*a) + 48*b \\
& *c*\tan(1/2*b*x)^2*\tan(1/2*a)^2 - 24*d*\tan(1/2*b*x)^3*\tan(1/2*a)^2 + 16*b*c* \\
& \tan(1/2*b*x)*\tan(1/2*a)^3 - 4*d*\log(2*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4*\tan \\
& (1/2*a)^2 + 2*\tan(1/2*b*x)^4*\tan(1/2*a) + 2*\tan(1/2*b*x)^3*\tan(1/2*a)^2 + \\
& \tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^2*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^3 + 2*\tan(1 \\
& /2*b*x)*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^2 + \tan(1/2*a)^2 - 2*\tan(1/2*b*x) - 2 \\
& *\tan(1/2*a) + 1))*\tan(1/2*b*x)*\tan(1/2*a)^3 + 4*d*\log(2*(\tan(1/2*a)^2 + 1)/ \\
& (\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^4*\tan(1/2*a) - 2*\tan(1/2*b*x) \\
& ^3*\tan(1/2*a)^2 + \tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^2*\tan(1/2*a)^2 + 2*\tan(1/ \\
& 2*b*x)^3 - 2*\tan(1/2*b*x)*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^2 + \tan(1/2*a)^2 + \\
& 2*\tan(1/2*b*x) + 2*\tan(1/2*a) + 1))*\tan(1/2*b*x)*\tan(1/2*a)^3 - 24*d*\tan(1/ \\
& 2*b*x)^2*\tan(1/2*a)^3 + 4*b*c*\tan(1/2*a)^4 - d*\log(2*(\tan(1/2*a)^2 + 1)/(\tan \\
& (1/2*b*x)^4*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^4*\tan(1/2*a) + 2*\tan(1/2*b*x)^3* \\
& \tan(1/2*a)^2 + \tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^2*\tan(1/2*a)^2 - 2*\tan(1/2*b \\
& *x)^3 + 2*\tan(1/2*b*x)*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^2 + \tan(1/2*a)^2 - 2*t \\
& \tan(1/2*b*x) - 2*\tan(1/2*a) + 1))*\tan(1/2*a)^4 + d*\log(2*(\tan(1/2*a)^2 + 1)/ \\
& (\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^4*\tan(1/2*a) - 2*\tan(1/2*b*x) \\
& ^3*\tan(1/2*a)^2 + \tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^2*\tan(1/2*a)^2 + 2*\tan(1/ \\
& 2*b*x)^3 - 2*\tan(1/2*b*x)*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^2 + \tan(1/2*a)^2 + \\
& 2*\tan(1/2*b*x) + 2*\tan(1/2*a) + 1))*\tan(1/2*a)^4 - 4*d*\tan(1/2*b*x)*\tan(1/2 \\
& *a)^4 - 16*b*d*x*\tan(1/2*b*x)*\tan(1/2*a) + 4*d*\tan(1/2*b*x)^3 - 16*b*c*\tan \\
& (1/2*b*x)*\tan(1/2*a) - 4*d*\log(2*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4*\tan(1/2* \\
& a)^2 + 2*\tan(1/2*b*x)^4*\tan(1/2*a) + 2*\tan(1/2*b*x)^3*\tan(1/2*a)^2 + \tan(1/ \\
& 2*b*x)^4 + 2*\tan(1/2*b*x)^2*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^3 + 2*\tan(1/2*b*x \\
&)*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^2 + \tan(1/2*a)^2 - 2*\tan(1/2*b*x) - 2*\tan(1 \\
& /2*a) + 1))*\tan(1/2*b*x)*\tan(1/2*a) + 4*d*\log(2*(\tan(1/2*a)^2 + 1)/(\tan(1/2 \\
& *b*x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^4*\tan(1/2*a) - 2*\tan(1/2*b*x)^3*\tan(1 \\
& /2*a)^2 + \tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^2*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^3 \\
& - 2*\tan(1/2*b*x)*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^2 + \tan(1/2*a)^2 + 2*\tan(1/ \\
& 2*b*x) + 2*\tan(1/2*a) + 1))*\tan(1/2*b*x)*\tan(1/2*a) + 24*d*\tan(1/2*b*x)^2*t \\
& \tan(1/2*a) + 24*d*\tan(1/2*b*x)*\tan(1/2*a)^2 + 4*d*\tan(1/2*a)^3 + 4*b*d*x + 4 \\
& *b*c + d*\log(2*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4*\tan(1/2*a)^2 + 2*\tan(1/2* \\
& b*x)^4*\tan(1/2*a) + 2*\tan(1/2*b*x)^3*\tan(1/2*a)^2 + \tan(1/2*b*x)^4 + 2*\tan \\
& (1/2*b*x)^2*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^3 + 2*\tan(1/2*b*x)*\tan(1/2*a)^2 + \\
& 2*\tan(1/2*b*x)^2 + \tan(1/2*a)^2 - 2*\tan(1/2*b*x) - 2*\tan(1/2*a) + 1)) - d*l \\
& \log(2*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^4*\tan \\
& (1/2*a) - 2*\tan(1/2*b*x)^3*\tan(1/2*a)^2 + \tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^2 \\
& *\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^3 - 2*\tan(1/2*b*x)*\tan(1/2*a)^2 + 2*\tan(1/2* \\
& b*x)^2 + \tan(1/2*a)^2 + 2*\tan(1/2*b*x) + 2*\tan(1/2*a) + 1)) - 4*d*\tan(1/2*b \\
& *x) - 4*d*\tan(1/2*a))/ (b^2*\tan(1/2*b*x)^4*\tan(1/2*a)^4 - 4*b^2*\tan(1/2*b*x) \\
& ^3*\tan(1/2*a)^3 - b^2*\tan(1/2*b*x)^4 - 4*b^2*\tan(1/2*b*x)^3*\tan(1/2*a) - 4* \\
& b^2*\tan(1/2*b*x)*\tan(1/2*a)^3 - b^2*\tan(1/2*a)^4 - 4*b^2*\tan(1/2*b*x)*\tan(1 \\
& /2*a) + b^2)
\end{aligned}$$

$$3.263 \quad \int \frac{\sin(a+bx) \tan^2(a+bx)}{c+dx} dx$$

Optimal. Leaf size=75

$$\text{CannotIntegrate}\left(\frac{\tan(a+bx) \sec(a+bx)}{c+dx}, x\right) - \frac{\sin\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{d} - \frac{\cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d}$$

[Out] CannotIntegrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x), x] - (CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d - (Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d

Rubi [A] time = 0.150145, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sin(a+bx) \tan^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x), x]

[Out] -((CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d - (Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d + Defer[Int][(Sec[a + b*x]*Tan[a + b*x])/(c + d*x), x]

Rubi steps

$$\begin{aligned} \int \frac{\sin(a+bx) \tan^2(a+bx)}{c+dx} dx &= - \int \frac{\sin(a+bx)}{c+dx} dx + \int \frac{\sec(a+bx) \tan(a+bx)}{c+dx} dx \\ &= - \left(\cos\left(a - \frac{bc}{d}\right) \int \frac{\sin\left(\frac{bc}{d} + bx\right)}{c+dx} dx \right) - \sin\left(a - \frac{bc}{d}\right) \int \frac{\cos\left(\frac{bc}{d} + bx\right)}{c+dx} dx + \int \frac{\sec(a+bx)}{c+dx} dx \\ &= - \frac{\text{Ci}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{d} - \frac{\cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d} + \int \frac{\sec(a+bx) \tan(a+bx)}{c+dx} dx \end{aligned}$$

Mathematica [A] time = 3.81839, size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx) \tan^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x), x]

[Out] Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x), x]

Maple [A] time = 0.299, size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a) (\tan(bx+a))^2}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x)`

[Out] `int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x)`

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="maxima")`

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin(bx+a)\tan(bx+a)^2}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="fricas")`

[Out] `integral(sin(b*x + a)*tan(b*x + a)^2/(d*x + c), x)`

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(b*x+a)*tan(b*x+a)**2/(d*x+c),x)`

[Out] `Integral(sin(a + b*x)*tan(a + b*x)**2/(c + d*x), x)`

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)\tan(bx+a)^2}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="giac")`

[Out] `integrate(sin(b*x + a)*tan(b*x + a)^2/(d*x + c), x)`

$$3.264 \quad \int \frac{\sin(a+bx) \tan^2(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=93

$$\text{CannotIntegrate}\left(\frac{\tan(a+bx) \sec(a+bx)}{(c+dx)^2}, x\right) - \frac{b \cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{d^2} + \frac{b \sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d^2}$$

[Out] CannotIntegrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x] - (b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2 + Sin[a + b*x]/(d*(c + d*x)) + (b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2

Rubi [A] time = 0.181405, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sin(a+bx) \tan^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2, x]

[Out] -((b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2) + Sin[a + b*x]/(d*(c + d*x)) + (b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Defer[Int] [(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]

Rubi steps

$$\begin{aligned} \int \frac{\sin(a+bx) \tan^2(a+bx)}{(c+dx)^2} dx &= - \int \frac{\sin(a+bx)}{(c+dx)^2} dx + \int \frac{\sec(a+bx) \tan(a+bx)}{(c+dx)^2} dx \\ &= \frac{\sin(a+bx)}{d(c+dx)} - \frac{b \int \frac{\cos(a+bx)}{c+dx} dx}{d} + \int \frac{\sec(a+bx) \tan(a+bx)}{(c+dx)^2} dx \\ &= \frac{\sin(a+bx)}{d(c+dx)} - \frac{\left(b \cos\left(a - \frac{bc}{d}\right)\right) \int \frac{\cos\left(\frac{bc}{d} + bx\right)}{c+dx} dx}{d} + \frac{\left(b \sin\left(a - \frac{bc}{d}\right)\right) \int \frac{\sin\left(\frac{bc}{d} + bx\right)}{c+dx} dx}{d} + \int \frac{\sec(a+bx) \tan(a+bx)}{(c+dx)^2} dx \\ &= -\frac{b \cos\left(a - \frac{bc}{d}\right) \text{Ci}\left(\frac{bc}{d} + bx\right)}{d^2} + \frac{\sin(a+bx)}{d(c+dx)} + \frac{b \sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d^2} + \int \frac{\sec(a+bx) \tan(a+bx)}{(c+dx)^2} dx \end{aligned}$$

Mathematica [A] time = 4.18268, size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx) \tan^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2, x]

[Out] Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2, x]

Maple [A] time = 0.378, size = 0, normalized size = 0.

$$\int \frac{\sin (bx+a) (\tan (bx+a))^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)

[Out] int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin (bx+a) \tan (bx+a)^2}{d^2 x^2+2 c d x+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(sin(b*x + a)*tan(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin (a+bx) \tan ^2(a+bx)}{(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)*tan(b*x+a)**2/(d*x+c)**2,x)

[Out] Integral(sin(a + b*x)*tan(a + b*x)**2/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin (bx+a) \tan (bx+a)^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)*tan(b*x + a)^2/(d*x + c)^2, x)
```

3.265 $\int (c + dx)^m \csc(a + bx) \sec^2(a + bx) dx$

Optimal. Leaf size=24

CannotIntegrate(csc(a + bx) sec^2(a + bx)(c + dx)^m, x)

[Out] CannotIntegrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2, x]

Rubi [A] time = 0.24846, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \csc(a + bx) \sec^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2, x]

[Out] Defer[Int] [(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2, x]

Rubi steps

$$\int (c + dx)^m \csc(a + bx) \sec^2(a + bx) dx = \int (c + dx)^m \csc(a + bx) \sec^2(a + bx) dx$$

Mathematica [A] time = 8.94658, size = 0, normalized size = 0.

$$\int (c + dx)^m \csc(a + bx) \sec^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2, x]

[Out] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2, x]

Maple [A] time = 0.16, size = 0, normalized size = 0.

$$\int (dx + c)^m \csc(bx + a) (\sec(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2, x)

[Out] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \csc(bx + a) \sec(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left((dx + c)^m \csc(bx + a) \sec(bx + a)^2, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \csc(bx + a) \sec(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^2, x)
```

3.266 $\int (c + dx)^4 \csc(a + bx) \sec^2(a + bx) dx$

Optimal. Leaf size=469

$$\frac{24d^3(c + dx)\text{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^4} - \frac{24d^3(c + dx)\text{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^4} - \frac{24id^3(c + dx)\text{PolyLog}\left(4, -e^{i(a+bx)}\right)}{b^4} + \frac{24id^3(c + dx)\text{PolyLog}\left(4, e^{i(a+bx)}\right)}{b^4}$$

```
[Out] ((8*I)*d*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b^2 - (2*(c + d*x)^4*ArcTanh[E^(I*(a + b*x))])/b + ((4*I)*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((12*I)*d^2*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((12*I)*d^2*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((4*I)*d*(c + d*x)^3*PolyLog[2, E^(I*(a + b*x))])/b^2 - (12*d^2*(c + d*x)^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (24*d^3*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (24*d^3*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^4 + (12*d^2*(c + d*x)^2*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((24*I)*d^3*(c + d*x)*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((24*I)*d^4*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^5 - ((24*I)*d^4*PolyLog[4, I*E^(I*(a + b*x))])/b^5 + ((24*I)*d^3*(c + d*x)*PolyLog[4, E^(I*(a + b*x))])/b^4 + (24*d^4*PolyLog[5, -E^(I*(a + b*x))])/b^5 - (24*d^4*PolyLog[5, E^(I*(a + b*x))])/b^5 + ((c + d*x)^4*Sec[a + b*x])/b
```

Rubi [A] time = 0.794557, antiderivative size = 469, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {2622, 321, 207, 4420, 6741, 12, 6742, 6273, 4183, 2531, 6609, 2282, 6589, 4181}

$$\frac{24d^3(c + dx)\text{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^4} - \frac{24d^3(c + dx)\text{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^4} - \frac{24id^3(c + dx)\text{PolyLog}\left(4, -e^{i(a+bx)}\right)}{b^4} + \frac{24id^3(c + dx)\text{PolyLog}\left(4, e^{i(a+bx)}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] ((8*I)*d*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b^2 - (2*(c + d*x)^4*ArcTanh[E^(I*(a + b*x))])/b + ((4*I)*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((12*I)*d^2*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((12*I)*d^2*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((4*I)*d*(c + d*x)^3*PolyLog[2, E^(I*(a + b*x))])/b^2 - (12*d^2*(c + d*x)^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (24*d^3*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (24*d^3*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^4 + (12*d^2*(c + d*x)^2*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((24*I)*d^3*(c + d*x)*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((24*I)*d^4*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^5 - ((24*I)*d^4*PolyLog[4, I*E^(I*(a + b*x))])/b^5 + ((24*I)*d^3*(c + d*x)*PolyLog[4, E^(I*(a + b*x))])/b^4 + (24*d^4*PolyLog[5, -E^(I*(a + b*x))])/b^5 - (24*d^4*PolyLog[5, E^(I*(a + b*x))])/b^5 + ((c + d*x)^4*Sec[a + b*x])/b
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

$x]$ /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[Rt[b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 4420

Int[Csc[(a_) + (b_)*(x_)]^(n_)*((c_) + (d_)*(x_))^(m_)*Sec[(a_) + (b_)*(x_)]^(p_), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]

Rule 6741

Int[u_, x_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rule 6273

Int[((a_) + ArcTanh[u_]*(b_))*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x]] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m + 1, x]]

Rule 4183

Int[csc[(e_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^4 \csc(a + bx) \sec^2(a + bx) dx &= -\frac{(c + dx)^4 \tanh^{-1}(\cos(a + bx))}{b} + \frac{(c + dx)^4 \sec(a + bx)}{b} - (4d) \int (c + dx)^3 \left(\right. \\
&= -\frac{(c + dx)^4 \tanh^{-1}(\cos(a + bx))}{b} + \frac{(c + dx)^4 \sec(a + bx)}{b} - (4d) \int \frac{(c + dx)^3}{b} \\
&= -\frac{(c + dx)^4 \tanh^{-1}(\cos(a + bx))}{b} + \frac{(c + dx)^4 \sec(a + bx)}{b} - \frac{(4d) \int (c + dx)^3}{b} \\
&= -\frac{(c + dx)^4 \tanh^{-1}(\cos(a + bx))}{b} + \frac{(c + dx)^4 \sec(a + bx)}{b} - \frac{(4d) \int (-c + dx)}{b} \\
&= -\frac{(c + dx)^4 \tanh^{-1}(\cos(a + bx))}{b} + \frac{(c + dx)^4 \sec(a + bx)}{b} + \frac{(4d) \int (c + dx)^3}{b} \\
&= \frac{8id(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{(c + dx)^4 \sec(a + bx)}{b} + \frac{\int b(c + dx)^4 \csc(a + bx)}{b} \\
&= \frac{8id(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{12id^2(c + dx)^2 \text{Li}_2(-ie^{i(a+bx)})}{b^3} + \frac{12id^2(c + dx)^2}{b} \\
&= \frac{8id(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{2(c + dx)^4 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{12id^2(c + dx)^2}{b} \\
&= \frac{8id(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{2(c + dx)^4 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{4id(c + dx)^3 \text{Li}_2(-ie^{i(a+bx)})}{b^2} \\
&= \frac{8id(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{2(c + dx)^4 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{4id(c + dx)^3 \text{Li}_2(-ie^{i(a+bx)})}{b^2} \\
&= \frac{8id(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{2(c + dx)^4 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{4id(c + dx)^3 \text{Li}_2(-ie^{i(a+bx)})}{b^2} \\
&= \frac{8id(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{2(c + dx)^4 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{4id(c + dx)^3 \text{Li}_2(-ie^{i(a+bx)})}{b^2} \\
&= \frac{8id(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{2(c + dx)^4 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{4id(c + dx)^3 \text{Li}_2(-ie^{i(a+bx)})}{b^2}
\end{aligned}$$

Mathematica [A] time = 3.62998, size = 694, normalized size = 1.48

$$-4d(3ib^2d(c + dx)^2 \text{PolyLog}(2, -ie^{i(a+bx)}) - 3ib^2d(c + dx)^2 \text{PolyLog}(2, ie^{i(a+bx)}) - 6bcd^2 \text{PolyLog}(3, -ie^{i(a+bx)}) + 6bcd^2 \text{PolyLog}(3, ie^{i(a+bx)}))$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^2,x]

[Out] (b^4*(c + d*x)^4*Log[1 - E^(I*(a + b*x))] - b^4*(c + d*x)^4*Log[1 + E^(I*(a + b*x))]) - 4*d*((-2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 - I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1 - I*E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + I*E^(I*(a + b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))] - (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, I*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))] + (4*I)*d*(b^3*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[3, -E^(I*(a + b*x))] - 6*d^2*(b*(c + d*x)*PolyLog[4, -E^(I*(a + b*x))] + I*d*PolyLog[5, -E^(I*(a + b*x))]) - (4*I)*d*(b^3*(c + d*x)^3*PolyLog[2, E^(I*(a + b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[3, E^(I*(a + b*x))] - 6*d^2*(b*(c + d*x)*PolyLog[4, E^(I*(a + b*x))] + I

*d*PolyLog[5, E^(I*(a + b*x))]] + b^4*(c + d*x)^4*Sec[a + b*x])/b^5

Maple [B] time = 0.758, size = 1866, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x)

[Out] $\frac{1}{b^5 d^4} \ln(1 - \exp(I(b*x+a))) x^4 - \frac{1}{b^5 d^4} \ln(1 - \exp(I(b*x+a))) a^4 + 24 I d^3 / b^3 c^* \text{polylog}(2, \exp(I(b*x+a))) x + 24 I d^3 / b^4 c^* a^2 \arctan(\exp(I(b*x+a))) - 24 I d^2 / b^3 c^2 a^* \arctan(\exp(I(b*x+a))) + 4 / b^3 c^* d^3 \ln(1 - \exp(I(b*x+a))) x^3 + 4 / b^4 c^* d^3 \ln(1 - \exp(I(b*x+a))) a^3 + 24 I / b^4 c^* d^3 \text{polylog}(2, \exp(I(b*x+a))) a - 4 / b^3 c^* d^3 \ln(\exp(I(b*x+a)) + 1) x^3 + 12 I / b^2 \text{polylog}(2, -\exp(I(b*x+a))) c^2 d^2 x + 12 I / b^2 c^* d^3 \text{polylog}(2, -\exp(I(b*x+a))) x^2 + 12 / b^3 d^4 \text{polylog}(3, \exp(I(b*x+a))) x^2 + 12 / b^3 c^2 d^2 \text{polylog}(3, \exp(I(b*x+a))) - 12 / b^3 c^2 d^2 \text{polylog}(3, -\exp(I(b*x+a))) - 12 / b^3 d^4 \text{polylog}(3, -\exp(I(b*x+a))) x^2 + 6 / b^3 c^2 d^2 \ln(1 - \exp(I(b*x+a))) x^2 - 1 / b^4 d^4 \ln(\exp(I(b*x+a)) + 1) x^4 - 6 / b^3 c^2 d^2 a^2 \ln(1 - \exp(I(b*x+a))) + 4 / b^3 c^3 d^* \ln(1 - \exp(I(b*x+a))) x + 4 / b^2 c^3 d^* \ln(1 - \exp(I(b*x+a))) a - 4 / b^3 c^3 d^* \ln(\exp(I(b*x+a)) + 1) x - 24 / b^3 c^* d^3 \text{polylog}(3, -\exp(I(b*x+a))) x - 6 / b^3 c^2 d^2 \ln(\exp(I(b*x+a)) + 1) x^2 + 24 / b^3 c^* d^3 \text{polylog}(3, \exp(I(b*x+a))) x + 24 I d^4 \text{polylog}(4, -\exp(I(b*x+a))) / b^5 + 2 \exp(I(b*x+a)) (d^4 x^4 + 4 c^* d^3 x^3 + 6 c^2 d^2 x^2 + 4 c^3 d^* x + c^4) / b / (\exp(2 I(b*x+a)) + 1) - 24 d^4 / b^4 \text{polylog}(3, \exp(I(b*x+a))) x + 24 d^4 / b^4 \text{polylog}(3, -\exp(I(b*x+a))) x - 4 d^4 / b^5 a^3 \ln(1 - \exp(I(b*x+a))) + 4 d^4 / b^2 \ln(1 + \exp(I(b*x+a))) x^3 - 4 d^4 / b^2 \ln(1 - \exp(I(b*x+a))) x^3 - 24 d^3 / b^4 c^* \text{polylog}(3, \exp(I(b*x+a))) + 4 d^4 / b^5 a^3 \ln(1 + \exp(I(b*x+a))) + 24 d^3 / b^4 c^* \text{polylog}(3, -\exp(I(b*x+a))) - 24 I d^4 \text{polylog}(4, \exp(I(b*x+a))) / b^5 + 4 I / b^2 c^3 d^* \text{polylog}(2, -\exp(I(b*x+a))) + 4 I / b^2 d^4 \text{polylog}(2, -\exp(I(b*x+a))) x^3 - 24 I / b^4 d^4 \text{polylog}(4, -\exp(I(b*x+a))) x - 24 I / b^4 c^* d^3 \text{polylog}(4, -\exp(I(b*x+a))) + 1 / b^5 d^4 a^4 \ln(\exp(I(b*x+a)) - 1) - 12 I / b^3 c^2 d^2 \text{dilog}(1 + \exp(I(b*x+a))) + 12 I / b^3 c^2 d^2 \text{dilog}(1 - \exp(I(b*x+a))) - 12 I / b^5 d^4 a^2 \text{dilog}(1 + \exp(I(b*x+a))) + 12 I / b^5 d^4 a^2 \text{dilog}(1 - \exp(I(b*x+a))) + 12 I / b^5 d^4 a^2 \text{polylog}(2, -\exp(I(b*x+a))) - 12 I / b^5 d^4 a^2 \text{polylog}(2, \exp(I(b*x+a))) + 6 / b^3 c^2 d^2 a^2 \ln(\exp(I(b*x+a)) - 1) - 24 I d^3 / b^3 c^* \text{polylog}(2, -\exp(I(b*x+a))) x - 4 / b^4 c^* d^3 a^3 \ln(\exp(I(b*x+a)) - 1) - 4 / b^2 c^3 d^* a^* \ln(\exp(I(b*x+a)) - 1) - 4 I / b^2 c^3 d^* \text{polylog}(2, \exp(I(b*x+a))) + 24 I / b^4 d^4 \text{polylog}(4, \exp(I(b*x+a))) x - 4 I / b^2 d^4 \text{polylog}(2, \exp(I(b*x+a))) x^3 + 24 I / b^4 c^* d^3 \text{polylog}(4, \exp(I(b*x+a))) - 12 d^2 / b^3 c^2 \ln(1 - \exp(I(b*x+a))) a + 12 d^3 / b^2 c^* \ln(1 + \exp(I(b*x+a))) x^2 - 12 d^3 / b^2 c^* \ln(1 - \exp(I(b*x+a))) x^2 - 12 I d^4 / b^3 \text{polylog}(2, -\exp(I(b*x+a))) x^2 + 12 I d^4 / b^3 \text{polylog}(2, \exp(I(b*x+a))) x^2 - 8 I d^4 / b^5 a^3 \arctan(\exp(I(b*x+a))) + 8 I d / b^2 c^3 \arctan(\exp(I(b*x+a))) + 12 d^3 / b^4 c^* a^2 \ln(1 - \exp(I(b*x+a))) + 12 d^2 / b^2 c^2 \ln(1 + \exp(I(b*x+a))) x + 12 d^2 / b^3 c^2 \ln(1 + \exp(I(b*x+a))) a - 12 d^3 / b^4 c^* a^2 \ln(1 + \exp(I(b*x+a))) - 12 d^2 / b^2 c^2 \ln(1 - \exp(I(b*x+a))) x + 24 d^4 \text{polylog}(5, -\exp(I(b*x+a))) / b^5 - 24 d^4 \text{polylog}(5, \exp(I(b*x+a))) / b^5 - 12 I / b^2 c^2 d^2 \text{polylog}(2, \exp(I(b*x+a))) x - 1 / b^4 c^4 \ln(\exp(I(b*x+a)) + 1) + 1 / b^4 c^4 \ln(\exp(I(b*x+a)) - 1) - 12 I / b^2 c^* d^3 \text{polylog}(2, \exp(I(b*x+a))) x^2 - 24 I / b^4 c^* d^3 \text{polylog}(2, -\exp(I(b*x+a))) a + 24 I / b^4 c^* d^3 a^* \text{dilog}(1 + \exp(I(b*x+a))) - 24 I / b^4 c^* d^3 a^* \text{dilog}(1 - \exp(I(b*x+a)))$

Maxima [B] time = 9.45849, size = 7688, normalized size = 16.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")

[Out] $\frac{1}{2}(c^4(2/\cos(bx + a) - \log(\cos(bx + a) + 1) + \log(\cos(bx + a) - 1)) - 4ac^3d(2/\cos(bx + a) - \log(\cos(bx + a) + 1) + \log(\cos(bx + a) - 1)) / b + 6a^2c^2d^2(2/\cos(bx + a) - \log(\cos(bx + a) + 1) + \log(\cos(bx + a) - 1)) / b^2 - 4a^3cd^3(2/\cos(bx + a) - \log(\cos(bx + a) + 1) + \log(\cos(bx + a) - 1)) / b^3 + a^4d^4(2/\cos(bx + a) - \log(\cos(bx + a) + 1) + \log(\cos(bx + a) - 1)) / b^4 + 2((8b^3c^3d - 24ab^2c^2d^2 + 24a^2b^2cd^3 + 8(bx + a)^3d^4 - 8a^3d^4 + 24(bcd^3 - ad^4)(bx + a)^2 + 24(b^2c^2d^2 - 2ab^2cd^3 + a^2d^4)(bx + a) + 8(b^3c^3d - 3ab^2c^2d^2 + 3a^2b^2cd^3 + (bx + a)^3d^4 - a^3d^4 + 3(bcd^3 - ad^4)(bx + a)^2 + 3(b^2c^2d^2 - 2ab^2cd^3 + a^2d^4)(bx + a))\cos(2bx + 2a) - (-8Ib^3c^3d + 24Iab^2c^2d^2 - 24Ia^2b^2cd^3 - 8I(bx + a)^3d^4 + 8Ia^3d^4 + (-24Ibcd^3 + 24Iad^4)(bx + a)^2 + (-24Ib^2c^2d^2 + 48Iab^2cd^3 - 24Ia^2d^4)(bx + a))\sin(2bx + 2a))\arctan2(\cos(bx + a), \sin(bx + a) + 1) + (8b^3c^3d - 24ab^2c^2d^2 + 24a^2b^2cd^3 + 8(bx + a)^3d^4 - 8a^3d^4 + 24(bcd^3 - ad^4)(bx + a)^2 + 24(b^2c^2d^2 - 2ab^2cd^3 + a^2d^4)(bx + a) + 8(b^3c^3d - 3ab^2c^2d^2 + 3a^2b^2cd^3 + (bx + a)^3d^4 - a^3d^4 + 3(bcd^3 - ad^4)(bx + a)^2 + 3(b^2c^2d^2 - 2ab^2cd^3 + a^2d^4)(bx + a))\cos(2bx + 2a) - (-8Ib^3c^3d + 24Iab^2c^2d^2 - 24Ia^2b^2cd^3 - 8I(bx + a)^3d^4 + 8Ia^3d^4 + (-24Ibcd^3 + 24Iad^4)(bx + a)^2 + (-24Ib^2c^2d^2 + 48Iab^2cd^3 - 24Ia^2d^4)(bx + a))\sin(2bx + 2a))\arctan2(\cos(bx + a), -\sin(bx + a) + 1) - (2(bx + a)^4d^4 + 8(bcd^3 - ad^4)(bx + a)^3 + 12(b^2c^2d^2 - 2ab^2cd^3 + a^2d^4)(bx + a)^2 + 8(b^3c^3d - 3ab^2c^2d^2 + 3a^2b^2cd^3 - a^3d^4)(bx + a) + 2((bx + a)^4d^4 + 4(bcd^3 - ad^4)(bx + a)^3 + 6(b^2c^2d^2 - 2ab^2cd^3 + a^2d^4)(bx + a)^2 + 4(b^3c^3d - 3ab^2c^2d^2 + 3a^2b^2cd^3 - a^3d^4)(bx + a))\cos(2bx + 2a) + (2I(bx + a)^4d^4 + (8Ibcd^3 - 8Iad^4)(bx + a)^3 + (12Ib^2c^2d^2 - 24Iab^2cd^3 + 12Ia^2d^4)(bx + a)^2 + (8Ib^3c^3d - 24Iab^2c^2d^2 + 24Ia^2b^2cd^3 - 8Ia^3d^4)(bx + a))\sin(2bx + 2a))\arctan2(\sin(bx + a), \cos(bx + a) + 1) - (2(bx + a)^4d^4 + 8(bcd^3 - ad^4)(bx + a)^3 + 12(b^2c^2d^2 - 2ab^2cd^3 + a^2d^4)(bx + a)^2 + 8(b^3c^3d - 3ab^2c^2d^2 + 3a^2b^2cd^3 - a^3d^4)(bx + a) + 2((bx + a)^4d^4 + 4(bcd^3 - ad^4)(bx + a)^3 + 6(b^2c^2d^2 - 2ab^2cd^3 + a^2d^4)(bx + a)^2 + 4(b^3c^3d - 3ab^2c^2d^2 + 3a^2b^2cd^3 - a^3d^4)(bx + a))\cos(2bx + 2a) + (2I(bx + a)^4d^4 + (8Ibcd^3 - 8Iad^4)(bx + a)^3 + (12Ib^2c^2d^2 - 24Iab^2cd^3 + 12Ia^2d^4)(bx + a)^2 + (8Ib^3c^3d - 24Iab^2c^2d^2 + 24Ia^2b^2cd^3 - 8Ia^3d^4)(bx + a))\sin(2bx + 2a))\arctan2(\sin(bx + a), -\cos(bx + a) + 1) - (4I(bx + a)^4d^4 + (16Ibcd^3 - 16Iad^4)(bx + a)^3 + (24Ib^2c^2d^2 - 48Iab^2cd^3 + 24Ia^2d^4)(bx + a)^2 + (16Ib^3c^3d - 48Iab^2c^2d^2 + 48Ia^2b^2cd^3 - 16Ia^3d^4)(bx + a))\cos(bx + a) + (24b^2c^2d^2 - 48ab^2cd^3 + 24(bx + a)^2d^4 + 24a^2d^4 + 48(bcd^3 - ad^4)(bx + a) + 24(b^2c^2d^2 - 2ab^2cd^3 + (bx + a)^2d^4 + a^2d^4 + 2(bcd^3 - ad^4)(bx + a))\cos(2bx + 2a) - (-24Ib^2c^2d^2 + 48Iab^2cd^3 - 24I(bx + a)^2d^4 - 24Ia^2d^4 + (-48Ibcd^3 + 48Iad^4)(bx + a))\sin(2bx + 2a))\operatorname{dilog}(Ie^{Ibx + Ia}) - (24b^2c^2d^2 - 48ab^2cd^3 + 24(bx + a)^2d^4 + 24a^2d^4 + 48(bcd^3 - ad^4)(bx + a) + 24(b^2c^2d^2 - 2ab^2cd^3 + (bx + a)^2d^4 + a^2d^4 + 2(bcd^3 - ad^4)(bx + a))\cos(2bx + 2a) + (24Ib^2c^2d^2 - 48Iab^2cd^3 + 24I(bx + a)^2d^4 + 24Ia^2d^4 + (48Ibcd^3 - 48Iad^4)(bx + a))\sin(2bx + 2a))\operatorname{dilog}(-Ie^{Ibx + Ia}) + (8b^3c^3d - 24ab^2c^2d^2 + 24a^2b^2cd^3 + 8(bx + a)^3d^4 - 8a^3d^4 + 24(bcd^3 - ad^4)(bx + a)^2 + 24(b^2c^2d^2 - 2ab^2cd^3 + a^2d^4)(bx + a) + 8(b^3c^3d - 3ab^2c^2d^2 + 3a^2b^2cd^3 + (bx + a)^3d^4$

$$\begin{aligned}
&^4 - a^3d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*\cos(2*b*x + 2*a) - (-8*I*b^3*c^3*d + 24*I*a*b^2*c^2*d^2 - 24*I*a^2*b*c*d^3 - 8*I*(b*x + a)^3*d^4 + 8*I*a^3*d^4 + (-24*I*b*c*d^3 + 24*I*a*d^4)*(b*x + a)^2 + (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*a^2*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{dilog}(-e^{(I*b*x + I*a)}) - (8*b^3*c^3*d - 24*a*b^2*c^2*d^2 + 24*a^2*b*c*d^3 + 8*(b*x + a)^3*d^4 - 8*a^3*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*\cos(2*b*x + 2*a) + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2 + 24*I*a^2*b*c*d^3 + 8*I*(b*x + a)^3*d^4 - 8*I*a^3*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a)^2 + (24*I*b^2*c^2*d^2 - 48*I*a*b*c*d^3 + 24*I*a^2*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{dilog}(e^{(I*b*x + I*a)}) - (-I*(b*x + a)^4*d^4 + (-4*I*b*c*d^3 + 4*I*a*d^4)*(b*x + a)^3 + (-6*I*b^2*c^2*d^2 + 12*I*a*b*c*d^3 - 6*I*a^2*d^4)*(b*x + a)^2 + (-4*I*b^3*c^3*d + 12*I*a*b^2*c^2*d^2 - 12*I*a^2*b*c*d^3 + 4*I*a^3*d^4)*(b*x + a) + (-I*(b*x + a)^4*d^4 + (-4*I*b*c*d^3 + 4*I*a*d^4)*(b*x + a)^3 + (-6*I*b^2*c^2*d^2 + 12*I*a*b*c*d^3 - 6*I*a^2*d^4)*(b*x + a)^2 + (-4*I*b^3*c^3*d + 12*I*a*b^2*c^2*d^2 - 12*I*a^2*b*c*d^3 + 4*I*a^3*d^4)*(b*x + a))*\cos(2*b*x + 2*a) + ((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - (I*(b*x + a)^4*d^4 + (4*I*b*c*d^3 - 4*I*a*d^4)*(b*x + a)^3 + (6*I*b^2*c^2*d^2 - 12*I*a*b*c*d^3 + 6*I*a^2*d^4)*(b*x + a)^2 + (4*I*b^3*c^3*d - 12*I*a*b^2*c^2*d^2 + 12*I*a^2*b*c*d^3 - 4*I*a^3*d^4)*(b*x + a) + (I*(b*x + a)^4*d^4 + (4*I*b*c*d^3 - 4*I*a*d^4)*(b*x + a)^3 + (6*I*b^2*c^2*d^2 - 12*I*a*b*c*d^3 + 6*I*a^2*d^4)*(b*x + a)^2 + (4*I*b^3*c^3*d - 12*I*a*b^2*c^2*d^2 + 12*I*a^2*b*c*d^3 - 4*I*a^3*d^4)*(b*x + a))*\cos(2*b*x + 2*a) - ((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) - (-4*I*b^3*c^3*d + 12*I*a*b^2*c^2*d^2 - 12*I*a^2*b*c*d^3 - 4*I*(b*x + a)^3*d^4 + 4*I*a^3*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2 + (-12*I*b^2*c^2*d^2 + 24*I*a*b*c*d^3 - 12*I*a^2*d^4)*(b*x + a) + (-4*I*b^3*c^3*d + 12*I*a*b^2*c^2*d^2 - 12*I*a^2*b*c*d^3 - 4*I*(b*x + a)^3*d^4 + 4*I*a^3*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2 + (-12*I*b^2*c^2*d^2 + 24*I*a*b*c*d^3 - 12*I*a^2*d^4)*(b*x + a))*\cos(2*b*x + 2*a) + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\sin(b*x + a) + 1) - (4*I*b^3*c^3*d - 12*I*a*b^2*c^2*d^2 + 12*I*a^2*b*c*d^3 + 4*I*(b*x + a)^3*d^4 - 4*I*a^3*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^2 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4)*(b*x + a) + (4*I*b^3*c^3*d - 12*I*a*b^2*c^2*d^2 + 12*I*a^2*b*c*d^3 + 4*I*(b*x + a)^3*d^4 - 4*I*a^3*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^2 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4)*(b*x + a))*\cos(2*b*x + 2*a) - 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\sin(b*x + a) + 1) - (48*I*d^4*\cos(2*b*x + 2*a) - 48*d^4*\sin(2*b*x + 2*a) + 48*I*d^4)*\operatorname{polylog}(5, -e^{(I*b*x + I*a)}) - (-48*I*d^4*\cos(2*b*x + 2*a) + 48*d^4*\sin(2*b*x + 2*a) - 48*I*d^4)*\operatorname{polylog}(5, e^{(I*b*x + I*a)}) - 48*(d^4*\cos(2*b*x + 2*a) + I*d^4*\sin(2*b*x + 2*a) + d^4)*\operatorname{polylog}(4, I*e^{(I*b*x + I*a)}) + 48*(d^4*\cos(2*b*x + 2*a) + I*d^4*\sin(2*b*x + 2*a) + d^4)*\operatorname{polylog}(4, -I*e^{(I*b*x + I*a)}) - (48*b*c*d^3 + 48*(b*x + a)*d^4 - 48*a*d^4 + 48*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*\cos(2*b*x + 2*a) + (48*I*b*c*d^3 + 48*I*(b*x + a)*d^4 - 48*I*a*d^4)*\sin(2*b*x + 2*a))*\operatorname{polylog}(4, -e^{(I*b*x + I*a)}) + (48*b*c*d^3 + 48*(b*x + a)*d^4 - 48*a*d^4 + 48*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*\cos(2*b*x + 2*a) - (-48*I*b*c*d^3 - 48*I*(b*x + a)*d^4 + 48*I*a*d^4)*\sin(2*b*x + 2*a))*\operatorname{polylog}(4, e^{(I*b*x + I*a)}) - (-48*I*b*c*d^
\end{aligned}$$

$$\begin{aligned}
& 3 - 48I*(b*x + a)*d^4 + 48I*a*d^4 + (-48I*b*c*d^3 - 48I*(b*x + a)*d^4 + \\
& 48I*a*d^4)*\cos(2*b*x + 2*a) + 48*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*\sin(2* \\
& b*x + 2*a))*\text{polylog}(3, I*e^{(I*b*x + I*a)}) - (48I*b*c*d^3 + 48I*(b*x + a)* \\
& d^4 - 48I*a*d^4 + (48I*b*c*d^3 + 48I*(b*x + a)*d^4 - 48I*a*d^4)*\cos(2*b* \\
& *x + 2*a) - 48*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*\sin(2*b*x + 2*a))*\text{polylog}(\\
& 3, -I*e^{(I*b*x + I*a)}) - (-24I*b^2*c^2*d^2 + 48I*a*b*c*d^3 - 24I*(b*x + \\
& a)^2*d^4 - 24I*a^2*d^4 + (-48I*b*c*d^3 + 48I*a*d^4)*(b*x + a) + (-24I*b \\
& ^2*c^2*d^2 + 48I*a*b*c*d^3 - 24I*(b*x + a)^2*d^4 - 24I*a^2*d^4 + (-48I* \\
& b*c*d^3 + 48I*a*d^4)*(b*x + a))*\cos(2*b*x + 2*a) + 24*(b^2*c^2*d^2 - 2*a*b \\
& *c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*\sin(2*b \\
& *x + 2*a))*\text{polylog}(3, -e^{(I*b*x + I*a)}) - (24I*b^2*c^2*d^2 - 48I*a*b*c*d^ \\
& 3 + 24I*(b*x + a)^2*d^4 + 24I*a^2*d^4 + (48I*b*c*d^3 - 48I*a*d^4)*(b*x \\
& + a) + (24I*b^2*c^2*d^2 - 48I*a*b*c*d^3 + 24I*(b*x + a)^2*d^4 + 24I*a^2 \\
& *d^4 + (48I*b*c*d^3 - 48I*a*d^4)*(b*x + a))*\cos(2*b*x + 2*a) - 24*(b^2*c^ \\
& 2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x \\
& + a))*\sin(2*b*x + 2*a))*\text{polylog}(3, e^{(I*b*x + I*a)}) + 4*((b*x + a)^4*d^4 + \\
& 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(\\
& b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x \\
& + a))*\sin(b*x + a))/(-2I*b^4*\cos(2*b*x + 2*a) + 2*b^4*\sin(2*b*x + 2*a) - \\
& 2I*b^4))/b
\end{aligned}$$

Fricas [C] time = 1.35805, size = 6263, normalized size = 13.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")

$$\begin{aligned}
\text{[Out]} & 1/2*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x + \\
& 2*b^4*c^4 - 24*d^4*\cos(b*x + a)*\text{polylog}(5, \cos(b*x + a) + I*\sin(b*x + a)) \\
& - 24*d^4*\cos(b*x + a)*\text{polylog}(5, \cos(b*x + a) - I*\sin(b*x + a)) + 24*d^4*\cos \\
& (b*x + a)*\text{polylog}(5, -\cos(b*x + a) + I*\sin(b*x + a)) + 24*d^4*\cos(b*x + a) \\
& *\text{polylog}(5, -\cos(b*x + a) - I*\sin(b*x + a)) - 24I*d^4*\cos(b*x + a)*\text{polylog} \\
& (4, I*\cos(b*x + a) + \sin(b*x + a)) - 24I*d^4*\cos(b*x + a)*\text{polylog}(4, I*\cos \\
& (b*x + a) - \sin(b*x + a)) + 24I*d^4*\cos(b*x + a)*\text{polylog}(4, -I*\cos(b*x + a) \\
&) + \sin(b*x + a)) + 24I*d^4*\cos(b*x + a)*\text{polylog}(4, -I*\cos(b*x + a) - \sin \\
& (b*x + a)) + (-4I*b^3*d^4*x^3 - 12I*b^3*c*d^3*x^2 - 12I*b^3*c^2*d^2*x - 4 \\
& *I*b^3*c^3*d)*\cos(b*x + a)*\text{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) + (4I*b^3* \\
& d^4*x^3 + 12I*b^3*c*d^3*x^2 + 12I*b^3*c^2*d^2*x + 4I*b^3*c^3*d)*\cos(b*x \\
& + a)*\text{dilog}(\cos(b*x + a) - I*\sin(b*x + a)) + (12I*b^2*d^4*x^2 + 24I*b^2*c* \\
& d^3*x + 12I*b^2*c^2*d^2)*\cos(b*x + a)*\text{dilog}(I*\cos(b*x + a) + \sin(b*x + a)) \\
& + (12I*b^2*d^4*x^2 + 24I*b^2*c*d^3*x + 12I*b^2*c^2*d^2)*\cos(b*x + a)*\text{di} \\
& \text{log}(I*\cos(b*x + a) - \sin(b*x + a)) + (-12I*b^2*d^4*x^2 - 24I*b^2*c*d^3*x \\
& - 12I*b^2*c^2*d^2)*\cos(b*x + a)*\text{dilog}(-I*\cos(b*x + a) + \sin(b*x + a)) + (- \\
& 12I*b^2*d^4*x^2 - 24I*b^2*c*d^3*x - 12I*b^2*c^2*d^2)*\cos(b*x + a)*\text{dilog} \\
& (-I*\cos(b*x + a) - \sin(b*x + a)) + (-4I*b^3*d^4*x^3 - 12I*b^3*c*d^3*x^2 - \\
& 12I*b^3*c^2*d^2*x - 4I*b^3*c^3*d)*\cos(b*x + a)*\text{dilog}(-\cos(b*x + a) + I*\sin \\
& (b*x + a)) + (4I*b^3*d^4*x^3 + 12I*b^3*c*d^3*x^2 + 12I*b^3*c^2*d^2*x + \\
& 4I*b^3*c^3*d)*\cos(b*x + a)*\text{dilog}(-\cos(b*x + a) - I*\sin(b*x + a)) - (b^4*d^ \\
& 4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*\cos \\
& (b*x + a)*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) - 4*(b^3*c^3*d - 3*a*b^2*c^ \\
& 2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*\cos(b*x + a)*\log(\cos(b*x + a) + I*\sin(b*x \\
& + a) + I) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3* \\
& d*x + b^4*c^4)*\cos(b*x + a)*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1) + 4*(b^3 \\
& *c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*\cos(b*x + a)*\log(\cos(b* \\
& x + a) - I*\sin(b*x + a) + I) - 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2
\end{aligned}$$

```

*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(I*cos(
b*x + a) + sin(b*x + a) + 1) + 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2
*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(I*cos(
b*x + a) - sin(b*x + a) + 1) - 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2
*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(-I*cos
(b*x + a) + sin(b*x + a) + 1) + 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^
2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(-I*co
s(b*x + a) - sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d
^2 - 4*a^3*b*c*d^3 + a^4*d^4)*cos(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*si
n(b*x + a) + 1/2) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*
c*d^3 + a^4*d^4)*cos(b*x + a)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) +
1/2) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x +
4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*cos(b*x + a)*
log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 +
3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) +
I) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4
*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*cos(b*x + a)*lo
g(-cos(b*x + a) - I*sin(b*x + a) + 1) + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*
a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I)
+ (24*I*b*d^4*x + 24*I*b*c*d^3)*cos(b*x + a)*polylog(4, cos(b*x + a) + I*s
in(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos(b*x + a)*polylog(4, cos(b
*x + a) - I*sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b*c*d^3)*cos(b*x + a)*poly
log(4, -cos(b*x + a) + I*sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos
(b*x + a)*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*
b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)*polylog(3, cos(b*x + a) + I*sin(b*x
+ a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)*polylo
g(3, cos(b*x + a) - I*sin(b*x + a)) + 24*(b*d^4*x + b*c*d^3)*cos(b*x + a)*p
olylog(3, I*cos(b*x + a) + sin(b*x + a)) - 24*(b*d^4*x + b*c*d^3)*cos(b*x +
a)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 24*(b*d^4*x + b*c*d^3)*cos(
b*x + a)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 24*(b*d^4*x + b*c*d^3
)*cos(b*x + a)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 12*(b^2*d^4*x^2
+ 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)*polylog(3, -cos(b*x + a) + I*s
in(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)*
polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^5*cos(b*x + a))

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**4*csc(b*x+a)*sec(b*x+a)**2,x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")

[Out] Timed out

3.267 $\int (c + dx)^3 \csc(a + bx) \sec^2(a + bx) dx$

Optimal. Leaf size=343

$$-\frac{6id^2(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} + \frac{6id^2(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} - \frac{6d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} + \frac{6d^2(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}$$

```
[Out] ((6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 - (2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b + ((3*I)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^2 - (6*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*PolyLog[3, I*E^(I*(a + b*x))])/b^4 + (6*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((6*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4 + ((c + d*x)^3*Sec[a + b*x])/b
```

Rubi [A] time = 0.572627, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {2622, 321, 207, 4420, 6741, 12, 6742, 6273, 4183, 2531, 6609, 2282, 6589, 4181}

$$-\frac{6id^2(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} + \frac{6id^2(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} - \frac{6d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} + \frac{6d^2(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] ((6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 - (2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b + ((3*I)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^2 - (6*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*PolyLog[3, I*E^(I*(a + b*x))])/b^4 + (6*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((6*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4 + ((c + d*x)^3*Sec[a + b*x])/b
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[Rt[b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_) + (b_)*(x_)]^(n_)*((c_) + (d_)*(x_))^(m_)*Sec[(a_) + (b_)*(x_)]^(p_), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6741

```
Int[u_, x_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rule 6273

```
Int[((a_) + ArcTanh[u_]*(b_))*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x]] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m + 1, x]]
```

Rule 4183

```
Int[csc[(e_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_) + (f_)*(x_))^(m_)*PolyLog[n_, (d_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(p_)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 \csc(a + bx) \sec^2(a + bx) dx &= -\frac{(c + dx)^3 \tanh^{-1}(\cos(a + bx))}{b} + \frac{(c + dx)^3 \sec(a + bx)}{b} - (3d) \int (c + dx)^2 \left(\right. \\
&= -\frac{(c + dx)^3 \tanh^{-1}(\cos(a + bx))}{b} + \frac{(c + dx)^3 \sec(a + bx)}{b} - (3d) \int \frac{(c + dx)^2}{b} \\
&= -\frac{(c + dx)^3 \tanh^{-1}(\cos(a + bx))}{b} + \frac{(c + dx)^3 \sec(a + bx)}{b} - \frac{(3d) \int (c + dx)^2}{b} \\
&= -\frac{(c + dx)^3 \tanh^{-1}(\cos(a + bx))}{b} + \frac{(c + dx)^3 \sec(a + bx)}{b} - \frac{(3d) \int (-(c + dx))}{b} \\
&= -\frac{(c + dx)^3 \tanh^{-1}(\cos(a + bx))}{b} + \frac{(c + dx)^3 \sec(a + bx)}{b} + \frac{(3d) \int (c + dx)^2}{b} \\
&= \frac{6id(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{(c + dx)^3 \sec(a + bx)}{b} - \frac{\int b(-c - dx)^3 \csc(a + bx)}{b} \\
&= \frac{6id(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{6id^2(c + dx)\text{Li}_2(-ie^{i(a+bx)})}{b^3} + \frac{6id^2(c + dx)\text{Li}_2(e^{i(a+bx)})}{b^3} \\
&= \frac{6id(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{2(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{6id^2(c + dx)\text{Li}_2(-ie^{i(a+bx)})}{b^3} \\
&= \frac{6id(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{2(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{3id(c + dx)^2 \text{Li}_2(e^{i(a+bx)})}{b^2} \\
&= \frac{6id(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{2(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{3id(c + dx)^2 \text{Li}_2(e^{i(a+bx)})}{b^2} \\
&= \frac{6id(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{2(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{3id(c + dx)^2 \text{Li}_2(e^{i(a+bx)})}{b^2} \\
&= \frac{6id(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{2(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{3id(c + dx)^2 \text{Li}_2(e^{i(a+bx)})}{b^2}
\end{aligned}$$

Mathematica [A] time = 1.43863, size = 473, normalized size = 1.38

$$-3d(2ibd(c + dx)\text{PolyLog}(2, -ie^{i(a+bx)}) - 2ibd(c + dx)\text{PolyLog}(2, ie^{i(a+bx)}) - 2d^2\text{PolyLog}(3, -ie^{i(a+bx)}) + 2d^2\text{PolyLog}(3, ie^{i(a+bx)})$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^2,x]

[Out] $(-2*b^3*(c + d*x)^3*\text{ArcTanh}[\text{Cos}[a + b*x] + I*\text{Sin}[a + b*x]] - 3*d*((-2*I)*b^2*c^2*\text{ArcTan}[E^{I*(a + b*x)}] + 2*b^2*c*d*x*\text{Log}[1 - I*E^{I*(a + b*x)}] + b^2*d^2*x^2*\text{Log}[1 - I*E^{I*(a + b*x)}] - 2*b^2*c*d*x*\text{Log}[1 + I*E^{I*(a + b*x)}]) - b^2*d^2*x^2*\text{Log}[1 + I*E^{I*(a + b*x)}] + (2*I)*b*d*(c + d*x)*\text{PolyLog}[2, (-I)*E^{I*(a + b*x)}] - (2*I)*b*d*(c + d*x)*\text{PolyLog}[2, I*E^{I*(a + b*x)}] - 2*d^2*\text{PolyLog}[3, (-I)*E^{I*(a + b*x)}] + 2*d^2*\text{PolyLog}[3, I*E^{I*(a + b*x)}]) + (3*I)*d*(b^2*(c + d*x)^2*\text{PolyLog}[2, -\text{Cos}[a + b*x] - I*\text{Sin}[a + b*x]] + (2*I)*b*d*(c + d*x)*\text{PolyLog}[3, -\text{Cos}[a + b*x] - I*\text{Sin}[a + b*x]] - 2*d^2*\text{PolyLog}[4, -\text{Cos}[a + b*x] - I*\text{Sin}[a + b*x]]) - (3*I)*d*(b^2*(c + d*x)^2*\text{PolyLog}[2, \text{Cos}[a + b*x] + I*\text{Sin}[a + b*x]] + (2*I)*b*d*(c + d*x)*\text{PolyLog}[3, \text{Cos}[a + b*x] + I*\text{Sin}[a + b*x]] - 2*d^2*\text{PolyLog}[4, \text{Cos}[a + b*x] + I*\text{Sin}[a + b*x]]) + b^3*(c + d*x)^3*\text{Sec}[a + b*x])/b^4$

Maple [B] time = 0.651, size = 1152, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x)

[Out] $-3/b*c*d^2*\ln(\exp(I*(b*x+a))+1)*x^2+3/b*c*d^2*\ln(1-\exp(I*(b*x+a)))*x^2-3*d^3/b^2*\ln(1-I*\exp(I*(b*x+a)))*x^2-3*d^3/b^4*a^2*\ln(1+I*\exp(I*(b*x+a)))+3*d^3/b^2*\ln(1+I*\exp(I*(b*x+a)))*x^2+3*d^3/b^4*a^2*\ln(1-I*\exp(I*(b*x+a)))+6*I*d^3*x*\text{polylog}(2, I*\exp(I*(b*x+a)))/b^3+6*d^3*\text{polylog}(3, -I*\exp(I*(b*x+a)))/b^4-6*d^3*\text{polylog}(3, I*\exp(I*(b*x+a)))/b^4+2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*\exp(I*(b*x+a))/b/(\exp(2*I*(b*x+a))+1)+6*I*d/b^2*c^2*\arctan(\exp(I*(b*x+a)))-6*I/b^2*d^2*c*\text{polylog}(2, \exp(I*(b*x+a)))*x-6*I*d^3*\text{polylog}(4, -\exp(I*(b*x+a)))/b^4+6*I*d^3*\text{polylog}(4, \exp(I*(b*x+a)))/b^4+1/b*d^3*\ln(1-\exp(I*(b*x+a)))*x^3+1/b^4*d^3*\ln(1-\exp(I*(b*x+a)))*a^3-1/b*d^3*\ln(\exp(I*(b*x+a))+1)*x^3+3/b^3*c*d^2*a^2*\ln(\exp(I*(b*x+a))-1)-3/b^2*c^2*d*a*\ln(\exp(I*(b*x+a))-1)-3/b^3*c*d^2*a^2*\ln(1-\exp(I*(b*x+a)))-3/b*c^2*d*\ln(\exp(I*(b*x+a))+1)*x+3/b*c^2*d*\ln(1-\exp(I*(b*x+a)))*x+3/b^2*c^2*d*\ln(1-\exp(I*(b*x+a)))*a+3*I/b^2*c^2*d*\text{polylog}(2, -\exp(I*(b*x+a)))+3*I/b^2*d^3*\text{polylog}(2, -\exp(I*(b*x+a)))*x^2-6*I*d^3*x*\text{polylog}(2, -I*\exp(I*(b*x+a)))/b^3+6*d^2/b^3*c*\ln(1+I*\exp(I*(b*x+a)))*a-6*d^2/b^2*c*\ln(1-I*\exp(I*(b*x+a)))*x-6*d^2/b^3*c*\ln(1-I*\exp(I*(b*x+a)))*a+6*d^2/b^2*c*\ln(1+I*\exp(I*(b*x+a)))*x+6*I*d^3/b^4*a^2*\arctan(\exp(I*(b*x+a)))-12*I*d^2/b^3*c*a*\arctan(\exp(I*(b*x+a)))+6/b^3*d^3*\text{polylog}(3, \exp(I*(b*x+a)))*x-6/b^3*d^3*\text{polylog}(3, -\exp(I*(b*x+a)))*x+6/b^3*c*d^2*\text{polylog}(3, \exp(I*(b*x+a)))-6/b^3*c*d^2*\text{polylog}(3, -\exp(I*(b*x+a)))-1/b^4*d^3*a^3*\ln(\exp(I*(b*x+a))-1)+6*I/b^2*\text{polylog}(2, -\exp(I*(b*x+a)))*c*d^2*x+6*I/b^4*d^3*\text{polylog}(2, I*\exp(I*(b*x+a)))*a+6*I/b^3*d^2*c*d*\text{dilog}(1-I*\exp(I*(b*x+a)))-6*I/b^3*d^2*c*d*\text{dilog}(1+I*\exp(I*(b*x+a)))+6*I/b^4*d^3*a*d*\text{dilog}(1+I*\exp(I*(b*x+a)))-3*I/b^2*d^3*\text{polylog}(2, \exp(I*(b*x+a)))*x^2-6*I/b^4*d^3*\text{polylog}(2, -I*\exp(I*(b*x+a)))*a-3*I/b^2*c^2*d*\text{polylog}(2, \exp(I*(b*x+a)))-6*I/b^4*d^3*a*d*\text{dilog}(1-I*\exp(I*(b*x+a)))+1/b*c^3*\ln(\exp(I*(b*x+a))-1)-1/b*c^3*\ln(\exp(I*(b*x+a))+1)$

Maxima [B] time = 4.01994, size = 4327, normalized size = 12.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")

[Out] $\frac{1}{2}(c^3(2/\cos(bx+a) - \log(\cos(bx+a) + 1) + \log(\cos(bx+a) - 1)) - 3ac^2d(2/\cos(bx+a) - \log(\cos(bx+a) + 1) + \log(\cos(bx+a) - 1)) / b + 3a^2cd^2(2/\cos(bx+a) - \log(\cos(bx+a) + 1) + \log(\cos(bx+a) - 1)) / b^2 - a^3d^3(2/\cos(bx+a) - \log(\cos(bx+a) + 1) + \log(\cos(bx+a) - 1)) / b^3 + 2((6b^2c^2d - 12a^2bcd^2 + 6(bx+a)^2d^3 + 6a^2d^3 + 12(bcd^2 - ad^3)(bx+a) + 6(b^2c^2d - 2abcd^2 + (bx+a)^2d^3 + a^2d^3 + 2(bcd^2 - ad^3)(bx+a))\cos(2bx+2a) - (-6Ib^2c^2d + 12Iabcd^2 - 6I(bx+a)^2d^3 - 6Ia^2d^3 + (-12Ibcd^2 + 12Iad^3)(bx+a))\sin(2bx+2a))\arctan2(\cos(bx+a), \sin(bx+a) + 1) + (6b^2c^2d - 12a^2bcd^2 + 6(bx+a)^2d^3 + 6a^2d^3 + 12(bcd^2 - ad^3)(bx+a) + 6(b^2c^2d - 2abcd^2 + (bx+a)^2d^3 + a^2d^3 + 2(bcd^2 - ad^3)(bx+a))\cos(2bx+2a) - (-6Ib^2c^2d + 12Iabcd^2 - 6I(bx+a)^2d^3 - 6Ia^2d^3 + (-12Ibcd^2 + 12Iad^3)(bx+a))\sin(2bx+2a))\arctan2(\cos(bx+a), -\sin(bx+a) + 1) - (2(bx+a)^3d^3 + 6(bcd^2 - ad^3)(bx+a)^2 + 6(b^2c^2d - 2abcd^2 + a^2d^3)(bx+a) + 2((bx+a)^3d^3 + 3(bcd^2 - ad^3)(bx+a)^2 + 3(b^2c^2d - 2abcd^2 + a^2d^3)(bx+a))\cos(2bx+2a) + (2I(bx+a)^3d^3 + (6Ibcd^2 - 6Iad^3)(bx+a)^2 + (6Ib^2c^2d - 12Iabcd^2 + 6Ia^2d^3)(bx+a))\sin(2bx+2a))\arctan2(\sin(bx+a), \cos(bx+a) + 1) - (2(bx+a)^3d^3 + 6(bcd^2 - ad^3)(bx+a)^2 + 6(b^2c^2d - 2abcd^2 + a^2d^3)(bx+a) + 2((bx+a)^3d^3 + 3(bcd^2 - ad^3)(bx+a)^2 + 3(b^2c^2d - 2abcd^2 + a^2d^3)(bx+a))\cos(2bx+2a) + (2I(bx+a)^3d^3 + (6Ibcd^2 - 6Iad^3)(bx+a)^2 + (6Ib^2c^2d - 12Iabcd^2 + 6Ia^2d^3)(bx+a))\sin(2bx+2a))\arctan2(\sin(bx+a), -\cos(bx+a) + 1) - (4I(bx+a)^3d^3 + (12Ibcd^2 - 12Iad^3)(bx+a)^2 + (12Ib^2c^2d - 24Iabcd^2 + 12Ia^2d^3)(bx+a))\cos(bx+a) + (12bcd^2 + 12(bx+a)d^3 - 12ad^3 + 12(bcd^2 + (bx+a)d^3 - ad^3)\cos(2bx+2a) - (-12Ibcd^2 - 12I(bx+a)d^3 + 12Ia^2d^3)\sin(2bx+2a))\operatorname{dilog}(e^{(Ibx+Ia)}) - (12bcd^2 + 12(bx+a)d^3 - 12ad^3 + 12(bcd^2 + (bx+a)d^3 - ad^3)\cos(2bx+2a) + (12Ibcd^2 + 12I(bx+a)d^3 - 12Ia^2d^3)\sin(2bx+2a))\operatorname{dilog}(-e^{(Ibx+Ia)}) + (6b^2c^2d - 12a^2bcd^2 + 6(bx+a)^2d^3 + 6a^2d^3 + 12(bcd^2 - ad^3)(bx+a) + 6(b^2c^2d - 2abcd^2 + (bx+a)^2d^3 + a^2d^3 + 2(bcd^2 - ad^3)(bx+a))\cos(2bx+2a) - (-6Ib^2c^2d + 12Iabcd^2 - 6I(bx+a)^2d^3 - 6Ia^2d^3 + (-12Ibcd^2 + 12Iad^3)(bx+a))\sin(2bx+2a))\operatorname{dilog}(-e^{(Ibx+Ia)}) - (6b^2c^2d - 12a^2bcd^2 + 6(bx+a)^2d^3 + 6a^2d^3 + 12(bcd^2 - ad^3)(bx+a) + 6(b^2c^2d - 2abcd^2 + (bx+a)^2d^3 + a^2d^3 + 2(bcd^2 - ad^3)(bx+a))\cos(2bx+2a) + (6Ib^2c^2d - 12Iabcd^2 + 6Ia^2d^3 + (12Ibcd^2 - 12Iad^3)(bx+a))\sin(2bx+2a))\operatorname{dilog}(e^{(Ibx+Ia)}) - (-I(bx+a)^3d^3 + (-3Ibcd^2 + 3Ia^2d^3)(bx+a) + (-3Ib^2c^2d + 6Iabcd^2 - 3Ia^2d^3)(bx+a) + (-I(bx+a)^3d^3 + (-3Ibcd^2 + 3Ia^2d^3)(bx+a)^2 + (-3Ib^2c^2d + 6Iabcd^2 - 3Ia^2d^3)(bx+a))\cos(2bx+2a) + ((bx+a)^3d^3 + 3(bcd^2 - ad^3)(bx+a)^2 + 3(b^2c^2d - 2abcd^2 + a^2d^3)(bx+a))\sin(2bx+2a))\log(\cos(bx+a)^2 + \sin(bx+a)^2 + 2\cos(bx+a) + 1) - (I(bx+a)^3d^3 + (3Ibcd^2 - 3Ia^2d^3)(bx+a)^2 + (3Ib^2c^2d - 6Iabcd^2 + 3Ia^2d^3)(bx+a) + (I(bx+a)^3d^3 + (3Ibcd^2 - 3Ia^2d^3)(bx+a)^2 + (3Ib^2c^2d - 6Iabcd^2 + 3Ia^2d^3)(bx+a))\cos(2bx+2a) - ((bx+a)^3d^3 + 3(bcd^2 - ad^3)(bx+a)^2 + 3(b^2c^2d - 2abcd^2 + a^2d^3)(bx+a))\sin(2bx+2a))\log(\cos(bx+a)^2 + \sin(bx+a)^2 - 2\cos(bx+a) + 1) - (-3Ib^2c^2d + 6Iabcd^2 - 3I(bx+a)^2d^3 - 3Ia^2d^3 + (-6Ibcd^2 + 6Iad^3)(bx+a) + (-3Ib^2c^2d$

$$\begin{aligned} &^2*d + 6*I*a*b*c*d^2 - 3*I*(b*x + a)^2*d^3 - 3*I*a^2*d^3 + (-6*I*b*c*d^2 + \\ &6*I*a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x \\ &+ a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\log \\ &(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\sin(b*x + a) + 1) - (3*I*b^2*c^2*d - 6 \\ &*I*a*b*c*d^2 + 3*I*(b*x + a)^2*d^3 + 3*I*a^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3 \\ &)*(b*x + a) + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*(b*x + a)^2*d^3 + 3*I*a^2 \\ &2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - 3*(b^2*c^2*d \\ &- 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) \\ &)*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\sin(b*x + a) + \\ &1) - 12*(d^3*\cos(2*b*x + 2*a) + I*d^3*\sin(2*b*x + 2*a) + d^3)*\text{polylog}(4, -e \\ &^(I*b*x + I*a)) + 12*(d^3*\cos(2*b*x + 2*a) + I*d^3*\sin(2*b*x + 2*a) + d^3)* \\ &\text{polylog}(4, e^(I*b*x + I*a)) - (-12*I*d^3*\cos(2*b*x + 2*a) + 12*d^3*\sin(2*b*x \\ &+ 2*a) - 12*I*d^3)*\text{polylog}(3, I*e^(I*b*x + I*a)) - (12*I*d^3*\cos(2*b*x + \\ &2*a) - 12*d^3*\sin(2*b*x + 2*a) + 12*I*d^3)*\text{polylog}(3, -I*e^(I*b*x + I*a)) - \\ &(-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3 + (-12*I*b*c*d^2 - 12*I*(\\ &b*x + a)*d^3 + 12*I*a*d^3)*\cos(2*b*x + 2*a) + 12*(b*c*d^2 + (b*x + a)*d^3 - \\ &a*d^3)*\sin(2*b*x + 2*a))*\text{polylog}(3, -e^(I*b*x + I*a)) - (12*I*b*c*d^2 + 12 \\ &*I*(b*x + a)*d^3 - 12*I*a*d^3 + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a \\ &*d^3)*\cos(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\sin(2*b*x + 2 \\ &2*a))*\text{polylog}(3, e^(I*b*x + I*a)) + 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3) \\ &*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*\sin(b*x + a \\ &))/(-2*I*b^3*\cos(2*b*x + 2*a) + 2*b^3*\sin(2*b*x + 2*a) - 2*I*b^3))/b \end{aligned}$$

Fricas [C] time = 1.00506, size = 4352, normalized size = 12.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")

[Out] $\frac{1}{2}*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*I*d^3*\cos(b*x + a)*\text{polylog}(4, \cos(b*x + a) + I*\sin(b*x + a)) - 6*I*d^3*\cos(b*x + a)*\text{polylog}(4, \cos(b*x + a) - I*\sin(b*x + a)) + 6*I*d^3*\cos(b*x + a)*\text{polylog}(4, -\cos(b*x + a) + I*\sin(b*x + a)) - 6*I*d^3*\cos(b*x + a)*\text{polylog}(4, -\cos(b*x + a) - I*\sin(b*x + a)) + 6*d^3*\cos(b*x + a)*\text{polylog}(3, I*\cos(b*x + a) + \sin(b*x + a)) - 6*d^3*\cos(b*x + a)*\text{polylog}(3, I*\cos(b*x + a) - \sin(b*x + a)) + 6*d^3*\cos(b*x + a)*\text{polylog}(3, -I*\cos(b*x + a) + \sin(b*x + a)) - 6*d^3*\cos(b*x + a)*\text{polylog}(3, -I*\cos(b*x + a) - \sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*\cos(b*x + a)*\text{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*\cos(b*x + a)*\text{dilog}(\cos(b*x + a) - I*\sin(b*x + a)) + (6*I*b*d^3*x + 6*I*b*c*d^2)*\cos(b*x + a)*\text{dilog}(I*\cos(b*x + a) + \sin(b*x + a)) + (6*I*b*d^3*x + 6*I*b*c*d^2)*\cos(b*x + a)*\text{dilog}(I*\cos(b*x + a) - \sin(b*x + a)) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*\cos(b*x + a)*\text{dilog}(-I*\cos(b*x + a) + \sin(b*x + a)) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*\cos(b*x + a)*\text{dilog}(-I*\cos(b*x + a) - \sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*\cos(b*x + a)*\text{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*\cos(b*x + a)*\text{dilog}(-\cos(b*x + a) - I*\sin(b*x + a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\cos(b*x + a)*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*\cos(b*x + a)*\log(\cos(b*x + a) + I*\sin(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\cos(b*x + a)*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*\cos(b*x + a)*\log(\cos(b*x + a) - I*\sin(b*x + a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*\cos(b*x + a)*\log(I*\cos(b*x + a) + \sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*\cos(b*x + a)*\log(I*\cos(b*x + a) - \sin(b*x + a) +$

```

1) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^4*cos(b*x + a))

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*csc(b*x+a)*sec(b*x+a)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \csc(bx + a) \sec(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^3*csc(b*x + a)*sec(b*x + a)^2, x)

3.268 $\int (c + dx)^2 \csc(a + bx) \sec^2(a + bx) dx$

Optimal. Leaf size=219

$$\frac{2id(c + dx)\text{PolyLog}(2, -e^{i(a+bx)})}{b^2} - \frac{2id(c + dx)\text{PolyLog}(2, e^{i(a+bx)})}{b^2} - \frac{2id^2\text{PolyLog}(2, -ie^{i(a+bx)})}{b^3} + \frac{2id^2\text{PolyLog}(2, ie^{i(a+bx)})}{b^3}$$

[Out] $((4*I)*d*(c + d*x)*\text{ArcTan}[E^{(I*(a + b*x))}])/b^2 - (2*(c + d*x)^2*\text{ArcTanh}[E^{(I*(a + b*x))}])/b + ((2*I)*d*(c + d*x)*\text{PolyLog}[2, -E^{(I*(a + b*x))}])/b^2 - ((2*I)*d^2*\text{PolyLog}[2, (-I)*E^{(I*(a + b*x))}])/b^3 + ((2*I)*d^2*\text{PolyLog}[2, I*E^{(I*(a + b*x))}])/b^3 - ((2*I)*d*(c + d*x)*\text{PolyLog}[2, E^{(I*(a + b*x))}])/b^2 - (2*d^2*\text{PolyLog}[3, -E^{(I*(a + b*x))}])/b^3 + (2*d^2*\text{PolyLog}[3, E^{(I*(a + b*x))}])/b^3 + ((c + d*x)^2*\text{Sec}[a + b*x])/b$

Rubi [A] time = 0.37738, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {2622, 321, 207, 4420, 6741, 12, 6742, 6273, 4183, 2531, 2282, 6589, 4181, 2279, 2391}

$$\frac{2id(c + dx)\text{PolyLog}(2, -e^{i(a+bx)})}{b^2} - \frac{2id(c + dx)\text{PolyLog}(2, e^{i(a+bx)})}{b^2} - \frac{2id^2\text{PolyLog}(2, -ie^{i(a+bx)})}{b^3} + \frac{2id^2\text{PolyLog}(2, ie^{i(a+bx)})}{b^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^2*\text{Csc}[a + b*x]*\text{Sec}[a + b*x]^2, x]$

[Out] $((4*I)*d*(c + d*x)*\text{ArcTan}[E^{(I*(a + b*x))}])/b^2 - (2*(c + d*x)^2*\text{ArcTanh}[E^{(I*(a + b*x))}])/b + ((2*I)*d*(c + d*x)*\text{PolyLog}[2, -E^{(I*(a + b*x))}])/b^2 - ((2*I)*d^2*\text{PolyLog}[2, (-I)*E^{(I*(a + b*x))}])/b^3 + ((2*I)*d^2*\text{PolyLog}[2, I*E^{(I*(a + b*x))}])/b^3 - ((2*I)*d*(c + d*x)*\text{PolyLog}[2, E^{(I*(a + b*x))}])/b^2 - (2*d^2*\text{PolyLog}[3, -E^{(I*(a + b*x))}])/b^3 + (2*d^2*\text{PolyLog}[3, E^{(I*(a + b*x))}])/b^3 + ((c + d*x)^2*\text{Sec}[a + b*x])/b$

Rule 2622

$\text{Int}[\csc[(e_.) + (f_.)*(x_.)]^{(n_.)}*((a_.)*\sec[(e_.) + (f_.)*(x_.)]^{(m_.)}, x_Symbol] \rightarrow \text{Dist}[1/(f*a^n), \text{Subst}[\text{Int}[x^{(m+n-1)}/(-1+x^2/a^2)^{((n+1)/2)}, x], x, a*\text{Sec}[e+f*x]], x] /; \text{FreeQ}\{a, e, f, m\}, x\} \ \&\& \ \text{IntegerQ}\{(n+1)/2\} \ \&\& \ !(\text{IntegerQ}\{(m+1)/2\} \ \&\& \ \text{LtQ}\{0, m, n\})$

Rule 321

$\text{Int}[((c_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a+b*x^n)^{(p+1)})/(b*(m+n*p+1)), x] - \text{Dist}[(a*c^{(n*(m-n+1))})/(b*(m+n*p+1)), \text{Int}[(c*x)^{(m-n)}*(a+b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, p\}, x\} \ \&\& \ \text{IGtQ}\{n, 0\} \ \&\& \ \text{GtQ}\{m, n-1\} \ \&\& \ \text{NeQ}\{m+n*p+1, 0\} \ \&\& \ \text{IntBinomialQ}\{a, b, c, n, m, p, x\}$

Rule 207

$\text{Int}[((a_.) + (b_.)*(x_.)^2)^{(-1)}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[(\text{Rt}[b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{NegQ}\{a/b\} \ \&\& \ (\text{LtQ}\{a, 0\} \ || \ \text{GtQ}\{b, 0\})$

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6741

```
Int[u_, x_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m + 1, x]]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_.))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \csc(a + bx) \sec^2(a + bx) dx &= -\frac{(c + dx)^2 \tanh^{-1}(\cos(a + bx))}{b} + \frac{(c + dx)^2 \sec(a + bx)}{b} - (2d) \int (c + dx) \left(-\frac{\tan(a + bx)}{b} \right) dx \\ &= -\frac{(c + dx)^2 \tanh^{-1}(\cos(a + bx))}{b} + \frac{(c + dx)^2 \sec(a + bx)}{b} - (2d) \int \frac{(c + dx) (-\tan(a + bx))}{b} dx \\ &= -\frac{(c + dx)^2 \tanh^{-1}(\cos(a + bx))}{b} + \frac{(c + dx)^2 \sec(a + bx)}{b} - \frac{(2d) \int (c + dx) (-\tan(a + bx)) dx}{b} \\ &= -\frac{(c + dx)^2 \tanh^{-1}(\cos(a + bx))}{b} + \frac{(c + dx)^2 \sec(a + bx)}{b} - \frac{(2d) \int (-(c + dx) \tan(a + bx)) dx}{b} \\ &= -\frac{(c + dx)^2 \tanh^{-1}(\cos(a + bx))}{b} + \frac{(c + dx)^2 \sec(a + bx)}{b} + \frac{(2d) \int (c + dx) \tanh(a + bx) dx}{b} \\ &= \frac{4id(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{(c + dx)^2 \sec(a + bx)}{b} + \frac{\int b(c + dx)^2 \csc(a + bx) dx}{b} \\ &= \frac{4id(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{(c + dx)^2 \sec(a + bx)}{b} - \frac{(2id^2) \text{Subst}\left(\int \frac{\log(1-ix)}{x} dx\right)}{b^3} \\ &= \frac{4id(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{2(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{2id^2 \text{Li}_2(-ie^{i(a+bx)})}{b^3} \\ &= \frac{4id(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{2(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{2id(c + dx) \text{Li}_2(-e^{i(a+bx)})}{b^2} \\ &= \frac{4id(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{2(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{2id(c + dx) \text{Li}_2(-e^{i(a+bx)})}{b^2} \\ &= \frac{4id(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{2(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{2id(c + dx) \text{Li}_2(-e^{i(a+bx)})}{b^2} \end{aligned}$$

Mathematica [A] time = 2.58814, size = 317, normalized size = 1.45

$$2id(b(c + dx)\text{PolyLog}(2, -e^{i(a+bx)}) + id\text{PolyLog}(3, -e^{i(a+bx)})) + 2d(d\text{PolyLog}(3, e^{i(a+bx)}) - ib(c + dx)\text{PolyLog}(2, e^{i(a+bx)}))$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] (-4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - 4*d^2*ArcTan[Cot[a]]*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + b^2*(c + d*x)^2*Log[1 - E^(I*(a + b*x))] - b^2*(c + d*x)^2*Log[1 + E^(I*(a + b*x))] + (2*d^2*Csc[a]*((b*x - ArcTan[Cot[a]])*(Log[1 - E^(I*(b*x - ArcTan[Cot[a]])]) - Log[1 + E^(I*(b*x - ArcTan[Cot[a]])])]) + I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]])]) - I*PolyLog[2, E^(I*(b*x - ArcTan[Cot[a]])])])/Sqrt[Csc[a]^2] + (2*I)*d*(b*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] + I*d*PolyLog[3, -E^(I*(a + b*x))] + 2*d*((-I)*b*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] + d*PolyLog[3, E^(I*(a + b*x))]) + b^2*(c + d*x)^2*Sec[a + b*x])/b^3
```

Maple [B] time = 0.448, size = 568, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x)
```

```
[Out] 2*d^2/b^2*ln(1+I*exp(I*(b*x+a)))*x+2*d^2/b^3*ln(1+I*exp(I*(b*x+a)))*a-2*d^2/b^2*ln(1-I*exp(I*(b*x+a)))*x-2*d^2/b^3*ln(1-I*exp(I*(b*x+a)))*a-2*I*d^2/b^3*dilog(1+I*exp(I*(b*x+a)))+2*I*d^2/b^3*dilog(1-I*exp(I*(b*x+a)))-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2-1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2+2*exp(I*(b*x+a))*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x+a))+1)+2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))+4*I*d/b^2*c*arctan(exp(I*(b*x+a)))-4*I*d^2/b^3*a*arctan(exp(I*(b*x+a)))-2/b*c*d*ln(exp(I*(b*x+a))+1)*x-1/b*c^2*ln(exp(I*(b*x+a))+1)+1/b*c^2*ln(exp(I*(b*x+a))-1)-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I*(b*x+a)))/b^3+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+2/b^2*c*d*ln(1-exp(I*(b*x+a)))*a+1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)-2/b^2*c*d*a*ln(exp(I*(b*x+a))-1)
```

Maxima [B] time = 2.26547, size = 2157, normalized size = 9.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) - 2*a*c*d*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b + a^2*d^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b^2 + 2*((4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*(b*x + a)^2*d^2 + 4*(b*c*d -
```

$$\begin{aligned}
& a*d^2*(b*x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\cos(2*b*x + 2*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*\sin(2*b*x + 2*a) \\
& * \arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) - (4*I*(b*x + a)^2*d^2 + (8*I*b*c*d - 8*I*a*d^2)*(b*x + a))*\cos(b*x + a) + 4*(d^2*\cos(2*b*x + 2*a) + I*d^2*\sin(2*b*x + 2*a) + d^2)*\operatorname{dilog}(I*e^{(I*b*x + I*a)}) - 4*(d^2*\cos(2*b*x + 2*a) + I*d^2*\sin(2*b*x + 2*a) + d^2)*\operatorname{dilog}(-I*e^{(I*b*x + I*a)}) + (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2))*\cos(2*b*x + 2*a) \\
& - (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*\sin(2*b*x + 2*a) * \operatorname{dilog}(-e^{(I*b*x + I*a)}) - (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2))*\cos(2*b*x + 2*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*\sin(2*b*x + 2*a) * \operatorname{dilog}(e^{(I*b*x + I*a)}) - (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a))*\cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\sin(2*b*x + 2*a) * \log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a) + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a))*\cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\sin(2*b*x + 2*a) * \log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) - (-2*I*b*c*d - 2*I*(b*x + a)*d^2 + 2*I*a*d^2 + (-2*I*b*c*d - 2*I*(b*x + a)*d^2 + 2*I*a*d^2))*\cos(2*b*x + 2*a) + 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*\sin(2*b*x + 2*a) * \log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\sin(b*x + a) + 1) - (2*I*b*c*d + 2*I*(b*x + a)*d^2 - 2*I*a*d^2 + (2*I*b*c*d + 2*I*(b*x + a)*d^2 - 2*I*a*d^2))*\cos(2*b*x + 2*a) - 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*\sin(2*b*x + 2*a) * \log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\sin(b*x + a) + 1) - (-4*I*d^2*\cos(2*b*x + 2*a) + 4*d^2*\sin(2*b*x + 2*a) - 4*I*d^2)*\operatorname{polylog}(3, -e^{(I*b*x + I*a)}) - (4*I*d^2*\cos(2*b*x + 2*a) - 4*d^2*\sin(2*b*x + 2*a) + 4*I*d^2)*\operatorname{polylog}(3, e^{(I*b*x + I*a)}) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\sin(b*x + a)/(-2*I*b^2*\cos(2*b*x + 2*a) + 2*b^2*\sin(2*b*x + 2*a) - 2*I*b^2)/b
\end{aligned}$$

Fricas [C] time = 0.776298, size = 2773, normalized size = 12.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")

[Out] $1/2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 2*I*d^2*\cos(b*x + a)*\operatorname{dilog}(I*\cos(b*x + a) + \sin(b*x + a)) + 2*I*d^2*\cos(b*x + a)*\operatorname{dilog}(I*\cos(b*x + a) - \sin(b*x + a)) - 2*I*d^2*\cos(b*x + a)*\operatorname{dilog}(-I*\cos(b*x + a) + \sin(b*x + a)) - 2*I*d^2*\cos(b*x + a)*\operatorname{dilog}(-I*\cos(b*x + a) - \sin(b*x + a)) + 2*d^2*\cos(b*x + a)*\operatorname{polylog}(3, \cos(b*x + a) + I*\sin(b*x + a)) + 2*d^2*\cos(b*x + a)*\operatorname{polylog}(3, \cos(b*x + a) - I*\sin(b*x + a)) - 2*d^2*\cos(b*x + a)*\operatorname{polylog}(3, -\cos(b*x + a) + I*\sin(b*x + a)) - 2*d^2*\cos(b*x + a)*\operatorname{polylog}(3, -\cos(b*x + a) - I*\sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*\cos(b*x + a)*\operatorname{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*\cos(b*x + a)*\operatorname{dilog}(\cos(b*x + a) - I*\sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*\cos(b*x + a)*\operatorname{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*\cos(b*x + a)*\operatorname{dilog}(-\cos(b*x + a) - I*\sin(b*x + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(b*x + a)*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) - 2*(b*c*d - a*d^2)*\cos(b*x + a)*\log(\cos(b*x + a) + I*\sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(b*x + a)*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1) + 2*(b*c*d - a*d^2)*\cos(b*x + a)*\log(\cos(b*x + a) - I*\sin(b*x + a) + I) - 2*(b*d^2*x + a*d^2)*\cos(b*x + a)*\log(I*\cos(b*x + a) + \sin(b*x + a) + 1) + 2*(b*d^2*x + a*d^2)*\cos(b*x + a)*\log(I*\cos(b*x + a) - \sin(b*x + a) + 1) - 2*(b*d^2*x + a*d^2)*\cos(b*x + a)*\log(-I*\cos(b*x + a) + \sin(b*x + a) + 1) + 2*(b*d^2*x + a*d^2)*\cos(b*x + a)*\log(-I*\cos(b*x + a) - \sin(b*x + a) + 1) + (b^2*c^2 - 2*a*$

$$\frac{b^2 c^2 d + a^2 d^2 \cos(bx + a) \log(-1/2 \cos(bx + a) + 1/2 I \sin(bx + a) + 1/2) + (b^2 c^2 - 2 a b c d + a^2 d^2) \cos(bx + a) \log(-1/2 \cos(bx + a) - 1/2 I \sin(bx + a) + 1/2) + (b^2 d^2 x^2 + 2 b^2 c d x + 2 a b c d - a^2 d^2) \cos(bx + a) \log(-\cos(bx + a) + I \sin(bx + a) + 1) - 2 (b^2 c d - a d^2) \cos(bx + a) \log(-\cos(bx + a) + I \sin(bx + a) + I) + (b^2 d^2 x^2 + 2 b^2 c d x + 2 a b c d - a^2 d^2) \cos(bx + a) \log(-\cos(bx + a) - I \sin(bx + a) + 1) + 2 (b^2 c d - a d^2) \cos(bx + a) \log(-\cos(bx + a) - I \sin(bx + a) + I)}{b^3 \cos(bx + a)}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*csc(b*x+a)*sec(b*x+a)**2,x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")

[Out] Timed out

3.269 $\int (c + dx) \csc(a + bx) \sec^2(a + bx) dx$

Optimal. Leaf size=113

$$\frac{idPolyLog(2, -e^{i(a+bx)})}{b^2} - \frac{idPolyLog(2, e^{i(a+bx)})}{b^2} - \frac{d \tanh^{-1}(\sin(a + bx))}{b^2} + \frac{c \sec(a + bx)}{b} - \frac{c \tanh^{-1}(\cos(a + bx))}{b} + \frac{d \tanh^{-1}(\cos(a + bx))}{b}$$

[Out] $(-2*d*x*ArcTanh[E^{I*(a + b*x)}])/b - (c*ArcTanh[Cos[a + b*x]])/b - (d*ArcTanh[Sin[a + b*x]])/b^2 + (I*d*PolyLog[2, -E^{I*(a + b*x)}])/b^2 - (I*d*PolyLog[2, E^{I*(a + b*x)}])/b^2 + (c*Sec[a + b*x])/b + (d*x*Sec[a + b*x])/b$

Rubi [A] time = 0.13045, antiderivative size = 122, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {2622, 321, 207, 4420, 6271, 12, 4183, 2279, 2391, 3770}

$$\frac{idPolyLog(2, -e^{i(a+bx)})}{b^2} - \frac{idPolyLog(2, e^{i(a+bx)})}{b^2} - \frac{d \tanh^{-1}(\sin(a + bx))}{b^2} + \frac{(c + dx) \sec(a + bx)}{b} - \frac{(c + dx) \tanh^{-1}(\cos(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^2,x]

[Out] $(-2*d*x*ArcTanh[E^{I*(a + b*x)}])/b + (d*x*ArcTanh[Cos[a + b*x]])/b - ((c + d*x)*ArcTanh[Cos[a + b*x]])/b - (d*ArcTanh[Sin[a + b*x]])/b^2 + (I*d*PolyLog[2, -E^{I*(a + b*x)}])/b^2 - (I*d*PolyLog[2, E^{I*(a + b*x)}])/b^2 + ((c + d*x)*Sec[a + b*x])/b$

Rule 2622

Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

Rule 321

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 207

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 4420

Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_)^(m_.))*Sec[(a_.) + (b_.)*(x_)^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]

Rule 6271

Int[ArcTanh[u_], x_Symbol] := Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand [(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match Q[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 4183

Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \int (c + dx) \csc(a + bx) \sec^2(a + bx) dx &= -\frac{(c + dx) \tanh^{-1}(\cos(a + bx))}{b} + \frac{(c + dx) \sec(a + bx)}{b} - d \int \left(-\frac{\tanh^{-1}(\cos(a + bx))}{b} \right. \\
 &= -\frac{(c + dx) \tanh^{-1}(\cos(a + bx))}{b} + \frac{(c + dx) \sec(a + bx)}{b} + \frac{d \int \tanh^{-1}(\cos(a + bx)) dx}{b} \\
 &= \frac{dx \tanh^{-1}(\cos(a + bx))}{b} - \frac{(c + dx) \tanh^{-1}(\cos(a + bx))}{b} - \frac{d \tanh^{-1}(\sin(a + bx))}{b^2} \\
 &= \frac{dx \tanh^{-1}(\cos(a + bx))}{b} - \frac{(c + dx) \tanh^{-1}(\cos(a + bx))}{b} - \frac{d \tanh^{-1}(\sin(a + bx))}{b^2} \\
 &= -\frac{2dx \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{dx \tanh^{-1}(\cos(a + bx))}{b} - \frac{(c + dx) \tanh^{-1}(\cos(a + bx))}{b} \\
 &= -\frac{2dx \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{dx \tanh^{-1}(\cos(a + bx))}{b} - \frac{(c + dx) \tanh^{-1}(\cos(a + bx))}{b} \\
 &= -\frac{2dx \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{dx \tanh^{-1}(\cos(a + bx))}{b} - \frac{(c + dx) \tanh^{-1}(\cos(a + bx))}{b}
 \end{aligned}$$

Mathematica [A] time = 0.504331, size = 212, normalized size = 1.88

$$\frac{d \left(i \left(\text{PolyLog} \left(2, -e^{i(a+bx)} \right) - \text{PolyLog} \left(2, e^{i(a+bx)} \right) \right) + (a + bx) \left(\log \left(1 - e^{i(a+bx)} \right) - \log \left(1 + e^{i(a+bx)} \right) \right) \right)}{b^2} - \frac{ad \log \left(\tan \left(\frac{1}{2} \arctan \left(\frac{\sin(a + bx)}{\cos(a + bx)} \right) \right) \right)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^2,x]

[Out] -((c*Log[Cos[(a + b*x)/2]])/b) + (d*Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]])/b^2 + (c*Log[Sin[(a + b*x)/2]])/b - (d*Log[Cos[(a + b*x)/2] + Sin[(a + b*x)/2]])/b^2 - (a*d*Log[Tan[(a + b*x)/2]])/b^2 + (d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a + b*x))])))/b^2 + (c*Sec[a + b*x])/b + (d*x*Sec[a + b*x])/b

Maple [A] time = 0.29, size = 160, normalized size = 1.4

$$\frac{2 e^{i(bx+a)} (dx + c)}{b (e^{2i(bx+a)} + 1)} - \frac{c \ln(e^{i(bx+a)} + 1)}{b} + \frac{c \ln(e^{i(bx+a)} - 1)}{b} + \frac{2 id \arctan(e^{i(bx+a)})}{b^2} + \frac{id \operatorname{dilog}(e^{i(bx+a)} + 1)}{b^2} - \frac{d \ln(e^{i(bx+a)})}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x)

[Out] 2*exp(I*(b*x+a))*(d*x+c)/b/(exp(2*I*(b*x+a))+1)-1/b*c*ln(exp(I*(b*x+a))+1)+1/b*c*ln(exp(I*(b*x+a))-1)+2*I/b^2*d*arctan(exp(I*(b*x+a)))+I/b^2*d*dilog(exp(I*(b*x+a))+1)-1/b*d*ln(exp(I*(b*x+a))+1)*x+I/b^2*d*dilog(exp(I*(b*x+a)))-1/b^2*d*a*ln(exp(I*(b*x+a))-1)

Maxima [B] time = 2.04559, size = 1085, normalized size = 9.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")

[Out] -(2*(d*cos(2*b*x + 2*a) + I*d*sin(2*b*x + 2*a) + d)*arctan2(2*(cos(b*x + 2*a)*cos(a) + sin(b*x + 2*a)*sin(a))/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2), (cos(b*x + 2*a)^2 - cos(a)^2 + sin(b*x + 2*a)^2 - sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) + (2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (-2*I*b*d*x - 2*I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(b*c*cos(2*b*x + 2*a) + I*b*c*sin(2*b*x + 2*a) + b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*b*d*x*cos(2*b*x + 2*a) + 2*I*b*d*x*sin(2*b*x + 2*a) + 2*b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (-4*I*b*d*x - 4*I*b*c)*cos(b*x + a) - 2*(d*cos(2*b*x + 2*a) + I*d*sin(2*b*x + 2*a) + d)*dilog(-e^(I*b*x + I*a)) + 2*(d*cos(2*b*x + 2*a) + I*d*sin(2*b*x + 2*a) + d)*dilog(e^(I*b*x + I*a)) - (I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(2*b*x + 2*a) - (b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-I*d*cos(2*b*x + 2*a) + d*sin(2*b*x + 2*a) - I*d)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(b*d*x + b*c)*sin(b*x + a)/(-2*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(2*b*x + 2*a) - 2*I*b^2)

Fricas [B] time = 0.581003, size = 1046, normalized size = 9.26

$2 b d x - i d \cos (b x + a) \operatorname{Li}_2(\cos (b x + a) + i \sin (b x + a)) + i d \cos (b x + a) \operatorname{Li}_2(\cos (b x + a) - i \sin (b x + a)) - i d \cos (b x + a)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")

[Out] $\frac{1}{2}*(2*b*d*x - I*d*\cos(b*x + a)*\operatorname{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) + I*d*\cos(b*x + a)*\operatorname{dilog}(\cos(b*x + a) - I*\sin(b*x + a)) - I*d*\cos(b*x + a)*\operatorname{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) + I*d*\cos(b*x + a)*\operatorname{dilog}(-\cos(b*x + a) - I*\sin(b*x + a)) - (b*d*x + b*c)*\cos(b*x + a)*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) - (b*d*x + b*c)*\cos(b*x + a)*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1) + (b*c - a*d)*\cos(b*x + a)*\log(-1/2*\cos(b*x + a) + 1/2*I*\sin(b*x + a) + 1/2) + (b*c - a*d)*\cos(b*x + a)*\log(-1/2*\cos(b*x + a) - 1/2*I*\sin(b*x + a) + 1/2) + (b*d*x + a*d)*\cos(b*x + a)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1) + (b*d*x + a*d)*\cos(b*x + a)*\log(-\cos(b*x + a) - I*\sin(b*x + a) + 1) - d*\cos(b*x + a)*\log(\sin(b*x + a) + 1) + d*\cos(b*x + a)*\log(-\sin(b*x + a) + 1) + 2*b*c)/(b^2*\cos(b*x + a))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \csc(a + bx) \sec^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)**2,x)

[Out] Integral((c + d*x)*csc(a + b*x)*sec(a + b*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \csc(bx + a) \sec(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)*csc(b*x + a)*sec(b*x + a)^2, x)

$$3.270 \quad \int \frac{\csc(a+bx) \sec^2(a+bx)}{c+dx} dx$$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\csc(a+bx) \sec^2(a+bx)}{c+dx}, x\right)$$

[Out] CannotIntegrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x), x]

Rubi [A] time = 0.153201, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc(a+bx) \sec^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x), x]

[Out] Defer[Int] [(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x), x]

Rubi steps

$$\int \frac{\csc(a+bx) \sec^2(a+bx)}{c+dx} dx = \int \frac{\csc(a+bx) \sec^2(a+bx)}{c+dx} dx$$

Mathematica [A] time = 9.84684, size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx) \sec^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x), x]

[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x), x]

Maple [A] time = 2.332, size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a) (\sec(bx+a))^2}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c), x)

[Out] int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c), x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc (bx+a) \sec (bx+a)^2}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")

[Out] integral(csc(b*x + a)*sec(b*x + a)^2/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (a+bx) \sec ^2(a+bx)}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sec(b*x+a)**2/(d*x+c),x)

[Out] Integral(csc(a + b*x)*sec(a + b*x)**2/(c + d*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (bx+a) \sec (bx+a)^2}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x, algorithm="giac")

[Out] integrate(csc(b*x + a)*sec(b*x + a)^2/(d*x + c), x)

$$3.271 \quad \int \frac{\csc(a+bx) \sec^2(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\csc(a+bx) \sec^2(a+bx)}{(c+dx)^2}, x\right)$$

[Out] CannotIntegrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2, x]

Rubi [A] time = 0.180374, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc(a+bx) \sec^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2, x]

[Out] Defer[Int] [(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\csc(a+bx) \sec^2(a+bx)}{(c+dx)^2} dx = \int \frac{\csc(a+bx) \sec^2(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 10.0946, size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx) \sec^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2, x]

[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2, x]

Maple [A] time = 2.663, size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a) (\sec(bx+a))^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2, x)

[Out] int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2, x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)\sec(bx+a)^2}{d^2x^2+2cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(csc(b*x + a)*sec(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sec(b*x+a)**2/(d*x+c)**2,x)

[Out] Integral(csc(a + b*x)*sec(a + b*x)**2/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

sage₀x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")

[Out] sage0*x

3.272 $\int (c + dx)^m \csc^2(a + bx) \sec^2(a + bx) dx$

Optimal. Leaf size=26

$$\text{CannotIntegrate}\left(\csc^2(a + bx) \sec^2(a + bx)(c + dx)^m, x\right)$$

[Out] CannotIntegrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2, x]

Rubi [A] time = 0.189649, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \csc^2(a + bx) \sec^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2, x]

[Out] Defer[Int] [(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2, x]

Rubi steps

$$\int (c + dx)^m \csc^2(a + bx) \sec^2(a + bx) dx = \int (c + dx)^m \csc^2(a + bx) \sec^2(a + bx) dx$$

Mathematica [A] time = 2.78455, size = 0, normalized size = 0.

$$\int (c + dx)^m \csc^2(a + bx) \sec^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2, x]

[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2, x]

Maple [A] time = 0.173, size = 0, normalized size = 0.

$$\int (dx + c)^m (\csc(bx + a))^2 (\sec(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2, x)

[Out] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \csc(bx + a)^2 \sec(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx + c)^m \csc(bx + a)^2 \sec(bx + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**2*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \csc(bx + a)^2 \sec(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

3.273 $\int (c + dx)^3 \csc^2(a + bx) \sec^2(a + bx) dx$

Optimal. Leaf size=118

$$-\frac{3id^2(c + dx)\text{PolyLog}\left(2, e^{4i(a+bx)}\right)}{2b^3} + \frac{3d^3\text{PolyLog}\left(3, e^{4i(a+bx)}\right)}{8b^4} + \frac{3d(c + dx)^2 \log\left(1 - e^{4i(a+bx)}\right)}{b^2} - \frac{2(c + dx)^3 \cot(2a + 2bx)}{b}$$

```
[Out] ((-2*I)*(c + d*x)^3)/b - (2*(c + d*x)^3*Cot[2*a + 2*b*x])/b + (3*d*(c + d*x)^2*Log[1 - E^((4*I)*(a + b*x))])/b^2 - (((3*I)/2)*d^2*(c + d*x)*PolyLog[2, E^((4*I)*(a + b*x))])/b^3 + (3*d^3*PolyLog[3, E^((4*I)*(a + b*x))])/(8*b^4)
```

Rubi [A] time = 0.280106, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4419, 4184, 3717, 2190, 2531, 2282, 6589}

$$-\frac{3id^2(c + dx)\text{PolyLog}\left(2, e^{4i(a+bx)}\right)}{2b^3} + \frac{3d^3\text{PolyLog}\left(3, e^{4i(a+bx)}\right)}{8b^4} + \frac{3d(c + dx)^2 \log\left(1 - e^{4i(a+bx)}\right)}{b^2} - \frac{2(c + dx)^3 \cot(2a + 2bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] ((-2*I)*(c + d*x)^3)/b - (2*(c + d*x)^3*Cot[2*a + 2*b*x])/b + (3*d*(c + d*x)^2*Log[1 - E^((4*I)*(a + b*x))])/b^2 - (((3*I)/2)*d^2*(c + d*x)*PolyLog[2, E^((4*I)*(a + b*x))])/b^3 + (3*d^3*PolyLog[3, E^((4*I)*(a + b*x))])/(8*b^4)
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*(a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*(a_.) + (b_.)*(x_)]^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^3 \csc^2(a + bx) \sec^2(a + bx) dx &= 4 \int (c + dx)^3 \csc^2(2a + 2bx) dx \\ &= -\frac{2(c + dx)^3 \cot(2a + 2bx)}{b} + \frac{(6d) \int (c + dx)^2 \cot(2a + 2bx) dx}{b} \\ &= -\frac{2i(c + dx)^3}{b} - \frac{2(c + dx)^3 \cot(2a + 2bx)}{b} - \frac{(12id) \int \frac{e^{2i(2a+2bx)}(c+dx)^2}{1-e^{2i(2a+2bx)}} dx}{b} \\ &= -\frac{2i(c + dx)^3}{b} - \frac{2(c + dx)^3 \cot(2a + 2bx)}{b} + \frac{3d(c + dx)^2 \log(1 - e^{4i(a+bx)})}{b^2} \\ &= -\frac{2i(c + dx)^3}{b} - \frac{2(c + dx)^3 \cot(2a + 2bx)}{b} + \frac{3d(c + dx)^2 \log(1 - e^{4i(a+bx)})}{b^2} \\ &= -\frac{2i(c + dx)^3}{b} - \frac{2(c + dx)^3 \cot(2a + 2bx)}{b} + \frac{3d(c + dx)^2 \log(1 - e^{4i(a+bx)})}{b^2} \\ &= -\frac{2i(c + dx)^3}{b} - \frac{2(c + dx)^3 \cot(2a + 2bx)}{b} + \frac{3d(c + dx)^2 \log(1 - e^{4i(a+bx)})}{b^2} \end{aligned}$$

Mathematica [B] time = 2.23132, size = 285, normalized size = 2.42

$$\frac{12ibd^2(c + dx)\text{PolyLog}\left(2, -e^{-i(a+bx)}\right) + 12ibd^2(c + dx)\text{PolyLog}\left(2, e^{-i(a+bx)}\right) + 6ibd^2(c + dx)\text{PolyLog}\left(2, -e^{-2i(a+bx)}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] (((-8*I)*b^3*(c + d*x)^3)/(-1 + E^((4*I)*a)) + 6*b^2*d*(c + d*x)^2*Log[1 -
E^((-I)*(a + b*x))] + 6*b^2*d*(c + d*x)^2*Log[1 + E^((-I)*(a + b*x))] + 6*b
^2*d*(c + d*x)^2*Log[1 + E^((-2*I)*(a + b*x))] + (12*I)*b*d^2*(c + d*x)*Pol
yLog[2, -E^((-I)*(a + b*x))] + (12*I)*b*d^2*(c + d*x)*PolyLog[2, E^((-I)*(a
+ b*x))] + (6*I)*b*d^2*(c + d*x)*PolyLog[2, -E^((-2*I)*(a + b*x))] + 12*d^
3*PolyLog[3, -E^((-I)*(a + b*x))] + 12*d^3*PolyLog[3, E^((-I)*(a + b*x))] +
3*d^3*PolyLog[3, -E^((-2*I)*(a + b*x))] + 4*b^3*(c + d*x)^3*Csc[2*a]*Csc[2
```

$*(a + b*x)]*Sin[2*b*x]]/(2*b^4)$

Maple [B] time = 0.336, size = 687, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x)`

[Out] $6*d^3*\text{polylog}(3,-\exp(I*(b*x+a)))/b^4+6*d^3*\text{polylog}(3,\exp(I*(b*x+a)))/b^4-3*d^3/b^4*\ln(1-\exp(I*(b*x+a)))*a^2+3*d^3/b^2*\ln(\exp(I*(b*x+a))+1)*x^2+3*d^3/b^2*\ln(1-\exp(I*(b*x+a)))*x^2-12*d^3/b^4*a^2*\ln(\exp(I*(b*x+a)))-12*d/b^2*c^2*\ln(\exp(I*(b*x+a)))+3*d/b^2*c^2*\ln(\exp(2*I*(b*x+a))+1)+3*d^3/b^2*\ln(\exp(2*I*(b*x+a))+1)*x^2+6*d^2/b^2*c*\ln(\exp(2*I*(b*x+a))+1)*x-6*I*d^2/b^3*c*\text{polylog}(2,-\exp(I*(b*x+a)))-12*I*d^2/b*c*x^2+12*I*d^3/b^3*a^2*x-12*I*d^2/b^3*c*a^2-3*I*d^2/b^3*c*\text{polylog}(2,-\exp(2*I*(b*x+a)))-3*I*d^3/b^3*\text{polylog}(2,-\exp(2*I*(b*x+a)))*x-6/b^3*c*d^2*a*\ln(\exp(I*(b*x+a))-1)+24*d^2/b^3*c*a*\ln(\exp(I*(b*x+a)))+3/2*d^3*\text{polylog}(3,-\exp(2*I*(b*x+a)))/b^4+8*I*d^3/b^4*a^3-4*I*d^3/b*x^3-24*I*d^2/b^2*c*a*x-6*I*d^3/b^3*\text{polylog}(2,-\exp(I*(b*x+a)))*x+6*d^2/b^2*c*\ln(\exp(I*(b*x+a))+1)*x+6*d^2/b^2*c*\ln(1-\exp(I*(b*x+a)))*x+6*d^2/b^3*c*\ln(1-\exp(I*(b*x+a)))*a-6*I*d^3/b^3*\text{polylog}(2,\exp(I*(b*x+a)))*x+3/b^2*c^2*d*\ln(\exp(I*(b*x+a))+1)+3/b^2*c^2*d*\ln(\exp(I*(b*x+a))-1)+3/b^4*d^3*a^2*\ln(\exp(I*(b*x+a)))-1)-4*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(\exp(2*I*(b*x+a))+1)/(\exp(2*I*(b*x+a))-1)-6*I*d^2/b^3*c*\text{polylog}(2,\exp(I*(b*x+a)))$

Maxima [B] time = 2.35505, size = 3179, normalized size = 26.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")`

[Out] $-1/2*(2*c^3*(1/\tan(b*x + a) - \tan(b*x + a)) - 6*a*c^2*d*(1/\tan(b*x + a) - \tan(b*x + a))/b + 6*a^2*c*d^2*(1/\tan(b*x + a) - \tan(b*x + a))/b^2 - 2*a^3*d^3*(1/\tan(b*x + a) - \tan(b*x + a))/b^3 - 3*((\cos(4*b*x + 4*a))^2 + \sin(4*b*x + 4*a))^2 - 2*\cos(4*b*x + 4*a) + 1)*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) + (\cos(4*b*x + 4*a))^2 + \sin(4*b*x + 4*a))^2 - 2*\cos(4*b*x + 4*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) + (\cos(4*b*x + 4*a))^2 + \sin(4*b*x + 4*a))^2 - 2*\cos(4*b*x + 4*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) - 8*(b*x + a)*\sin(4*b*x + 4*a))*c^2*d/((\cos(4*b*x + 4*a))^2 + \sin(4*b*x + 4*a))^2 - 2*\cos(4*b*x + 4*a) + 1)*b) + 6*((\cos(4*b*x + 4*a))^2 + \sin(4*b*x + 4*a))^2 - 2*\cos(4*b*x + 4*a) + 1)*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) + (\cos(4*b*x + 4*a))^2 + \sin(4*b*x + 4*a))^2 - 2*\cos(4*b*x + 4*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) + (\cos(4*b*x + 4*a))^2 + \sin(4*b*x + 4*a))^2 - 2*\cos(4*b*x + 4*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) - 8*(b*x + a)*\sin(4*b*x + 4*a))*a*c*d^2/((\cos(4*b*x + 4*a))^2 + \sin(4*b*x + 4*a))^2 - 2*\cos(4*b*x + 4*a) + 1)*b^2) - 3*((\cos(4*b*x + 4*a))^2 + \sin(4*b*x + 4*a))^2 - 2*\cos(4*b*x + 4*a) + 1)*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) + (\cos(4*b*x + 4*a))^2 + \sin(4*b*x + 4*a))^2 - 2*\cos(4*b*x + 4*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) + (\cos(4*b*x + 4*a))^2 + \sin(4$

$$\begin{aligned}
& *b*x + 4*a)^2 - 2*\cos(4*b*x + 4*a) + 1)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 \\
& - 2*\cos(b*x + a) + 1) - 8*(b*x + a)*\sin(4*b*x + 4*a))*a^2*d^3/((\cos(4*b*x \\
& + 4*a)^2 + \sin(4*b*x + 4*a)^2 - 2*\cos(4*b*x + 4*a) + 1)*b^3) + 2*((6*(b*x + \\
& a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 \\
& - a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) - (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 \\
& - 12*I*a*d^3)*(b*x + a))*\sin(4*b*x + 4*a))*\arctan2(\sin(2*b*x + 2*a), \cos \\
& (2*b*x + 2*a) + 1) + (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - \\
& 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) - (6*I \\
& *(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*\sin(4*b*x + 4*a)) \\
& *\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) - (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 \\
& - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\c \\
& \cos(4*b*x + 4*a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x \\
& + a))*\sin(4*b*x + 4*a))*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) + 8*((b*x \\
& + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2)*\cos(4*b*x + 4*a) - (6*b*c*d^2 \\
& + 6*(b*x + a)*d^3 - 6*a*d^3 - 6*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\cos(4*b \\
& *x + 4*a) + (-6*I*b*c*d^2 - 6*I*(b*x + a)*d^3 + 6*I*a*d^3)*\sin(4*b*x + 4*a) \\
&)*\operatorname{dilog}(-e^{(2*I*b*x + 2*I*a)}) - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 - \\
& 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\cos(4*b*x + 4*a) + (-12*I*b*c*d^2 - 1 \\
& 2*I*(b*x + a)*d^3 + 12*I*a*d^3)*\sin(4*b*x + 4*a))*\operatorname{dilog}(-e^{(I*b*x + I*a)}) - \\
& (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 - 12*(b*c*d^2 + (b*x + a)*d^3 - \\
& a*d^3)*\cos(4*b*x + 4*a) + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3) \\
& *\sin(4*b*x + 4*a))*\operatorname{dilog}(e^{(I*b*x + I*a)}) - (3*I*(b*x + a)^2*d^3 + (6*I*b*c \\
& *d^2 - 6*I*a*d^3)*(b*x + a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a \\
& *d^3)*(b*x + a))*\cos(4*b*x + 4*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3 \\
&)*(b*x + a))*\sin(4*b*x + 4*a))*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 \\
& + 2*\cos(2*b*x + 2*a) + 1) - (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3 \\
&)*(b*x + a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)) \\
& *\cos(4*b*x + 4*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\sin \\
& (4*b*x + 4*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - \\
& (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a) + (-3*I*(b*x + a \\
&)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + 3*((b*x \\
& + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\sin(4*b*x + 4*a))*\log(\cos(b*x + \\
& a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) - (-3*I*d^3*\cos(4*b*x + 4*a) + \\
& 3*d^3*\sin(4*b*x + 4*a) + 3*I*d^3)*\operatorname{polylog}(3, -e^{(2*I*b*x + 2*I*a)}) - (-12* \\
& I*d^3*\cos(4*b*x + 4*a) + 12*d^3*\sin(4*b*x + 4*a) + 12*I*d^3)*\operatorname{polylog}(3, -e^{ \\
& (I*b*x + I*a)}) - (-12*I*d^3*\cos(4*b*x + 4*a) + 12*d^3*\sin(4*b*x + 4*a) + 12 \\
& *I*d^3)*\operatorname{polylog}(3, e^{(I*b*x + I*a)}) - (-8*I*(b*x + a)^3*d^3 + (-24*I*b*c*d^2 \\
& + 24*I*a*d^3)*(b*x + a)^2)*\sin(4*b*x + 4*a))/(-2*I*b^3*\cos(4*b*x + 4*a) + \\
& 2*b^3*\sin(4*b*x + 4*a) + 2*I*b^3))/b
\end{aligned}$$

Fricas [C] time = 0.898168, size = 4263, normalized size = 36.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")

[Out] $1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*\cos(b*x + a)*\operatorname{polylog}(3, \cos(b*x + a) + I*\sin(b*x + a))*\sin(b*x + a) + 6*d^3*\cos(b*x + a)*\operatorname{polylog}(3, \cos(b*x + a) - I*\sin(b*x + a))*\sin(b*x + a) + 6*d^3*\cos(b*x + a)*\operatorname{polylog}(3, I*\cos(b*x + a) + \sin(b*x + a))*\sin(b*x + a) + 6*d^3*\cos(b*x + a)*\operatorname{polylog}(3, I*\cos(b*x + a) - \sin(b*x + a))*\sin(b*x + a) + 6*d^3*\cos(b*x + a)*\operatorname{polylog}(3, -I*\cos(b*x + a) + \sin(b*x + a))*\sin(b*x + a) + 6*d^3*\cos(b*x + a)*\operatorname{polylog}(3, -I*\cos(b*x + a) - \sin(b*x + a))*\sin(b*x + a) + 6*d^3*\cos(b*x + a)*\operatorname{polylog}(3, -\cos(b*x + a) + I*\sin(b*x + a))*\sin(b*x + a) + 6*d^3*\cos(b*x + a)*\operatorname{polylog}(3, -\cos(b*x + a) - I*\sin(b*x + a))*\sin(b*x + a)$

```

) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*cos(b*x + a)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b*x + a)*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b*x + a)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*cos(b*x + a)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)^2)/(b^4*cos(b*x + a)*sin(b*x + a))

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*csc(b*x+a)**2*sec(b*x+a)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \csc(bx + a)^2 \sec(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^3*csc(b*x + a)^2*sec(b*x + a)^2, x)

3.274 $\int (c + dx)^2 \csc^2(a + bx) \sec^2(a + bx) dx$

Optimal. Leaf size=88

$$-\frac{id^2 \text{PolyLog}\left(2, e^{4i(a+bx)}\right)}{2b^3} + \frac{2d(c+dx) \log\left(1 - e^{4i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^2 \cot(2a+2bx)}{b} - \frac{2i(c+dx)^2}{b}$$

[Out] $((-2*I)*(c + d*x)^2)/b - (2*(c + d*x)^2*\text{Cot}[2*a + 2*b*x])/b + (2*d*(c + d*x)*\text{Log}[1 - E^{((4*I)*(a + b*x))}])/b^2 - ((I/2)*d^2*\text{PolyLog}[2, E^{((4*I)*(a + b*x))}])/b^3$

Rubi [A] time = 0.191444, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {4419, 4184, 3717, 2190, 2279, 2391}

$$-\frac{id^2 \text{PolyLog}\left(2, e^{4i(a+bx)}\right)}{2b^3} + \frac{2d(c+dx) \log\left(1 - e^{4i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^2 \cot(2a+2bx)}{b} - \frac{2i(c+dx)^2}{b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^2*\text{Csc}[a + b*x]^2*\text{Sec}[a + b*x]^2, x]$

[Out] $((-2*I)*(c + d*x)^2)/b - (2*(c + d*x)^2*\text{Cot}[2*a + 2*b*x])/b + (2*d*(c + d*x)*\text{Log}[1 - E^{((4*I)*(a + b*x))}])/b^2 - ((I/2)*d^2*\text{PolyLog}[2, E^{((4*I)*(a + b*x))}])/b^3$

Rule 4419

$\text{Int}[\text{Csc}[(a_.) + (b_.)*(x_.)]^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sec}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \text{ :> } \text{Dist}[2^n, \text{Int}[(c + d*x)^m*\text{Csc}[2*a + 2*b*x]^n, x], x] \text{ /; } \text{FreeQ}\{a, b, c, d, m\}, x\} \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{RationalQ}[m]$

Rule 4184

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]^2*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \text{ :> } -\text{Simp}[(c + d*x)^m*\text{Cot}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cot}[e + f*x], x], x] \text{ /; } \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{GtQ}[m, 0]$

Rule 3717

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\tan[(e_.) + \text{Pi}*(k_.) + (f_.)*(x_.)], x_Symbol] \text{ :> } \text{Simp}[(I*(c + d*x)^{(m+1)})/(d*(m+1)), x] - \text{Dist}[2*I, \text{Int}[(c + d*x)^m*\text{E}^{(2*I*k*Pi)}*\text{E}^{(2*I*(e + f*x))}/(1 + \text{E}^{(2*I*k*Pi)}*\text{E}^{(2*I*(e + f*x))}), x], x] \text{ /; } \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{IntegerQ}[4*k] \ \&\& \ \text{IGtQ}[m, 0]$

Rule 2190

$\text{Int}[(F_.)^{((g_.)*((e_.) + (f_.)*(x_.)))^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}}/((a_.) + (b_.)*(F_.)^{((g_.)*((e_.) + (f_.)*(x_.)))^{(n_.)}), x_Symbol] \text{ :> } \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F_.)^{(g*(e + f*x))})^n/a]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + (b*(F_.)^{(g*(e + f*x))})^n/a], x], x] \text{ /; } \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \ \&\& \ \text{IGtQ}[m, 0]$

Rule 2279

$\text{Int}[\text{Log}[(a_.) + (b_.)*((F_.)^{((e_.)*((c_.) + (d_.)*(x_.)))^{(n_.)}})], x_Symbol] \text{ :> } \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F_.)^{(e*(c + d*x))}]$

)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*(d_) + (e_.)*(x_)^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \csc^2(a + bx) \sec^2(a + bx) dx &= 4 \int (c + dx)^2 \csc^2(2a + 2bx) dx \\ &= -\frac{2(c + dx)^2 \cot(2a + 2bx)}{b} + \frac{(4d) \int (c + dx) \cot(2a + 2bx) dx}{b} \\ &= -\frac{2i(c + dx)^2}{b} - \frac{2(c + dx)^2 \cot(2a + 2bx)}{b} - \frac{(8id) \int \frac{e^{2i(2a+2bx)(c+dx)}}{1-e^{2i(2a+2bx)}} dx}{b} \\ &= -\frac{2i(c + dx)^2}{b} - \frac{2(c + dx)^2 \cot(2a + 2bx)}{b} + \frac{2d(c + dx) \log(1 - e^{4i(a+bx)})}{b^2} - \frac{(2a + dx) \log(1 - e^{4i(a+bx)})}{b^2} \\ &= -\frac{2i(c + dx)^2}{b} - \frac{2(c + dx)^2 \cot(2a + 2bx)}{b} + \frac{2d(c + dx) \log(1 - e^{4i(a+bx)})}{b^2} + \frac{(id)^2 \log(1 - e^{4i(a+bx)})}{b^2} \\ &= -\frac{2i(c + dx)^2}{b} - \frac{2(c + dx)^2 \cot(2a + 2bx)}{b} + \frac{2d(c + dx) \log(1 - e^{4i(a+bx)})}{b^2} - \frac{id^2 \log(1 - e^{4i(a+bx)})}{b^2} \end{aligned}$$

Mathematica [B] time = 1.75939, size = 277, normalized size = 3.15

$$\frac{2b^2 \csc(2a) \sin(2bx)(c + dx)^2 \csc(2(a + bx)) - \frac{ie^{4ia}(-2(1-e^{-4ia})d^2 \text{PolyLog}(2, -e^{-i(a+bx)}) - 2(1-e^{-4ia})d^2 \text{PolyLog}(2, e^{-i(a+bx)}) - (1-e^{-4ia})d^2 \text{PolyLog}(2, -e^{-i(a+bx)}) - 2(1-e^{-4ia})d^2 \text{PolyLog}(2, e^{-i(a+bx)})}{b^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^2,x]

[Out] (((-I)*E^((4*I)*a))*((4*b^2*(c + d*x)^2)/E^((4*I)*a) + (2*I)*b*d*(1 - E^((-4*I)*a)))*(c + d*x)*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b*d*(1 - E^((-4*I)*a))*(c + d*x)*Log[1 + E^((-I)*(a + b*x))] + (2*I)*b*d*(1 - E^((-4*I)*a))*(c + d*x)*Log[1 + E^((-2*I)*(a + b*x))] - 2*d^2*(1 - E^((-4*I)*a))*PolyLog[2, -E^((-I)*(a + b*x))] - 2*d^2*(1 - E^((-4*I)*a))*PolyLog[2, E^((-I)*(a + b*x))] - d^2*(1 - E^((-4*I)*a))*PolyLog[2, -E^((-2*I)*(a + b*x))])/(-1 + E^((4*I)*a)) + 2*b^2*(c + d*x)^2*Csc[2*a]*Csc[2*(a + b*x)]*Sin[2*b*x])/b^3

Maple [B] time = 0.332, size = 351, normalized size = 4.

$$\frac{-4id^2x^2}{b} + 2 \frac{cd \ln(e^{i(bx+a)} - 1)}{b^2} + 2 \frac{cd \ln(e^{i(bx+a)} + 1)}{b^2} + 2 \frac{cd \ln(e^{2i(bx+a)} + 1)}{b^2} - 8 \frac{cd \ln(e^{i(bx+a)})}{b^2} - \frac{2id^2 \text{polylog}(2, -e^{i(bx+a)})}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x)

[Out] -4*I*d^2/b*x^2+2*d/b^2*c*ln(exp(I*(b*x+a))-1)+2*d/b^2*c*ln(exp(I*(b*x+a))+1)+2*d/b^2*c*ln(exp(2*I*(b*x+a))+1)-8*d/b^2*c*ln(exp(I*(b*x+a)))-2*I*d^2/b^3*polylog(2,-exp(I*(b*x+a)))+2*d^2/b^2*ln(exp(2*I*(b*x+a))+1)*x-I*d^2*polylo

$$g(2, -\exp(2I*(b*x+a)))/b^3+2*d^2/b^2*\ln(1-\exp(I*(b*x+a)))*x+2*d^2/b^3*\ln(1-\exp(I*(b*x+a)))*a-8*I*d^2/b^2*a*x-4*I*d^2/b^3*a^2-4*I*(d^2*x^2+2*c*d*x+c^2)/b/(\exp(2*I*(b*x+a))+1)/(\exp(2*I*(b*x+a))-1)-2*I*d^2*polylog(2, \exp(I*(b*x+a)))/b^3+2*d^2/b^2*\ln(\exp(I*(b*x+a))+1)*x-2*d^2/b^3*a*\ln(\exp(I*(b*x+a))-1)+8*d^2/b^3*a*\ln(\exp(I*(b*x+a)))$$

Maxima [B] time = 2.12905, size = 1049, normalized size = 11.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")

[Out] $-(4*b^2*c^2 + (2*b*d^2*x + 2*b*c*d - 2*(b*d^2*x + b*c*d)*\cos(4*b*x + 4*a) - (2*I*b*d^2*x + 2*I*b*c*d)*\sin(4*b*x + 4*a))*\arctan2(\sin(2*b*x + 2*a), \cos(2*b*x + 2*a) + 1) + (2*b*d^2*x + 2*b*c*d - 2*(b*d^2*x + b*c*d)*\cos(4*b*x + 4*a) - (2*I*b*d^2*x + 2*I*b*c*d)*\sin(4*b*x + 4*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) - (2*b*c*d*\cos(4*b*x + 4*a) + 2*I*b*c*d*\sin(4*b*x + 4*a) - 2*b*c*d)*\arctan2(\sin(b*x + a), \cos(b*x + a) - 1) + (2*b*d^2*x*\cos(4*b*x + 4*a) + 2*I*b*d^2*x*\sin(4*b*x + 4*a) - 2*b*d^2*x)*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x)*\cos(4*b*x + 4*a) + (d^2*\cos(4*b*x + 4*a) + I*d^2*\sin(4*b*x + 4*a) - d^2)*\operatorname{dilog}(-e^{(2*I*b*x + 2*I*a)}) + 2*(d^2*\cos(4*b*x + 4*a) + I*d^2*\sin(4*b*x + 4*a) - d^2)*\operatorname{dilog}(-e^{(I*b*x + I*a)}) + 2*(d^2*\cos(4*b*x + 4*a) + I*d^2*\sin(4*b*x + 4*a) - d^2)*\operatorname{dilog}(e^{(I*b*x + I*a)}) - (I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*\cos(4*b*x + 4*a) + (b*d^2*x + b*c*d)*\sin(4*b*x + 4*a))*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) - (I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*\cos(4*b*x + 4*a) + (b*d^2*x + b*c*d)*\sin(4*b*x + 4*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - (I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*\cos(4*b*x + 4*a) + (b*d^2*x + b*c*d)*\sin(4*b*x + 4*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) - (-4*I*b^2*d^2*x^2 - 8*I*b^2*c*d*x)*\sin(4*b*x + 4*a))/(-I*b^3*\cos(4*b*x + 4*a) + b^3*\sin(4*b*x + 4*a) + I*b^3)$

Fricas [B] time = 0.746377, size = 2573, normalized size = 29.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")

[Out] $(b^2*d^2*x^2 + 2*b^2*c*d*x - I*d^2*\cos(b*x + a)*\operatorname{dilog}(\cos(b*x + a) + I*\sin(b*x + a))*\sin(b*x + a) + I*d^2*\cos(b*x + a)*\operatorname{dilog}(\cos(b*x + a) - I*\sin(b*x + a))*\sin(b*x + a) + I*d^2*\cos(b*x + a)*\operatorname{dilog}(I*\cos(b*x + a) + \sin(b*x + a))*\sin(b*x + a) - I*d^2*\cos(b*x + a)*\operatorname{dilog}(I*\cos(b*x + a) - \sin(b*x + a))*\sin(b*x + a) - I*d^2*\cos(b*x + a)*\operatorname{dilog}(-I*\cos(b*x + a) + \sin(b*x + a))*\sin(b*x + a) + I*d^2*\cos(b*x + a)*\operatorname{dilog}(-I*\cos(b*x + a) - \sin(b*x + a))*\sin(b*x + a) + I*d^2*\cos(b*x + a)*\operatorname{dilog}(-\cos(b*x + a) + I*\sin(b*x + a))*\sin(b*x + a) - I*d^2*\cos(b*x + a)*\operatorname{dilog}(-\cos(b*x + a) - I*\sin(b*x + a))*\sin(b*x + a) + b^2*c^2 + (b*d^2*x + b*c*d)*\cos(b*x + a)*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1)*\sin(b*x + a) + (b*c*d - a*d^2)*\cos(b*x + a)*\log(\cos(b*x + a) + I*\sin(b*x + a) + I)*\sin(b*x + a) + (b*d^2*x + b*c*d)*\cos(b*x + a)*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1)*\sin(b*x + a) + (b*c*d - a*d^2)*\cos(b*x + a)*\log(\cos$

```
(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + (b*d^2*x + a*d^2)*cos(b*x +
a)*log(I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + (b*d^2*x + a*d^2)*
cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) + (b*d^2*x
+ a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a)
+ (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)*s
in(b*x + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*si
n(b*x + a) + 1/2)*sin(b*x + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(-1/2*cos(
b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + (b*d^2*x + a*d^2)*cos(b
*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + (b*c*d - a*d
^2)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + (b
d^2*x + a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x
+ a) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I
)*sin(b*x + a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a)^2)/(b
^3*cos(b*x + a)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*csc(b*x+a)**2*sec(b*x+a)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \csc(bx + a)^2 \sec(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^2*csc(b*x + a)^2*sec(b*x + a)^2, x)

3.275 $\int (c + dx) \csc^2(a + bx) \sec^2(a + bx) dx$

Optimal. Leaf size=35

$$\frac{d \log(\sin(2a + 2bx))}{b^2} - \frac{2(c + dx) \cot(2a + 2bx)}{b}$$

[Out] $(-2*(c + d*x)*\text{Cot}[2*a + 2*b*x])/b + (d*\text{Log}[\text{Sin}[2*a + 2*b*x]])/b^2$

Rubi [A] time = 0.0594156, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4419, 4184, 3475}

$$\frac{d \log(\sin(2a + 2bx))}{b^2} - \frac{2(c + dx) \cot(2a + 2bx)}{b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)*\text{Csc}[a + b*x]^2*\text{Sec}[a + b*x]^2, x]$

[Out] $(-2*(c + d*x)*\text{Cot}[2*a + 2*b*x])/b + (d*\text{Log}[\text{Sin}[2*a + 2*b*x]])/b^2$

Rule 4419

$\text{Int}[\text{Csc}[(a_.) + (b_.)*(x_.)]^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sec}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] := \text{Dist}[2^n, \text{Int}[(c + d*x)^m*\text{Csc}[2*a + 2*b*x]^n, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]

Rule 4184

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]^2*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] := -\text{Simp}[\text{Dist}[(c + d*x)^m*\text{Cot}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cot}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3475

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x_Symbol] := -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (c + dx) \csc^2(a + bx) \sec^2(a + bx) dx &= 4 \int (c + dx) \csc^2(2a + 2bx) dx \\ &= -\frac{2(c + dx) \cot(2a + 2bx)}{b} + \frac{(2d) \int \cot(2a + 2bx) dx}{b} \\ &= -\frac{2(c + dx) \cot(2a + 2bx)}{b} + \frac{d \log(\sin(2a + 2bx))}{b^2} \end{aligned}$$

Mathematica [A] time = 0.203898, size = 32, normalized size = 0.91

$$\frac{d \log(\sin(2(a + bx))) - 2b(c + dx) \cot(2(a + bx))}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^2,x]

[Out] (-2*b*(c + d*x)*Cot[2*(a + b*x)] + d*Log[Sin[2*(a + b*x)]])/b^2

Maple [B] time = 0.074, size = 182, normalized size = 5.2

$$\left(\frac{c}{2b} - 3 \frac{c(\tan(1/2bx + a/2))^2}{b} + \frac{c}{2b} \left(\tan\left(\frac{bx}{2} + \frac{a}{2}\right)\right)^4 + \frac{dx}{2b} - 3 \frac{dx(\tan(1/2bx + a/2))^2}{b} + \frac{dx}{2b} \left(\tan\left(\frac{bx}{2} + \frac{a}{2}\right)\right)^4\right) \left(\tan\left(\frac{bx}{2} + \frac{a}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x)

[Out] (1/2/b*c-3/b*c*tan(1/2*b*x+1/2*a)^2+1/2/b*c*tan(1/2*b*x+1/2*a)^4+1/2*d*x/b-3*d/b*x*tan(1/2*b*x+1/2*a)^2+1/2*d/b*x*tan(1/2*b*x+1/2*a)^4)/tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1)+d/b^2*ln(tan(1/2*b*x+1/2*a))+d/b^2*ln(tan(1/2*b*x+1/2*a)-1)+d/b^2*ln(tan(1/2*b*x+1/2*a)+1)-2*d/b^2*ln(1+tan(1/2*b*x+1/2*a)^2)

Maxima [B] time = 1.50139, size = 416, normalized size = 11.89

$$2c\left(\frac{1}{\tan(bx+a)} - \tan(bx+a)\right) - \frac{2ad\left(\frac{1}{\tan(bx+a)} - \tan(bx+a)\right)}{b} - \frac{((\cos(4bx+4a)^2 + \sin(4bx+4a)^2 - 2\cos(4bx+4a) + 1)\log(\cos(2bx+2a)^2 + \sin(2bx+2a)^2 + 2\cos(2bx+2a) + 1) + (\cos(4bx+4a)^2 + \sin(4bx+4a)^2 - 2\cos(4bx+4a) + 1)\log(\cos(bx+a)^2 + \sin(bx+a)^2 + 2\cos(bx+a) + 1) + (\cos(4bx+4a)^2 + \sin(4bx+4a)^2 - 2\cos(4bx+4a) + 1)\log(\cos(bx+a)^2 + \sin(bx+a)^2 - 2\cos(bx+a) + 1) - 8*(bx+a)*\sin(4bx+4a))*d/((\cos(4bx+4a)^2 + \sin(4bx+4a)^2 - 2\cos(4bx+4a) + 1)*b))/b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")

[Out] -1/2*(2*c*(1/tan(b*x + a) - tan(b*x + a)) - 2*a*d*(1/tan(b*x + a) - tan(b*x + a))/b - ((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 8*(b*x + a)*sin(4*b*x + 4*a))*d/((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*b))/b

Fricas [B] time = 0.49963, size = 197, normalized size = 5.63

$$\frac{d \cos(bx + a) \log\left(-\frac{1}{2} \cos(bx + a) \sin(bx + a)\right) \sin(bx + a) + bdx - 2(bdx + bc) \cos(bx + a)^2 + bc}{b^2 \cos(bx + a) \sin(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")

[Out] (d*cos(b*x + a)*log(-1/2*cos(b*x + a)*sin(b*x + a))*sin(b*x + a) + b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + b*c)/(b^2*cos(b*x + a)*sin(b*x + a))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**2*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.276 \quad \int \frac{\csc^2(a+bx) \sec^2(a+bx)}{c+dx} dx$$

Optimal. Leaf size=23

$$4\text{Unintegrable}\left(\frac{\csc^2(2a+2bx)}{c+dx}, x\right)$$

[Out] 4*Unintegrable[Csc[2*a + 2*b*x]^2/(c + d*x), x]

Rubi [A] time = 0.0936632, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^2(a+bx) \sec^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x), x]

[Out] 4*Defer[Int][Csc[2*a + 2*b*x]^2/(c + d*x), x]

Rubi steps

$$\int \frac{\csc^2(a+bx) \sec^2(a+bx)}{c+dx} dx = 4 \int \frac{\csc^2(2a+2bx)}{c+dx} dx$$

Mathematica [A] time = 7.3347, size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx) \sec^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x), x]

[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x), x]

Maple [A] time = 0.518, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx+a))^2 (\sec(bx+a))^2}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c), x)

[Out] int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c), x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)^2 \sec(bx+a)^2}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")

[Out] integral(csc(b*x + a)^2*sec(b*x + a)^2/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx) \sec^2(a+bx)}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)**2*sec(b*x+a)**2/(d*x+c),x)

[Out] Integral(csc(a + b*x)**2*sec(a + b*x)**2/(c + d*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^2 \sec(bx+a)^2}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x, algorithm="giac")

[Out] integrate(csc(b*x + a)^2*sec(b*x + a)^2/(d*x + c), x)

$$3.277 \quad \int \frac{\csc^2(a+bx) \sec^2(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=23

$$4\text{Unintegrable}\left(\frac{\csc^2(2a+2bx)}{(c+dx)^2}, x\right)$$

[Out] 4*Unintegrable[Csc[2*a + 2*b*x]^2/(c + d*x)^2, x]

Rubi [A] time = 0.0857983, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^2(a+bx) \sec^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x)^2, x]

[Out] 4*Defer[Int][Csc[2*a + 2*b*x]^2/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\csc^2(a+bx) \sec^2(a+bx)}{(c+dx)^2} dx = 4 \int \frac{\csc^2(2a+2bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 7.58574, size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx) \sec^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x)^2, x]

[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x)^2, x]

Maple [A] time = 0.811, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx+a))^2 (\sec(bx+a))^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2, x)

[Out] int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2, x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)^2 \sec(bx+a)^2}{d^2x^2 + 2cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(csc(b*x + a)^2*sec(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx) \sec^2(a+bx)}{(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)**2*sec(b*x+a)**2/(d*x+c)**2,x)

[Out] Integral(csc(a + b*x)**2*sec(a + b*x)**2/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^2 \sec(bx+a)^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(csc(b*x + a)^2*sec(b*x + a)^2/(d*x + c)^2, x)

3.278 $\int (c + dx)^m \csc^3(a + bx) \sec^2(a + bx) dx$

Optimal. Leaf size=26

$$\text{CannotIntegrate}\left(\csc^3(a + bx) \sec^2(a + bx)(c + dx)^m, x\right)$$

[Out] CannotIntegrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]

Rubi [A] time = 0.215213, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \csc^3(a + bx) \sec^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]

[Out] Defer[Int] [(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]

Rubi steps

$$\int (c + dx)^m \csc^3(a + bx) \sec^2(a + bx) dx = \int (c + dx)^m \csc^3(a + bx) \sec^2(a + bx) dx$$

Mathematica [A] time = 11.7618, size = 0, normalized size = 0.

$$\int (c + dx)^m \csc^3(a + bx) \sec^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]

[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]

Maple [A] time = 0.21, size = 0, normalized size = 0.

$$\int (dx + c)^m (\csc(bx + a))^3 (\sec(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2, x)

[Out] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \csc(bx + a)^3 \sec(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx + c)^m \csc(bx + a)^3 \sec(bx + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \csc(bx + a)^3 \sec(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

3.279 $\int (c + dx)^3 \csc^3(a + bx) \sec^2(a + bx) dx$

Optimal. Leaf size=601

$$-\frac{6icd^2 \text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} + \frac{6icd^2 \text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3} - \frac{9d^2(c+dx) \text{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^3} + \frac{9d^2(c+dx) \text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3}$$

[Out] $((12*I)*c*d^2*x*ArcTan[E^(I*(a + b*x))])/b^2 + ((6*I)*d^3*x^2*ArcTan[E^(I*(a + b*x))])/b^2 - (6*d^3*x*ArcTanh[E^(I*(a + b*x))])/b^3 - (3*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b - (3*c*d^2*ArcTanh[Cos[a + b*x]])/b^3 - (3*c^2*d*ArcTanh[Sin[a + b*x]])/b^2 - (3*c^2*d*Csc[a + b*x])/(2*b^2) - (3*c*d^2*x*Csc[a + b*x])/b^2 - (3*d^3*x^2*Csc[a + b*x])/(2*b^2) + ((3*I)*d^3*PolyLog[2, -E^(I*(a + b*x))])/b^4 + (((9*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((6*I)*c*d^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*x*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*c*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 + ((6*I)*d^3*x*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*d^3*PolyLog[2, E^(I*(a + b*x))])/b^4 - (((9*I)/2)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^2 - (9*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*PolyLog[3, I*E^(I*(a + b*x))])/b^4 + (9*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((9*I)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((9*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4 + (3*(c + d*x)^3*Sec[a + b*x])/(2*b) - ((c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)$

Rubi [A] time = 2.31269, antiderivative size = 601, normalized size of antiderivative = 1., number of steps used = 64, number of rules used = 24, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 1.$, Rules used = {2622, 288, 321, 207, 4420, 6688, 12, 6742, 6273, 4183, 2531, 6609, 2282, 6589, 4133, 453, 206, 4181, 2279, 2391, 2621, 6271, 3770, 14}

$$-\frac{6icd^2 \text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} + \frac{6icd^2 \text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3} - \frac{9d^2(c+dx) \text{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^3} + \frac{9d^2(c+dx) \text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]

[Out] $((12*I)*c*d^2*x*ArcTan[E^(I*(a + b*x))])/b^2 + ((6*I)*d^3*x^2*ArcTan[E^(I*(a + b*x))])/b^2 - (6*d^3*x*ArcTanh[E^(I*(a + b*x))])/b^3 - (3*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b - (3*c*d^2*ArcTanh[Cos[a + b*x]])/b^3 - (3*c^2*d*ArcTanh[Sin[a + b*x]])/b^2 - (3*c^2*d*Csc[a + b*x])/(2*b^2) - (3*c*d^2*x*Csc[a + b*x])/b^2 - (3*d^3*x^2*Csc[a + b*x])/(2*b^2) + ((3*I)*d^3*PolyLog[2, -E^(I*(a + b*x))])/b^4 + (((9*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((6*I)*c*d^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*x*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*c*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 + ((6*I)*d^3*x*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*d^3*PolyLog[2, E^(I*(a + b*x))])/b^4 - (((9*I)/2)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^2 - (9*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*PolyLog[3, I*E^(I*(a + b*x))])/b^4 + (9*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((9*I)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((9*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4 + (3*(c + d*x)^3*Sec[a + b*x])/(2*b) - ((c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)$

Rule 2622

Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)]^(n + 1)/2

), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 4420

Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]

Rule 6688

Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifierIntegrandQ[v, u, x]]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rule 6273

Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m + 1, x]]

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/(ff*x)^(n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_))^(n_)]^(p_.)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```


Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Dist[
(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))],
x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_.))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2,
-(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol]
:> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x],
x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] &&
!(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rule 14

```
Int[(u_.)*((c_.)*(x_.))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x],
x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 \csc^3(a + bx) \sec^2(a + bx) dx &= -\frac{3(c + dx)^3 \tanh^{-1}(\cos(a + bx))}{2b} + \frac{3(c + dx)^3 \sec(a + bx)}{2b} - \frac{(c + dx)^3 \csc^2(a + bx)}{2b} \\
&= -\frac{3(c + dx)^3 \tanh^{-1}(\cos(a + bx))}{2b} + \frac{3(c + dx)^3 \sec(a + bx)}{2b} - \frac{(c + dx)^3 \csc^2(a + bx)}{2b} \\
&= -\frac{3(c + dx)^3 \tanh^{-1}(\cos(a + bx))}{2b} + \frac{3(c + dx)^3 \sec(a + bx)}{2b} - \frac{(c + dx)^3 \csc^2(a + bx)}{2b} \\
&= -\frac{3(c + dx)^3 \tanh^{-1}(\cos(a + bx))}{2b} + \frac{3(c + dx)^3 \sec(a + bx)}{2b} - \frac{(c + dx)^3 \csc^2(a + bx)}{2b} \\
&= -\frac{3(c + dx)^3 \tanh^{-1}(\cos(a + bx))}{2b} + \frac{3(c + dx)^3 \sec(a + bx)}{2b} - \frac{(c + dx)^3 \csc^2(a + bx)}{2b} \\
&= \frac{3(c + dx)^3 \sec(a + bx)}{2b} - \frac{(c + dx)^3 \csc^2(a + bx) \sec(a + bx)}{2b} - \frac{3 \int b(-c - dx)^3 \csc^3(a + bx) dx}{2b} \\
&= \frac{3(c + dx)^3 \sec(a + bx)}{2b} - \frac{(c + dx)^3 \csc^2(a + bx) \sec(a + bx)}{2b} - \frac{3}{2} \int (-c - dx)^3 \csc^3(a + bx) dx \\
&= -\frac{3(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{3(c + dx)^3 \sec(a + bx)}{2b} - \frac{(c + dx)^3 \csc^2(a + bx)}{2b} \\
&= -\frac{3(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{3c^2 d \csc(a + bx)}{2b^2} + \frac{9id(c + dx)^2 \text{Li}_2(-e^{i(a+bx)})}{2b^2} \\
&= \frac{18icd^2 x \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{9id^3 x^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{3(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b} \\
&= \frac{18icd^2 x \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{9id^3 x^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{3(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b} \\
&= \frac{18icd^2 x \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{9id^3 x^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{3(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b} \\
&= \frac{18icd^2 x \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{9id^3 x^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{6d^3 x \tanh^{-1}(e^{i(a+bx)})}{b^3} - \frac{3(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b} \\
&= \frac{12icd^2 x \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{9id^3 x^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{6d^3 x \tanh^{-1}(e^{i(a+bx)})}{b^3} - \frac{3(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b} \\
&= \frac{12icd^2 x \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{6id^3 x^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{6d^3 x \tanh^{-1}(e^{i(a+bx)})}{b^3} - \frac{3(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b} \\
&= \frac{12icd^2 x \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{6id^3 x^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{6d^3 x \tanh^{-1}(e^{i(a+bx)})}{b^3} - \frac{3(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b} \\
&= \frac{12icd^2 x \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{6id^3 x^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{6d^3 x \tanh^{-1}(e^{i(a+bx)})}{b^3} - \frac{3(c + dx)^3 \tanh^{-1}(e^{i(a+bx)})}{b}
\end{aligned}$$

Mathematica [A] time = 8.31608, size = 907, normalized size = 1.51

$$\frac{\sec(a + bx) (-bc^3 + 3b \cos(2a + 2bx)c^3 - 3bdc^2 + 9bdx \cos(2a + 2bx)c^2 + 3d \sin(2a + 2bx)c^2 - 3bd^2x^2c + 9bd^2x^2 \cos(2a + 2bx)c - 3bd^3x^3)}{4b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]

[Out] (-3*d*((-2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))]) + 2*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] + b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - 2*b^2*c*d*x*Log[1 +

$$\begin{aligned}
& I * E^{(I * (a + b * x))} - b^2 * d^2 * x^2 * \text{Log}[1 + I * E^{(I * (a + b * x))}] + (2 * I) * b * d * (c \\
& + d * x) * \text{PolyLog}[2, (-I) * E^{(I * (a + b * x))}] - (2 * I) * b * d * (c + d * x) * \text{PolyLog}[2, I * \\
& E^{(I * (a + b * x))}] - 2 * d^2 * \text{PolyLog}[3, (-I) * E^{(I * (a + b * x))}] + 2 * d^2 * \text{PolyLog}[3 \\
& , I * E^{(I * (a + b * x))}]) / b^4 + (3 * (b^3 * c^3 * \text{Log}[1 - E^{(I * (a + b * x))}] + 2 * b * c * d \\
& ^2 * \text{Log}[1 - E^{(I * (a + b * x))}] + 3 * b^3 * c^2 * d * x * \text{Log}[1 - E^{(I * (a + b * x))}] + 2 * b * \\
& d^3 * x * \text{Log}[1 - E^{(I * (a + b * x))}] + 3 * b^3 * c * d^2 * x^2 * \text{Log}[1 - E^{(I * (a + b * x))}] + \\
& b^3 * d^3 * x^3 * \text{Log}[1 - E^{(I * (a + b * x))}] - b^3 * c^3 * \text{Log}[1 + E^{(I * (a + b * x))}] - \\
& 2 * b * c * d^2 * \text{Log}[1 + E^{(I * (a + b * x))}] - 3 * b^3 * c^2 * d * x * \text{Log}[1 + E^{(I * (a + b * x))}] \\
& - 2 * b * d^3 * x * \text{Log}[1 + E^{(I * (a + b * x))}] - 3 * b^3 * c * d^2 * x^2 * \text{Log}[1 + E^{(I * (a + b * \\
& * x))}] - b^3 * d^3 * x^3 * \text{Log}[1 + E^{(I * (a + b * x))}] + I * d * (2 * d^2 + 3 * b^2 * (c + d * x) \\
& ^2) * \text{PolyLog}[2, -E^{(I * (a + b * x))}] - I * d * (2 * d^2 + 3 * b^2 * (c + d * x)^2) * \text{PolyLog}[\\
& 2, E^{(I * (a + b * x))}] - 6 * b * c * d^2 * \text{PolyLog}[3, -E^{(I * (a + b * x))}] - 6 * b * d^3 * x * \text{Po \\
& lyLog}[3, -E^{(I * (a + b * x))}] + 6 * b * c * d^2 * \text{PolyLog}[3, E^{(I * (a + b * x))}] + 6 * b * d^ \\
& 3 * x * \text{PolyLog}[3, E^{(I * (a + b * x))}] - (6 * I) * d^3 * \text{PolyLog}[4, -E^{(I * (a + b * x))}] + \\
& (6 * I) * d^3 * \text{PolyLog}[4, E^{(I * (a + b * x))}]) / (2 * b^4) - (\text{Csc}[a + b * x]^2 * \text{Sec}[a + b \\
& * x] * (- (b * c^3) - 3 * b * c^2 * d * x - 3 * b * c * d^2 * x^2 - b * d^3 * x^3 + 3 * b * c^3 * \text{Cos}[2 * a + \\
& 2 * b * x] + 9 * b * c^2 * d * x * \text{Cos}[2 * a + 2 * b * x] + 9 * b * c * d^2 * x^2 * \text{Cos}[2 * a + 2 * b * x] + 3 \\
& * b * d^3 * x^3 * \text{Cos}[2 * a + 2 * b * x] + 3 * c^2 * d * \text{Sin}[2 * a + 2 * b * x] + 6 * c * d^2 * x * \text{Sin}[2 * a \\
& + 2 * b * x] + 3 * d^3 * x^2 * \text{Sin}[2 * a + 2 * b * x])) / (4 * b^2)
\end{aligned}$$

Maple [B] time = 0.727, size = 1613, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x)

[Out]
$$\begin{aligned}
& -9/2/b * c * d^2 * \ln(\exp(I * (b * x + a)) + 1) * x^2 + 9/2/b * c * d^2 * \ln(1 - \exp(I * (b * x + a))) * x^2 - \\
& 9 * I / b^2 * d^2 * c * \text{polylog}(2, \exp(I * (b * x + a))) * x + 9 * I / b^2 * d^2 * c * \text{polylog}(2, -\exp(I * (b \\
& * x + a))) * x + 3 * I * d^3 * \text{polylog}(2, -\exp(I * (b * x + a))) / b^4 - 3 * d^3 / b^2 * \ln(1 - I * \exp(I * (b * \\
& x + a))) * x^2 - 3 * d^3 / b^4 * a^2 * \ln(1 + I * \exp(I * (b * x + a))) + 3 * d^3 / b^2 * \ln(1 + I * \exp(I * (b * x \\
& + a))) * x^2 + 3 * d^3 / b^4 * a^2 * \ln(1 - I * \exp(I * (b * x + a))) + 6 * I * d^3 * x * \text{polylog}(2, I * \exp(I * \\
& (b * x + a))) / b^3 + 9 * I * d^3 * \text{polylog}(4, \exp(I * (b * x + a))) / b^4 - 3 / b^3 * d^3 * \ln(\exp(I * (b * x \\
& + a)) + 1) * x + 3 / b^3 * d^3 * \ln(1 - \exp(I * (b * x + a))) * x + 3 / b^4 * d^3 * \ln(1 - \exp(I * (b * x + a))) * a \\
& + 6 * d^3 * \text{polylog}(3, -I * \exp(I * (b * x + a))) / b^4 - 6 * d^3 * \text{polylog}(3, I * \exp(I * (b * x + a))) / b \\
& ^4 + 6 * I * d / b^2 * c^2 * \arctan(\exp(I * (b * x + a))) - 9 * I * d^3 * \text{polylog}(4, -\exp(I * (b * x + a))) / \\
& b^4 - 3 / b^4 * d^3 * a * \ln(\exp(I * (b * x + a)) - 1) - 3 / b^3 * d^2 * c * \ln(\exp(I * (b * x + a)) + 1) + 3 / b^3 \\
& * d^2 * c * \ln(\exp(I * (b * x + a)) - 1) + 3 / 2 / b * d^3 * \ln(1 - \exp(I * (b * x + a))) * x^3 + 3 / 2 / b^4 * d^3 * \\
& \ln(1 - \exp(I * (b * x + a))) * a^3 - 3 / 2 / b * d^3 * \ln(\exp(I * (b * x + a)) + 1) * x^3 + 9 / 2 / b^3 * c * d^2 * a \\
& ^2 * \ln(\exp(I * (b * x + a)) - 1) - 9 / 2 / b^2 * c^2 * d * a * \ln(\exp(I * (b * x + a)) - 1) - 9 / 2 / b^3 * c * d^2 * \\
& a^2 * \ln(1 - \exp(I * (b * x + a))) - 9 / 2 / b * c^2 * d * \ln(\exp(I * (b * x + a)) + 1) * x + 9 / 2 / b * c^2 * d * \ln(\\
& 1 - \exp(I * (b * x + a))) * x + 9 / 2 / b^2 * c^2 * d * \ln(1 - \exp(I * (b * x + a))) * a - 6 * I * d^3 * x * \text{polylog}(\\
& 2, -I * \exp(I * (b * x + a))) / b^3 + 6 * d^2 / b^3 * c * \ln(1 + I * \exp(I * (b * x + a))) * a - 6 * d^2 / b^2 * c * \ln \\
& (1 - I * \exp(I * (b * x + a))) * x - 6 * d^2 / b^3 * c * \ln(1 - I * \exp(I * (b * x + a))) * a + 6 * d^2 / b^2 * c * \ln \\
& (1 + I * \exp(I * (b * x + a))) * x + 6 * I * d^3 / b^4 * a^2 * \arctan(\exp(I * (b * x + a))) - 12 * I * d^2 / b^3 * \\
& c * a * \arctan(\exp(I * (b * x + a))) + 9 / b^3 * d^3 * \text{polylog}(3, \exp(I * (b * x + a))) * x - 9 / b^3 * d^3 * \\
& \text{polylog}(3, -\exp(I * (b * x + a))) * x + 9 / b^3 * c * d^2 * \text{polylog}(3, \exp(I * (b * x + a))) - 9 / b^3 * c * \\
& d^2 * \text{polylog}(3, -\exp(I * (b * x + a))) - 3 / 2 / b^4 * d^3 * a^3 * \ln(\exp(I * (b * x + a)) - 1) - 9 / 2 * I / b \\
& ^2 * d^3 * \text{polylog}(2, \exp(I * (b * x + a))) * x^2 + 9 / 2 * I / b^2 * d^3 * \text{polylog}(2, -\exp(I * (b * x + a) \\
&)) * x^2 - 9 / 2 * I / b^2 * c^2 * d * \text{polylog}(2, \exp(I * (b * x + a))) + 9 / 2 * I / b^2 * c^2 * d * \text{polylog}(2, \\
& -\exp(I * (b * x + a))) + 6 * I / b^4 * d^3 * \text{polylog}(2, I * \exp(I * (b * x + a))) * a + 6 * I / b^3 * d^2 * c * d \\
& \text{ilog}(1 - I * \exp(I * (b * x + a))) - 6 * I / b^3 * d^2 * c * d \text{ilog}(1 + I * \exp(I * (b * x + a))) + 6 * I / b^4 * d^3 \\
& * a * d \text{ilog}(1 + I * \exp(I * (b * x + a))) - 6 * I / b^4 * d^3 * \text{polylog}(2, -I * \exp(I * (b * x + a))) * a - 6 * I \\
& / b^4 * d^3 * a * d \text{ilog}(1 - I * \exp(I * (b * x + a))) + 3 / 2 / b * c^3 * \ln(\exp(I * (b * x + a)) - 1) - 3 / 2 / b * c \\
& ^3 * \ln(\exp(I * (b * x + a)) + 1) + 1 / b^2 / (\exp(2 * I * (b * x + a)) - 1)^2 / (\exp(2 * I * (b * x + a)) + 1) * (\\
& 3 * d^3 * x^3 * b * \exp(5 * I * (b * x + a)) + 9 * c * d^2 * x^2 * b * \exp(5 * I * (b * x + a)) + 9 * c^2 * d * x * b * \exp
\end{aligned}$$

$$\frac{(5I*(b*x+a))-2*d^3*x^3*b*exp(3*I*(b*x+a))+3*c^3*b*exp(5*I*(b*x+a))-6*c*d^2*x^2*b*exp(3*I*(b*x+a))+6*I*c*d^2*x*exp(I*(b*x+a))-6*c^2*d*x*b*exp(3*I*(b*x+a))+3*d^3*x^3*b*exp(I*(b*x+a))-6*I*c*d^2*x*exp(5*I*(b*x+a))-2*c^3*b*exp(3*I*(b*x+a))+9*c*d^2*x^2*b*exp(I*(b*x+a))-3*I*d^3*x^2*exp(5*I*(b*x+a))+9*c^2*d*x*b*exp(I*(b*x+a))+3*c^3*b*exp(I*(b*x+a))+3*I*d^3*x^2*exp(I*(b*x+a))-3*I*c^2*d*exp(5*I*(b*x+a))+3*I*c^2*d*exp(I*(b*x+a)))-3*I*d^3*polylog(2,exp(I*(b*x+a)))}{b^4}$$

Maxima [B] time = 18.6152, size = 10858, normalized size = 18.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")

[Out] $\frac{1}{4}*(c^3*(2*(3*\cos(b*x + a)^2 - 2)/(\cos(b*x + a)^3 - \cos(b*x + a)) - 3*\log(\cos(b*x + a) + 1) + 3*\log(\cos(b*x + a) - 1)) - 3*a*c^2*d*(2*(3*\cos(b*x + a)^2 - 2)/(\cos(b*x + a)^3 - \cos(b*x + a)) - 3*\log(\cos(b*x + a) + 1) + 3*\log(\cos(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*(3*\cos(b*x + a)^2 - 2)/(\cos(b*x + a)^3 - \cos(b*x + a)) - 3*\log(\cos(b*x + a) + 1) + 3*\log(\cos(b*x + a) - 1))/b^2 - a^3*d^3*(2*(3*\cos(b*x + a)^2 - 2)/(\cos(b*x + a)^3 - \cos(b*x + a)) - 3*\log(\cos(b*x + a) + 1) + 3*\log(\cos(b*x + a) - 1))/b^3 + 4*((12*b^2*c^2*d - 24*a*b*c*d^2 + 12*(b*x + a)^2*d^3 + 12*a^2*d^3 + 24*(b*c*d^2 - a*d^3)*(b*x + a) + 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(6*b*x + 6*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 12*I*a^2*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*\sin(6*b*x + 6*a) - (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*d^3 + 12*I*a^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*\sin(4*b*x + 4*a) - (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*d^3 + 12*I*a^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\cos(b*x + a), \sin(b*x + a) + 1) + (12*b^2*c^2*d - 24*a*b*c*d^2 + 12*(b*x + a)^2*d^3 + 12*a^2*d^3 + 24*(b*c*d^2 - a*d^3)*(b*x + a) + 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(6*b*x + 6*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 12*I*a^2*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*\sin(6*b*x + 6*a) - (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*d^3 + 12*I*a^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*\sin(4*b*x + 4*a) - (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*d^3 + 12*I*a^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\cos(b*x + a), -\sin(b*x + a) + 1) - (6*(b*x + a)^3*d^3 + 12*b*c*d^2 - 12*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a) + 6*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*\cos(6*b*x + 6*a) - 6*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*\cos(4*b*x + 4*a) - 6*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (6*I*(b*x + a)^3*d^3 + 12*I*b*c*d^2 - 12*I*a*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + (18*I*a^2 + 12*I)*d^3)*(b*x + a))*\sin(6*b*x + 6*a) + (-6*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*$

$$\begin{aligned}
& a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 12*I) \\
& *d^3)*(b*x + a))*\sin(4*b*x + 4*a) + (-6*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + \\
& 12*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + \\
& 36*I*a*b*c*d^2 + (-18*I*a^2 - 12*I)*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan \\
& 2(\sin(b*x + a), \cos(b*x + a) + 1) + (12*b*c*d^2 - 12*a*d^3 + 12*(b*c*d^2 - \\
& a*d^3))*\cos(6*b*x + 6*a) - 12*(b*c*d^2 - a*d^3))*\cos(4*b*x + 4*a) - 12*(b*c* \\
& d^2 - a*d^3))*\cos(2*b*x + 2*a) - (-12*I*b*c*d^2 + 12*I*a*d^3))*\sin(6*b*x + 6* \\
& a) - (12*I*b*c*d^2 - 12*I*a*d^3))*\sin(4*b*x + 4*a) - (12*I*b*c*d^2 - 12*I*a* \\
& d^3))*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) - 1) - (6*(b*x + \\
& a)^3*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(3*b^2*c^2*d - 6*a*b*c*d^2 \\
& + (3*a^2 + 2)*d^3)*(b*x + a) + 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b* \\
& x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*\cos(6*b \\
& *x + 6*a) - 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c \\
& ^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*\cos(4*b*x + 4*a) - 6*((b*x \\
& + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 \\
& + (3*a^2 + 2)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (6*I*(b*x + a)^3*d^3 + (18 \\
& *I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + (\\
& 18*I*a^2 + 12*I)*d^3)*(b*x + a))*\sin(6*b*x + 6*a) + (-6*I*(b*x + a)^3*d^3 + \\
& (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d \\
& ^2 + (-18*I*a^2 - 12*I)*d^3)*(b*x + a))*\sin(4*b*x + 4*a) + (-6*I*(b*x + a)^ \\
& 3*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I* \\
& a*b*c*d^2 + (-18*I*a^2 - 12*I)*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin \\
& (b*x + a), -\cos(b*x + a) + 1) - (12*I*(b*x + a)^3*d^3 + 12*b^2*c^2*d - 24* \\
& a*b*c*d^2 + 12*a^2*d^3 + (36*I*b*c*d^2 - 12*(3*I*a - 1)*d^3)*(b*x + a)^2 + \\
& (36*I*b^2*c^2*d - 24*(3*I*a - 1)*b*c*d^2 + (36*I*a^2 - 24*a)*d^3)*(b*x + a) \\
&)*\cos(5*b*x + 5*a) - (-8*I*(b*x + a)^3*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(\\
& b*x + a)^2 + (-24*I*b^2*c^2*d + 48*I*a*b*c*d^2 - 24*I*a^2*d^3)*(b*x + a))*\cos \\
& (3*b*x + 3*a) - (12*I*(b*x + a)^3*d^3 - 12*b^2*c^2*d + 24*a*b*c*d^2 - 12* \\
& a^2*d^3 + (36*I*b*c*d^2 - 12*(3*I*a + 1)*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d \\
& - 24*(3*I*a + 1)*b*c*d^2 + (36*I*a^2 + 24*a)*d^3)*(b*x + a))*\cos(b*x + a) \\
& + (24*b*c*d^2 + 24*(b*x + a)*d^3 - 24*a*d^3 + 24*(b*c*d^2 + (b*x + a)*d^3 - \\
& a*d^3))*\cos(6*b*x + 6*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3))*\cos(4*b*x + \\
& 4*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3))*\cos(2*b*x + 2*a) - (-24*I*b*c* \\
& d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3))*\sin(6*b*x + 6*a) - (24*I*b*c*d^2 + 2 \\
& 4*I*(b*x + a)*d^3 - 24*I*a*d^3))*\sin(4*b*x + 4*a) - (24*I*b*c*d^2 + 24*I*(b* \\
& x + a)*d^3 - 24*I*a*d^3))*\sin(2*b*x + 2*a))*\operatorname{dilog}(I*e^{(I*b*x + I*a)}) - (24*b \\
& *c*d^2 + 24*(b*x + a)*d^3 - 24*a*d^3 + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3) \\
&)*\cos(6*b*x + 6*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3))*\cos(4*b*x + 4*a) - \\
& 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3))*\cos(2*b*x + 2*a) + (24*I*b*c*d^2 + 24 \\
& *I*(b*x + a)*d^3 - 24*I*a*d^3))*\sin(6*b*x + 6*a) + (-24*I*b*c*d^2 - 24*I*(b* \\
& x + a)*d^3 + 24*I*a*d^3))*\sin(4*b*x + 4*a) + (-24*I*b*c*d^2 - 24*I*(b*x + a) \\
&)*d^3 + 24*I*a*d^3))*\sin(2*b*x + 2*a))*\operatorname{dilog}(-I*e^{(I*b*x + I*a)}) + (18*b^2*c^ \\
& 2*d - 36*a*b*c*d^2 + 18*(b*x + a)^2*d^3 + 6*(3*a^2 + 2)*d^3 + 36*(b*c*d^2 - \\
& a*d^3)*(b*x + a) + 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (3*a \\
& ^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(6*b*x + 6*a) - 6*(3*b^2*c^ \\
& 2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^ \\
& 3)*(b*x + a))*\cos(4*b*x + 4*a) - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a) \\
& ^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) \\
& - (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 1 \\
& 2*I)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*\sin(6*b*x + 6*a) - (18*I \\
& *b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I*a^2 + 12*I)*d^3 \\
& + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*\sin(4*b*x + 4*a) - (18*I*b^2*c^2*d \\
& - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I*a^2 + 12*I)*d^3 + (36*I*b* \\
& c*d^2 - 36*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{dilog}(-e^{(I*b*x + I*a)}) - \\
& (18*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b*x + a)^2*d^3 + 6*(3*a^2 + 2)*d^3 + 36* \\
& (b*c*d^2 - a*d^3)*(b*x + a) + 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2* \\
& d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(6*b*x + 6*a) - 6 \\
& *(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c* \\
& d^2 - a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3
\end{aligned}$$

$$\begin{aligned}
&*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)*\cos(2*b*x + 2*a) + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I*a^2 + 12*I)*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*\sin(6*b*x + 6*a) \\
&+ (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 12*I)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*\sin(4*b*x + 4*a) + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 12*I)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{dilog}(e^{(I*b*x + I*a)}) - (-3*I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 + (-9*I*a^2 - 6*I)*d^3)*(b*x + a) + (-3*I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 + (-9*I*a^2 - 6*I)*d^3)*(b*x + a))*\cos(6*b*x + 6*a) + (3*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 6*I)*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + (3*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 6*I)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*\sin(6*b*x + 6*a) - 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*\sin(4*b*x + 4*a) - 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - (3*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 6*I)*d^3)*(b*x + a) + (3*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 6*I)*d^3)*(b*x + a))*\cos(6*b*x + 6*a) + (-3*I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 + (-9*I*a^2 - 6*I)*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + (-3*I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 + (-9*I*a^2 - 6*I)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*\sin(6*b*x + 6*a) + 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*\sin(4*b*x + 4*a) + 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) - (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*\cos(6*b*x + 6*a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\sin(6*b*x + 6*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\sin(4*b*x + 4*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\sin(b*x + a) + 1) - (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*\cos(6*b*x + 6*a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\sin(6*b*x + 6*a)
\end{aligned}$$

$$\begin{aligned}
& + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\sin(4*b*x + 4*a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\sin(b*x + a) + 1) - (36*d^3*\cos(6*b*x + 6*a) - 36*d^3*\cos(4*b*x + 4*a) - 36*d^3*\cos(2*b*x + 2*a) + 36*I*d^3*\sin(6*b*x + 6*a) - 36*I*d^3*\sin(4*b*x + 4*a) - 36*I*d^3*\sin(2*b*x + 2*a) + 36*d^3)*\text{polylog}(4, -e^{(I*b*x + I*a)}) + (36*d^3*\cos(6*b*x + 6*a) - 36*d^3*\cos(4*b*x + 4*a) - 36*d^3*\cos(2*b*x + 2*a) + 36*I*d^3*\sin(6*b*x + 6*a) - 36*I*d^3*\sin(4*b*x + 4*a) - 36*I*d^3*\sin(2*b*x + 2*a) + 36*d^3)*\text{polylog}(4, e^{(I*b*x + I*a)}) - (-24*I*d^3*\cos(6*b*x + 6*a) + 24*I*d^3*\cos(4*b*x + 4*a) + 24*I*d^3*\cos(2*b*x + 2*a) + 24*d^3*\sin(6*b*x + 6*a) - 24*d^3*\sin(4*b*x + 4*a) - 24*d^3*\sin(2*b*x + 2*a) - 24*I*d^3)*\text{polylog}(3, I*e^{(I*b*x + I*a)}) - (24*I*d^3*\cos(6*b*x + 6*a) - 24*I*d^3*\cos(4*b*x + 4*a) - 24*I*d^3*\cos(2*b*x + 2*a) - 24*d^3*\sin(6*b*x + 6*a) + 24*d^3*\sin(4*b*x + 4*a) + 24*d^3*\sin(2*b*x + 2*a) + 24*I*d^3)*\text{polylog}(3, -I*e^{(I*b*x + I*a)}) - (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^3 + 36*I*a*d^3)*\cos(6*b*x + 6*a) + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3)*\cos(4*b*x + 4*a) + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3)*\cos(2*b*x + 2*a) + 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\sin(6*b*x + 6*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\sin(4*b*x + 4*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\sin(2*b*x + 2*a))*\text{polylog}(3, -e^{(I*b*x + I*a)}) - (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3 + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3)*\cos(6*b*x + 6*a) + (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^3 + 36*I*a*d^3)*\cos(4*b*x + 4*a) + (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^3 + 36*I*a*d^3)*\cos(2*b*x + 2*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\sin(6*b*x + 6*a) + 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\sin(4*b*x + 4*a) + 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\sin(2*b*x + 2*a))*\text{polylog}(3, e^{(I*b*x + I*a)}) + (12*(b*x + a)^3*d^3 - 12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*a^2*d^3 + (36*b*c*d^2 - (36*a + 12*I)*d^3)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a + 24*I)*b*c*d^2 + 12*(3*a^2 + 2*I*a)*d^3)*(b*x + a))*\sin(5*b*x + 5*a) - 8*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*\sin(3*b*x + 3*a) + (12*(b*x + a)^3*d^3 + 12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^3 + (36*b*c*d^2 - (36*a - 12*I)*d^3)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a - 24*I)*b*c*d^2 + 12*(3*a^2 - 2*I*a)*d^3)*(b*x + a))*\sin(b*x + a))/(-4*I*b^3*\cos(6*b*x + 6*a) + 4*I*b^3*\cos(4*b*x + 4*a) + 4*I*b^3*\cos(2*b*x + 2*a) + 4*b^3*\sin(6*b*x + 6*a) - 4*b^3*\sin(4*b*x + 4*a) - 4*b^3*\sin(2*b*x + 2*a) - 4*I*b^3))/b
\end{aligned}$$

Fricas [C] time = 1.3474, size = 7588, normalized size = 12.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned}
& -1/4*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x + 4*b^3*c^3 - 6*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\cos(b*x + a)^2 - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*\cos(b*x + a)*\sin(b*x + a) - ((-9*I*b^2*d^3*x^2 - 18*I*b^2*c*d^2*x - 9*I*b^2*c^2*d - 6*I*d^3)*\cos(b*x + a)^3 + (9*I*b^2*d^3*x^2 + 18*I*b^2*c*d^2*x + 9*I*b^2*c^2*d + 6*I*d^3)*\cos(b*x + a))*\text{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) - ((9*I*b^2*d^3*x^2 + 18*I*b^2*c*d^2*x + 9*I*b^2*c^2*d + 6*I*d^3)*\cos(b*x + a)^3 + (-9*I*b^2*d^3*x^2 - 18*I*b^2*c*d^2*x - 9*I*b^2*c^2*d - 6*I*d^3)*\cos(b*x + a))*\text{dilog}(\cos(b*x + a) - I*\sin(b*x + a)) - ((12*I*b*d^3*x + 12*I*b*c*d^2)*\cos(b*x + a)^3 + (-12*I*b*d^3*x - 12*I*b*c*d^2)*\cos(b*x + a))*\text{dilog}(I*\cos(b*x + a) + \sin(b*x + a)) - ((12*I*b*d^3*x + 12*I*b*c*d^2)*\cos(b*x + a)^3 + (-12*I*b*d^3*x - 12*I*b*c*d^2)
\end{aligned}$$

$$\begin{aligned}
& * \cos(b*x + a)) * \operatorname{dilog}(I * \cos(b*x + a) - \sin(b*x + a)) - ((-12 * I * b * d^3 * x - 12 * \\
& I * b * c * d^2) * \cos(b*x + a)^3 + (12 * I * b * d^3 * x + 12 * I * b * c * d^2) * \cos(b*x + a)) * \operatorname{dilog} \\
& \operatorname{og}(-I * \cos(b*x + a) + \sin(b*x + a)) - ((-12 * I * b * d^3 * x - 12 * I * b * c * d^2) * \cos(b * \\
& x + a)^3 + (12 * I * b * d^3 * x + 12 * I * b * c * d^2) * \cos(b*x + a)) * \operatorname{dilog}(-I * \cos(b*x + a) \\
&) - \sin(b*x + a)) - ((-9 * I * b^2 * d^3 * x^2 - 18 * I * b^2 * c * d^2 * x - 9 * I * b^2 * c^2 * d - \\
& 6 * I * d^3) * \cos(b*x + a)^3 + (9 * I * b^2 * d^3 * x^2 + 18 * I * b^2 * c * d^2 * x + 9 * I * b^2 * c^2 * \\
& 2 * d + 6 * I * d^3) * \cos(b*x + a)) * \operatorname{dilog}(-\cos(b*x + a) + I * \sin(b*x + a)) - ((9 * I * \\
& b^2 * d^3 * x^2 + 18 * I * b^2 * c * d^2 * x + 9 * I * b^2 * c^2 * d + 6 * I * d^3) * \cos(b*x + a)^3 + \\
& (-9 * I * b^2 * d^3 * x^2 - 18 * I * b^2 * c * d^2 * x - 9 * I * b^2 * c^2 * d - 6 * I * d^3) * \cos(b*x + a) \\
&)) * \operatorname{dilog}(-\cos(b*x + a) - I * \sin(b*x + a)) + 3 * ((b^3 * d^3 * x^3 + 3 * b^3 * c * d^2 * x^2 + \\
& b^3 * c^3 + 2 * b * c * d^2 + (3 * b^3 * c^2 * d + 2 * b * d^3) * x) * \cos(b*x + a)^3 - (b^3 * \\
& d^3 * x^3 + 3 * b^3 * c * d^2 * x^2 + b^3 * c^3 + 2 * b * c * d^2 + (3 * b^3 * c^2 * d + 2 * b * d^3) * x \\
&) * \cos(b*x + a)) * \log(\cos(b*x + a) + I * \sin(b*x + a) + 1) + 6 * ((b^2 * c^2 * d - 2 * \\
& a * b * c * d^2 + a^2 * d^3) * \cos(b*x + a)^3 - (b^2 * c^2 * d - 2 * a * b * c * d^2 + a^2 * d^3) * c \\
& \cos(b*x + a)) * \log(\cos(b*x + a) + I * \sin(b*x + a) + I) + 3 * ((b^3 * d^3 * x^3 + 3 * b \\
& ^3 * c * d^2 * x^2 + b^3 * c^3 + 2 * b * c * d^2 + (3 * b^3 * c^2 * d + 2 * b * d^3) * x) * \cos(b*x + a) \\
&)^3 - (b^3 * d^3 * x^3 + 3 * b^3 * c * d^2 * x^2 + b^3 * c^3 + 2 * b * c * d^2 + (3 * b^3 * c^2 * d + \\
& 2 * b * d^3) * x) * \cos(b*x + a)) * \log(\cos(b*x + a) - I * \sin(b*x + a) + 1) - 6 * ((b^2 \\
& * c^2 * d - 2 * a * b * c * d^2 + a^2 * d^3) * \cos(b*x + a)^3 - (b^2 * c^2 * d - 2 * a * b * c * d^2 + \\
& a^2 * d^3) * \cos(b*x + a)) * \log(\cos(b*x + a) - I * \sin(b*x + a) + I) + 6 * ((b^2 * d^3 * x^2 + \\
& 2 * b^2 * c * d^2 * x + 2 * a * b * c * d^2 - a^2 * d^3) * \cos(b*x + a)^3 - (b^2 * d^3 * x^2 + \\
& 2 * b^2 * c * d^2 * x + 2 * a * b * c * d^2 - a^2 * d^3) * \cos(b*x + a)) * \log(I * \cos(b*x + a) \\
& + \sin(b*x + a) + 1) - 6 * ((b^2 * d^3 * x^2 + 2 * b^2 * c * d^2 * x + 2 * a * b * c * d^2 - a^2 * \\
& d^3) * \cos(b*x + a)^3 - (b^2 * d^3 * x^2 + 2 * b^2 * c * d^2 * x + 2 * a * b * c * d^2 - a^2 * d^3) \\
& * \cos(b*x + a)) * \log(I * \cos(b*x + a) - \sin(b*x + a) + 1) + 6 * ((b^2 * d^3 * x^2 + 2 \\
& * b^2 * c * d^2 * x + 2 * a * b * c * d^2 - a^2 * d^3) * \cos(b*x + a)^3 - (b^2 * d^3 * x^2 + 2 * b^2 \\
& * c * d^2 * x + 2 * a * b * c * d^2 - a^2 * d^3) * \cos(b*x + a)) * \log(-I * \cos(b*x + a) + \sin(b \\
& * x + a) + 1) - 6 * ((b^2 * d^3 * x^2 + 2 * b^2 * c * d^2 * x + 2 * a * b * c * d^2 - a^2 * d^3) * \cos \\
& (b*x + a)^3 - (b^2 * d^3 * x^2 + 2 * b^2 * c * d^2 * x + 2 * a * b * c * d^2 - a^2 * d^3) * \cos(b*x \\
& + a)) * \log(-I * \cos(b*x + a) - \sin(b*x + a) + 1) - 3 * ((b^3 * c^3 - 3 * a * b^2 * c^2 * \\
& d + (3 * a^2 + 2) * b * c * d^2 - (a^3 + 2 * a) * d^3) * \cos(b*x + a)^3 - (b^3 * c^3 - 3 * a * \\
& b^2 * c^2 * d + (3 * a^2 + 2) * b * c * d^2 - (a^3 + 2 * a) * d^3) * \cos(b*x + a)) * \log(-1/2 * c \\
& \cos(b*x + a) + 1/2 * I * \sin(b*x + a) + 1/2) - 3 * ((b^3 * c^3 - 3 * a * b^2 * c^2 * d + (3 * \\
& a^2 + 2) * b * c * d^2 - (a^3 + 2 * a) * d^3) * \cos(b*x + a)^3 - (b^3 * c^3 - 3 * a * b^2 * c^2 * \\
& d + (3 * a^2 + 2) * b * c * d^2 - (a^3 + 2 * a) * d^3) * \cos(b*x + a)) * \log(-1/2 * \cos(b*x \\
& + a) - 1/2 * I * \sin(b*x + a) + 1/2) - 3 * ((b^3 * d^3 * x^3 + 3 * b^3 * c * d^2 * x^2 + 3 * a * \\
& b^2 * c^2 * d - 3 * a^2 * b * c * d^2 + (a^3 + 2 * a) * d^3 + (3 * b^3 * c^2 * d + 2 * b * d^3) * x) * c \\
& \cos(b*x + a)^3 - (b^3 * d^3 * x^3 + 3 * b^3 * c * d^2 * x^2 + 3 * a * b^2 * c^2 * d - 3 * a^2 * b * c * d \\
& ^2 + (a^3 + 2 * a) * d^3 + (3 * b^3 * c^2 * d + 2 * b * d^3) * x) * \cos(b*x + a)) * \log(-\cos(b * \\
& x + a) + I * \sin(b*x + a) + 1) + 6 * ((b^2 * c^2 * d - 2 * a * b * c * d^2 + a^2 * d^3) * \cos(b \\
& * x + a)^3 - (b^2 * c^2 * d - 2 * a * b * c * d^2 + a^2 * d^3) * \cos(b*x + a)) * \log(-\cos(b*x \\
& + a) + I * \sin(b*x + a) + I) - 3 * ((b^3 * d^3 * x^3 + 3 * b^3 * c * d^2 * x^2 + 3 * a * b^2 * c^2 * \\
& 2 * d - 3 * a^2 * b * c * d^2 + (a^3 + 2 * a) * d^3 + (3 * b^3 * c^2 * d + 2 * b * d^3) * x) * \cos(b*x \\
& + a)^3 - (b^3 * d^3 * x^3 + 3 * b^3 * c * d^2 * x^2 + 3 * a * b^2 * c^2 * d - 3 * a^2 * b * c * d^2 + (\\
& a^3 + 2 * a) * d^3 + (3 * b^3 * c^2 * d + 2 * b * d^3) * x) * \cos(b*x + a)) * \log(-\cos(b*x + a) \\
& - I * \sin(b*x + a) + 1) - 6 * ((b^2 * c^2 * d - 2 * a * b * c * d^2 + a^2 * d^3) * \cos(b*x + a) \\
&)^3 - (b^2 * c^2 * d - 2 * a * b * c * d^2 + a^2 * d^3) * \cos(b*x + a)) * \log(-\cos(b*x + a) - \\
& I * \sin(b*x + a) + I) - (18 * I * d^3 * \cos(b*x + a)^3 - 18 * I * d^3 * \cos(b*x + a)) * \operatorname{poly} \\
& \operatorname{log}(4, \cos(b*x + a) + I * \sin(b*x + a)) - (-18 * I * d^3 * \cos(b*x + a)^3 + 18 * I * \\
& d^3 * \cos(b*x + a)) * \operatorname{polylog}(4, \cos(b*x + a) - I * \sin(b*x + a)) - (18 * I * d^3 * \cos \\
& (b*x + a)^3 - 18 * I * d^3 * \cos(b*x + a)) * \operatorname{polylog}(4, -\cos(b*x + a) + I * \sin(b*x + \\
& a)) - (-18 * I * d^3 * \cos(b*x + a)^3 + 18 * I * d^3 * \cos(b*x + a)) * \operatorname{polylog}(4, -\cos(b \\
& * x + a) - I * \sin(b*x + a)) - 18 * ((b * d^3 * x + b * c * d^2) * \cos(b*x + a)^3 - (b * d^3 \\
& * x + b * c * d^2) * \cos(b*x + a)) * \operatorname{polylog}(3, \cos(b*x + a) + I * \sin(b*x + a)) - 18 * \\
& ((b * d^3 * x + b * c * d^2) * \cos(b*x + a)^3 - (b * d^3 * x + b * c * d^2) * \cos(b*x + a)) * \operatorname{pol} \\
& \operatorname{ylog}(3, \cos(b*x + a) - I * \sin(b*x + a)) - 12 * (d^3 * \cos(b*x + a)^3 - d^3 * \cos(b \\
& * x + a)) * \operatorname{polylog}(3, I * \cos(b*x + a) + \sin(b*x + a)) + 12 * (d^3 * \cos(b*x + a)^3 \\
& - d^3 * \cos(b*x + a)) * \operatorname{polylog}(3, I * \cos(b*x + a) - \sin(b*x + a)) - 12 * (d^3 * c \\
& \cos(b*x + a)^3 - d^3 * \cos(b*x + a)) * \operatorname{polylog}(3, -I * \cos(b*x + a) + \sin(b*x + a))
\end{aligned}$$

$$+ 12*(d^3*\cos(b*x + a)^3 - d^3*\cos(b*x + a))*\text{polylog}(3, -I*\cos(b*x + a) - \sin(b*x + a)) + 18*((b*d^3*x + b*c*d^2)*\cos(b*x + a)^3 - (b*d^3*x + b*c*d^2)*\cos(b*x + a))*\text{polylog}(3, -\cos(b*x + a) + I*\sin(b*x + a)) + 18*((b*d^3*x + b*c*d^2)*\cos(b*x + a)^3 - (b*d^3*x + b*c*d^2)*\cos(b*x + a))*\text{polylog}(3, -\cos(b*x + a) - I*\sin(b*x + a))/(b^4*\cos(b*x + a)^3 - b^4*\cos(b*x + a))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*csc(b*x+a)**3*sec(b*x+a)**2,x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")

[Out] Timed out

3.280 $\int (c + dx)^2 \csc^3(a + bx) \sec^2(a + bx) dx$

Optimal. Leaf size=305

$$\frac{3id(c + dx)\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^2} - \frac{3id(c + dx)\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^2} - \frac{2id^2\text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} + \frac{2id^2\text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3}$$

```
[Out] ((4*I)*d^2*x*ArcTan[E^(I*(a + b*x))])/b^2 - (3*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (d^2*ArcTanh[Cos[a + b*x]]/b^3 - (2*c*d*ArcTanh[Sin[a + b*x]])/b^2 - (c*d*Csc[a + b*x])/b^2 - (d^2*x*Csc[a + b*x])/b^2 + ((3*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (3*d^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (3*d^2*PolyLog[3, E^(I*(a + b*x))])/b^3 + (3*(c + d*x)^2*Sec[a + b*x])/(2*b) - ((c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

Rubi [A] time = 0.865244, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 22, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {2622, 288, 321, 207, 4420, 6688, 12, 6742, 6273, 4183, 2531, 2282, 6589, 4133, 453, 206, 4181, 2279, 2391, 2621, 6271, 3770}

$$\frac{3id(c + dx)\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^2} - \frac{3id(c + dx)\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^2} - \frac{2id^2\text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} + \frac{2id^2\text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] ((4*I)*d^2*x*ArcTan[E^(I*(a + b*x))])/b^2 - (3*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (d^2*ArcTanh[Cos[a + b*x]]/b^3 - (2*c*d*ArcTanh[Sin[a + b*x]])/b^2 - (c*d*Csc[a + b*x])/b^2 - (d^2*x*Csc[a + b*x])/b^2 + ((3*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (3*d^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (3*d^2*PolyLog[3, E^(I*(a + b*x))])/b^3 + (3*(c + d*x)^2*Sec[a + b*x])/(2*b) - ((c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

$x] /;$ FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[Rt[b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 4420

Int[Csc[(a_) + (b_)*(x_)]^(n_)*((c_) + (d_)*(x_))^(m_)*Sec[(a_) + (b_)*(x_)]^(p_), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]

Rule 6688

Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifierIntegrandQ[v, u, x]]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rule 6273

Int[((a_) + ArcTanh[u_]*(b_))*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x]] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m + 1, x]]

Rule 4183

Int[csc[(e_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/ (b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/(ff*x)^(n*p), x], x, Cos[e + f*x]/ff, x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_))^(n_)]^(p_.)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e^(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
```

+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

Rule 6271

Int[ArcTanh[u_], x_Symbol] := Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand [(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \int (c + dx)^2 \csc^3(a + bx) \sec^2(a + bx) dx &= -\frac{3(c + dx)^2 \tanh^{-1}(\cos(a + bx))}{2b} + \frac{3(c + dx)^2 \sec(a + bx)}{2b} - \frac{(c + dx)^2 \csc^2(a + bx)}{2b} \\
 &= -\frac{3(c + dx)^2 \tanh^{-1}(\cos(a + bx))}{2b} + \frac{3(c + dx)^2 \sec(a + bx)}{2b} - \frac{(c + dx)^2 \csc^2(a + bx)}{2b} \\
 &= -\frac{3(c + dx)^2 \tanh^{-1}(\cos(a + bx))}{2b} + \frac{3(c + dx)^2 \sec(a + bx)}{2b} - \frac{(c + dx)^2 \csc^2(a + bx)}{2b} \\
 &= -\frac{3(c + dx)^2 \tanh^{-1}(\cos(a + bx))}{2b} + \frac{3(c + dx)^2 \sec(a + bx)}{2b} - \frac{(c + dx)^2 \csc^2(a + bx)}{2b} \\
 &= -\frac{3(c + dx)^2 \tanh^{-1}(\cos(a + bx))}{2b} + \frac{3(c + dx)^2 \sec(a + bx)}{2b} - \frac{(c + dx)^2 \csc^2(a + bx)}{2b} \\
 &= \frac{3(c + dx)^2 \sec(a + bx)}{2b} - \frac{(c + dx)^2 \csc^2(a + bx) \sec(a + bx)}{2b} + \frac{3 \int b(c + dx) \csc^2(a + bx) dx}{2} \\
 &= \frac{3(c + dx)^2 \sec(a + bx)}{2b} - \frac{(c + dx)^2 \csc^2(a + bx) \sec(a + bx)}{2b} + \frac{3}{2} \int (c + dx) \csc^2(a + bx) dx \\
 &= -\frac{3(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{3(c + dx)^2 \sec(a + bx)}{2b} - \frac{(c + dx)^2 \csc^2(a + bx)}{2b} \\
 &= -\frac{3(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{cd \csc(a + bx)}{b^2} + \frac{3id(c + dx) \text{Li}_2(-e^{i(a+bx)})}{b^2} \\
 &= \frac{6id^2x \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{3(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{2cd \tanh^{-1}(\sin(a + bx))}{b^2} \\
 &= \frac{6id^2x \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{3(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{2cd \tanh^{-1}(\sin(a + bx))}{b^2} \\
 &= \frac{6id^2x \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{3(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{d^2 \tanh^{-1}(\cos(a + bx))}{b^3} \\
 &= \frac{6id^2x \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{3(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{d^2 \tanh^{-1}(\cos(a + bx))}{b^3} \\
 &= \frac{4id^2x \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{3(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{d^2 \tanh^{-1}(\cos(a + bx))}{b^3} \\
 &= \frac{4id^2x \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{3(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{d^2 \tanh^{-1}(\cos(a + bx))}{b^3} \\
 &= \frac{4id^2x \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{3(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{d^2 \tanh^{-1}(\cos(a + bx))}{b^3}
 \end{aligned}$$

Mathematica [B] time = 7.9231, size = 889, normalized size = 2.91

$$2 \left[\frac{2 \tan^{-1}(\cot(a)) \tanh^{-1}\left(\frac{\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)}{\sqrt{\cos^2(a)+\sin^2(a)}}\right)}{\sqrt{\cos^2(a)+\sin^2(a)}} - \frac{\csc(a)\left(bx-\tan^{-1}(\cot(a))\right)\left(\log\left(1-e^{i(bx-\tan^{-1}(\cot(a)))}\right)\right)-\log\left(1+e^{i(bx-\tan^{-1}(\cot(a)))}\right)\right)}{\sqrt{\cot^2(a)+1}} + i \operatorname{PolyLog}\left(2,-e^{i(bx-\tan^{-1}(\cot(a)))}\right) \right] b^3$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] ((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a/2 + (b*x)/2]^2)/(8*b) + (3*b^2*c^2*Log[1 - E^(I*(a + b*x))] + 2*d^2*Log[1 - E^(I*(a + b*x))] + 6*b^2*c*d*x*Log[1 - E^(I*(a + b*x))] + 3*b^2*d^2*x^2*Log[1 - E^(I*(a + b*x))] - 3*b^2*c^2*Log[1 + E^(I*(a + b*x))] - 2*d^2*Log[1 + E^(I*(a + b*x))] - 6*b^2*c*d*x*Log[1 + E^(I*(a + b*x))] - 3*b^2*d^2*x^2*Log[1 + E^(I*(a + b*x))] + (6*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] - (6*I)*b*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] - 6*d^2*PolyLog[3, -E^(I*(a + b*x))] + 6*d^2*PolyLog[3, E^(I*(a + b*x))])/(2*b^3) + ((c^2 + 2*c*d*x + d^2*x^2)*Sec[a/2 + (b*x)/2]^2)/(8*b) + ((c + d*x)*Csc[a]*Sec[a]*(-d*Cos[a] + b*c*Sin[a] + b*d*x*Sin[a]))/b^2 - ((4*I)*c*d*ArcTan[(-I)*Sin[a] - I*Cos[a]*Tan[(b*x)/2]]/Sqrt[Cos[a]^2 + Sin[a]^2])/b^2*Sqrt[Cos[a]^2 + Sin[a]^2] - (2*d^2*(-((Csc[a]*((b*x - ArcTan[Cot[a]]))*(Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))]) - Log[1 + E^(I*(b*x - ArcTan[Cot[a]]))]) + I*(PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))]) - PolyLog[2, E^(I*(b*x - ArcTan[Cot[a]]))])])/Sqrt[1 + Cot[a]^2] + (2*ArcTan[Cot[a]]*ArcTanh[(Sin[a] + Cos[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]]/Sqrt[Cos[a]^2 + Sin[a]^2])/b^3 + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-c*d*Sin[(b*x)/2] - d^2*x*Sin[(b*x)/2]))/(2*b^2) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*(c*d*Sin[(b*x)/2] + d^2*x*Sin[(b*x)/2]))/(2*b^2) + (c^2*Sin[(b*x)/2] + 2*c*d*x*Sin[(b*x)/2] + d^2*x^2*Sin[(b*x)/2])/(b*(Cos[a/2] - Sin[a/2])*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) + (-c^2*Sin[(b*x)/2] - 2*c*d*x*Sin[(b*x)/2] - d^2*x^2*Sin[(b*x)/2])/(b*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2]))
```

Maple [B] time = 0.508, size = 802, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

```
[Out] 2*d^2/b^2*ln(1+I*exp(I*(b*x+a)))*x+2*d^2/b^3*ln(1+I*exp(I*(b*x+a)))*a-2*d^2/b^2*ln(1-I*exp(I*(b*x+a)))*x-2*d^2/b^3*ln(1-I*exp(I*(b*x+a)))*a-2*I*d^2/b^3*dilog(1+I*exp(I*(b*x+a)))+2*I*d^2/b^3*dilog(1-I*exp(I*(b*x+a)))-3/2/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2-3/2/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/2/b*d^2*ln(1-exp(I*(b*x+a)))*x^2+4*I*d/b^2*c*arctan(exp(I*(b*x+a)))-4*I*d^2/b^3*a*arctan(exp(I*(b*x+a)))-1/b^3*d^2*ln(exp(I*(b*x+a))+1)+1/b^3*d^2*ln(exp(I*(b*x+a)))-1)-3/b*c*d*ln(exp(I*(b*x+a))+1)*x-3/2/b*c^2*ln(exp(I*(b*x+a))+1)+3/2/b*c^2*ln(exp(I*(b*x+a))-1)-3*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+3*d^2*polylog(3,exp(I*(b*x+a)))/b^3+3/b*c*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*c*d*ln(1-exp(I*(b*x+a)))*a+1/b^2/(exp(2*I*(b*x+a))-1)^2/(exp(2*I*(b*x+a))+1)*(3*d^2*x^2*b*exp(5*I*(b*x+a))+6*c*d*x*b*exp(5*I*(b*x+a))+3*c^2*b*exp(5*I*(b*x+a))-2*d^2*x^2*b*exp(3*I*(b*x+a))-4*c*d*x*b*exp(3*I*(b*x+a))-2*I*d^2*x*exp(5*I*(b*x+a))
```

$$\begin{aligned}
& -2*c^2*b*exp(3*I*(b*x+a))+3*d^2*x^2*b*exp(I*(b*x+a))-2*I*d*c*exp(5*I*(b*x+a)) \\
& +6*c*d*x*b*exp(I*(b*x+a))+3*c^2*b*exp(I*(b*x+a))+2*I*d^2*x*exp(I*(b*x+a)) \\
& +2*I*d*c*exp(I*(b*x+a))+3/2/b^3*d^2*a^2*\ln(\exp(I*(b*x+a))-1)-3/b^2*c*d*a*\ln(\exp(I*(b*x+a))-1) \\
& -3*I/b^2*c*d*polylog(2,\exp(I*(b*x+a)))+3*I/b^2*d^2*polylog(2,-\exp(I*(b*x+a)))*x-3*I/b^2*d^2*polylog(2,\exp(I*(b*x+a)))*x \\
& +3*I/b^2*c*d*polylog(2,-\exp(I*(b*x+a)))
\end{aligned}$$

Maxima [B] time = 5.04162, size = 5157, normalized size = 16.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")

[Out]
$$\begin{aligned}
& 1/4*(c^2*(2*(3*\cos(b*x + a)^2 - 2)/(\cos(b*x + a)^3 - \cos(b*x + a)) - 3*\log(\cos(b*x + a) + 1) + 3*\log(\cos(b*x + a) - 1)) - 2*a*c*d*(2*(3*\cos(b*x + a)^2 - 2)/(\cos(b*x + a)^3 - \cos(b*x + a)) - 3*\log(\cos(b*x + a) + 1) + 3*\log(\cos(b*x + a) - 1))/b + a^2*d^2*(2*(3*\cos(b*x + a)^2 - 2)/(\cos(b*x + a)^3 - \cos(b*x + a)) - 3*\log(\cos(b*x + a) + 1) + 3*\log(\cos(b*x + a) - 1))/b^2 + 4*((8*b*c*d + 8*(b*x + a)*d^2 - 8*a*d^2 + 8*(b*c*d + (b*x + a)*d^2 - a*d^2))*\cos(6*b*x + 6*a) - 8*(b*c*d + (b*x + a)*d^2 - a*d^2))*\cos(4*b*x + 4*a) - 8*(b*c*d + (b*x + a)*d^2 - a*d^2))*\cos(2*b*x + 2*a) - (-8*I*b*c*d - 8*I*(b*x + a)*d^2 + 8*I*a*d^2)*\sin(6*b*x + 6*a) - (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*\sin(4*b*x + 4*a) - (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*\sin(2*b*x + 2*a))*\arctan2(\cos(b*x + a), \sin(b*x + a) + 1) + (8*b*c*d + 8*(b*x + a)*d^2 - 8*a*d^2 + 8*(b*c*d + (b*x + a)*d^2 - a*d^2))*\cos(6*b*x + 6*a) - 8*(b*c*d + (b*x + a)*d^2 - a*d^2))*\cos(4*b*x + 4*a) - 8*(b*c*d + (b*x + a)*d^2 - a*d^2))*\cos(2*b*x + 2*a) - (-8*I*b*c*d - 8*I*(b*x + a)*d^2 + 8*I*a*d^2)*\sin(6*b*x + 6*a) - (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*\sin(4*b*x + 4*a) - (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*\sin(2*b*x + 2*a))*\arctan2(\cos(b*x + a), -\sin(b*x + a) + 1) - (6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2))*(b*x + a) + 4*d^2 + 2*(3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2))*(b*x + a) + 2*d^2)*\cos(6*b*x + 6*a) - 2*(3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2))*(b*x + a) + 2*d^2)*\cos(4*b*x + 4*a) - 2*(3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2))*(b*x + a) + 2*d^2)*\cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a) + 4*I*d^2)*\sin(6*b*x + 6*a) + (-6*I*(b*x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a) - 4*I*d^2)*\sin(4*b*x + 4*a) + (-6*I*(b*x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a) - 4*I*d^2)*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) + (4*d^2*\cos(6*b*x + 6*a) - 4*d^2*\cos(4*b*x + 4*a) - 4*d^2*\cos(2*b*x + 2*a) + 4*I*d^2*\sin(6*b*x + 6*a) - 4*I*d^2*\sin(4*b*x + 4*a) - 4*I*d^2*\sin(2*b*x + 2*a) + 4*d^2)*\arctan2(\sin(b*x + a), \cos(b*x + a) - 1) - (6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2))*(b*x + a) + 6*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\cos(6*b*x + 6*a) - 6*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\cos(4*b*x + 4*a) - 6*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a))*\sin(6*b*x + 6*a) + (-6*I*(b*x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a))*\sin(4*b*x + 4*a) + (-6*I*(b*x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) - (12*I*(b*x + a)^2*d^2 + 8*b*c*d - 8*a*d^2 + (24*I*b*c*d - 8*(3*I*a - 1)*d^2)*(b*x + a))*\cos(5*b*x + 5*a) - (-8*I*(b*x + a)^2*d^2 + (-16*I*b*c*d + 16*I*a*d^2)*(b*x + a))*\cos(3*b*x + 3*a) - (12*I*(b*x + a)^2*d^2 - 8*b*c*d + 8*a*d^2 + (24*I*b*c*d - 8*(3*I*a + 1)*d^2)*(b*x + a))*\cos(b*x + a) + (8*d^2*\cos(6*b*x + 6*a) - 8*d^2*\cos(4*b*x + 4*a) - 8*d^2*\cos(2*b*x + 2*a) + 8*I*d^2*\sin(6*b*x + 6*a) - 8*I*d^2*\sin(4*b*x + 4*a) - 8*I*d^2*\sin(2*b*x + 2*a) + 8*d^2)*\operatorname{dilog}(I*\exp(I*b*x + I*a)) - (8*d^2*\cos(6*b*x + 6*a) - 8*d^2*\cos(4*b*x + 4*a) - 8*d^2*\cos(2*b*x + 2*a) + 8*I
\end{aligned}$$

$$\begin{aligned}
& *d^2*\sin(6*b*x + 6*a) - 8*I*d^2*\sin(4*b*x + 4*a) - 8*I*d^2*\sin(2*b*x + 2*a) \\
& + 8*d^2)*\operatorname{dilog}(-I*e^{(I*b*x + I*a)}) + (12*b*c*d + 12*(b*x + a)*d^2 - 12*a*d^2 \\
& ^2 + 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*\cos(6*b*x + 6*a) - 12*(b*c*d + (b*x \\
& + a)*d^2 - a*d^2)*\cos(4*b*x + 4*a) - 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*\cos \\
& s(2*b*x + 2*a) - (-12*I*b*c*d - 12*I*(b*x + a)*d^2 + 12*I*a*d^2)*\sin(6*b*x \\
& + 6*a) - (12*I*b*c*d + 12*I*(b*x + a)*d^2 - 12*I*a*d^2)*\sin(4*b*x + 4*a) - \\
& (12*I*b*c*d + 12*I*(b*x + a)*d^2 - 12*I*a*d^2)*\sin(2*b*x + 2*a))*\operatorname{dilog}(-e^{(\\
& I*b*x + I*a)}) - (12*b*c*d + 12*(b*x + a)*d^2 - 12*a*d^2 + 12*(b*c*d + (b*x \\
& + a)*d^2 - a*d^2)*\cos(6*b*x + 6*a) - 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*\cos \\
& (4*b*x + 4*a) - 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*\cos(2*b*x + 2*a) + (12*I \\
& *b*c*d + 12*I*(b*x + a)*d^2 - 12*I*a*d^2)*\sin(6*b*x + 6*a) + (-12*I*b*c*d - \\
& 12*I*(b*x + a)*d^2 + 12*I*a*d^2)*\sin(4*b*x + 4*a) + (-12*I*b*c*d - 12*I*(b \\
& *x + a)*d^2 + 12*I*a*d^2)*\sin(2*b*x + 2*a))*\operatorname{dilog}(e^{(I*b*x + I*a)}) - (-3*I* \\
& (b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) - 2*I*d^2 + (-3*I*(b*x \\
& + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) - 2*I*d^2)*\cos(6*b*x + 6*a \\
&) + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a) + 2*I*d^2)*\cos \\
& (4*b*x + 4*a) + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a) + \\
& 2*I*d^2)*\cos(2*b*x + 2*a) + (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a \\
&) + 2*d^2)*\sin(6*b*x + 6*a) - (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + \\
& a) + 2*d^2)*\sin(4*b*x + 4*a) - (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x \\
& + a) + 2*d^2)*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos \\
& s(b*x + a) + 1) - (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a) \\
& + 2*I*d^2 + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a) + 2*I* \\
& d^2)*\cos(6*b*x + 6*a) + (-3*I*(b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b \\
& *x + a) - 2*I*d^2)*\cos(4*b*x + 4*a) + (-3*I*(b*x + a)^2*d^2 + (-6*I*b*c*d + \\
& 6*I*a*d^2)*(b*x + a) - 2*I*d^2)*\cos(2*b*x + 2*a) - (3*(b*x + a)^2*d^2 + 6* \\
& (b*c*d - a*d^2)*(b*x + a) + 2*d^2)*\sin(6*b*x + 6*a) + (3*(b*x + a)^2*d^2 + \\
& 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*\sin(4*b*x + 4*a) + (3*(b*x + a)^2*d^2 \\
& + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 \\
& + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + \\
& 4*I*a*d^2 + (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*\cos(6*b*x + 6*a) \\
& + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*\cos(4*b*x + 4*a) + (4*I*b*c*d \\
& + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*\cos(2*b*x + 2*a) + 4*(b*c*d + (b*x + a)*d^2 \\
& ^2 - a*d^2)*\sin(6*b*x + 6*a) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*\sin(4*b*x \\
& + 4*a) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*\sin(2*b*x + 2*a))*\log(\cos(b*x + \\
& a)^2 + \sin(b*x + a)^2 + 2*\sin(b*x + a) + 1) - (4*I*b*c*d + 4*I*(b*x + a)*d^2 \\
& ^2 - 4*I*a*d^2 + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*\cos(6*b*x + 6*a \\
&) + (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*\cos(4*b*x + 4*a) + (-4*I*b \\
& *c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*\cos(2*b*x + 2*a) - 4*(b*c*d + (b*x + \\
& a)*d^2 - a*d^2)*\sin(6*b*x + 6*a) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*\sin(4* \\
& b*x + 4*a) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*\sin(2*b*x + 2*a))*\log(\cos(b* \\
& x + a)^2 + \sin(b*x + a)^2 - 2*\sin(b*x + a) + 1) - (-12*I*d^2*\cos(6*b*x + 6* \\
& a) + 12*I*d^2*\cos(4*b*x + 4*a) + 12*I*d^2*\cos(2*b*x + 2*a) + 12*d^2*\sin(6*b \\
& *x + 6*a) - 12*d^2*\sin(4*b*x + 4*a) - 12*d^2*\sin(2*b*x + 2*a) - 12*I*d^2)*\operatorname{p} \\
& olylog(3, -e^{(I*b*x + I*a)}) - (12*I*d^2*\cos(6*b*x + 6*a) - 12*I*d^2*\cos(4*b \\
& *x + 4*a) - 12*I*d^2*\cos(2*b*x + 2*a) - 12*d^2*\sin(6*b*x + 6*a) + 12*d^2*\operatorname{si} \\
& n(4*b*x + 4*a) + 12*d^2*\sin(2*b*x + 2*a) + 12*I*d^2)*\operatorname{polylog}(3, e^{(I*b*x + \\
& I*a)}) + (12*(b*x + a)^2*d^2 - 8*I*b*c*d + 8*I*a*d^2 + (24*b*c*d - (24*a + 8 \\
& *I)*d^2)*(b*x + a))*\sin(5*b*x + 5*a) - 8*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2) \\
& *(b*x + a))*\sin(3*b*x + 3*a) + (12*(b*x + a)^2*d^2 + 8*I*b*c*d - 8*I*a*d^2 \\
& + (24*b*c*d - (24*a - 8*I)*d^2)*(b*x + a))*\sin(b*x + a))/(-4*I*b^2*\cos(6* \\
& b*x + 6*a) + 4*I*b^2*\cos(4*b*x + 4*a) + 4*I*b^2*\cos(2*b*x + 2*a) + 4*b^2*\operatorname{si} \\
& n(6*b*x + 6*a) - 4*b^2*\sin(4*b*x + 4*a) - 4*b^2*\sin(2*b*x + 2*a) - 4*I*b^2) \\
&)/b
\end{aligned}$$

Fricas [C] time = 0.958072, size = 4454, normalized size = 14.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")

[Out]
$$-1/4*(4*b^2*d^2*x^2 + 8*b^2*c*d*x + 4*b^2*c^2 - 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(b*x + a)^2 - 4*(b*d^2*x + b*c*d)*\cos(b*x + a)*\sin(b*x + a) - ((-6*I*b*d^2*x - 6*I*b*c*d)*\cos(b*x + a)^3 + (6*I*b*d^2*x + 6*I*b*c*d)*\cos(b*x + a))*\operatorname{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) - ((6*I*b*d^2*x + 6*I*b*c*d)*\cos(b*x + a)^3 + (-6*I*b*d^2*x - 6*I*b*c*d)*\cos(b*x + a))*\operatorname{dilog}(\cos(b*x + a) - I*\sin(b*x + a)) - (4*I*d^2*\cos(b*x + a)^3 - 4*I*d^2*\cos(b*x + a))*\operatorname{dilog}(I*\cos(b*x + a) + \sin(b*x + a)) - (4*I*d^2*\cos(b*x + a)^3 - 4*I*d^2*\cos(b*x + a))*\operatorname{dilog}(I*\cos(b*x + a) - \sin(b*x + a)) - (-4*I*d^2*\cos(b*x + a)^3 + 4*I*d^2*\cos(b*x + a))*\operatorname{dilog}(-I*\cos(b*x + a) + \sin(b*x + a)) - (-4*I*d^2*\cos(b*x + a)^3 + 4*I*d^2*\cos(b*x + a))*\operatorname{dilog}(-I*\cos(b*x + a) - \sin(b*x + a)) - ((-6*I*b*d^2*x - 6*I*b*c*d)*\cos(b*x + a)^3 + (6*I*b*d^2*x + 6*I*b*c*d)*\cos(b*x + a))*\operatorname{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) - ((6*I*b*d^2*x + 6*I*b*c*d)*\cos(b*x + a)^3 + (-6*I*b*d^2*x - 6*I*b*c*d)*\cos(b*x + a))*\operatorname{dilog}(-\cos(b*x + a) - I*\sin(b*x + a)) + ((3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*\cos(b*x + a)^3 - (3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*\cos(b*x + a))*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) + 4*((b*c*d - a*d^2)*\cos(b*x + a)^3 - (b*c*d - a*d^2)*\cos(b*x + a))*\log(\cos(b*x + a) + I*\sin(b*x + a) + I) + ((3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*\cos(b*x + a)^3 - (3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*\cos(b*x + a))*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1) - 4*((b*c*d - a*d^2)*\cos(b*x + a)^3 - (b*c*d - a*d^2)*\cos(b*x + a))*\log(\cos(b*x + a) - I*\sin(b*x + a) + I) + 4*((b*d^2*x + a*d^2)*\cos(b*x + a)^3 - (b*d^2*x + a*d^2)*\cos(b*x + a))*\log(I*\cos(b*x + a) + \sin(b*x + a) + 1) - 4*((b*d^2*x + a*d^2)*\cos(b*x + a)^3 - (b*d^2*x + a*d^2)*\cos(b*x + a))*\log(I*\cos(b*x + a) - \sin(b*x + a) + 1) + 4*((b*d^2*x + a*d^2)*\cos(b*x + a)^3 - (b*d^2*x + a*d^2)*\cos(b*x + a))*\log(-I*\cos(b*x + a) + \sin(b*x + a) + 1) - 4*((b*d^2*x + a*d^2)*\cos(b*x + a)^3 - (b*d^2*x + a*d^2)*\cos(b*x + a))*\log(-I*\cos(b*x + a) - \sin(b*x + a) + 1) - ((3*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*\cos(b*x + a)^3 - (3*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*\cos(b*x + a))*\log(-1/2*\cos(b*x + a) + 1/2*I*\sin(b*x + a) + 1/2) - ((3*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*\cos(b*x + a)^3 - (3*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*\cos(b*x + a))*\log(-1/2*\cos(b*x + a) - 1/2*I*\sin(b*x + a) + 1/2) - 3*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\cos(b*x + a)^3 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\cos(b*x + a))*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1) + 4*((b*c*d - a*d^2)*\cos(b*x + a)^3 - (b*c*d - a*d^2)*\cos(b*x + a))*\log(-\cos(b*x + a) + I*\sin(b*x + a) + I) - 3*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\cos(b*x + a)^3 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\cos(b*x + a))*\log(-\cos(b*x + a) - I*\sin(b*x + a) + 1) - 4*((b*c*d - a*d^2)*\cos(b*x + a)^3 - (b*c*d - a*d^2)*\cos(b*x + a))*\log(-\cos(b*x + a) - I*\sin(b*x + a) + I) - 6*(d^2*\cos(b*x + a)^3 - d^2*\cos(b*x + a))*\operatorname{polylog}(3, \cos(b*x + a) + I*\sin(b*x + a)) - 6*(d^2*\cos(b*x + a)^3 - d^2*\cos(b*x + a))*\operatorname{polylog}(3, \cos(b*x + a) - I*\sin(b*x + a)) + 6*(d^2*\cos(b*x + a)^3 - d^2*\cos(b*x + a))*\operatorname{polylog}(3, -\cos(b*x + a) + I*\sin(b*x + a)) + 6*(d^2*\cos(b*x + a)^3 - d^2*\cos(b*x + a))*\operatorname{polylog}(3, -\cos(b*x + a) - I*\sin(b*x + a)))/(b^3*\cos(b*x + a)^3 - b^3*\cos(b*x + a))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*csc(b*x+a)**3*sec(b*x+a)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \csc(bx + a)^3 \sec(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^2*csc(b*x + a)^3*sec(b*x + a)^2, x)

3.281 $\int (c + dx) \csc^3(a + bx) \sec^2(a + bx) dx$

Optimal. Leaf size=154

$$\frac{3 \operatorname{IdPolyLog}\left(2, -e^{i(a+bx)}\right)}{2b^2} - \frac{3 \operatorname{IdPolyLog}\left(2, e^{i(a+bx)}\right)}{2b^2} - \frac{d \csc(a+bx)}{2b^2} - \frac{d \tanh^{-1}(\sin(a+bx))}{b^2} + \frac{3(c+dx) \sec(a+bx)}{2b}$$

[Out] $(-3*d*x*ArcTanh[E^{I*(a + b*x)}])/b - (3*c*ArcTanh[Cos[a + b*x]])/(2*b) - (d*ArcTanh[Sin[a + b*x]])/b^2 - (d*Csc[a + b*x])/(2*b^2) + (((3*I)/2)*d*PolyLog[2, -E^{I*(a + b*x)}])/b^2 - (((3*I)/2)*d*PolyLog[2, E^{I*(a + b*x)}])/b^2 + (3*(c + d*x)*Sec[a + b*x])/(2*b) - ((c + d*x)*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)$

Rubi [A] time = 0.191378, antiderivative size = 174, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {2622, 288, 321, 207, 4420, 6271, 12, 4183, 2279, 2391, 3770, 2621}

$$\frac{3 \operatorname{IdPolyLog}\left(2, -e^{i(a+bx)}\right)}{2b^2} - \frac{3 \operatorname{IdPolyLog}\left(2, e^{i(a+bx)}\right)}{2b^2} - \frac{d \csc(a+bx)}{2b^2} - \frac{d \tanh^{-1}(\sin(a+bx))}{b^2} + \frac{3(c+dx) \sec(a+bx)}{2b}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^2, x]$

[Out] $(-3*d*x*ArcTanh[E^{I*(a + b*x)}])/b + (3*d*x*ArcTanh[Cos[a + b*x]])/(2*b) - (3*(c + d*x)*ArcTanh[Cos[a + b*x]])/(2*b) - (d*ArcTanh[Sin[a + b*x]])/b^2 - (d*Csc[a + b*x])/(2*b^2) + (((3*I)/2)*d*PolyLog[2, -E^{I*(a + b*x)}])/b^2 - (((3*I)/2)*d*PolyLog[2, E^{I*(a + b*x)}])/b^2 + (3*(c + d*x)*Sec[a + b*x])/(2*b) - ((c + d*x)*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)$

Rule 2622

$\operatorname{Int}[\csc[(e_{.}) + (f_{.})*(x_{.})]^{(n_{.})} * ((a_{.}) * \sec[(e_{.}) + (f_{.})*(x_{.})])^{(m_{.})}, x_Symbol] \rightarrow \operatorname{Dist}[1/(f*a^n), \operatorname{Subst}[\operatorname{Int}[x^{(m+n-1)} / (-1 + x^2/a^2)^{((n+1)/2)}, x], x, a*Sec[e + f*x]], x] /;$ $\operatorname{FreeQ}\{a, e, f, m\}, x$ && $\operatorname{IntegerQ}[(n+1)/2]$ && $\operatorname{IntegerQ}[(m+1)/2]$ && $\operatorname{LtQ}[0, m, n]$

Rule 288

$\operatorname{Int}[(c_{.}) * (x_{.})^{(m_{.})} * ((a_{.}) + (b_{.}) * (x_{.})^{(n_{.})})^{(p_{.})}, x_Symbol] \rightarrow \operatorname{Simp}[(c^{(n-1)} * (c*x)^{(m-n+1)} * (a + b*x^n)^{(p+1)}) / (b*n*(p+1)), x] - \operatorname{Dist}[(c^n * (m-n+1)) / (b*n*(p+1)), \operatorname{Int}[(c*x)^{(m-n)} * (a + b*x^n)^{(p+1)}, x], x] /;$ $\operatorname{FreeQ}\{a, b, c\}, x$ && $\operatorname{IGtQ}[n, 0]$ && $\operatorname{LtQ}[p, -1]$ && $\operatorname{GtQ}[m+1, n]$ && $\operatorname{LtQ}[(m+n*(p+1)+1)/n, 0]$ && $\operatorname{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 321

$\operatorname{Int}[(c_{.}) * (x_{.})^{(m_{.})} * ((a_{.}) + (b_{.}) * (x_{.})^{(n_{.})})^{(p_{.})}, x_Symbol] \rightarrow \operatorname{Simp}[(c^{(n-1)} * (c*x)^{(m-n+1)} * (a + b*x^n)^{(p+1)}) / (b*(m+n*p+1)), x] - \operatorname{Dist}[(a*c^n * (m-n+1)) / (b*(m+n*p+1)), \operatorname{Int}[(c*x)^{(m-n)} * (a + b*x^n)^p, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, p\}, x$ && $\operatorname{IGtQ}[n, 0]$ && $\operatorname{GtQ}[m, n-1]$ && $\operatorname{NeQ}[m+n*p+1, 0]$ && $\operatorname{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 207

$\operatorname{Int}[(a_{.}) + (b_{.}) * (x_{.})^2]^{(-1)}, x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[(\operatorname{Rt}[b, 2]*x) / \operatorname{Rt}[-a, 2]] / (\operatorname{Rt}[-a, 2] * \operatorname{Rt}[b, 2]), x] /;$ $\operatorname{FreeQ}\{a, b\}, x$ && $\operatorname{NegQ}[a/b]$ && $\operatorname{LtQ}[a$

, 0] || GtQ[b, 0])

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] := Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned} \int (c + dx) \csc^3(a + bx) \sec^2(a + bx) dx &= -\frac{3(c + dx) \tanh^{-1}(\cos(a + bx))}{2b} + \frac{3(c + dx) \sec(a + bx)}{2b} - \frac{(c + dx) \csc^2(a + bx)}{2b} \\ &= -\frac{3(c + dx) \tanh^{-1}(\cos(a + bx))}{2b} + \frac{3(c + dx) \sec(a + bx)}{2b} - \frac{(c + dx) \csc^2(a + bx)}{2b} \\ &= \frac{3dx \tanh^{-1}(\cos(a + bx))}{2b} - \frac{3(c + dx) \tanh^{-1}(\cos(a + bx))}{2b} - \frac{3d \tanh^{-1}(\sin(a + bx))}{2b^2} \\ &= \frac{3dx \tanh^{-1}(\cos(a + bx))}{2b} - \frac{3(c + dx) \tanh^{-1}(\cos(a + bx))}{2b} - \frac{3d \tanh^{-1}(\sin(a + bx))}{2b^2} \\ &= -\frac{3dx \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{3dx \tanh^{-1}(\cos(a + bx))}{2b} - \frac{3(c + dx) \tanh^{-1}(\cos(a + bx))}{2b} \\ &= -\frac{3dx \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{3dx \tanh^{-1}(\cos(a + bx))}{2b} - \frac{3(c + dx) \tanh^{-1}(\cos(a + bx))}{2b} \\ &= -\frac{3dx \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{3dx \tanh^{-1}(\cos(a + bx))}{2b} - \frac{3(c + dx) \tanh^{-1}(\cos(a + bx))}{2b} \end{aligned}$$

Mathematica [B] time = 5.32919, size = 520, normalized size = 3.38

$$\frac{3d \left(i \left(\text{PolyLog} \left(2, -e^{i(a+bx)} \right) - \text{PolyLog} \left(2, e^{i(a+bx)} \right) \right) + (a + bx) \left(\log \left(1 - e^{i(a+bx)} \right) - \log \left(1 + e^{i(a+bx)} \right) \right) \right)}{2b^2} - \frac{d \tan \left(\frac{1}{2}(a + bx) \right)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (d*x)/b - (d*Cot[(a + b*x)/2])/(4*b^2) - (c*Csc[(a + b*x)/2]^2)/(8*b) - (d*x*Csc[(a + b*x)/2]^2)/(8*b) - (3*c*Log[Cos[(a + b*x)/2]])/(2*b) + (d*Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]])/b^2 + (3*c*Log[Sin[(a + b*x)/2]])/(2*b) - (d*Log[Cos[(a + b*x)/2] + Sin[(a + b*x)/2]])/b^2 - (3*a*d*Log[Tan[(a + b*x)/2]])/(2*b^2) + (3*d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a + b*x))]))/(2*b^2) + (c*Sec[(a + b*x)/2]^2)/(8*b) + (d*x*Sec[(a + b*x)/2]^2)/(8*b) + (c*Sin[(a + b*x)/2])/(b*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])) - (c*Sin[(a + b*x)/2])/(b*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])) + (d*(a*Sin[(a + b*x)/2] - (a + b*x)*Sin[(a + b*x)/2]))/(b^2*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])) + (d*(-a*Sin[(a + b*x)/2] + (a + b*x)*Sin[(a + b*x)/2]))/(b^2*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])) - (d*Tan[(a + b*x)/2])/(4*b^2)
```

Maple [A] time = 0.352, size = 267, normalized size = 1.7

$$\frac{3 dxbe^{5i(bx+a)} + 3 bce^{5i(bx+a)} - 2 dxbe^{3i(bx+a)} - 2 bce^{3i(bx+a)} - ide^{5i(bx+a)} + 3 dxbe^{i(bx+a)} + 3 bce^{i(bx+a)} + ide^{i(bx+a)}}{b^2 (e^{2i(bx+a)} - 1)^2 (e^{2i(bx+a)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

```
[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2/(exp(2*I*(b*x+a))+1)*(3*d*x*b*exp(5*I*(b*x+a))+3*b*c*exp(5*I*(b*x+a))-2*d*x*b*exp(3*I*(b*x+a))-2*b*c*exp(3*I*(b*x+a))-I*d
```

```
*exp(5*I*(b*x+a))+3*d*x*b*exp(I*(b*x+a))+3*b*c*exp(I*(b*x+a))+I*d*exp(I*(b*x+a)))-3/2/b*c*ln(exp(I*(b*x+a))+1)+3/2/b*c*ln(exp(I*(b*x+a))-1)-3/2/b^2*d*a*ln(exp(I*(b*x+a))-1)+2*I/b^2*d*arctan(exp(I*(b*x+a)))+3/2*I/b^2*d*dilog(exp(I*(b*x+a))+1)-3/2/b*d*ln(exp(I*(b*x+a))+1)*x+3/2*I/b^2*d*dilog(exp(I*(b*x+a)))
```

Maxima [B] time = 2.80594, size = 2029, normalized size = 13.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -((4*d*cos(6*b*x + 6*a) - 4*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) + 4*I*d*sin(6*b*x + 6*a) - 4*I*d*sin(4*b*x + 4*a) - 4*I*d*sin(2*b*x + 2*a) + 4*d)*arctan2(2*(cos(b*x + 2*a)*cos(a) + sin(b*x + 2*a)*sin(a))/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2), (cos(b*x + 2*a)^2 - cos(a)^2 + sin(b*x + 2*a)^2 - sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) + (6*b*d*x + 6*b*c + 6*(b*d*x + b*c)*cos(6*b*x + 6*a) - 6*(b*d*x + b*c)*cos(4*b*x + 4*a) - 6*(b*d*x + b*c)*cos(2*b*x + 2*a) - (-6*I*b*d*x - 6*I*b*c)*sin(6*b*x + 6*a) - (6*I*b*d*x + 6*I*b*c)*sin(4*b*x + 4*a) - (6*I*b*d*x + 6*I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (6*b*c*cos(6*b*x + 6*a) - 6*b*c*cos(4*b*x + 4*a) - 6*b*c*cos(2*b*x + 2*a) + 6*I*b*c*sin(6*b*x + 6*a) - 6*I*b*c*sin(4*b*x + 4*a) - 6*I*b*c*sin(2*b*x + 2*a) + 6*b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (6*b*d*x*cos(6*b*x + 6*a) - 6*b*d*x*cos(4*b*x + 4*a) - 6*b*d*x*cos(2*b*x + 2*a) + 6*I*b*d*x*sin(6*b*x + 6*a) - 6*I*b*d*x*sin(4*b*x + 4*a) - 6*I*b*d*x*sin(2*b*x + 2*a) + 6*b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (-12*I*b*d*x - 12*I*b*c - 4*d)*cos(5*b*x + 5*a) - (8*I*b*d*x + 8*I*b*c)*cos(3*b*x + 3*a) - (-12*I*b*d*x - 12*I*b*c + 4*d)*cos(b*x + a) - (6*d*cos(6*b*x + 6*a) - 6*d*cos(4*b*x + 4*a) - 6*d*cos(2*b*x + 2*a) + 6*I*d*sin(6*b*x + 6*a) - 6*I*d*sin(4*b*x + 4*a) - 6*I*d*sin(2*b*x + 2*a) + 6*d)*dilog(-e^(I*b*x + I*a)) + (6*d*cos(6*b*x + 6*a) - 6*d*cos(4*b*x + 4*a) - 6*d*cos(2*b*x + 2*a) + 6*I*d*sin(6*b*x + 6*a) - 6*I*d*sin(4*b*x + 4*a) - 6*I*d*sin(2*b*x + 2*a) + 6*d)*dilog(e^(I*b*x + I*a)) - (3*I*b*d*x + 3*I*b*c + (3*I*b*d*x + 3*I*b*c)*cos(6*b*x + 6*a) + (-3*I*b*d*x - 3*I*b*c)*cos(4*b*x + 4*a) + (-3*I*b*d*x - 3*I*b*c)*cos(2*b*x + 2*a) - 3*(b*d*x + b*c)*sin(6*b*x + 6*a) + 3*(b*d*x + b*c)*sin(4*b*x + 4*a) + 3*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-3*I*b*d*x - 3*I*b*c + (-3*I*b*d*x - 3*I*b*c)*cos(6*b*x + 6*a) + (3*I*b*d*x + 3*I*b*c)*cos(4*b*x + 4*a) + (3*I*b*d*x + 3*I*b*c)*cos(2*b*x + 2*a) + 3*(b*d*x + b*c)*sin(6*b*x + 6*a) - 3*(b*d*x + b*c)*sin(4*b*x + 4*a) - 3*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-2*I*d*cos(6*b*x + 6*a) + 2*I*d*cos(4*b*x + 4*a) + 2*I*d*cos(2*b*x + 2*a) + 2*d*sin(6*b*x + 6*a) - 2*d*sin(4*b*x + 4*a) - 2*d*sin(2*b*x + 2*a) - 2*I*d)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(3*b*d*x + 3*b*c - I*d)*sin(5*b*x + 5*a) + 8*(b*d*x + b*c)*sin(3*b*x + 3*a) - 4*(3*b*d*x + 3*b*c + I*d)*sin(b*x + a))/(-4*I*b^2*cos(6*b*x + 6*a) + 4*I*b^2*cos(4*b*x + 4*a) + 4*I*b^2*cos(2*b*x + 2*a) + 4*b^2*sin(6*b*x + 6*a) - 4*b^2*sin(4*b*x + 4*a) - 4*b^2*sin(2*b*x + 2*a) - 4*I*b^2)
```

Fricas [B] time = 0.633759, size = 1652, normalized size = 10.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/4*(4*b*d*x - 6*(b*d*x + b*c)*\cos(b*x + a)^2 - 2*d*\cos(b*x + a)*\sin(b*x + a) + 4*b*c - (-3*I*d*\cos(b*x + a)^3 + 3*I*d*\cos(b*x + a))*\operatorname{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) - (3*I*d*\cos(b*x + a)^3 - 3*I*d*\cos(b*x + a))*\operatorname{dilog}(\cos(b*x + a) - I*\sin(b*x + a)) - (-3*I*d*\cos(b*x + a)^3 + 3*I*d*\cos(b*x + a))*\operatorname{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) - (3*I*d*\cos(b*x + a)^3 - 3*I*d*\cos(b*x + a))*\operatorname{dilog}(-\cos(b*x + a) - I*\sin(b*x + a)) + 3*((b*d*x + b*c)*\cos(b*x + a)^3 - (b*d*x + b*c)*\cos(b*x + a))*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) + 3*((b*d*x + b*c)*\cos(b*x + a)^3 - (b*d*x + b*c)*\cos(b*x + a))*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1) - 3*((b*c - a*d)*\cos(b*x + a)^3 - (b*c - a*d)*\cos(b*x + a))*\log(-1/2*\cos(b*x + a) + 1/2*I*\sin(b*x + a) + 1/2) - 3*((b*c - a*d)*\cos(b*x + a)^3 - (b*c - a*d)*\cos(b*x + a))*\log(-1/2*\cos(b*x + a) - 1/2*I*\sin(b*x + a) + 1/2) - 3*((b*d*x + a*d)*\cos(b*x + a)^3 - (b*d*x + a*d)*\cos(b*x + a))*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1) - 3*((b*d*x + a*d)*\cos(b*x + a)^3 - (b*d*x + a*d)*\cos(b*x + a))*\log(-\cos(b*x + a) - I*\sin(b*x + a) + 1) + 2*(d*\cos(b*x + a)^3 - d*\cos(b*x + a))*\log(\sin(b*x + a) + 1) - 2*(d*\cos(b*x + a)^3 - d*\cos(b*x + a))*\log(-\sin(b*x + a) + 1))/(b^2*\cos(b*x + a)^3 - b^2*\cos(b*x + a)) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)**3*sec(b*x+a)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \csc(bx + a)^3 \sec(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)*csc(b*x + a)^3*sec(b*x + a)^2, x)

$$3.282 \quad \int \frac{\csc^3(a+bx) \sec^2(a+bx)}{c+dx} dx$$

Optimal. Leaf size=26

$$\text{CannotIntegrate}\left(\frac{\csc^3(a+bx) \sec^2(a+bx)}{c+dx}, x\right)$$

[Out] CannotIntegrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x), x]

Rubi [A] time = 0.201175, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^3(a+bx) \sec^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x), x]

[Out] Defer[Int] [(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x), x]

Rubi steps

$$\int \frac{\csc^3(a+bx) \sec^2(a+bx)}{c+dx} dx = \int \frac{\csc^3(a+bx) \sec^2(a+bx)}{c+dx} dx$$

Mathematica [A] time = 23.0601, size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx) \sec^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x), x]

[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x), x]

Maple [A] time = 2.755, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx+a))^3 (\sec(bx+a))^2}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c), x)

[Out] int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c), x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)^3 \sec(bx+a)^2}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")

[Out] integral(csc(b*x + a)^3*sec(b*x + a)^2/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx) \sec^2(a+bx)}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)**3*sec(b*x+a)**2/(d*x+c),x)

[Out] Integral(csc(a + b*x)**3*sec(a + b*x)**2/(c + d*x), x)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x, algorithm="giac")

[Out] Timed out

$$3.283 \quad \int \frac{\csc^3(a+bx) \sec^2(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=26

$$\text{CannotIntegrate}\left(\frac{\csc^3(a+bx) \sec^2(a+bx)}{(c+dx)^2}, x\right)$$

[Out] CannotIntegrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2, x]

Rubi [A] time = 0.224429, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^3(a+bx) \sec^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2,x]

[Out] Defer[Int] [(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\csc^3(a+bx) \sec^2(a+bx)}{(c+dx)^2} dx = \int \frac{\csc^3(a+bx) \sec^2(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 31.8605, size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx) \sec^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2,x]

[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2, x]

Maple [A] time = 4.368, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx+a))^3 (\sec(bx+a))^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x)

[Out] int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)^3 \sec(bx+a)^2}{d^2x^2 + 2cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(csc(b*x + a)^3*sec(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)**3*sec(b*x+a)**2/(d*x+c)**2,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^3 \sec(bx+a)^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(csc(b*x + a)^3*sec(b*x + a)^2/(d*x + c)^2, x)

$$3.284 \quad \int x^m \csc^3(a + bx) \sec^2(a + bx) dx$$

Optimal. Leaf size=22

$$\text{CannotIntegrate}(x^m \csc^3(a + bx) \sec^2(a + bx), x)$$

[Out] CannotIntegrate[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]

Rubi [A] time = 0.923652, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^m \csc^3(a + bx) \sec^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]

[Out] Defer[Int][x^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]

Rubi steps

$$\int x^m \csc^3(a + bx) \sec^2(a + bx) dx = \int x^m \csc^3(a + bx) \sec^2(a + bx) dx$$

Mathematica [A] time = 11.3338, size = 0, normalized size = 0.

$$\int x^m \csc^3(a + bx) \sec^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]

[Out] Integrate[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]

Maple [A] time = 0.102, size = 0, normalized size = 0.

$$\int x^m (\csc(bx + a))^3 (\sec(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*csc(b*x+a)^3*sec(b*x+a)^2, x)

[Out] int(x^m*csc(b*x+a)^3*sec(b*x+a)^2, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \csc(bx + a)^3 \sec(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^m \csc(bx + a)^3 \sec(bx + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \csc(bx + a)^3 \sec(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

3.285 $\int x^3 \csc^3(a + bx) \sec^2(a + bx) dx$

Optimal. Leaf size=387

$$\frac{9ix^2 \text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{2b^2} - \frac{9ix^2 \text{PolyLog}\left(2, e^{i(a+bx)}\right)}{2b^2} - \frac{6ix \text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} + \frac{6ix \text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3} - \frac{9x \text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^2} + \frac{9x \text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^2}$$

```
[Out] ((6*I)*x^2*ArcTan[E^(I*(a + b*x))])/b^2 - (6*x*ArcTanh[E^(I*(a + b*x))])/b^3 - (3*x^3*ArcTanh[E^(I*(a + b*x))])/b - (3*x^2*Csc[a + b*x])/(2*b^2) + ((3*I)*PolyLog[2, -E^(I*(a + b*x))])/b^4 + (((9*I)/2)*x^2*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((6*I)*x*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*x*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*PolyLog[2, E^(I*(a + b*x))])/b^4 - (((9*I)/2)*x^2*PolyLog[2, E^(I*(a + b*x))])/b^2 - (9*x*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (6*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*PolyLog[3, I*E^(I*(a + b*x))])/b^4 + (9*x*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((9*I)*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((9*I)*PolyLog[4, E^(I*(a + b*x))])/b^4 + (3*x^3*Sec[a + b*x])/(2*b) - (x^3*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

Rubi [A] time = 0.960072, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 40, number of rules used = 18, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.9$, Rules used = {2622, 288, 321, 207, 4420, 14, 6273, 12, 4183, 2531, 6609, 2282, 6589, 6742, 4181, 2621, 2279, 2391}

$$\frac{9ix^2 \text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{2b^2} - \frac{9ix^2 \text{PolyLog}\left(2, e^{i(a+bx)}\right)}{2b^2} - \frac{6ix \text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} + \frac{6ix \text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3} - \frac{9x \text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^2} + \frac{9x \text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] ((6*I)*x^2*ArcTan[E^(I*(a + b*x))])/b^2 - (6*x*ArcTanh[E^(I*(a + b*x))])/b^3 - (3*x^3*ArcTanh[E^(I*(a + b*x))])/b - (3*x^2*Csc[a + b*x])/(2*b^2) + ((3*I)*PolyLog[2, -E^(I*(a + b*x))])/b^4 + (((9*I)/2)*x^2*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((6*I)*x*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*x*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*PolyLog[2, E^(I*(a + b*x))])/b^4 - (((9*I)/2)*x^2*PolyLog[2, E^(I*(a + b*x))])/b^2 - (9*x*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (6*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*PolyLog[3, I*E^(I*(a + b*x))])/b^4 + (9*x*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((9*I)*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((9*I)*PolyLog[4, E^(I*(a + b*x))])/b^4 + (3*x^3*Sec[a + b*x])/(2*b) - (x^3*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] := Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 4420

Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 6273

Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(c + d*x)^(m + 1)*(a + b*ArcTanh[u])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m + 1, x]]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 4183

Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x]) /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x]
, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x)
)], x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] := -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n +
1)/2], x], x, a*Csc[e + f*x], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int x^3 \csc^3(a + bx) \sec^2(a + bx) dx &= -\frac{3x^3 \tanh^{-1}(\cos(a + bx))}{2b} + \frac{3x^3 \sec(a + bx)}{2b} - \frac{x^3 \csc^2(a + bx) \sec(a + bx)}{2b} - 3 \int \dots \\
&= -\frac{3x^3 \tanh^{-1}(\cos(a + bx))}{2b} + \frac{3x^3 \sec(a + bx)}{2b} - \frac{x^3 \csc^2(a + bx) \sec(a + bx)}{2b} - 3 \int \dots \\
&= -\frac{3x^3 \tanh^{-1}(\cos(a + bx))}{2b} + \frac{3x^3 \sec(a + bx)}{2b} - \frac{x^3 \csc^2(a + bx) \sec(a + bx)}{2b} + \frac{3 \int \dots}{2b} \\
&= \frac{3x^3 \sec(a + bx)}{2b} - \frac{x^3 \csc^2(a + bx) \sec(a + bx)}{2b} + \frac{3 \int bx^3 \csc(a + bx) dx}{2b} + \frac{3 \int (-3 \dots)}{2b} \\
&= \frac{3x^3 \sec(a + bx)}{2b} - \frac{x^3 \csc^2(a + bx) \sec(a + bx)}{2b} + \frac{3}{2} \int x^3 \csc(a + bx) dx + \frac{3 \int x^2 \dots}{2b} \\
&= \frac{9ix^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{3x^3 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{3x^2 \tanh^{-1}(\sin(a + bx))}{2b^2} - \frac{3x^2 \csc(a + bx)}{2b} \\
&= \frac{9ix^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{3x^3 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{3x^2 \tanh^{-1}(\sin(a + bx))}{2b^2} - \frac{3x^2 \csc(a + bx)}{2b} \\
&= \frac{9ix^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{3x^3 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{3x^2 \tanh^{-1}(\sin(a + bx))}{2b^2} - \frac{3x^2 \csc(a + bx)}{2b} \\
&= \frac{9ix^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{6x \tanh^{-1}(e^{i(a+bx)})}{b^3} - \frac{3x^3 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{3x^2 \csc(a + bx)}{2b^2} \\
&= \frac{9ix^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{6x \tanh^{-1}(e^{i(a+bx)})}{b^3} - \frac{3x^3 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{3x^2 \csc(a + bx)}{2b^2} \\
&= \frac{6ix^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{6x \tanh^{-1}(e^{i(a+bx)})}{b^3} - \frac{3x^3 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{3x^2 \csc(a + bx)}{2b^2} \\
&= \frac{6ix^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{6x \tanh^{-1}(e^{i(a+bx)})}{b^3} - \frac{3x^3 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{3x^2 \csc(a + bx)}{2b^2} \\
&= \frac{6ix^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{6x \tanh^{-1}(e^{i(a+bx)})}{b^3} - \frac{3x^3 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{3x^2 \csc(a + bx)}{2b^2} \\
&= \frac{6ix^2 \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{6x \tanh^{-1}(e^{i(a+bx)})}{b^3} - \frac{3x^3 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{3x^2 \csc(a + bx)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 7.19335, size = 672, normalized size = 1.74

$$3 \left(i \left(3b^2 x^2 + 2 \right) \text{PolyLog} \left(2, -\cos(a + bx) - i \sin(a + bx) \right) - i \left(3b^2 x^2 + 2 \right) \text{PolyLog} \left(2, \cos(a + bx) + i \sin(a + bx) \right) \right) - 6$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]

[Out] $-(x^3 \text{Csc}[a/2 + (b*x)/2]^2)/(8*b) + (6*(I*b^2*x^2*\text{ArcTan}[\text{Cos}[a + b*x] + I*\text{Sin}[a + b*x]] + I*b*x*\text{PolyLog}[2, I*\text{Cos}[a + b*x] - \text{Sin}[a + b*x]] - I*b*x*\text{PolyLog}[2, (-I)*\text{Cos}[a + b*x] + \text{Sin}[a + b*x]] - \text{PolyLog}[3, I*\text{Cos}[a + b*x] - \text{Sin}[a + b*x]] + \text{PolyLog}[3, (-I)*\text{Cos}[a + b*x] + \text{Sin}[a + b*x]]))/b^4 + (3*(2*b*x*\text{Log}[1 - \text{Cos}[a + b*x] - I*\text{Sin}[a + b*x]] + b^3*x^3*\text{Log}[1 - \text{Cos}[a + b*x] - I*\text{Sin}[a + b*x]] - 2*b*x*\text{Log}[1 + \text{Cos}[a + b*x] + I*\text{Sin}[a + b*x]] - b^3*x^3*\text{Log}[1 + \text{Cos}[a + b*x] + I*\text{Sin}[a + b*x]] + I*(2 + 3*b^2*x^2)*\text{PolyLog}[2, -\text{Cos}[a + b*x] - I*\text{Sin}[a + b*x]] - I*(2 + 3*b^2*x^2)*\text{PolyLog}[2, \text{Cos}[a + b*x] + I*\text{Sin}[a + b*x]] - 6*b*x*\text{PolyLog}[3, -\text{Cos}[a + b*x] - I*\text{Sin}[a + b*x]] + 6*b*x*\text{PolyLog}[3, \text{Cos}[a + b*x] + I*\text{Sin}[a + b*x]] - (6*I)*\text{PolyLog}[4, -\text{Cos}[a + b*x] - I*\text{Sin}[a + b*x]] + 6*I*\text{PolyLog}[4, \text{Cos}[a + b*x] + I*\text{Sin}[a + b*x]]))/b^4$

$$\begin{aligned}
& a))\cos(4bx + 4a) - 6((bx + a)^3 - 3(bx + a)^2a + (3a^2 + 2)(bx + a))\cos(2bx + 2a) + (6I(bx + a)^3 - 18I(bx + a)^2a + (18Ia^2 + 12I)(bx + a))\sin(6bx + 6a) + (-6I(bx + a)^3 + 18I(bx + a)^2a + (-18Ia^2 - 12I)(bx + a))\sin(4bx + 4a) + (-6I(bx + a)^3 + 18I(bx + a)^2a + (-18Ia^2 - 12I)(bx + a))\sin(2bx + 2a))\arctan2(\sin(bx + a), -\cos(bx + a) + 1) - (12I(bx + a)^3 - 12(bx + a)^2(3Ia - 1) + (36Ia^2 - 24a)(bx + a) + 12a^2)\cos(5bx + 5a) - (-8I(bx + a)^3 + 24I(bx + a)^2a - 24I(bx + a)a^2)\cos(3bx + 3a) - (12I(bx + a)^3 - 12(bx + a)^2(3Ia + 1) + (36Ia^2 + 24a)(bx + a) - 12a^2)\cos(bx + a) + (24bxx\cos(6bx + 6a) - 24bxx\cos(4bx + 4a) - 24bxx\cos(2bx + 2a) + 24Ibxx\sin(6bx + 6a) - 24Ibxx\sin(4bx + 4a) - 24Ibxx\sin(2bx + 2a) + 24bxx)\operatorname{dilog}(Ie^{(Ibx + Ia)}) - (24bxx\cos(6bx + 6a) - 24bxx\cos(4bx + 4a) - 24bxx\cos(2bx + 2a) + 24Ibxx\sin(6bx + 6a) - 24Ibxx\sin(4bx + 4a) - 24Ibxx\sin(2bx + 2a) + 24bxx)\operatorname{dilog}(-Ie^{(Ibx + Ia)}) + (18(bx + a)^2 - 36(bx + a)a + 18a^2 + 6(3(bx + a)^2 - 6(bx + a)a + 3a^2 + 2)\cos(6bx + 6a) - 6(3(bx + a)^2 - 6(bx + a)a + 3a^2 + 2)\cos(4bx + 4a) - 6(3(bx + a)^2 - 6(bx + a)a + 3a^2 + 2)\cos(2bx + 2a) - (-18I(bx + a)^2 + 36I(bx + a)a - 18Ia^2 - 12I)\sin(6bx + 6a) - (18I(bx + a)^2 - 36I(bx + a)a + 18Ia^2 + 12I)\sin(4bx + 4a) - (18I(bx + a)^2 - 36I(bx + a)a + 18Ia^2 + 12I)\sin(2bx + 2a) + 12)\operatorname{dilog}(-e^{(Ibx + Ia)}) - (18(bx + a)^2 - 36(bx + a)a + 18a^2 + 6(3(bx + a)^2 - 6(bx + a)a + 3a^2 + 2)\cos(6bx + 6a) - 6(3(bx + a)^2 - 6(bx + a)a + 3a^2 + 2)\cos(4bx + 4a) - 6(3(bx + a)^2 - 6(bx + a)a + 3a^2 + 2)\cos(2bx + 2a) + (18I(bx + a)^2 - 36I(bx + a)a + 18Ia^2 + 12I)\sin(6bx + 6a) + (-18I(bx + a)^2 + 36I(bx + a)a - 18Ia^2 - 12I)\sin(4bx + 4a) + (-18I(bx + a)^2 + 36I(bx + a)a - 18Ia^2 - 12I)\sin(2bx + 2a) + 12)\operatorname{dilog}(e^{(Ibx + Ia)}) - (-3I(bx + a)^3 + 9I(bx + a)^2a + (-9Ia^2 - 6I)(bx + a) + (-3I(bx + a)^3 + 9I(bx + a)^2a + (-9Ia^2 - 6I)(bx + a) + 6Ia)\cos(6bx + 6a) + (3I(bx + a)^3 - 9I(bx + a)^2a + (9Ia^2 + 6I)(bx + a) - 6Ia)\cos(4bx + 4a) + (3I(bx + a)^3 - 9I(bx + a)^2a + (9Ia^2 + 6I)(bx + a) - 6Ia)\cos(2bx + 2a) + 3((bx + a)^3 - 3(bx + a)^2a + (3a^2 + 2)(bx + a) - 2a)\sin(6bx + 6a) - 3((bx + a)^3 - 3(bx + a)^2a + (3a^2 + 2)(bx + a) - 2a)\sin(4bx + 4a) - 3((bx + a)^3 - 3(bx + a)^2a + (3a^2 + 2)(bx + a) - 2a)\sin(2bx + 2a) + 6Ia)\log(\cos(bx + a)^2 + \sin(bx + a)^2 + 2\cos(bx + a) + 1) - (3I(bx + a)^3 - 9I(bx + a)^2a + (9Ia^2 + 6I)(bx + a) + (3I(bx + a)^3 - 9I(bx + a)^2a + (9Ia^2 + 6I)(bx + a) - 6Ia)\cos(6bx + 6a) + (-3I(bx + a)^3 + 9I(bx + a)^2a + (-9Ia^2 - 6I)(bx + a) + 6Ia)\cos(4bx + 4a) + (-3I(bx + a)^3 + 9I(bx + a)^2a + (-9Ia^2 - 6I)(bx + a) + 6Ia)\cos(2bx + 2a) - 3((bx + a)^3 - 3(bx + a)^2a + (3a^2 + 2)(bx + a) - 2a)\sin(6bx + 6a) + 3((bx + a)^3 - 3(bx + a)^2a + (3a^2 + 2)(bx + a) - 2a)\sin(4bx + 4a) + 3((bx + a)^3 - 3(bx + a)^2a + (3a^2 + 2)(bx + a) - 2a)\sin(2bx + 2a) - 6Ia)\log(\cos(bx + a)^2 + \sin(bx + a)^2 - 2\cos(bx + a) + 1) - (-6I(bx + a)^2 + 12I(bx + a)a - 6Ia^2 + (-6I(bx + a)^2 + 12I(bx + a)a - 6Ia^2)\cos(6bx + 6a) + (6I(bx + a)^2 - 12I(bx + a)a + 6Ia^2)\cos(4bx + 4a) + (6I(bx + a)^2 - 12I(bx + a)a + 6Ia^2)\cos(2bx + 2a) + 6((bx + a)^2 - 2(bx + a)a + a^2)\sin(6bx + 6a) - 6((bx + a)^2 - 2(bx + a)a + a^2)\sin(4bx + 4a) - 6((bx + a)^2 - 2(bx + a)a + a^2)\sin(2bx + 2a))\log(\cos(bx + a)^2 + \sin(bx + a)^2 + 2\sin(bx + a) + 1) - (6I(bx + a)^2 - 12I(bx + a)a + 6Ia^2 + (6I(bx + a)^2 - 12I(bx + a)a + 6Ia^2)\cos(6bx + 6a) + (-6I(bx + a)^2 + 12I(bx + a)a - 6Ia^2)\cos(4bx + 4a) + (-6I(bx + a)^2 + 12I(bx + a)a - 6Ia^2)\cos(2bx + 2a) - 6((bx + a)^2 - 2(bx + a)a + a^2)\sin(6bx + 6a) + 6((bx + a)^2 - 2(bx + a)a + a^2)\sin(4bx + 4a) + 6((bx + a)^2 - 2(bx + a)a + a^2)\sin(2bx + 2a))\log(\cos(bx + a)^2 + \sin(bx + a)^2 - 2\sin(bx + a) + 1) - (36\cos(6bx + 6a) - 36\cos(4bx + 4a) - 36\cos
\end{aligned}$$

$$\begin{aligned}
& (2*b*x + 2*a) + 36*I*\sin(6*b*x + 6*a) - 36*I*\sin(4*b*x + 4*a) - 36*I*\sin(2* \\
& b*x + 2*a) + 36)*\text{polylog}(4, -e^{(I*b*x + I*a)}) + (36*\cos(6*b*x + 6*a) - 36*\cos \\
& \cos(4*b*x + 4*a) - 36*\cos(2*b*x + 2*a) + 36*I*\sin(6*b*x + 6*a) - 36*I*\sin(4* \\
& b*x + 4*a) - 36*I*\sin(2*b*x + 2*a) + 36)*\text{polylog}(4, e^{(I*b*x + I*a)}) - (-24 \\
& *I*\cos(6*b*x + 6*a) + 24*I*\cos(4*b*x + 4*a) + 24*I*\cos(2*b*x + 2*a) + 24*\sin \\
& \sin(6*b*x + 6*a) - 24*\sin(4*b*x + 4*a) - 24*\sin(2*b*x + 2*a) - 24*I)*\text{polylog}(\\
& 3, I*e^{(I*b*x + I*a)}) - (24*I*\cos(6*b*x + 6*a) - 24*I*\cos(4*b*x + 4*a) - 24 \\
& *I*\cos(2*b*x + 2*a) - 24*\sin(6*b*x + 6*a) + 24*\sin(4*b*x + 4*a) + 24*\sin(2* \\
& b*x + 2*a) + 24*I)*\text{polylog}(3, -I*e^{(I*b*x + I*a)}) - (-36*I*b*x*\cos(6*b*x + \\
& 6*a) + 36*I*b*x*\cos(4*b*x + 4*a) + 36*I*b*x*\cos(2*b*x + 2*a) + 36*b*x*\sin(6 \\
& *b*x + 6*a) - 36*b*x*\sin(4*b*x + 4*a) - 36*b*x*\sin(2*b*x + 2*a) - 36*I*b*x) \\
& *\text{polylog}(3, -e^{(I*b*x + I*a)}) - (36*I*b*x*\cos(6*b*x + 6*a) - 36*I*b*x*\cos(4 \\
& *b*x + 4*a) - 36*I*b*x*\cos(2*b*x + 2*a) - 36*b*x*\sin(6*b*x + 6*a) + 36*b*x* \\
& \sin(4*b*x + 4*a) + 36*b*x*\sin(2*b*x + 2*a) + 36*I*b*x)*\text{polylog}(3, e^{(I*b*x \\
& + I*a)}) + (12*(b*x + a)^3 - (b*x + a)^2*(36*a + 12*I) + 12*(3*a^2 + 2*I*a)* \\
& (b*x + a) - 12*I*a^2)*\sin(5*b*x + 5*a) - 8*((b*x + a)^3 - 3*(b*x + a)^2*a + \\
& 3*(b*x + a)*a^2)*\sin(3*b*x + 3*a) + (12*(b*x + a)^3 - (b*x + a)^2*(36*a - \\
& 12*I) + 12*(3*a^2 - 2*I*a)*(b*x + a) + 12*I*a^2)*\sin(b*x + a))/(-4*I*\cos(6* \\
& b*x + 6*a) + 4*I*\cos(4*b*x + 4*a) + 4*I*\cos(2*b*x + 2*a) + 4*\sin(6*b*x + 6* \\
& a) - 4*\sin(4*b*x + 4*a) - 4*\sin(2*b*x + 2*a) - 4*I))/b^4
\end{aligned}$$

Fricas [C] time = 0.911414, size = 4625, normalized size = 11.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")

[Out] $1/4*(6*b^3*x^3*\cos(b*x + a)^2 - 4*b^3*x^3 + 6*b^2*x^2*\cos(b*x + a)*\sin(b*x + a) + ((-9*I*b^2*x^2 - 6*I)*\cos(b*x + a)^3 + (9*I*b^2*x^2 + 6*I)*\cos(b*x + a))*\text{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) + ((9*I*b^2*x^2 + 6*I)*\cos(b*x + a)^3 + (-9*I*b^2*x^2 - 6*I)*\cos(b*x + a))*\text{dilog}(\cos(b*x + a) - I*\sin(b*x + a)) + (12*I*b*x*\cos(b*x + a)^3 - 12*I*b*x*\cos(b*x + a))*\text{dilog}(I*\cos(b*x + a) + \sin(b*x + a)) + (12*I*b*x*\cos(b*x + a)^3 - 12*I*b*x*\cos(b*x + a))*\text{dilog}(I*\cos(b*x + a) - \sin(b*x + a)) + (-12*I*b*x*\cos(b*x + a)^3 + 12*I*b*x*\cos(b*x + a))*\text{dilog}(-I*\cos(b*x + a) + \sin(b*x + a)) + (-12*I*b*x*\cos(b*x + a)^3 + 12*I*b*x*\cos(b*x + a))*\text{dilog}(-I*\cos(b*x + a) - \sin(b*x + a)) + ((-9*I*b^2*x^2 - 6*I)*\cos(b*x + a)^3 + (9*I*b^2*x^2 + 6*I)*\cos(b*x + a))*\text{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) + ((9*I*b^2*x^2 + 6*I)*\cos(b*x + a)^3 + (-9*I*b^2*x^2 - 6*I)*\cos(b*x + a))*\text{dilog}(-\cos(b*x + a) - I*\sin(b*x + a)) - 3*((b^3*x^3 + 2*b*x)*\cos(b*x + a)^3 - (b^3*x^3 + 2*b*x)*\cos(b*x + a))*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) - 6*(a^2*\cos(b*x + a)^3 - a^2*\cos(b*x + a))*\log(\cos(b*x + a) + I*\sin(b*x + a) + I) - 3*((b^3*x^3 + 2*b*x)*\cos(b*x + a)^3 - (b^3*x^3 + 2*b*x)*\cos(b*x + a))*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1) + 6*(a^2*\cos(b*x + a)^3 - a^2*\cos(b*x + a))*\log(\cos(b*x + a) - I*\sin(b*x + a) + I) - 6*((b^2*x^2 - a^2)*\cos(b*x + a)^3 - (b^2*x^2 - a^2)*\cos(b*x + a))*\log(I*\cos(b*x + a) + \sin(b*x + a) + 1) + 6*((b^2*x^2 - a^2)*\cos(b*x + a)^3 - (b^2*x^2 - a^2)*\cos(b*x + a))*\log(I*\cos(b*x + a) - \sin(b*x + a) + 1) - 6*((b^2*x^2 - a^2)*\cos(b*x + a)^3 - (b^2*x^2 - a^2)*\cos(b*x + a))*\log(-I*\cos(b*x + a) + \sin(b*x + a) + 1) + 6*((b^2*x^2 - a^2)*\cos(b*x + a)^3 - (b^2*x^2 - a^2)*\cos(b*x + a))*\log(-I*\cos(b*x + a) - \sin(b*x + a) + 1) - 3*((a^3 + 2*a)*\cos(b*x + a)^3 - (a^3 + 2*a)*\cos(b*x + a))*\log(-1/2*\cos(b*x + a) + 1/2*I*\sin(b*x + a) + 1/2) - 3*((a^3 + 2*a)*\cos(b*x + a)^3 - (a^3 + 2*a)*\cos(b*x + a))*\log(-1/2*\cos(b*x + a) - 1/2*I*\sin(b*x + a) + 1/2) + 3*((b^3*x^3 + a^3 + 2*b*x + 2*a)*\cos(b*x + a)^3 - (b^3*x^3 + a^3 + 2*b*x + 2*a)*\cos(b*x + a))*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1) - 6*(a^2*\cos(b*x + a)^3 - a^2*\cos(b*x + a))$

```

x + a))*log(-cos(b*x + a) + I*sin(b*x + a) + I) + 3*((b^3*x^3 + a^3 + 2*b*x
+ 2*a)*cos(b*x + a)^3 - (b^3*x^3 + a^3 + 2*b*x + 2*a)*cos(b*x + a))*log(-c
os(b*x + a) - I*sin(b*x + a) + 1) + 6*(a^2*cos(b*x + a)^3 - a^2*cos(b*x + a
))*log(-cos(b*x + a) - I*sin(b*x + a) + I) + (18*I*cos(b*x + a)^3 - 18*I*co
s(b*x + a))*polylog(4, cos(b*x + a) + I*sin(b*x + a)) + (-18*I*cos(b*x + a)
^3 + 18*I*cos(b*x + a))*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + (18*I*c
os(b*x + a)^3 - 18*I*cos(b*x + a))*polylog(4, -cos(b*x + a) + I*sin(b*x + a
)) + (-18*I*cos(b*x + a)^3 + 18*I*cos(b*x + a))*polylog(4, -cos(b*x + a) -
I*sin(b*x + a)) + 18*(b*x*cos(b*x + a)^3 - b*x*cos(b*x + a))*polylog(3, cos
(b*x + a) + I*sin(b*x + a)) + 18*(b*x*cos(b*x + a)^3 - b*x*cos(b*x + a))*po
lylog(3, cos(b*x + a) - I*sin(b*x + a)) + 12*(cos(b*x + a)^3 - cos(b*x + a
))*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 12*(cos(b*x + a)^3 - cos(b*x
+ a))*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 12*(cos(b*x + a)^3 - cos(
b*x + a))*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 12*(cos(b*x + a)^3 -
cos(b*x + a))*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 18*(b*x*cos(b*x
+ a)^3 - b*x*cos(b*x + a))*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 18
*(b*x*cos(b*x + a)^3 - b*x*cos(b*x + a))*polylog(3, -cos(b*x + a) - I*sin(b
*x + a)))/(b^4*cos(b*x + a)^3 - b^4*cos(b*x + a))

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*csc(b*x+a)**3*sec(b*x+a)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \csc(bx + a)^3 \sec(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")

[Out] integrate(x^3*csc(b*x + a)^3*sec(b*x + a)^2, x)

3.286 $\int x^2 \csc^3(a + bx) \sec^2(a + bx) dx$

Optimal. Leaf size=235

$$\frac{3ix \operatorname{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^2} - \frac{3ix \operatorname{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^2} - \frac{2i \operatorname{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} + \frac{2i \operatorname{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3} - \frac{3 \operatorname{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3}$$

```
[Out] ((4*I)*x*ArcTan[E^(I*(a + b*x))]/b^2 - (3*x^2*ArcTanh[E^(I*(a + b*x))]/b - ArcTanh[Cos[a + b*x]]/b^3 - (x*Csc[a + b*x])/b^2 + ((3*I)*x*PolyLog[2, -E^(I*(a + b*x))]/b^2 - ((2*I)*PolyLog[2, (-I)*E^(I*(a + b*x))]/b^3 + ((2*I)*PolyLog[2, I*E^(I*(a + b*x))]/b^3 - ((3*I)*x*PolyLog[2, E^(I*(a + b*x))]/b^2 - (3*PolyLog[3, -E^(I*(a + b*x))]/b^3 + (3*PolyLog[3, E^(I*(a + b*x))]/b^3 + (3*x^2*Sec[a + b*x])/(2*b) - (x^2*Csc[a + b*x]^2*Sec[a + b*x])/(2*b))
```

Rubi [A] time = 0.53745, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 19, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.95$, Rules used = {2622, 288, 321, 207, 4420, 14, 6273, 12, 4183, 2531, 2282, 6589, 6742, 4181, 2279, 2391, 2621, 6271, 3770}

$$\frac{3ix \operatorname{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^2} - \frac{3ix \operatorname{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^2} - \frac{2i \operatorname{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} + \frac{2i \operatorname{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3} - \frac{3 \operatorname{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] ((4*I)*x*ArcTan[E^(I*(a + b*x))]/b^2 - (3*x^2*ArcTanh[E^(I*(a + b*x))]/b - ArcTanh[Cos[a + b*x]]/b^3 - (x*Csc[a + b*x])/b^2 + ((3*I)*x*PolyLog[2, -E^(I*(a + b*x))]/b^2 - ((2*I)*PolyLog[2, (-I)*E^(I*(a + b*x))]/b^3 + ((2*I)*PolyLog[2, I*E^(I*(a + b*x))]/b^3 - ((3*I)*x*PolyLog[2, E^(I*(a + b*x))]/b^2 - (3*PolyLog[3, -E^(I*(a + b*x))]/b^3 + (3*PolyLog[3, E^(I*(a + b*x))]/b^3 + (3*x^2*Sec[a + b*x])/(2*b) - (x^2*Csc[a + b*x]^2*Sec[a + b*x])/(2*b))
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] := Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 4420

Int[Csc[(a_) + (b_)*(x_)]^(n_)*((c_) + (d_)*(x_))^(m_)*Sec[(a_) + (b_)*(x_)]^(p_), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

Rule 6273

Int[((a_) + ArcTanh[u]*(b_))*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m + 1, x]]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 4183

Int[csc[(e_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:= Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:= Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f,
Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f,
Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x)) /; FreeQ[{c, d, e, f}, x]
&& IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x]
/; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x]
/; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_), x_Symbol]
:= -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a*Csc[e + f*x]], x]
/; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] := Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand[(x*D[u, x])/(1 - u^2), x], x]
/; InverseFunctionFreeQ[u, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^2 \csc^3(a + bx) \sec^2(a + bx) dx &= -\frac{3x^2 \tanh^{-1}(\cos(a + bx))}{2b} + \frac{3x^2 \sec(a + bx)}{2b} - \frac{x^2 \csc^2(a + bx) \sec(a + bx)}{2b} - 2 \int x \csc(a + bx) \sec^2(a + bx) dx \\
&= -\frac{3x^2 \tanh^{-1}(\cos(a + bx))}{2b} + \frac{3x^2 \sec(a + bx)}{2b} - \frac{x^2 \csc^2(a + bx) \sec(a + bx)}{2b} - 2 \int x \csc(a + bx) \sec^2(a + bx) dx \\
&= -\frac{3x^2 \tanh^{-1}(\cos(a + bx))}{2b} + \frac{3x^2 \sec(a + bx)}{2b} - \frac{x^2 \csc^2(a + bx) \sec(a + bx)}{2b} + \frac{\int x \csc(a + bx) \sec^2(a + bx) dx}{b} \\
&= \frac{3x^2 \sec(a + bx)}{2b} - \frac{x^2 \csc^2(a + bx) \sec(a + bx)}{2b} + \frac{\int (-3x \sec(a + bx) + x \csc^2(a + bx)) dx}{b} \\
&= \frac{3x^2 \sec(a + bx)}{2b} - \frac{x^2 \csc^2(a + bx) \sec(a + bx)}{2b} + \frac{3}{2} \int x^2 \csc(a + bx) dx + \frac{\int x \csc(a + bx) \sec^2(a + bx) dx}{b} \\
&= \frac{6ix \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{3x^2 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{x \tanh^{-1}(\sin(a + bx))}{b^2} - \frac{x \csc(a + bx)}{b^2} \\
&= \frac{6ix \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{3x^2 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{x \tanh^{-1}(\sin(a + bx))}{b^2} - \frac{x \csc(a + bx)}{b^2} \\
&= \frac{6ix \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{3x^2 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{\tanh^{-1}(\cos(a + bx))}{b^3} - \frac{x \csc(a + bx)}{b^2} \\
&= \frac{6ix \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{3x^2 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{\tanh^{-1}(\cos(a + bx))}{b^3} - \frac{x \csc(a + bx)}{b^2} \\
&= \frac{4ix \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{3x^2 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{\tanh^{-1}(\cos(a + bx))}{b^3} - \frac{x \csc(a + bx)}{b^2} \\
&= \frac{4ix \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{3x^2 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{\tanh^{-1}(\cos(a + bx))}{b^3} - \frac{x \csc(a + bx)}{b^2} \\
&= \frac{4ix \tan^{-1}(e^{i(a+bx)})}{b^2} - \frac{3x^2 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{\tanh^{-1}(\cos(a + bx))}{b^3} - \frac{x \csc(a + bx)}{b^2}
\end{aligned}$$

Mathematica [B] time = 6.66329, size = 613, normalized size = 2.61

$$\frac{6ibx \text{PolyLog}(2, -\cos(a + bx) - i \sin(a + bx)) - 6ibx \text{PolyLog}(2, \cos(a + bx) + i \sin(a + bx)) - 6 \text{PolyLog}(3, -\cos(a + bx) + i \sin(a + bx))}{b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]

[Out]
$$\begin{aligned}
&-(x^2 \text{Csc}[a/2 + (b*x)/2]^2)/(8*b) - (2*((-a + \text{Pi}/2 - b*x)*(\text{Log}[1 - \text{E}^{(I*(-a + \text{Pi}/2 - b*x))}] - \text{Log}[1 + \text{E}^{(I*(-a + \text{Pi}/2 - b*x))}]) - (-a + \text{Pi}/2)*\text{Log}[\text{Tan}[-a + \text{Pi}/2 - b*x]/2]) + I*(\text{PolyLog}[2, -\text{E}^{(I*(-a + \text{Pi}/2 - b*x))}] - \text{PolyLog}[2, \text{E}^{(I*(-a + \text{Pi}/2 - b*x))}]))/b^3 + (2*\text{Log}[1 - \text{Cos}[a + b*x] - I*\text{Sin}[a + b*x]] + 3*b^2*x^2*\text{Log}[1 - \text{Cos}[a + b*x] - I*\text{Sin}[a + b*x]] - 2*\text{Log}[1 + \text{Cos}[a + b*x] + I*\text{Sin}[a + b*x]] - 3*b^2*x^2*\text{Log}[1 + \text{Cos}[a + b*x] + I*\text{Sin}[a + b*x]] + (6*I)*b*x*\text{PolyLog}[2, -\text{Cos}[a + b*x] - I*\text{Sin}[a + b*x]] - (6*I)*b*x*\text{PolyLog}[2, \text{Cos}[a + b*x] + I*\text{Sin}[a + b*x]] - 6*\text{PolyLog}[3, -\text{Cos}[a + b*x] - I*\text{Sin}[a + b*x]] + 6*\text{PolyLog}[3, \text{Cos}[a + b*x] + I*\text{Sin}[a + b*x]])/(2*b^3) + (x^2*\text{Sec}[a/2 + (b*x)/2]^2)/(8*b) + (x*\text{Csc}[a]*\text{Sec}[a]*(-\text{Cos}[a] + b*x*\text{Sin}[a]))/b^2 + (x*\text{Csc}[a/2]*\text{Csc}[a/2 + (b*x)/2]*\text{Sin}[(b*x)/2])/(2*b^2) - (x*\text{Sec}[a/2]*\text{Sec}[a/2 + (b*x)/2]*\text{Sin}[(b*x)/2])/(2*b^2) + (x^2*\text{Sin}[(b*x)/2])/(b*(\text{Cos}[a/2] - \text{Sin}[a/2]))*(\text{Cos}[a/2 + (b*x)/2] - \text{Sin}[a/2 + (b*x)/2]) - (x^2*\text{Sin}[(b*x)/2])/(b*(\text{Cos}[a/2] + \text{Sin}[a/2]))*(\text{Cos}[a/2 + (b*x)/2] + \text{Sin}[a/2 + (b*x)/2])
\end{aligned}$$

Maple [B] time = 0.402, size = 429, normalized size = 1.8

$$\frac{x(3bx e^{5i(bx+a)} - 2bx e^{3i(bx+a)} - 2ie^{5i(bx+a)} + 3bx e^{i(bx+a)} + 2ie^{i(bx+a)})}{b^2(e^{2i(bx+a)} - 1)^2(e^{2i(bx+a)} + 1)} + \frac{3a^2 \ln(e^{i(bx+a)} - 1)}{2b^3} - \frac{2i \operatorname{dilog}(1 + ie^{i(bx+a)})}{b^3} - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x)`

[Out] $x/b^2/(\exp(2I*(b*x+a))-1)^2/(\exp(2I*(b*x+a))+1)*(3*b*x*\exp(5*I*(b*x+a))-2*b*x*\exp(3*I*(b*x+a))-2*I*\exp(5*I*(b*x+a))+3*b*x*\exp(I*(b*x+a))+2*I*\exp(I*(b*x+a)))+3/2/b^3*a^2*\ln(\exp(I*(b*x+a))-1)-2*I/b^3*\operatorname{dilog}(1+I*\exp(I*(b*x+a)))-3/2/b^3*a^2*\ln(1-\exp(I*(b*x+a)))+2*I/b^3*\operatorname{dilog}(1-I*\exp(I*(b*x+a)))+3*I*x*\operatorname{polylog}(2,-\exp(I*(b*x+a)))/b^2+3*\operatorname{polylog}(3,\exp(I*(b*x+a)))/b^3+2/b^3*\ln(1+I*\exp(I*(b*x+a)))*a-2/b^3*\ln(1-I*\exp(I*(b*x+a)))*a-3*I*x*\operatorname{polylog}(2,\exp(I*(b*x+a)))/b^2+2/b^2*\ln(1+I*\exp(I*(b*x+a)))*x-2/b^2*\ln(1-I*\exp(I*(b*x+a)))*x-4*I/b^3*a*\arctan(\exp(I*(b*x+a)))+3/2/b*\ln(1-\exp(I*(b*x+a)))*x^2-3/2/b*\ln(\exp(I*(b*x+a))+1)*x^2+1/b^3*\ln(\exp(I*(b*x+a))-1)-1/b^3*\ln(\exp(I*(b*x+a))+1)-3*\operatorname{polylog}(3,-\exp(I*(b*x+a)))/b^3$

Maxima [B] time = 2.52132, size = 2996, normalized size = 12.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")`

[Out] $1/4*(a^2*(2*(3*\cos(b*x + a)^2 - 2)/(\cos(b*x + a)^3 - \cos(b*x + a)) - 3*\log(\cos(b*x + a) + 1) + 3*\log(\cos(b*x + a) - 1)) + 4*((8*b*x*\cos(6*b*x + 6*a) - 8*b*x*\cos(4*b*x + 4*a) - 8*b*x*\cos(2*b*x + 2*a) + 8*I*b*x*\sin(6*b*x + 6*a) - 8*I*b*x*\sin(4*b*x + 4*a) - 8*I*b*x*\sin(2*b*x + 2*a) + 8*b*x)*\arctan2(\cos(b*x + a), \sin(b*x + a) + 1) + (8*b*x*\cos(6*b*x + 6*a) - 8*b*x*\cos(4*b*x + 4*a) - 8*b*x*\cos(2*b*x + 2*a) + 8*I*b*x*\sin(6*b*x + 6*a) - 8*I*b*x*\sin(4*b*x + 4*a) - 8*I*b*x*\sin(2*b*x + 2*a) + 8*b*x)*\arctan2(\cos(b*x + a), -\sin(b*x + a) + 1) - (6*(b*x + a)^2 - 12*(b*x + a)*a + 2*(3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*\cos(6*b*x + 6*a) - 2*(3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*\cos(4*b*x + 4*a) - 2*(3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*\cos(2*b*x + 2*a) + (6*I*(b*x + a)^2 - 12*I*(b*x + a)*a + 4*I)*\sin(6*b*x + 6*a) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a - 4*I)*\sin(4*b*x + 4*a) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a - 4*I)*\sin(2*b*x + 2*a) + 4)*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) + (4*\cos(6*b*x + 6*a) - 4*\cos(4*b*x + 4*a) - 4*\cos(2*b*x + 2*a) + 4*I*\sin(6*b*x + 6*a) - 4*I*\sin(4*b*x + 4*a) - 4*I*\sin(2*b*x + 2*a) + 4)*\arctan2(\sin(b*x + a), \cos(b*x + a) - 1) - (6*(b*x + a)^2 - 12*(b*x + a)*a + 6*((b*x + a)^2 - 2*(b*x + a)*a)*\cos(6*b*x + 6*a) - 6*((b*x + a)^2 - 2*(b*x + a)*a)*\cos(4*b*x + 4*a) - 6*((b*x + a)^2 - 2*(b*x + a)*a)*\cos(2*b*x + 2*a) + (6*I*(b*x + a)^2 - 12*I*(b*x + a)*a)*\sin(6*b*x + 6*a) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a)*\sin(4*b*x + 4*a) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a)*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) - (12*I*(b*x + a)^2 - 8*(b*x + a)*(3*I*a - 1) - 8*a)*\cos(5*b*x + 5*a) - (-8*I*(b*x + a)^2 + 16*I*(b*x + a)*a)*\cos(3*b*x + 3*a) - (12*I*(b*x + a)^2 - 8*(b*x + a)*(3*I*a + 1) + 8*a)*\cos(b*x + a) + (8*\cos(6*b*x + 6*a) - 8*\cos(4*b*x + 4*a) - 8*\cos(2*b*x + 2*a) + 8*I*\sin(6*b*x + 6*a) - 8*I*\sin(4*b*x + 4*a) - 8*I*\sin(2*b*x + 2*a) + 8)*\operatorname{dilog}(I*e^{I*b*x + I*a}) - (8*\cos(6*b*x + 6*a) - 8*\cos(4*b*x + 4*a)$

$$\begin{aligned}
& - 8\cos(2bx + 2a) + 8I\sin(6bx + 6a) - 8I\sin(4bx + 4a) - 8I\sin(2bx + 2a) + 8\operatorname{dilog}(-Ie^{(Ibx + I)a}) + (12bx\cos(6bx + 6a) - \\
& 12bx\cos(4bx + 4a) - 12bx\cos(2bx + 2a) + 12Ibx\sin(6bx + 6a) - 12Ibx\sin(4bx + 4a) - 12Ibx\sin(2bx + 2a) + 12bx)\operatorname{dilog}(- \\
& e^{(Ibx + I)a}) - (12bx\cos(6bx + 6a) - 12bx\cos(4bx + 4a) - 12bx\cos(2bx + 2a) + 12Ibx\sin(6bx + 6a) - 12Ibx\sin(4bx + 4a) \\
& - 12Ibx\sin(2bx + 2a) + 12bx)\operatorname{dilog}(e^{(Ibx + I)a}) - (-3I(bx + a)^2 + 6I(bx + a)a + (-3I(bx + a)^2 + 6I(bx + a)a - 2I)\cos \\
& (6bx + 6a) + (3I(bx + a)^2 - 6I(bx + a)a + 2I)\cos(4bx + 4a) + (3I(bx + a)^2 - 6I(bx + a)a + 2I)\cos(2bx + 2a) + (3(bx + a) \\
& ^2 - 6(bx + a)a + 2)\sin(6bx + 6a) - (3(bx + a)^2 - 6(bx + a)a + 2)\sin(4bx + 4a) - (3(bx + a)^2 - 6(bx + a)a + 2)\sin(2bx + 2a) \\
& - 2I)\log(\cos(bx + a)^2 + \sin(bx + a)^2 + 2\cos(bx + a) + 1) - (3I(bx + a)^2 - 6I(bx + a)a + (3I(bx + a)^2 - 6I(bx + a)a + 2I)\cos \\
& (6bx + 6a) + (-3I(bx + a)^2 + 6I(bx + a)a - 2I)\cos(4bx + 4a) + (-3I(bx + a)^2 + 6I(bx + a)a - 2I)\cos(2bx + 2a) - (3(bx + \\
& a)^2 - 6(bx + a)a + 2)\sin(6bx + 6a) + (3(bx + a)^2 - 6(bx + a)a + 2)\sin(4bx + 4a) + (3(bx + a)^2 - 6(bx + a)a + 2)\sin(2bx + 2a) \\
& a + 2I)\log(\cos(bx + a)^2 + \sin(bx + a)^2 - 2\cos(bx + a) + 1) - (-4Ibx\cos(6bx + 6a) + 4Ibx\cos(4bx + 4a) + 4Ibx\cos(2bx + 2a) \\
& + 4bx\sin(6bx + 6a) - 4bx\sin(4bx + 4a) - 4bx\sin(2bx + 2a) - 4Ibx)\log(\cos(bx + a)^2 + \sin(bx + a)^2 + 2\sin(bx + a) + 1) - (4I \\
& Ibx\cos(6bx + 6a) - 4Ibx\cos(4bx + 4a) - 4Ibx\cos(2bx + 2a) - 4bx\sin(6bx + 6a) + 4bx\sin(4bx + 4a) + 4bx\sin(2bx + 2a) \\
&) + 4Ibx)\log(\cos(bx + a)^2 + \sin(bx + a)^2 - 2\sin(bx + a) + 1) - (-12I\cos(6bx + 6a) + 12I\cos(4bx + 4a) + 12I\cos(2bx + 2a) + 12\sin \\
& (6bx + 6a) - 12\sin(4bx + 4a) - 12\sin(2bx + 2a) - 12I)\operatorname{polylog}(3, -e^{(Ibx + I)a}) - (12I\cos(6bx + 6a) - 12I\cos(4bx + 4a) - 12I\cos(2bx + 2a) - 12\sin(6bx + 6a) + 12\sin(4bx + 4a) + 12\sin(2bx + 2a) + 12I)\operatorname{polylog}(3, e^{(Ibx + I)a}) + (12(bx + a)^2 - (bx + a)(24a + 8I) + 8Ia)\sin(5bx + 5a) - 8((bx + a)^2 - 2(bx + a)a)\sin(3bx + 3a) + (12(bx + a)^2 - (bx + a)(24a - 8I) - 8Ia)\sin(bx + a))/(-4I\cos(6bx + 6a) + 4I\cos(4bx + 4a) + 4I\cos(2bx + 2a) + 4\sin(6bx + 6a) - 4\sin(4bx + 4a) - 4\sin(2bx + 2a) - 4I))/b^3
\end{aligned}$$

Fricas [C] time = 0.770386, size = 3302, normalized size = 14.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")

[Out] $1/4*(6b^2x^2\cos(bx + a)^2 - 4b^2x^2 + 4bx\cos(bx + a)\sin(bx + a) + (-6Ibx\cos(bx + a)^3 + 6Ibx\cos(bx + a))\operatorname{dilog}(\cos(bx + a) + I\sin(bx + a)) + (6Ibx\cos(bx + a)^3 - 6Ibx\cos(bx + a))\operatorname{dilog}(\cos(bx + a) - I\sin(bx + a)) + (4I\cos(bx + a)^3 - 4I\cos(bx + a))\operatorname{dilog}(I\cos(bx + a) + \sin(bx + a)) + (4I\cos(bx + a)^3 - 4I\cos(bx + a))\operatorname{dilog}(I\cos(bx + a) - \sin(bx + a)) + (-4I\cos(bx + a)^3 + 4I\cos(bx + a))\operatorname{dilog}(-I\cos(bx + a) + \sin(bx + a)) + (-4I\cos(bx + a)^3 + 4I\cos(bx + a))\operatorname{dilog}(-I\cos(bx + a) - \sin(bx + a)) + (-6Ibx\cos(bx + a)^3 + 6Ibx\cos(bx + a))\operatorname{dilog}(-\cos(bx + a) + I\sin(bx + a)) + (6Ibx\cos(bx + a)^3 - 6Ibx\cos(bx + a))\operatorname{dilog}(-\cos(bx + a) - I\sin(bx + a)) - ((3b^2x^2 + 2)\cos(bx + a)^3 - (3b^2x^2 + 2)\cos(bx + a))\log(\cos(bx + a) + I\sin(bx + a) + 1) + 4(a\cos(bx + a)^3 - a\cos(bx + a))\log(\cos(bx + a) + I\sin(bx + a) + I) - ((3b^2x^2 + 2)\cos(bx + a)^3 - (3b^2x^2 + 2)\cos(bx + a))\log(\cos(bx + a) - I\sin(bx + a) + 1) + 4(a\cos(bx + a)^3 - a\cos(bx + a))\log(\cos(bx + a) - I\sin(bx + a) + I) - ((3b^2x^2 + 2)\cos(bx + a)^3 - (3b^2x^2 + 2)\cos(bx + a))\log(\cos(bx + a) - I\sin(bx + a) - 1) + 4(a\cos(bx + a)^3 - a\cos(bx + a))\log(\cos(bx + a) - I\sin(bx + a) - I) - ((3b^2x^2 + 2)\cos(bx + a)^3 - (3b^2x^2 + 2)\cos(bx + a))\log(\cos(bx + a) + I\sin(bx + a) - 1) + 4(a\cos(bx + a)^3 - a\cos(bx + a))\log(\cos(bx + a) + I\sin(bx + a) - I) - ((3b^2x^2 + 2)\cos(bx + a)^3 - (3b^2x^2 + 2)\cos(bx + a))\log(\cos(bx + a) - I\sin(bx + a) - 1) + 4(a\cos(bx + a)^3 - a\cos(bx + a))\log(\cos(bx + a) - I\sin(bx + a) - I)$

```

x^2 + 2)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 4*(a*cos(b*
x + a)^3 - a*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + I) - 4*((b*x
+ a)*cos(b*x + a)^3 - (b*x + a)*cos(b*x + a))*log(I*cos(b*x + a) + sin(b*x
+ a) + 1) + 4*((b*x + a)*cos(b*x + a)^3 - (b*x + a)*cos(b*x + a))*log(I*co
s(b*x + a) - sin(b*x + a) + 1) - 4*((b*x + a)*cos(b*x + a)^3 - (b*x + a)*co
s(b*x + a))*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 4*((b*x + a)*cos(b*x
+ a)^3 - (b*x + a)*cos(b*x + a))*log(-I*cos(b*x + a) - sin(b*x + a) + 1) +
((3*a^2 + 2)*cos(b*x + a)^3 - (3*a^2 + 2)*cos(b*x + a))*log(-1/2*cos(b*x +
a) + 1/2*I*sin(b*x + a) + 1/2) + ((3*a^2 + 2)*cos(b*x + a)^3 - (3*a^2 + 2)*
cos(b*x + a))*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + 3*((b^2*x
^2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 - a^2)*cos(b*x + a))*log(-cos(b*x + a)
+ I*sin(b*x + a) + 1) + 4*(a*cos(b*x + a)^3 - a*cos(b*x + a))*log(-cos(b*x
+ a) + I*sin(b*x + a) + I) + 3*((b^2*x^2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 -
a^2)*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 4*(a*cos(b*x
+ a)^3 - a*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 6*(cos(b
*x + a)^3 - cos(b*x + a))*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(co
s(b*x + a)^3 - cos(b*x + a))*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 6*
(cos(b*x + a)^3 - cos(b*x + a))*polylog(3, -cos(b*x + a) + I*sin(b*x + a))
- 6*(cos(b*x + a)^3 - cos(b*x + a))*polylog(3, -cos(b*x + a) - I*sin(b*x +
a)))/(b^3*cos(b*x + a)^3 - b^3*cos(b*x + a))

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*csc(b*x+a)**3*sec(b*x+a)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \csc (bx + a)^3 \sec (bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")

[Out] integrate(x^2*csc(b*x + a)^3*sec(b*x + a)^2, x)

3.287 $\int x \csc^3(a + bx) \sec^2(a + bx) dx$

Optimal. Leaf size=126

$$\frac{3i \operatorname{PolyLog}\left(2, -e^{i(a+bx)}\right)}{2b^2} - \frac{3i \operatorname{PolyLog}\left(2, e^{i(a+bx)}\right)}{2b^2} - \frac{\csc(a+bx)}{2b^2} - \frac{\tanh^{-1}(\sin(a+bx))}{b^2} + \frac{3x \sec(a+bx)}{2b} - \frac{3x \tanh^{-1}(\sin(a+bx))}{2b}$$

```
[Out] (-3*x*ArcTanh[E^(I*(a + b*x))])/b - ArcTanh[Sin[a + b*x]]/b^2 - Csc[a + b*x]/(2*b^2) + (((3*I)/2)*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (((3*I)/2)*PolyLog[2, E^(I*(a + b*x))])/b^2 + (3*x*Sec[a + b*x])/(2*b) - (x*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

Rubi [A] time = 0.1677, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2622, 288, 321, 207, 4420, 6271, 12, 4183, 2279, 2391, 3770, 2621}

$$\frac{3i \operatorname{PolyLog}\left(2, -e^{i(a+bx)}\right)}{2b^2} - \frac{3i \operatorname{PolyLog}\left(2, e^{i(a+bx)}\right)}{2b^2} - \frac{\csc(a+bx)}{2b^2} - \frac{\tanh^{-1}(\sin(a+bx))}{b^2} + \frac{3x \sec(a+bx)}{2b} - \frac{3x \tanh^{-1}(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (-3*x*ArcTanh[E^(I*(a + b*x))])/b - ArcTanh[Sin[a + b*x]]/b^2 - Csc[a + b*x]/(2*b^2) + (((3*I)/2)*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (((3*I)/2)*PolyLog[2, E^(I*(a + b*x))])/b^2 + (3*x*Sec[a + b*x])/(2*b) - (x*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] := Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] := Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a*Csc[e + f*x], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\int x \csc^3(a+bx) \sec^2(a+bx) dx &= -\frac{3x \tanh^{-1}(\cos(a+bx))}{2b} + \frac{3x \sec(a+bx)}{2b} - \frac{x \csc^2(a+bx) \sec(a+bx)}{2b} - \int \left(-\frac{3}{2b} \right. \\
&= -\frac{3x \tanh^{-1}(\cos(a+bx))}{2b} + \frac{3x \sec(a+bx)}{2b} - \frac{x \csc^2(a+bx) \sec(a+bx)}{2b} + \int \csc^2(a+bx) dx \\
&= -\frac{3 \tanh^{-1}(\sin(a+bx))}{2b^2} + \frac{3x \sec(a+bx)}{2b} - \frac{x \csc^2(a+bx) \sec(a+bx)}{2b} - \frac{\text{Subst}\left(\int \csc^2(u) du, u = a+bx\right)}{b} \\
&= -\frac{3 \tanh^{-1}(\sin(a+bx))}{2b^2} - \frac{\csc(a+bx)}{2b^2} + \frac{3x \sec(a+bx)}{2b} - \frac{x \csc^2(a+bx) \sec(a+bx)}{2b} \\
&= -\frac{3x \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{\tanh^{-1}(\sin(a+bx))}{b^2} - \frac{\csc(a+bx)}{2b^2} + \frac{3x \sec(a+bx)}{2b} - \frac{x \csc^2(a+bx) \sec(a+bx)}{2b} \\
&= -\frac{3x \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{\tanh^{-1}(\sin(a+bx))}{b^2} - \frac{\csc(a+bx)}{2b^2} + \frac{3x \sec(a+bx)}{2b} - \frac{x \csc^2(a+bx) \sec(a+bx)}{2b} \\
&= -\frac{3x \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{\tanh^{-1}(\sin(a+bx))}{b^2} - \frac{\csc(a+bx)}{2b^2} + \frac{3i \text{Li}_2(-e^{i(a+bx)})}{2b^2} - \frac{3x \csc^2(a+bx) \sec(a+bx)}{2b}
\end{aligned}$$

Mathematica [B] time = 2.56348, size = 282, normalized size = 2.24

$$12i \left(\text{PolyLog}\left(2, -e^{i(a+bx)}\right) - \text{PolyLog}\left(2, e^{i(a+bx)}\right) \right) + 12(a+bx) \left(\log\left(1 - e^{i(a+bx)}\right) - \log\left(1 + e^{i(a+bx)}\right) \right) - 2 \tan\left(\frac{1}{2}(a+bx)\right)$$

Antiderivative was successfully verified.

[In] Integrate[x*Csc[a + b*x]^3*Sec[a + b*x]^2,x]

[Out] (8*b*x - 2*Cot[(a + b*x)/2] - b*x*Csc[(a + b*x)/2]^2 + 12*(a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + 8*Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]] - 8*Log[Cos[(a + b*x)/2] + Sin[(a + b*x)/2]] - 12*a*Log[Tan[(a + b*x)/2]] + (12*I)*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a + b*x))]) + b*x*Sec[(a + b*x)/2]^2 + (8*b*x*Sin[(a + b*x)/2])/(Cos[(a + b*x)/2] - Sin[(a + b*x)/2]) - (8*b*x*Sin[(a + b*x)/2])/(Cos[(a + b*x)/2] + Sin[(a + b*x)/2]) - 2*Tan[(a + b*x)/2])/(8*b^2)

Maple [A] time = 0.299, size = 182, normalized size = 1.4

$$\frac{3bx e^{5i(bx+a)} - 2bx e^{3i(bx+a)} - i e^{5i(bx+a)} + 3bx e^{i(bx+a)} + i e^{i(bx+a)}}{b^2 (e^{2i(bx+a)} - 1)^2 (e^{2i(bx+a)} + 1)} - \frac{3a \ln(e^{i(bx+a)} - 1)}{2b^2} + \frac{2i \arctan(e^{i(bx+a)})}{b^2} + \frac{3i}{2} \text{dilog}\left(\frac{e^{i(bx+a)} - 1}{e^{i(bx+a)} + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*csc(b*x+a)^3*sec(b*x+a)^2,x)

[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2/(exp(2*I*(b*x+a))+1)*(3*b*x*exp(5*I*(b*x+a))-2*b*x*exp(3*I*(b*x+a))-I*exp(5*I*(b*x+a))+3*b*x*exp(I*(b*x+a))+I*exp(I*(b*x+a)))-3/2/b^2*a*ln(exp(I*(b*x+a))-1)+2*I/b^2*arctan(exp(I*(b*x+a)))+3/2*I/b^2*dilog(exp(I*(b*x+a)))+3/2*I/b^2*dilog(exp(I*(b*x+a))+1)-3/2/b*ln(exp(I*(b*x+a))+1)*x

Maxima [B] time = 2.34442, size = 1598, normalized size = 12.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")

[Out] $(8*I*b*x*\cos(3*b*x + 3*a) - 8*b*x*\sin(3*b*x + 3*a) - (4*\cos(6*b*x + 6*a) - 4*\cos(4*b*x + 4*a) - 4*\cos(2*b*x + 2*a) + 4*I*\sin(6*b*x + 6*a) - 4*I*\sin(4*b*x + 4*a) - 4*I*\sin(2*b*x + 2*a) + 4)*\arctan2(2*(\cos(b*x + 2*a)*\cos(a) + \sin(b*x + 2*a)*\sin(a))/(\cos(b*x + 2*a)^2 + \cos(a)^2 + 2*\cos(a)*\sin(b*x + 2*a) + \sin(b*x + 2*a)^2 - 2*\cos(b*x + 2*a)*\sin(a) + \sin(a)^2), (\cos(b*x + 2*a)^2 - \cos(a)^2 + \sin(b*x + 2*a)^2 - \sin(a)^2)/(\cos(b*x + 2*a)^2 + \cos(a)^2 + 2*\cos(a)*\sin(b*x + 2*a) + \sin(b*x + 2*a)^2 - 2*\cos(b*x + 2*a)*\sin(a) + \sin(a)^2)) - (6*b*x*\cos(6*b*x + 6*a) - 6*b*x*\cos(4*b*x + 4*a) - 6*b*x*\cos(2*b*x + 2*a) + 6*I*b*x*\sin(6*b*x + 6*a) - 6*I*b*x*\sin(4*b*x + 4*a) - 6*I*b*x*\sin(2*b*x + 2*a) + 6*b*x)*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) - (6*b*x*\cos(6*b*x + 6*a) - 6*b*x*\cos(4*b*x + 4*a) - 6*b*x*\cos(2*b*x + 2*a) + 6*I*b*x*\sin(6*b*x + 6*a) - 6*I*b*x*\sin(4*b*x + 4*a) - 6*I*b*x*\sin(2*b*x + 2*a) + 6*b*x)*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) - 4*(3*I*b*x + 1)*\cos(5*b*x + 5*a) - 4*(3*I*b*x - 1)*\cos(b*x + a) + (6*\cos(6*b*x + 6*a) - 6*\cos(4*b*x + 4*a) - 6*\cos(2*b*x + 2*a) + 6*I*\sin(6*b*x + 6*a) - 6*I*\sin(4*b*x + 4*a) - 6*I*\sin(2*b*x + 2*a) + 6)*\operatorname{dilog}(-e^{(I*b*x + I*a)}) - (6*\cos(6*b*x + 6*a) - 6*\cos(4*b*x + 4*a) - 6*\cos(2*b*x + 2*a) + 6*I*\sin(6*b*x + 6*a) - 6*I*\sin(4*b*x + 4*a) - 6*I*\sin(2*b*x + 2*a) + 6)*\operatorname{dilog}(e^{(I*b*x + I*a)}) + (3*I*b*x*\cos(6*b*x + 6*a) - 3*I*b*x*\cos(4*b*x + 4*a) - 3*I*b*x*\cos(2*b*x + 2*a) - 3*b*x*\sin(6*b*x + 6*a) + 3*b*x*\sin(4*b*x + 4*a) + 3*b*x*\sin(2*b*x + 2*a) + 3*I*b*x)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) + (-3*I*b*x*\cos(6*b*x + 6*a) + 3*I*b*x*\cos(4*b*x + 4*a) + 3*I*b*x*\cos(2*b*x + 2*a) + 3*b*x*\sin(6*b*x + 6*a) - 3*b*x*\sin(4*b*x + 4*a) - 3*b*x*\sin(2*b*x + 2*a) - 3*I*b*x)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) + (-2*I*\cos(6*b*x + 6*a) + 2*I*\cos(4*b*x + 4*a) + 2*I*\cos(2*b*x + 2*a) + 2*\sin(6*b*x + 6*a) - 2*\sin(4*b*x + 4*a) - 2*\sin(2*b*x + 2*a) - 2*I)*\log((\cos(b*x + 2*a)^2 + \cos(a)^2 - 2*\cos(a)*\sin(b*x + 2*a) + \sin(b*x + 2*a)^2 + 2*\cos(b*x + 2*a)*\sin(a) + \sin(a)^2)/(\cos(b*x + 2*a)^2 + \cos(a)^2 + 2*\cos(a)*\sin(b*x + 2*a) + \sin(b*x + 2*a)^2 - 2*\cos(b*x + 2*a)*\sin(a) + \sin(a)^2)) + (12*b*x - 4*I)*\sin(5*b*x + 5*a) + (12*b*x + 4*I)*\sin(b*x + a))/(-4*I*b^2*\cos(6*b*x + 6*a) + 4*I*b^2*\cos(4*b*x + 4*a) + 4*I*b^2*\cos(2*b*x + 2*a) + 4*b^2*\sin(6*b*x + 6*a) - 4*b^2*\sin(4*b*x + 4*a) - 4*b^2*\sin(2*b*x + 2*a) - 4*I*b^2)$

Fricas [B] time = 0.607694, size = 1459, normalized size = 11.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")

[Out] $1/4*(6*b*x*\cos(b*x + a)^2 - 4*b*x + (-3*I*\cos(b*x + a)^3 + 3*I*\cos(b*x + a))*\operatorname{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) + (3*I*\cos(b*x + a)^3 - 3*I*\cos(b*x + a))*\operatorname{dilog}(\cos(b*x + a) - I*\sin(b*x + a)) + (-3*I*\cos(b*x + a)^3 + 3*I*\cos(b*x + a))*\operatorname{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) + (3*I*\cos(b*x + a)^3 - 3*I*\cos(b*x + a))*\operatorname{dilog}(-\cos(b*x + a) - I*\sin(b*x + a)) - 3*(b*x*\cos(b*x + a)^3 - b*x*\cos(b*x + a))*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) - 3*(b*x*\cos(b*x + a)^3 - b*x*\cos(b*x + a))*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1) - 3*(a*\cos(b*x + a)^3 - a*\cos(b*x + a))*\log(-1/2*\cos(b*x + a) + 1/2*I*\sin(b*x +$

$a) + 1/2) - 3*(a*\cos(b*x + a)^3 - a*\cos(b*x + a))*\log(-1/2*\cos(b*x + a) - 1/2*I*\sin(b*x + a) + 1/2) + 3*((b*x + a)*\cos(b*x + a)^3 - (b*x + a)*\cos(b*x + a))*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1) + 3*((b*x + a)*\cos(b*x + a)^3 - (b*x + a)*\cos(b*x + a))*\log(-\cos(b*x + a) - I*\sin(b*x + a) + 1) - 2*(\cos(b*x + a)^3 - \cos(b*x + a))*\log(\sin(b*x + a) + 1) + 2*(\cos(b*x + a)^3 - \cos(b*x + a))*\log(-\sin(b*x + a) + 1) + 2*\cos(b*x + a)*\sin(b*x + a))/(b^2*\cos(b*x + a)^3 - b^2*\cos(b*x + a))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(b*x+a)**3*sec(b*x+a)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \csc(bx + a)^3 \sec(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")

[Out] integrate(x*csc(b*x + a)^3*sec(b*x + a)^2, x)

$$3.288 \quad \int \frac{\csc^3(a+bx) \sec^2(a+bx)}{x} dx$$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\csc^3(a+bx) \sec^2(a+bx)}{x}, x\right)$$

[Out] CannotIntegrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x, x]

Rubi [A] time = 0.499808, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^3(a+bx) \sec^2(a+bx)}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x, x]

[Out] Defer[Int] [(Csc[a + b*x]^3*Sec[a + b*x]^2)/x, x]

Rubi steps

$$\int \frac{\csc^3(a+bx) \sec^2(a+bx)}{x} dx = \int \frac{\csc^3(a+bx) \sec^2(a+bx)}{x} dx$$

Mathematica [A] time = 49.5159, size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx) \sec^2(a+bx)}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x, x]

[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x, x]

Maple [A] time = 0.784, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx+a))^3 (\sec(bx+a))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^3*sec(b*x+a)^2/x, x)

[Out] int(csc(b*x+a)^3*sec(b*x+a)^2/x, x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)^3 \sec(bx+a)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x, algorithm="fricas")

[Out] integral(csc(b*x + a)^3*sec(b*x + a)^2/x, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)**3*sec(b*x+a)**2/x,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^3 \sec(bx+a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x, algorithm="giac")

[Out] integrate(csc(b*x + a)^3*sec(b*x + a)^2/x, x)

$$3.289 \quad \int \frac{\csc^3(a+bx) \sec^2(a+bx)}{x^2} dx$$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\csc^3(a+bx) \sec^2(a+bx)}{x^2}, x\right)$$

[Out] CannotIntegrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2, x]

Rubi [A] time = 0.513283, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^3(a+bx) \sec^2(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2,x]

[Out] Defer[Int] [(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2, x]

Rubi steps

$$\int \frac{\csc^3(a+bx) \sec^2(a+bx)}{x^2} dx = \int \frac{\csc^3(a+bx) \sec^2(a+bx)}{x^2} dx$$

Mathematica [A] time = 20.9802, size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx) \sec^2(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2,x]

[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2, x]

Maple [A] time = 0.884, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx+a))^3 (\sec(bx+a))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x)

[Out] int(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)^3 \sec(bx+a)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x, algorithm="fricas")

[Out] integral(csc(b*x + a)^3*sec(b*x + a)^2/x^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)**3*sec(b*x+a)**2/x**2,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^3 \sec(bx+a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x, algorithm="giac")

[Out] integrate(csc(b*x + a)^3*sec(b*x + a)^2/x^2, x)

3.290 $\int (c + dx)^m \sec^2(a + bx) \tan(a + bx) dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}(\tan(a + bx) \sec^2(a + bx)(c + dx)^m, x)$$

[Out] CannotIntegrate[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x], x]

Rubi [A] time = 0.152441, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \sec^2(a + bx) \tan(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x], x]

[Out] Defer[Int] [(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x], x]

Rubi steps

$$\int (c + dx)^m \sec^2(a + bx) \tan(a + bx) dx = \int (c + dx)^m \sec^2(a + bx) \tan(a + bx) dx$$

Mathematica [A] time = 2.41648, size = 0, normalized size = 0.

$$\int (c + dx)^m \sec^2(a + bx) \tan(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x], x]

[Out] Integrate[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x], x]

Maple [A] time = 0.17, size = 0, normalized size = 0.

$$\int (dx + c)^m (\sec(bx + a))^2 \tan(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a), x)

[Out] int((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \sec(bx + a)^2 \tan(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)^2*tan(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx + c)^m \sec(bx + a)^2 \tan(bx + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*sec(b*x + a)^2*tan(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)**2*tan(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \sec(bx + a)^2 \tan(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)^2*tan(b*x + a), x)
```

3.291 $\int (c + dx)^4 \sec^2(a + bx) \tan(a + bx) dx$

Optimal. Leaf size=139

$$\frac{6id^3(c + dx)\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^4} - \frac{3d^4\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{b^5} - \frac{6d^2(c + dx)^2 \log\left(1 + e^{2i(a+bx)}\right)}{b^3} - \frac{2d(c + dx)^3 \tan(a + bx)}{b^2}$$

```
[Out] ((2*I)*d*(c + d*x)^3)/b^2 - (6*d^2*(c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b^3 + ((6*I)*d^3*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 - (3*d^4*PolyLog[3, -E^((2*I)*(a + b*x))])/b^5 + ((c + d*x)^4*Sec[a + b*x]^2)/(2*b) - (2*d*(c + d*x)^3*Tan[a + b*x])/b^2
```

Rubi [A] time = 0.257889, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {4409, 4184, 3719, 2190, 2531, 2282, 6589}

$$\frac{6id^3(c + dx)\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^4} - \frac{3d^4\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{b^5} - \frac{6d^2(c + dx)^2 \log\left(1 + e^{2i(a+bx)}\right)}{b^3} - \frac{2d(c + dx)^3 \tan(a + bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sec[a + b*x]^2*Tan[a + b*x], x]
```

```
[Out] ((2*I)*d*(c + d*x)^3)/b^2 - (6*d^2*(c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b^3 + ((6*I)*d^3*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 - (3*d^4*PolyLog[3, -E^((2*I)*(a + b*x))])/b^5 + ((c + d*x)^4*Sec[a + b*x]^2)/(2*b) - (2*d*(c + d*x)^3*Tan[a + b*x])/b^2
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] - Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```


Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_ /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^4 \sec^2(a + bx) \tan(a + bx) dx &= \frac{(c + dx)^4 \sec^2(a + bx)}{2b} - \frac{(2d) \int (c + dx)^3 \sec^2(a + bx) dx}{b} \\
&= \frac{(c + dx)^4 \sec^2(a + bx)}{2b} - \frac{2d(c + dx)^3 \tan(a + bx)}{b^2} + \frac{(6d^2) \int (c + dx)^2 \tan(a + bx) dx}{b^2} \\
&= \frac{2id(c + dx)^3}{b^2} + \frac{(c + dx)^4 \sec^2(a + bx)}{2b} - \frac{2d(c + dx)^3 \tan(a + bx)}{b^2} - \frac{(12id^2) \int (c + dx) \tan(a + bx) dx}{b^2} \\
&= \frac{2id(c + dx)^3}{b^2} - \frac{6d^2(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b^3} + \frac{(c + dx)^4 \sec^2(a + bx)}{2b} - \frac{2d(c + dx)^3 \tan(a + bx)}{b^2} \\
&= \frac{2id(c + dx)^3}{b^2} - \frac{6d^2(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b^3} + \frac{6id^3(c + dx) \text{Li}_2(-e^{2i(a+bx)})}{b^4} \\
&= \frac{2id(c + dx)^3}{b^2} - \frac{6d^2(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b^3} + \frac{6id^3(c + dx) \text{Li}_2(-e^{2i(a+bx)})}{b^4} \\
&= \frac{2id(c + dx)^3}{b^2} - \frac{6d^2(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b^3} + \frac{6id^3(c + dx) \text{Li}_2(-e^{2i(a+bx)})}{b^4}
\end{aligned}$$

Mathematica [B] time = 6.58545, size = 418, normalized size = 3.01

$$\frac{6cd^3 \csc(a) \sec(a) \left(b^2 x^2 e^{-i \tan^{-1}(\cot(a))} - \frac{\cot(a) \left(i \text{PolyLog} \left(2, e^{2i(bx - \tan^{-1}(\cot(a)))} \right) + ibx(-2 \tan^{-1}(\cot(a)) - \pi) - 2(bx - \tan^{-1}(\cot(a))) \log \left(1 - e^{2i(bx - \tan^{-1}(\cot(a)))} \right) \right)}{\sqrt{\cot^2(a)}} \right)}{b^4 \sqrt{\csc^2(a) (\sin^2(a) + \cos^2(a))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Sec[a + b*x]^2*Tan[a + b*x], x]
```

```
[Out] ((-I/2)*d^4*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(
a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (
3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a]/(b^5*E^(I
```

*a)) + ((c + d*x)^4*Sec[a + b*x]^2)/(2*b) - (6*c^2*d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) - (6*c*d^3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]])) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]])) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]])])]) + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]]) + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]])])])]/Sqrt[1 + Cot[a]^2])*Sec[a))/(b^4*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (2*Sec[a]*Sec[a + b*x]*(c^3*d*Sin[b*x] + 3*c^2*d^2*x*Sin[b*x] + 3*c*d^3*x^2*Sin[b*x] + d^4*x^3*Sin[b*x]))/b^2

Maple [B] time = 0.193, size = 489, normalized size = 3.5

$$\frac{2bd^4x^4e^{2i(bx+a)} + 4bcd^3x^3e^{2i(bx+a)} + 6bc^2d^2x^2e^{2i(bx+a)} + 4bc^3dx^2e^{2i(bx+a)} - 2id^4x^3e^{2i(bx+a)} + bc^4e^{2i(bx+a)} - 6icd^3x^2e^{2i(bx+a)}}{b^2(e^{2i(bx+a)} + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a), x)

[Out] 2*(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^3*exp(2*I*(b*x+a))+6*b*c^2*d^2*x^2*exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*x+a))-2*I*d^4*x^3*exp(2*I*(b*x+a))+b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp(2*I*(b*x+a))-6*I*c^2*d^2*x*exp(2*I*(b*x+a))-2*I*d^4*x^3-2*I*c^3*d*exp(2*I*(b*x+a))-6*I*c*d^3*x^2-6*I*c^2*d^2*x-2*I*c^3*d)/b^2/(exp(2*I*(b*x+a))+1)^2-6*d^2/b^3*c^2*ln(exp(2*I*(b*x+a))+1)+12*d^2/b^3*c^2*ln(exp(I*(b*x+a)))+12*d^4/b^5*a^2*ln(exp(I*(b*x+a)))+24*I*d^3/b^3*c*a*x+12*I*d^3/b^4*c*a^2-8*I*d^4/b^5*a^3-6*d^4/b^3*ln(exp(2*I*(b*x+a))+1)*x^2+6*I*d^4/b^4*polylog(2,-exp(2*I*(b*x+a)))*x-3*d^4*polylog(3,-exp(2*I*(b*x+a)))/b^5-24*d^3/b^4*c*a*ln(exp(I*(b*x+a)))-12*d^3/b^3*c*ln(exp(2*I*(b*x+a))+1)*x+12*I*d^3/b^2*c*x^2+6*I*d^3/b^4*c*polylog(2,-exp(2*I*(b*x+a)))+4*I*d^4/b^2*x^3-12*I*d^4/b^4*a^2*x

Maxima [B] time = 2.31788, size = 4641, normalized size = 33.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a), x, algorithm="maxima")

[Out] 1/2*(c^4*tan(b*x + a)^2 - 4*a*c^3*d*tan(b*x + a)^2/b + 6*a^2*c^2*d^2*tan(b*x + a)^2/b^2 - 4*a^3*c*d^3*tan(b*x + a)^2/b^3 + a^4*d^4*tan(b*x + a)^2/b^4 + 8*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*c^3*d/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b) - 24*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*a*c^2*d^2/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2) + 24*(4*(b*x + a)*cos(2*b*x +

$$\begin{aligned}
& 2*a)^2 + 4*(b*x + a)*\sin(2*b*x + 2*a)^2 + (2*(b*x + a)*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a))*\cos(4*b*x + 4*a) + 2*(b*x + a)*\cos(2*b*x + 2*a) + (2*(b*x + a)*\sin(2*b*x + 2*a) - \cos(2*b*x + 2*a) - 1)*\sin(4*b*x + 4*a) - \sin(2*b*x + 2*a))*a^2*c*d^3/((2*(2*\cos(2*b*x + 2*a) + 1)*\cos(4*b*x + 4*a) + \cos(4*b*x + 4*a))^2 + 4*\cos(2*b*x + 2*a)^2 + \sin(4*b*x + 4*a)^2 + 4*\sin(4*b*x + 4*a)*\sin(2*b*x + 2*a) + 4*\sin(2*b*x + 2*a)^2 + 4*\cos(2*b*x + 2*a) + 1)*b^3) - 8*(4*(b*x + a)*\cos(2*b*x + 2*a)^2 + 4*(b*x + a)*\sin(2*b*x + 2*a)^2 + (2*(b*x + a)*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a))*\cos(4*b*x + 4*a) + 2*(b*x + a)*\cos(2*b*x + 2*a) + (2*(b*x + a)*\sin(2*b*x + 2*a) - \cos(2*b*x + 2*a) - 1)*\sin(4*b*x + 4*a) - \sin(2*b*x + 2*a))*a^3*d^4/((2*(2*\cos(2*b*x + 2*a) + 1)*\cos(4*b*x + 4*a) + \cos(4*b*x + 4*a))^2 + 4*\cos(2*b*x + 2*a)^2 + \sin(4*b*x + 4*a)^2 + 4*\sin(4*b*x + 4*a)*\sin(2*b*x + 2*a) + 4*\sin(2*b*x + 2*a)^2 + 4*\cos(2*b*x + 2*a) + 1)*b^4) + 6*(8*(b*x + a)^2*\cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*\sin(2*b*x + 2*a)^2 + 4*(b*x + a)^2*\cos(2*b*x + 2*a) + 4*((b*x + a)^2*\cos(2*b*x + 2*a) + (b*x + a)*\sin(2*b*x + 2*a))*\cos(4*b*x + 4*a) - (2*(2*\cos(2*b*x + 2*a) + 1)*\cos(4*b*x + 4*a) + \cos(4*b*x + 4*a))^2 + 4*\cos(2*b*x + 2*a)^2 + \sin(4*b*x + 4*a)^2 + 4*\sin(4*b*x + 4*a)*\sin(2*b*x + 2*a) + 4*\sin(2*b*x + 2*a)^2 + 4*\cos(2*b*x + 2*a) + 1)*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) + 4*((b*x + a)^2*\sin(2*b*x + 2*a) - b*x - (b*x + a)*\cos(2*b*x + 2*a) - a)*\sin(4*b*x + 4*a) - 4*(b*x + a)*\sin(2*b*x + 2*a))*c^2*d^2/((2*(2*\cos(2*b*x + 2*a) + 1)*\cos(4*b*x + 4*a) + \cos(4*b*x + 4*a))^2 + 4*\cos(2*b*x + 2*a)^2 + \sin(4*b*x + 4*a)^2 + 4*\sin(4*b*x + 4*a)*\sin(2*b*x + 2*a) + 4*\sin(2*b*x + 2*a)^2 + 4*\cos(2*b*x + 2*a) + 1)*b^2) - 12*(8*(b*x + a)^2*\cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*\sin(2*b*x + 2*a)^2 + 4*(b*x + a)^2*\cos(2*b*x + 2*a) + 4*((b*x + a)^2*\cos(2*b*x + 2*a) + (b*x + a)*\sin(2*b*x + 2*a))*\cos(4*b*x + 4*a) - (2*(2*\cos(2*b*x + 2*a) + 1)*\cos(4*b*x + 4*a) + \cos(4*b*x + 4*a))^2 + 4*\cos(2*b*x + 2*a)^2 + \sin(4*b*x + 4*a)^2 + 4*\sin(4*b*x + 4*a)*\sin(2*b*x + 2*a) + 4*\sin(2*b*x + 2*a)^2 + 4*\cos(2*b*x + 2*a) + 1)*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) + 4*((b*x + a)^2*\sin(2*b*x + 2*a) - b*x - (b*x + a)*\cos(2*b*x + 2*a) - a)*\sin(4*b*x + 4*a) - 4*(b*x + a)*\sin(2*b*x + 2*a))*a*c*d^3/((2*(2*\cos(2*b*x + 2*a) + 1)*\cos(4*b*x + 4*a) + \cos(4*b*x + 4*a))^2 + 4*\cos(2*b*x + 2*a)^2 + \sin(4*b*x + 4*a)^2 + 4*\sin(4*b*x + 4*a)*\sin(2*b*x + 2*a) + 4*\sin(2*b*x + 2*a)^2 + 4*\cos(2*b*x + 2*a) + 1)*b^3) + 6*(8*(b*x + a)^2*\cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*\sin(2*b*x + 2*a)^2 + 4*(b*x + a)^2*\cos(2*b*x + 2*a) + 4*((b*x + a)^2*\cos(2*b*x + 2*a) + (b*x + a)*\sin(2*b*x + 2*a))*\cos(4*b*x + 4*a) - (2*(2*\cos(2*b*x + 2*a) + 1)*\cos(4*b*x + 4*a) + \cos(4*b*x + 4*a))^2 + 4*\cos(2*b*x + 2*a)^2 + \sin(4*b*x + 4*a)^2 + 4*\sin(4*b*x + 4*a)*\sin(2*b*x + 2*a) + 4*\sin(2*b*x + 2*a)^2 + 4*\cos(2*b*x + 2*a) + 1)*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) + 4*((b*x + a)^2*\sin(2*b*x + 2*a) - b*x - (b*x + a)*\cos(2*b*x + 2*a) - a)*\sin(4*b*x + 4*a) - 4*(b*x + a)*\sin(2*b*x + 2*a))*a^2*d^4/((2*(2*\cos(2*b*x + 2*a) + 1)*\cos(4*b*x + 4*a) + \cos(4*b*x + 4*a))^2 + 4*\cos(2*b*x + 2*a)^2 + \sin(4*b*x + 4*a)^2 + 4*\sin(4*b*x + 4*a)*\sin(2*b*x + 2*a) + 4*\sin(2*b*x + 2*a)^2 + 4*\cos(2*b*x + 2*a) + 1)*b^4) - 2*((6*(b*x + a)^2*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a) + 6*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*\cos(4*b*x + 4*a) + 12*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*\cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a))*\sin(4*b*x + 4*a) + (12*I*(b*x + a)^2*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(2*b*x + 2*a), \cos(2*b*x + 2*a) + 1) - 4*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2)*\cos(4*b*x + 4*a) + (2*I*(b*x + a)^4*d^4 + (8*I*b*c*d^3 - 4*(2*I*a + 1)*d^4)*(b*x + a)^3 - 12*(b*c*d^3 - a*d^4)*(b*x + a)^2)*\cos(2*b*x + 2*a) - (6*b*c*d^3 + 6*(b*x + a)*d^4 - 6*a*d^4 + 6*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*\cos(4*b*x + 4*a) + 12*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*\cos(2*b*x + 2*a) - (-6*I*b*c*d^3 - 6*I*(b*x + a)*d^4 + 6*I*a*d^4)*\sin(4*b*x + 4*a) - (-12*I*b*c*d^3 - 12*I*(b*x + a)*d^4 + 12*I*a*d^4)*\sin(2*b*x + 2*a))*\operatorname{dilog}(-e^(2*I*b*x + 2*I*a)) + (-3*I*(b*x + a)^2*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x + a) + (-3*I*(b*x + a)^2*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x + a))*\cos(4*b*x + 4*a) + (-6*I*(b*x + a)^2*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a))*
\end{aligned}$$

```

cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(
4*b*x + 4*a) + 6*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*
x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)
+ 1) + (-3*I*d^4*cos(4*b*x + 4*a) - 6*I*d^4*cos(2*b*x + 2*a) + 3*d^4*sin(4*
b*x + 4*a) + 6*d^4*sin(2*b*x + 2*a) - 3*I*d^4)*polylog(3, -e^(2*I*b*x + 2*I
*a)) + (-4*I*(b*x + a)^3*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2)*si
n(4*b*x + 4*a) - (2*(b*x + a)^4*d^4 + (8*b*c*d^3 - (8*a - 4*I)*d^4)*(b*x +
a)^3 - (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2)*sin(2*b*x + 2*a))/(-I*b^4*
cos(4*b*x + 4*a) - 2*I*b^4*cos(2*b*x + 2*a) + b^4*sin(4*b*x + 4*a) + 2*b^4*
sin(2*b*x + 2*a) - I*b^4))/b

```

Fricas [C] time = 0.776583, size = 2211, normalized size = 15.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^
4*c^4 - 12*d^4*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 1
2*d^4*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 12*d^4*cos
(b*x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 12*d^4*cos(b*x + a
)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + (-12*I*b*d^4*x - 12*I*b*c*
d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + (12*I*b*d^4*x +
12*I*b*c*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + (12*I*b
*d^4*x + 12*I*b*c*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a))
+ (-12*I*b*d^4*x - 12*I*b*c*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - si
n(b*x + a)) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2*log(co
s(b*x + a) + I*sin(b*x + a) + I) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*
cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b^2*d^4*x^2 + 2*
b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*cos(b*x + a)^2*log(I*cos(b*x + a) + si
n(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*c
os(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b
^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*cos(b*x + a)^2*log(-I*cos(b*x + a) + si
n(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*c
os(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - 6*(b^2*c^2*d^2 - 2*
a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I)
- 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2*log(-cos(b*x + a)
- I*sin(b*x + a) + I) - 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x
+ b^3*c^3*d)*cos(b*x + a)*sin(b*x + a))/(b^5*cos(b*x + a)^2)

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^4 \tan(a + bx) \sec^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sec(b*x+a)**2*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)**4*tan(a + b*x)*sec(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^4 \sec (bx + a)^2 \tan (bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*sec(b*x + a)^2*tan(b*x + a), x)
```

3.292 $\int (c + dx)^3 \sec^2(a + bx) \tan(a + bx) dx$

Optimal. Leaf size=115

$$\frac{3id^3 \text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^4} - \frac{3d^2(c+dx) \log\left(1 + e^{2i(a+bx)}\right)}{b^3} - \frac{3d(c+dx)^2 \tan(a+bx)}{2b^2} + \frac{(c+dx)^3 \sec^2(a+bx)}{2b} + \frac{3id(c+dx)^2 \tan(a+bx)}{2b}$$

[Out] (((3*I)/2)*d*(c + d*x)^2)/b^2 - (3*d^2*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b^3 + (((3*I)/2)*d^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 + ((c + d*x)^3*Sec[a + b*x]^2)/(2*b) - (3*d*(c + d*x)^2*Tan[a + b*x])/(2*b^2)

Rubi [A] time = 0.173672, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4409, 4184, 3719, 2190, 2279, 2391}

$$\frac{3id^3 \text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^4} - \frac{3d^2(c+dx) \log\left(1 + e^{2i(a+bx)}\right)}{b^3} - \frac{3d(c+dx)^2 \tan(a+bx)}{2b^2} + \frac{(c+dx)^3 \sec^2(a+bx)}{2b} + \frac{3id(c+dx)^2 \tan(a+bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^3*Sec[a + b*x]^2*Tan[a + b*x], x]

[Out] (((3*I)/2)*d*(c + d*x)^2)/b^2 - (3*d^2*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b^3 + (((3*I)/2)*d^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 + ((c + d*x)^3*Sec[a + b*x]^2)/(2*b) - (3*d*(c + d*x)^2*Tan[a + b*x])/(2*b^2)

Rule 4409

Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] - Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

Rule 4184

Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3719

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^3 \sec^2(a + bx) \tan(a + bx) dx &= \frac{(c + dx)^3 \sec^2(a + bx)}{2b} - \frac{(3d) \int (c + dx)^2 \sec^2(a + bx) dx}{2b} \\ &= \frac{(c + dx)^3 \sec^2(a + bx)}{2b} - \frac{3d(c + dx)^2 \tan(a + bx)}{2b^2} + \frac{(3d^2) \int (c + dx) \tan(a + bx) dx}{b^2} \\ &= \frac{3id(c + dx)^2}{2b^2} + \frac{(c + dx)^3 \sec^2(a + bx)}{2b} - \frac{3d(c + dx)^2 \tan(a + bx)}{2b^2} - \frac{(6id^2) \int (c + dx) dx}{b^2} \\ &= \frac{3id(c + dx)^2}{2b^2} - \frac{3d^2(c + dx) \log(1 + e^{2i(a+bx)})}{b^3} + \frac{(c + dx)^3 \sec^2(a + bx)}{2b} - \frac{3d^2(c + dx)}{b^2} \\ &= \frac{3id(c + dx)^2}{2b^2} - \frac{3d^2(c + dx) \log(1 + e^{2i(a+bx)})}{b^3} + \frac{(c + dx)^3 \sec^2(a + bx)}{2b} - \frac{3d^2(c + dx)}{b^2} \\ &= \frac{3id(c + dx)^2}{2b^2} - \frac{3d^2(c + dx) \log(1 + e^{2i(a+bx)})}{b^3} + \frac{3id^3 \text{Li}_2(-e^{2i(a+bx)})}{2b^4} + \frac{(c + dx)^3 \sec^2(a + bx)}{2b} \end{aligned}$$

Mathematica [B] time = 6.39472, size = 286, normalized size = 2.49

$$\frac{3d^3 \csc(a) \sec(a) \left(b^2 x^2 e^{-i \tan^{-1}(\cot(a))} - \frac{\cot(a) \left(i \text{PolyLog} \left(2, e^{2i(bx - \tan^{-1}(\cot(a)))} \right) \right) + ibx(-2 \tan^{-1}(\cot(a)) - \pi) - 2(bx - \tan^{-1}(\cot(a))) \log \left(1 - e^{2i(bx - \tan^{-1}(\cot(a)))} \right)}{\sqrt{\cot^2(a) + 1}} \right)}{2b^4 \sqrt{\csc^2(a) (\sin^2(a) + \cos^2(a))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]^2*Tan[a + b*x], x]
```

```
[Out] ((c + d*x)^3*Sec[a + b*x]^2)/(2*b) - (3*c*d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos
[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) - (3*d^
3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[C
ot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^
(2*I)*(b*x - ArcTan[Cot[a]])]) + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Si
n[b*x - ArcTan[Cot[a]]]) + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]])]))
/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^4*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) -
(3*Sec[a]*Sec[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3*x^2*Sin[
b*x]))/(2*b^2)
```

Maple [B] time = 0.158, size = 301, normalized size = 2.6

$$\frac{2bd^3x^3e^{2i(bx+a)} - 3id^3x^2e^{2i(bx+a)} + 6bcd^2x^2e^{2i(bx+a)} - 6icd^2xe^{2i(bx+a)} + 6bc^2dxe^{2i(bx+a)} - 3ic^2de^{2i(bx+a)} - 3id^3x^2 + 2c^2}{b^2(e^{2i(bx+a)} + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x)`

[Out] $(2*b*d^3*x^3*\exp(2*I*(b*x+a))-3*I*d^3*x^2*\exp(2*I*(b*x+a))+6*b*c*d^2*x^2*\exp(2*I*(b*x+a))-6*I*c*d^2*x*\exp(2*I*(b*x+a))+6*b*c^2*d*x*\exp(2*I*(b*x+a))-3*I*c^2*d*\exp(2*I*(b*x+a))-3*I*d^3*x^2+2*b*c^3*\exp(2*I*(b*x+a))-6*I*c*d^2*x-3*I*c^2*d)/b^2/(\exp(2*I*(b*x+a))+1)^2+6/b^3*d^2*c*\ln(\exp(I*(b*x+a)))-3*d^2/b^3*c*\ln(\exp(2*I*(b*x+a))+1)+3*I*d^3/b^2*x^2+6*I*d^3/b^3*a*x+3*I*d^3/b^4*a^2-3*d^3/b^3*\ln(\exp(2*I*(b*x+a))+1)*x+3/2*I*d^3*\text{polylog}(2,-\exp(2*I*(b*x+a)))/b^4-6/b^4*d^3*a*\ln(\exp(I*(b*x+a)))$

Maxima [B] time = 2.24113, size = 900, normalized size = 7.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")`

[Out] $-(6*b^2*c^2*d + (6*b*d^3*x + 6*b*c*d^2 + 6*(b*d^3*x + b*c*d^2))*\cos(4*b*x + 4*a) + 12*(b*d^3*x + b*c*d^2))*\cos(2*b*x + 2*a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*\sin(4*b*x + 4*a) + (12*I*b*d^3*x + 12*I*b*c*d^2))*\sin(2*b*x + 2*a))*\arctan(2(\sin(2*b*x + 2*a), \cos(2*b*x + 2*a) + 1) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x)*\cos(4*b*x + 4*a) + (4*I*b^3*d^3*x^3 + 4*I*b^3*c^3 + 6*b^2*c^2*d + (12*I*b^3*c*d^2 - 6*b^2*d^3))*x^2 - 12*(-I*b^3*c^2*d + b^2*c*d^2)*x)*\cos(2*b*x + 2*a) - (3*d^3*\cos(4*b*x + 4*a) + 6*d^3*\cos(2*b*x + 2*a) + 3*I*d^3*\sin(4*b*x + 4*a) + 6*I*d^3*\sin(2*b*x + 2*a) + 3*d^3)*\text{dilog}(-e^{(2*I*b*x + 2*I*a)}) + (-3*I*b*d^3*x - 3*I*b*c*d^2 + (-3*I*b*d^3*x - 3*I*b*c*d^2))*\cos(4*b*x + 4*a) + (-6*I*b*d^3*x - 6*I*b*c*d^2))*\cos(2*b*x + 2*a) + 3*(b*d^3*x + b*c*d^2))*\sin(4*b*x + 4*a) + 6*(b*d^3*x + b*c*d^2))*\sin(2*b*x + 2*a))*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*x)*\sin(4*b*x + 4*a) - (4*b^3*d^3*x^3 + 4*b^3*c^3 - 6*I*b^2*c^2*d + 6*(2*b^3*c*d^2 + I*b^2*d^3))*x^2 + (12*b^3*c^2*d + 12*I*b^2*c*d^2)*x)*\sin(2*b*x + 2*a))/(-2*I*b^4*\cos(4*b*x + 4*a) - 4*I*b^4*\cos(2*b*x + 2*a) + 2*b^4*\sin(4*b*x + 4*a) + 4*b^4*\sin(2*b*x + 2*a) - 2*I*b^4)$

Fricas [B] time = 0.690775, size = 1374, normalized size = 11.95

$b^3 d^3 x^3 + 3 b^3 c d^2 x^2 + 3 b^3 c^2 d x + b^3 c^3 - 3 i d^3 \cos(bx + a)^2 \text{Li}_2(i \cos(bx + a) + \sin(bx + a)) + 3 i d^3 \cos(bx + a)^2 \text{Li}_2(i \cos(bx + a) - \sin(bx + a))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")`

[Out] $1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 3*I*d^3*\cos(b*x + a)^2*\text{dilog}(I*\cos(b*x + a) + \sin(b*x + a)) + 3*I*d^3*\cos(b*x + a)^2*\text{dilog}(I*\cos(b*x + a) - \sin(b*x + a)) + 3*I*d^3*\cos(b*x + a)^2*\text{dilog}(-I*\cos(b*x + a) + \sin(b*x + a)) - 3*I*d^3*\cos(b*x + a)^2*\text{dilog}(-I*\cos(b*x + a) - \sin(b*x + a)) - 3*(b*c*d^2 - a*d^3))*\cos(b*x + a)^2*\log(\cos(b*x + a) + I*\sin(b*x + a) + I) - 3*(b*c*d^2 - a*d^3))*\cos(b*x + a)^2*\log(\cos(b*x + a) - I*\sin(b*x + a) + I) - 3*(b*d^3*x + a*d^3))*\cos(b*x + a)^2*\log(I*\cos(b*x + a) + \sin(b*x + a) + 1) - 3*(b*d^3*x + a*d^3))*\cos(b*x + a)^2*\log(I*\cos(b*x + a) - \sin(b*x + a) + 1)$

$(b*x + a) + 1) - 3*(b*d^3*x + a*d^3)*\cos(b*x + a)^2*\log(-I*\cos(b*x + a) + \sin(b*x + a) + 1) - 3*(b*d^3*x + a*d^3)*\cos(b*x + a)^2*\log(-I*\cos(b*x + a) - \sin(b*x + a) + 1) - 3*(b*c*d^2 - a*d^3)*\cos(b*x + a)^2*\log(-\cos(b*x + a) + I*\sin(b*x + a) + I) - 3*(b*c*d^2 - a*d^3)*\cos(b*x + a)^2*\log(-\cos(b*x + a) - I*\sin(b*x + a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*\cos(b*x + a)*\sin(b*x + a))/(b^4*\cos(b*x + a)^2)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \tan(a + bx) \sec^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*sec(b*x+a)**2*tan(b*x+a),x)

[Out] Integral((c + d*x)**3*tan(a + b*x)*sec(a + b*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \sec(bx + a)^2 \tan(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^3*sec(b*x + a)^2*tan(b*x + a), x)

3.293 $\int (c + dx)^2 \sec^2(a + bx) \tan(a + bx) dx$

Optimal. Leaf size=55

$$-\frac{d(c + dx) \tan(a + bx)}{b^2} - \frac{d^2 \log(\cos(a + bx))}{b^3} + \frac{(c + dx)^2 \sec^2(a + bx)}{2b}$$

[Out] $-\left(\frac{d^2 \text{Log}[\text{Cos}[a + b*x]]}{b^3}\right) + \left(\frac{(c + d*x)^2 \text{Sec}[a + b*x]^2}{2*b}\right) - \left(\frac{d*(c + d*x)*\text{Tan}[a + b*x]}{b^2}\right)$

Rubi [A] time = 0.0615893, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4409, 4184, 3475}

$$-\frac{d(c + dx) \tan(a + bx)}{b^2} - \frac{d^2 \log(\cos(a + bx))}{b^3} + \frac{(c + dx)^2 \sec^2(a + bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2*Sec[a + b*x]^2*Tan[a + b*x], x]

[Out] $-\left(\frac{d^2 \text{Log}[\text{Cos}[a + b*x]]}{b^3}\right) + \left(\frac{(c + d*x)^2 \text{Sec}[a + b*x]^2}{2*b}\right) - \left(\frac{d*(c + d*x)*\text{Tan}[a + b*x]}{b^2}\right)$

Rule 4409

Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] - Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

Rule 4184

Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \sec^2(a + bx) \tan(a + bx) dx &= \frac{(c + dx)^2 \sec^2(a + bx)}{2b} - \frac{d \int (c + dx) \sec^2(a + bx) dx}{b} \\ &= \frac{(c + dx)^2 \sec^2(a + bx)}{2b} - \frac{d(c + dx) \tan(a + bx)}{b^2} + \frac{d^2 \int \tan(a + bx) dx}{b^2} \\ &= -\frac{d^2 \log(\cos(a + bx))}{b^3} + \frac{(c + dx)^2 \sec^2(a + bx)}{2b} - \frac{d(c + dx) \tan(a + bx)}{b^2} \end{aligned}$$

Mathematica [A] time = 0.524199, size = 66, normalized size = 1.2

$$\frac{b^2(c + dx)^2 \sec^2(a + bx) - 2bd \sec(a) \sin(bx)(c + dx) \sec(a + bx) - 2d^2(bx \tan(a) + \log(\cos(a + bx)))}{2b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2*Sec[a + b*x]^2*Tan[a + b*x], x]

[Out] (b^2*(c + d*x)^2*Sec[a + b*x]^2 - 2*b*d*(c + d*x)*Sec[a]*Sec[a + b*x]*Sin[b*x] - 2*d^2*(Log[Cos[a + b*x]] + b*x*Tan[a]))/(2*b^3)

Maple [A] time = 0.029, size = 95, normalized size = 1.7

$$\frac{d^2 x^2}{2b(\cos(bx+a))^2} - \frac{d^2 \tan(bx+a)x}{b^2} - \frac{d^2 \ln(\cos(bx+a))}{b^3} + \frac{cdx}{b(\cos(bx+a))^2} - \frac{cd \tan(bx+a)}{b^2} + \frac{c^2}{2b(\cos(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a), x)

[Out] 1/2/b*d^2/cos(b*x+a)^2*x^2-1/b^2*d^2*tan(b*x+a)*x-d^2*ln(cos(b*x+a))/b^3+1/b*c*d/cos(b*x+a)^2*x-1/b^2*c*d*tan(b*x+a)+1/2/b*c^2/cos(b*x+a)^2

Maxima [B] time = 1.57501, size = 1334, normalized size = 24.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a), x, algorithm="maxima")

[Out] 1/2*(c^2*tan(b*x + a)^2 - 2*a*c*d*tan(b*x + a)^2/b + a^2*d^2*tan(b*x + a)^2/b^2 + 4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*c*d/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b) - 4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*a*d^2/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2) + (8*(b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 + 4*(b*x + a)^2*cos(2*b*x + 2*a) + 4*((b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - (2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*((b*x + a)^2*sin(2*b*x + 2*a) - b*x - (b*x + a)*cos(2*b*x + 2*a) - a)*sin(4*b*x + 4*a) - 4*(b*x + a)*sin(2*b*x + 2*a))*d^2/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2))/b

Fricas [A] time = 0.508424, size = 208, normalized size = 3.78

$$\frac{b^2 d^2 x^2 + 2 b^2 c d x - 2 d^2 \cos(bx + a)^2 \log(-\cos(bx + a)) + b^2 c^2 - 2 (bd^2 x + bcd) \cos(bx + a) \sin(bx + a)}{2 b^3 \cos(bx + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")

[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x - 2*d^2*cos(b*x + a)^2*log(-cos(b*x + a)) + b^2*c^2 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3*cos(b*x + a)^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \tan(a + bx) \sec^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*sec(b*x+a)**2*tan(b*x+a),x)

[Out] Integral((c + d*x)**2*tan(a + b*x)*sec(a + b*x)**2, x)

Giac [B] time = 2.83337, size = 6040, normalized size = 109.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")

[Out] 1/2*(b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a)^4 + b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b^2*c*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 4*b*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + 4*b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 + 4*b*d^2*x*tan(1/2*b*x)^3*tan(1/2*a)^4 - d^2*log(4*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^4 + b^2*d^2*x^2*tan(1/2*b*x)^4 + 4*b^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 4*b*c*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + b^2*d^2*x^2*tan(1/2*a)^4 + 2*b^2*c^2*tan(1/2*b*x)^2*tan(1/2*a)^4 + 4*b*c*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4 - 4*b*d^2*x*tan(1/2*b*x)^4*tan(1/2*a) + 8*b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - 24*b*d^2*x*tan(1/2*b*x)^3*tan(1/2*a)^2 + 2*d^2*log(4*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2

$$\begin{aligned}
& /2*a)^4 + 2*\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x)^8*\tan(1/2*a)^4 - 2*\tan(1/2*b*x) \\
& ^8*\tan(1/2*a)^2 - 8*\tan(1/2*b*x)^7*\tan(1/2*a)^3 + \tan(1/2*b*x)^8 + 8*\tan(1/ \\
& 2*b*x)^7*\tan(1/2*a) + 16*\tan(1/2*b*x)^6*\tan(1/2*a)^2 - 8*\tan(1/2*b*x)^5*\tan \\
& (1/2*a)^3 - 2*\tan(1/2*b*x)^4*\tan(1/2*a)^4 + 8*\tan(1/2*b*x)^5*\tan(1/2*a) + 3 \\
& 6*\tan(1/2*b*x)^4*\tan(1/2*a)^2 + 8*\tan(1/2*b*x)^3*\tan(1/2*a)^3 - 2*\tan(1/2*b \\
& *x)^4 - 8*\tan(1/2*b*x)^3*\tan(1/2*a) + 16*\tan(1/2*b*x)^2*\tan(1/2*a)^2 + 8*ta \\
& n(1/2*b*x)*\tan(1/2*a)^3 + \tan(1/2*a)^4 - 8*\tan(1/2*b*x)*\tan(1/2*a) - 2*\tan(\\
& 1/2*a)^2 + 1))*\tan(1/2*a)^4 + b^2*d^2*x^2 + 2*b^2*c^2*\tan(1/2*b*x)^2 + 4*b* \\
& c*d*\tan(1/2*b*x)^3 + 24*b*c*d*\tan(1/2*b*x)^2*\tan(1/2*a) + 2*b^2*c^2*\tan(1/2 \\
& *a)^2 + 24*b*c*d*\tan(1/2*b*x)*\tan(1/2*a)^2 + 4*b*c*d*\tan(1/2*a)^3 + 2*b^2*c \\
& *d*x - 4*b*d^2*x*\tan(1/2*b*x) + 2*d^2*\log(4*(\tan(1/2*a)^4 + 2*\tan(1/2*a)^2 \\
& + 1)/(\tan(1/2*b*x)^8*\tan(1/2*a)^4 - 2*\tan(1/2*b*x)^8*\tan(1/2*a)^2 - 8*\tan(1 \\
& /2*b*x)^7*\tan(1/2*a)^3 + \tan(1/2*b*x)^8 + 8*\tan(1/2*b*x)^7*\tan(1/2*a) + 16* \\
& \tan(1/2*b*x)^6*\tan(1/2*a)^2 - 8*\tan(1/2*b*x)^5*\tan(1/2*a)^3 - 2*\tan(1/2*b*x) \\
&)^4*\tan(1/2*a)^4 + 8*\tan(1/2*b*x)^5*\tan(1/2*a) + 36*\tan(1/2*b*x)^4*\tan(1/2* \\
& a)^2 + 8*\tan(1/2*b*x)^3*\tan(1/2*a)^3 - 2*\tan(1/2*b*x)^4 - 8*\tan(1/2*b*x)^3* \\
& \tan(1/2*a) + 16*\tan(1/2*b*x)^2*\tan(1/2*a)^2 + 8*\tan(1/2*b*x)*\tan(1/2*a)^3 + \\
& \tan(1/2*a)^4 - 8*\tan(1/2*b*x)*\tan(1/2*a) - 2*\tan(1/2*a)^2 + 1))*\tan(1/2*b*x) \\
& ^2 - 4*b*d^2*x*\tan(1/2*a) + 8*d^2*\log(4*(\tan(1/2*a)^4 + 2*\tan(1/2*a)^2 + \\
& 1)/(\tan(1/2*b*x)^8*\tan(1/2*a)^4 - 2*\tan(1/2*b*x)^8*\tan(1/2*a)^2 - 8*\tan(1/2 \\
& *b*x)^7*\tan(1/2*a)^3 + \tan(1/2*b*x)^8 + 8*\tan(1/2*b*x)^7*\tan(1/2*a) + 16*ta \\
& n(1/2*b*x)^6*\tan(1/2*a)^2 - 8*\tan(1/2*b*x)^5*\tan(1/2*a)^3 - 2*\tan(1/2*b*x)^ \\
& 4*\tan(1/2*a)^4 + 8*\tan(1/2*b*x)^5*\tan(1/2*a) + 36*\tan(1/2*b*x)^4*\tan(1/2*a) \\
& ^2 + 8*\tan(1/2*b*x)^3*\tan(1/2*a)^3 - 2*\tan(1/2*b*x)^4 - 8*\tan(1/2*b*x)^3*ta \\
& n(1/2*a) + 16*\tan(1/2*b*x)^2*\tan(1/2*a)^2 + 8*\tan(1/2*b*x)*\tan(1/2*a)^3 + t \\
& an(1/2*a)^4 - 8*\tan(1/2*b*x)*\tan(1/2*a) - 2*\tan(1/2*a)^2 + 1))*\tan(1/2*b*x) \\
& *\tan(1/2*a) + 2*d^2*\log(4*(\tan(1/2*a)^4 + 2*\tan(1/2*a)^2 + 1)/(\tan(1/2*b*x) \\
& ^8*\tan(1/2*a)^4 - 2*\tan(1/2*b*x)^8*\tan(1/2*a)^2 - 8*\tan(1/2*b*x)^7*\tan(1/2* \\
& a)^3 + \tan(1/2*b*x)^8 + 8*\tan(1/2*b*x)^7*\tan(1/2*a) + 16*\tan(1/2*b*x)^6*\tan \\
& (1/2*a)^2 - 8*\tan(1/2*b*x)^5*\tan(1/2*a)^3 - 2*\tan(1/2*b*x)^4*\tan(1/2*a)^4 + \\
& 8*\tan(1/2*b*x)^5*\tan(1/2*a) + 36*\tan(1/2*b*x)^4*\tan(1/2*a)^2 + 8*\tan(1/2*b \\
& *x)^3*\tan(1/2*a)^3 - 2*\tan(1/2*b*x)^4 - 8*\tan(1/2*b*x)^3*\tan(1/2*a) + 16*ta \\
& n(1/2*b*x)^2*\tan(1/2*a)^2 + 8*\tan(1/2*b*x)*\tan(1/2*a)^3 + \tan(1/2*a)^4 - 8* \\
& \tan(1/2*b*x)*\tan(1/2*a) - 2*\tan(1/2*a)^2 + 1))*\tan(1/2*a)^2 + b^2*c^2 - 4*b \\
& *c*d*\tan(1/2*b*x) - 4*b*c*d*\tan(1/2*a) - d^2*\log(4*(\tan(1/2*a)^4 + 2*\tan(1/ \\
& 2*a)^2 + 1)/(\tan(1/2*b*x)^8*\tan(1/2*a)^4 - 2*\tan(1/2*b*x)^8*\tan(1/2*a)^2 - \\
& 8*\tan(1/2*b*x)^7*\tan(1/2*a)^3 + \tan(1/2*b*x)^8 + 8*\tan(1/2*b*x)^7*\tan(1/2*a) \\
&) + 16*\tan(1/2*b*x)^6*\tan(1/2*a)^2 - 8*\tan(1/2*b*x)^5*\tan(1/2*a)^3 - 2*\tan(\\
& 1/2*b*x)^4*\tan(1/2*a)^4 + 8*\tan(1/2*b*x)^5*\tan(1/2*a) + 36*\tan(1/2*b*x)^4*t \\
& an(1/2*a)^2 + 8*\tan(1/2*b*x)^3*\tan(1/2*a)^3 - 2*\tan(1/2*b*x)^4 - 8*\tan(1/2* \\
& b*x)^3*\tan(1/2*a) + 16*\tan(1/2*b*x)^2*\tan(1/2*a)^2 + 8*\tan(1/2*b*x)*\tan(1/2 \\
& *a)^3 + \tan(1/2*a)^4 - 8*\tan(1/2*b*x)*\tan(1/2*a) - 2*\tan(1/2*a)^2 + 1))))/(b \\
& ^3*\tan(1/2*b*x)^4*\tan(1/2*a)^4 - 2*b^3*\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 8*b^3* \\
& \tan(1/2*b*x)^3*\tan(1/2*a)^3 - 2*b^3*\tan(1/2*b*x)^2*\tan(1/2*a)^4 + b^3*\tan(1 \\
& /2*b*x)^4 + 8*b^3*\tan(1/2*b*x)^3*\tan(1/2*a) + 20*b^3*\tan(1/2*b*x)^2*\tan(1/2 \\
& *a)^2 + 8*b^3*\tan(1/2*b*x)*\tan(1/2*a)^3 + b^3*\tan(1/2*a)^4 - 2*b^3*\tan(1/2* \\
& b*x)^2 - 8*b^3*\tan(1/2*b*x)*\tan(1/2*a) - 2*b^3*\tan(1/2*a)^2 + b^3)
\end{aligned}$$

3.294 $\int (c + dx) \sec^2(a + bx) \tan(a + bx) dx$

Optimal. Leaf size=35

$$\frac{(c + dx) \sec^2(a + bx)}{2b} - \frac{d \tan(a + bx)}{2b^2}$$

[Out] $((c + d*x)*\text{Sec}[a + b*x]^2)/(2*b) - (d*\text{Tan}[a + b*x])/(2*b^2)$

Rubi [A] time = 0.0316128, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {4409, 3767, 8}

$$\frac{(c + dx) \sec^2(a + bx)}{2b} - \frac{d \tan(a + bx)}{2b^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)*\text{Sec}[a + b*x]^2*\text{Tan}[a + b*x], x]$

[Out] $((c + d*x)*\text{Sec}[a + b*x]^2)/(2*b) - (d*\text{Tan}[a + b*x])/(2*b^2)$

Rule 4409

$\text{Int}[(c + d*x)^m * \text{Sec}[a + b*x]^n * \text{Tan}[a + b*x]^p, x] := \text{Simp}[(c + d*x)^m * \text{Sec}[a + b*x]^n / (b*n), x] - \text{Dist}[(d*m)/(b*n), \text{Int}[(c + d*x)^{m-1} * \text{Sec}[a + b*x]^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

Rule 3767

$\text{Int}[\text{csc}[c + d*x]^n, x] := -\text{Dist}[d^{-1}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{n/2 - 1}], x], x, \text{Cot}[c + d*x]], x] /;$ FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

$\text{Int}[a*x, x] := \text{Simp}[a*x, x] /;$ FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int (c + dx) \sec^2(a + bx) \tan(a + bx) dx &= \frac{(c + dx) \sec^2(a + bx)}{2b} - \frac{d \int \sec^2(a + bx) dx}{2b} \\ &= \frac{(c + dx) \sec^2(a + bx)}{2b} + \frac{d \text{Subst}(\int 1 dx, x, -\tan(a + bx))}{2b^2} \\ &= \frac{(c + dx) \sec^2(a + bx)}{2b} - \frac{d \tan(a + bx)}{2b^2} \end{aligned}$$

Mathematica [A] time = 0.0621442, size = 48, normalized size = 1.37

$$-\frac{d \tan(a + bx)}{2b^2} + \frac{c \sec^2(a + bx)}{2b} + \frac{dx \sec^2(a + bx)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Sec[a + b*x]^2*Tan[a + b*x], x]

[Out] (c*Sec[a + b*x]^2)/(2*b) + (d*x*Sec[a + b*x]^2)/(2*b) - (d*Tan[a + b*x])/(2*b^2)

Maple [A] time = 0.027, size = 61, normalized size = 1.7

$$\frac{1}{b} \left(\frac{d}{b} \left(\frac{bx+a}{2(\cos(bx+a))^2} - \frac{\tan(bx+a)}{2} \right) - \frac{ad}{2b(\cos(bx+a))^2} + \frac{c}{2(\cos(bx+a))^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*sec(b*x+a)^2*tan(b*x+a), x)

[Out] 1/b*(d/b*(1/2*(b*x+a)/cos(b*x+a)^2-1/2*tan(b*x+a))-1/2/b*d*a/cos(b*x+a)^2+1/2*c/cos(b*x+a)^2)

Maxima [B] time = 0.976842, size = 382, normalized size = 10.91

$$\frac{c \tan(bx+a)^2 - \frac{ad \tan(bx+a)^2}{b} + \frac{2(4(bx+a)\cos(2bx+2a)^2+4(bx+a)\sin(2bx+2a)^2+(2(bx+a)\cos(2bx+2a)+\sin(2bx+2a))\cos(4bx+4a)+2(bx+a)\cos(4bx+4a)}{(2(2\cos(2bx+2a)+1)\cos(4bx+4a)+\cos(4bx+4a)^2+4\cos(2bx+2a)^2+\sin(4bx+4a)^2+4\sin(4bx+4a)\cos(2bx+2a))}{2b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*sec(b*x+a)^2*tan(b*x+a), x, algorithm="maxima")

[Out] 1/2*(c*tan(b*x + a)^2 - a*d*tan(b*x + a)^2/b + 2*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*d/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b))/b

Fricas [A] time = 0.464679, size = 95, normalized size = 2.71

$$\frac{bdx - d \cos(bx+a) \sin(bx+a) + bc}{2b^2 \cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*sec(b*x+a)^2*tan(b*x+a), x, algorithm="fricas")

[Out] 1/2*(b*d*x - d*cos(b*x + a)*sin(b*x + a) + b*c)/(b^2*cos(b*x + a)^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \tan(a + bx) \sec^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)**2*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)*tan(a + b*x)*sec(a + b*x)**2, x)
```

Giac [B] time = 1.23716, size = 771, normalized size = 22.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4 +
2*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 +
2*b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + 2*b*
c*tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + b*d*x*tan
(1/2*b*x)^4 + 4*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*d*x*tan(1/2*a)^4 + b*
c*tan(1/2*b*x)^4 - 2*d*tan(1/2*b*x)^4*tan(1/2*a) + 4*b*c*tan(1/2*b*x)^2*tan
(1/2*a)^2 - 12*d*tan(1/2*b*x)^3*tan(1/2*a)^2 - 12*d*tan(1/2*b*x)^2*tan(1/2*
a)^3 + b*c*tan(1/2*a)^4 - 2*d*tan(1/2*b*x)*tan(1/2*a)^4 + 2*b*d*x*tan(1/2*b
*x)^2 + 2*b*d*x*tan(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2 + 2*d*tan(1/2*b*x)^3 +
12*d*tan(1/2*b*x)^2*tan(1/2*a) + 2*b*c*tan(1/2*a)^2 + 12*d*tan(1/2*b*x)*tan
(1/2*a)^2 + 2*d*tan(1/2*a)^3 + b*d*x + b*c - 2*d*tan(1/2*b*x) - 2*d*tan(1/2
*a))/(b^2*tan(1/2*b*x)^4*tan(1/2*a)^4 - 2*b^2*tan(1/2*b*x)^4*tan(1/2*a)^2 -
8*b^2*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*b^2*tan(1/2*b*x)^2*tan(1/2*a)^4 + b^
2*tan(1/2*b*x)^4 + 8*b^2*tan(1/2*b*x)^3*tan(1/2*a) + 20*b^2*tan(1/2*b*x)^2*
tan(1/2*a)^2 + 8*b^2*tan(1/2*b*x)*tan(1/2*a)^3 + b^2*tan(1/2*a)^4 - 2*b^2*t
an(1/2*b*x)^2 - 8*b^2*tan(1/2*b*x)*tan(1/2*a) - 2*b^2*tan(1/2*a)^2 + b^2)
```

$$3.295 \quad \int \frac{\sec^2(a+bx) \tan(a+bx)}{c+dx} dx$$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\tan(a+bx)\sec^2(a+bx)}{c+dx}, x\right)$$

[Out] CannotIntegrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]

Rubi [A] time = 0.106442, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sec^2(a+bx) \tan(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]

[Out] Defer[Int] [(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]

Rubi steps

$$\int \frac{\sec^2(a+bx) \tan(a+bx)}{c+dx} dx = \int \frac{\sec^2(a+bx) \tan(a+bx)}{c+dx} dx$$

Mathematica [A] time = 7.37083, size = 0, normalized size = 0.

$$\int \frac{\sec^2(a+bx) \tan(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]

[Out] Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]

Maple [A] time = 0.367, size = 0, normalized size = 0.

$$\int \frac{(\sec(bx+a))^2 \tan(bx+a)}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c), x)

[Out] int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c), x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(bx+a)^2 \tan(bx+a)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="fricas")

[Out] integral(sec(b*x + a)^2*tan(b*x + a)/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(a+bx) \sec^2(a+bx)}{c+dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)**2*tan(b*x+a)/(d*x+c),x)

[Out] Integral(tan(a + b*x)*sec(a + b*x)**2/(c + d*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)^2 \tan(bx+a)}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="giac")

[Out] integrate(sec(b*x + a)^2*tan(b*x + a)/(d*x + c), x)

$$3.296 \quad \int \frac{\sec^2(a+bx) \tan(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\tan(a+bx) \sec^2(a+bx)}{(c+dx)^2}, x\right)$$

[Out] CannotIntegrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]

Rubi [A] time = 0.128311, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sec^2(a+bx) \tan(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]

[Out] Defer[Int] [(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\sec^2(a+bx) \tan(a+bx)}{(c+dx)^2} dx = \int \frac{\sec^2(a+bx) \tan(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 10.436, size = 0, normalized size = 0.

$$\int \frac{\sec^2(a+bx) \tan(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]

[Out] Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]

Maple [A] time = 0.382, size = 0, normalized size = 0.

$$\int \frac{(\sec(bx+a))^2 \tan(bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2, x)

[Out] int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2, x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec (b x+a)^2 \tan (b x+a)}{d^2 x^2+2 c d x+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(sec(b*x + a)^2*tan(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan (a+b x) \sec ^2(a+b x)}{(c+d x)^2} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)**2*tan(b*x+a)/(d*x+c)**2,x)

[Out] Integral(tan(a + b*x)*sec(a + b*x)**2/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (b x+a)^2 \tan (b x+a)}{(d x+c)^2} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(sec(b*x + a)^2*tan(b*x + a)/(d*x + c)^2, x)

3.297 $\int (c + dx)^m \sec(a + bx) \tan^2(a + bx) dx$

Optimal. Leaf size=37

$$\text{Unintegrable}(\sec^3(a + bx)(c + dx)^m, x) - \text{Unintegrable}(\sec(a + bx)(c + dx)^m, x)$$

[Out] -Unintegrable[(c + d*x)^m*Sec[a + b*x], x] + Unintegrable[(c + d*x)^m*Sec[a + b*x]^3, x]

Rubi [A] time = 0.0839489, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \sec(a + bx) \tan^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^2, x]

[Out] -Defer[Int] [(c + d*x)^m*Sec[a + b*x], x] + Defer[Int] [(c + d*x)^m*Sec[a + b*x]^3, x]

Rubi steps

$$\int (c + dx)^m \sec(a + bx) \tan^2(a + bx) dx = - \int (c + dx)^m \sec(a + bx) dx + \int (c + dx)^m \sec^3(a + bx) dx$$

Mathematica [A] time = 8.98918, size = 0, normalized size = 0.

$$\int (c + dx)^m \sec(a + bx) \tan^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^2, x]

[Out] Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^2, x]

Maple [A] time = 0.174, size = 0, normalized size = 0.

$$\int (dx + c)^m \sec(bx + a) (\tan(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2, x)

[Out] int((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \sec(bx + a) \tan(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")

[Out] integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a)^2, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx + c)^m \sec(bx + a) \tan(bx + a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")

[Out] integral((d*x + c)^m*sec(b*x + a)*tan(b*x + a)^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**m*sec(b*x+a)*tan(b*x+a)**2,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \sec(bx + a) \tan(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a)^2, x)

3.298 $\int (c + dx)^3 \sec(a + bx) \tan^2(a + bx) dx$

Optimal. Leaf size=337

$$\frac{3d^2(c + dx)\text{PolyLog}(3, -ie^{i(a+bx)})}{b^3} - \frac{3d^2(c + dx)\text{PolyLog}(3, ie^{i(a+bx)})}{b^3} - \frac{3id(c + dx)^2\text{PolyLog}(2, -ie^{i(a+bx)})}{2b^2} + \frac{3id(c + dx)^2\text{PolyLog}(2, ie^{i(a+bx)})}{2b^2}$$

```
[Out] ((-6*I)*d^2*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^3 + (I*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + ((3*I)*d^3*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^4 - ((3*I)/2)*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))]/b^2 - ((3*I)*d^3*PolyLog[2, I*E^(I*(a + b*x))])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 - (3*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + ((3*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^4 - ((3*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4 - (3*d*(c + d*x)^2*Sec[a + b*x])/(2*b^2) + ((c + d*x)^3*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rubi [A] time = 0.408332, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {4413, 4181, 2531, 6609, 2282, 6589, 4186, 2279, 2391}

$$\frac{3d^2(c + dx)\text{PolyLog}(3, -ie^{i(a+bx)})}{b^3} - \frac{3d^2(c + dx)\text{PolyLog}(3, ie^{i(a+bx)})}{b^3} - \frac{3id(c + dx)^2\text{PolyLog}(2, -ie^{i(a+bx)})}{2b^2} + \frac{3id(c + dx)^2\text{PolyLog}(2, ie^{i(a+bx)})}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x]^2, x]
```

```
[Out] ((-6*I)*d^2*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^3 + (I*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + ((3*I)*d^3*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^4 - ((3*I)/2)*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))]/b^2 - ((3*I)*d^3*PolyLog[2, I*E^(I*(a + b*x))])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 - (3*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + ((3*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^4 - ((3*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4 - (3*d*(c + d*x)^2*Sec[a + b*x])/(2*b^2) + ((c + d*x)^3*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rule 4413

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))]^(n_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))], x])
```


)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 6609

Int[((e_.) + (f_.)*(x_.))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_.)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rule 4186

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(n_)*((c_.) + (d_.)*(x_.))^(m_), x_Symbol] := -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

Rule 2279

Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_.)))^(n_.))], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_.) + (e_.)*(x_.)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 \sec(a + bx) \tan^2(a + bx) dx &= - \int (c + dx)^3 \sec(a + bx) dx + \int (c + dx)^3 \sec^3(a + bx) dx \\
&= \frac{2i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{3d(c + dx)^2 \sec(a + bx)}{2b^2} + \frac{(c + dx)^3 \sec(a + bx) \tan(a + bx)}{2b} \\
&= -\frac{6id^2(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^3} + \frac{i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{3id(c + dx)^2 \operatorname{Li}_2(-e^{i(a+bx)})}{b^2} \\
&= -\frac{6id^2(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^3} + \frac{i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{3id(c + dx)^2 \operatorname{Li}_2(-e^{i(a+bx)})}{2b^2} \\
&= -\frac{6id^2(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^3} + \frac{i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} + \frac{3id^3 \operatorname{Li}_2(-ie^{i(a+bx)})}{b^4} \\
&= -\frac{6id^2(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^3} + \frac{i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} + \frac{3id^3 \operatorname{Li}_2(-ie^{i(a+bx)})}{b^4} \\
&= -\frac{6id^2(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^3} + \frac{i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} + \frac{3id^3 \operatorname{Li}_2(-ie^{i(a+bx)})}{b^4}
\end{aligned}$$

Mathematica [A] time = 3.6347, size = 530, normalized size = 1.57

$$-3id(b^2(c + dx)^2 - 2d^2) \operatorname{PolyLog}(2, -ie^{i(a+bx)}) + 3id(b^2(c + dx)^2 - 2d^2) \operatorname{PolyLog}(2, ie^{i(a+bx)}) + 6bcd^2 \operatorname{PolyLog}(3, -ie^{i(a+bx)})$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x]^2,x]

[Out] ((2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] - (12*I)*b*c*d^2*ArcTan[E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 - I*E^(I*(a + b*x))] + 6*b*d^3*x*Log[1 - I*E^(I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 - I*E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] - 6*b*d^3*x*Log[1 + I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1 + I*E^(I*(a + b*x))] - (3*I)*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, (-I)*E^(I*(a + b*x))] + (3*I)*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, I*E^(I*(a + b*x))] + 6*b*c*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, I*E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))] + b^2*(c + d*x)^2*Sec[a + b*x]*(-3*d + b*(c + d*x)*Tan[a + b*x])/(2*b^4)

Maple [B] time = 0.355, size = 1127, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x)

[Out] 3*I*d^3*polylog(4, -I*exp(I*(b*x+a)))/b^4 + 3/b^4*d^3*ln(1 - I*exp(I*(b*x+a)))*a + 3/b^3*d^3*polylog(3, -I*exp(I*(b*x+a)))*x - 3/b^3*d^3*ln(1 + I*exp(I*(b*x+a)))*x - 3/b^4*d^3*ln(1 + I*exp(I*(b*x+a)))*a + 1/2/b^4*d^3*a^3*ln(1 + I*exp(I*(b*x+a))) - 3/b^3*d^3*polylog(3, I*exp(I*(b*x+a)))*x + 1/2/b*d^3*ln(1 + I*exp(I*(b*x+a)))*x^3 - 1/2/b*d^3*ln(1 - I*exp(I*(b*x+a)))*x^3 - 6*I/b^3*d^2*c*arctan(exp(I*(b*x+a))) + 3/2*I/b^2*c^2*d*polylog(2, I*exp(I*(b*x+a))) - I/b^4*d^3*a^3*arctan(exp(I*(b*x+a)))

$$\begin{aligned} & *x+a)) - 3*I/b^2*c^2*d*a*\arctan(\exp(I*(b*x+a))) + 3*I/b^3*c*d^2*a^2*\arctan(\exp \\ & (I*(b*x+a))) - 3*I/b^2*d^2*c*polylog(2, -I*\exp(I*(b*x+a)))*x + 3*I/b^2*d^2*c*pol \\ & ylog(2, I*\exp(I*(b*x+a)))*x - 3*I*d^3*polylog(2, I*\exp(I*(b*x+a)))/b^4 - 3*I*d^3* \\ & polylog(4, I*\exp(I*(b*x+a)))/b^4 + I/b*c^3*\arctan(\exp(I*(b*x+a))) + 3/b^3*d^2*c* \\ & polylog(3, -I*\exp(I*(b*x+a))) - 3/b^3*d^2*c*polylog(3, I*\exp(I*(b*x+a))) - 1/2/b^ \\ & 4*d^3*a^3*\ln(1 - I*\exp(I*(b*x+a))) + 3/b^3*d^3*\ln(1 - I*\exp(I*(b*x+a)))*x - I/b^2/(\\ & \exp(2*I*(b*x+a)) + 1)^2*(d^3*x^3*b*\exp(3*I*(b*x+a)) + 3*c*d^2*x^2*b*\exp(3*I*(b* \\ & x+a)) + 3*c^2*d*x*b*\exp(3*I*(b*x+a)) - d^3*x^3*b*\exp(I*(b*x+a)) + c^3*b*\exp(3*I*(\\ & b*x+a)) - 3*c*d^2*x^2*b*\exp(I*(b*x+a)) - 3*I*d^3*x^2*\exp(3*I*(b*x+a)) - 3*c^2*d*x \\ & *b*\exp(I*(b*x+a)) - 6*I*c*d^2*x*\exp(3*I*(b*x+a)) - c^3*b*\exp(I*(b*x+a)) - 3*I*c^2 \\ & *d*\exp(3*I*(b*x+a)) - 3*I*d^3*x^2*\exp(I*(b*x+a)) - 6*I*c*d^2*x*\exp(I*(b*x+a)) - 3 \\ & *I*c^2*d*\exp(I*(b*x+a)) + 3*I*d^3*polylog(2, -I*\exp(I*(b*x+a)))/b^4 - 3/2/b*d^2 \\ & *c*\ln(1 - I*\exp(I*(b*x+a)))*x^2 + 3/2/b*d^2*c*\ln(1 + I*\exp(I*(b*x+a)))*x^2 + 3/2/b* \\ & c^2*d*\ln(1 + I*\exp(I*(b*x+a)))*x + 3/2/b^2*c^2*d*\ln(1 + I*\exp(I*(b*x+a)))*a - 3/2/b \\ & *c^2*d*\ln(1 - I*\exp(I*(b*x+a)))*x - 3/2/b^2*c^2*d*\ln(1 - I*\exp(I*(b*x+a)))*a - 3/2/ \\ & b^3*d^2*c*a^2*\ln(1 + I*\exp(I*(b*x+a))) + 3/2/b^3*d^2*c*a^2*\ln(1 - I*\exp(I*(b*x+a) \\ &)) + 6*I/b^4*d^3*a*\arctan(\exp(I*(b*x+a))) - 3/2*I/b^2*c^2*d*polylog(2, -I*\exp(I* \\ & (b*x+a))) - 3/2*I/b^2*d^3*polylog(2, -I*\exp(I*(b*x+a)))*x^2 + 3/2*I/b^2*d^3*poly \\ & log(2, I*\exp(I*(b*x+a)))*x^2 \end{aligned}$$

Maxima [B] time = 6.80647, size = 5168, normalized size = 15.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/4*(c^3*(2*\sin(b*x + a)/(\sin(b*x + a)^2 - 1) + \log(\sin(b*x + a) + 1) - \log \\ & (\sin(b*x + a) - 1)) - 3*a*c^2*d*(2*\sin(b*x + a)/(\sin(b*x + a)^2 - 1) + \log \\ & (\sin(b*x + a) + 1) - \log(\sin(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*\sin(b*x + a) \\ & /(\sin(b*x + a)^2 - 1) + \log(\sin(b*x + a) + 1) - \log(\sin(b*x + a) - 1))/b^2 \\ & - a^3*d^3*(2*\sin(b*x + a)/(\sin(b*x + a)^2 - 1) + \log(\sin(b*x + a) + 1) - \log \\ & (\sin(b*x + a) - 1))/b^3 - 4*((2*(b*x + a)^3*d^3 - 12*b*c*d^2 + 12*a*d^3 + \\ & 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^ \\ & 3)*(b*x + a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^ \\ & 3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*\cos \\ & (4*b*x + 4*a) + 4*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d \\ & ^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*\co \\ & s(2*b*x + 2*a) - (-2*I*(b*x + a)^3*d^3 + 12*I*b*c*d^2 - 12*I*a*d^3 + (-6*I* \\ & b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I \\ & *a^2 + 12*I)*d^3)*(b*x + a))*\sin(4*b*x + 4*a) - (-4*I*(b*x + a)^3*d^3 + 24* \\ & I*b*c*d^2 - 24*I*a*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2 + (-12*I* \\ & b^2*c^2*d + 24*I*a*b*c*d^2 + (-12*I*a^2 + 24*I)*d^3)*(b*x + a))*\sin(2*b*x + \\ & 2*a))*\arctan2(\cos(b*x + a), \sin(b*x + a) + 1) + (2*(b*x + a)^3*d^3 - 12*b* \\ & c*d^2 + 12*a*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c \\ & *d^2 + (a^2 - 2)*d^3)*(b*x + a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 \\ & + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)* \\ & d^3)*(b*x + a))*\cos(4*b*x + 4*a) + 4*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 \\ & + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2) \\ & *d^3)*(b*x + a))*\cos(2*b*x + 2*a) - (-2*I*(b*x + a)^3*d^3 + 12*I*b*c*d^2 - \\ & 12*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12* \\ & I*a*b*c*d^2 + (-6*I*a^2 + 12*I)*d^3)*(b*x + a))*\sin(4*b*x + 4*a) - (-4*I*(b \\ & *x + a)^3*d^3 + 24*I*b*c*d^2 - 24*I*a*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b \\ & *x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 + (-12*I*a^2 + 24*I)*d^3)*(b* \\ & x + a))*\sin(2*b*x + 2*a))*\arctan2(\cos(b*x + a), -\sin(b*x + a) + 1) - (4*(b \\ & x + a)^3*d^3 - 12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*a^2*d^3 + (12*b*c*d^2 \end{aligned}$$

$$\begin{aligned}
& - (12*a + 12*I)*d^3*(b*x + a)^2 + (12*b^2*c^2*d - (24*a + 24*I)*b*c*d^2 + \\
& 12*(a^2 + 2*I*a)*d^3)*(b*x + a)*\cos(3*b*x + 3*a) + (4*(b*x + a)^3*d^3 + 1 \\
& 2*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^3 + (12*b*c*d^2 - (12*a - 12*I) \\
& *d^3)*(b*x + a)^2 + (12*b^2*c^2*d - (24*a - 24*I)*b*c*d^2 + 12*(a^2 - 2*I*a) \\
&)*d^3)*(b*x + a)*\cos(b*x + a) + (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^ \\
& 2*d^3 + 6*(a^2 - 2)*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2 \\
& *a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a \\
&))*\cos(4*b*x + 4*a) + 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 \\
& - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - (-6*I*b^2*c^2* \\
& d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 + (-6*I*a^2 + 12*I)*d^3 + (-12*I*b \\
& *c*d^2 + 12*I*a*d^3)*(b*x + a)*\sin(4*b*x + 4*a) - (-12*I*b^2*c^2*d + 24*I* \\
& a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 + (-12*I*a^2 + 24*I)*d^3 + (-24*I*b*c*d^2 \\
& + 24*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{dilog}(I*e^{(I*b*x + I*a)}) - (6*b^ \\
& 2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*(a^2 - 2)*d^3 + 12*(b*c*d^2 \\
& - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - \\
& 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + 12*(b^2*c^2*d - \\
& 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + \\
& a))*\cos(2*b*x + 2*a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^ \\
& 3 + (6*I*a^2 - 12*I)*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*\sin(4*b*x \\
& + 4*a) + (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*d^3 + (12*I*a \\
& ^2 - 24*I)*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{d} \\
& \operatorname{ilog}(-I*e^{(I*b*x + I*a)}) - (-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + \\
& (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 + \\
& (-3*I*a^2 + 6*I)*d^3)*(b*x + a) + (-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a \\
& *d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c \\
& *d^2 + (-3*I*a^2 + 6*I)*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + (-2*I*(b*x + a)^ \\
& 3*d^3 + 12*I*b*c*d^2 - 12*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 \\
& + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 12*I)*d^3)*(b*x + a))*\cos(\\
& 2*b*x + 2*a) + ((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3) \\
& *(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*\sin(4 \\
& *b*x + 4*a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3) \\
&)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*\sin(\\
& 2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\sin(b*x + a) + 1) - (\\
& I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*(b* \\
& x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + (3*I*a^2 - 6*I)*d^3)*(b*x + a) \\
& + (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)* \\
& (b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + (3*I*a^2 - 6*I)*d^3)*(b*x + \\
& a))*\cos(4*b*x + 4*a) + (2*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (\\
& 6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + (6 \\
& *I*a^2 - 12*I)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - ((b*x + a)^3*d^3 - 6*b*c* \\
& d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^ \\
& 2 + (a^2 - 2)*d^3)*(b*x + a))*\sin(4*b*x + 4*a) - 2*((b*x + a)^3*d^3 - 6*b*c \\
& *d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d \\
& ^2 + (a^2 - 2)*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b \\
& *x + a)^2 - 2*\sin(b*x + a) + 1) - (12*d^3*\cos(4*b*x + 4*a) + 24*d^3*\cos(2*b \\
& *x + 2*a) + 12*I*d^3*\sin(4*b*x + 4*a) + 24*I*d^3*\sin(2*b*x + 2*a) + 12*d^3) \\
& *polylog(4, I*e^{(I*b*x + I*a)}) + (12*d^3*\cos(4*b*x + 4*a) + 24*d^3*\cos(2*b* \\
& x + 2*a) + 12*I*d^3*\sin(4*b*x + 4*a) + 24*I*d^3*\sin(2*b*x + 2*a) + 12*d^3)* \\
& polylog(4, -I*e^{(I*b*x + I*a)}) - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I \\
& *a*d^3 + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*\cos(4*b*x + 4*a) \\
& + (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*\cos(2*b*x + 2*a) + 12 \\
& *(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\sin(4*b*x + 4*a) + 24*(b*c*d^2 + (b*x + a) \\
&)*d^3 - a*d^3)*\sin(2*b*x + 2*a))*polylog(3, I*e^{(I*b*x + I*a)}) - (12*I*b*c* \\
& d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3 + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 \\
& - 12*I*a*d^3)*\cos(4*b*x + 4*a) + (24*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 24*I* \\
& a*d^3)*\cos(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\sin(4*b*x + \\
& 4*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\sin(2*b*x + 2*a))*polylog(3, -I \\
& *e^{(I*b*x + I*a)}) - (4*I*(b*x + a)^3*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12 \\
& *a^2*d^3 + (12*I*b*c*d^2 - 12*(I*a - 1)*d^3)*(b*x + a)^2 + (12*I*b^2*c^2*d
\end{aligned}$$

$$- 24*(I*a - 1)*b*c*d^2 + (12*I*a^2 - 24*a)*d^3*(b*x + a))*\sin(3*b*x + 3*a) - (-4*I*(b*x + a)^3*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^2*d^3 + (-12*I*b*c*d^2 - 12*(-I*a - 1)*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d - 24*(-I*a - 1)*b*c*d^2 + (-12*I*a^2 - 24*a)*d^3)*(b*x + a))*\sin(b*x + a))/(-4*I*b^3*\cos(4*b*x + 4*a) - 8*I*b^3*\cos(2*b*x + 2*a) + 4*b^3*\sin(4*b*x + 4*a) + 8*b^3*\sin(2*b*x + 2*a) - 4*I*b^3))/b$$

Fricas [C] time = 0.945466, size = 3214, normalized size = 9.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")

[Out] $\frac{1}{4}*(-6*I*d^3*\cos(b*x + a)^2*\text{polylog}(4, I*\cos(b*x + a) + \sin(b*x + a)) - 6*I*d^3*\cos(b*x + a)^2*\text{polylog}(4, I*\cos(b*x + a) - \sin(b*x + a)) + 6*I*d^3*\cos(b*x + a)^2*\text{polylog}(4, -I*\cos(b*x + a) + \sin(b*x + a)) + 6*I*d^3*\cos(b*x + a)^2*\text{polylog}(4, -I*\cos(b*x + a) - \sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d - 6*I*d^3)*\cos(b*x + a)^2*\text{dilog}(I*\cos(b*x + a) + \sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d - 6*I*d^3)*\cos(b*x + a)^2*\text{dilog}(I*\cos(b*x + a) - \sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + 6*I*d^3)*\cos(b*x + a)^2*\text{dilog}(-I*\cos(b*x + a) + \sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + 6*I*d^3)*\cos(b*x + a)^2*\text{dilog}(-I*\cos(b*x + a) - \sin(b*x + a)) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*\cos(b*x + a)^2*\log(\cos(b*x + a) + I*\sin(b*x + a) + I) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*\cos(b*x + a)^2*\log(\cos(b*x + a) - I*\sin(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*\cos(b*x + a)^2*\log(I*\cos(b*x + a) + \sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*\cos(b*x + a)^2*\log(-I*\cos(b*x + a) + \sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*\cos(b*x + a)^2*\log(-I*\cos(b*x + a) - \sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*\cos(b*x + a)^2*\log(-\cos(b*x + a) + I*\sin(b*x + a) + I) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*\cos(b*x + a)^2*\log(-\cos(b*x + a) - I*\sin(b*x + a) + I) + 6*(b*d^3*x + b*c*d^2)*\cos(b*x + a)^2*\text{polylog}(3, I*\cos(b*x + a) + \sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*\cos(b*x + a)^2*\text{polylog}(3, I*\cos(b*x + a) - \sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*\cos(b*x + a)^2*\text{polylog}(3, -I*\cos(b*x + a) + \sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*\cos(b*x + a)^2*\text{polylog}(3, -I*\cos(b*x + a) - \sin(b*x + a)) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*\cos(b*x + a) + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\sin(b*x + a))/(b^4*\cos(b*x + a)^2)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \tan^2(a + bx) \sec(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*tan(a + b*x)**2*sec(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \sec(bx + a) \tan(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)*tan(b*x + a)^2, x)
```

3.299 $\int (c + dx)^2 \sec(a + bx) \tan^2(a + bx) dx$

Optimal. Leaf size=193

$$\frac{id(c + dx)\text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^2} + \frac{id(c + dx)\text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^2} + \frac{d^2\text{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^3} - \frac{d^2\text{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^3}$$

```
[Out] (I*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (d^2*ArcTanh[Sin[a + b*x]])/b^3
- (I*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 + (I*d*(c + d*x)*Po
lyLog[2, I*E^(I*(a + b*x))])/b^2 + (d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])/b
^3 - (d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - (d*(c + d*x)*Sec[a + b*x])/b
^2 + ((c + d*x)^2*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rubi [A] time = 0.271143, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {4413, 4181, 2531, 2282, 6589, 4186, 3770}

$$\frac{id(c + dx)\text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^2} + \frac{id(c + dx)\text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^2} + \frac{d^2\text{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^3} - \frac{d^2\text{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (I*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (d^2*ArcTanh[Sin[a + b*x]])/b^3
- (I*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 + (I*d*(c + d*x)*Po
lyLog[2, I*E^(I*(a + b*x))])/b^2 + (d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])/b
^3 - (d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - (d*(c + d*x)*Sec[a + b*x])/b
^2 + ((c + d*x)^2*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rule 4413

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/ (b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
```

```
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x]
+ (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*
(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^(m - 2)*
(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x]
&& GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \sec(a + bx) \tan^2(a + bx) dx &= - \int (c + dx)^2 \sec(a + bx) dx + \int (c + dx)^2 \sec^3(a + bx) dx \\ &= \frac{2i(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{d(c + dx) \sec(a + bx)}{b^2} + \frac{(c + dx)^2 \sec(a + bx) \tan(a + bx)}{2b} \\ &= \frac{i(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b} + \frac{d^2 \tanh^{-1}(\sin(a + bx))}{b^3} - \frac{2id(c + dx) \operatorname{Li}_2(-ie^{i(a+bx)})}{b^2} \\ &= \frac{i(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b} + \frac{d^2 \tanh^{-1}(\sin(a + bx))}{b^3} - \frac{id(c + dx) \operatorname{Li}_2(-ie^{i(a+bx)})}{b^2} \\ &= \frac{i(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b} + \frac{d^2 \tanh^{-1}(\sin(a + bx))}{b^3} - \frac{id(c + dx) \operatorname{Li}_2(-ie^{i(a+bx)})}{b^2} \\ &= \frac{i(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b} + \frac{d^2 \tanh^{-1}(\sin(a + bx))}{b^3} - \frac{id(c + dx) \operatorname{Li}_2(-ie^{i(a+bx)})}{b^2} \end{aligned}$$

Mathematica [B] time = 7.22457, size = 526, normalized size = 2.73

$$\frac{-id(c + dx) \operatorname{PolyLog}(2, -ie^{i(a+bx)}) + id(c + dx) \operatorname{PolyLog}(2, ie^{i(a+bx)}) + \frac{d^2 \operatorname{PolyLog}(3, -ie^{i(a+bx)})}{b} - \frac{d^2 \operatorname{PolyLog}(3, ie^{i(a+bx)})}{b} + ibc^2 \tan(a + bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x]^2, x]
```

```
[Out] (I*b*c^2*ArcTan[E^(I*(a + b*x))] - ((2*I)*d^2*ArcTan[E^(I*(a + b*x))])/b -
b*c*d*x*Log[1 - I*E^(I*(a + b*x))] - (b*d^2*x^2*Log[1 - I*E^(I*(a + b*x))])
/2 + b*c*d*x*Log[1 + I*E^(I*(a + b*x))] + (b*d^2*x^2*Log[1 + I*E^(I*(a + b*x))])
/2 - I*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] + I*d*(c + d*x)*Po
```



```
lyLog[2, I*E^(I*(a + b*x))] + (d^2*PolyLog[3, (-I)*E^(I*(a + b*x))]/b - (d
^2*PolyLog[3, I*E^(I*(a + b*x))]/b)/b^2 - (d*(c + d*x)*Sec[a])/b^2 + (c^2
+ 2*c*d*x + d^2*x^2)/(4*b*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])^2) + (-
(c*d*Sin[(b*x)/2]) - d^2*x*Sin[(b*x)/2])/(b^2*(Cos[a/2] - Sin[a/2])*(Cos[a/
2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) + (-c^2 - 2*c*d*x - d^2*x^2)/(4*b*(Cos[
a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2])^2) + (c*d*Sin[(b*x)/2] + d^2*x*Sin[(b*
x)/2])/(b^2*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2]
)
```

Maple [B] time = 0.31, size = 584, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] I/b^3*d^2*a^2*arctan(exp(I*(b*x+a)))-I/b^2*d^2*polylog(2,-I*exp(I*(b*x+a)))
*x+I/b^2*c*d*polylog(2,I*exp(I*(b*x+a)))-1/2/b*d^2*ln(1-I*exp(I*(b*x+a)))*x
^2+1/b*c*d*ln(1+I*exp(I*(b*x+a)))*x-d^2*polylog(3,I*exp(I*(b*x+a)))/b^3-1/2
/b^3*d^2*a^2*ln(1+I*exp(I*(b*x+a)))+1/b^2*c*d*ln(1+I*exp(I*(b*x+a)))*a+1/2/
b^3*d^2*a^2*ln(1-I*exp(I*(b*x+a)))+I/b*c^2*arctan(exp(I*(b*x+a)))-2*I/b^3*d
^2*arctan(exp(I*(b*x+a)))-2*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))+1/2/b*d^2*ln
(1+I*exp(I*(b*x+a)))*x^2-1/b*c*d*ln(1-I*exp(I*(b*x+a)))*x+I/b^2*d^2*polylog
(2,I*exp(I*(b*x+a)))*x-1/b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a+d^2*polylog(3,-I*
exp(I*(b*x+a)))/b^3-I/b^2*c*d*polylog(2,-I*exp(I*(b*x+a)))-I/b^2/(exp(2*I*(
b*x+a))+1)^2*(d^2*x^2*b*exp(3*I*(b*x+a))+2*c*d*x*b*exp(3*I*(b*x+a))+c^2*b*exp
(3*I*(b*x+a))-d^2*x^2*b*exp(I*(b*x+a))-2*c*d*x*b*exp(I*(b*x+a))-2*I*d^2*x
*exp(3*I*(b*x+a))-c^2*b*exp(I*(b*x+a))-2*I*d*c*exp(3*I*(b*x+a))-2*I*d^2*x*exp
(I*(b*x+a))-2*I*d*c*exp(I*(b*x+a))
```

Maxima [B] time = 2.89778, size = 2556, normalized size = 13.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(c^2*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - lo
g(sin(b*x + a) - 1)) - 2*a*c*d*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(s
in(b*x + a) + 1) - log(sin(b*x + a) - 1))/b + a^2*d^2*(2*sin(b*x + a)/(sin(
b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b^2 - 4*((
2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - 4*d^2 + 2*((b*x + a)^2*d^
2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(4*b*x + 4*a) + 4*((b*x + a)^2*
d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(2*b*x + 2*a) - (-2*I*(b*x +
a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a) + 4*I*d^2)*sin(4*b*x + 4*a) -
(-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) + 8*I*d^2)*sin(
2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (2*(b*x + a)^2*d^2
+ 4*(b*c*d - a*d^2)*(b*x + a) - 4*d^2 + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a
*d^2)*(b*x + a) - 2*d^2)*cos(4*b*x + 4*a) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a
*d^2)*(b*x + a) - 2*d^2)*cos(2*b*x + 2*a) - (-2*I*(b*x + a)^2*d^2 + (-4*I*b
*c*d + 4*I*a*d^2)*(b*x + a) + 4*I*d^2)*sin(4*b*x + 4*a) - (-4*I*(b*x + a)^2
*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) + 8*I*d^2)*sin(2*b*x + 2*a))*arct
```

```

an2(cos(b*x + a), -sin(b*x + a) + 1) - (4*(b*x + a)^2*d^2 - 8*I*b*c*d + 8*I
*a*d^2 + (8*b*c*d - (8*a + 8*I)*d^2)*(b*x + a))*cos(3*b*x + 3*a) + (4*(b*x
+ a)^2*d^2 + 8*I*b*c*d - 8*I*a*d^2 + (8*b*c*d - (8*a - 8*I)*d^2)*(b*x + a))
*cos(b*x + a) + (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)
*d^2 - a*d^2))*cos(4*b*x + 4*a) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*
x + 2*a) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin(4*b*x + 4*a) -
(-8*I*b*c*d - 8*I*(b*x + a)*d^2 + 8*I*a*d^2)*sin(2*b*x + 2*a))*dilog(I*e^(I
*b*x + I*a)) - (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*
d^2 - a*d^2))*cos(4*b*x + 4*a) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x
+ 2*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*sin(4*b*x + 4*a) + (8
*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-I*e^(I*b
*x + I*a)) - (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) + 2*I
*d^2 + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) + 2*I*d^2))*
cos(4*b*x + 4*a) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x +
a) + 4*I*d^2))*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
+ a) - 2*d^2)*sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*
x + a) - 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*s
in(b*x + a) + 1) - (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a) -
2*I*d^2 + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a) - 2*I*d^2)
)*cos(4*b*x + 4*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x +
a) - 4*I*d^2))*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
+ a) - 2*d^2)*sin(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*
x + a) - 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*s
in(b*x + a) + 1) - (-4*I*d^2*cos(4*b*x + 4*a) - 8*I*d^2*cos(2*b*x + 2*a) +
4*d^2*sin(4*b*x + 4*a) + 8*d^2*sin(2*b*x + 2*a) - 4*I*d^2)*polylog(3, I*e^(
I*b*x + I*a)) - (4*I*d^2*cos(4*b*x + 4*a) + 8*I*d^2*cos(2*b*x + 2*a) - 4*d^
2*sin(4*b*x + 4*a) - 8*d^2*sin(2*b*x + 2*a) + 4*I*d^2)*polylog(3, -I*e^(I*b
*x + I*a)) - (4*I*(b*x + a)^2*d^2 + 8*b*c*d - 8*a*d^2 + (8*I*b*c*d - 8*(I*a
- 1)*d^2)*(b*x + a))*sin(3*b*x + 3*a) - (-4*I*(b*x + a)^2*d^2 + 8*b*c*d -
8*a*d^2 + (-8*I*b*c*d - 8*(-I*a - 1)*d^2)*(b*x + a))*sin(b*x + a))/(-4*I*b^
2*cos(4*b*x + 4*a) - 8*I*b^2*cos(2*b*x + 2*a) + 4*b^2*sin(4*b*x + 4*a) + 8*
b^2*sin(2*b*x + 2*a) - 4*I*b^2))/b

```

Fricas [C] time = 0.747209, size = 2033, normalized size = 10.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```

[Out] 1/4*(2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*d^2
*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*cos(b*x +
a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*cos(b*x + a)^2*pol
ylog(3, -I*cos(b*x + a) - sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x
+ a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*co
s(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c
*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-2*I*b*d^2*x -
2*I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^2*c^2
- 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x +
a) + I) + (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2*log(cos(b*x
+ a) - I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d
^2)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 +
2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(
b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x +
a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x
+ 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) +

```

1) $-(b^2c^2 - 2ab^2cd + (a^2 - 2)d^2)\cos(bx + a)^2 \log(-\cos(bx + a) + I\sin(bx + a) + I) + (b^2c^2 - 2ab^2cd + (a^2 - 2)d^2)\cos(bx + a)^2 \log(-\cos(bx + a) - I\sin(bx + a) + I) - 4(bd^2x + bcd)\cos(bx + a) + 2(b^2d^2x^2 + 2b^2cdx + b^2c^2)\sin(bx + a) / (b^3\cos(bx + a)^2)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \tan^2(a + bx) \sec(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*sec(b*x+a)*tan(b*x+a)**2,x)

[Out] Integral((c + d*x)**2*tan(a + b*x)**2*sec(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \sec(bx + a) \tan(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^2*sec(b*x + a)*tan(b*x + a)^2, x)

3.300 $\int (c + dx) \sec(a + bx) \tan^2(a + bx) dx$

Optimal. Leaf size=117

$$-\frac{id\text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{2b^2} + \frac{id\text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{2b^2} - \frac{d\sec(a+bx)}{2b^2} + \frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)\tan(a+bx)}{2b}$$

[Out] (I*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b - ((I/2)*d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 + ((I/2)*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (d*Sec[a + b*x])/(2*b^2) + ((c + d*x)*Sec[a + b*x]*Tan[a + b*x])/(2*b)

Rubi [A] time = 0.12925, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {4413, 4181, 2279, 2391, 4185}

$$-\frac{id\text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{2b^2} + \frac{id\text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{2b^2} - \frac{d\sec(a+bx)}{2b^2} + \frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)\tan(a+bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Sec[a + b*x]*Tan[a + b*x]^2, x]

[Out] (I*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b - ((I/2)*d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 + ((I/2)*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (d*Sec[a + b*x])/(2*b^2) + ((c + d*x)*Sec[a + b*x]*Tan[a + b*x])/(2*b)

Rule 4413

Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x_)]^(p_), x_Symbol] := -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p/2, 0]

Rule 4181

Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_.) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 4185

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_)), x_Symbol] := -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]

, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]

Rubi steps

$$\begin{aligned} \int (c + dx) \sec(a + bx) \tan^2(a + bx) dx &= - \int (c + dx) \sec(a + bx) dx + \int (c + dx) \sec^3(a + bx) dx \\ &= \frac{2i(c + dx) \tan^{-1}(e^{i(a+bx)})}{b} - \frac{d \sec(a + bx)}{2b^2} + \frac{(c + dx) \sec(a + bx) \tan(a + bx)}{2b} \\ &= \frac{i(c + dx) \tan^{-1}(e^{i(a+bx)})}{b} - \frac{d \sec(a + bx)}{2b^2} + \frac{(c + dx) \sec(a + bx) \tan(a + bx)}{2b} \\ &= \frac{i(c + dx) \tan^{-1}(e^{i(a+bx)})}{b} - \frac{id\text{Li}_2(-ie^{i(a+bx)})}{b^2} + \frac{id\text{Li}_2(ie^{i(a+bx)})}{b^2} - \frac{d \sec(a + bx)}{2b^2} \\ &= \frac{i(c + dx) \tan^{-1}(e^{i(a+bx)})}{b} - \frac{id\text{Li}_2(-ie^{i(a+bx)})}{2b^2} + \frac{id\text{Li}_2(ie^{i(a+bx)})}{2b^2} - \frac{d \sec(a + bx)}{2b^2} \end{aligned}$$

Mathematica [B] time = 6.5224, size = 555, normalized size = 4.74

$$dx \left(-i\text{PolyLog} \left(2, \frac{1}{2} \left((1 + i) - (1 - i) \tan \left(\frac{1}{2}(a + bx) \right) \right) \right) + i\text{PolyLog} \left(2, \left(-\frac{1}{2} - \frac{i}{2} \right) \left(\tan \left(\frac{1}{2}(a + bx) \right) + i \right) \right) - i\text{PolyLog} \right.$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)*Sec[a + b*x]*Tan[a + b*x]^2,x]

[Out] -(c*ArcTanh[Sin[a + b*x]])/(2*b) + (d*x*(a*Log[1 - Tan[(a + b*x)/2]] + I*Log[1 + I*Tan[(a + b*x)/2]]*Log[(-1/2 - I/2)*(-1 + Tan[(a + b*x)/2]]) - I*Log[1 - I*Tan[(a + b*x)/2]]*Log[(-1/2 + I/2)*(-1 + Tan[(a + b*x)/2]]) - I*Log[1 + I*Tan[(a + b*x)/2]]*Log[(1/2 - I/2)*(1 + Tan[(a + b*x)/2]]) + I*Log[1 - I*Tan[(a + b*x)/2]]*Log[(1/2 + I/2)*(1 + Tan[(a + b*x)/2]]) - a*Log[1 + Tan[(a + b*x)/2]] - I*PolyLog[2, ((1 + I) - (1 - I)*Tan[(a + b*x)/2])/2] + I*PolyLog[2, (-1/2 - I/2)*(I + Tan[(a + b*x)/2])] - I*PolyLog[2, ((1 + I) + (1 - I)*Tan[(a + b*x)/2])/2] + I*PolyLog[2, ((1 - I) + (1 + I)*Tan[(a + b*x)/2])/2]))/(2*b*(a - I*Log[1 - I*Tan[(a + b*x)/2]] + I*Log[1 + I*Tan[(a + b*x)/2]])) + (d*x)/(4*b*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])^2) - (d*Sin[(a + b*x)/2])/(2*b^2*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])) - (d*x)/(4*b*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])^2) + (d*Sin[(a + b*x)/2])/(2*b^2*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])) + (c*Sec[a + b*x]*Tan[a + b*x])/(2*b)

Maple [B] time = 0.162, size = 267, normalized size = 2.3

$$\frac{-i(dxbe^{3i(bx+a)} - ide^{3i(bx+a)} + bce^{3i(bx+a)} - dxbe^{i(bx+a)} - ide^{i(bx+a)} - bce^{i(bx+a)})}{b^2(e^{2i(bx+a)} + 1)^2} + \frac{ic \arctan(e^{i(bx+a)})}{b} + \frac{d \ln(1 + ie^{i(bx+a)})}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x)

[Out] -I/b^2/(exp(2*I*(b*x+a))+1)^2*(d*x*b*exp(3*I*(b*x+a))-I*d*exp(3*I*(b*x+a))+b*c*exp(3*I*(b*x+a))-d*x*b*exp(I*(b*x+a))-I*d*exp(I*(b*x+a))-b*c*exp(I*(b*x

+a)))+I/b*c*arctan(exp(I*(b*x+a)))+1/2/b*d*ln(1+I*exp(I*(b*x+a)))*x+1/2/b^2*d*ln(1+I*exp(I*(b*x+a)))*a-1/2/b*d*ln(1-I*exp(I*(b*x+a)))*x-1/2/b^2*d*ln(1-I*exp(I*(b*x+a)))*a-1/2*I/b^2*d*dilog(1+I*exp(I*(b*x+a)))+1/2*I/b^2*d*dilog(1-I*exp(I*(b*x+a)))-I/b^2*d*a*arctan(exp(I*(b*x+a)))

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [B] time = 0.624285, size = 1169, normalized size = 9.99

$i d \cos (b x+a)^2 \operatorname{Li}_2(i \cos (b x+a)+\sin (b x+a))+i d \cos (b x+a)^2 \operatorname{Li}_2(i \cos (b x+a)-\sin (b x+a))-i d \cos (b x+a)^2$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")

[Out] $\frac{1}{4} * (I * d * \cos (b * x + a)^2 * \operatorname{dilog}(I * \cos (b * x + a) + \sin (b * x + a)) + I * d * \cos (b * x + a)^2 * \operatorname{dilog}(I * \cos (b * x + a) - \sin (b * x + a)) - I * d * \cos (b * x + a)^2 * \operatorname{dilog}(-I * \cos (b * x + a) + \sin (b * x + a)) - I * d * \cos (b * x + a)^2 * \operatorname{dilog}(-I * \cos (b * x + a) - \sin (b * x + a)) - (b * c - a * d) * \cos (b * x + a)^2 * \log(\cos (b * x + a) + I * \sin (b * x + a) + I) + (b * c - a * d) * \cos (b * x + a)^2 * \log(\cos (b * x + a) - I * \sin (b * x + a) + I) - (b * d * x + a * d) * \cos (b * x + a)^2 * \log(I * \cos (b * x + a) + \sin (b * x + a) + 1) + (b * d * x + a * d) * \cos (b * x + a)^2 * \log(I * \cos (b * x + a) - \sin (b * x + a) + 1) - (b * d * x + a * d) * \cos (b * x + a)^2 * \log(-I * \cos (b * x + a) + \sin (b * x + a) + 1) + (b * d * x + a * d) * \cos (b * x + a)^2 * \log(-I * \cos (b * x + a) - \sin (b * x + a) + 1) - (b * c - a * d) * \cos (b * x + a)^2 * \log(-\cos (b * x + a) + I * \sin (b * x + a) + I) + (b * c - a * d) * \cos (b * x + a)^2 * \log(-\cos (b * x + a) - I * \sin (b * x + a) + I) - 2 * d * \cos (b * x + a) + 2 * (b * d * x + b * c) * \sin (b * x + a)) / (b^2 * \cos (b * x + a)^2)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \tan^2(a + bx) \sec(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)**2,x)

[Out] Integral((c + d*x)*tan(a + b*x)**2*sec(a + b*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \sec (bx + a) \tan (bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a)*tan(b*x + a)^2, x)
```

$$3.301 \quad \int \frac{\sec(a+bx) \tan^2(a+bx)}{c+dx} dx$$

Optimal. Leaf size=37

$$\text{Unintegrable}\left(\frac{\sec^3(a+bx)}{c+dx}, x\right) - \text{Unintegrable}\left(\frac{\sec(a+bx)}{c+dx}, x\right)$$

[Out] -Unintegrable[Sec[a + b*x]/(c + d*x), x] + Unintegrable[Sec[a + b*x]^3/(c + d*x), x]

Rubi [A] time = 0.0911012, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sec(a+bx) \tan^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x), x]

[Out] -Defer[Int][Sec[a + b*x]/(c + d*x), x] + Defer[Int][Sec[a + b*x]^3/(c + d*x), x]

Rubi steps

$$\int \frac{\sec(a+bx) \tan^2(a+bx)}{c+dx} dx = - \int \frac{\sec(a+bx)}{c+dx} dx + \int \frac{\sec^3(a+bx)}{c+dx} dx$$

Mathematica [A] time = 26.6999, size = 0, normalized size = 0.

$$\int \frac{\sec(a+bx) \tan^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x), x]

[Out] Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x), x]

Maple [A] time = 1.455, size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a) (\tan(bx+a))^2}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c), x)

[Out] int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c), x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec (b x+a) \tan (b x+a)^2}{d x+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="fricas")

[Out] integral(sec(b*x + a)*tan(b*x + a)^2/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan ^2(a+b x) \sec (a+b x)}{c+d x} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*tan(b*x+a)**2/(d*x+c),x)

[Out] Integral(tan(a + b*x)**2*sec(a + b*x)/(c + d*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (b x+a) \tan (b x+a)^2}{d x+c} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="giac")

[Out] integrate(sec(b*x + a)*tan(b*x + a)^2/(d*x + c), x)

$$3.302 \quad \int \frac{\sec(a+bx) \tan^2(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=37

$$\text{Unintegrable}\left(\frac{\sec^3(a+bx)}{(c+dx)^2}, x\right) - \text{Unintegrable}\left(\frac{\sec(a+bx)}{(c+dx)^2}, x\right)$$

[Out] -Unintegrable[Sec[a + b*x]/(c + d*x)^2, x] + Unintegrable[Sec[a + b*x]^3/(c + d*x)^2, x]

Rubi [A] time = 0.0888488, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sec(a+bx) \tan^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2, x]

[Out] -Defer[Int][Sec[a + b*x]/(c + d*x)^2, x] + Defer[Int][Sec[a + b*x]^3/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\sec(a+bx) \tan^2(a+bx)}{(c+dx)^2} dx = - \int \frac{\sec(a+bx)}{(c+dx)^2} dx + \int \frac{\sec^3(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 29.522, size = 0, normalized size = 0.

$$\int \frac{\sec(a+bx) \tan^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2, x]

[Out] Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2, x]

Maple [A] time = 2.21, size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a) (\tan(bx+a))^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2, x)

[Out] `int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)`

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")`

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(bx+a)\tan(bx+a)^2}{d^2x^2+2cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")`

[Out] `integral(sec(b*x + a)*tan(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)`

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(a+bx)\sec(a+bx)}{(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(b*x+a)*tan(b*x+a)**2/(d*x+c)**2,x)`

[Out] `Integral(tan(a + b*x)**2*sec(a + b*x)/(c + d*x)**2, x)`

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)\tan(bx+a)^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")`

[Out] `integrate(sec(b*x + a)*tan(b*x + a)^2/(d*x + c)^2, x)`

3.303 $\int (c + dx)^m \tan^3(a + bx) dx$

Optimal. Leaf size=18

Unintegrable($\tan^3(a + bx)(c + dx)^m, x$)

[Out] Unintegrable[(c + d*x)^m*Tan[a + b*x]^3, x]

Rubi [A] time = 0.0348048, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \tan^3(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Tan[a + b*x]^3, x]

[Out] Defer[Int] [(c + d*x)^m*Tan[a + b*x]^3, x]

Rubi steps

$$\int (c + dx)^m \tan^3(a + bx) dx = \int (c + dx)^m \tan^3(a + bx) dx$$

Mathematica [A] time = 5.54264, size = 0, normalized size = 0.

$$\int (c + dx)^m \tan^3(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Tan[a + b*x]^3, x]

[Out] Integrate[(c + d*x)^m*Tan[a + b*x]^3, x]

Maple [A] time = 0.197, size = 0, normalized size = 0.

$$\int (dx + c)^m (\tan(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*tan(b*x+a)^3, x)

[Out] int((d*x+c)^m*tan(b*x+a)^3, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \tan(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*tan(b*x+a)^3,x, algorithm="maxima")

[Out] integrate((d*x + c)^m*tan(b*x + a)^3, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx + c)^m \tan(bx + a)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*tan(b*x+a)^3,x, algorithm="fricas")

[Out] integral((d*x + c)^m*tan(b*x + a)^3, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^m \tan^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**m*tan(b*x+a)**3,x)

[Out] Integral((c + d*x)**m*tan(a + b*x)**3, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \tan(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*tan(b*x+a)^3,x, algorithm="giac")

[Out] integrate((d*x + c)^m*tan(b*x + a)^3, x)

3.304 $\int (c + dx)^3 \tan^3(a + bx) dx$

Optimal. Leaf size=259

$$\frac{3d^2(c + dx)\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} - \frac{3id(c + dx)^2\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^2} + \frac{3id^3\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^4} + \frac{3id^3\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{4b^4}$$

[Out] $\left(\frac{(3I)}{2}d(c + dx)^2/b^2 + (c + dx)^3/(2b) - (I/4)(c + dx)^4/d - (3d^2(c + dx)\text{Log}[1 + E^{(2I)(a + bx)}])/b^3 + ((c + dx)^3\text{Log}[1 + E^{(2I)(a + bx)}])/b + ((3I)/2)d^3\text{PolyLog}[2, -E^{(2I)(a + bx)}]/b^4 - ((3I)/2)d(c + dx)^2\text{PolyLog}[2, -E^{(2I)(a + bx)}]/b^2 + (3d^2(c + dx)\text{PolyLog}[3, -E^{(2I)(a + bx)}])/(2b^3) + ((3I)/4)d^3\text{PolyLog}[4, -E^{(2I)(a + bx)}]/b^4 - (3d(c + dx)^2\text{Tan}[a + bx])/(2b^2) + ((c + dx)^3\text{Tan}[a + bx]^2)/(2b)\right)$

Rubi [A] time = 0.355707, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3720, 3719, 2190, 2279, 2391, 32, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c + dx)\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} - \frac{3id(c + dx)^2\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^2} + \frac{3id^3\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^4} + \frac{3id^3\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{4b^4}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^3*Tan[a + b*x]^3,x]

[Out] $\left(\frac{(3I)}{2}d(c + dx)^2/b^2 + (c + dx)^3/(2b) - (I/4)(c + dx)^4/d - (3d^2(c + dx)\text{Log}[1 + E^{(2I)(a + bx)}])/b^3 + ((c + dx)^3\text{Log}[1 + E^{(2I)(a + bx)}])/b + ((3I)/2)d^3\text{PolyLog}[2, -E^{(2I)(a + bx)}]/b^4 - ((3I)/2)d(c + dx)^2\text{PolyLog}[2, -E^{(2I)(a + bx)}]/b^2 + (3d^2(c + dx)\text{PolyLog}[3, -E^{(2I)(a + bx)}])/(2b^3) + ((3I)/4)d^3\text{PolyLog}[4, -E^{(2I)(a + bx)}]/b^4 - (3d(c + dx)^2\text{Tan}[a + bx])/(2b^2) + ((c + dx)^3\text{Tan}[a + bx]^2)/(2b)\right)$

Rule 3720

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 3719

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_)^(m_)), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.)), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_)^(m_.))*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 \tan^3(a + bx) dx &= \frac{(c + dx)^3 \tan^2(a + bx)}{2b} - \frac{(3d) \int (c + dx)^2 \tan^2(a + bx) dx}{2b} - \int (c + dx)^3 \tan(a + bx) dx \\
&= -\frac{i(c + dx)^4}{4d} - \frac{3d(c + dx)^2 \tan(a + bx)}{2b^2} + \frac{(c + dx)^3 \tan^2(a + bx)}{2b} + 2i \int \frac{e^{2i(a+bx)}(c + dx)^3}{1 + e^{2i(a+bx)}} dx \\
&= \frac{3id(c + dx)^2}{2b^2} + \frac{(c + dx)^3}{2b} - \frac{i(c + dx)^4}{4d} + \frac{(c + dx)^3 \log(1 + e^{2i(a+bx)})}{b} - \frac{3d(c + dx)^2 \tan(a + bx)}{2b^2} \\
&= \frac{3id(c + dx)^2}{2b^2} + \frac{(c + dx)^3}{2b} - \frac{i(c + dx)^4}{4d} - \frac{3d^2(c + dx) \log(1 + e^{2i(a+bx)})}{b^3} + \frac{(c + dx)^3 \log(1 + e^{2i(a+bx)})}{b} \\
&= \frac{3id(c + dx)^2}{2b^2} + \frac{(c + dx)^3}{2b} - \frac{i(c + dx)^4}{4d} - \frac{3d^2(c + dx) \log(1 + e^{2i(a+bx)})}{b^3} + \frac{(c + dx)^3 \log(1 + e^{2i(a+bx)})}{b} \\
&= \frac{3id(c + dx)^2}{2b^2} + \frac{(c + dx)^3}{2b} - \frac{i(c + dx)^4}{4d} - \frac{3d^2(c + dx) \log(1 + e^{2i(a+bx)})}{b^3} + \frac{(c + dx)^3 \log(1 + e^{2i(a+bx)})}{b} \\
&= \frac{3id(c + dx)^2}{2b^2} + \frac{(c + dx)^3}{2b} - \frac{i(c + dx)^4}{4d} - \frac{3d^2(c + dx) \log(1 + e^{2i(a+bx)})}{b^3} + \frac{(c + dx)^3 \log(1 + e^{2i(a+bx)})}{b}
\end{aligned}$$

Mathematica [B] time = 6.9231, size = 803, normalized size = 3.1

$$\frac{\sec(a)(\cos(a) \log(\cos(a) \cos(bx) - \sin(a) \sin(bx)) + bx \sin(a))c^3}{b(\cos^2(a) + \sin^2(a))} + \frac{3d \csc(a) \left(b^2 e^{-i \tan^{-1}(\cot(a)) x^2} - \frac{\cot(a) \left(ibx(-2 \tan^{-1}(\cot(a))) \right)}{\dots} \right)}{\dots}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^3*Tan[a + b*x]^3,x]

[Out] ((I/4)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))]) - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))]*Sec[a]/(b^3*E^(I*a)) + (I/8)*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a) - ((4*I)*(1 + E^((-2*I)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1 + E^((2*I)*a))*(2*b^2*x^2*PolyLog[2, -E^((-2*I)*(a + b*x))]) - (2*I)*b*x*PolyLog[3, -E^((-2*I)*(a + b*x))]) - PolyLog[4, -E^((-2*I)*(a + b*x))])/b^4*E^((2*I)*a))*Sec[a] + ((c + d*x)^3*Sec[a + b*x]^2)/(2*b) + (c^3*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (3*c*d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x]) - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]])]) + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]) + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]])])]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (3*d^3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x]) - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]])]) + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]) + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]])])]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^4*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (3*Sec[a]*Sec[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3*x^2*Sin[b*x]))/(2*b^2) - (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Tan[a])/4

Maple [B] time = 0.368, size = 720, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (d*x+c)^3*\tan(b*x+a)^3, x$

[Out] $\frac{3}{4}I*d^3*\text{polylog}(4, -\exp(2*I*(b*x+a)))/b^4+1/b*c^3*\ln(\exp(2*I*(b*x+a))+1)+3/2/b^3*c*d^2*\text{polylog}(3, -\exp(2*I*(b*x+a)))+3/2/b^3*d^3*\text{polylog}(3, -\exp(2*I*(b*x+a)))*x+(2*b*d^3*x^3*\exp(2*I*(b*x+a))-3*I*d^3*x^2*\exp(2*I*(b*x+a))+6*b*c*d^2*x^2*\exp(2*I*(b*x+a))-6*I*c*d^2*x*\exp(2*I*(b*x+a))+6*b*c^2*d*x*\exp(2*I*(b*x+a))-3*I*c^2*d*\exp(2*I*(b*x+a))-3*I*d^3*x^2+2*b*c^3*\exp(2*I*(b*x+a))-6*I*c*d^2*x-3*I*c^2*d)/b^2/(\exp(2*I*(b*x+a))+1)^2-3*d^2/b^3*c*\ln(\exp(2*I*(b*x+a))+1)-3*d^3/b^3*\ln(\exp(2*I*(b*x+a))+1)*x+3*I*d^3/b^2*x^2+3*I*d^3/b^4*a^2-3/2*I/b^4*d^3*a^4+3/2*I*d^3*\text{polylog}(2, -\exp(2*I*(b*x+a)))/b^4-3/2*I/b^2*c^2*d*\text{polylog}(2, -\exp(2*I*(b*x+a)))-3/2*I/b^2*d^3*\text{polylog}(2, -\exp(2*I*(b*x+a)))*x^2+4*I/b^3*a^3*c*d^2-3*I/b^2*a^2*c^2*d+3/b*c^2*d*\ln(\exp(2*I*(b*x+a))+1)*x+3/b*c*d^2*\ln(\exp(2*I*(b*x+a))+1)*x^2+1/b*d^3*\ln(\exp(2*I*(b*x+a))+1)*x^3-2*I/b^3*d^3*a^3*x-6/b^4*d^3*a*\ln(\exp(I*(b*x+a)))+6/b^3*d^2*c*\ln(\exp(I*(b*x+a)))+6*I*d^3/b^3*a*x-6/b^3*c*d^2*a^2*\ln(\exp(I*(b*x+a)))+6/b^2*c^2*d*a*\ln(\exp(I*(b*x+a)))+I*c^3*x-I*c*d^2*x^3-3/2*I*c^2*d*x^2+2/b^4*d^3*a^3*\ln(\exp(I*(b*x+a)))-3*I/b^2*\text{polylog}(2, -\exp(2*I*(b*x+a)))*c*d^2*x-6*I/b*a*c^2*d*x+6*I/b^2*a^2*c*d^2*x-2/b*c^3*\ln(\exp(I*(b*x+a)))-1/4*I*d^3*x^4$

Maxima [B] time = 4.48058, size = 3247, normalized size = 12.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((d*x+c)^3*\tan(b*x+a)^3, x, \text{algorithm}="maxima")$

[Out] $-1/2*(c^3*(1/(\sin(b*x + a)^2 - 1) - \log(\sin(b*x + a)^2 - 1)) - 3*a*c^2*d*(1/(\sin(b*x + a)^2 - 1) - \log(\sin(b*x + a)^2 - 1))/b + 3*a^2*c*d^2*(1/(\sin(b*x + a)^2 - 1) - \log(\sin(b*x + a)^2 - 1))/b^2 - a^3*d^3*(1/(\sin(b*x + a)^2 - 1) - \log(\sin(b*x + a)^2 - 1))/b^3 + 2*(3*(b*x + a)^4*d^3 + 36*b^2*c^2*d - 72*a*b*c*d^2 + 36*a^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a)^3 + 18*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a)^2 - (16*(b*x + a)^3*d^3 - 36*b*c*d^2 + 36*a*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 36*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a) + 4*(4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + 8*(4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (16*I*(b*x + a)^3*d^3 - 36*I*b*c*d^2 + 36*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + (36*I*a^2 - 36*I)*d^3)*(b*x + a))*\sin(4*b*x + 4*a) + (32*I*(b*x + a)^3*d^3 - 72*I*b*c*d^2 + 72*I*a*d^3 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*x + a)^2 + (72*I*b^2*c^2*d - 144*I*a*b*c*d^2 + (72*I*a^2 - 72*I)*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(2*b*x + 2*a), \cos(2*b*x + 2*a) + 1) + 3*((b*x + a)^4*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a)^3 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a)^2 - 24*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + (6*(b*x + a)^4*d^3 + 36*b^2*c^2*d - 72*a*b*c*d^2 + 36*a^2*d^3 + (24*b*c*d^2 - (24*a - 24*I)*d^3)*(b*x + a)^3 + (36*b^2*c^2*d - (72*a - 72*I)*b*c*d^2 + 36*(a^2 - 2*I*a - 1)*d^3)*(b*x + a)^2 - (-72*I*b^2*c^2*d - 72*(-2*I*a - 1)*b*c*d^2 + (-72*I*a^2 - 72*a)*d^3)*(b*x + a))*\cos(2*b*x + 2*a)$

$$\begin{aligned}
& + (18*b^2*c^2*d - 36*a*b*c*d^2 + 24*(b*x + a)^2*d^3 + 18*(a^2 - 1)*d^3 + 3 \\
& 6*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^2 \\
& 2*d^3 + 3*(a^2 - 1)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + \\
& 12*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^2*d^3 + 3*(a^2 - 1)*d^3 + 6*(b \\
& *c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - (-18*I*b^2*c^2*d + 36*I*a*b*c \\
& *d^2 - 24*I*(b*x + a)^2*d^3 + (-18*I*a^2 + 18*I)*d^3 + (-36*I*b*c*d^2 + 36* \\
& I*a*d^3)*(b*x + a))*\sin(4*b*x + 4*a) - (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2 - \\
& 48*I*(b*x + a)^2*d^3 + (-36*I*a^2 + 36*I)*d^3 + (-72*I*b*c*d^2 + 72*I*a*d^3 \\
&)*(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{dilog}(-e^{(2*I*b*x + 2*I*a)}) - (-8*I*(b*x + a) \\
&)^3*d^3 + 18*I*b*c*d^2 - 18*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a) \\
&)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 + 18*I)*d^3)*(b*x + a) \\
& + (-8*I*(b*x + a)^3*d^3 + 18*I*b*c*d^2 - 18*I*a*d^3 + (-18*I*b*c*d^2 + 18* \\
& I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 + 18* \\
& I)*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + (-16*I*(b*x + a)^3*d^3 + 36*I*b*c*d^2 \\
& - 36*I*a*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a)^2 + (-36*I*b^2*c^2*d \\
& + 72*I*a*b*c*d^2 + (-36*I*a^2 + 36*I)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + 2 \\
& *(4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 \\
& + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*\sin(4*b*x + 4*a) \\
& + 4*(4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a) \\
&)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*\sin(2*b*x + 2* \\
& a))*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) - \\
& (12*d^3*\cos(4*b*x + 4*a) + 24*d^3*\cos(2*b*x + 2*a) + 12*I*d^3*\sin(4*b*x + \\
& 4*a) + 24*I*d^3*\sin(2*b*x + 2*a) + 12*d^3)*\operatorname{polylog}(4, -e^{(2*I*b*x + 2*I*a)}) \\
& - (-18*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 18*I*a*d^3 + (-18*I*b*c*d^2 - 24*I \\
& *(b*x + a)*d^3 + 18*I*a*d^3)*\cos(4*b*x + 4*a) + (-36*I*b*c*d^2 - 48*I*(b*x \\
& + a)*d^3 + 36*I*a*d^3)*\cos(2*b*x + 2*a) + 6*(3*b*c*d^2 + 4*(b*x + a)*d^3 - \\
& 3*a*d^3)*\sin(4*b*x + 4*a) + 12*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*\sin(\\
& 2*b*x + 2*a))*\operatorname{polylog}(3, -e^{(2*I*b*x + 2*I*a)}) - (-3*I*(b*x + a)^4*d^3 + (- \\
& 12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^3 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 \\
& + (-18*I*a^2 + 36*I)*d^3)*(b*x + a)^2 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*x + \\
& a))*\sin(4*b*x + 4*a) - (-6*I*(b*x + a)^4*d^3 - 36*I*b^2*c^2*d + 72*I*a*b*c* \\
& d^2 - 36*I*a^2*d^3 + (-24*I*b*c*d^2 - 24*(-I*a - 1)*d^3)*(b*x + a)^3 + (-36 \\
& *I*b^2*c^2*d - 72*(-I*a - 1)*b*c*d^2 + (-36*I*a^2 - 72*a + 36*I)*d^3)*(b*x \\
& + a)^2 + (72*b^2*c^2*d - (144*a - 72*I)*b*c*d^2 + 72*(a^2 - I*a)*d^3)*(b*x \\
& + a))*\sin(2*b*x + 2*a))/(-12*I*b^3*\cos(4*b*x + 4*a) - 24*I*b^3*\cos(2*b*x + \\
& 2*a) + 12*b^3*\sin(4*b*x + 4*a) + 24*b^3*\sin(2*b*x + 2*a) - 12*I*b^3))/b
\end{aligned}$$

Fricas [C] time = 0.537343, size = 1443, normalized size = 5.57

$$4b^3d^3x^3 + 12b^3cd^2x^2 + 12b^3c^2dx - 3id^3\operatorname{polylog}\left(4, \frac{\tan(bx+a)^2+2i\tan(bx+a)-1}{\tan(bx+a)^2+1}\right) + 3id^3\operatorname{polylog}\left(4, \frac{\tan(bx+a)^2-2i\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*tan(b*x+a)^3,x, algorithm="fricas")

[Out] 1/8*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x - 3*I*d^3*polylog(4, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 3*I*d^3*polylog(4, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*tan(b*x + a)^2 + (6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d - 6*I*d^3)*dilog(2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d + 6*I*d^3)*dilog(2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 3*b*c*d^2 + 3*(b^3*c^2*d - b*d^3)*x)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 3*b*c*d^2 + 3*(b^3*c^2*d - b*d^3)*x)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 6*(b*d^3*x + b

```
*c*d^2)*polylog(3, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 12*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*tan(b*x + a))/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \tan^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*tan(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**3*tan(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \tan(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*tan(b*x + a)^3, x)
```

3.305 $\int (c + dx)^2 \tan^3(a + bx) dx$

Optimal. Leaf size=169

$$-\frac{id(c + dx)\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^2} + \frac{d^2\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} - \frac{d(c + dx)\tan(a + bx)}{b^2} - \frac{d^2 \log(\cos(a + bx))}{b^3} + \frac{(c + dx)^2}{2b}$$

[Out] (c*d*x)/b + (d^2*x^2)/(2*b) - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b - (d^2*Log[Cos[a + b*x]])/b^3 - (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) - (d*(c + d*x)*Tan[a + b*x])/b^2 + ((c + d*x)^2*Tan[a + b*x]^2)/(2*b)

Rubi [A] time = 0.221749, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3720, 3475, 3719, 2190, 2531, 2282, 6589}

$$-\frac{id(c + dx)\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^2} + \frac{d^2\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} - \frac{d(c + dx)\tan(a + bx)}{b^2} - \frac{d^2 \log(\cos(a + bx))}{b^3} + \frac{(c + dx)^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2*Tan[a + b*x]^3,x]

[Out] (c*d*x)/b + (d^2*x^2)/(2*b) - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b - (d^2*Log[Cos[a + b*x]])/b^3 - (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) - (d*(c + d*x)*Tan[a + b*x])/b^2 + ((c + d*x)^2*Tan[a + b*x]^2)/(2*b)

Rule 3720

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3719

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]], x]

)^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*(a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \tan^3(a + bx) dx &= \frac{(c + dx)^2 \tan^2(a + bx)}{2b} - \frac{d \int (c + dx) \tan^2(a + bx) dx}{b} - \int (c + dx)^2 \tan(a + bx) dx \\ &= -\frac{i(c + dx)^3}{3d} - \frac{d(c + dx) \tan(a + bx)}{b^2} + \frac{(c + dx)^2 \tan^2(a + bx)}{2b} + 2i \int \frac{e^{2i(a+bx)}(c + dx)^2}{1 + e^{2i(a+bx)}} \\ &= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{i(c + dx)^3}{3d} + \frac{(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b} - \frac{d^2 \log(\cos(a + bx))}{b^3} - \frac{d(c + dx)}{b^2} \\ &= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{i(c + dx)^3}{3d} + \frac{(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b} - \frac{d^2 \log(\cos(a + bx))}{b^3} - \frac{id(c + dx)}{b^2} \\ &= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{i(c + dx)^3}{3d} + \frac{(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b} - \frac{d^2 \log(\cos(a + bx))}{b^3} - \frac{id(c + dx)}{b^2} \\ &= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{i(c + dx)^3}{3d} + \frac{(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b} - \frac{d^2 \log(\cos(a + bx))}{b^3} - \frac{id(c + dx)}{b^2} \end{aligned}$$

Mathematica [B] time = 6.64045, size = 454, normalized size = 2.69

$$cd \operatorname{csc}(a) \operatorname{sec}(a) \left(b^2 x^2 e^{-i \tan^{-1}(\cot(a))} - \frac{\cot(a) \left(i \operatorname{PolyLog} \left(2, e^{2i(bx - \tan^{-1}(\cot(a)))} \right) \right) + ibx(-2 \tan^{-1}(\cot(a)) - \pi) - 2(bx - \tan^{-1}(\cot(a))) \log \left(1 - e^{2i(bx - \tan^{-1}(\cot(a)))} \right)}{\sqrt{\cot^2(a) + 1}} \right)}{b^2 \sqrt{\operatorname{csc}^2(a) (\sin^2(a) + \cos^2(a))}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^2*Tan[a + b*x]^3,x]

[Out] ((I/12)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a]/(b^3*E^((I

*a)) + ((c + d*x)^2*Sec[a + b*x]^2)/(2*b) + (c^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]])) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]])])]) + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]]) + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]])]))/Sqrt[1 + Cot[a]^2]*Sec[a]/(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Sec[a]*Sec[a + b*x]*(-(c*d*Sin[b*x]) - d^2*x*Sin[b*x]))/b^2 - (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Tan[a])/3

Maple [B] time = 0.321, size = 400, normalized size = 2.4

$$\frac{-id^2 \operatorname{polylog}\left(2, -e^{2i(bx+a)}\right)x}{b^2} - icdx^2 + ic^2x + 2 \frac{bd^2x^2e^{2i(bx+a)} - id^2xe^{2i(bx+a)} + 2bcdxe^{2i(bx+a)} - icde^{2i(bx+a)} + bc^2e^{2i(bx+a)}}{b^2 \left(e^{2i(bx+a)} + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*tan(b*x+a)^3,x)

[Out] -I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-I*c*d*x^2+I*c^2*x+2*(b*d^2*x^2*exp(2*I*(b*x+a))-I*d^2*x*exp(2*I*(b*x+a))+2*b*c*d*x*exp(2*I*(b*x+a))-I*c*d*exp(2*I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^2*x-I*d*c)/b^2/(exp(2*I*(b*x+a))+1)^2-4*I/b*a*c*d*x-I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))-1/3*I*d^2*x^3+2*I/b^2*a^2*d^2*x+1/b*d^2*ln(exp(2*I*(b*x+a))+1)*x^2-2*I/b^2*a^2*c*d-2/b*c^2*ln(exp(I*(b*x+a)))+1/b*c^2*ln(exp(2*I*(b*x+a))+1)-2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))+4/3*I/b^3*a^3*d^2+2/b*c*d*ln(exp(2*I*(b*x+a))+1)*x+1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+2/b^3*d^2*ln(exp(I*(b*x+a)))-1/b^3*d^2*ln(exp(2*I*(b*x+a))+1)+4/b^2*c*d*a*ln(exp(I*(b*x+a)))

Maxima [B] time = 2.44249, size = 1655, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*tan(b*x+a)^3,x, algorithm="maxima")

[Out] -1/2*(c^2*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1)) - 2*a*c*d*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b + a^2*d^2*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b^2 + 2*(2*(b*x + a)^3*d^2 + 6*(b*c*d - a*d^2)*(b*x + a)^2 + 12*b*c*d - 12*a*d^2 - (6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2)*(b*x + a) - 6*d^2 + 6*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(4*b*x + 4*a) + 12*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a) - 6*I*d^2)*sin(4*b*x + 4*a) + (12*I*(b*x + a)^2*d^2 + (24*I*b*c*d - 24*I*a*d^2)*(b*x + a) - 12*I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + 2*((b*x + a)^3*d^2 + 3*(b*c*d - a*d^2)*(b*x + a)^2 - 6*(b*x + a)*d^2)*cos(4*b*x + 4*a) + (4*(b*x + a)^3*d^2 + (12*b*c*d - (12*a - 12*I)*d^2)*(b*x + a)^2 + 12*b*c*d - 12*a*d^2 - (-24*I*b*c*d - 12*(-2*I*a - 1)*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (6*b*c*d + 6*(b*x + a)*d^2 - 6*a*d^2 + 6*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-6*I*b*c*d - 6*I*(b*x + a)*d^2 + 6*I*a*d^2)*sin(4*b*x + 4*a) - (-12*I*b*c*d - 12*I*(b*x + a)*d^2 + 12*

$$I*a*d^2*\sin(2*b*x + 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) - (-3*I*(b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) + 3*I*d^2 + (-3*I*(b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) + 3*I*d^2)*\cos(4*b*x + 4*a) + (-6*I*(b*x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a) + 6*I*d^2)*\cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*\sin(4*b*x + 4*a) + 6*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*\sin(2*b*x + 2*a))*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) - (-3*I*d^2*\cos(4*b*x + 4*a) - 6*I*d^2*\cos(2*b*x + 2*a) + 3*d^2*\sin(4*b*x + 4*a) + 6*d^2*\sin(2*b*x + 2*a) - 3*I*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) - (-2*I*(b*x + a)^3*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a)^2 + 12*I*(b*x + a)*d^2)*\sin(4*b*x + 4*a) - (-4*I*(b*x + a)^3*d^2 + (-12*I*b*c*d - 12*(-I*a - 1)*d^2)*(b*x + a)^2 - 12*I*b*c*d + 12*I*a*d^2 + (24*b*c*d - (24*a - 12*I)*d^2)*(b*x + a))*\sin(2*b*x + 2*a))/(-6*I*b^2*\cos(4*b*x + 4*a) - 12*I*b^2*\cos(2*b*x + 2*a) + 6*b^2*\sin(4*b*x + 4*a) + 12*b^2*\sin(2*b*x + 2*a) - 6*I*b^2))/b$$

Fricas [C] time = 0.518016, size = 883, normalized size = 5.22

$$2b^2d^2x^2 + 4b^2cdx + d^2\text{polylog}\left(3, \frac{\tan(bx+a)^2+2i\tan(bx+a)-1}{\tan(bx+a)^2+1}\right) + d^2\text{polylog}\left(3, \frac{\tan(bx+a)^2-2i\tan(bx+a)-1}{\tan(bx+a)^2+1}\right) + 2(b^2d^2x^2 + 2b^2cdx + d^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*tan(b*x+a)^3,x, algorithm="fricas")

[Out] 1/4*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + d^2*polylog(3, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + d^2*polylog(3, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*tan(b*x + a)^2 + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - d^2)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - d^2)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 4*(b*d^2*x + b*c*d)*tan(b*x + a))/b^3

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \tan^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*tan(b*x+a)**3,x)

[Out] Integral((c + d*x)**2*tan(a + b*x)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \tan(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*tan(b*x + a)^3, x)
```


3.306 $\int (c + dx) \tan^3(a + bx) dx$

Optimal. Leaf size=108

$$\frac{id\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^2} - \frac{d \tan(a + bx)}{2b^2} + \frac{(c + dx) \log\left(1 + e^{2i(a+bx)}\right)}{b} + \frac{(c + dx) \tan^2(a + bx)}{2b} + \frac{dx}{2b} - \frac{i(c + dx)^2}{2d}$$

[Out] (d*x)/(2*b) - ((I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b - ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (d*Tan[a + b*x])/(2*b^2) + ((c + d*x)*Tan[a + b*x]^2)/(2*b)

Rubi [A] time = 0.116911, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3720, 3473, 8, 3719, 2190, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^2} - \frac{d \tan(a + bx)}{2b^2} + \frac{(c + dx) \log\left(1 + e^{2i(a+bx)}\right)}{b} + \frac{(c + dx) \tan^2(a + bx)}{2b} + \frac{dx}{2b} - \frac{i(c + dx)^2}{2d}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Tan[a + b*x]^3, x]

[Out] (d*x)/(2*b) - ((I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b - ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (d*Tan[a + b*x])/(2*b^2) + ((c + d*x)*Tan[a + b*x]^2)/(2*b)

Rule 3720

Int[((c_.) + (d_.)*(x_.))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_), x_Symbol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 3473

Int[(b_.)*tan[(c_.) + (d_.)*(x_.)]^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 3719

Int[((c_.) + (d_.)*(x_.))^(m_.)*tan[(e_.) + (f_.)*(x_.)], x_Symbol] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[(c + d*x)^m*E^(2*I*(e + f*x))]/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2190

Int[(F_)^((g_.)*((e_.) + (f_.)*(x_.)))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.)/((a_.) + (b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_.)))^(n_.)), x_Symbol] :> Simp[(c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x]

))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned} \int (c + dx) \tan^3(a + bx) dx &= \frac{(c + dx) \tan^2(a + bx)}{2b} - \frac{d \int \tan^2(a + bx) dx}{2b} - \int (c + dx) \tan(a + bx) dx \\ &= -\frac{i(c + dx)^2}{2d} - \frac{d \tan(a + bx)}{2b^2} + \frac{(c + dx) \tan^2(a + bx)}{2b} + 2i \int \frac{e^{2i(a+bx)}(c + dx)}{1 + e^{2i(a+bx)}} dx + \frac{d \int 1 dx}{2b} \\ &= \frac{dx}{2b} - \frac{i(c + dx)^2}{2d} + \frac{(c + dx) \log(1 + e^{2i(a+bx)})}{b} - \frac{d \tan(a + bx)}{2b^2} + \frac{(c + dx) \tan^2(a + bx)}{2b} - \frac{d \int 1 dx}{2b} \\ &= \frac{dx}{2b} - \frac{i(c + dx)^2}{2d} + \frac{(c + dx) \log(1 + e^{2i(a+bx)})}{b} - \frac{d \tan(a + bx)}{2b^2} + \frac{(c + dx) \tan^2(a + bx)}{2b} + \frac{d \int 1 dx}{2b} \\ &= \frac{dx}{2b} - \frac{i(c + dx)^2}{2d} + \frac{(c + dx) \log(1 + e^{2i(a+bx)})}{b} - \frac{id \text{Li}_2(-e^{2i(a+bx)})}{2b^2} - \frac{d \tan(a + bx)}{2b^2} + \frac{(c + dx) \tan^2(a + bx)}{2b} \end{aligned}$$

Mathematica [B] time = 6.15395, size = 240, normalized size = 2.22

$$\frac{d \csc(a) \sec(a) \left(b^2 x^2 e^{-i \tan^{-1}(\cot(a))} - \frac{\cot(a) \left(i \text{PolyLog} \left(2, e^{2i(bx - \tan^{-1}(\cot(a)))} \right) \right) + ibx(-2 \tan^{-1}(\cot(a)) - \pi) - 2(bx - \tan^{-1}(\cot(a))) \log \left(1 - e^{2i(bx - \tan^{-1}(\cot(a)))} \right)}{\sqrt{\cot^2(a) + 1}} \right)}{2b^2 \sqrt{\csc^2(a) (\sin^2(a) + \cos^2(a))}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)*Tan[a + b*x]^3,x]

[Out] (d*x*Sec[a + b*x]^2)/(2*b) + (d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]])) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]])])]) + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]]) + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]])]))/Sqrt[1 + Cot[a]^2])*Sec[a]/(2*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)) - (d*Sec[a]*Sec[a + b*x]*Sin[b*x])/(2*b^2) - (d*x^2*Tan[a])/2 + (c*(2*Log[Cos[a + b*x]] + Tan[a + b*x]^2))/(2*b)

Maple [A] time = 0.159, size = 183, normalized size = 1.7

$$-\frac{i}{2} dx^2 + icx + \frac{2bdxe^{2i(bx+a)} + 2bce^{2i(bx+a)} - ide^{2i(bx+a)} - id}{b^2(e^{2i(bx+a)} + 1)^2} - 2 \frac{c \ln(e^{i(bx+a)})}{b} + \frac{c \ln(e^{2i(bx+a)} + 1)}{b} - \frac{2idax}{b} - \frac{ida^2}{b^2} + \frac{d}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)*tan(b*x+a)^3,x)`

[Out]
$$-1/2*I*d*x^2+I*c*x+(2*b*d*x*\exp(2*I*(b*x+a))+2*b*c*\exp(2*I*(b*x+a))-I*d*\exp(2*I*(b*x+a))-I*d)/b^2/(\exp(2*I*(b*x+a))+1)^2-2/b*c*\ln(\exp(I*(b*x+a)))+1/b*c*\ln(\exp(2*I*(b*x+a))+1)-2*I/b*d*a*x-I/b^2*d*a^2+1/b*d*\ln(\exp(2*I*(b*x+a)))+1)*x-1/2*I*d*polylog(2,-\exp(2*I*(b*x+a)))/b^2+2/b^2*d*a*\ln(\exp(I*(b*x+a)))$$

Maxima [B] time = 2.04237, size = 701, normalized size = 6.49

$$b^2 dx^2 + 2 b^2 cx - (2 b dx + 2 bc + 2 (bdx + bc) \cos(4 bx + 4 a) + 4 (bdx + bc) \cos(2 bx + 2 a) + (2i b dx + 2i bc) \sin(4 bx + 4 a))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*tan(b*x+a)^3,x, algorithm="maxima")`

[Out]
$$-(b^2*d*x^2 + 2*b^2*c*x - (2*b*d*x + 2*b*c + 2*(b*d*x + b*c))*\cos(4*b*x + 4*a) + 4*(b*d*x + b*c))*\cos(2*b*x + 2*a) + (2*I*b*d*x + 2*I*b*c)*\sin(4*b*x + 4*a) + (4*I*b*d*x + 4*I*b*c)*\sin(2*b*x + 2*a))*\arctan2(\sin(2*b*x + 2*a), \cos(2*b*x + 2*a) + 1) + (b^2*d*x^2 + 2*b^2*c*x)*\cos(4*b*x + 4*a) + (2*b^2*d*x^2 + 4*I*b*c + (4*b^2*c + 4*I*b*d))*x + 2*d)*\cos(2*b*x + 2*a) + (d*\cos(4*b*x + 4*a) + 2*d*\cos(2*b*x + 2*a) + I*d*\sin(4*b*x + 4*a) + 2*I*d*\sin(2*b*x + 2*a) + d)*\operatorname{dilog}(-e^{(2*I*b*x + 2*I*a)}) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c))*\cos(4*b*x + 4*a) + (-2*I*b*d*x - 2*I*b*c)*\cos(2*b*x + 2*a) + (b*d*x + b*c)*\sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*\sin(2*b*x + 2*a))*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) - (-I*b^2*d*x^2 - 2*I*b^2*c*x)*\sin(4*b*x + 4*a) - (-2*I*b^2*d*x^2 + 4*b*c - 4*(I*b^2*c - b*d))*x - 2*I*d*\sin(2*b*x + 2*a) + 2*d)/(-2*I*b^2*\cos(4*b*x + 4*a) - 4*I*b^2*\cos(2*b*x + 2*a) + 2*b^2*\sin(4*b*x + 4*a) + 4*b^2*\sin(2*b*x + 2*a) - 2*I*b^2)$$

Fricas [A] time = 0.492585, size = 447, normalized size = 4.14

$$\frac{2 b dx + 2 (bdx + bc) \tan (bx + a)^2 + i d \operatorname{Li}_2\left(\frac{2(i \tan (bx+a)-1)}{\tan (bx+a)^2+1}+1\right)-i d \operatorname{Li}_2\left(\frac{2(-i \tan (bx+a)-1)}{\tan (bx+a)^2+1}+1\right)+2(b dx + bc) \log\left(-\frac{2(i \tan (bx+a)-1)}{\tan (bx+a)^2+1}\right)}{4 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*tan(b*x+a)^3,x, algorithm="fricas")`

[Out]
$$1/4*(2*b*d*x + 2*(b*d*x + b*c))*\tan(b*x + a)^2 + I*d*\operatorname{dilog}(2*(I*\tan(b*x + a) - 1)/(\tan(b*x + a)^2 + 1) + 1) - I*d*\operatorname{dilog}(2*(-I*\tan(b*x + a) - 1)/(\tan(b*x + a)^2 + 1) + 1) + 2*(b*d*x + b*c)*\log(-2*(I*\tan(b*x + a) - 1)/(\tan(b*x + a)^2 + 1)) + 2*(b*d*x + b*c)*\log(-2*(-I*\tan(b*x + a) - 1)/(\tan(b*x + a)^2 + 1)) - 2*d*\tan(b*x + a))/b^2$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \tan^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*tan(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \tan(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*tan(b*x + a)^3, x)
```

$$3.307 \quad \int \frac{\tan^3(a+bx)}{c+dx} dx$$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\tan^3(a+bx)}{c+dx}, x\right)$$

[Out] Unintegrable[Tan[a + b*x]^3/(c + d*x), x]

Rubi [A] time = 0.0385794, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\tan^3(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[Tan[a + b*x]^3/(c + d*x), x]

[Out] Defer[Int][Tan[a + b*x]^3/(c + d*x), x]

Rubi steps

$$\int \frac{\tan^3(a+bx)}{c+dx} dx = \int \frac{\tan^3(a+bx)}{c+dx} dx$$

Mathematica [A] time = 6.45732, size = 0, normalized size = 0.

$$\int \frac{\tan^3(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[Tan[a + b*x]^3/(c + d*x), x]

[Out] Integrate[Tan[a + b*x]^3/(c + d*x), x]

Maple [A] time = 1.853, size = 0, normalized size = 0.

$$\int \frac{(\tan(bx+a))^3}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(b*x+a)^3/(d*x+c), x)

[Out] int(tan(b*x+a)^3/(d*x+c), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a)^3/(d*x+c),x, algorithm="maxima")

[Out] $(4*(b*d*x + b*c)*\cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*\sin(2*b*x + 2*a)^2 + (2*(b*d*x + b*c)*\cos(2*b*x + 2*a) - d*\sin(2*b*x + 2*a))*\cos(4*b*x + 4*a) + 2*(b*d*x + b*c)*\cos(2*b*x + 2*a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2))*\cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\sin(4*b*x + 4*a)*\sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2))*\cos(2*b*x + 2*a))*\cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(2*b*x + 2*a))*\integrate(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - d^2)*\sin(2*b*x + 2*a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3))*\cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cos(2*b*x + 2*a)), x) + (d*\cos(2*b*x + 2*a) + 2*(b*d*x + b*c)*\sin(2*b*x + 2*a) + d)*\sin(4*b*x + 4*a) + d*\sin(2*b*x + 2*a))/(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2))*\cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2))*\cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\sin(4*b*x + 4*a)*\sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2))*\cos(2*b*x + 2*a))*\cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2))*\cos(2*b*x + 2*a))$

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(bx + a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a)^3/(d*x+c),x, algorithm="fricas")

[Out] integral(tan(b*x + a)^3/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a)**3/(d*x+c),x)

[Out] Integral(tan(a + b*x)**3/(c + d*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(bx + a)^3}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a)^3/(d*x+c),x, algorithm="giac")

[Out] integrate(tan(b*x + a)^3/(d*x + c), x)

$$3.308 \quad \int \frac{\tan^3(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\tan^3(a+bx)}{(c+dx)^2}, x\right)$$

[Out] Unintegrable[Tan[a + b*x]^3/(c + d*x)^2, x]

Rubi [A] time = 0.0366005, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[Tan[a + b*x]^3/(c + d*x)^2,x]

[Out] Defer[Int][Tan[a + b*x]^3/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2} dx = \int \frac{\tan^3(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 7.03406, size = 0, normalized size = 0.

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Tan[a + b*x]^3/(c + d*x)^2,x]

[Out] Integrate[Tan[a + b*x]^3/(c + d*x)^2, x]

Maple [A] time = 2.576, size = 0, normalized size = 0.

$$\int \frac{(\tan(bx+a))^3}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(b*x+a)^3/(d*x+c)^2,x)

[Out] int(tan(b*x+a)^3/(d*x+c)^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")

[Out] $(4*(b*d*x + b*c)*\cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*\sin(2*b*x + 2*a)^2 + 2*((b*d*x + b*c)*\cos(2*b*x + 2*a) - d*\sin(2*b*x + 2*a))*\cos(4*b*x + 4*a) + 2*(b*d*x + b*c)*\cos(2*b*x + 2*a) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\sin(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\sin(4*b*x + 4*a)*\sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3) + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cos(2*b*x + 2*a))*\cos(4*b*x + 4*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cos(2*b*x + 2*a))*\integrate(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 3*d^2)*\sin(2*b*x + 2*a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*\cos(2*b*x + 2*a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*\sin(2*b*x + 2*a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*\cos(2*b*x + 2*a)), x) + 2*(d*\cos(2*b*x + 2*a) + (b*d*x + b*c)*\sin(2*b*x + 2*a) + d)*\sin(4*b*x + 4*a) + 2*d*\sin(2*b*x + 2*a))/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\sin(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\sin(4*b*x + 4*a)*\sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3) + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cos(2*b*x + 2*a))*\cos(4*b*x + 4*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cos(2*b*x + 2*a))$

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(bx + a)}{d^2x^2 + 2cdx + c^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(tan(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(tan(a + b*x)**3/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(bx + a)^3}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(tan(b*x + a)^3/(d*x + c)^2, x)
```

$$3.309 \quad \int (c + dx)^m \csc(a + bx) \sec^3(a + bx) dx$$

Optimal. Leaf size=24

$$\text{CannotIntegrate}(\csc(a + bx) \sec^3(a + bx)(c + dx)^m, x)$$

[Out] CannotIntegrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3, x]

Rubi [A] time = 0.268662, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \csc(a + bx) \sec^3(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3, x]

[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3, x]

Rubi steps

$$\int (c + dx)^m \csc(a + bx) \sec^3(a + bx) dx = \int (c + dx)^m \csc(a + bx) \sec^3(a + bx) dx$$

Mathematica [A] time = 9.56064, size = 0, normalized size = 0.

$$\int (c + dx)^m \csc(a + bx) \sec^3(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3, x]

[Out] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3, x]

Maple [A] time = 0.185, size = 0, normalized size = 0.

$$\int (dx + c)^m \csc(bx + a) (\sec(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3, x)

[Out] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \csc(bx + a) \sec(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")

[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^3, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx + c)^m \csc(bx + a) \sec(bx + a)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")

[Out] integral((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**m*csc(b*x+a)*sec(b*x+a)**3,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \csc(bx + a) \sec(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")

[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^3, x)

3.310 $\int (c + dx)^4 \csc(a + bx) \sec^3(a + bx) dx$

Optimal. Leaf size=399

$$\frac{6id^3(c + dx)\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^4} - \frac{3id^3(c + dx)\text{PolyLog}\left(4, -e^{2i(a+bx)}\right)}{b^4} + \frac{3id^3(c + dx)\text{PolyLog}\left(4, e^{2i(a+bx)}\right)}{b^4} - \frac{3d^2}{b^4}$$

```
[Out] ((2*I)*d*(c + d*x)^3)/b^2 + (c + d*x)^4/(2*b) - (2*(c + d*x)^4*ArcTanh[E^((2*I)*(a + b*x))])/b - (6*d^2*(c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b^3 + ((6*I)*d^3*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 + ((2*I)*d*(c + d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((2*I)*d*(c + d*x)^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^4*PolyLog[3, -E^((2*I)*(a + b*x))])/b^5 - (3*d^2*(c + d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))])/b^3 + (3*d^2*(c + d*x)^2*PolyLog[3, E^((2*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(c + d*x)*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + ((3*I)*d^3*(c + d*x)*PolyLog[4, E^((2*I)*(a + b*x))])/b^4 + (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/(2*b^5) - (3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5) - (2*d*(c + d*x)^3*Tan[a + b*x])/b^2 + ((c + d*x)^4*Tan[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.971034, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 16, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {2620, 14, 4420, 6741, 12, 6742, 2551, 4183, 2531, 6609, 2282, 6589, 3720, 3719, 2190, 32}

$$\frac{6id^3(c + dx)\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^4} - \frac{3id^3(c + dx)\text{PolyLog}\left(4, -e^{2i(a+bx)}\right)}{b^4} + \frac{3id^3(c + dx)\text{PolyLog}\left(4, e^{2i(a+bx)}\right)}{b^4} - \frac{3d^2}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] ((2*I)*d*(c + d*x)^3)/b^2 + (c + d*x)^4/(2*b) - (2*(c + d*x)^4*ArcTanh[E^((2*I)*(a + b*x))])/b - (6*d^2*(c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b^3 + ((6*I)*d^3*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 + ((2*I)*d*(c + d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((2*I)*d*(c + d*x)^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^4*PolyLog[3, -E^((2*I)*(a + b*x))])/b^5 - (3*d^2*(c + d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))])/b^3 + (3*d^2*(c + d*x)^2*PolyLog[3, E^((2*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(c + d*x)*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + ((3*I)*d^3*(c + d*x)*PolyLog[4, E^((2*I)*(a + b*x))])/b^4 + (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/(2*b^5) - (3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5) - (2*d*(c + d*x)^3*Tan[a + b*x])/b^2 + ((c + d*x)^4*Tan[a + b*x]^2)/(2*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x],
x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6741

```
Int[u_, x_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rule 2551

```
Int[Log[u_*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[((a + b*x)^(m + 1)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a + b*x)^(m + 1)*D[u, x])/u, x], x] /; FreeQ[{a, b, m}, x] && InverseFunctionFreeQ[u, x] && NeQ[m, -1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/ (b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/ (b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
:> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[(c + d*x)^m*E^(2*I*(e + f*x))]/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol]
:> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol]
:> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^4 \csc(a + bx) \sec^3(a + bx) dx &= \frac{(c + dx)^4 \log(\tan(a + bx))}{b} + \frac{(c + dx)^4 \tan^2(a + bx)}{2b} - (4d) \int (c + dx)^3 \left(\frac{\log(\tan(a + bx))}{b} + \frac{\tan^2(a + bx)}{2b} \right) dx \\
&= \frac{(c + dx)^4 \log(\tan(a + bx))}{b} + \frac{(c + dx)^4 \tan^2(a + bx)}{2b} - (4d) \int \frac{(c + dx)^3 (2 \log(\tan(a + bx)) + \tan^2(a + bx))}{2b} dx \\
&= \frac{(c + dx)^4 \log(\tan(a + bx))}{b} + \frac{(c + dx)^4 \tan^2(a + bx)}{2b} - \frac{(2d) \int (c + dx)^3 (2 \log(\tan(a + bx)) + \tan^2(a + bx)) dx}{2b} \\
&= \frac{(c + dx)^4 \log(\tan(a + bx))}{b} + \frac{(c + dx)^4 \tan^2(a + bx)}{2b} - \frac{(2d) \int (2(c + dx)^3 \log(\tan(a + bx)) + (c + dx)^3 \tan^2(a + bx)) dx}{b} \\
&= \frac{(c + dx)^4 \log(\tan(a + bx))}{b} + \frac{(c + dx)^4 \tan^2(a + bx)}{2b} - \frac{(2d) \int (c + dx)^3 \tan^2(a + bx) dx}{b} \\
&= -\frac{2d(c + dx)^3 \tan(a + bx)}{b^2} + \frac{(c + dx)^4 \tan^2(a + bx)}{2b} + \frac{\int 2b(c + dx)^4 \csc(2a + 2bx) dx}{b} \\
&= \frac{2id(c + dx)^3}{b^2} + \frac{(c + dx)^4}{2b} - \frac{2d(c + dx)^3 \tan(a + bx)}{b^2} + \frac{(c + dx)^4 \tan^2(a + bx)}{2b} + \frac{\int 2b(c + dx)^4 \csc(2a + 2bx) dx}{b} \\
&= \frac{2id(c + dx)^3}{b^2} + \frac{(c + dx)^4}{2b} - \frac{2(c + dx)^4 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{6d^2(c + dx)^2 \log(1 - e^{2i(a+bx)})}{b^3} \\
&= \frac{2id(c + dx)^3}{b^2} + \frac{(c + dx)^4}{2b} - \frac{2(c + dx)^4 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{6d^2(c + dx)^2 \log(1 - e^{2i(a+bx)})}{b^3} \\
&= \frac{2id(c + dx)^3}{b^2} + \frac{(c + dx)^4}{2b} - \frac{2(c + dx)^4 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{6d^2(c + dx)^2 \log(1 - e^{2i(a+bx)})}{b^3} \\
&= \frac{2id(c + dx)^3}{b^2} + \frac{(c + dx)^4}{2b} - \frac{2(c + dx)^4 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{6d^2(c + dx)^2 \log(1 - e^{2i(a+bx)})}{b^3} \\
&= \frac{2id(c + dx)^3}{b^2} + \frac{(c + dx)^4}{2b} - \frac{2(c + dx)^4 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{6d^2(c + dx)^2 \log(1 - e^{2i(a+bx)})}{b^3} \\
&= \frac{2id(c + dx)^3}{b^2} + \frac{(c + dx)^4}{2b} - \frac{2(c + dx)^4 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{6d^2(c + dx)^2 \log(1 - e^{2i(a+bx)})}{b^3} \\
&= \frac{2id(c + dx)^3}{b^2} + \frac{(c + dx)^4}{2b} - \frac{2(c + dx)^4 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{6d^2(c + dx)^2 \log(1 - e^{2i(a+bx)})}{b^3}
\end{aligned}$$

Mathematica [B] time = 7.50809, size = 2090, normalized size = 5.24

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^3,x]

[Out] $-\left(\frac{c^2 d^2 E^{Ia} \text{Csc}[a] \left(\frac{2 b^3 x^3}{E^{(2I)a}} + (3I) b^2 (1 - E^{(-2I)a})\right) x^2 \text{Log}[1 - E^{(-I)(a + b x)}] + (3I) b^2 (1 - E^{(-2I)a}) x^2 \text{Log}[1 + E^{(-I)(a + b x)}] - (6(-1 + E^{(2I)a}) (b x \text{PolyLog}[2, -E^{(-I)(a + b x)}] - I \text{PolyLog}[3, -E^{(-I)(a + b x)}])]/E^{(2I)a} - (6(-1 + E^{(2I)a}) (b x \text{PolyLog}[2, E^{(-I)(a + b x)}] - I \text{PolyLog}[3, E^{(-I)(a + b x)}])]/E^{(2I)a})/b^3 - (c d^3 E^{Ia} \text{Csc}[a] \left(\frac{b^4 x^4}{E^{(2I)a}} + (2I) b^3 (1 - E^{(-2I)a}) x^3 \text{Log}[1 - E^{(-I)(a + b x)}] + (2I) b^3 (1 - E^{(-2I)a}) x^3 \text{Log}[1 + E^{(-I)(a + b x)}] - (6(-1 + E^{(2I)a}) (b^2 x^2 \text{PolyLog}[2, -E^{(-I)(a + b x)}] - (2I) b x \text{PolyLog}[3, -E^{(-I)(a + b x)}]) - 2 \text{PolyLog}[4, -E^{(-I)(a + b x)}])]/E^{(2I)a} - (6(-1 + E^{(2I)a}) (b^2 x^2 \text{PolyLog}[2, E^{(-I)(a + b x)}] - (2I) b x \text{PolyLog}[3, E^{(-I)(a + b x)}] - 2 \text{PolyLog}[4, E^{(-I)(a + b x)}])]/E^{(2I)a})/b^4 - (d^4 E^{Ia} \text{Csc}[a] \left(\frac{2 b^5 x^5}{E^{(2I)a}} + (5I) b^4 (1 - E^{(-2I)a}) x^4 \text{Log}[1 - E^{(-I)(a + b x)}] + (5I) b^4 (1 - E^{(-2I)a}) x^4 \text{Log}[1 + E^{(-I)(a + b x)}] - (20(-1 + E^{(2I)a}) (b^3 x^3 \text{PolyLog}[2, -E^{(-I)(a + b x)}] - (3I) b^2 x^2 \text{PolyLog}[3, -E^{(-I)(a + b x)}]) - 6 b x \text{PolyLog}$

$$\begin{aligned}
& [4, -E^{(-1)*(a + b*x)}] + (6*I)*PolyLog[5, -E^{(-1)*(a + b*x)}] / E^{(2*I)*a} \\
& - (20*(-1 + E^{(2*I)*a})*(b^3*x^3*PolyLog[2, E^{(-1)*(a + b*x)}] - (3*I) \\
&) * b^2*x^2*PolyLog[3, E^{(-1)*(a + b*x)}] - 6*b*x*PolyLog[4, E^{(-1)*(a + b*x)}] \\
& + (6*I)*PolyLog[5, E^{(-1)*(a + b*x)}]) / E^{(2*I)*a} / (10*b^5) + (x*(5*c^4 \\
& + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4)*Csc[a]*Sec[a]) / 5 \\
& - ((I/2)*c^2*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^{(2*I)*a}))*Log[1 + E^{(-2*I)*(a + b*x)}] \\
&) + 6*b*(1 + E^{(2*I)*a}))*x*PolyLog[2, -E^{(-2*I)*(a + b*x)}] - (3*I)*(1 + E^{(2*I)*a}))* \\
& PolyLog[3, -E^{(-2*I)*(a + b*x)}])*Sec[a] / (b^3*E^{I*a}) - ((I/2)*d^4*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^{(2*I)*a}))*Log[1 \\
& + E^{(-2*I)*(a + b*x)}] + 6*b*(1 + E^{(2*I)*a}))*x*PolyLog[2, -E^{(-2*I)*(a + b*x)}] \\
& - (3*I)*(1 + E^{(2*I)*a}))*PolyLog[3, -E^{(-2*I)*(a + b*x)}])*Sec[a] / (b^5*E^{I*a}) - (I/2)*c*d^3*E^{I*a}*(\\
& (2*x^4)/E^{(2*I)*a} - ((4*I)*(1 + E^{(-2*I)*a}))*x^3*Log[1 + E^{(-2*I)*(a + b*x)}] / b + (3*(1 + E^{(2*I)*a}))* \\
& (2*b^2*x^2*PolyLog[2, -E^{(-2*I)*(a + b*x)}] - (2*I)*b*x*PolyLog[3, -E^{(-2*I)*(a + b*x)}] \\
& - PolyLog[4, -E^{(-2*I)*(a + b*x)}])) / (b^4*E^{(2*I)*a}))*Sec[a] + (d^4*((-4*I)*x^5 - (10*(1 + E^{(2*I)*a}))*x^4*Log[1 + E^{(-2*I)*(a + b*x)}] \\
&) / b + (5*(1 + E^{(2*I)*a}))*((-4*I)*b^3*x^3*PolyLog[2, -E^{(-2*I)*(a + b*x)}] - 6*b^2*x^2*PolyLog[3, \\
& -E^{(-2*I)*(a + b*x)}] + (6*I)*b*x*PolyLog[4, -E^{(-2*I)*(a + b*x)}] + 3*PolyLog[5, -E^{(-2*I)*(a + b*x)}] \\
&)) / (20*E^{I*a}) + ((c + d*x)^4*Sec[a + b*x]^2) / (2*b) - (c^4*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x] \\
& + b*x*Sin[a])) / (b*(Cos[a]^2 + Sin[a]^2)) - (6*c^2*d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x] \\
& + b*x*Sin[a])) / (b^3*(Cos[a]^2 + Sin[a]^2)) + (c^4*Csc[a]*(-b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x] \\
&]*Sin[a])) / (b*(Cos[a]^2 + Sin[a]^2)) - (2*c^3*d*Csc[a]*((b^2*x^2)/E^{I*ArcTan[Cot[a]}) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a] \\
&])) - Pi*Log[1 + E^{(-2*I)*b*x}] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^{(2*I)*(b*x - ArcTan[Cot[a] \\
&])}]) + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a] \\
&]])) / Sqrt[1 + Cot[a]^2])*Sec[a] / (b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2))) - (6*c*d^3*Csc[a]*((b^2*x^2)/E^{I*ArcTan[Cot[a]}) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a] \\
&])) - Pi*Log[1 + E^{(-2*I)*b*x}] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^{(2*I)*(b*x - ArcTan[Cot[a] \\
&])}]) + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a] \\
&]])) / Sqrt[1 + Cot[a]^2])*Sec[a] / (b^4*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2))) - (2*Sec[a]*Sec[a + b*x]*(c^3*d*Sin[b*x] + 3*c^2*d^2*x*Sin[b*x] + 3*c*d^3*x^2*Sin[b*x] + d^4*x^3*Sin[b*x])) / b^2 - (2*c^3*d*Csc[a]*Sec[a]*(b^2*E^{I*ArcTan[Tan[a] \\
&]}) * x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^{(-2*I)*b*x}] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^{(2*I)*(b*x + ArcTan[Tan[a] \\
&])}]) + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a] \\
&]])) * I*PolyLog[2, E^{(2*I)*(b*x + ArcTan[Tan[a] \\
&])}]) * Tan[a]) / Sqrt[1 + Tan[a]^2]) / (b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
\end{aligned}$$

Maple [B] time = 0.483, size = 1729, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x)

[Out] $24*I*d^3/b^3*c*a*x-4/b*c*d^3*\ln(\exp(2*I*(b*x+a))+1)*x^3-6*d^2/b^3*c^2*\ln(\exp(2*I*(b*x+a))+1)-6*d^4/b^3*\ln(\exp(2*I*(b*x+a))+1)*x^2+4*I*d^4/b^2*x^3-8*I*d^4/b^5*a^3+3/2*d^4*polylog(5,-\exp(2*I*(b*x+a)))/b^5-4/b*\ln(\exp(2*I*(b*x+a))+1)*c^3*d*x-6/b*\ln(\exp(2*I*(b*x+a))+1)*c^2*d^2*x^2+2*I/b^2*c^3*d*polylog(2,-\exp(2*I*(b*x+a)))-3*I/b^4*c*d^3*polylog(4,-\exp(2*I*(b*x+a)))+2*I/b^2*d^4*polylog(2,-\exp(2*I*(b*x+a)))*x^3-3*I/b^4*d^4*polylog(4,-\exp(2*I*(b*x+a)))*x+1/b*d^4*\ln(1-\exp(I*(b*x+a)))*x^4-1/b^5*d^4*\ln(1-\exp(I*(b*x+a)))*a^4-12*d^3$

$$\begin{aligned} & /b^3*c*\ln(\exp(2*I*(b*x+a))+1)*x-3*d^4*polylog(3,-\exp(2*I*(b*x+a)))/b^5+4/b* \\ & c*d^3*\ln(1-\exp(I*(b*x+a)))*x^3+6*I*d^3/b^4*c*polylog(2,-\exp(2*I*(b*x+a)))+1 \\ & 2*d^4/b^5*a^2*\ln(\exp(I*(b*x+a)))+12*d^2/b^3*c^2*\ln(\exp(I*(b*x+a)))+4/b^4*c* \\ & d^3*\ln(1-\exp(I*(b*x+a)))*a^3-12*I*d^4/b^4*a^2*x+12*I*d^3/b^2*c*x^2+6*I*d^4/ \\ & b^4*polylog(2,-\exp(2*I*(b*x+a)))*x+6*I/b^2*c*d^3*polylog(2,-\exp(2*I*(b*x+a) \\ &))*x^2-6/b^3*c*d^3*polylog(3,-\exp(2*I*(b*x+a)))*x-1/b*d^4*\ln(\exp(2*I*(b*x+a) \\ &))+1)*x^4+12*I*d^3/b^4*c*a^2+4/b*c*d^3*\ln(\exp(I*(b*x+a))+1)*x^3+6*I/b^2*pol \\ & ylog(2,-\exp(2*I*(b*x+a)))*c^2*d^2*x-1/b*c^4*\ln(\exp(2*I*(b*x+a))+1)+12/b^3*d \\ & ^4*polylog(3,\exp(I*(b*x+a)))*x^2+12/b^3*c^2*d^2*polylog(3,\exp(I*(b*x+a)))+1 \\ & 2/b^3*c^2*d^2*polylog(3,-\exp(I*(b*x+a)))+12/b^3*d^4*polylog(3,-\exp(I*(b*x+a) \\ &))*x^2+6/b*c^2*d^2*\ln(1-\exp(I*(b*x+a)))*x^2+1/b*d^4*\ln(\exp(I*(b*x+a))+1)*x \\ & ^4-6/b^3*c^2*d^2*a^2*\ln(1-\exp(I*(b*x+a)))+4/b*c^3*d*\ln(1-\exp(I*(b*x+a)))*x+ \\ & 4/b^2*c^3*d*\ln(1-\exp(I*(b*x+a)))*a+4/b*c^3*d*\ln(\exp(I*(b*x+a))+1)*x+24/b^3* \\ & c*d^3*polylog(3,-\exp(I*(b*x+a)))*x+6/b*c^2*d^2*\ln(\exp(I*(b*x+a))+1)*x^2+24/ \\ & b^3*c*d^3*polylog(3,\exp(I*(b*x+a)))*x-24*d^3/b^4*c*a*\ln(\exp(I*(b*x+a)))+1/b \\ & ^5*d^4*a^4*\ln(\exp(I*(b*x+a))-1)+6/b^3*c^2*d^2*a^2*\ln(\exp(I*(b*x+a))-1)-4/b^ \\ & 4*c*d^3*a^3*\ln(\exp(I*(b*x+a))-1)-4/b^2*c^3*d*a*\ln(\exp(I*(b*x+a))-1)-4*I/b^2 \\ & *c^3*d*polylog(2,\exp(I*(b*x+a)))-4*I/b^2*c^3*d*polylog(2,-\exp(I*(b*x+a)))+2 \\ & 4*I/b^4*d^4*polylog(4,\exp(I*(b*x+a)))*x-4*I/b^2*d^4*polylog(2,\exp(I*(b*x+a) \\ &))*x^3-4*I/b^2*d^4*polylog(2,-\exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*polylog(4,-e \\ & xp(I*(b*x+a)))*x+24*I/b^4*c*d^3*polylog(4,\exp(I*(b*x+a)))+24*I/b^4*c*d^3*po \\ & lylog(4,-\exp(I*(b*x+a)))-24*d^4*polylog(5,-\exp(I*(b*x+a)))/b^5-24*d^4*polyl \\ & og(5,\exp(I*(b*x+a)))/b^5+2*(b*d^4*x^4*\exp(2*I*(b*x+a))+4*b*c*d^3*x^3*\exp(2* \\ & I*(b*x+a))+6*b*c^2*d^2*x^2*\exp(2*I*(b*x+a))+4*b*c^3*d*x*\exp(2*I*(b*x+a))-2* \\ & I*d^4*x^3*\exp(2*I*(b*x+a))+b*c^4*\exp(2*I*(b*x+a))-6*I*c*d^3*x^2*\exp(2*I*(b* \\ & x+a))-6*I*c^2*d^2*x*\exp(2*I*(b*x+a))-2*I*d^4*x^3-2*I*c^3*d*\exp(2*I*(b*x+a)) \\ & -6*I*c*d^3*x^2-6*I*c^2*d^2*x-2*I*c^3*d)/b^2/(\exp(2*I*(b*x+a))+1)^2-12*I/b^2 \\ & *c*d^3*polylog(2,-\exp(I*(b*x+a)))*x^2-12*I/b^2*c^2*d^2*polylog(2,\exp(I*(b*x \\ & +a)))*x-12*I/b^2*c^2*d^2*polylog(2,-\exp(I*(b*x+a)))*x+1/b*c^4*\ln(\exp(I*(b*x \\ & +a))+1)+1/b*c^4*\ln(\exp(I*(b*x+a))-1)-3/b^3*c^2*d^2*polylog(3,-\exp(2*I*(b*x+ \\ & a)))-3/b^3*d^4*polylog(3,-\exp(2*I*(b*x+a)))*x^2-12*I/b^2*c*d^3*polylog(2,ex \\ & p(I*(b*x+a)))*x^2 \end{aligned}$$

Maxima [B] time = 23.4574, size = 11840, normalized size = 29.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/2*(c^4*(1/(\sin(b*x + a)^2 - 1) + \log(\sin(b*x + a)^2 - 1) - \log(\sin(b*x + \\ & a)^2)) - 4*a*c^3*d*(1/(\sin(b*x + a)^2 - 1) + \log(\sin(b*x + a)^2 - 1) - \log \\ & (\sin(b*x + a)^2))/b + 6*a^2*c^2*d^2*(1/(\sin(b*x + a)^2 - 1) + \log(\sin(b*x + \\ & a)^2 - 1) - \log(\sin(b*x + a)^2))/b^2 - 4*a^3*c*d^3*(1/(\sin(b*x + a)^2 - 1) \\ & + \log(\sin(b*x + a)^2 - 1) - \log(\sin(b*x + a)^2))/b^3 + a^4*d^4*(1/(\sin(b*x \\ & + a)^2 - 1) + \log(\sin(b*x + a)^2 - 1) - \log(\sin(b*x + a)^2))/b^4 + 2*(24*b \\ & ^3*c^3*d - 72*a*b^2*c^2*d^2 + 72*a^2*b*c*d^3 - 24*a^3*d^4 + (12*(b*x + a)^4 \\ & *d^4 + 36*b^2*c^2*d^2 - 72*a*b*c*d^3 + 36*a^2*d^4 + 32*(b*c*d^3 - a*d^4)*(b \\ & *x + a)^3 + 36*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a)^2 + 24 \\ & *(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4)*(b*x \\ & + a) + 4*(3*(b*x + a)^4*d^4 + 9*b^2*c^2*d^2 - 18*a*b*c*d^3 + 9*a^2*d^4 + 8 \\ & *(b*c*d^3 - a*d^4)*(b*x + a)^3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d \\ & ^4)*(b*x + a)^2 + 6*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d^3 - (a \\ & ^3 + 3*a)*d^4)*(b*x + a))*\cos(4*b*x + 4*a) + 8*(3*(b*x + a)^4*d^4 + 9*b^2*c \\ & ^2*d^2 - 18*a*b*c*d^3 + 9*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 9*(b^ \\ & 2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a)^2 + 6*(b^3*c^3*d - 3*a*b \end{aligned}$$

$$\begin{aligned}
& ^2c^2d^2 + 3*(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4)*(b*x + a))*\cos(2*b*x + \\
& 2*a) + (12*I*(b*x + a)^4*d^4 + 36*I*b^2*c^2*d^2 - 72*I*a*b*c*d^3 + 36*I*a^2 \\
& *d^4 + (32*I*b*c*d^3 - 32*I*a*d^4)*(b*x + a)^3 + (36*I*b^2*c^2*d^2 - 72*I*a \\
& *b*c*d^3 + (36*I*a^2 + 36*I)*d^4)*(b*x + a)^2 + (24*I*b^3*c^3*d - 72*I*a*b^ \\
& 2*c^2*d^2 + (72*I*a^2 + 72*I)*b*c*d^3 + (-24*I*a^3 - 72*I*a)*d^4)*(b*x + a) \\
&)*\sin(4*b*x + 4*a) + (24*I*(b*x + a)^4*d^4 + 72*I*b^2*c^2*d^2 - 144*I*a*b*c \\
& *d^3 + 72*I*a^2*d^4 + (64*I*b*c*d^3 - 64*I*a*d^4)*(b*x + a)^3 + (72*I*b^2*c \\
& ^2*d^2 - 144*I*a*b*c*d^3 + (72*I*a^2 + 72*I)*d^4)*(b*x + a)^2 + (48*I*b^3*c \\
& ^3*d - 144*I*a*b^2*c^2*d^2 + (144*I*a^2 + 144*I)*b*c*d^3 + (-48*I*a^3 - 144 \\
& *I*a)*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(2*b*x + 2*a), \cos(2*b*x \\
& + 2*a) + 1) - (6*(b*x + a)^4*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 36*(\\
& b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 24*(b^3*c^3*d - 3*a*b^2* \\
& c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a) + 6*((b*x + a)^4*d^4 + 4*(b*c* \\
& d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) \\
&)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))* \\
& \cos(4*b*x + 4*a) + 12*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + \\
& 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^ \\
& 2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*\cos(2*b*x + 2*a) - (-6*I*(b \\
& *x + a)^4*d^4 + (-24*I*b*c*d^3 + 24*I*a*d^4)*(b*x + a)^3 + (-36*I*b^2*c^2*d \\
& ^2 + 72*I*a*b*c*d^3 - 36*I*a^2*d^4)*(b*x + a)^2 + (-24*I*b^3*c^3*d + 72*I*a \\
& *b^2*c^2*d^2 - 72*I*a^2*b*c*d^3 + 24*I*a^3*d^4)*(b*x + a))*\sin(4*b*x + 4*a) \\
& - (-12*I*(b*x + a)^4*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*x + a)^3 + (-72 \\
& *I*b^2*c^2*d^2 + 144*I*a*b*c*d^3 - 72*I*a^2*d^4)*(b*x + a)^2 + (-48*I*b^3*c \\
& ^3*d + 144*I*a*b^2*c^2*d^2 - 144*I*a^2*b*c*d^3 + 48*I*a^3*d^4)*(b*x + a))*s \\
& \sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) + (6*(b*x + a)^4*d \\
& ^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 36*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2 \\
& *d^4)*(b*x + a)^2 + 24*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d \\
& ^4)*(b*x + a) + 6*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b \\
& ^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^ \\
& 2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*\cos(4*b*x + 4*a) + 12*((b*x + a) \\
&)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + \\
& a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3 \\
& *d^4)*(b*x + a))*\cos(2*b*x + 2*a) + (6*I*(b*x + a)^4*d^4 + (24*I*b*c*d^3 - \\
& 24*I*a*d^4)*(b*x + a)^3 + (36*I*b^2*c^2*d^2 - 72*I*a*b*c*d^3 + 36*I*a^2*d^4) \\
&)*(b*x + a)^2 + (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 - 2 \\
& 4*I*a^3*d^4)*(b*x + a))*\sin(4*b*x + 4*a) + (12*I*(b*x + a)^4*d^4 + (48*I*b* \\
& c*d^3 - 48*I*a*d^4)*(b*x + a)^3 + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72* \\
& I*a^2*d^4)*(b*x + a)^2 + (48*I*b^3*c^3*d - 144*I*a*b^2*c^2*d^2 + 144*I*a^2* \\
& b*c*d^3 - 48*I*a^3*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), \\
& -\cos(b*x + a) + 1) - 24*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 \\
& + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*\cos(4*b*x + 4*a) + (12 \\
& *I*(b*x + a)^4*d^4 + 24*b^3*c^3*d - 72*a*b^2*c^2*d^2 + 72*a^2*b*c*d^3 - 24* \\
& a^3*d^4 + (48*I*b*c*d^3 - 24*(2*I*a + 1)*d^4)*(b*x + a)^3 + (72*I*b^2*c^2*d \\
& ^2 - 72*(2*I*a + 1)*b*c*d^3 + (72*I*a^2 + 72*a)*d^4)*(b*x + a)^2 + (48*I*b^ \\
& 3*c^3*d - 72*(2*I*a + 1)*b^2*c^2*d^2 + (144*I*a^2 + 144*a)*b*c*d^3 + (-48*I \\
& *a^3 - 72*a^2)*d^4)*(b*x + a))*\cos(2*b*x + 2*a) - (12*b^3*c^3*d - 36*a*b^2* \\
& c^2*d^2 + 24*(b*x + a)^3*d^4 + 36*(a^2 + 1)*b*c*d^3 - 12*(a^3 + 3*a)*d^4 + \\
& 48*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 36*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1) \\
&)*d^4)*(b*x + a) + 12*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 2*(b*x + a)^3*d^4 + 3* \\
& (a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(\\
& b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a))*\cos(4*b*x + 4*a) + 24 \\
& *(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 2*(b*x + a)^3*d^4 + 3*(a^2 + 1)*b*c*d^3 - (\\
& a^3 + 3*a)*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c \\
& *d^3 + (a^2 + 1)*d^4)*(b*x + a))*\cos(2*b*x + 2*a) - (-12*I*b^3*c^3*d + 36*I \\
& *a*b^2*c^2*d^2 - 24*I*(b*x + a)^3*d^4 + (-36*I*a^2 - 36*I)*b*c*d^3 + (12*I* \\
& a^3 + 36*I*a)*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*x + a)^2 + (-36*I*b^2*c \\
& ^2*d^2 + 72*I*a*b*c*d^3 + (-36*I*a^2 - 36*I)*d^4)*(b*x + a))*\sin(4*b*x + 4* \\
& a) - (-24*I*b^3*c^3*d + 72*I*a*b^2*c^2*d^2 - 48*I*(b*x + a)^3*d^4 + (-72*I* \\
& a^2 - 72*I)*b*c*d^3 + (24*I*a^3 + 72*I*a)*d^4 + (-96*I*b*c*d^3 + 96*I*a*d^4
\end{aligned}$$

$$\begin{aligned}
&)*(b*x + a)^2 + (-72*I*b^2*c^2*d^2 + 144*I*a*b*c*d^3 + (-72*I*a^2 - 72*I)*d \\
& ^4)*(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{dilog}(-e^{(2*I*b*x + 2*I*a)}) + (24*b^3*c^3* \\
& d - 72*a*b^2*c^2*d^2 + 72*a^2*b*c*d^3 + 24*(b*x + a)^3*d^4 - 24*a^3*d^4 + 7 \\
& 2*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 72*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)* \\
& (b*x + a) + 24*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d \\
& ^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d \\
& ^3 + a^2*d^4)*(b*x + a))*\cos(4*b*x + 4*a) + 48*(b^3*c^3*d - 3*a*b^2*c^2*d^2 \\
& + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a \\
&)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*\cos(2*b*x + 2*a) + \\
& (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 + 24*I*(b*x + a)^3 \\
& *d^4 - 24*I*a^3*d^4 + (72*I*b*c*d^3 - 72*I*a*d^4)*(b*x + a)^2 + (72*I*b^2*c \\
& ^2*d^2 - 144*I*a*b*c*d^3 + 72*I*a^2*d^4)*(b*x + a))*\sin(4*b*x + 4*a) + (48* \\
& I*b^3*c^3*d - 144*I*a*b^2*c^2*d^2 + 144*I*a^2*b*c*d^3 + 48*I*(b*x + a)^3*d^4 \\
& - 48*I*a^3*d^4 + (144*I*b*c*d^3 - 144*I*a*d^4)*(b*x + a)^2 + (144*I*b^2*c \\
& ^2*d^2 - 288*I*a*b*c*d^3 + 144*I*a^2*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{dilo} \\
& g(-e^{(I*b*x + I*a)}) + (24*b^3*c^3*d - 72*a*b^2*c^2*d^2 + 72*a^2*b*c*d^3 + 2 \\
& 4*(b*x + a)^3*d^4 - 24*a^3*d^4 + 72*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 72*(b^2 \\
& *c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) + 24*(b^3*c^3*d - 3*a*b^2*c^2*d \\
& ^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + \\
& a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*\cos(4*b*x + 4*a) \\
& + 48*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3* \\
& d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2* \\
& d^4)*(b*x + a))*\cos(2*b*x + 2*a) + (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 7 \\
& 2*I*a^2*b*c*d^3 + 24*I*(b*x + a)^3*d^4 - 24*I*a^3*d^4 + (72*I*b*c*d^3 - 72* \\
& I*a*d^4)*(b*x + a)^2 + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*a^2*d^4)* \\
& (b*x + a))*\sin(4*b*x + 4*a) + (48*I*b^3*c^3*d - 144*I*a*b^2*c^2*d^2 + 144*I \\
& *a^2*b*c*d^3 + 48*I*(b*x + a)^3*d^4 - 48*I*a^3*d^4 + (144*I*b*c*d^3 - 144*I \\
& *a*d^4)*(b*x + a)^2 + (144*I*b^2*c^2*d^2 - 288*I*a*b*c*d^3 + 144*I*a^2*d^4) \\
& *(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{dilog}(e^{(I*b*x + I*a)}) + (-6*I*(b*x + a)^4*d^ \\
& 4 - 18*I*b^2*c^2*d^2 + 36*I*a*b*c*d^3 - 18*I*a^2*d^4 + (-16*I*b*c*d^3 + 16* \\
& I*a*d^4)*(b*x + a)^3 + (-18*I*b^2*c^2*d^2 + 36*I*a*b*c*d^3 + (-18*I*a^2 - 1 \\
& 8*I)*d^4)*(b*x + a)^2 + (-12*I*b^3*c^3*d + 36*I*a*b^2*c^2*d^2 + (-36*I*a^2 \\
& - 36*I)*b*c*d^3 + (12*I*a^3 + 36*I*a)*d^4)*(b*x + a) + (-6*I*(b*x + a)^4*d^ \\
& 4 - 18*I*b^2*c^2*d^2 + 36*I*a*b*c*d^3 - 18*I*a^2*d^4 + (-16*I*b*c*d^3 + 16* \\
& I*a*d^4)*(b*x + a)^3 + (-18*I*b^2*c^2*d^2 + 36*I*a*b*c*d^3 + (-18*I*a^2 - 1 \\
& 8*I)*d^4)*(b*x + a)^2 + (-12*I*b^3*c^3*d + 36*I*a*b^2*c^2*d^2 + (-36*I*a^2 \\
& - 36*I)*b*c*d^3 + (12*I*a^3 + 36*I*a)*d^4)*(b*x + a))*\cos(4*b*x + 4*a) + (- \\
& 12*I*(b*x + a)^4*d^4 - 36*I*b^2*c^2*d^2 + 72*I*a*b*c*d^3 - 36*I*a^2*d^4 + (\\
& -32*I*b*c*d^3 + 32*I*a*d^4)*(b*x + a)^3 + (-36*I*b^2*c^2*d^2 + 72*I*a*b*c*d \\
& ^3 + (-36*I*a^2 - 36*I)*d^4)*(b*x + a)^2 + (-24*I*b^3*c^3*d + 72*I*a*b^2*c^ \\
& 2*d^2 + (-72*I*a^2 - 72*I)*b*c*d^3 + (24*I*a^3 + 72*I*a)*d^4)*(b*x + a))*\co \\
& s(2*b*x + 2*a) + 2*(3*(b*x + a)^4*d^4 + 9*b^2*c^2*d^2 - 18*a*b*c*d^3 + 9*a^ \\
& 2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a \\
& ^2 + 1)*d^4)*(b*x + a)^2 + 6*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c \\
& *d^3 - (a^3 + 3*a)*d^4)*(b*x + a))*\sin(4*b*x + 4*a) + 4*(3*(b*x + a)^4*d^4 \\
& + 9*b^2*c^2*d^2 - 18*a*b*c*d^3 + 9*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^ \\
& 3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a)^2 + 6*(b^3*c^3* \\
& d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4)*(b*x + a))*\sin \\
& (2*b*x + 2*a))*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + \\
& 2*a) + 1) + (3*I*(b*x + a)^4*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^3 \\
& + (18*I*b^2*c^2*d^2 - 36*I*a*b*c*d^3 + 18*I*a^2*d^4)*(b*x + a)^2 + (12*I*b^ \\
& 3*c^3*d - 36*I*a*b^2*c^2*d^2 + 36*I*a^2*b*c*d^3 - 12*I*a^3*d^4)*(b*x + a) + \\
& (3*I*(b*x + a)^4*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^3 + (18*I*b^2 \\
& *c^2*d^2 - 36*I*a*b*c*d^3 + 18*I*a^2*d^4)*(b*x + a)^2 + (12*I*b^3*c^3*d - 3 \\
& 6*I*a*b^2*c^2*d^2 + 36*I*a^2*b*c*d^3 - 12*I*a^3*d^4)*(b*x + a))*\cos(4*b*x + \\
& 4*a) + (6*I*(b*x + a)^4*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a)^3 + (3 \\
& 6*I*b^2*c^2*d^2 - 72*I*a*b*c*d^3 + 36*I*a^2*d^4)*(b*x + a)^2 + (24*I*b^3*c^ \\
& 3*d - 72*I*a*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 - 24*I*a^3*d^4)*(b*x + a))*\cos(\\
& 2*b*x + 2*a) - 3*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^
\end{aligned}$$

$$\begin{aligned}
& 2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a)*\sin(4*b*x + 4*a) - 6*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a)*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) + (3*I*(b*x + a)^4*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^3 + (18*I*b^2*c^2*d^2 - 36*I*a*b*c*d^3 + 18*I*a^2*d^4)*(b*x + a)^2 + (12*I*b^3*c^3*d - 36*I*a*b^2*c^2*d^2 + 36*I*a^2*b*c*d^3 - 12*I*a^3*d^4)*(b*x + a) + (3*I*(b*x + a)^4*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^3 + (18*I*b^2*c^2*d^2 - 36*I*a*b*c*d^3 + 18*I*a^2*d^4)*(b*x + a)^2 + (12*I*b^3*c^3*d - 36*I*a*b^2*c^2*d^2 + 36*I*a^2*b*c*d^3 - 12*I*a^3*d^4)*(b*x + a))*\cos(4*b*x + 4*a) + (6*I*(b*x + a)^4*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a)^3 + (36*I*b^2*c^2*d^2 - 72*I*a*b*c*d^3 + 36*I*a^2*d^4)*(b*x + a)^2 + (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 - 24*I*a^3*d^4)*(b*x + a))*\cos(2*b*x + 2*a) - 3*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*\sin(4*b*x + 4*a) - 6*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) + (18*I*d^4*\cos(4*b*x + 4*a) + 36*I*d^4*\cos(2*b*x + 2*a) - 18*d^4*\sin(4*b*x + 4*a) - 36*d^4*\sin(2*b*x + 2*a) + 18*I*d^4)*\text{polylog}(5, -e^{(2*I*b*x + 2*I*a)}) + (-144*I*d^4*\cos(4*b*x + 4*a) - 288*I*d^4*\cos(2*b*x + 2*a) + 144*d^4*\sin(4*b*x + 4*a) + 288*d^4*\sin(2*b*x + 2*a) - 144*I*d^4)*\text{polylog}(5, -e^{(I*b*x + I*a)}) + (-144*I*d^4*\cos(4*b*x + 4*a) - 288*I*d^4*\cos(2*b*x + 2*a) + 144*d^4*\sin(4*b*x + 4*a) + 288*d^4*\sin(2*b*x + 2*a) - 144*I*d^4)*\text{polylog}(5, e^{(I*b*x + I*a)}) + (24*b*c*d^3 + 36*(b*x + a)*d^4 - 24*a*d^4 + 12*(2*b*c*d^3 + 3*(b*x + a)*d^4 - 2*a*d^4)*\cos(4*b*x + 4*a) + 24*(2*b*c*d^3 + 3*(b*x + a)*d^4 - 2*a*d^4)*\cos(2*b*x + 2*a) + (24*I*b*c*d^3 + 36*I*(b*x + a)*d^4 - 24*I*a*d^4)*\sin(4*b*x + 4*a) + (48*I*b*c*d^3 + 72*I*(b*x + a)*d^4 - 48*I*a*d^4)*\sin(2*b*x + 2*a))*\text{polylog}(4, -e^{(2*I*b*x + 2*I*a)}) - (144*b*c*d^3 + 144*(b*x + a)*d^4 - 144*a*d^4 + 144*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*\cos(4*b*x + 4*a) + 288*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*\cos(2*b*x + 2*a) - (-144*I*b*c*d^3 - 144*I*(b*x + a)*d^4 + 144*I*a*d^4)*\sin(4*b*x + 4*a) - (-288*I*b*c*d^3 - 288*I*(b*x + a)*d^4 + 288*I*a*d^4)*\sin(2*b*x + 2*a))*\text{polylog}(4, -e^{(I*b*x + I*a)}) - (144*b*c*d^3 + 144*(b*x + a)*d^4 - 144*a*d^4 + 144*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*\cos(4*b*x + 4*a) + 288*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*\cos(2*b*x + 2*a) - (-144*I*b*c*d^3 - 144*I*(b*x + a)*d^4 + 144*I*a*d^4)*\sin(4*b*x + 4*a) - (-288*I*b*c*d^3 - 288*I*(b*x + a)*d^4 + 288*I*a*d^4)*\sin(2*b*x + 2*a))*\text{polylog}(4, e^{(I*b*x + I*a)}) + (-18*I*b^2*c^2*d^2 + 36*I*a*b*c*d^3 - 36*I*(b*x + a)^2*d^4 + (-18*I*a^2 - 18*I)*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*x + a) + (-18*I*b^2*c^2*d^2 + 36*I*a*b*c*d^3 - 36*I*(b*x + a)^2*d^4 + (-18*I*a^2 - 18*I)*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*x + a))*\cos(4*b*x + 4*a) + (-36*I*b^2*c^2*d^2 + 72*I*a*b*c*d^3 - 72*I*(b*x + a)^2*d^4 + (-36*I*a^2 - 36*I)*d^4 + (-96*I*b*c*d^3 + 96*I*a*d^4)*(b*x + a))*\cos(2*b*x + 2*a) + 6*(3*b^2*c^2*d^2 - 6*a*b*c*d^3 + 6*(b*x + a)^2*d^4 + 3*(a^2 + 1)*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a))*\sin(4*b*x + 4*a) + 12*(3*b^2*c^2*d^2 - 6*a*b*c*d^3 + 6*(b*x + a)^2*d^4 + 3*(a^2 + 1)*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\text{polylog}(3, -e^{(2*I*b*x + 2*I*a)}) + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*(b*x + a)^2*d^4 + 72*I*a^2*d^4 + (144*I*b*c*d^3 - 144*I*a*d^4)*(b*x + a) + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*(b*x + a)^2*d^4 + 72*I*a^2*d^4 + (144*I*b*c*d^3 - 144*I*a*d^4)*(b*x + a))*\cos(4*b*x + 4*a) + (144*I*b^2*c^2*d^2 - 288*I*a*b*c*d^3 + 144*I*(b*x + a)^2*d^4 + 144*I*a^2*d^4 + (288*I*b*c*d^3 - 288*I*a*d^4)*(b*x + a))*\cos(2*b*x + 2*a) - 72*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*\sin(4*b*x + 4*a) - 144*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\text{polylog}(3, -e^{(I*b*x + I*a)}) + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*(b*x + a)^2*d^4 + 72*I*a^2*d^4 + (144*I*b*c*d^3
\end{aligned}$$

$$\begin{aligned}
& - 144*I*a*d^4)*(b*x + a) + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*(b*x \\
& + a)^2*d^4 + 72*I*a^2*d^4 + (144*I*b*c*d^3 - 144*I*a*d^4)*(b*x + a))*\cos(4* \\
& b*x + 4*a) + (144*I*b^2*c^2*d^2 - 288*I*a*b*c*d^3 + 144*I*(b*x + a)^2*d^4 + \\
& 144*I*a^2*d^4 + (288*I*b*c*d^3 - 288*I*a*d^4)*(b*x + a))*\cos(2*b*x + 2*a) \\
& - 72*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - \\
& a*d^4)*(b*x + a))*\sin(4*b*x + 4*a) - 144*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x \\
& + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*\sin(2*b*x + 2*a))*\text{poly} \\
& \text{ylog}(3, e^{(I*b*x + I*a)}) + (-24*I*(b*x + a)^3*d^4 + (-72*I*b*c*d^3 + 72*I*a \\
& *d^4)*(b*x + a)^2 + (-72*I*b^2*c^2*d^2 + 144*I*a*b*c*d^3 - 72*I*a^2*d^4)*(b \\
& *x + a))*\sin(4*b*x + 4*a) - (12*(b*x + a)^4*d^4 - 24*I*b^3*c^3*d + 72*I*a*b \\
& ^2*c^2*d^2 - 72*I*a^2*b*c*d^3 + 24*I*a^3*d^4 + (48*b*c*d^3 - (48*a - 24*I)* \\
& d^4)*(b*x + a)^3 + (72*b^2*c^2*d^2 - (144*a - 72*I)*b*c*d^3 + 72*(a^2 - I*a \\
&)*d^4)*(b*x + a)^2 + (48*b^3*c^3*d - (144*a - 72*I)*b^2*c^2*d^2 + 144*(a^2 \\
& - I*a)*b*c*d^3 - 24*(2*a^3 - 3*I*a^2)*d^4)*(b*x + a))*\sin(2*b*x + 2*a))/(-6 \\
& *I*b^4*\cos(4*b*x + 4*a) - 12*I*b^4*\cos(2*b*x + 2*a) + 6*b^4*\sin(4*b*x + 4*a \\
&) + 12*b^4*\sin(2*b*x + 2*a) - 6*I*b^4))/b
\end{aligned}$$

Fricas [C] time = 1.96768, size = 8008, normalized size = 20.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")

[Out] $1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 24*d^4*\cos(b*x + a)^2*\text{polylog}(5, \cos(b*x + a) + I*\sin(b*x + a)) - 24*d^4*\cos(b*x + a)^2*\text{polylog}(5, \cos(b*x + a) - I*\sin(b*x + a)) + 24*d^4*\cos(b*x + a)^2*\text{polylog}(5, I*\cos(b*x + a) + \sin(b*x + a)) + 24*d^4*\cos(b*x + a)^2*\text{polylog}(5, I*\cos(b*x + a) - \sin(b*x + a)) + 24*d^4*\cos(b*x + a)^2*\text{polylog}(5, -I*\cos(b*x + a) + \sin(b*x + a)) + 24*d^4*\cos(b*x + a)^2*\text{polylog}(5, -I*\cos(b*x + a) - \sin(b*x + a)) - 24*d^4*\cos(b*x + a)^2*\text{polylog}(5, -\cos(b*x + a) + I*\sin(b*x + a)) - 24*d^4*\cos(b*x + a)^2*\text{polylog}(5, -\cos(b*x + a) - I*\sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*\cos(b*x + a)^2*\text{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*\cos(b*x + a)^2*\text{dilog}(\cos(b*x + a) - I*\sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d - 12*I*b*c*d^3 - 12*I*(b^3*c^2*d^2 + b*d^4)*x)*\cos(b*x + a)^2*\text{dilog}(I*\cos(b*x + a) + \sin(b*x + a)) + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d + 12*I*b*c*d^3 + 12*I*(b^3*c^2*d^2 + b*d^4)*x)*\cos(b*x + a)^2*\text{dilog}(I*\cos(b*x + a) - \sin(b*x + a)) + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d + 12*I*b*c*d^3 + 12*I*(b^3*c^2*d^2 + b*d^4)*x)*\cos(b*x + a)^2*\text{dilog}(-I*\cos(b*x + a) + \sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d - 12*I*b*c*d^3 - 12*I*(b^3*c^2*d^2 + b*d^4)*x)*\cos(b*x + a)^2*\text{dilog}(-I*\cos(b*x + a) - \sin(b*x + a)) + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*\cos(b*x + a)^2*\text{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*\cos(b*x + a)^2*\text{dilog}(-\cos(b*x + a) - I*\sin(b*x + a)) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*\cos(b*x + a)^2*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 + 1)*b^2*c^2*d^2 - 4*(a^3 + 3*a)*b*c*d^3 + (a^4 + 6*a^2)*d^4)*\cos(b*x + a)^2*\log(\cos(b*x + a) + I*\sin(b*x + a) + I) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*\cos(b*x + a)^2*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 + 1)*b^2*c^2*d^2 - 4*(a^3 + 3*a)*b*c*d^3 + (a^4 + 6*a^2)*d^4)*\cos(b*x + a)^2*\log(\cos(b*x + a) - I*\sin(b*x + a) + I) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2$

$$\begin{aligned}
& 2 + 4*(a^3 + 3*a)*b*c*d^3 - (a^4 + 6*a^2)*d^4 + 6*(b^4*c^2*d^2 + b^2*d^4)*x \\
& ^2 + 4*(b^4*c^3*d + 3*b^2*c*d^3)*x)*\cos(b*x + a)^2*\log(I*\cos(b*x + a) + \sin \\
& (b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2 \\
& *c^2*d^2 + 4*(a^3 + 3*a)*b*c*d^3 - (a^4 + 6*a^2)*d^4 + 6*(b^4*c^2*d^2 + b^2 \\
& *d^4)*x^2 + 4*(b^4*c^3*d + 3*b^2*c*d^3)*x)*\cos(b*x + a)^2*\log(I*\cos(b*x + a \\
&) - \sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6* \\
& a^2*b^2*c^2*d^2 + 4*(a^3 + 3*a)*b*c*d^3 - (a^4 + 6*a^2)*d^4 + 6*(b^4*c^2*d^2 \\
& + b^2*d^4)*x^2 + 4*(b^4*c^3*d + 3*b^2*c*d^3)*x)*\cos(b*x + a)^2*\log(-I*\cos \\
& (b*x + a) + \sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3 \\
& *d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 + 3*a)*b*c*d^3 - (a^4 + 6*a^2)*d^4 + 6*(b^4 \\
& *c^2*d^2 + b^2*d^4)*x^2 + 4*(b^4*c^3*d + 3*b^2*c*d^3)*x)*\cos(b*x + a)^2*lo \\
& g(-I*\cos(b*x + a) - \sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^ \\
& 2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*\cos(b*x + a)^2*\log(-1/2*\cos(b*x + a) + \\
& 1/2*I*\sin(b*x + a) + 1/2) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - \\
& 4*a^3*b*c*d^3 + a^4*d^4)*\cos(b*x + a)^2*\log(-1/2*\cos(b*x + a) - 1/2*I*\sin \\
& (b*x + a) + 1/2) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^ \\
& 4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*co \\
& s(b*x + a)^2*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c \\
& ^3*d + 6*(a^2 + 1)*b^2*c^2*d^2 - 4*(a^3 + 3*a)*b*c*d^3 + (a^4 + 6*a^2)*d^4) \\
& *\cos(b*x + a)^2*\log(-\cos(b*x + a) + I*\sin(b*x + a) + I) + (b^4*d^4*x^4 + 4* \\
& b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^ \\
& ^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*\cos(b*x + a)^2*\log(-\cos(b*x + a) - I* \\
& \sin(b*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 + 1)*b^2*c^2*d^2 - 4* \\
& (a^3 + 3*a)*b*c*d^3 + (a^4 + 6*a^2)*d^4)*\cos(b*x + a)^2*\log(-\cos(b*x + a) - \\
& I*\sin(b*x + a) + I) + (24*I*b*d^4*x + 24*I*b*c*d^3)*\cos(b*x + a)^2*\text{polylog} \\
& (4, \cos(b*x + a) + I*\sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*\cos(b*x \\
& + a)^2*\text{polylog}(4, \cos(b*x + a) - I*\sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b* \\
& c*d^3)*\cos(b*x + a)^2*\text{polylog}(4, I*\cos(b*x + a) + \sin(b*x + a)) + (-24*I*b* \\
& d^4*x - 24*I*b*c*d^3)*\cos(b*x + a)^2*\text{polylog}(4, I*\cos(b*x + a) - \sin(b*x + \\
& a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*\cos(b*x + a)^2*\text{polylog}(4, -I*\cos(b*x + \\
& a) + \sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b*c*d^3)*\cos(b*x + a)^2*\text{polylog}(\\
& 4, -I*\cos(b*x + a) - \sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*\cos(b*x \\
& + a)^2*\text{polylog}(4, -\cos(b*x + a) + I*\sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b \\
& *c*d^3)*\cos(b*x + a)^2*\text{polylog}(4, -\cos(b*x + a) - I*\sin(b*x + a)) + 12*(b^2 \\
& *d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*\cos(b*x + a)^2*\text{polylog}(3, \cos(b*x + \\
& a) + I*\sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*\cos(\\
& b*x + a)^2*\text{polylog}(3, \cos(b*x + a) - I*\sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2* \\
& b^2*c*d^3*x + b^2*c^2*d^2 + d^4)*\cos(b*x + a)^2*\text{polylog}(3, I*\cos(b*x + a) + \\
& \sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 + d^4)*\cos(b \\
& *x + a)^2*\text{polylog}(3, I*\cos(b*x + a) - \sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b \\
& ^2*c*d^3*x + b^2*c^2*d^2 + d^4)*\cos(b*x + a)^2*\text{polylog}(3, -I*\cos(b*x + a) + \\
& \sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 + d^4)*\cos(b \\
& *x + a)^2*\text{polylog}(3, -I*\cos(b*x + a) - \sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2* \\
& b^2*c*d^3*x + b^2*c^2*d^2)*\cos(b*x + a)^2*\text{polylog}(3, -\cos(b*x + a) + I*\sin(\\
& b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*\cos(b*x + a)^2*p \\
& olylog(3, -\cos(b*x + a) - I*\sin(b*x + a)) - 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 \\
& + 3*b^3*c^2*d^2*x + b^3*c^3*d)*\cos(b*x + a)*\sin(b*x + a))/(b^5*\cos(b*x + \\
& a)^2)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**4*csc(b*x+a)*sec(b*x+a)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^4 \csc(bx + a) \sec(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")

[Out] integrate((d*x + c)^4*csc(b*x + a)*sec(b*x + a)^3, x)

3.311 $\int (c + dx)^3 \csc(a + bx) \sec^3(a + bx) dx$

Optimal. Leaf size=325

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} + \frac{3d^2(c+dx)\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3} + \frac{3id(c+dx)^2\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^2} - \frac{3id(c+dx)^2\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2}$$

```
[Out] (((3*I)/2)*d*(c + d*x)^2)/b^2 + (c + d*x)^3/(2*b) - (2*(c + d*x)^3*ArcTanh[E^((2*I)*(a + b*x))])/b - (3*d^2*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b^3 + (((3*I)/2)*d^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/((2*b^3) + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/((2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/4)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4 - (3*d*(c + d*x)^2*Tan[a + b*x])/((2*b^2) + ((c + d*x)^3*Tan[a + b*x]^2)/(2*b))
```

Rubi [A] time = 0.640141, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 18, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {2620, 14, 4420, 6741, 12, 6742, 2551, 4183, 2531, 6609, 2282, 6589, 3720, 3719, 2190, 2279, 2391, 32}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} + \frac{3d^2(c+dx)\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3} + \frac{3id(c+dx)^2\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^2} - \frac{3id(c+dx)^2\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] (((3*I)/2)*d*(c + d*x)^2)/b^2 + (c + d*x)^3/(2*b) - (2*(c + d*x)^3*ArcTanh[E^((2*I)*(a + b*x))])/b - (3*d^2*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b^3 + (((3*I)/2)*d^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/((2*b^3) + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/((2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/4)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4 - (3*d*(c + d*x)^2*Tan[a + b*x])/((2*b^2) + ((c + d*x)^3*Tan[a + b*x]^2)/(2*b))
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x],
x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)
*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*x]^p, x]},
Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
```

x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]

Rule 6741

Int[u_, x_Symbol] :=> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]

Rule 12

Int[(a_)*(u_), x_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 6742

Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rule 2551

Int[Log[u_*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :=> Simp[((a + b*x)^(m + 1)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a + b*x)^(m + 1)*D[u, x])/u, x], x] /; FreeQ[{a, b, m}, x] && InverseFunctionFreeQ[u, x] && NeQ[m, -1]

Rule 4183

Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :=> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :=> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/((b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 6609

Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.), x_Symbol] :=> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/((b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_), x_Symbol]
:> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^(m)*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*tan[(e_.) + (f_.)*(x_.)], x_Symbol]
:> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[(c + d*x)^m*E^(2*I*(e + f*x))]/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_.))))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_.))))^(n_.)), x_Symbol]
:> Simp[(((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n]/a)]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_.))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]/(x_), x_Symbol]
:> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_.))^(m_), x_Symbol]
:> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 \csc(a + bx) \sec^3(a + bx) dx &= \frac{(c + dx)^3 \log(\tan(a + bx))}{b} + \frac{(c + dx)^3 \tan^2(a + bx)}{2b} - (3d) \int (c + dx)^2 \left(\frac{\log(\tan(a + bx))}{b} + \frac{\tan^2(a + bx)}{2b} \right) dx \\
&= \frac{(c + dx)^3 \log(\tan(a + bx))}{b} + \frac{(c + dx)^3 \tan^2(a + bx)}{2b} - (3d) \int \frac{(c + dx)^2 (2 \log(\tan(a + bx)) + \tan^2(a + bx))}{2b} dx \\
&= \frac{(c + dx)^3 \log(\tan(a + bx))}{b} + \frac{(c + dx)^3 \tan^2(a + bx)}{2b} - \frac{(3d) \int (c + dx)^2 (2 \log(\tan(a + bx)) + \tan^2(a + bx)) dx}{2b} \\
&= \frac{(c + dx)^3 \log(\tan(a + bx))}{b} + \frac{(c + dx)^3 \tan^2(a + bx)}{2b} - \frac{(3d) \int (2(c + dx)^2 \log(\tan(a + bx)) + (c + dx)^2 \tan^2(a + bx)) dx}{2b} \\
&= \frac{(c + dx)^3 \log(\tan(a + bx))}{b} + \frac{(c + dx)^3 \tan^2(a + bx)}{2b} - \frac{(3d) \int (c + dx)^2 \tan^2(a + bx) dx}{2b} \\
&= -\frac{3d(c + dx)^2 \tan(a + bx)}{2b^2} + \frac{(c + dx)^3 \tan^2(a + bx)}{2b} + \frac{\int 2b(c + dx)^3 \csc(2a + 2bx) dx}{b} \\
&= \frac{3id(c + dx)^2}{2b^2} + \frac{(c + dx)^3}{2b} - \frac{3d(c + dx)^2 \tan(a + bx)}{2b^2} + \frac{(c + dx)^3 \tan^2(a + bx)}{2b} + \frac{\int 2b(c + dx)^3 \csc(2a + 2bx) dx}{b} \\
&= \frac{3id(c + dx)^2}{2b^2} + \frac{(c + dx)^3}{2b} - \frac{2(c + dx)^3 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{3d^2(c + dx) \log(1 + e^{2i(a+bx)})}{b^3} \\
&= \frac{3id(c + dx)^2}{2b^2} + \frac{(c + dx)^3}{2b} - \frac{2(c + dx)^3 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{3d^2(c + dx) \log(1 + e^{2i(a+bx)})}{b^3} \\
&= \frac{3id(c + dx)^2}{2b^2} + \frac{(c + dx)^3}{2b} - \frac{2(c + dx)^3 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{3d^2(c + dx) \log(1 + e^{2i(a+bx)})}{b^3} \\
&= \frac{3id(c + dx)^2}{2b^2} + \frac{(c + dx)^3}{2b} - \frac{2(c + dx)^3 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{3d^2(c + dx) \log(1 + e^{2i(a+bx)})}{b^3} \\
&= \frac{3id(c + dx)^2}{2b^2} + \frac{(c + dx)^3}{2b} - \frac{2(c + dx)^3 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{3d^2(c + dx) \log(1 + e^{2i(a+bx)})}{b^3}
\end{aligned}$$

Mathematica [B] time = 6.92562, size = 1486, normalized size = 4.57

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^3,x]

[Out] $-(c*d^2*E^{I*a}*Csc[a]*((2*b^3*x^3)/E^{((2*I)*a)} + (3*I)*b^2*(1 - E^{((-2*I)*a)})) * x^2 * \log[1 - E^{((-I)*(a + b*x))}] + (3*I)*b^2*(1 - E^{((-2*I)*a)}) * x^2 * \log[1 + E^{((-I)*(a + b*x))}] - (6*(-1 + E^{((2*I)*a)}) * (b*x * \text{PolyLog}[2, -E^{((-I)*(a + b*x))}] - I * \text{PolyLog}[3, -E^{((-I)*(a + b*x))}])) / E^{((2*I)*a)} - (6*(-1 + E^{((2*I)*a)}) * (b*x * \text{PolyLog}[2, E^{((-I)*(a + b*x))}] - I * \text{PolyLog}[3, E^{((-I)*(a + b*x))}])) / E^{((2*I)*a)}) / (2*b^3) - (d^3 * E^{I*a} * Csc[a] * ((b^4 * x^4) / E^{((2*I)*a)} + (2*I) * b^3 * (1 - E^{((-2*I)*a)}) * x^3 * \log[1 - E^{((-I)*(a + b*x))}] + (2*I) * b^3 * (1 - E^{((-2*I)*a)}) * x^3 * \log[1 + E^{((-I)*(a + b*x))}] - (6*(-1 + E^{((2*I)*a)}) * (b^2 * x^2 * \text{PolyLog}[2, -E^{((-I)*(a + b*x))}] - (2*I) * b * x * \text{PolyLog}[3, -E^{((-I)*(a + b*x))}] - 2 * \text{PolyLog}[4, -E^{((-I)*(a + b*x))}])) / E^{((2*I)*a)} - (6*(-1 + E^{((2*I)*a)}) * (b^2 * x^2 * \text{PolyLog}[2, E^{((-I)*(a + b*x))}] - (2*I) * b * x * \text{PolyLog}[3, E^{((-I)*(a + b*x))}] - 2 * \text{PolyLog}[4, E^{((-I)*(a + b*x))}])) / E^{((2*I)*a)}) / (4*b^4) + (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) * Csc[a] * Sec[a]) / 4 - ((I/4) * c*d^2 * (2*b^2 * x^2 * (2*b*x - (3*I) * (1 + E^{((2*I)*a)}) * \log[1 + E^{((-2*I)*a)} * (a + b*x)])) + 6*b * (1 + E^{((2*I)*a)}) * x * \text{PolyLog}[2, -E^{((-2*I)*a)} * (a + b*x)] - (3*I) * (1 + E^{((2*I)*a)}) * \text{PolyLog}[3, -E^{((-2*I)*a)} * (a + b*x)])) * Sec[a]) / (b^3 * E^{I*a}) - (I/8) * d^3 * E^{I*a} * ((2*x^4) / E^{((2*I)*a)} - ((4*I) * (1 + E^{((-2*I)*a)}) * x^3 * \log[1 + E^{((-2*I)*a)} * (a + b*x)])) / b + (3 * (1 + E^{((2*I)*a)}) * (2*b^2 * x^2 * \text{PolyLog}[2, -E^{((-2*I)*a)} * (a + b*x)] - (2*I) * b * x * \text{PolyLog}[3, -E^{((-2*I)*a)} * (a + b*x)] - \text{PolyLog}[4, -E^{((-2*I)*a)} * (a + b*x)])) / b^3$

$$\begin{aligned} & \text{Log}[4, -E^{((-2*I)*(a + b*x))}]/(b^4 * E^{((2*I)*a)}) * \text{Sec}[a] + ((c + d*x)^3 * \text{Sec}[a + b*x]^2)/(2*b) - (c^3 * \text{Sec}[a] * (\text{Cos}[a] * \text{Log}[\text{Cos}[a] * \text{Cos}[b*x] - \text{Sin}[a] * \text{Sin}[b*x]] + b*x * \text{Sin}[a]))/(b * (\text{Cos}[a]^2 + \text{Sin}[a]^2)) - (3*c*d^2 * \text{Sec}[a] * (\text{Cos}[a] * \text{Log}[\text{Cos}[a] * \text{Cos}[b*x] - \text{Sin}[a] * \text{Sin}[b*x]] + b*x * \text{Sin}[a]))/(b^3 * (\text{Cos}[a]^2 + \text{Sin}[a]^2)) + (c^3 * \text{Csc}[a] * (-(b*x * \text{Cos}[a]) + \text{Log}[\text{Cos}[b*x] * \text{Sin}[a] + \text{Cos}[a] * \text{Sin}[b*x]] * \text{Sin}[a]))/(b * (\text{Cos}[a]^2 + \text{Sin}[a]^2)) - (3*c^2*d * \text{Csc}[a] * ((b^2*x^2)/E^{(I * \text{ArcTan}[\text{Cot}[a]])}) - (\text{Cot}[a] * (I*b*x * (-\text{Pi} - 2 * \text{ArcTan}[\text{Cot}[a]]) - \text{Pi} * \text{Log}[1 + E^{((-2*I)*b*x)}] - 2*(b*x - \text{ArcTan}[\text{Cot}[a]]) * \text{Log}[1 - E^{((2*I)*(b*x - \text{ArcTan}[\text{Cot}[a])}]))) + \text{Pi} * \text{Log}[\text{Cos}[b*x]] - 2 * \text{ArcTan}[\text{Cot}[a]] * \text{Log}[\text{Sin}[b*x - \text{ArcTan}[\text{Cot}[a]]]] + I * \text{PolyLog}[2, E^{((2*I)*(b*x - \text{ArcTan}[\text{Cot}[a])}]))) / \text{Sqrt}[1 + \text{Cot}[a]^2]) * \text{Sec}[a]) / (2 * b^2 * \text{Sqrt}[\text{Csc}[a]^2 * (\text{Cos}[a]^2 + \text{Sin}[a]^2)]) - (3*d^3 * \text{Csc}[a] * ((b^2*x^2)/E^{(I * \text{ArcTan}[\text{Cot}[a]])}) - (\text{Cot}[a] * (I*b*x * (-\text{Pi} - 2 * \text{ArcTan}[\text{Cot}[a]]) - \text{Pi} * \text{Log}[1 + E^{((-2*I)*b*x)}] - 2*(b*x - \text{ArcTan}[\text{Cot}[a]]) * \text{Log}[1 - E^{((2*I)*(b*x - \text{ArcTan}[\text{Cot}[a])}]))) + \text{Pi} * \text{Log}[\text{Cos}[b*x]] - 2 * \text{ArcTan}[\text{Cot}[a]] * \text{Log}[\text{Sin}[b*x - \text{ArcTan}[\text{Cot}[a]]]] + I * \text{PolyLog}[2, E^{((2*I)*(b*x - \text{ArcTan}[\text{Cot}[a])}]))) / \text{Sqrt}[1 + \text{Cot}[a]^2]) * \text{Sec}[a]) / (2 * b^4 * \text{Sqrt}[\text{Csc}[a]^2 * (\text{Cos}[a]^2 + \text{Sin}[a]^2)]) - (3 * \text{Sec}[a] * \text{Sec}[a + b*x] * (c^2*d * \text{Sin}[b*x] + 2*c*d^2*x * \text{Sin}[b*x] + d^3*x^2 * \text{Sin}[b*x]))/(2*b^2) - (3*c^2*d * \text{Csc}[a] * \text{Sec}[a] * (b^2 * E^{(I * \text{ArcTan}[\text{Tan}[a])}] * x^2 + ((I*b*x * (-\text{Pi} + 2 * \text{ArcTan}[\text{Tan}[a])]) - \text{Pi} * \text{Log}[1 + E^{((-2*I)*b*x)}] - 2*(b*x + \text{ArcTan}[\text{Tan}[a]]) * \text{Log}[1 - E^{((2*I)*(b*x + \text{ArcTan}[\text{Tan}[a])}]))) + \text{Pi} * \text{Log}[\text{Cos}[b*x]] + 2 * \text{ArcTan}[\text{Tan}[a]] * \text{Log}[\text{Sin}[b*x + \text{ArcTan}[\text{Tan}[a]]]] + I * \text{PolyLog}[2, E^{((2*I)*(b*x + \text{ArcTan}[\text{Tan}[a])}]))) * \text{Tan}[a]) / \text{Sqrt}[1 + \text{Tan}[a]^2]) / (2*b^2 * \text{Sqrt}[\text{Sec}[a]^2 * (\text{Cos}[a]^2 + \text{Sin}[a]^2)]) \end{aligned}$$

Maple [B] time = 0.4, size = 1115, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x)

[Out]
$$\begin{aligned} & 3/b*c*d^2*\ln(\exp(I*(b*x+a))+1)*x^2+3/b*c*d^2*\ln(1-\exp(I*(b*x+a)))*x^2-1/b*c \\ & ^3*\ln(\exp(2*I*(b*x+a))+1)-3/2/b^3*c*d^2*\text{polylog}(3,-\exp(2*I*(b*x+a)))-3/2/b^3 \\ & *d^3*\text{polylog}(3,-\exp(2*I*(b*x+a)))*x+(2*b*d^3*x^3*\exp(2*I*(b*x+a))-3*I*d^3*x \\ & x^2*\exp(2*I*(b*x+a))+6*b*c*d^2*x^2*\exp(2*I*(b*x+a))-6*I*c*d^2*x*\exp(2*I*(b \\ & x+a))+6*b*c^2*d*x*\exp(2*I*(b*x+a))-3*I*c^2*d*\exp(2*I*(b*x+a))-3*I*d^3*x^2+2 \\ & *b*c^3*\exp(2*I*(b*x+a))-6*I*c*d^2*x-3*I*c^2*d)/b^2/(\exp(2*I*(b*x+a))+1)^2-3 \\ & *d^2/b^3*c*\ln(\exp(2*I*(b*x+a))+1)-3*d^3/b^3*\ln(\exp(2*I*(b*x+a))+1)*x+3*I*d^3 \\ & /b^2*x^2+3*I*d^3/b^4*a^2+3/2*I*d^3*\text{polylog}(2,-\exp(2*I*(b*x+a)))/b^4-6*I/b^2 \\ & *2*\text{polylog}(2,-\exp(I*(b*x+a)))*c*d^2*x-6*I/b^2*d^2*c*\text{polylog}(2,\exp(I*(b*x+a))) \\ &)*x-3*I/b^2*c^2*d*\text{polylog}(2,-\exp(I*(b*x+a)))-3*I/b^2*d^3*\text{polylog}(2,-\exp(I*(\\ & b*x+a)))*x^2+6*I*d^3*\text{polylog}(4,\exp(I*(b*x+a)))/b^4+3/2*I/b^2*c^2*d*\text{polylog}(\\ & 2,-\exp(2*I*(b*x+a)))-3/b*c^2*d*\ln(\exp(2*I*(b*x+a))+1)*x-3/b*c*d^2*\ln(\exp(2* \\ & I*(b*x+a))+1)*x^2-1/b*d^3*\ln(\exp(2*I*(b*x+a))+1)*x^3+3/2*I/b^2*d^3*\text{polylog}(\\ & 2,-\exp(2*I*(b*x+a)))*x^2-6/b^4*d^3*a*\ln(\exp(I*(b*x+a)))+6/b^3*d^2*c*\ln(\exp(\\ & I*(b*x+a)))+6*I*d^3/b^3*a*x+1/b*d^3*\ln(1-\exp(I*(b*x+a)))*x^3+1/b^4*d^3*\ln(1 \\ & -\exp(I*(b*x+a)))*a^3+1/b*d^3*\ln(\exp(I*(b*x+a))+1)*x^3+3/b^3*c*d^2*a^2*\ln(\exp \\ & (I*(b*x+a))-1)-3/b^2*c^2*d*a*\ln(\exp(I*(b*x+a))-1)-3/b^3*c*d^2*a^2*\ln(1-\exp \\ & (I*(b*x+a)))+3/b*c^2*d*\ln(\exp(I*(b*x+a))+1)*x+3/b*c^2*d*\ln(1-\exp(I*(b*x+a))) \\ &)*x+3/b^2*c^2*d*\ln(1-\exp(I*(b*x+a)))*a+6*I/b^4*d^3*\text{polylog}(4,-\exp(I*(b*x+a) \\ &))-3/4*I*d^3*\text{polylog}(4,-\exp(2*I*(b*x+a)))/b^4+3*I/b^2*c*d^2*\text{polylog}(2,-\exp(\\ & 2*I*(b*x+a)))*x+6/b^3*d^3*\text{polylog}(3,\exp(I*(b*x+a)))*x+6/b^3*d^3*\text{polylog}(3,- \\ & \exp(I*(b*x+a)))*x+6/b^3*c*d^2*\text{polylog}(3,\exp(I*(b*x+a)))+6/b^3*c*d^2*\text{polylog} \\ & (3,-\exp(I*(b*x+a)))-1/b^4*d^3*a^3*\ln(\exp(I*(b*x+a))-1)-3*I/b^2*d^3*\text{polylog}(\\ & 2,\exp(I*(b*x+a)))*x^2-3*I/b^2*c^2*d*\text{polylog}(2,\exp(I*(b*x+a)))+1/b*c^3*\ln(\exp \\ & (I*(b*x+a))-1)+1/b*c^3*\ln(\exp(I*(b*x+a))+1) \end{aligned}$$

Maxima [B] time = 7.39027, size = 6639, normalized size = 20.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")

[Out]
$$-1/2*(c^3*(1/(\sin(b*x + a)^2 - 1) + \log(\sin(b*x + a)^2 - 1) - \log(\sin(b*x + a)^2)) - 3*a*c^2*d*(1/(\sin(b*x + a)^2 - 1) + \log(\sin(b*x + a)^2 - 1) - \log(\sin(b*x + a)^2))/b + 3*a^2*c*d^2*(1/(\sin(b*x + a)^2 - 1) + \log(\sin(b*x + a)^2 - 1) - \log(\sin(b*x + a)^2))/b^2 - a^3*d^3*(1/(\sin(b*x + a)^2 - 1) + \log(\sin(b*x + a)^2 - 1) - \log(\sin(b*x + a)^2))/b^3 + 2*(18*b^2*c^2*d - 36*a*b*c*d^2 + 18*a^2*d^3 + (8*(b*x + a)^3*d^3 + 18*b*c*d^2 - 18*a*d^3 + 18*(b*c*d^2 - a*d^3))*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a) + 2*(4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + 4*(4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (8*I*(b*x + a)^3*d^3 + 18*I*b*c*d^2 - 18*I*a*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + (18*I*a^2 + 18*I)*d^3)*(b*x + a))*\sin(4*b*x + 4*a) + (16*I*(b*x + a)^3*d^3 + 36*I*b*c*d^2 - 36*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + (36*I*a^2 + 36*I)*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(2*b*x + 2*a), \cos(2*b*x + 2*a) + 1) - (6*(b*x + a)^3*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a) + 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + 12*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - (-6*I*(b*x + a)^3*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*a^2*d^3)*(b*x + a))*\sin(4*b*x + 4*a) - (-12*I*(b*x + a)^3*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a)^2 + (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2 - 36*I*a^2*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) + (6*(b*x + a)^3*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a) + 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + 12*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (6*I*(b*x + a)^3*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*a^2*d^3)*(b*x + a))*\sin(4*b*x + 4*a) + (12*I*(b*x + a)^3*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 36*I*a^2*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) - 18*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + (12*I*(b*x + a)^3*d^3 + 18*b^2*c^2*d - 36*a*b*c*d^2 + 18*a^2*d^3 + (36*I*b*c*d^2 - 18*(2*I*a + 1)*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 36*(2*I*a + 1)*b*c*d^2 + (36*I*a^2 + 36*a)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - (9*b^2*c^2*d - 18*a*b*c*d^2 + 12*(b*x + a)^2*d^3 + 9*(a^2 + 1)*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a) + 3*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^2*d^3 + 3*(a^2 + 1)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^2*d^3 + 3*(a^2 + 1)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) - (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 + (-9*I*a^2 - 9*I)*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a))*\sin(4*b*x + 4*a) - (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 24*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 18*I)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{dilog}(-e^{(2*I*b*x + 2*I*a)}) + (18*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b*x + a)^2*d^3 + 18*a^2*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a) + 18*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*$$

$$\begin{aligned} & \cos(4bx + 4a) + 36(b^2c^2d - 2abc^2d^2 + (bx + a)^2d^3 + a^2d^3 \\ & + 2(bc^2d^2 - ad^3)(bx + a))\cos(2bx + 2a) + (18Ib^2c^2d - 36I \\ & abc^2d^2 + 18I(bx + a)^2d^3 + 18Ia^2d^3 + (36Iabc^2d^2 - 36Ia^2d^3 \\ & 3)(bx + a))\sin(4bx + 4a) + (36Ib^2c^2d - 72Iaabc^2d^2 + 36I(b \\ & x + a)^2d^3 + 36Ia^2d^3 + (72Iabc^2d^2 - 72Ia^2d^3)(bx + a))\sin(2 \\ & bx + 2a))\operatorname{dilog}(-e^{Ibx + Ia})) + (18b^2c^2d - 36abc^2d^2 + 18(b \\ & x + a)^2d^3 + 18a^2d^3 + 36(bc^2d^2 - ad^3)(bx + a) + 18(b^2c^2d \\ & - 2abc^2d^2 + (bx + a)^2d^3 + a^2d^3 + 2(bc^2d^2 - ad^3)(bx + a)) \\ & \cos(4bx + 4a) + 36(b^2c^2d - 2abc^2d^2 + (bx + a)^2d^3 + a^2d^3 \\ & + 2(bc^2d^2 - ad^3)(bx + a))\cos(2bx + 2a) + (18Ib^2c^2d - 36I \\ & abc^2d^2 + 18I(bx + a)^2d^3 + 18Ia^2d^3 + (36Iabc^2d^2 - 36Ia^2d^3 \\ & 3)(bx + a))\sin(4bx + 4a) + (36Ib^2c^2d - 72Iaabc^2d^2 + 36I(b \\ & x + a)^2d^3 + 36Ia^2d^3 + (72Iabc^2d^2 - 72Ia^2d^3)(bx + a))\sin(\\ & 2bx + 2a))\operatorname{dilog}(e^{Ibx + Ia})) + (-4I(bx + a)^3d^3 - 9Iabc^2d^2 \\ & + 9Ia^2d^3 + (-9Iabc^2d^2 + 9Ia^2d^3)(bx + a)^2 + (-9Ib^2c^2d + 18 \\ & Iaabc^2d^2 + (-9Ia^2 - 9I)d^3)(bx + a) + (-4I(bx + a)^3d^3 - 9I \\ & abc^2d^2 + 9Ia^2d^3 + (-9Iabc^2d^2 + 9Ia^2d^3)(bx + a)^2 + (-9Ib^2c^2 \\ & c^2d + 18Iaabc^2d^2 + (-9Ia^2 - 9I)d^3)(bx + a))\cos(4bx + 4a) \\ & + (-8I(bx + a)^3d^3 - 18Iabc^2d^2 + 18Ia^2d^3 + (-18Iabc^2d^2 + 18I \\ & a^2d^3)(bx + a)^2 + (-18Ib^2c^2d + 36Iaabc^2d^2 + (-18Ia^2 - 18I \\ &)d^3)(bx + a))\cos(2bx + 2a) + (4(bx + a)^3d^3 + 9abc^2d^2 - 9a^2 \\ & d^3 + 9(bc^2d^2 - ad^3)(bx + a)^2 + 9(b^2c^2d - 2abc^2d^2 + (a^2 + \\ & 1)d^3)(bx + a))\sin(4bx + 4a) + 2(4(bx + a)^3d^3 + 9abc^2d^2 - 9a \\ & a^2d^3 + 9(bc^2d^2 - ad^3)(bx + a)^2 + 9(b^2c^2d - 2abc^2d^2 + (a^2 \\ & + 1)d^3)(bx + a))\sin(2bx + 2a))\log(\cos(2bx + 2a)^2 + \sin(2bx \\ & + 2a)^2 + 2\cos(2bx + 2a) + 1) + (3I(bx + a)^3d^3 + (9Iabc^2d^2 - \\ & 9Ia^2d^3)(bx + a)^2 + (9Ib^2c^2d - 18Iaabc^2d^2 + 9Ia^2d^3)(b \\ & x + a) + (3I(bx + a)^3d^3 + (9Iabc^2d^2 - 9Ia^2d^3)(bx + a)^2 + (9I \\ & b^2c^2d - 18Iaabc^2d^2 + 9Ia^2d^3)(bx + a))\cos(4bx + 4a) + (\\ & 6I(bx + a)^3d^3 + (18Iabc^2d^2 - 18Ia^2d^3)(bx + a)^2 + (18Ib^2c^2 \\ & ^2d - 36Iaabc^2d^2 + 18Ia^2d^3)(bx + a))\cos(2bx + 2a) - 3((bx \\ & + a)^3d^3 + 3(bc^2d^2 - ad^3)(bx + a)^2 + 3(b^2c^2d - 2abc^2d^2 \\ & + a^2d^3)(bx + a))\sin(4bx + 4a) - 6((bx + a)^3d^3 + 3(bc^2d^2 - \\ & ad^3)(bx + a)^2 + 3(b^2c^2d - 2abc^2d^2 + a^2d^3)(bx + a))\sin(2 \\ & bx + 2a))\log(\cos(bx + a)^2 + \sin(bx + a)^2 + 2\cos(bx + a) + 1) + (3 \\ & I(bx + a)^3d^3 + (9Iabc^2d^2 - 9Ia^2d^3)(bx + a)^2 + (9Ib^2c^2d \\ & - 18Iaabc^2d^2 + 9Ia^2d^3)(bx + a) + (3I(bx + a)^3d^3 + (9Iabc^2 \\ & c^2d^2 - 9Ia^2d^3)(bx + a)^2 + (9Ib^2c^2d - 18Iaabc^2d^2 + 9Ia^2d^3 \\ & d^3)(bx + a))\cos(4bx + 4a) + (6I(bx + a)^3d^3 + (18Iabc^2d^2 - 1 \\ & 8Ia^2d^3)(bx + a)^2 + (18Ib^2c^2d - 36Iaabc^2d^2 + 18Ia^2d^3)(b \\ & x + a))\cos(2bx + 2a) - 3((bx + a)^3d^3 + 3(bc^2d^2 - ad^3)(bx \\ & + a)^2 + 3(b^2c^2d - 2abc^2d^2 + a^2d^3)(bx + a))\sin(4bx + 4a) \\ & - 6((bx + a)^3d^3 + 3(bc^2d^2 - ad^3)(bx + a)^2 + 3(b^2c^2d - 2a \\ & abc^2d^2 + a^2d^3)(bx + a))\sin(2bx + 2a))\log(\cos(bx + a)^2 + \sin(b \\ & x + a)^2 - 2\cos(bx + a) + 1) + (6d^3\cos(4bx + 4a) + 12d^3\cos(2bx \\ & x + 2a) + 6Id^3\sin(4bx + 4a) + 12Id^3\sin(2bx + 2a) + 6d^3)\operatorname{poly} \\ & \log(4, -e^{(2Ibx + 2Ia)}) - (36d^3\cos(4bx + 4a) + 72d^3\cos(2bx \\ & x + 2a) + 36Id^3\sin(4bx + 4a) + 72Id^3\sin(2bx + 2a) + 36d^3)\operatorname{poly} \\ & \log(4, -e^{Ibx + Ia}) - (36d^3\cos(4bx + 4a) + 72d^3\cos(2bx \\ & + 2a) + 36Id^3\sin(4bx + 4a) + 72Id^3\sin(2bx + 2a) + 36d^3)\operatorname{poly} \\ & \log(4, e^{Ibx + Ia}) + (-9Iabc^2d^2 - 12I(bx + a)d^3 + 9Ia^2d^3 \\ & + (-9Iabc^2d^2 - 12I(bx + a)d^3 + 9Ia^2d^3)\cos(4bx + 4a) + (-18I \\ & abc^2d^2 - 24I(bx + a)d^3 + 18Ia^2d^3)\cos(2bx + 2a) + 3(3abc^2d^2 \\ & + 4(bx + a)d^3 - 3a^2d^3)\sin(4bx + 4a) + 6(3abc^2d^2 + 4(bx + a) \\ & d^3 - 3a^2d^3)\sin(2bx + 2a))\operatorname{polylog}(3, -e^{(2Ibx + 2Ia)}) + (36I \\ & abc^2d^2 + 36I(bx + a)d^3 - 36Ia^2d^3 + (36Iabc^2d^2 + 36I(bx + a) \\ & d^3 - 36Ia^2d^3)\cos(4bx + 4a) + (72Iabc^2d^2 + 72I(bx + a)d^3 - 7 \\ & 2Ia^2d^3)\cos(2bx + 2a) - 36(bc^2d^2 + (bx + a)d^3 - ad^3)\sin(4bx \\ & x + 4a) - 72(bc^2d^2 + (bx + a)d^3 - ad^3)\sin(2bx + 2a))\operatorname{polylog}(3 \end{aligned}$$

$$\begin{aligned} & , -e^{(I*b*x + I*a)} + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3 + (36 \\ & *I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3)*\cos(4*b*x + 4*a) + (72*I*b*c* \\ & d^2 + 72*I*(b*x + a)*d^3 - 72*I*a*d^3)*\cos(2*b*x + 2*a) - 36*(b*c*d^2 + (b* \\ & x + a)*d^3 - a*d^3)*\sin(4*b*x + 4*a) - 72*(b*c*d^2 + (b*x + a)*d^3 - a*d^3) \\ & *\sin(2*b*x + 2*a))*\text{polylog}(3, e^{(I*b*x + I*a)}) + (-18*I*(b*x + a)^2*d^3 + (\\ & -36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*\sin(4*b*x + 4*a) - (12*(b*x + a)^3*d \\ & ^3 - 18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*a^2*d^3 + (36*b*c*d^2 - (36*a - \\ & 18*I)*d^3)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a - 36*I)*b*c*d^2 + 36*(a^2 - \\ & I*a)*d^3)*(b*x + a))*\sin(2*b*x + 2*a))/(-6*I*b^3*\cos(4*b*x + 4*a) - 12*I*b \\ & ^3*\cos(2*b*x + 2*a) + 6*b^3*\sin(4*b*x + 4*a) + 12*b^3*\sin(2*b*x + 2*a) - 6* \\ & I*b^3))/b \end{aligned}$$

Fricas [C] time = 1.41763, size = 5605, normalized size = 17.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")

[Out] $\frac{1}{2}*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 + 6*I*d^3*\cos(b*x + a)^2*\text{polylog}(4, \cos(b*x + a) + I*\sin(b*x + a)) - 6*I*d^3*\cos(b*x + a)^2*\text{polylog}(4, \cos(b*x + a) - I*\sin(b*x + a)) + 6*I*d^3*\cos(b*x + a)^2*\text{polylog}(4, I*\cos(b*x + a) + \sin(b*x + a)) - 6*I*d^3*\cos(b*x + a)^2*\text{polylog}(4, I*\cos(b*x + a) - \sin(b*x + a)) - 6*I*d^3*\cos(b*x + a)^2*\text{polylog}(4, -I*\cos(b*x + a) + \sin(b*x + a)) + 6*I*d^3*\cos(b*x + a)^2*\text{polylog}(4, -I*\cos(b*x + a) - \sin(b*x + a)) - 6*I*d^3*\cos(b*x + a)^2*\text{polylog}(4, -\cos(b*x + a) + I*\sin(b*x + a)) + 6*I*d^3*\cos(b*x + a)^2*\text{polylog}(4, -\cos(b*x + a) - I*\sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*\cos(b*x + a)^2*\text{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*\cos(b*x + a)^2*\text{dilog}(\cos(b*x + a) - I*\sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 3*I*d^3)*\cos(b*x + a)^2*\text{dilog}(I*\cos(b*x + a) + \sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 3*I*d^3)*\cos(b*x + a)^2*\text{dilog}(I*\cos(b*x + a) - \sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 3*I*d^3)*\cos(b*x + a)^2*\text{dilog}(-I*\cos(b*x + a) + \sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 3*I*d^3)*\cos(b*x + a)^2*\text{dilog}(-I*\cos(b*x + a) - \sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*\cos(b*x + a)^2*\text{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*\cos(b*x + a)^2*\text{dilog}(-\cos(b*x + a) - I*\sin(b*x + a)) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\cos(b*x + a)^2*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3)*\cos(b*x + a)^2*\log(\cos(b*x + a) + I*\sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\cos(b*x + a)^2*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3)*\cos(b*x + a)^2*\log(\cos(b*x + a) - I*\sin(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d^3)*x)*\cos(b*x + a)^2*\log(I*\cos(b*x + a) + \sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d^3)*x)*\cos(b*x + a)^2*\log(I*\cos(b*x + a) - \sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d^3)*x)*\cos(b*x + a)^2*\log(-I*\cos(b*x + a) + \sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d^3)*x)*\cos(b*x + a)^2*\log(-I*\cos(b*x + a) - \sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\cos(b*x + a)^2*\log(-1/2*\cos(b*x + a) + 1/2*I*\sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d +$


```

3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(b
*x + a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c
^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*
x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*
d^3)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^3*d^3*x^3
+ 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3
)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b
^2*c^2*d + 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3)*cos(b*x + a)^2*log(-cos(b
*x + a) - I*sin(b*x + a) + I) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylo
g(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*
polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x +
a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos
(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c*d^
2)*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x +
b*c*d^2)*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 6*(b*
d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, -cos(b*x + a) + I*sin(b*x + a))
+ 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, -cos(b*x + a) - I*sin(b*x
+ a)) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x +
a))/(b^4*cos(b*x + a)^2)

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*csc(b*x+a)*sec(b*x+a)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \csc(bx + a) \sec(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")

[Out] integrate((d*x + c)^3*csc(b*x + a)*sec(b*x + a)^3, x)

3.312 $\int (c + dx)^2 \csc(a + bx) \sec^3(a + bx) dx$

Optimal. Leaf size=201

$$\frac{id(c + dx)\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^2} - \frac{id(c + dx)\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^2} - \frac{d^2\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} + \frac{d^2\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3}$$

```
[Out] (c*d*x)/b + (d^2*x^2)/(2*b) - (2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/
b - (d^2*Log[Cos[a + b*x]])/b^3 + (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a +
b*x))])/b^2 - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d^2*Po
lyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (d^2*PolyLog[3, E^((2*I)*(a + b*x
))])/(2*b^3) - (d*(c + d*x)*Tan[a + b*x])/b^2 + ((c + d*x)^2*Tan[a + b*x]^2
)/(2*b)
```

Rubi [A] time = 0.413906, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {2620, 14, 4420, 6741, 12, 6742, 2551, 4183, 2531, 2282, 6589, 3720, 3475}

$$\frac{id(c + dx)\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^2} - \frac{id(c + dx)\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^2} - \frac{d^2\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} + \frac{d^2\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^3, x]
```

```
[Out] (c*d*x)/b + (d^2*x^2)/(2*b) - (2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/
b - (d^2*Log[Cos[a + b*x]])/b^3 + (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a +
b*x))])/b^2 - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d^2*Po
lyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (d^2*PolyLog[3, E^((2*I)*(a + b*x
))])/(2*b^3) - (d*(c + d*x)*Tan[a + b*x])/b^2 + ((c + d*x)^2*Tan[a + b*x]^2
)/(2*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^(m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v !=
= u]
```

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rule 2551

Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[((a + b*x)^(m + 1)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a + b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunctionFreeQ[u, x] && NeQ[m, -1]

Rule 4183

Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/b*c*n*Log[F], x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rule 3720

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (c + dx)^2 \csc(a + bx) \sec^3(a + bx) dx &= \frac{(c + dx)^2 \log(\tan(a + bx))}{b} + \frac{(c + dx)^2 \tan^2(a + bx)}{2b} - (2d) \int (c + dx) \left(\frac{\log(\tan(a + bx))}{b} + \frac{\tan^2(a + bx)}{2b} \right) dx \\
&= \frac{(c + dx)^2 \log(\tan(a + bx))}{b} + \frac{(c + dx)^2 \tan^2(a + bx)}{2b} - (2d) \int \frac{(c + dx) (2 \log(\tan(a + bx)) + \tan^2(a + bx))}{b} dx \\
&= \frac{(c + dx)^2 \log(\tan(a + bx))}{b} + \frac{(c + dx)^2 \tan^2(a + bx)}{2b} - \frac{d \int (c + dx) (2 \log(\tan(a + bx)) + \tan^2(a + bx)) dx}{b} \\
&= \frac{(c + dx)^2 \log(\tan(a + bx))}{b} + \frac{(c + dx)^2 \tan^2(a + bx)}{2b} - \frac{d \int (2(c + dx) \log(\tan(a + bx)) + (c + dx) \tan^2(a + bx)) dx}{b} \\
&= \frac{(c + dx)^2 \log(\tan(a + bx))}{b} + \frac{(c + dx)^2 \tan^2(a + bx)}{2b} - \frac{d \int (c + dx) \tan^2(a + bx) dx}{b} \\
&= -\frac{d(c + dx) \tan(a + bx)}{b^2} + \frac{(c + dx)^2 \tan^2(a + bx)}{2b} + \frac{\int 2b(c + dx)^2 \csc(2a + 2bx) dx}{b} \\
&= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{d^2 \log(\cos(a + bx))}{b^3} - \frac{d(c + dx) \tan(a + bx)}{b^2} + \frac{(c + dx)^2 \tan^2(a + bx)}{2b} \\
&= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{2(c + dx)^2 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{d^2 \log(\cos(a + bx))}{b^3} - \frac{d(c + dx) \tan(a + bx)}{b^2} \\
&= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{2(c + dx)^2 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{d^2 \log(\cos(a + bx))}{b^3} + \frac{id(c + dx) \tan(a + bx)}{b^2} \\
&= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{2(c + dx)^2 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{d^2 \log(\cos(a + bx))}{b^3} + \frac{id(c + dx) \tan(a + bx)}{b^2} \\
&= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{2(c + dx)^2 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{d^2 \log(\cos(a + bx))}{b^3} + \frac{id(c + dx) \tan(a + bx)}{b^2}
\end{aligned}$$

Mathematica [B] time = 6.74202, size = 875, normalized size = 4.35

$$\frac{\sec(a)(\cos(a) \log(\cos(a) \cos(bx) - \sin(a) \sin(bx)) + bx \sin(a))c^2}{b(\cos^2(a) + \sin^2(a))} + \frac{\csc(a)(\log(\cos(bx) \sin(a) + \cos(a) \sin(bx)) \sin(a) - \cos(a) \log(\cos(a) \cos(bx) - \sin(a) \sin(bx)))}{b(\cos^2(a) + \sin^2(a))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^3,x]

[Out] $-(d^2 E^{I a} \text{Csc}[a] ((2 b^3 x^3)/E^{(2 I) a} + (3 I) b^2 (1 - E^{(-2 I) a}) x^2 \text{Log}[1 - E^{(-I) (a + b x)}]) + (3 I) b^2 (1 - E^{(-2 I) a}) x^2 \text{Log}[1 + E^{(-I) (a + b x)}]) - (6 (-1 + E^{(2 I) a}) (b x \text{PolyLog}[2, -E^{(-I) (a + b x)}]) - I \text{PolyLog}[3, -E^{(-I) (a + b x)}]))/E^{(2 I) a} - (6 (-1 + E^{(2 I) a}) (b x \text{PolyLog}[2, E^{(-I) (a + b x)}]) - I \text{PolyLog}[3, E^{(-I) (a + b x)}]))/E^{(2 I) a})/(6 b^3) + (x (3 c^2 + 3 c d x + d^2 x^2) \text{Csc}[a] \text{Sec}[a])/3 - ((I/12) d^2 (2 b^2 x^2 (2 b x - (3 I) (1 + E^{(2 I) a})) \text{Log}[1 + E^{(-2 I) (a + b x)}]) + 6 b (1 + E^{(2 I) a}) x \text{PolyLog}[2, -E^{(-2 I) (a + b x)}]) - (3 I) (1 + E^{(2 I) a}) \text{PolyLog}[3, -E^{(-2 I) (a + b x)}]) \text{Sec}[a])/(b^3 E^{I a}) + ((c + d x)^2 \text{Sec}[a + b x]^2)/(2 b) - (c^2 \text{Sec}[a] (\text{Cos}[a] \text{Log}[\text{Cos}[a] \text{Cos}[b x] - \text{Sin}[a] \text{Sin}[b x]] + b x \text{Sin}[a]))/(b (\text{Cos}[a]^2 + \text{Sin}[a]^2)) - (d^2 \text{Sec}[a] (\text{Cos}[a] \text{Log}[\text{Cos}[a] \text{Cos}[b x] - \text{Sin}[a] \text{Sin}[b x]] + b x \text{Sin}[a]))/(b^3 (\text{Cos}[a]^2 + \text{Sin}[a]^2)) + (c^2 \text{Csc}[a] (-(b x \text{Cos}[a]) + \text{Log}[\text{Cos}[b x] \text{Sin}[a] + \text{Cos}[a] \text{Sin}[b x]] \text{Sin}[a]))/(b (\text{Cos}[a]^2 + \text{Sin}[a]^2)) - (c d \text{Csc}[a] ((b^2 x^2)/E^{I \text{ArcTan}[\text{Cot}[a]}) - (\text{Cot}[a] (I b x (-\text{Pi} - 2 \text{ArcTan}[\text{Cot}[a]]) - \text{Pi} \text{Log}[1 + E^{(-2 I) b x}] - 2 (b x - \text{ArcTan}[\text{Cot}[a]]) \text{Log}[1 - E^{(2 I) b x}] -$

$$\begin{aligned} & \text{ArcTan}[\text{Cot}[a]]) + \text{Pi} * \text{Log}[\text{Cos}[b*x]] - 2 * \text{ArcTan}[\text{Cot}[a]] * \text{Log}[\text{Sin}[b*x - \text{ArcTan}[\text{Cot}[a]]]] \\ & + \text{I} * \text{PolyLog}[2, \text{E}^{\text{((2*I)*(b*x - \text{ArcTan}[\text{Cot}[a])])}})] / \text{Sqrt}[1 + \text{Cot}[a]^2] * \text{Sec}[a] / (b^2 * \text{Sqrt}[\text{Csc}[a]^2 * (\text{Cos}[a]^2 + \text{Sin}[a]^2)]) + (\text{Sec}[a] * \text{Sec}[a + b*x] * \\ & (-c*d*\text{Sin}[b*x]) - d^2*x*\text{Sin}[b*x]) / b^2 - (c*d*\text{Csc}[a] * \text{Sec}[a] * (b^2 * \text{E}^{\text{(I*ArcTan}[\text{Tan}[a]] * x^2 + ((\text{I}*b*x*(-\text{Pi} + 2*\text{ArcTan}[\text{Tan}[a])) - \text{Pi} * \text{Log}[1 + \text{E}^{\text{((-2*I)*b*x])}}] - 2*(b*x + \text{ArcTan}[\text{Tan}[a]]) * \text{Log}[1 - \text{E}^{\text{((2*I)*(b*x + \text{ArcTan}[\text{Tan}[a])])}}] + \text{Pi} * \text{Log}[\text{Cos}[b*x]] + 2 * \text{ArcTan}[\text{Tan}[a]] * \text{Log}[\text{Sin}[b*x + \text{ArcTan}[\text{Tan}[a]]]] + \text{I} * \text{PolyLog}[2, \text{E}^{\text{((2*I)*(b*x + \text{ArcTan}[\text{Tan}[a])])}}] * \text{Tan}[a]) / \text{Sqrt}[1 + \text{Tan}[a]^2]) / (b^2 * \text{Sqrt}[\text{Sec}[a]^2 * (\text{Cos}[a]^2 + \text{Sin}[a]^2)]) \end{aligned}$$

Maple [B] time = 0.339, size = 614, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x)

[Out]
$$\begin{aligned} & -1/2*d^2*\text{polylog}(3, -\exp(2*I*(b*x+a))) / b^3 - 1/b*d^2*\ln(\exp(2*I*(b*x+a))+1)*x^2 - 1/b^3*d^2*\ln(1-\exp(I*(b*x+a)))*a^2 + 1/b*d^2*\ln(\exp(I*(b*x+a))+1)*x^2 + 1/b*d^2*\ln(1-\exp(I*(b*x+a)))*x^2 - 1/b*c^2*\ln(\exp(2*I*(b*x+a))+1) + I/b^2*c*d*\text{polylog}(2, -\exp(2*I*(b*x+a))) - 2*I/b^2*d^2*\text{polylog}(2, \exp(I*(b*x+a)))*x - 2*I/b^2*d^2*\text{polylog}(2, -\exp(I*(b*x+a)))*x - 2*I/b^2*c*d*\text{polylog}(2, \exp(I*(b*x+a))) - 2*I/b^2*c*d*\text{polylog}(2, -\exp(I*(b*x+a)))+ 2/b^3*d^2*\ln(\exp(I*(b*x+a)))+ 2/b*c*d*\ln(\exp(I*(b*x+a))+1)*x + 1/b*c^2*\ln(\exp(I*(b*x+a))+1) + 1/b*c^2*\ln(\exp(I*(b*x+a))-1) + 2*d^2*\text{polylog}(3, -\exp(I*(b*x+a))) / b^3 + 2*d^2*\text{polylog}(3, \exp(I*(b*x+a))) / b^3 - 1/b^3*d^2*\ln(\exp(2*I*(b*x+a))+1) + I/b^2*d^2*\text{polylog}(2, -\exp(2*I*(b*x+a)))*x - 2/b*c*d*\ln(\exp(2*I*(b*x+a))+1)*x + 2/b*c*d*\ln(1-\exp(I*(b*x+a)))*x + 2/b^2*c*d*\ln(1-\exp(I*(b*x+a)))*a^2 + 2*(b*d^2*x^2*\exp(2*I*(b*x+a)) - I*d^2*x*\exp(2*I*(b*x+a)) + 2*b*c*d*x*\exp(2*I*(b*x+a)) - I*c*d*\exp(2*I*(b*x+a)) + b*c^2*\exp(2*I*(b*x+a)) - I*d^2*x - I*d*c) / b^2 / (\exp(2*I*(b*x+a))+1)^2 + 1/b^3*d^2*a^2*\ln(\exp(I*(b*x+a))-1) - 2/b^2*c*d*a*\ln(\exp(I*(b*x+a))-1) \end{aligned}$$

Maxima [B] time = 2.8488, size = 3297, normalized size = 16.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/2*(c^2*(1/(\text{sin}(b*x + a)^2 - 1) + \log(\text{sin}(b*x + a)^2 - 1) - \log(\text{sin}(b*x + a)^2)) - 2*a*c*d*(1/(\text{sin}(b*x + a)^2 - 1) + \log(\text{sin}(b*x + a)^2 - 1) - \log(\text{sin}(b*x + a)^2)) / b + a^2*d^2*(1/(\text{sin}(b*x + a)^2 - 1) + \log(\text{sin}(b*x + a)^2 - 1) - \log(\text{sin}(b*x + a)^2)) / b^2 - 2*(4*(b*x + a)*d^2*\cos(4*b*x + 4*a) + 4*I*(b*x + a)*d^2*\sin(4*b*x + 4*a) - 4*b*c*d + 4*a*d^2 - (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2)*\cos(4*b*x + 4*a) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2)*\cos(2*b*x + 2*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a) + 2*I*d^2)*\sin(4*b*x + 4*a) + (4*I*(b*x + a)^2*d^2 + (8*I*b*c*d - 8*I*a*d^2)*(b*x + a) + 4*I*d^2)*\sin(2*b*x + 2*a)) * \arctan2(\text{sin}(2*b*x + 2*a), \cos(2*b*x + 2*a) + 1) + (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\cos(4*b*x + 4*a) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\cos(2*b*x + 2*a) - (- \end{aligned}$$

$$\begin{aligned}
& 2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a))*\sin(4*b*x + 4*a) \\
& - (-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a))*\sin(2*b*x + 2*a)) \\
& *\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) - (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) \\
& + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\cos(4*b*x + 4*a) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\cos(2*b*x + 2*a) \\
& + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*\sin(4*b*x + 4*a) + (4*I*(b*x + a)^2*d^2 + (8*I*b*c*d - 8*I*a*d^2)*(b*x + a))*\sin(2*b*x + 2*a)) \\
& *\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) - (4*I*(b*x + a)^2*d^2 + 4*b*c*d - 4*a*d^2 + (8*I*b*c*d - 4*(2*I*a + 1)*d^2)*(b*x + a))*\cos(2*b*x + 2*a) \\
& + (2*b*c*d + 2*(b*x + a)*d^2 - 2*a*d^2 + 2*(b*c*d + (b*x + a)*d^2 - a*d^2))*\cos(4*b*x + 4*a) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2))*\cos(2*b*x + 2*a) \\
& - (-2*I*b*c*d - 2*I*(b*x + a)*d^2 + 2*I*a*d^2))*\sin(4*b*x + 4*a) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2))*\sin(2*b*x + 2*a))*\operatorname{dilog}(-e^{(2*I*b*x + 2*I*a)}) \\
& - (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2))*\cos(4*b*x + 4*a) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2))*\cos(2*b*x + 2*a) \\
& + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2))*\sin(4*b*x + 4*a) + (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2))*\sin(2*b*x + 2*a))*\operatorname{dilog}(-e^{(I*b*x + I*a)}) \\
& - (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2))*\cos(4*b*x + 4*a) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2))*\cos(2*b*x + 2*a) \\
& + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2))*\sin(4*b*x + 4*a) + (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2))*\sin(2*b*x + 2*a))*\operatorname{dilog}(e^{(I*b*x + I*a)}) \\
& - (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) - I*d^2 + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) - I*d^2))*\cos(4*b*x + 4*a) \\
& + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a) - 2*I*d^2))*\cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2))*\sin(4*b*x + 4*a) \\
& + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2))*\sin(2*b*x + 2*a))*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) \\
& - (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a) + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a))*\cos(4*b*x + 4*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*\cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\sin(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) \\
& - (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a) + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a))*\cos(4*b*x + 4*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*\cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\sin(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) \\
& - (-I*d^2*\cos(4*b*x + 4*a) - 2*I*d^2*\cos(2*b*x + 2*a) + d^2*\sin(4*b*x + 4*a) + 2*d^2*\sin(2*b*x + 2*a) - I*d^2))*\operatorname{polylog}(3, -e^{(2*I*b*x + 2*I*a)}) - (4*I*d^2*\cos(4*b*x + 4*a) + 8*I*d^2*\cos(2*b*x + 2*a) - 4*d^2*\sin(4*b*x + 4*a) - 8*d^2*\sin(2*b*x + 2*a) + 4*I*d^2))*\operatorname{polylog}(3, -e^{(I*b*x + I*a)}) - (4*I*d^2*\cos(4*b*x + 4*a) + 8*I*d^2*\cos(2*b*x + 2*a) - 4*d^2*\sin(4*b*x + 4*a) - 8*d^2*\sin(2*b*x + 2*a) + 4*I*d^2))*\operatorname{polylog}(3, e^{(I*b*x + I*a)}) \\
& + (4*(b*x + a)^2*d^2 - 4*I*b*c*d + 4*I*a*d^2 + (8*b*c*d - (8*a - 4*I)*d^2)*(b*x + a))*\sin(2*b*x + 2*a))/(-2*I*b^2*\cos(4*b*x + 4*a) - 4*I*b^2*\cos(2*b*x + 2*a) + 2*b^2*\sin(4*b*x + 4*a) + 4*b^2*\sin(2*b*x + 2*a) - 2*I*b^2))/b
\end{aligned}$$

Fricas [C] time = 0.963077, size = 3629, normalized size = 18.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")

[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*d^2*cos(b*x + a)^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*cos(b*x + a)^2*polylog(3, cos(b*x + a) - I*sin

```
(b*x + a)) - 2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a))
- 2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 2*d^2*cos
os(b*x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*cos(b*x +
a)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*cos(b*x + a)^2*poly
log(3, -cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*cos(b*x + a)^2*polylog(3, -c
os(b*x + a) - I*sin(b*x + a)) + b^2*c^2 + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*
x + a)^2*dilog(cos(b*x + a) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*c
os(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*
c*d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + (2*I*b*d^2*x + 2
*I*b*c*d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + (2*I*b*d^2*x
+ 2*I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-2*I
*b*d^2*x - 2*I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a))
+ (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2*dilog(-cos(b*x + a) + I*sin(b*x
+ a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x + a)^2*dilog(-cos(b*x + a) - I*s
in(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a)^2*log(cos
(b*x + a) + I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos
(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b^
2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(
b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x +
a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x +
2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1)
- (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(-I*
cos(b*x + a) + sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d -
a^2*d^2)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*c^2
- 2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*
x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2*log(-1/2*cos
(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*
c*d - a^2*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^
2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin
(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x
+ a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + (a^
2 + 1)*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 2*(b*d
^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3*cos(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*csc(b*x+a)*sec(b*x+a)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \csc(bx + a) \sec(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")

[Out] integrate((d*x + c)^2*csc(b*x + a)*sec(b*x + a)^3, x)

3.313 $\int (c + dx) \csc(a + bx) \sec^3(a + bx) dx$

Optimal. Leaf size=139

$$\frac{idPolyLog(2, -e^{2i(a+bx)})}{2b^2} - \frac{idPolyLog(2, e^{2i(a+bx)})}{2b^2} - \frac{d \tan(a + bx)}{2b^2} + \frac{c \tan^2(a + bx)}{2b} + \frac{c \log(\tan(a + bx))}{b} + \frac{dx \tan^2(a + bx)}{2b}$$

[Out] (d*x)/(2*b) - (2*d*x*ArcTanh[E^((2*I)*a + (2*I)*b*x))]/b + (c*Log[Tan[a + b*x]])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d*Tan[a + b*x])/(2*b^2) + (c*Tan[a + b*x]^2)/(2*b) + (d*x*Tan[a + b*x]^2)/(2*b)

Rubi [A] time = 0.135266, antiderivative size = 141, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {2620, 14, 4420, 2548, 12, 4183, 2279, 2391, 3473, 8}

$$\frac{idPolyLog(2, -e^{2i(a+bx)})}{2b^2} - \frac{idPolyLog(2, e^{2i(a+bx)})}{2b^2} - \frac{d \tan(a + bx)}{2b^2} + \frac{(c + dx) \tan^2(a + bx)}{2b} + \frac{(c + dx) \log(\tan(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^3,x]

[Out] (d*x)/(2*b) - (2*d*x*ArcTanh[E^((2*I)*(a + b*x))])/b - (d*x*Log[Tan[a + b*x]])/b + ((c + d*x)*Log[Tan[a + b*x]])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d*Tan[a + b*x])/(2*b^2) + ((c + d*x)*Tan[a + b*x]^2)/(2*b)

Rule 2620

Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Dist[1/f, Subst[Int[(1 + x^2)^(m + n)/2 - 1/x^m, x], x, Tan[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 4420

Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]

Rule 2548

Int[Log[u_], x_Symbol] := Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u, x])/u, x], x] /; InverseFunctionFreeQ[u, x]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_.)*(v_)] /; FreeQ[b, x]

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_.))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int (c + dx) \csc(a + bx) \sec^3(a + bx) dx &= \frac{(c + dx) \log(\tan(a + bx))}{b} + \frac{(c + dx) \tan^2(a + bx)}{2b} - d \int \left(\frac{\log(\tan(a + bx))}{b} \right. \\ &= \frac{(c + dx) \log(\tan(a + bx))}{b} + \frac{(c + dx) \tan^2(a + bx)}{2b} - \frac{d \int \tan^2(a + bx) dx}{2b} - \frac{d \int \tan(a + bx) dx}{2b} \\ &= -\frac{dx \log(\tan(a + bx))}{b} + \frac{(c + dx) \log(\tan(a + bx))}{b} - \frac{d \tan(a + bx)}{2b^2} + \frac{(c + dx) \tan(a + bx)}{2b} \\ &= \frac{dx}{2b} - \frac{dx \log(\tan(a + bx))}{b} + \frac{(c + dx) \log(\tan(a + bx))}{b} - \frac{d \tan(a + bx)}{2b^2} + \frac{(c + dx) \tan(a + bx)}{2b} \\ &= \frac{dx}{2b} - \frac{2dx \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{dx \log(\tan(a + bx))}{b} + \frac{(c + dx) \log(\tan(a + bx))}{b} \\ &= \frac{dx}{2b} - \frac{2dx \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{dx \log(\tan(a + bx))}{b} + \frac{(c + dx) \log(\tan(a + bx))}{b} \\ &= \frac{dx}{2b} - \frac{2dx \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{dx \log(\tan(a + bx))}{b} + \frac{(c + dx) \log(\tan(a + bx))}{b} \end{aligned}$$

Mathematica [A] time = 0.565265, size = 212, normalized size = 1.53

$$\frac{d \left(\frac{1}{2} i \text{PolyLog} \left(2, -e^{2i(a+bx)} \right) + \frac{1}{2} i (a + bx)^2 - (a + bx) \log \left(1 + e^{2i(a+bx)} \right) \right)}{b^2} + \frac{d \left((a + bx) \log \left(1 - e^{2i(a+bx)} \right) - \frac{1}{2} i ((a + bx)^2) \right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] (a*d*Log[Cos[a + b*x]]/b^2 - (a*d*Log[Sin[a + b*x]]/b^2 + (d*((I/2)*(a +
b*x)^2 - (a + b*x)*Log[1 + E^((2*I)*(a + b*x))] + (I/2)*PolyLog[2, -E^((2*I
)*(a + b*x))]))/b^2 + (d*((a + b*x)*Log[1 - E^((2*I)*(a + b*x))] - (I/2)*((
a + b*x)^2 + PolyLog[2, E^((2*I)*(a + b*x))])))/b^2 + (d*x*Sec[a + b*x]^2)/
(2*b) - (c*(2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]] - Sec[a + b*x]^2))/(2
*b) - (d*Tan[a + b*x])/(2*b^2)
```

Maple [B] time = 0.158, size = 270, normalized size = 1.9

$$\frac{2bdxe^{2i(bx+a)} + 2bce^{2i(bx+a)} - ide^{2i(bx+a)} - id}{b^2(e^{2i(bx+a)} + 1)^2} - \frac{c \ln(e^{2i(bx+a)} + 1)}{b} + \frac{c \ln(e^{i(bx+a)} - 1)}{b} + \frac{c \ln(e^{i(bx+a)} + 1)}{b} + \frac{d \ln(1 - e^{i(bx+a)})}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x)
```

```
[Out] (2*b*d*x*exp(2*I*(b*x+a))+2*b*c*exp(2*I*(b*x+a))-I*d*exp(2*I*(b*x+a))-I*d)/
b^2/(exp(2*I*(b*x+a))+1)^2-1/b*c*ln(exp(2*I*(b*x+a))+1)+1/b*c*ln(exp(I*(b*x
+a))-1)+1/b*c*ln(exp(I*(b*x+a))+1)+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(
1-exp(I*(b*x+a)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2-1/b*d*ln(exp(2*I*(b*x
+a))+1)*x+1/2*I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+1/b*d*ln(exp(I*(b*x+a))+
1)*x-I*d*polylog(2,-exp(I*(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

Maxima [B] time = 2.22434, size = 1397, normalized size = 10.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -((2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*a) + 4*(b*d*x + b*c)*cos
(2*b*x + 2*a) + (2*I*b*d*x + 2*I*b*c)*sin(4*b*x + 4*a) + (4*I*b*d*x + 4*I*b
*c)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - (2*
b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*a) + 4*(b*d*x + b*c)*cos(2*b*
x + 2*a) - (-2*I*b*d*x - 2*I*b*c)*sin(4*b*x + 4*a) - (-4*I*b*d*x - 4*I*b*c)
*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*b*c*cos(4*b
*x + 4*a) + 4*b*c*cos(2*b*x + 2*a) + 2*I*b*c*sin(4*b*x + 4*a) + 4*I*b*c*sin
(2*b*x + 2*a) + 2*b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*b*d*x*c
os(4*b*x + 4*a) + 4*b*d*x*cos(2*b*x + 2*a) + 2*I*b*d*x*sin(4*b*x + 4*a) + 4
*I*b*d*x*sin(2*b*x + 2*a) + 2*b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) +
1) + (4*I*b*d*x + 4*I*b*c + 2*d)*cos(2*b*x + 2*a) - (d*cos(4*b*x + 4*a) + 2
*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) + 2*I*d*sin(2*b*x + 2*a) + d)*di
log(-e^(2*I*b*x + 2*I*a)) + (2*d*cos(4*b*x + 4*a) + 4*d*cos(2*b*x + 2*a) +
2*I*d*sin(4*b*x + 4*a) + 4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(-e^(I*b*x + I*
a)) + (2*d*cos(4*b*x + 4*a) + 4*d*cos(2*b*x + 2*a) + 2*I*d*sin(4*b*x + 4*a)
+ 4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c
+ (-I*b*d*x - I*b*c)*cos(4*b*x + 4*a) + (-2*I*b*d*x - 2*I*b*c)*cos(2*b*x +
2*a) + (b*d*x + b*c)*sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*
log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (I*
b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*
cos(2*b*x + 2*a) - (b*d*x + b*c)*sin(4*b*x + 4*a) - 2*(b*d*x + b*c)*sin(2*b
*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*b
*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*c
```

$$\cos(2bx + 2a) - (bdx + bc)\sin(4bx + 4a) - 2(bdx + bc)\sin(2bx + 2a) \log(\cos(bx + a)^2 + \sin(bx + a)^2 - 2\cos(bx + a) + 1) - 2(2bdx + 2bc - Id)\sin(2bx + 2a) + 2d / (-2Ib^2\cos(4bx + 4a) - 4Ib^2\cos(2bx + 2a) + 2b^2\sin(4bx + 4a) + 4b^2\sin(2bx + 2a) - 2Ib^2)$$

Fricas [B] time = 0.751633, size = 2072, normalized size = 14.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((dx+c)*csc(bx+a)*sec(bx+a)^3,x, algorithm="fricas")

[Out] $\frac{1}{2}(-Id\cos(bx + a)^2\operatorname{dilog}(\cos(bx + a) + I\sin(bx + a)) + Id\cos(bx + a)^2\operatorname{dilog}(\cos(bx + a) - I\sin(bx + a)) - Id\cos(bx + a)^2\operatorname{dilog}(I\cos(bx + a) + \sin(bx + a)) + Id\cos(bx + a)^2\operatorname{dilog}(I\cos(bx + a) - \sin(bx + a)) + Id\cos(bx + a)^2\operatorname{dilog}(-I\cos(bx + a) + \sin(bx + a)) - Id\cos(bx + a)^2\operatorname{dilog}(-I\cos(bx + a) - \sin(bx + a)) + Id\cos(bx + a)^2\operatorname{dilog}(-\cos(bx + a) + I\sin(bx + a)) - Id\cos(bx + a)^2\operatorname{dilog}(-\cos(bx + a) - I\sin(bx + a)) + (bdx + bc)\cos(bx + a)^2\log(\cos(bx + a) + I\sin(bx + a) + 1) - (bc - ad)\cos(bx + a)^2\log(\cos(bx + a) + I\sin(bx + a) + I) + (bdx + bc)\cos(bx + a)^2\log(\cos(bx + a) - I\sin(bx + a) + 1) - (bc - ad)\cos(bx + a)^2\log(\cos(bx + a) - I\sin(bx + a) + I) - (bdx + ad)\cos(bx + a)^2\log(I\cos(bx + a) + \sin(bx + a) + 1) - (bdx + ad)\cos(bx + a)^2\log(I\cos(bx + a) - \sin(bx + a) + 1) - (bdx + ad)\cos(bx + a)^2\log(-I\cos(bx + a) + \sin(bx + a) + 1) - (bdx + ad)\cos(bx + a)^2\log(-I\cos(bx + a) - \sin(bx + a) + 1) + (bc - ad)\cos(bx + a)^2\log(-\frac{1}{2}\cos(bx + a) + \frac{1}{2}I\sin(bx + a) + \frac{1}{2}) + (bc - ad)\cos(bx + a)^2\log(-\frac{1}{2}\cos(bx + a) - \frac{1}{2}I\sin(bx + a) + \frac{1}{2}) + (bdx + ad)\cos(bx + a)^2\log(-\cos(bx + a) + I\sin(bx + a) + 1) - (bc - ad)\cos(bx + a)^2\log(-\cos(bx + a) + I\sin(bx + a) + I) + (bdx + ad)\cos(bx + a)^2\log(-\cos(bx + a) - I\sin(bx + a) + 1) - (bc - ad)\cos(bx + a)^2\log(-\cos(bx + a) - I\sin(bx + a) + I) + bdx - d\cos(bx + a)\sin(bx + a) + bc) / (b^2\cos(bx + a)^2)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((dx+c)*csc(bx+a)*sec(bx+a)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \csc(bx + a) \sec(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)*sec(b*x + a)^3, x)
```

$$3.314 \quad \int \frac{\csc(a+bx) \sec^3(a+bx)}{c+dx} dx$$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\csc(a+bx) \sec^3(a+bx)}{c+dx}, x\right)$$

[Out] CannotIntegrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x), x]

Rubi [A] time = 0.188899, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc(a+bx) \sec^3(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x), x]

[Out] Defer[Int] [(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x), x]

Rubi steps

$$\int \frac{\csc(a+bx) \sec^3(a+bx)}{c+dx} dx = \int \frac{\csc(a+bx) \sec^3(a+bx)}{c+dx} dx$$

Mathematica [A] time = 9.06757, size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx) \sec^3(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x), x]

[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x), x]

Maple [A] time = 3.113, size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a) (\sec(bx+a))^3}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c), x)

[Out] int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x, algorithm="maxima")

[Out] $(4*(b*d*x + b*c)*\cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*\sin(2*b*x + 2*a)^2 + (2*(b*d*x + b*c)*\cos(2*b*x + 2*a) - d*\sin(2*b*x + 2*a))*\cos(4*b*x + 4*a) + 2*(b*d*x + b*c)*\cos(2*b*x + 2*a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\sin(4*b*x + 4*a)*\sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(2*b*x + 2*a))*\cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(2*b*x + 2*a))*\integrate(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + d^2)*\sin(2*b*x + 2*a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cos(2*b*x + 2*a)), x) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\sin(4*b*x + 4*a)*\sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(2*b*x + 2*a))*\cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(2*b*x + 2*a))*\integrate(\sin(b*x + a)/((d*x + c)*\cos(b*x + a)^2 + (d*x + c)*\sin(b*x + a)^2 + d*x + 2*(d*x + c)*\cos(b*x + a) + c), x) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\sin(4*b*x + 4*a)*\sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(2*b*x + 2*a))*\cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(2*b*x + 2*a))*\integrate(\sin(b*x + a)/((d*x + c)*\cos(b*x + a)^2 + (d*x + c)*\sin(b*x + a)^2 + d*x - 2*(d*x + c)*\cos(b*x + a) + c), x) + (d*\cos(2*b*x + 2*a) + 2*(b*d*x + b*c)*\sin(2*b*x + 2*a) + d)*\sin(4*b*x + 4*a) + d*\sin(2*b*x + 2*a))/(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\sin(4*b*x + 4*a)*\sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(2*b*x + 2*a))*\cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\cos(2*b*x + 2*a))$

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{\csc(bx + a) \sec(bx + a)^3}{dx + c}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)*sec(b*x + a)^3/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(a + bx) \sec^3(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)*sec(a + b*x)**3/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx + a) \sec(bx + a)^3}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)*sec(b*x + a)^3/(d*x + c), x)
```

$$3.315 \quad \int \frac{\csc(a+bx) \sec^3(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\csc(a+bx) \sec^3(a+bx)}{(c+dx)^2}, x\right)$$

[Out] CannotIntegrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2, x]

Rubi [A] time = 0.208693, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc(a+bx) \sec^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2, x]

[Out] Defer[Int] [(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\csc(a+bx) \sec^3(a+bx)}{(c+dx)^2} dx = \int \frac{\csc(a+bx) \sec^3(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 6.70905, size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx) \sec^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2, x]

[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2, x]

Maple [A] time = 4.859, size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a) (\sec(bx+a))^3}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2, x)

[Out] int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2, x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)\sec(bx+a)^3}{d^2x^2+2cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(csc(b*x + a)*sec(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sec(b*x+a)**3/(d*x+c)**2,x)

[Out] Integral(csc(a + b*x)*sec(a + b*x)**3/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)\sec(bx+a)^3}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(csc(b*x + a)*sec(b*x + a)^3/(d*x + c)^2, x)

3.316 $\int (c + dx)^m \csc^2(a + bx) \sec^3(a + bx) dx$

Optimal. Leaf size=26

$$\text{CannotIntegrate}\left(\csc^2(a + bx) \sec^3(a + bx)(c + dx)^m, x\right)$$

[Out] CannotIntegrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3, x]

Rubi [A] time = 0.232894, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \csc^2(a + bx) \sec^3(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3, x]

[Out] Defer[Int] [(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3, x]

Rubi steps

$$\int (c + dx)^m \csc^2(a + bx) \sec^3(a + bx) dx = \int (c + dx)^m \csc^2(a + bx) \sec^3(a + bx) dx$$

Mathematica [A] time = 11.9623, size = 0, normalized size = 0.

$$\int (c + dx)^m \csc^2(a + bx) \sec^3(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3, x]

[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3, x]

Maple [A] time = 0.215, size = 0, normalized size = 0.

$$\int (dx + c)^m (\csc(bx + a))^2 (\sec(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3, x)

[Out] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \csc(bx + a)^2 \sec(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")

[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^3, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx + c)^m \csc(bx + a)^2 \sec(bx + a)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")

[Out] integral((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**m*csc(b*x+a)**2*sec(b*x+a)**3,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \csc(bx + a)^2 \sec(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")

[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^3, x)

3.317 $\int (c + dx)^3 \csc^2(a + bx) \sec^3(a + bx) dx$

Optimal. Leaf size=486

$$\frac{6id^2(c + dx)\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} - \frac{6id^2(c + dx)\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3} - \frac{9d^2(c + dx)\text{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^3} + \frac{9d^2(c + dx)\text{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^3}$$

```
[Out] ((-6*I)*d^2*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^3 - ((3*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b - (6*d*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^2 - (3*(c + d*x)^3*Csc[a + b*x])/(2*b) + ((6*I)*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^3 + ((3*I)*d^3*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^4 + (((9*I)/2)*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((3*I)*d^3*PolyLog[2, I*E^(I*(a + b*x))])/b^4 - (((9*I)/2)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^3 - (6*d^3*PolyLog[3, -E^(I*(a + b*x))])/b^4 - (9*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 + (9*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + (6*d^3*PolyLog[3, E^(I*(a + b*x))])/b^4 - ((9*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^4 + ((9*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4 - (3*d*(c + d*x)^2*Sec[a + b*x])/(2*b^2) + ((c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^2)/(2*b)
```

Rubi [A] time = 1.20635, antiderivative size = 486, normalized size of antiderivative = 1., number of steps used = 44, number of rules used = 19, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.792$, Rules used = {2621, 288, 321, 207, 4420, 6688, 12, 6742, 6273, 4181, 2531, 6609, 2282, 6589, 4183, 2622, 6741, 2279, 2391}

$$\frac{6id^2(c + dx)\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} - \frac{6id^2(c + dx)\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3} - \frac{9d^2(c + dx)\text{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^3} + \frac{9d^2(c + dx)\text{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^3, x]
```

```
[Out] ((-6*I)*d^2*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^3 - ((3*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b - (6*d*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^2 - (3*(c + d*x)^3*Csc[a + b*x])/(2*b) + ((6*I)*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^3 + ((3*I)*d^3*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^4 + (((9*I)/2)*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((3*I)*d^3*PolyLog[2, I*E^(I*(a + b*x))])/b^4 - (((9*I)/2)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^3 - (6*d^3*PolyLog[3, -E^(I*(a + b*x))])/b^4 - (9*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 + (9*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + (6*d^3*PolyLog[3, E^(I*(a + b*x))])/b^4 - ((9*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^4 + ((9*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4 - (3*d*(c + d*x)^2*Sec[a + b*x])/(2*b^2) + ((c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^2)/(2*b)
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*x
]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6688

```
Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&& !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:= Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_)^(m_.))*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_)^(m_.), x_Symbol] := Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(
m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_.), x_S
ymbol] := Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 6741

```
Int[u_, x_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v !=
= u]
```

Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 \csc^2(a + bx) \sec^3(a + bx) dx &= \frac{3(c + dx)^3 \tanh^{-1}(\sin(a + bx))}{2b} - \frac{3(c + dx)^3 \csc(a + bx)}{2b} + \frac{(c + dx)^3 \csc(a + bx)}{2b} \\
&= \frac{3(c + dx)^3 \tanh^{-1}(\sin(a + bx))}{2b} - \frac{3(c + dx)^3 \csc(a + bx)}{2b} + \frac{(c + dx)^3 \csc(a + bx)}{2b} \\
&= \frac{3(c + dx)^3 \tanh^{-1}(\sin(a + bx))}{2b} - \frac{3(c + dx)^3 \csc(a + bx)}{2b} + \frac{(c + dx)^3 \csc(a + bx)}{2b} \\
&= \frac{3(c + dx)^3 \tanh^{-1}(\sin(a + bx))}{2b} - \frac{3(c + dx)^3 \csc(a + bx)}{2b} + \frac{(c + dx)^3 \csc(a + bx)}{2b} \\
&= \frac{3(c + dx)^3 \tanh^{-1}(\sin(a + bx))}{2b} - \frac{3(c + dx)^3 \csc(a + bx)}{2b} + \frac{(c + dx)^3 \csc(a + bx)}{2b} \\
&= \frac{3d(c + dx)^2 \tanh^{-1}(\cos(a + bx))}{2b^2} + \frac{3(c + dx)^3 \tanh^{-1}(\sin(a + bx))}{2b} - \frac{3(c + dx)^3 \csc(a + bx)}{2b} \\
&= \frac{3d(c + dx)^2 \tanh^{-1}(\cos(a + bx))}{2b^2} + \frac{3(c + dx)^3 \tanh^{-1}(\sin(a + bx))}{2b} - \frac{3(c + dx)^3 \csc(a + bx)}{2b} \\
&= -\frac{9d(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^2} + \frac{3d(c + dx)^2 \tanh^{-1}(\cos(a + bx))}{2b^2} - \frac{3(c + dx)^3 \csc(a + bx)}{2b} \\
&= -\frac{9d(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^2} + \frac{3d(c + dx)^2 \tanh^{-1}(\cos(a + bx))}{2b^2} - \frac{3(c + dx)^3 \csc(a + bx)}{2b} \\
&= -\frac{3i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{9d(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b^2} + \frac{3d(c + dx)^2 \tanh^{-1}(\cos(a + bx))}{2b^2} \\
&= -\frac{6id^2(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^3} - \frac{3i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{9d(c + dx)^2 \tanh^{-1}(\cos(a + bx))}{2b^2} \\
&= -\frac{6id^2(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^3} - \frac{3i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{9d(c + dx)^2 \tanh^{-1}(\cos(a + bx))}{2b^2} \\
&= -\frac{6id^2(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^3} - \frac{3i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{6d(c + dx)^2 \tanh^{-1}(\cos(a + bx))}{2b^2} \\
&= -\frac{6id^2(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^3} - \frac{3i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{6d(c + dx)^2 \tanh^{-1}(\cos(a + bx))}{2b^2} \\
&= -\frac{6id^2(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^3} - \frac{3i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{6d(c + dx)^2 \tanh^{-1}(\cos(a + bx))}{2b^2} \\
&= -\frac{6id^2(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^3} - \frac{3i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{6d(c + dx)^2 \tanh^{-1}(\cos(a + bx))}{2b^2} \\
&= -\frac{6id^2(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^3} - \frac{3i(c + dx)^3 \tan^{-1}(e^{i(a+bx)})}{b} - \frac{6d(c + dx)^2 \tanh^{-1}(\cos(a + bx))}{2b^2}
\end{aligned}$$

Mathematica [A] time = 7.9131, size = 819, normalized size = 1.69

$$\frac{\csc(a + bx) (bc^3 + 3b \cos(2a + 2bx)c^3 + 3bdxc^2 + 9bdx \cos(2a + 2bx)c^2 + 3d \sin(2a + 2bx)c^2 + 3bd^2x^2c + 9bd^2x^2c^2)}{4b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^3,x]

[Out] (3*d*((c + d*x)^2*Log[1 - E^(I*(a + b*x))] - (c + d*x)^2*Log[1 + E^(I*(a + b*x))]) + ((2*I)*d*(b*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] + I*d*PolyLog[3, -E^(I*(a + b*x))]))/b^2 + (2*d*((-I)*b*(c + d*x)*PolyLog[2, E^(I*(a + b*x))]))/b^2

$$\begin{aligned} &))] + d \cdot \text{PolyLog}[3, E^{(I \cdot (a + b \cdot x))}] / b^2) / b^2 - (3 \cdot ((2 \cdot I) \cdot b^3 \cdot c^3 \cdot \text{ArcTan}[\\ & E^{(I \cdot (a + b \cdot x))}] + (4 \cdot I) \cdot b \cdot c \cdot d^2 \cdot \text{ArcTan}[E^{(I \cdot (a + b \cdot x))}] - 3 \cdot b^3 \cdot c^2 \cdot d \cdot x \cdot \text{Lo} \\ & \text{g}[1 - I \cdot E^{(I \cdot (a + b \cdot x))}] - 2 \cdot b \cdot d^3 \cdot x \cdot \text{Log}[1 - I \cdot E^{(I \cdot (a + b \cdot x))}] - 3 \cdot b^3 \cdot c \cdot d \\ & ^2 \cdot x^2 \cdot \text{Log}[1 - I \cdot E^{(I \cdot (a + b \cdot x))}] - b^3 \cdot d^3 \cdot x^3 \cdot \text{Log}[1 - I \cdot E^{(I \cdot (a + b \cdot x))}] \\ & + 3 \cdot b^3 \cdot c^2 \cdot d \cdot x \cdot \text{Log}[1 + I \cdot E^{(I \cdot (a + b \cdot x))}] + 2 \cdot b \cdot d^3 \cdot x \cdot \text{Log}[1 + I \cdot E^{(I \cdot (a + \\ & b \cdot x))}] + 3 \cdot b^3 \cdot c \cdot d^2 \cdot x^2 \cdot \text{Log}[1 + I \cdot E^{(I \cdot (a + b \cdot x))}] + b^3 \cdot d^3 \cdot x^3 \cdot \text{Log}[1 + I \\ & \cdot E^{(I \cdot (a + b \cdot x))}] - I \cdot d \cdot (2 \cdot d^2 + 3 \cdot b^2 \cdot (c + d \cdot x)^2) \cdot \text{PolyLog}[2, (-I) \cdot E^{(I \cdot (a \\ & + b \cdot x))}] + I \cdot d \cdot (2 \cdot d^2 + 3 \cdot b^2 \cdot (c + d \cdot x)^2) \cdot \text{PolyLog}[2, I \cdot E^{(I \cdot (a + b \cdot x))}] + \\ & 6 \cdot b \cdot c \cdot d^2 \cdot \text{PolyLog}[3, (-I) \cdot E^{(I \cdot (a + b \cdot x))}] + 6 \cdot b \cdot d^3 \cdot x \cdot \text{PolyLog}[3, (-I) \cdot E^{(\\ & I \cdot (a + b \cdot x))}] - 6 \cdot b \cdot c \cdot d^2 \cdot \text{PolyLog}[3, I \cdot E^{(I \cdot (a + b \cdot x))}] - 6 \cdot b \cdot d^3 \cdot x \cdot \text{PolyLog} \\ & [3, I \cdot E^{(I \cdot (a + b \cdot x))}] + (6 \cdot I) \cdot d^3 \cdot \text{PolyLog}[4, (-I) \cdot E^{(I \cdot (a + b \cdot x))}] - (6 \cdot I) \\ & \cdot d^3 \cdot \text{PolyLog}[4, I \cdot E^{(I \cdot (a + b \cdot x))}]) / (2 \cdot b^4) - (\text{Csc}[a + b \cdot x] \cdot \text{Sec}[a + b \cdot x]^2 \\ & \cdot (b \cdot c^3 + 3 \cdot b \cdot c^2 \cdot d \cdot x + 3 \cdot b \cdot c \cdot d^2 \cdot x^2 + b \cdot d^3 \cdot x^3 + 3 \cdot b \cdot c^3 \cdot \text{Cos}[2 \cdot a + 2 \cdot b \cdot x] \\ &] + 9 \cdot b \cdot c^2 \cdot d \cdot x \cdot \text{Cos}[2 \cdot a + 2 \cdot b \cdot x] + 9 \cdot b \cdot c \cdot d^2 \cdot x^2 \cdot \text{Cos}[2 \cdot a + 2 \cdot b \cdot x] + 3 \cdot b \cdot d^3 \\ & \cdot x^3 \cdot \text{Cos}[2 \cdot a + 2 \cdot b \cdot x] + 3 \cdot c^2 \cdot d \cdot \text{Sin}[2 \cdot a + 2 \cdot b \cdot x] + 6 \cdot c \cdot d^2 \cdot x \cdot \text{Sin}[2 \cdot a + 2 \cdot b \cdot \\ & x] + 3 \cdot d^3 \cdot x^2 \cdot \text{Sin}[2 \cdot a + 2 \cdot b \cdot x])) / (4 \cdot b^2) \end{aligned}$$

Maple [B] time = 0.738, size = 1629, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x)`

[Out]
$$\begin{aligned} & -6 \cdot d^3 \cdot \text{polylog}(3, -\exp(I \cdot (b \cdot x + a))) / b^4 + 6 \cdot d^3 \cdot \text{polylog}(3, \exp(I \cdot (b \cdot x + a))) / b^4 + 3 \\ & \cdot d^3 / b^4 \cdot \ln(1 - \exp(I \cdot (b \cdot x + a))) \cdot a^2 - 3 \cdot d^3 / b^2 \cdot \ln(\exp(I \cdot (b \cdot x + a)) + 1) \cdot x^2 + 3 \cdot d^3 / \\ & b^2 \cdot \ln(1 - \exp(I \cdot (b \cdot x + a))) \cdot x^2 + 3 / b^4 \cdot d^3 \cdot \ln(1 - I \cdot \exp(I \cdot (b \cdot x + a))) \cdot a - 9 / b^3 \cdot d^3 \cdot \text{p} \\ & \text{olylog}(3, -I \cdot \exp(I \cdot (b \cdot x + a))) \cdot x - 3 / b^3 \cdot d^3 \cdot \ln(1 + I \cdot \exp(I \cdot (b \cdot x + a))) \cdot x - 3 / b^4 \cdot d^3 \cdot \\ & \ln(1 + I \cdot \exp(I \cdot (b \cdot x + a))) \cdot a - 3 / 2 / b^4 \cdot d^3 \cdot a^3 \cdot \ln(1 + I \cdot \exp(I \cdot (b \cdot x + a))) + 9 / b^3 \cdot d^3 \cdot \text{p} \\ & \text{olylog}(3, I \cdot \exp(I \cdot (b \cdot x + a))) \cdot x - 3 / 2 / b \cdot d^3 \cdot \ln(1 + I \cdot \exp(I \cdot (b \cdot x + a))) \cdot x^3 + 3 / 2 / b \cdot d^3 \\ & \cdot \ln(1 - I \cdot \exp(I \cdot (b \cdot x + a))) \cdot x^3 + 3 / b^2 \cdot c^2 \cdot d \cdot \ln(\exp(I \cdot (b \cdot x + a)) - 1) - 3 / b^2 \cdot c^2 \cdot d \cdot \ln \\ & (\exp(I \cdot (b \cdot x + a)) + 1) + 3 / b^4 \cdot d^3 \cdot a^2 \cdot \ln(\exp(I \cdot (b \cdot x + a)) - 1) - 3 \cdot I / b \cdot c^3 \cdot \arctan(\exp(\\ & I \cdot (b \cdot x + a))) - 6 \cdot I / b^3 \cdot d^2 \cdot c \cdot \arctan(\exp(I \cdot (b \cdot x + a))) - 9 \cdot I \cdot d^3 \cdot \text{polylog}(4, -I \cdot \exp(I \\ & \cdot (b \cdot x + a))) / b^4 + 6 / b^3 \cdot d^3 \cdot \ln(1 - \exp(I \cdot (b \cdot x + a))) \cdot a \cdot x - 6 / b^3 \cdot d^2 \cdot c \cdot a \cdot \ln(\exp(I \cdot (b \\ & \cdot x + a)) - 1) - 6 \cdot d^2 / b^2 \cdot c \cdot \ln(\exp(I \cdot (b \cdot x + a)) + 1) \cdot x - 6 \cdot I \cdot d^3 / b^3 \cdot \text{polylog}(2, \exp(I \cdot (b \\ & \cdot x + a))) \cdot x - 3 \cdot I \cdot d^3 \cdot \text{polylog}(2, I \cdot \exp(I \cdot (b \cdot x + a))) / b^4 - I / b^2 / (\exp(2 \cdot I \cdot (b \cdot x + a)) + 1 \\ &)^2 / (\exp(2 \cdot I \cdot (b \cdot x + a)) - 1) \cdot (3 \cdot d^3 \cdot x^3 \cdot b \cdot \exp(5 \cdot I \cdot (b \cdot x + a)) + 9 \cdot c \cdot d^2 \cdot x^2 \cdot b \cdot \exp(5 \cdot \\ & I \cdot (b \cdot x + a)) + 9 \cdot c^2 \cdot d \cdot x \cdot b \cdot \exp(5 \cdot I \cdot (b \cdot x + a)) + 2 \cdot d^3 \cdot x^3 \cdot b \cdot \exp(3 \cdot I \cdot (b \cdot x + a)) + 3 \cdot c^3 \cdot \\ & b \cdot \exp(5 \cdot I \cdot (b \cdot x + a)) + 6 \cdot c \cdot d^2 \cdot x^2 \cdot b \cdot \exp(3 \cdot I \cdot (b \cdot x + a)) + 3 \cdot I \cdot d^3 \cdot x^2 \cdot \exp(I \cdot (b \cdot x + a)) \\ &) + 6 \cdot c^2 \cdot d \cdot x \cdot b \cdot \exp(3 \cdot I \cdot (b \cdot x + a)) + 3 \cdot d^3 \cdot x^3 \cdot b \cdot \exp(I \cdot (b \cdot x + a)) + 6 \cdot I \cdot c \cdot d^2 \cdot x \cdot \exp(I \\ & \cdot (b \cdot x + a)) + 2 \cdot c^3 \cdot b \cdot \exp(3 \cdot I \cdot (b \cdot x + a)) + 9 \cdot c \cdot d^2 \cdot x^2 \cdot b \cdot \exp(I \cdot (b \cdot x + a)) - 3 \cdot I \cdot c^2 \cdot d \cdot e \\ & \text{xp}(5 \cdot I \cdot (b \cdot x + a)) + 9 \cdot c^2 \cdot d \cdot x \cdot b \cdot \exp(I \cdot (b \cdot x + a)) + 3 \cdot c^3 \cdot b \cdot \exp(I \cdot (b \cdot x + a)) - 6 \cdot I \cdot c \cdot d^2 \\ & \cdot x \cdot \exp(5 \cdot I \cdot (b \cdot x + a)) - 3 \cdot I \cdot d^3 \cdot x^2 \cdot \exp(5 \cdot I \cdot (b \cdot x + a)) + 3 \cdot I \cdot c^2 \cdot d \cdot \exp(I \cdot (b \cdot x + a)) - \\ & 9 / b^3 \cdot d^2 \cdot c \cdot \text{polylog}(3, -I \cdot \exp(I \cdot (b \cdot x + a))) + 9 / b^3 \cdot d^2 \cdot c \cdot \text{polylog}(3, I \cdot \exp(I \cdot (b \cdot x \\ & + a))) + 3 / 2 / b^4 \cdot d^3 \cdot a^3 \cdot \ln(1 - I \cdot \exp(I \cdot (b \cdot x + a))) + 3 / b^3 \cdot d^3 \cdot \ln(1 - I \cdot \exp(I \cdot (b \cdot x + a) \\ &)) \cdot x - 9 \cdot I / b^2 \cdot c \cdot d^2 \cdot \text{polylog}(2, I \cdot \exp(I \cdot (b \cdot x + a))) \cdot x + 9 \cdot I / b^2 \cdot c \cdot d^2 \cdot \text{polylog}(2, -I \\ & \cdot \exp(I \cdot (b \cdot x + a))) \cdot x - 9 \cdot I / b^3 \cdot d^2 \cdot c \cdot a^2 \cdot \arctan(\exp(I \cdot (b \cdot x + a))) + 9 \cdot I / b^2 \cdot c^2 \cdot d \cdot a \\ & \cdot \arctan(\exp(I \cdot (b \cdot x + a))) + 6 \cdot I \cdot d^3 / b^3 \cdot \text{polylog}(2, -\exp(I \cdot (b \cdot x + a))) \cdot x + 3 \cdot I \cdot d^3 \cdot \text{p} \\ & \text{olylog}(2, -I \cdot \exp(I \cdot (b \cdot x + a))) / b^4 + 9 / 2 / b \cdot d^2 \cdot c \cdot \ln(1 - I \cdot \exp(I \cdot (b \cdot x + a))) \cdot x^2 - 9 / 2 / b \\ & \cdot d^2 \cdot c \cdot \ln(1 + I \cdot \exp(I \cdot (b \cdot x + a))) \cdot x^2 - 9 / 2 / b \cdot c^2 \cdot d \cdot \ln(1 + I \cdot \exp(I \cdot (b \cdot x + a))) \cdot x - 9 / 2 / \\ & b^2 \cdot c^2 \cdot d \cdot \ln(1 + I \cdot \exp(I \cdot (b \cdot x + a))) \cdot a + 9 / 2 / b \cdot c^2 \cdot d \cdot \ln(1 - I \cdot \exp(I \cdot (b \cdot x + a))) \cdot x + 9 / 2 \\ & / b^2 \cdot c^2 \cdot d \cdot \ln(1 - I \cdot \exp(I \cdot (b \cdot x + a))) \cdot a + 9 / 2 / b^3 \cdot d^2 \cdot c \cdot a^2 \cdot \ln(1 + I \cdot \exp(I \cdot (b \cdot x + a) \\ &)) - 9 / 2 / b^3 \cdot d^2 \cdot c \cdot a^2 \cdot \ln(1 - I \cdot \exp(I \cdot (b \cdot x + a))) + 6 \cdot I / b^4 \cdot d^3 \cdot a \cdot \arctan(\exp(I \cdot (b \cdot x + \\ & a))) + 9 \cdot I \cdot d^3 \cdot \text{polylog}(4, I \cdot \exp(I \cdot (b \cdot x + a))) / b^4 - 6 \cdot I / b^4 \cdot d^3 \cdot \text{polylog}(2, \exp(I \cdot (b \\ & \cdot x + a))) \cdot a - 9 / 2 \cdot I / b^2 \cdot d^3 \cdot \text{polylog}(2, I \cdot \exp(I \cdot (b \cdot x + a))) \cdot x^2 + 9 / 2 \cdot I / b^2 \cdot d^3 \cdot \text{polyl} \\ & \text{og}(2, -I \cdot \exp(I \cdot (b \cdot x + a))) \cdot x^2 - 6 \cdot I / b^4 \cdot d^3 \cdot a \cdot \text{dilog}(\exp(I \cdot (b \cdot x + a))) + 9 / 2 \cdot I / b^2 \cdot c \end{aligned}$$

$$\begin{aligned} &^2*d*polylog(2,-I*\exp(I*(b*x+a)))-9/2*I/b^2*c^2*d*polylog(2,I*\exp(I*(b*x+a) \\ &)))+6*I/b^3*dilog(\exp(I*(b*x+a))+1)*c*d^2-6*I/b^4*d^3*a*dilog(\exp(I*(b*x+a) \\ &)+1)+6*I/b^3*d^2*c*dilog(\exp(I*(b*x+a)))+3*I/b^4*d^3*a^3*\arctan(\exp(I*(b*x+a) \\ &)))+6*I/b^4*d^3*polylog(2,-\exp(I*(b*x+a)))*a \end{aligned}$$

Maxima [B] time = 20.6871, size = 10843, normalized size = 22.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} &-1/4*(c^3*(2*(3*\sin(b*x + a)^2 - 2)/(\sin(b*x + a)^3 - \sin(b*x + a)) - 3*\log \\ &(\sin(b*x + a) + 1) + 3*\log(\sin(b*x + a) - 1)) - 3*a*c^2*d*(2*(3*\sin(b*x + a) \\ &)^2 - 2)/(\sin(b*x + a)^3 - \sin(b*x + a)) - 3*\log(\sin(b*x + a) + 1) + 3*\log(\\ &\sin(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*(3*\sin(b*x + a)^2 - 2)/(\sin(b*x + a)^ \\ &3 - \sin(b*x + a)) - 3*\log(\sin(b*x + a) + 1) + 3*\log(\sin(b*x + a) - 1))/b^2 \\ &- a^3*d^3*(2*(3*\sin(b*x + a)^2 - 2)/(\sin(b*x + a)^3 - \sin(b*x + a)) - 3*\log \\ &(\sin(b*x + a) + 1) + 3*\log(\sin(b*x + a) - 1))/b^3 - 4*((6*(b*x + a)^3*d^3 + \\ &12*b*c*d^2 - 12*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(3*b^2*c^2*d \\ &- 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a) - 6*((b*x + a)^3*d^3 + 2*b*c*d^2 \\ &- 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + \\ &(3*a^2 + 2)*d^3)*(b*x + a))*\cos(6*b*x + 6*a) - 6*((b*x + a)^3*d^3 + 2*b*c* \\ &d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^ \\ &2 + (3*a^2 + 2)*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + 6*((b*x + a)^3*d^3 + 2*b \\ &*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c \\ &*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (-6*I*(b*x + a)^3*d^3 \\ &- 12*I*b*c*d^2 + 12*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (\\ &-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 12*I)*d^3)*(b*x + a))*\sin(6 \\ &*b*x + 6*a) + (-6*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (-18*I*b* \\ &c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18* \\ &I*a^2 - 12*I)*d^3)*(b*x + a))*\sin(4*b*x + 4*a) + (6*I*(b*x + a)^3*d^3 + 12* \\ &I*b*c*d^2 - 12*I*a*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^ \\ &2*c^2*d - 36*I*a*b*c*d^2 + (18*I*a^2 + 12*I)*d^3)*(b*x + a))*\sin(2*b*x + 2* \\ &a))*\arctan2(\cos(b*x + a), \sin(b*x + a) + 1) + (6*(b*x + a)^3*d^3 + 12*b*c*d \\ &^2 - 12*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(3*b^2*c^2*d - 6*a*b*c \\ &*d^2 + (3*a^2 + 2)*d^3)*(b*x + a) - 6*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^ \\ &3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + \\ &2)*d^3)*(b*x + a))*\cos(6*b*x + 6*a) - 6*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a \\ &*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^ \\ &2 + 2)*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + 6*((b*x + a)^3*d^3 + 2*b*c*d^2 - \\ &2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3 \\ &*a^2 + 2)*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (-6*I*(b*x + a)^3*d^3 - 12*I*b \\ &*c*d^2 + 12*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2 \\ &*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 12*I)*d^3)*(b*x + a))*\sin(6*b*x + 6* \\ &a) + (-6*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (-18*I*b*c*d^2 + 1 \\ &8*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 1 \\ &2*I)*d^3)*(b*x + a))*\sin(4*b*x + 4*a) + (6*I*(b*x + a)^3*d^3 + 12*I*b*c*d^2 \\ &- 12*I*a*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - \\ &36*I*a*b*c*d^2 + (18*I*a^2 + 12*I)*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\arcta \\ &n2(\cos(b*x + a), -\sin(b*x + a) + 1) + (12*b^2*c^2*d - 24*a*b*c*d^2 + 12*(b \\ &x + a)^2*d^3 + 12*a^2*d^3 + 24*(b*c*d^2 - a*d^3)*(b*x + a) - 12*(b^2*c^2*d \\ &- 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))* \\ &\cos(6*b*x + 6*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 \\ &+ 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + 12*(b^2*c^2*d - 2*a*b*c \\ &*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x \end{aligned}$$

$$\begin{aligned}
& + 2*a) + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 12*I*a \\
& ^2*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*\sin(6*b*x + 6*a) + (-12*I* \\
& b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 12*I*a^2*d^3 + (-24*I*b \\
& *c*d^2 + 24*I*a*d^3)*(b*x + a))*\sin(4*b*x + 4*a) + (12*I*b^2*c^2*d - 24*I*a \\
& *b*c*d^2 + 12*I*(b*x + a)^2*d^3 + 12*I*a^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3 \\
&)*(b*x + a))*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) - (1 \\
& 2*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^2*d^3 - 12*(b^2*c^2*d - 2*a*b*c*d^2 + a^2 \\
& *d^3))*\cos(6*b*x + 6*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3))*\cos(4*b*x + \\
& 4*a) + 12*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3))*\cos(2*b*x + 2*a) - (12*I*b^2 \\
& *c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^3))*\sin(6*b*x + 6*a) - (12*I*b^2*c^2*d \\
& - 24*I*a*b*c*d^2 + 12*I*a^2*d^3))*\sin(4*b*x + 4*a) - (-12*I*b^2*c^2*d + 24*I \\
& *a*b*c*d^2 - 12*I*a^2*d^3))*\sin(2*b*x + 2*a))*\arctan2(\sin(b*x + a), \cos(b*x \\
& + a) - 1) + (12*(b*x + a)^2*d^3 + 24*(b*c*d^2 - a*d^3)*(b*x + a) - 12*((b*x \\
& + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(6*b*x + 6*a) - 12*((b*x + \\
& a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + 12*((b*x + a)^ \\
& 2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + 2*a) + (-12*I*(b*x + a)^ \\
& 2*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*\sin(6*b*x + 6*a) + (-12*I*(\\
& b*x + a)^2*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*\sin(4*b*x + 4*a) + \\
& (12*I*(b*x + a)^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*\sin(2*b*x + \\
& 2*a))*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) - (12*(b*x + a)^3*d^3 - 12* \\
& I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*a^2*d^3 + (36*b*c*d^2 - (36*a + 12*I)*d \\
& ^3)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a + 24*I)*b*c*d^2 + 12*(3*a^2 + 2*I*a \\
&)*d^3)*(b*x + a))*\cos(5*b*x + 5*a) - 8*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^ \\
& 3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*\cos(3*b*x \\
& + 3*a) - (12*(b*x + a)^3*d^3 + 12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2* \\
& d^3 + (36*b*c*d^2 - (36*a - 12*I)*d^3)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a \\
& - 24*I)*b*c*d^2 + 12*(3*a^2 - 2*I*a)*d^3)*(b*x + a))*\cos(b*x + a) + (18*b^2 \\
& *c^2*d - 36*a*b*c*d^2 + 18*(b*x + a)^2*d^3 + 6*(3*a^2 + 2)*d^3 + 36*(b*c*d^ \\
& 2 - a*d^3)*(b*x + a) - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (\\
& 3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(6*b*x + 6*a) - 6*(3*b^2 \\
& *c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a \\
& *d^3)*(b*x + a))*\cos(4*b*x + 4*a) + 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + \\
& a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(2*b*x + 2* \\
& a) + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2 \\
& - 12*I)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*\sin(6*b*x + 6*a) + (- \\
& 18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 12*I) \\
& *d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*\sin(4*b*x + 4*a) + (18*I*b^2 \\
& *c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I*a^2 + 12*I)*d^3 + (3 \\
& 6*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{dilog}(I*e^{(I*b*x + I \\
& *a)}) - (18*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b*x + a)^2*d^3 + 6*(3*a^2 + 2)*d^ \\
& 3 + 36*(b*c*d^2 - a*d^3)*(b*x + a) - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x \\
& + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(6*b*x + 6 \\
& *a) - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + \\
& 6*(b*c*d^2 - a*d^3)*(b*x + a))*\cos(4*b*x + 4*a) + 6*(3*b^2*c^2*d - 6*a*b*c* \\
& d^2 + 3*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))* \\
& \cos(2*b*x + 2*a) - (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 \\
& + (18*I*a^2 + 12*I)*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*\sin(6*b*x \\
& + 6*a) - (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I*a^ \\
& 2 + 12*I)*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*\sin(4*b*x + 4*a) - (\\
& -18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 12*I) \\
&)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*\sin(2*b*x + 2*a))*\operatorname{dilog}(-I* \\
& e^{(I*b*x + I*a)}) - (24*b*c*d^2 + 24*(b*x + a)*d^3 - 24*a*d^3 - 24*(b*c*d^2 \\
& + (b*x + a)*d^3 - a*d^3))*\cos(6*b*x + 6*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a \\
& *d^3))*\cos(4*b*x + 4*a) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3))*\cos(2*b*x + 2 \\
& *a) - (24*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 24*I*a*d^3))*\sin(6*b*x + 6*a) - (\\
& 24*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 24*I*a*d^3))*\sin(4*b*x + 4*a) - (-24*I*b \\
& *c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3))*\sin(2*b*x + 2*a))*\operatorname{dilog}(-e^{(I*b*x \\
& + I*a)}) + (24*b*c*d^2 + 24*(b*x + a)*d^3 - 24*a*d^3 - 24*(b*c*d^2 + (b*x + \\
& a)*d^3 - a*d^3))*\cos(6*b*x + 6*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3))*\operatorname{co}
\end{aligned}$$

$$\begin{aligned}
& s(4bx + 4a) + 24*(b*c*d^2 + (bx + a)*d^3 - a*d^3)*\cos(2*bx + 2*a) + (- \\
& 24*I*b*c*d^2 - 24*I*(bx + a)*d^3 + 24*I*a*d^3)*\sin(6*bx + 6*a) + (-24*I*b \\
& *c*d^2 - 24*I*(bx + a)*d^3 + 24*I*a*d^3)*\sin(4*bx + 4*a) + (24*I*b*c*d^2 \\
& + 24*I*(bx + a)*d^3 - 24*I*a*d^3)*\sin(2*bx + 2*a))*\operatorname{dilog}(e^{(I*bx + I*a)}) \\
& + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(bx + a)^2*d^3 - 6*I*a^2*d^3 + (\\
& -12*I*b*c*d^2 + 12*I*a*d^3)*(bx + a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6 \\
& *I*(bx + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(bx + a))*\cos \\
& \cos(6*bx + 6*a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(bx + a)^2*d^3 + 6 \\
& *I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(bx + a))*\cos(4*bx + 4*a) + (-6*I \\
& *b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(bx + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b \\
& *c*d^2 + 12*I*a*d^3)*(bx + a))*\cos(2*bx + 2*a) - 6*(b^2*c^2*d - 2*a*b*c*d \\
& ^2 + (bx + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(bx + a))*\sin(6*bx + \\
& 6*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (bx + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 \\
& - a*d^3)*(bx + a))*\sin(4*bx + 4*a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (bx + \\
& a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(bx + a))*\sin(2*bx + 2*a))*\log(\\
& \cos(bx + a)^2 + \sin(bx + a)^2 + 2*\cos(bx + a) + 1) + (6*I*b^2*c^2*d - 12 \\
& *I*a*b*c*d^2 + 6*I*(bx + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d \\
& ^3)*(bx + a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(bx + a)^2*d^3 - 6*I \\
& *a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(bx + a))*\cos(6*bx + 6*a) + (-6*I \\
& *b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(bx + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b \\
& *c*d^2 + 12*I*a*d^3)*(bx + a))*\cos(4*bx + 4*a) + (6*I*b^2*c^2*d - 12*I*a* \\
& b*c*d^2 + 6*I*(bx + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(\\
& bx + a))*\cos(2*bx + 2*a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (bx + a)^2*d^3 + \\
& a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(bx + a))*\sin(6*bx + 6*a) + 6*(b^2*c^2*d - \\
& 2*a*b*c*d^2 + (bx + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(bx + a))*\sin \\
& \sin(4*bx + 4*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (bx + a)^2*d^3 + a^2*d^3 + \\
& 2*(b*c*d^2 - a*d^3)*(bx + a))*\sin(2*bx + 2*a))*\log(\cos(bx + a)^2 + \sin(b \\
& *x + a)^2 - 2*\cos(bx + a) + 1) + (3*I*(bx + a)^3*d^3 + 6*I*b*c*d^2 - 6*I* \\
& a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(bx + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c \\
& *d^2 + (9*I*a^2 + 6*I)*d^3)*(bx + a) + (-3*I*(bx + a)^3*d^3 - 6*I*b*c*d^2 \\
& + 6*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(bx + a)^2 + (-9*I*b^2*c^2*d + 1 \\
& 8*I*a*b*c*d^2 + (-9*I*a^2 - 6*I)*d^3)*(bx + a))*\cos(6*bx + 6*a) + (-3*I*(\\
& bx + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(bx \\
& + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 + (-9*I*a^2 - 6*I)*d^3)*(bx + a) \\
&)*\cos(4*bx + 4*a) + (3*I*(bx + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (9*I* \\
& b*c*d^2 - 9*I*a*d^3)*(bx + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a \\
& ^2 + 6*I)*d^3)*(bx + a))*\cos(2*bx + 2*a) + 3*((bx + a)^3*d^3 + 2*b*c*d^2 \\
& - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(bx + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + \\
& (3*a^2 + 2)*d^3)*(bx + a))*\sin(6*bx + 6*a) + 3*((bx + a)^3*d^3 + 2*b*c* \\
& d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(bx + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 \\
& + (3*a^2 + 2)*d^3)*(bx + a))*\sin(4*bx + 4*a) - 3*((bx + a)^3*d^3 + 2*b \\
& *c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(bx + a)^2 + (3*b^2*c^2*d - 6*a*b*c \\
& *d^2 + (3*a^2 + 2)*d^3)*(bx + a))*\sin(2*bx + 2*a))*\log(\cos(bx + a)^2 + \sin(b \\
& *x + a)^2 + 2*\sin(bx + a) + 1) + (-3*I*(bx + a)^3*d^3 - 6*I*b*c*d^2 + \\
& 6*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(bx + a)^2 + (-9*I*b^2*c^2*d + 18* \\
& I*a*b*c*d^2 + (-9*I*a^2 - 6*I)*d^3)*(bx + a) + (3*I*(bx + a)^3*d^3 + 6*I* \\
& b*c*d^2 - 6*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(bx + a)^2 + (9*I*b^2*c^2* \\
& d - 18*I*a*b*c*d^2 + (9*I*a^2 + 6*I)*d^3)*(bx + a))*\cos(6*bx + 6*a) + (3* \\
& I*(bx + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b \\
& x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 6*I)*d^3)*(bx + a) \\
&)*\cos(4*bx + 4*a) + (-3*I*(bx + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (-9* \\
& I*b*c*d^2 + 9*I*a*d^3)*(bx + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 + (-9 \\
& *I*a^2 - 6*I)*d^3)*(bx + a))*\cos(2*bx + 2*a) - 3*((bx + a)^3*d^3 + 2*b*c \\
& *d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(bx + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d \\
& ^2 + (3*a^2 + 2)*d^3)*(bx + a))*\sin(6*bx + 6*a) - 3*((bx + a)^3*d^3 + 2* \\
& b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(bx + a)^2 + (3*b^2*c^2*d - 6*a*b* \\
& c*d^2 + (3*a^2 + 2)*d^3)*(bx + a))*\sin(4*bx + 4*a) + 3*((bx + a)^3*d^3 + \\
& 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(bx + a)^2 + (3*b^2*c^2*d - 6*a \\
& *b*c*d^2 + (3*a^2 + 2)*d^3)*(bx + a))*\sin(2*bx + 2*a))*\log(\cos(bx + a)^2
\end{aligned}$$

$$\begin{aligned}
& + \sin(b*x + a)^2 - 2*\sin(b*x + a) + 1) + (36*d^3*\cos(6*b*x + 6*a) + 36*d^3 \\
& * \cos(4*b*x + 4*a) - 36*d^3*\cos(2*b*x + 2*a) + 36*I*d^3*\sin(6*b*x + 6*a) + 3 \\
& 6*I*d^3*\sin(4*b*x + 4*a) - 36*I*d^3*\sin(2*b*x + 2*a) - 36*d^3)*\text{polylog}(4, I \\
& *e^{(I*b*x + I*a)}) - (36*d^3*\cos(6*b*x + 6*a) + 36*d^3*\cos(4*b*x + 4*a) - 36 \\
& *d^3*\cos(2*b*x + 2*a) + 36*I*d^3*\sin(6*b*x + 6*a) + 36*I*d^3*\sin(4*b*x + 4* \\
& a) - 36*I*d^3*\sin(2*b*x + 2*a) - 36*d^3)*\text{polylog}(4, -I*e^{(I*b*x + I*a)}) + (\\
& 36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3 + (-36*I*b*c*d^2 - 36*I*(b*x \\
& + a)*d^3 + 36*I*a*d^3)*\cos(6*b*x + 6*a) + (-36*I*b*c*d^2 - 36*I*(b*x + a)* \\
& d^3 + 36*I*a*d^3)*\cos(4*b*x + 4*a) + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 3 \\
& 6*I*a*d^3)*\cos(2*b*x + 2*a) + 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\sin(6*b* \\
& x + 6*a) + 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\sin(4*b*x + 4*a) - 36*(b*c* \\
& d^2 + (b*x + a)*d^3 - a*d^3)*\sin(2*b*x + 2*a))*\text{polylog}(3, I*e^{(I*b*x + I*a)} \\
&) + (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^3 + 36*I*a*d^3 + (36*I*b*c*d^2 + 36*I \\
& *(b*x + a)*d^3 - 36*I*a*d^3)*\cos(6*b*x + 6*a) + (36*I*b*c*d^2 + 36*I*(b*x + \\
& a)*d^3 - 36*I*a*d^3)*\cos(4*b*x + 4*a) + (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^ \\
& 3 + 36*I*a*d^3)*\cos(2*b*x + 2*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\sin \\
& (6*b*x + 6*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\sin(4*b*x + 4*a) + 36* \\
& (b*c*d^2 + (b*x + a)*d^3 - a*d^3)*\sin(2*b*x + 2*a))*\text{polylog}(3, -I*e^{(I*b*x \\
& + I*a)}) + (24*I*d^3*\cos(6*b*x + 6*a) + 24*I*d^3*\cos(4*b*x + 4*a) - 24*I*d^3 \\
& * \cos(2*b*x + 2*a) - 24*d^3*\sin(6*b*x + 6*a) - 24*d^3*\sin(4*b*x + 4*a) + 24* \\
& d^3*\sin(2*b*x + 2*a) - 24*I*d^3)*\text{polylog}(3, -e^{(I*b*x + I*a)}) + (-24*I*d^3* \\
& \cos(6*b*x + 6*a) - 24*I*d^3*\cos(4*b*x + 4*a) + 24*I*d^3*\cos(2*b*x + 2*a) + \\
& 24*d^3*\sin(6*b*x + 6*a) + 24*d^3*\sin(4*b*x + 4*a) - 24*d^3*\sin(2*b*x + 2*a) \\
& + 24*I*d^3)*\text{polylog}(3, e^{(I*b*x + I*a)}) + (-12*I*(b*x + a)^3*d^3 - 12*b^2*c^2*d \\
& + 24*a*b*c*d^2 - 12*a^2*d^3 + (-36*I*b*c*d^2 - 12*(-3*I*a + 1)*d^3)*(\\
& b*x + a)^2 + (-36*I*b^2*c^2*d - 24*(-3*I*a + 1)*b*c*d^2 + (-36*I*a^2 + 24*a \\
&)*d^3)*(b*x + a))*\sin(5*b*x + 5*a) + (-8*I*(b*x + a)^3*d^3 + (-24*I*b*c*d^2 \\
& + 24*I*a*d^3)*(b*x + a)^2 + (-24*I*b^2*c^2*d + 48*I*a*b*c*d^2 - 24*I*a^2*d \\
& ^3)*(b*x + a))*\sin(3*b*x + 3*a) + (-12*I*(b*x + a)^3*d^3 + 12*b^2*c^2*d - 2 \\
& 4*a*b*c*d^2 + 12*a^2*d^3 + (-36*I*b*c*d^2 - 12*(-3*I*a - 1)*d^3)*(b*x + a)^ \\
& 2 + (-36*I*b^2*c^2*d - 24*(-3*I*a - 1)*b*c*d^2 + (-36*I*a^2 - 24*a)*d^3)*(b \\
& *x + a))*\sin(b*x + a))/(-4*I*b^3*\cos(6*b*x + 6*a) - 4*I*b^3*\cos(4*b*x + 4*a \\
&) + 4*I*b^3*\cos(2*b*x + 2*a) + 4*b^3*\sin(6*b*x + 6*a) + 4*b^3*\sin(4*b*x + 4 \\
& *a) - 4*b^3*\sin(2*b*x + 2*a) + 4*I*b^3))/b
\end{aligned}$$

Fricas [C] time = 1.45295, size = 5646, normalized size = 11.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")

[Out] $1/4*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 18*I*d^3*\cos(b*x + a)^2*\text{polylog}(4, I*\cos(b*x + a) + \sin(b*x + a))*\sin(b*x + a) + 18*I*d^3*\cos(b*x + a)^2*\text{polylog}(4, I*\cos(b*x + a) - \sin(b*x + a))*\sin(b*x + a) - 18*I*d^3*\cos(b*x + a)^2*\text{polylog}(4, -I*\cos(b*x + a) + \sin(b*x + a))*\sin(b*x + a) - 18*I*d^3*\cos(b*x + a)^2*\text{polylog}(4, -I*\cos(b*x + a) - \sin(b*x + a))*\sin(b*x + a) + 12*d^3*\cos(b*x + a)^2*\text{polylog}(3, \cos(b*x + a) + I*\sin(b*x + a))*\sin(b*x + a) + 12*d^3*\cos(b*x + a)^2*\text{polylog}(3, \cos(b*x + a) - I*\sin(b*x + a))*\sin(b*x + a) - 12*d^3*\cos(b*x + a)^2*\text{polylog}(3, -\cos(b*x + a) + I*\sin(b*x + a))*\sin(b*x + a) - 12*d^3*\cos(b*x + a)^2*\text{polylog}(3, -\cos(b*x + a) - I*\sin(b*x + a))*\sin(b*x + a) + 2*b^3*c^3 + (-12*I*b*d^3*x - 12*I*b*c*d^2)*\cos(b*x + a)^2*\text{dilog}(\cos(b*x + a) + I*\sin(b*x + a))*\sin(b*x + a) + (12*I*b*d^3*x + 12*I*b*c*d^2)*\cos(b*x + a)^2*\text{dilog}(\cos(b*x + a) - I*\sin(b*x + a))*\sin(b*x + a) + (-9*I*b^2*d^3*x^2 - 18*I*b^2*c*d^2*x - 9*I*b^2*c^2*d - 6*I*d^3)*\cos(b*x + a)^2*\text{dilog}(I*\cos(b*x + a) + \sin(b*x + a))*\sin(b*x + a) + (-9*I*b^2*d^3$

$$\begin{aligned}
& x^2 - 18Ib^2cd^2x - 9Ib^2c^2d - 6Id^3) \cos(bx + a)^2 \operatorname{dilog}(I \cos(bx + a) - \sin(bx + a)) \sin(bx + a) + (9Ib^2d^3x^2 + 18Ib^2cd^2x + 9Ib^2c^2d + 6Id^3) \cos(bx + a)^2 \operatorname{dilog}(-I \cos(bx + a) + \sin(bx + a)) \sin(bx + a) + (9Ib^2d^3x^2 + 18Ib^2cd^2x + 9Ib^2c^2d + 6Id^3) \cos(bx + a)^2 \operatorname{dilog}(-I \cos(bx + a) - \sin(bx + a)) \sin(bx + a) + (-12Ibd^3x - 12Ib^2cd^2) \cos(bx + a)^2 \operatorname{dilog}(-\cos(bx + a) + I \sin(bx + a)) \sin(bx + a) + (12Ibd^3x + 12Ib^2cd^2) \cos(bx + a)^2 \operatorname{dilog}(-\cos(bx + a) - I \sin(bx + a)) \sin(bx + a) - 6(b^2d^3x^2 + 2b^2cd^2x + b^2c^2d) \cos(bx + a)^2 \log(\cos(bx + a) + I \sin(bx + a) + 1) \sin(bx + a) + 3(b^3c^3 - 3ab^2c^2d + (3a^2 + 2)b^2cd^2 - (a^3 + 2a)d^3) \cos(bx + a)^2 \log(\cos(bx + a) + I \sin(bx + a) + I) \sin(bx + a) - 6(b^2d^3x^2 + 2b^2cd^2x + b^2c^2d) \cos(bx + a)^2 \log(\cos(bx + a) - I \sin(bx + a) + 1) \sin(bx + a) - 3(b^3c^3 - 3ab^2c^2d + (3a^2 + 2)b^2cd^2 - (a^3 + 2a)d^3) \cos(bx + a)^2 \log(\cos(bx + a) - I \sin(bx + a) + I) \sin(bx + a) + 3(b^3d^3x^3 + 3b^3cd^2x^2 + 3ab^2c^2d - 3a^2b^2cd^2 + (a^3 + 2a)d^3 + (3b^3c^2d + 2bd^3)x) \cos(bx + a)^2 \log(I \cos(bx + a) + \sin(bx + a) + 1) \sin(bx + a) - 3(b^3d^3x^3 + 3b^3cd^2x^2 + 3ab^2c^2d - 3a^2b^2cd^2 + (a^3 + 2a)d^3 + (3b^3c^2d + 2bd^3)x) \cos(bx + a)^2 \log(I \cos(bx + a) - \sin(bx + a) + 1) \sin(bx + a) + 3(b^3d^3x^3 + 3b^3cd^2x^2 + 3ab^2c^2d - 3a^2b^2cd^2 + (a^3 + 2a)d^3 + (3b^3c^2d + 2bd^3)x) \cos(bx + a)^2 \log(-I \cos(bx + a) + \sin(bx + a) + 1) \sin(bx + a) - 3(b^3d^3x^3 + 3b^3cd^2x^2 + 3ab^2c^2d - 3a^2b^2cd^2 + (a^3 + 2a)d^3 + (3b^3c^2d + 2bd^3)x) \cos(bx + a)^2 \log(-I \cos(bx + a) - \sin(bx + a) + 1) \sin(bx + a) + 6(b^2c^2d - 2ab^2cd^2 + a^2d^3) \cos(bx + a)^2 \log(-1/2 \cos(bx + a) + 1/2 I \sin(bx + a) + 1/2) \sin(bx + a) + 1/2 I \sin(bx + a) + 1/2) \sin(bx + a) + 6(b^2c^2d - 2ab^2cd^2 + a^2d^3) \cos(bx + a)^2 \log(-1/2 \cos(bx + a) - 1/2 I \sin(bx + a) + 1/2) \sin(bx + a) + 6(b^2d^3x^2 + 2b^2cd^2x + 2ab^2cd^2 - a^2d^3) \cos(bx + a)^2 \log(-\cos(bx + a) + I \sin(bx + a) + 1) \sin(bx + a) + 3(b^3c^3 - 3ab^2c^2d + (3a^2 + 2)b^2cd^2 - (a^3 + 2a)d^3) \cos(bx + a)^2 \log(-\cos(bx + a) + I \sin(bx + a) + I) \sin(bx + a) + 6(b^2d^3x^2 + 2b^2cd^2x + 2ab^2cd^2 - a^2d^3) \cos(bx + a)^2 \log(-\cos(bx + a) - I \sin(bx + a) + 1) \sin(bx + a) - 3(b^3c^3 - 3ab^2c^2d + (3a^2 + 2)b^2cd^2 - (a^3 + 2a)d^3) \cos(bx + a)^2 \log(-\cos(bx + a) - I \sin(bx + a) + I) \sin(bx + a) - 18(bd^3x + b^2cd^2) \cos(bx + a)^2 \operatorname{polylog}(3, I \cos(bx + a) + \sin(bx + a)) \sin(bx + a) + 18(bd^3x + b^2cd^2) \cos(bx + a)^2 \operatorname{polylog}(3, I \cos(bx + a) - \sin(bx + a)) \sin(bx + a) - 18(bd^3x + b^2cd^2) \cos(bx + a)^2 \operatorname{polylog}(3, -I \cos(bx + a) + \sin(bx + a)) \sin(bx + a) + 18(bd^3x + b^2cd^2) \cos(bx + a)^2 \operatorname{polylog}(3, -I \cos(bx + a) - \sin(bx + a)) \sin(bx + a) - 6(b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2d^2x + b^3c^3) \cos(bx + a)^2 - 6(b^2d^3x^2 + 2b^2cd^2x + b^2c^2d) \cos(bx + a) \sin(bx + a) / (b^4 \cos(bx + a)^2 \sin(bx + a))
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*csc(b*x+a)**2*sec(b*x+a)**3,x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.318 $\int (c + dx)^2 \csc^2(a + bx) \sec^3(a + bx) dx$

Optimal. Leaf size=341

$$\frac{3id(c + dx)\text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^2} - \frac{3id(c + dx)\text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^2} + \frac{2id^2\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} - \frac{2id^2\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3}$$

[Out] $((-3I)(c + dx)^2 \text{ArcTan}[E^{I(a + bx)}])/b + (2d^2 x \text{ArcTanh}[E^{I(a + bx)}])/b^2 - (6d(c + dx) \text{ArcTanh}[E^{I(a + bx)}])/b^2 - (d^2 x \text{ArcTanh}[\text{Cos}[a + bx]])/b^2 + (d(c + dx) \text{ArcTanh}[\text{Cos}[a + bx]])/b^2 + (d^2 \text{ArcTanh}[\text{Sin}[a + bx]])/b^3 - (3(c + dx)^2 \text{Csc}[a + bx])/(2b) + ((2I)d^2 \text{PolyLog}[2, -E^{I(a + bx)}])/b^3 + ((3I)d(c + dx) \text{PolyLog}[2, (-I)E^{I(a + bx)}])/b^2 - ((3I)d(c + dx) \text{PolyLog}[2, IE^{I(a + bx)}])/b^2 - ((2I)d^2 \text{PolyLog}[2, E^{I(a + bx)}])/b^3 - (3d^2 \text{PolyLog}[3, (-I)E^{I(a + bx)}])/b^3 + (3d^2 \text{PolyLog}[3, IE^{I(a + bx)}])/b^3 - (d(c + dx) \text{Sec}[a + bx])/b^2 + ((c + dx)^2 \text{Csc}[a + bx] \text{Sec}[a + bx]^2)/(2b)$

Rubi [A] time = 0.64765, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 19, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.792$, Rules used = {2621, 288, 321, 207, 4420, 6688, 12, 6742, 6273, 4181, 2531, 2282, 6589, 4183, 2279, 2391, 2622, 6271, 3770}

$$\frac{3id(c + dx)\text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^2} - \frac{3id(c + dx)\text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^2} + \frac{2id^2\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} - \frac{2id^2\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + dx)^2 \text{Csc}[a + bx]^2 \text{Sec}[a + bx]^3, x]$

[Out] $((-3I)(c + dx)^2 \text{ArcTan}[E^{I(a + bx)}])/b + (2d^2 x \text{ArcTanh}[E^{I(a + bx)}])/b^2 - (6d(c + dx) \text{ArcTanh}[E^{I(a + bx)}])/b^2 - (d^2 x \text{ArcTanh}[\text{Cos}[a + bx]])/b^2 + (d(c + dx) \text{ArcTanh}[\text{Cos}[a + bx]])/b^2 + (d^2 \text{ArcTanh}[\text{Sin}[a + bx]])/b^3 - (3(c + dx)^2 \text{Csc}[a + bx])/(2b) + ((2I)d^2 \text{PolyLog}[2, -E^{I(a + bx)}])/b^3 + ((3I)d(c + dx) \text{PolyLog}[2, (-I)E^{I(a + bx)}])/b^2 - ((3I)d(c + dx) \text{PolyLog}[2, IE^{I(a + bx)}])/b^2 - ((2I)d^2 \text{PolyLog}[2, E^{I(a + bx)}])/b^3 - (3d^2 \text{PolyLog}[3, (-I)E^{I(a + bx)}])/b^3 + (3d^2 \text{PolyLog}[3, IE^{I(a + bx)}])/b^3 - (d(c + dx) \text{Sec}[a + bx])/b^2 + ((c + dx)^2 \text{Csc}[a + bx] \text{Sec}[a + bx]^2)/(2b)$

Rule 2621

$\text{Int}[(\text{csc}[e.] + (f.)(x.))(a.)^{(m.)} \text{sec}[(e.) + (f.)(x.)]^{(n.)}, x_Symbol] :> -\text{Dist}[(f \cdot a^n)^{-1}, \text{Subst}[\text{Int}[x^{(m+n-1)} / (-1 + x^2/a^2)^{(n+1)/2}, x], x, a \cdot \text{Csc}[e + f \cdot x], x] /; \text{FreeQ}\{a, e, f, m\}, x] \&\& \text{IntegerQ}[(n+1)/2] \&\& !(\text{IntegerQ}[(m+1)/2] \&\& \text{LtQ}[0, m, n])$

Rule 288

$\text{Int}[(c.)(x.)^{(m.)}((a.) + (b.)(x.)^{(n.)})^{(p.)}, x_Symbol] :> \text{Simp}[(c^{(n-1)}(c \cdot x)^{(m-n+1)}(a + b \cdot x^n)^{(p+1)}) / (b \cdot n \cdot (p+1)), x] - \text{Dist}[(c^{(n-m)}(m-n+1) / (b \cdot n \cdot (p+1)), \text{Int}[(c \cdot x)^{(m-n)}(a + b \cdot x^n)^{(p+1)}, x], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& \text{GtQ}[m+1, n] \&\& !\text{LtQ}[(m+n \cdot (p+1)+1)/n, 0] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 321

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[Rt[b, 2]*x]/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6688

```
Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&& !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```


Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] := Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] := Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \int (c + dx)^2 \csc^2(a + bx) \sec^3(a + bx) dx &= \frac{3(c + dx)^2 \tanh^{-1}(\sin(a + bx))}{2b} - \frac{3(c + dx)^2 \csc(a + bx)}{2b} + \frac{(c + dx)^2 \csc(a + bx)}{2b} \\
 &= \frac{3(c + dx)^2 \tanh^{-1}(\sin(a + bx))}{2b} - \frac{3(c + dx)^2 \csc(a + bx)}{2b} + \frac{(c + dx)^2 \csc(a + bx)}{2b} \\
 &= \frac{3(c + dx)^2 \tanh^{-1}(\sin(a + bx))}{2b} - \frac{3(c + dx)^2 \csc(a + bx)}{2b} + \frac{(c + dx)^2 \csc(a + bx)}{2b} \\
 &= \frac{3(c + dx)^2 \tanh^{-1}(\sin(a + bx))}{2b} - \frac{3(c + dx)^2 \csc(a + bx)}{2b} + \frac{(c + dx)^2 \csc(a + bx)}{2b} \\
 &= \frac{3(c + dx)^2 \tanh^{-1}(\sin(a + bx))}{2b} - \frac{3(c + dx)^2 \csc(a + bx)}{2b} + \frac{(c + dx)^2 \csc(a + bx)}{2b} \\
 &= \frac{d(c + dx) \tanh^{-1}(\cos(a + bx))}{b^2} + \frac{3(c + dx)^2 \tanh^{-1}(\sin(a + bx))}{2b} - \frac{3(c + dx)^2 \csc(a + bx)}{2b} \\
 &= \frac{d(c + dx) \tanh^{-1}(\cos(a + bx))}{b^2} + \frac{3(c + dx)^2 \tanh^{-1}(\sin(a + bx))}{2b} - \frac{3(c + dx)^2 \csc(a + bx)}{2b} \\
 &= -\frac{6d(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{d^2x \tanh^{-1}(\cos(a + bx))}{b^2} + \frac{d(c + dx) \tanh^{-1}(\cos(a + bx))}{b^2} \\
 &= -\frac{6d(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{d^2x \tanh^{-1}(\cos(a + bx))}{b^2} + \frac{d(c + dx) \tanh^{-1}(\cos(a + bx))}{b^2} \\
 &= -\frac{3i(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b} + \frac{2d^2x \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{6d(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^2} \\
 &= -\frac{3i(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b} + \frac{2d^2x \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{6d(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^2} \\
 &= -\frac{3i(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b} + \frac{2d^2x \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{6d(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^2} \\
 &= -\frac{3i(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b} + \frac{2d^2x \tanh^{-1}(e^{i(a+bx)})}{b^2} - \frac{6d(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b^2}
 \end{aligned}$$

Mathematica [B] time = 7.56174, size = 889, normalized size = 2.61

$$2 \frac{\left(\left((bx + \tan^{-1}(\tan(a))) \left(\log\left(1 - e^{i(bx + \tan^{-1}(\tan(a)))} \right) - \log\left(1 + e^{i(bx + \tan^{-1}(\tan(a)))} \right) \right) \right) + i \left(\text{PolyLog}\left(2, -e^{i(bx + \tan^{-1}(\tan(a)))} \right) - \text{PolyLog}\left(2, e^{i(bx + \tan^{-1}(\tan(a)))} \right) \right) \right) \sec(a)}{\sqrt{\tan^2(a) + 1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] -((6*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + (4*I)*d^2*ArcTan[E^(I*(a + b*x))]
- 6*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] - 3*b^2*d^2*x^2*Log[1 - I*E^(I*(a
+ b*x))] + 6*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x))] + 3*b^2*d^2*x^2*Log[1 +
I*E^(I*(a + b*x))] - (6*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] +
(6*I)*b*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] + 6*d^2*PolyLog[3, (-I)*
E^(I*(a + b*x))] - 6*d^2*PolyLog[3, I*E^(I*(a + b*x))]/(2*b^3) - ((c + d*x)
)*Csc[a]*Sec[a]*(b*c*Cos[a] + b*d*x*Cos[a] + d*Sin[a])/b^2 + ((4*I)*c*d*Ar
cTan[(I*Cos[a] - I*Sin[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2])/b^2*Sq
rt[Cos[a]^2 + Sin[a]^2] + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c^2*Sin[(b*x)/2]
) - 2*c*d*x*Sin[(b*x)/2] - d^2*x^2*Sin[(b*x)/2]))/(2*b) + (Csc[a/2]*Csc[a/2
```

$$\begin{aligned}
& + (b*x)/2*(c^2*\sin[(b*x)/2] + 2*c*d*x*\sin[(b*x)/2] + d^2*x^2*\sin[(b*x)/2]) \\
&)/(2*b) + (c^2 + 2*c*d*x + d^2*x^2)/(4*b*(\cos[a/2 + (b*x)/2] - \sin[a/2 + (b*x)/2])^2) \\
& + (-c*d*\sin[(b*x)/2]) - d^2*x*\sin[(b*x)/2])/(b^2*(\cos[a/2] - \sin[a/2])*(\cos[a/2 + (b*x)/2] - \sin[a/2 + (b*x)/2])) \\
& + (-c^2 - 2*c*d*x - d^2*x^2)/(4*b*(\cos[a/2 + (b*x)/2] + \sin[a/2 + (b*x)/2])^2) + (c*d*\sin[(b*x)/2] \\
& + d^2*x*\sin[(b*x)/2])/(b^2*(\cos[a/2] + \sin[a/2])*(\cos[a/2 + (b*x)/2] + \sin[a/2 + (b*x)/2])) \\
& + (2*d^2*((-2*\text{ArcTan}[\text{Tan}[a]]*\text{ArcTanh}[(-\cos[a] + \sin[a]*\text{Tan}[(b*x)/2])/\text{sqrt}[\cos[a]^2 + \sin[a]^2]])/\text{sqrt}[\cos[a]^2 + \sin[a]^2] + ((b*x + \text{ArcTan}[\text{Tan}[a]])*(\log[1 - E^{(I*(b*x + \text{ArcTan}[\text{Tan}[a]))})]) - \log[1 + E^{(I*(b*x + \text{ArcTan}[\text{Tan}[a]))})]) + I*(\text{PolyLog}[2, -E^{(I*(b*x + \text{ArcTan}[\text{Tan}[a]))})]) - \text{PolyLog}[2, E^{(I*(b*x + \text{ArcTan}[\text{Tan}[a]))})])]*\text{Sec}[a])/\text{sqrt}[1 + \text{Tan}[a]^2]))/b^3
\end{aligned}$$

Maple [B] time = 0.499, size = 770, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x)

[Out] $2*d/b^2*c*\ln(\exp(I*(b*x+a))-1)-2*d/b^2*c*\ln(\exp(I*(b*x+a))+1)-2*d^2/b^2*\ln(\exp(I*(b*x+a))+1)*x-2*d^2/b^3*a*\ln(\exp(I*(b*x+a))-1)-3/b*c*d*\ln(1+I*\exp(I*(b*x+a)))*x+3/2/b*d^2*\ln(1-I*\exp(I*(b*x+a)))*x^2+3/2/b^3*d^2*a^2*\ln(1+I*\exp(I*(b*x+a)))-3/2/b^3*d^2*a^2*\ln(1-I*\exp(I*(b*x+a)))-3/2/b*d^2*\ln(1+I*\exp(I*(b*x+a)))*x^2-2*I/b^3*d^2*\arctan(\exp(I*(b*x+a)))-I/b^2/(\exp(2*I*(b*x+a))+1)^2/(\exp(2*I*(b*x+a))-1)*(3*d^2*x^2*b*\exp(5*I*(b*x+a))+6*c*d*x*b*\exp(5*I*(b*x+a))+3*c^2*b*\exp(5*I*(b*x+a))+2*d^2*x^2*b*\exp(3*I*(b*x+a))+4*c*d*x*b*\exp(3*I*(b*x+a))-2*I*d^2*x*\exp(5*I*(b*x+a))+2*c^2*b*\exp(3*I*(b*x+a))+3*d^2*x^2*b*\exp(I*(b*x+a))-2*I*d*c*\exp(5*I*(b*x+a))+6*c*d*x*b*\exp(I*(b*x+a))+3*c^2*b*\exp(I*(b*x+a))+2*I*d^2*x*\exp(I*(b*x+a))+2*I*d*c*\exp(I*(b*x+a)))-3*d^2*\text{polylog}(3,-I*\exp(I*(b*x+a)))/b^3+3*d^2*\text{polylog}(3,I*\exp(I*(b*x+a)))/b^3-3/b^2*c*d*\ln(1+I*\exp(I*(b*x+a)))*a+3/b*c*d*\ln(1-I*\exp(I*(b*x+a)))*x+3/b^2*c*d*\ln(1-I*\exp(I*(b*x+a)))*a+6*I/b^2*a*c*d*\arctan(\exp(I*(b*x+a)))+2*I/b^3*\text{dilog}(\exp(I*(b*x+a))+1)*d^2-3*I/b*c^2*\arctan(\exp(I*(b*x+a)))+2*I/b^3*d^2*\text{dilog}(\exp(I*(b*x+a)))-3*I/b^2*d^2*\text{polylog}(2,I*\exp(I*(b*x+a)))*x-3*I/b^3*d^2*a^2*\arctan(\exp(I*(b*x+a)))+3*I/b^2*d^2*\text{polylog}(2,-I*\exp(I*(b*x+a)))*x+3*I/b^2*c*d*\text{polylog}(2,-I*\exp(I*(b*x+a)))-3*I/b^2*c*d*\text{polylog}(2,I*\exp(I*(b*x+a)))$

Maxima [B] time = 5.44259, size = 5154, normalized size = 15.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")

[Out] $-1/4*(c^2*(2*(3*\sin(b*x + a)^2 - 2)/(\sin(b*x + a)^3 - \sin(b*x + a)) - 3*\log(\sin(b*x + a) + 1) + 3*\log(\sin(b*x + a) - 1)) - 2*a*c*d*(2*(3*\sin(b*x + a)^2 - 2)/(\sin(b*x + a)^3 - \sin(b*x + a)) - 3*\log(\sin(b*x + a) + 1) + 3*\log(\sin(b*x + a) - 1))/b + a^2*d^2*(2*(3*\sin(b*x + a)^2 - 2)/(\sin(b*x + a)^3 - \sin(b*x + a)) - 3*\log(\sin(b*x + a) + 1) + 3*\log(\sin(b*x + a) - 1))/b^2 - 4*((6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2)*(b*x + a) + 4*d^2 - 2*(3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*\cos(6*b*x + 6*a) - 2*(3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*\cos(4*b*x + 4*a) + 2*(3*(b$

$$\begin{aligned}
& x + a)^2 d^2 + 6(bcd - ad^2)(bx + a) + 2d^2) \cos(2bx + 2a) + (-6I(bx + a)^2 d^2 + (-12Ib^2 cd + 12Ia^2 d^2)(bx + a) - 4I^2 d^2) \sin(6bx + 6a) \\
& + (-6I^2(bx + a)^2 d^2 + (-12I^2 b^2 cd + 12I^2 a^2 d^2)(bx + a) - 4I^2 d^2) \sin(4bx + 4a) + (6I^2(bx + a)^2 d^2 + (12I^2 b^2 cd - 12I^2 a^2 d^2) \\
&) (bx + a) + 4I^2 d^2) \sin(2bx + 2a)) \arctan2(\cos(bx + a), \sin(bx + a) + 1) + (6(bx + a)^2 d^2 + 12(bcd - ad^2)(bx + a) + 4d^2 - 2(3(bx + a)^2 d^2 + 6(bcd - ad^2)(bx + a) + 2d^2) \cos(6bx + 6a) - 2(3(3(bx + a)^2 d^2 + 6(bcd - ad^2)(bx + a) + 2d^2) \cos(4bx + 4a) + 2(3(bx + a)^2 d^2 + 6(bcd - ad^2)(bx + a) + 2d^2) \cos(2bx + 2a) + (-6I^2(bx + a)^2 d^2 + (-12I^2 b^2 cd + 12I^2 a^2 d^2)(bx + a) - 4I^2 d^2) \sin(6bx + 6a) + (-6I^2(bx + a)^2 d^2 + (-12I^2 b^2 cd + 12I^2 a^2 d^2)(bx + a) - 4I^2 d^2) \sin(4bx + 4a) + (6I^2(bx + a)^2 d^2 + (12I^2 b^2 cd - 12I^2 a^2 d^2)(bx + a) + 4I^2 d^2) \sin(2bx + 2a)) \arctan2(\cos(bx + a), -\sin(bx + a) + 1) + (8bcd + 8(bx + a)d^2 - 8a^2 d^2 - 8(bcd + (bx + a)d^2 - ad^2) \cos(6bx + 6a) - 8(bcd + (bx + a)d^2 - ad^2) \cos(4bx + 4a) + 8(bcd + (bx + a)d^2 - ad^2) \cos(2bx + 2a) + (-8I^2 b^2 cd - 8I^2(bx + a)d^2 + 8I^2 a^2 d^2) \sin(6bx + 6a) + (-8I^2 b^2 cd - 8I^2(bx + a)d^2 + 8I^2 a^2 d^2) \sin(4bx + 4a) + (8I^2 b^2 cd + 8I^2(bx + a)d^2 - 8I^2 a^2 d^2) \sin(2bx + 2a)) \arctan2(\sin(bx + a), \cos(bx + a) + 1) - (8bcd - 8a^2 d^2 - 8(bcd - ad^2) \cos(6bx + 6a) - 8(bcd - ad^2) \cos(4bx + 4a) + 8(bcd - ad^2) \cos(2bx + 2a) - (8I^2 b^2 cd - 8I^2 a^2 d^2) \sin(6bx + 6a) - (8I^2 b^2 cd - 8I^2 a^2 d^2) \sin(4bx + 4a) - (-8I^2 b^2 cd + 8I^2 a^2 d^2) \sin(2bx + 2a)) \arctan2(\sin(bx + a), \cos(bx + a) - 1) - (8(bx + a)d^2 \cos(6bx + 6a) + 8(bx + a)d^2 \cos(4bx + 4a) - 8(bx + a)d^2 \cos(2bx + 2a) + 8I^2(bx + a)d^2 \sin(6bx + 6a) + 8I^2(bx + a)d^2 \sin(4bx + 4a) - 8I^2(bx + a)d^2 \sin(2bx + 2a) - 8(bx + a)d^2) \arctan2(\sin(bx + a), -\cos(bx + a) + 1) - (12(bx + a)^2 d^2 - 8I^2 b^2 cd + 8I^2 a^2 d^2 + (24bcd - (24a + 8I)d^2)(bx + a)) \cos(5bx + 5a) - 8((bx + a)^2 d^2 + 2(bcd - ad^2)(bx + a)) \cos(3bx + 3a) - (12(bx + a)^2 d^2 + 8I^2 b^2 cd - 8I^2 a^2 d^2 + (24bcd - (24a - 8I)d^2)(bx + a)) \cos(bx + a) + (12b^2 cd + 12(bx + a)d^2 - 12a^2 d^2 - 12(bcd + (bx + a)d^2 - ad^2) \cos(6bx + 6a) - 12(bcd + (bx + a)d^2 - ad^2) \cos(4bx + 4a) + 12(bcd + (bx + a)d^2 - ad^2) \cos(2bx + 2a) + (-12I^2 b^2 cd - 12I^2(bx + a)d^2 + 12I^2 a^2 d^2) \sin(6bx + 6a) + (-12I^2 b^2 cd - 12I^2(bx + a)d^2 + 12I^2 a^2 d^2) \sin(4bx + 4a) + (12I^2 b^2 cd + 12I^2(bx + a)d^2 - 12I^2 a^2 d^2) \sin(2bx + 2a)) \operatorname{dilog}(Ie^{(Ibx + Ia)}) - (12b^2 cd + 12(bx + a)d^2 - 12a^2 d^2 - 12(bcd + (bx + a)d^2 - ad^2) \cos(6bx + 6a) - 12(bcd + (bx + a)d^2 - ad^2) \cos(4bx + 4a) + 12(bcd + (bx + a)d^2 - ad^2) \cos(2bx + 2a) - (12I^2 b^2 cd + 12I^2(bx + a)d^2 - 12I^2 a^2 d^2) \sin(6bx + 6a) - (12I^2 b^2 cd + 12I^2(bx + a)d^2 - 12I^2 a^2 d^2) \sin(4bx + 4a) - (-12I^2 b^2 cd - 12I^2(bx + a)d^2 + 12I^2 a^2 d^2) \sin(2bx + 2a)) \operatorname{dilog}(-Ie^{(Ibx + Ia)}) + (8d^2 \cos(6bx + 6a) + 8d^2 \cos(4bx + 4a) - 8d^2 \cos(2bx + 2a) + 8I^2 d^2 \sin(6bx + 6a) + 8I^2 d^2 \sin(4bx + 4a) - 8I^2 d^2 \sin(2bx + 2a) - 8d^2) \operatorname{dilog}(-e^{(Ibx + Ia)}) - (8d^2 \cos(6bx + 6a) + 8d^2 \cos(4bx + 4a) - 8d^2 \cos(2bx + 2a) + 8I^2 d^2 \sin(6bx + 6a) + 8I^2 d^2 \sin(4bx + 4a) - 8I^2 d^2 \sin(2bx + 2a) - 8d^2) \operatorname{dilog}(e^{(Ibx + Ia)}) + (-4I^2 b^2 cd - 4I^2(bx + a)d^2 + 4I^2 a^2 d^2 + (4I^2 b^2 cd + 4I^2(bx + a)d^2 - 4I^2 a^2 d^2) \cos(6bx + 6a) + (4I^2 b^2 cd + 4I^2(bx + a)d^2 - 4I^2 a^2 d^2) \cos(4bx + 4a) + (-4I^2 b^2 cd - 4I^2(bx + a)d^2 + 4I^2 a^2 d^2) \cos(2bx + 2a) - 4(bcd + (bx + a)d^2 - ad^2) \sin(6bx + 6a) - 4(bcd + (bx + a)d^2 - ad^2) \sin(4bx + 4a) + 4(bcd + (bx + a)d^2 - ad^2) \sin(2bx + 2a)) \log(\cos(bx + a)^2 + \sin(bx + a)^2 + 2\cos(bx + a) + 1) + (4I^2 b^2 cd + 4I^2(bx + a)d^2 - 4I^2 a^2 d^2 + (-4I^2 b^2 cd - 4I^2(bx + a)d^2 + 4I^2 a^2 d^2) \cos(6bx + 6a) + (-4I^2 b^2 cd - 4I^2(bx + a)d^2 + 4I^2 a^2 d^2) \cos(4bx + 4a) + (4I^2 b^2 cd + 4I^2(bx + a)d^2 - 4I^2 a^2 d^2) \cos(2bx + 2a) + 4(bcd + (bx + a)d^2 - ad^2) \sin(6bx + 6a) + 4(bcd + (bx + a)d^2 - ad^2) \sin(4bx + 4a) - 4(bcd + (bx + a)d^2 - ad^2) \sin(2bx + 2a)) \log(\cos(bx + a)^2 + \sin(bx + a)^2 - 2\cos(bx + a) + 1)
\end{aligned}$$

$$\begin{aligned}
& + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a) + 2*I*d^2 + (-3*I*(b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) - 2*I*d^2)*\cos(6*b*x + 6*a) \\
& + (-3*I*(b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) - 2*I*d^2)*\cos(4*b*x + 4*a) + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a) + 2*I*d^2)*\cos(2*b*x + 2*a) \\
& + (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*\sin(6*b*x + 6*a) + (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*\sin(4*b*x + 4*a) \\
& - (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\sin(b*x + a) + 1) \\
& + (-3*I*(b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) - 2*I*d^2 + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a) + 2*I*d^2)*\cos(6*b*x + 6*a) \\
& + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a) + 2*I*d^2)*\cos(4*b*x + 4*a) + (-3*I*(b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) - 2*I*d^2)*\cos(2*b*x + 2*a) \\
& - (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*\sin(6*b*x + 6*a) - (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*\sin(4*b*x + 4*a) \\
& + (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*\sin(2*b*x + 2*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\sin(b*x + a) + 1) \\
& + (-12*I*d^2*\cos(6*b*x + 6*a) - 12*I*d^2*\cos(4*b*x + 4*a) + 12*I*d^2*\cos(2*b*x + 2*a) + 12*d^2*\sin(6*b*x + 6*a) + 12*d^2*\sin(4*b*x + 4*a) - 12*d^2*\sin(2*b*x + 2*a) + 12*I*d^2)*\text{polylog}(3, I*e^{(I*b*x + I*a)}) \\
& + (12*I*d^2*\cos(6*b*x + 6*a) + 12*I*d^2*\cos(4*b*x + 4*a) - 12*I*d^2*\cos(2*b*x + 2*a) - 12*d^2*\sin(6*b*x + 6*a) - 12*d^2*\sin(4*b*x + 4*a) + 12*d^2*\sin(2*b*x + 2*a) - 12*I*d^2)*\text{polylog}(3, -I*e^{(I*b*x + I*a)}) \\
& + (-12*I*(b*x + a)^2*d^2 - 8*b*c*d + 8*a*d^2 + (-24*I*b*c*d - 8*(-3*I*a + 1)*d^2)*(b*x + a))*\sin(5*b*x + 5*a) + (-8*I*(b*x + a)^2*d^2 + (-16*I*b*c*d + 16*I*a*d^2)*(b*x + a))*\sin(3*b*x + 3*a) \\
& + (-12*I*(b*x + a)^2*d^2 + 8*b*c*d - 8*a*d^2 + (-24*I*b*c*d - 8*(-3*I*a - 1)*d^2)*(b*x + a))*\sin(b*x + a))/(-4*I*b^2*\cos(6*b*x + 6*a) - 4*I*b^2*\cos(4*b*x + 4*a) + 4*I*b^2*\cos(2*b*x + 2*a) + 4*b^2*\sin(6*b*x + 6*a) + 4*b^2*\sin(4*b*x + 4*a) - 4*b^2*\sin(2*b*x + 2*a) + 4*I*b^2))/b
\end{aligned}$$

Fricas [C] time = 0.972988, size = 3596, normalized size = 10.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")

[Out] $1/4*(2*b^2*d^2*x^2 - 4*I*d^2*\cos(b*x + a)^2*\text{dilog}(\cos(b*x + a) + I*\sin(b*x + a))*\sin(b*x + a) + 4*I*d^2*\cos(b*x + a)^2*\text{dilog}(\cos(b*x + a) - I*\sin(b*x + a))*\sin(b*x + a) - 4*I*d^2*\cos(b*x + a)^2*\text{dilog}(-\cos(b*x + a) + I*\sin(b*x + a))*\sin(b*x + a) + 4*I*d^2*\cos(b*x + a)^2*\text{dilog}(-\cos(b*x + a) - I*\sin(b*x + a))*\sin(b*x + a) - 6*d^2*\cos(b*x + a)^2*\text{polylog}(3, I*\cos(b*x + a) + \sin(b*x + a))*\sin(b*x + a) + 6*d^2*\cos(b*x + a)^2*\text{polylog}(3, I*\cos(b*x + a) - \sin(b*x + a))*\sin(b*x + a) - 6*d^2*\cos(b*x + a)^2*\text{polylog}(3, -I*\cos(b*x + a) + \sin(b*x + a))*\sin(b*x + a) + 6*d^2*\cos(b*x + a)^2*\text{polylog}(3, -I*\cos(b*x + a) - \sin(b*x + a))*\sin(b*x + a) + 4*b^2*c*d*x + (-6*I*b*d^2*x - 6*I*b*c*d)*\cos(b*x + a)^2*\text{dilog}(I*\cos(b*x + a) + \sin(b*x + a))*\sin(b*x + a) + (-6*I*b*d^2*x - 6*I*b*c*d)*\cos(b*x + a)^2*\text{dilog}(I*\cos(b*x + a) - \sin(b*x + a))*\sin(b*x + a) + (6*I*b*d^2*x + 6*I*b*c*d)*\cos(b*x + a)^2*\text{dilog}(-I*\cos(b*x + a) + \sin(b*x + a))*\sin(b*x + a) + (6*I*b*d^2*x + 6*I*b*c*d)*\cos(b*x + a)^2*\text{dilog}(-I*\cos(b*x + a) - \sin(b*x + a))*\sin(b*x + a) - 4*(b*d^2*x + b*c*d)*\cos(b*x + a)^2*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1)*\sin(b*x + a) + (3*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*\cos(b*x + a)^2*\log(\cos(b*x + a) + I*\sin(b*x + a) + I)*\sin(b*x + a) - 4*(b*d^2*x + b*c*d)*\cos(b*x + a)^2*\log(\cos(b*x + a) - I*\sin(b*x + a) + 1)*\sin(b*x + a) - (3*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*\cos(b*x + a)^2*\log(\cos(b*x + a) - I*\sin(b*x + a) + I)*\sin(b*x + a)$

```

+ 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*
cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*
x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) +
1)*sin(b*x + a) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(
b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*d^
2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(-I*cos(b*x +
a) - sin(b*x + a) + 1)*sin(b*x + a) + 4*(b*c*d - a*d^2)*cos(b*x + a)^2*log(
-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + 4*(b*c*d - a*d
^2)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*
x + a) + 4*(b*d^2*x + a*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x +
a) + 1)*sin(b*x + a) + (3*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x +
a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 4*(b*d^2*x + a
*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) -
(3*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x + a)^2*log(-cos(b*x + a)
- I*sin(b*x + a) + I)*sin(b*x + a) + 2*b^2*c^2 - 6*(b^2*d^2*x^2 + 2*b^2*c*
d*x + b^2*c^2)*cos(b*x + a)^2 - 4*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x +
a))/(b^3*cos(b*x + a)^2*sin(b*x + a))

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \csc(bx + a)^2 \sec(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

3.319 $\int (c + dx) \csc^2(a + bx) \sec^3(a + bx) dx$

Optimal. Leaf size=162

$$\frac{3 \operatorname{idPolyLog}\left(2, -ie^{i(a+bx)}\right)}{2b^2} - \frac{3 \operatorname{idPolyLog}\left(2, ie^{i(a+bx)}\right)}{2b^2} - \frac{d \sec(a+bx)}{2b^2} - \frac{d \tanh^{-1}(\cos(a+bx))}{b^2} - \frac{3(c+dx) \csc(a+bx)}{2b}$$

[Out] $((-3I)*d*x*ArcTan[E^{I*(a + b*x)}])/b - (d*ArcTanh[Cos[a + b*x]])/b^2 + (3*c*ArcTanh[Sin[a + b*x]])/(2*b) - (3*(c + d*x)*Csc[a + b*x])/(2*b) + (((3I)/2)*d*PolyLog[2, (-I)*E^{I*(a + b*x)}])/b^2 - (((3I)/2)*d*PolyLog[2, I*E^{I*(a + b*x)}])/b^2 - (d*Sec[a + b*x])/(2*b^2) + ((c + d*x)*Csc[a + b*x]*Sec[a + b*x]^2)/(2*b)$

Rubi [A] time = 0.195867, antiderivative size = 182, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {2621, 288, 321, 207, 4420, 6271, 12, 4181, 2279, 2391, 3770, 2622}

$$\frac{3 \operatorname{idPolyLog}\left(2, -ie^{i(a+bx)}\right)}{2b^2} - \frac{3 \operatorname{idPolyLog}\left(2, ie^{i(a+bx)}\right)}{2b^2} - \frac{d \sec(a+bx)}{2b^2} - \frac{d \tanh^{-1}(\cos(a+bx))}{b^2} - \frac{3(c+dx) \csc(a+bx)}{2b}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^3, x]$

[Out] $((-3I)*d*x*ArcTan[E^{I*(a + b*x)}])/b - (d*ArcTanh[Cos[a + b*x]])/b^2 - (3*d*x*ArcTanh[Sin[a + b*x]])/(2*b) + (3*(c + d*x)*ArcTanh[Sin[a + b*x]])/(2*b) - (3*(c + d*x)*Csc[a + b*x])/(2*b) + (((3I)/2)*d*PolyLog[2, (-I)*E^{I*(a + b*x)}])/b^2 - (((3I)/2)*d*PolyLog[2, I*E^{I*(a + b*x)}])/b^2 - (d*Sec[a + b*x])/(2*b^2) + ((c + d*x)*Csc[a + b*x]*Sec[a + b*x]^2)/(2*b)$

Rule 2621

$\operatorname{Int}[(\csc[(e_{.}) + (f_{.})*(x_{.})]*(a_{.}))^{(m_{.})} \sec[(e_{.}) + (f_{.})*(x_{.})]^{(n_{.})}, x_Symbol] :> -\operatorname{Dist}[(f*a^n)^{-1}, \operatorname{Subst}[\operatorname{Int}[x^{(m+n-1)} / (-1 + x^2/a^2)]^{((n+1)/2)}, x], x, a*Csc[e + f*x]], x] /; \operatorname{FreeQ}\{a, e, f, m\}, x \ \&\& \operatorname{IntegerQ}[(n+1)/2] \ \&\& \operatorname{IntegerQ}[(m+1)/2] \ \&\& \operatorname{LtQ}[0, m, n]$

Rule 288

$\operatorname{Int}[(c_{.})*(x_{.})^{(m_{.})} * ((a_{.}) + (b_{.})*(x_{.})^{(n_{.})})^{(p_{.})}, x_Symbol] :> \operatorname{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)}) / (b*n*(p+1)), x] - \operatorname{Dist}[(c^n*(m-n+1)) / (b*n*(p+1)), \operatorname{Int}[(c*x)^{(m-n)}*(a + b*x^n)^{(p+1)}, x], x] /; \operatorname{FreeQ}\{a, b, c\}, x \ \&\& \operatorname{IGtQ}[n, 0] \ \&\& \operatorname{LtQ}[p, -1] \ \&\& \operatorname{GtQ}[m+1, n] \ \&\& \operatorname{LtQ}[(m+n*(p+1)+1)/n, 0] \ \&\& \operatorname{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 321

$\operatorname{Int}[(c_{.})*(x_{.})^{(m_{.})} * ((a_{.}) + (b_{.})*(x_{.})^{(n_{.})})^{(p_{.})}, x_Symbol] :> \operatorname{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)}) / (b*(m+n*p+1)), x] - \operatorname{Dist}[(a*c^n*(m-n+1)) / (b*(m+n*p+1)), \operatorname{Int}[(c*x)^{(m-n)}*(a + b*x^n)^p, x], x] /; \operatorname{FreeQ}\{a, b, c, p\}, x \ \&\& \operatorname{IGtQ}[n, 0] \ \&\& \operatorname{GtQ}[m, n-1] \ \&\& \operatorname{NeQ}[m+n*p+1, 0] \ \&\& \operatorname{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 207

$\operatorname{Int}[(a_{.}) + (b_{.})*(x_{.})^2]^{-1}, x_Symbol] :> -\operatorname{Simp}[\operatorname{ArcTanh}[(\operatorname{Rt}[b, 2]*x) / \operatorname{Rt}[-a, 2]] / (\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& \operatorname{LtQ}[a$

, 0] || GtQ[b, 0])

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] := Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] := Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\int (c + dx) \csc^2(a + bx) \sec^3(a + bx) dx &= \frac{3(c + dx) \tanh^{-1}(\sin(a + bx))}{2b} - \frac{3(c + dx) \csc(a + bx)}{2b} + \frac{(c + dx) \csc(a + bx)}{2b} \\
&= \frac{3(c + dx) \tanh^{-1}(\sin(a + bx))}{2b} - \frac{3(c + dx) \csc(a + bx)}{2b} + \frac{(c + dx) \csc(a + bx)}{2b} \\
&= -\frac{3d \tanh^{-1}(\cos(a + bx))}{2b^2} - \frac{3dx \tanh^{-1}(\sin(a + bx))}{2b} + \frac{3(c + dx) \tanh^{-1}(\sin(a + bx))}{2b} \\
&= -\frac{3d \tanh^{-1}(\cos(a + bx))}{2b^2} - \frac{3dx \tanh^{-1}(\sin(a + bx))}{2b} + \frac{3(c + dx) \tanh^{-1}(\sin(a + bx))}{2b} \\
&= -\frac{3idx \tan^{-1}(e^{i(a+bx)})}{b} - \frac{d \tanh^{-1}(\cos(a + bx))}{b^2} - \frac{3dx \tanh^{-1}(\sin(a + bx))}{2b} \\
&= -\frac{3idx \tan^{-1}(e^{i(a+bx)})}{b} - \frac{d \tanh^{-1}(\cos(a + bx))}{b^2} - \frac{3dx \tanh^{-1}(\sin(a + bx))}{2b} \\
&= -\frac{3idx \tan^{-1}(e^{i(a+bx)})}{b} - \frac{d \tanh^{-1}(\cos(a + bx))}{b^2} - \frac{3dx \tanh^{-1}(\sin(a + bx))}{2b}
\end{aligned}$$

Mathematica [C] time = 6.57654, size = 669, normalized size = 4.13

$$\frac{c \csc(a + bx) \operatorname{Hypergeometric2F1}\left(-\frac{1}{2}, 2, \frac{1}{2}, \sin^2(a + bx)\right)}{b} - \frac{3dx \left(-i \left(\operatorname{PolyLog}\left(2, \frac{1}{2} \left((1 + i) - (1 - i) \tan\left(\frac{1}{2}(a + bx)\right)\right)\right)\right)}{b}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^3,x]

[Out] (d*(a*Cos[(a + b*x)/2] - (a + b*x)*Cos[(a + b*x)/2])*Csc[(a + b*x)/2])/(2*b^2) - (c*Csc[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, Sin[a + b*x]^2])/b - (d*Log[Cos[(a + b*x)/2]])/b^2 + (d*Log[Sin[(a + b*x)/2]])/b^2 - (3*d*x*(a*Log[1 - Tan[(a + b*x)/2]] - a*Log[1 + Tan[(a + b*x)/2]] - I*(Log[1 + I*Tan[(a + b*x)/2]]*Log[(1/2 - I/2)*(1 + Tan[(a + b*x)/2]]) + PolyLog[2, ((1 + I) - (1 - I)*Tan[(a + b*x)/2])/2]) + I*(Log[1 - I*Tan[(a + b*x)/2]]*Log[(1/2 + I/2)*(1 + Tan[(a + b*x)/2]]) + PolyLog[2, (-1/2 - I/2)*(I + Tan[(a + b*x)/2]]) - I*(Log[1 - I*Tan[(a + b*x)/2]]*Log[(-1/2 + I/2)*(-1 + Tan[(a + b*x)/2]]) + PolyLog[2, ((1 + I) + (1 - I)*Tan[(a + b*x)/2])/2]) + I*(Log[1 + I*Tan[(a + b*x)/2]]*Log[(-1/2 - I/2)*(-1 + Tan[(a + b*x)/2]]) + PolyLog[2, ((1 - I) + (1 + I)*Tan[(a + b*x)/2])/2]))/(2*b*(a - I*Log[1 - I*Tan[(a + b*x)/2]] + I*Log[1 + I*Tan[(a + b*x)/2]])) + (d*x)/(4*b*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])^2) - (d*Sin[(a + b*x)/2])/(2*b^2*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])) - (d*x)/(4*b*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])^2) + (d*Sin[(a + b*x)/2])/(2*b^2*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])) + (d*Sec[(a + b*x)/2]*(a*Sin[(a + b*x)/2] - (a + b*x)*Sin[(a + b*x)/2]))/(2*b^2)

Maple [B] time = 0.355, size = 344, normalized size = 2.1

$$\frac{-i(3 dxbe^{5i(bx+a)} + 3 bce^{5i(bx+a)} + 2 dxbe^{3i(bx+a)} + 2 bce^{3i(bx+a)} - ide^{5i(bx+a)} + 3 dxbe^{i(bx+a)} + 3 bce^{i(bx+a)} + ide^{i(bx+a)})}{b^2 (e^{2i(bx+a)} + 1)^2 (e^{2i(bx+a)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

```
[Out] -I/b^2/(exp(2*I*(b*x+a))+1)^2/(exp(2*I*(b*x+a))-1)*(3*d*x*b*exp(5*I*(b*x+a))
)+3*b*c*exp(5*I*(b*x+a))+2*d*x*b*exp(3*I*(b*x+a))+2*b*c*exp(3*I*(b*x+a))-I*
d*exp(5*I*(b*x+a))+3*d*x*b*exp(I*(b*x+a))+3*b*c*exp(I*(b*x+a))+I*d*exp(I*(b
*x+a))-3*I/b*c*arctan(exp(I*(b*x+a)))+3*I/b^2*d*a*arctan(exp(I*(b*x+a)))+d
/b^2*ln(exp(I*(b*x+a))-1)-d/b^2*ln(exp(I*(b*x+a))+1)+3/2*I/b^2*d*dilog(1+I*
exp(I*(b*x+a)))-3/2/b*d*ln(1+I*exp(I*(b*x+a)))*x-3/2/b^2*d*ln(1+I*exp(I*(b*
x+a)))*a+3/2/b*d*ln(1-I*exp(I*(b*x+a)))*x+3/2/b^2*d*ln(1-I*exp(I*(b*x+a)))*
a-3/2*I/b^2*d*dilog(1-I*exp(I*(b*x+a)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 0.696318, size = 1644, normalized size = 10.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(-3*I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a
) - 3*I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a)
+ 3*I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) +
3*I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) +
3*(b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x
+ a) - 3*(b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I)
*sin(b*x + a) - 2*d*cos(b*x + a)^2*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a)
+ 3*(b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1)*si
n(b*x + a) - 3*(b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x +
a) + 1)*sin(b*x + a) + 3*(b*d*x + a*d)*cos(b*x + a)^2*log(-I*cos(b*x + a)
+ sin(b*x + a) + 1)*sin(b*x + a) - 3*(b*d*x + a*d)*cos(b*x + a)^2*log(-I*cos
(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) + 2*d*cos(b*x + a)^2*log(-1/2*co
s(b*x + a) + 1/2)*sin(b*x + a) + 3*(b*c - a*d)*cos(b*x + a)^2*log(-cos(b*x
+ a) + I*sin(b*x + a) + I)*sin(b*x + a) - 3*(b*c - a*d)*cos(b*x + a)^2*log(
-cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + 2*b*d*x - 6*(b*d*x + b*c
)*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x + a) + 2*b*c)/(b^2*cos(b*x + a)
^2*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \csc(bx + a)^2 \sec(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

$$3.320 \quad \int \frac{\csc^2(a+bx) \sec^3(a+bx)}{c+dx} dx$$

Optimal. Leaf size=26

$$\text{CannotIntegrate}\left(\frac{\csc^2(a+bx) \sec^3(a+bx)}{c+dx}, x\right)$$

[Out] CannotIntegrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x), x]

Rubi [A] time = 0.18662, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^2(a+bx) \sec^3(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x), x]

[Out] Defer[Int] [(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x), x]

Rubi steps

$$\int \frac{\csc^2(a+bx) \sec^3(a+bx)}{c+dx} dx = \int \frac{\csc^2(a+bx) \sec^3(a+bx)}{c+dx} dx$$

Mathematica [A] time = 21.1897, size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx) \sec^3(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x), x]

[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x), x]

Maple [A] time = 2.754, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx+a))^2 (\sec(bx+a))^3}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c), x)

[Out] int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c), x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)^2 \sec(bx+a)^3}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x, algorithm="fricas")

[Out] integral(csc(b*x + a)^2*sec(b*x + a)^3/(d*x + c), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)**2*sec(b*x+a)**3/(d*x+c),x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x, algorithm="giac")

[Out] Timed out

$$3.321 \quad \int \frac{\csc^2(a+bx) \sec^3(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=26

$$\text{CannotIntegrate}\left(\frac{\csc^2(a+bx) \sec^3(a+bx)}{(c+dx)^2}, x\right)$$

[Out] CannotIntegrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2, x]

Rubi [A] time = 0.197133, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^2(a+bx) \sec^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2,x]

[Out] Defer[Int] [(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\csc^2(a+bx) \sec^3(a+bx)}{(c+dx)^2} dx = \int \frac{\csc^2(a+bx) \sec^3(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 26.4328, size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx) \sec^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2,x]

[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2, x]

Maple [A] time = 4.312, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx+a))^2 (\sec(bx+a))^3}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x)

[Out] int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc (b x+a)^2 \sec (b x+a)^3}{d^2 x^2+2 c d x+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(csc(b*x + a)^2*sec(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)**2*sec(b*x+a)**3/(d*x+c)**2,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (b x+a)^2 \sec (b x+a)^3}{(d x+c)^2} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(csc(b*x + a)^2*sec(b*x + a)^3/(d*x + c)^2, x)

3.322 $\int (c + dx)^m \csc^3(a + bx) \sec^3(a + bx) dx$

Optimal. Leaf size=26

$$\text{CannotIntegrate}\left(\csc^3(a + bx) \sec^3(a + bx)(c + dx)^m, x\right)$$

[Out] CannotIntegrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3, x]

Rubi [A] time = 0.264578, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m \csc^3(a + bx) \sec^3(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3, x]

[Out] Defer[Int] [(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3, x]

Rubi steps

$$\int (c + dx)^m \csc^3(a + bx) \sec^3(a + bx) dx = \int (c + dx)^m \csc^3(a + bx) \sec^3(a + bx) dx$$

Mathematica [A] time = 10.558, size = 0, normalized size = 0.

$$\int (c + dx)^m \csc^3(a + bx) \sec^3(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3, x]

[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3, x]

Maple [A] time = 0.187, size = 0, normalized size = 0.

$$\int (dx + c)^m (\csc(bx + a))^3 (\sec(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3, x)

[Out] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \csc(bx + a)^3 \sec(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")

[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^3, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx + c)^m \csc(bx + a)^3 \sec(bx + a)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")

[Out] integral((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**m*csc(b*x+a)**3*sec(b*x+a)**3,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \csc(bx + a)^3 \sec(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")

[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^3, x)

3.323 $\int (c + dx)^3 \csc^3(a + bx) \sec^3(a + bx) dx$

Optimal. Leaf size=318

$$-\frac{3d^2(c + dx)\text{PolyLog}(3, -e^{2i(a+bx)})}{b^3} + \frac{3d^2(c + dx)\text{PolyLog}(3, e^{2i(a+bx)})}{b^3} + \frac{3id(c + dx)^2\text{PolyLog}(2, -e^{2i(a+bx)})}{b^2} - \frac{3id(c + dx)^2\text{PolyLog}(2, e^{2i(a+bx)})}{b^2}$$

[Out] $(-6*d^2*(c + d*x)*\text{ArcTanh}[E^{((2*I)*(a + b*x))}])/b^3 - (4*(c + d*x)^3*\text{ArcTanh}[E^{((2*I)*(a + b*x))}])/b - (3*d*(c + d*x)^2*\text{Csc}[2*a + 2*b*x])/b^2 - (2*(c + d*x)^3*\text{Cot}[2*a + 2*b*x]*\text{Csc}[2*a + 2*b*x])/b + (((3*I)/2)*d^3*\text{PolyLog}[2, -E^{((2*I)*(a + b*x))}])/b^4 + ((3*I)*d*(c + d*x)^2*\text{PolyLog}[2, -E^{((2*I)*(a + b*x))}])/b^2 - (((3*I)/2)*d^3*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^4 - ((3*I)*d*(c + d*x)^2*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^2 - (3*d^2*(c + d*x)*\text{PolyLog}[3, -E^{((2*I)*(a + b*x))}])/b^3 + (3*d^2*(c + d*x)*\text{PolyLog}[3, E^{((2*I)*(a + b*x))}])/b^3 - (((3*I)/2)*d^3*\text{PolyLog}[4, -E^{((2*I)*(a + b*x))}])/b^4 + (((3*I)/2)*d^3*\text{PolyLog}[4, E^{((2*I)*(a + b*x))}])/b^4$

Rubi [A] time = 0.32147, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4419, 4186, 4183, 2279, 2391, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c + dx)\text{PolyLog}(3, -e^{2i(a+bx)})}{b^3} + \frac{3d^2(c + dx)\text{PolyLog}(3, e^{2i(a+bx)})}{b^3} + \frac{3id(c + dx)^2\text{PolyLog}(2, -e^{2i(a+bx)})}{b^2} - \frac{3id(c + dx)^2\text{PolyLog}(2, e^{2i(a+bx)})}{b^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^3*\text{Csc}[a + b*x]^3*\text{Sec}[a + b*x]^3, x]$

[Out] $(-6*d^2*(c + d*x)*\text{ArcTanh}[E^{((2*I)*(a + b*x))}])/b^3 - (4*(c + d*x)^3*\text{ArcTanh}[E^{((2*I)*(a + b*x))}])/b - (3*d*(c + d*x)^2*\text{Csc}[2*a + 2*b*x])/b^2 - (2*(c + d*x)^3*\text{Cot}[2*a + 2*b*x]*\text{Csc}[2*a + 2*b*x])/b + (((3*I)/2)*d^3*\text{PolyLog}[2, -E^{((2*I)*(a + b*x))}])/b^4 + ((3*I)*d*(c + d*x)^2*\text{PolyLog}[2, -E^{((2*I)*(a + b*x))}])/b^2 - (((3*I)/2)*d^3*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^4 - ((3*I)*d*(c + d*x)^2*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^2 - (3*d^2*(c + d*x)*\text{PolyLog}[3, -E^{((2*I)*(a + b*x))}])/b^3 + (3*d^2*(c + d*x)*\text{PolyLog}[3, E^{((2*I)*(a + b*x))}])/b^3 - (((3*I)/2)*d^3*\text{PolyLog}[4, -E^{((2*I)*(a + b*x))}])/b^4 + (((3*I)/2)*d^3*\text{PolyLog}[4, E^{((2*I)*(a + b*x))}])/b^4$

Rule 4419

$\text{Int}[\text{Csc}[(a_.) + (b_.)*(x_.)]^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sec}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[2^n, \text{Int}[(c + d*x)^m*\text{Csc}[2*a + 2*b*x]^n, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]

Rule 4186

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.))^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow -\text{Simp}[(b^2*(c + d*x)^m*\text{Cot}[e + f*x]*(b*\text{Csc}[e + f*x])^{(n-2)})/(f*(n-1)), x] + (\text{Dist}[(b^2*d^2*m*(m-1))/(f^2*(n-1)*(n-2)), \text{Int}[(c + d*x)^{(m-2)}*(b*\text{Csc}[e + f*x])^{(n-2)}, x], x] + \text{Dist}[(b^2*(n-2))/(n-1), \text{Int}[(c + d*x)^m*(b*\text{Csc}[e + f*x])^{(n-2)}, x], x] - \text{Simp}[(b^2*d*m*(c + d*x)^{(m-1)}*(b*\text{Csc}[e + f*x])^{(n-2)})/(f^2*(n-1)*(n-2)), x]) /;$ FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/((b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_)^(m_.))*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/((b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 \csc^3(a + bx) \sec^3(a + bx) dx &= 8 \int (c + dx)^3 \csc^3(2a + 2bx) dx \\
&= -\frac{3d(c + dx)^2 \csc(2a + 2bx)}{b^2} - \frac{2(c + dx)^3 \cot(2a + 2bx) \csc(2a + 2bx)}{b} + 4 \int (c + dx)^3 \csc^3(2a + 2bx) dx \\
&= -\frac{6d^2(c + dx) \tanh^{-1}(e^{2i(a+bx)})}{b^3} - \frac{4(c + dx)^3 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{3d(c + dx)^2 \csc(2a + 2bx)}{b} + 4 \int (c + dx)^3 \csc^3(2a + 2bx) dx \\
&= -\frac{6d^2(c + dx) \tanh^{-1}(e^{2i(a+bx)})}{b^3} - \frac{4(c + dx)^3 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{3d(c + dx)^2 \csc(2a + 2bx)}{b} + 4 \int (c + dx)^3 \csc^3(2a + 2bx) dx \\
&= -\frac{6d^2(c + dx) \tanh^{-1}(e^{2i(a+bx)})}{b^3} - \frac{4(c + dx)^3 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{3d(c + dx)^2 \csc(2a + 2bx)}{b} + 4 \int (c + dx)^3 \csc^3(2a + 2bx) dx \\
&= -\frac{6d^2(c + dx) \tanh^{-1}(e^{2i(a+bx)})}{b^3} - \frac{4(c + dx)^3 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{3d(c + dx)^2 \csc(2a + 2bx)}{b} + 4 \int (c + dx)^3 \csc^3(2a + 2bx) dx \\
&= -\frac{6d^2(c + dx) \tanh^{-1}(e^{2i(a+bx)})}{b^3} - \frac{4(c + dx)^3 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{3d(c + dx)^2 \csc(2a + 2bx)}{b} + 4 \int (c + dx)^3 \csc^3(2a + 2bx) dx
\end{aligned}$$

Mathematica [A] time = 9.18225, size = 582, normalized size = 1.83

$$8 \left(\frac{\csc^2(2a + 2bx) (3c^2 d \sin(2a + 2bx) + 6bc^2 dx \cos(2a + 2bx) + 2bc^3 \cos(2a + 2bx) + 6bcd^2 x^2 \cos(2a + 2bx) + 6cd^2 x \sin(2a + 2bx))}{8b^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^3,x]

[Out] 8*(-(8*b^3*c^3*ArcTanh[E^((2*I)*(a + b*x))]) + 12*b*c*d^2*ArcTanh[E^((2*I)*(a + b*x))]) - 12*b^3*c^2*d*x*Log[1 - E^((2*I)*(a + b*x))] - 6*b*d^3*x*Log[1 - E^((2*I)*(a + b*x))] - 12*b^3*c*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] - 4*b^3*d^3*x^3*Log[1 - E^((2*I)*(a + b*x))] + 12*b^3*c^2*d*x*Log[1 + E^((2*I)*(a + b*x))] + 6*b*d^3*x*Log[1 + E^((2*I)*(a + b*x))] + 12*b^3*c*d^2*x^2*Log[1 + E^((2*I)*(a + b*x))] + 4*b^3*d^3*x^3*Log[1 + E^((2*I)*(a + b*x))] - (3*I)*d*(d^2 + 2*b^2*(c + d*x)^2)*PolyLog[2, -E^((2*I)*(a + b*x))] + (3*I)*d*(d^2 + 2*b^2*(c + d*x)^2)*PolyLog[2, E^((2*I)*(a + b*x))] + 6*b*c*d^2*PolyLog[3, -E^((2*I)*(a + b*x))] + 6*b*d^3*x*PolyLog[3, -E^((2*I)*(a + b*x))] - 6*b*c*d^2*PolyLog[3, E^((2*I)*(a + b*x))] - 6*b*d^3*x*PolyLog[3, E^((2*I)*(a + b*x))] + (3*I)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))] - (3*I)*d^3*PolyLog[4, E^((2*I)*(a + b*x))]/(16*b^4) - (Csc[2*a + 2*b*x]^2*(2*b*c^3*Cos[2*a + 2*b*x] + 6*b*c^2*d*x*Cos[2*a + 2*b*x] + 6*b*c*d^2*x^2*Cos[2*a + 2*b*x] + 2*b*d^3*x^3*Cos[2*a + 2*b*x] + 3*c^2*d*Sin[2*a + 2*b*x] + 6*c*d^2*x*Sin[2*a + 2*b*x] + 3*d^3*x^2*Sin[2*a + 2*b*x]))/(8*b^2))

Maple [B] time = 0.295, size = 1329, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x)

[Out] 6/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+6/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-2/b*c^3*ln(exp(2*I*(b*x+a))+1)-3/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))-3/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x+3/b^3*d^3*ln(exp(I*(b*x+a))+1)*x+3/b^3*d^3

```

*ln(1-exp(I*(b*x+a)))*x+3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a-3*I/b^4*d^3*polylo
g(2,-exp(I*(b*x+a)))-3*d^2/b^3*c*ln(exp(2*I*(b*x+a))+1)-3*d^3/b^3*ln(exp(2*
I*(b*x+a))+1)*x+3/2*I*d^3*polylog(2,-exp(2*I*(b*x+a)))/b^4+12*I/b^4*d^3*pol
ylog(4,exp(I*(b*x+a)))+12*I/b^4*d^3*polylog(4,-exp(I*(b*x+a)))-3/2*I*d^3*po
lylog(4,-exp(2*I*(b*x+a)))/b^4-6/b*c^2*d*ln(exp(2*I*(b*x+a))+1)*x-6/b*c*d^2
*ln(exp(2*I*(b*x+a))+1)*x^2-2/b*d^3*ln(exp(2*I*(b*x+a))+1)*x^3-3/b^4*d^3*a*
ln(exp(I*(b*x+a))-1)+3/b^3*d^2*c*ln(exp(I*(b*x+a))+1)+3/b^3*d^2*c*ln(exp(I*
(b*x+a))-1)+2/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+2/b^4*d^3*ln(1-exp(I*(b*x+a)))
*a^3+2/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)-
6/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)-6/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))+6/b
*c^2*d*ln(exp(I*(b*x+a))+1)*x+6/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+6/b^2*c^2*d*
ln(1-exp(I*(b*x+a)))*a+2/b^2/(exp(2*I*(b*x+a))+1)^2/(exp(2*I*(b*x+a))-1)^2*
(2*d^3*x^3*b*exp(6*I*(b*x+a))+6*c*d^2*x^2*b*exp(6*I*(b*x+a))+6*c^2*d*x*b*ex
p(6*I*(b*x+a))+2*c^3*b*exp(6*I*(b*x+a))-3*I*d^3*x^2*exp(6*I*(b*x+a))+2*b*d^
3*x^3*exp(2*I*(b*x+a))-6*I*c*d^2*x*exp(6*I*(b*x+a))+6*b*c*d^2*x^2*exp(2*I*(
b*x+a))-3*I*c^2*d*exp(6*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))+2*b*c^3*exp
(2*I*(b*x+a))+3*I*d^3*x^2*exp(2*I*(b*x+a))+6*I*c*d^2*x*exp(2*I*(b*x+a))+3*I
*c^2*d*exp(2*I*(b*x+a))+12/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x+12/b^3*d^3*
polylog(3,-exp(I*(b*x+a)))*x+12/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+12/b^3*
c*d^2*polylog(3,-exp(I*(b*x+a)))-2/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+2/b*c^3
*ln(exp(I*(b*x+a))-1)+2/b*c^3*ln(exp(I*(b*x+a))+1)+3*I/b^2*c^2*d*polylog(2,
-exp(2*I*(b*x+a)))-6*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))+3*I/b^2*d^3*pol
ylog(2,-exp(2*I*(b*x+a)))*x^2-6*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))-3*I*d
^3*polylog(2,exp(I*(b*x+a)))/b^4-12*I/b^2*polylog(2,-exp(I*(b*x+a)))*c*d^2*
x-12*I/b^2*polylog(2,exp(I*(b*x+a)))*c*d^2*x+6*I/b^2*polylog(2,-exp(2*I*(b*
x+a)))*c*d^2*x-6*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-6*I/b^2*d^3*polyl
og(2,exp(I*(b*x+a)))*x^2

```

Maxima [B] time = 11.6935, size = 7574, normalized size = 23.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

```

[Out] -1/2*(c^3*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log
(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b + 3*a^2*c*d^2*((2*sin(b*x + a)^
2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*log
(sin(b*x + a)^2))/b^2 - a^3*d^3*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log
(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b^3 + 2*((16*(b*x + a)^3*d^3
+ 18*b*c*d^2 - 18*a*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(2*b^2*c^2*
d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a) + 2*(8*(b*x + a)^3*d^3 + 9*b*c
*d^2 - 9*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(2*b^2*c^2*d - 4*a*b*
c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*cos(8*b*x + 8*a) - 4*(8*(b*x + a)^3*d^3
+ 9*b*c*d^2 - 9*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(2*b^2*c^2*d
- 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (16*I*(b*x +
a)^3*d^3 + 18*I*b*c*d^2 - 18*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x +
a)^2 + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + (36*I*a^2 + 18*I)*d^3)*(b*x + a))
*sin(8*b*x + 8*a) + (-32*I*(b*x + a)^3*d^3 - 36*I*b*c*d^2 + 36*I*a*d^3 + (-
72*I*b*c*d^2 + 72*I*a*d^3)*(b*x + a)^2 + (-72*I*b^2*c^2*d + 144*I*a*b*c*d^2
+ (-72*I*a^2 - 36*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a)*arctan2(sin(2*b*x +
2*a), cos(2*b*x + 2*a) + 1) - (12*(b*x + a)^3*d^3 + 18*b*c*d^2 - 18*a*d^3
+ 36*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2
+ 1)*d^3)*(b*x + a) + 6*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*(b*c*

```

$$\begin{aligned}
& d^2 - a^d^3) * (b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3) * \\
& (b*x + a)) * \cos(8*b*x + 8*a) - 12*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + \\
& 6*(b*c*d^2 - a*d^3) * (b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + \\
& 1)*d^3) * (b*x + a)) * \cos(4*b*x + 4*a) - (-12*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2 \\
& + 18*I*a*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3) * (b*x + a)^2 + (-36*I*b^2*c^2*d \\
& + 72*I*a*b*c*d^2 + (-36*I*a^2 - 18*I)*d^3) * (b*x + a)) * \sin(8*b*x + 8*a) - (\\
& 24*I*(b*x + a)^3*d^3 + 36*I*b*c*d^2 - 36*I*a*d^3 + (72*I*b*c*d^2 - 72*I*a*d \\
& ^3) * (b*x + a)^2 + (72*I*b^2*c^2*d - 144*I*a*b*c*d^2 + (72*I*a^2 + 36*I)*d^3 \\
&) * (b*x + a)) * \sin(4*b*x + 4*a)) * \arctan2(\sin(b*x + a), \cos(b*x + a) + 1) - (1 \\
& 8*b*c*d^2 - 18*a*d^3 + 18*(b*c*d^2 - a*d^3) * \cos(8*b*x + 8*a) - 36*(b*c*d^2 \\
& - a*d^3) * \cos(4*b*x + 4*a) - (-18*I*b*c*d^2 + 18*I*a*d^3) * \sin(8*b*x + 8*a) - \\
& (36*I*b*c*d^2 - 36*I*a*d^3) * \sin(4*b*x + 4*a)) * \arctan2(\sin(b*x + a), \cos(b* \\
& x + a) - 1) + (12*(b*x + a)^3*d^3 + 36*(b*c*d^2 - a*d^3) * (b*x + a)^2 + 18*(\\
& 2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3) * (b*x + a) + 6*(2*(b*x + a)^3*d \\
& ^3 + 6*(b*c*d^2 - a*d^3) * (b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^ \\
& 2 + 1)*d^3) * (b*x + a)) * \cos(8*b*x + 8*a) - 12*(2*(b*x + a)^3*d^3 + 6*(b*c*d^ \\
& 2 - a*d^3) * (b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3) * (b \\
& *x + a)) * \cos(4*b*x + 4*a) + (12*I*(b*x + a)^3*d^3 + (36*I*b*c*d^2 - 36*I*a* \\
& d^3) * (b*x + a)^2 + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + (36*I*a^2 + 18*I)*d^3 \\
&) * (b*x + a)) * \sin(8*b*x + 8*a) + (-24*I*(b*x + a)^3*d^3 + (-72*I*b*c*d^2 + 7 \\
& 2*I*a*d^3) * (b*x + a)^2 + (-72*I*b^2*c^2*d + 144*I*a*b*c*d^2 + (-72*I*a^2 - \\
& 36*I)*d^3) * (b*x + a)) * \sin(4*b*x + 4*a)) * \arctan2(\sin(b*x + a), -\cos(b*x + a) \\
& + 1) + (24*I*(b*x + a)^3*d^3 + 36*b^2*c^2*d - 72*a*b*c*d^2 + 36*a^2*d^3 + \\
& (72*I*b*c*d^2 - 36*(2*I*a - 1)*d^3) * (b*x + a)^2 + (72*I*b^2*c^2*d - 72*(2*I \\
& *a - 1)*b*c*d^2 + (72*I*a^2 - 72*a)*d^3) * (b*x + a)) * \cos(6*b*x + 6*a) + (24* \\
& I*(b*x + a)^3*d^3 - 36*b^2*c^2*d + 72*a*b*c*d^2 - 36*a^2*d^3 + (72*I*b*c*d^ \\
& 2 - 36*(2*I*a + 1)*d^3) * (b*x + a)^2 + (72*I*b^2*c^2*d - 72*(2*I*a + 1)*b*c* \\
& d^2 + (72*I*a^2 + 72*a)*d^3) * (b*x + a)) * \cos(2*b*x + 2*a) - (18*b^2*c^2*d - \\
& 36*a*b*c*d^2 + 24*(b*x + a)^2*d^3 + 9*(2*a^2 + 1)*d^3 + 36*(b*c*d^2 - a*d^3 \\
&) * (b*x + a) + 3*(6*b^2*c^2*d - 12*a*b*c*d^2 + 8*(b*x + a)^2*d^3 + 3*(2*a^2 \\
& + 1)*d^3 + 12*(b*c*d^2 - a*d^3) * (b*x + a)) * \cos(8*b*x + 8*a) - 6*(6*b^2*c^2* \\
& d - 12*a*b*c*d^2 + 8*(b*x + a)^2*d^3 + 3*(2*a^2 + 1)*d^3 + 12*(b*c*d^2 - a* \\
& d^3) * (b*x + a)) * \cos(4*b*x + 4*a) - (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 24*I \\
& *(b*x + a)^2*d^3 + (-18*I*a^2 - 9*I)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3) * (b* \\
& x + a)) * \sin(8*b*x + 8*a) - (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 48*I*(b*x + a) \\
&)^2*d^3 + (36*I*a^2 + 18*I)*d^3 + (72*I*b*c*d^2 - 72*I*a*d^3) * (b*x + a)) * \si \\
& n(4*b*x + 4*a)) * \operatorname{dilog}(-e^{(2*I*b*x + 2*I*a)}) + (36*b^2*c^2*d - 72*a*b*c*d^2 \\
& + 36*(b*x + a)^2*d^3 + 18*(2*a^2 + 1)*d^3 + 72*(b*c*d^2 - a*d^3) * (b*x + a) \\
& + 18*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^2*d^3 + (2*a^2 + 1)*d^3 + 4*(\\
& b*c*d^2 - a*d^3) * (b*x + a)) * \cos(8*b*x + 8*a) - 36*(2*b^2*c^2*d - 4*a*b*c*d^ \\
& 2 + 2*(b*x + a)^2*d^3 + (2*a^2 + 1)*d^3 + 4*(b*c*d^2 - a*d^3) * (b*x + a)) * \co \\
& s(4*b*x + 4*a) + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 36*I*(b*x + a)^2*d^3 + \\
& (36*I*a^2 + 18*I)*d^3 + (72*I*b*c*d^2 - 72*I*a*d^3) * (b*x + a)) * \sin(8*b*x + \\
& 8*a) + (-72*I*b^2*c^2*d + 144*I*a*b*c*d^2 - 72*I*(b*x + a)^2*d^3 + (-72*I*a \\
& ^2 - 36*I)*d^3 + (-144*I*b*c*d^2 + 144*I*a*d^3) * (b*x + a)) * \sin(4*b*x + 4*a) \\
&) * \operatorname{dilog}(-e^{(I*b*x + I*a)}) + (36*b^2*c^2*d - 72*a*b*c*d^2 + 36*(b*x + a)^2*d \\
& ^3 + 18*(2*a^2 + 1)*d^3 + 72*(b*c*d^2 - a*d^3) * (b*x + a) + 18*(2*b^2*c^2*d \\
& - 4*a*b*c*d^2 + 2*(b*x + a)^2*d^3 + (2*a^2 + 1)*d^3 + 4*(b*c*d^2 - a*d^3) * (\\
& b*x + a)) * \cos(8*b*x + 8*a) - 36*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^2* \\
& d^3 + (2*a^2 + 1)*d^3 + 4*(b*c*d^2 - a*d^3) * (b*x + a)) * \cos(4*b*x + 4*a) + (\\
& 36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 36*I*(b*x + a)^2*d^3 + (36*I*a^2 + 18*I)* \\
& d^3 + (72*I*b*c*d^2 - 72*I*a*d^3) * (b*x + a)) * \sin(8*b*x + 8*a) + (-72*I*b^2* \\
& c^2*d + 144*I*a*b*c*d^2 - 72*I*(b*x + a)^2*d^3 + (-72*I*a^2 - 36*I)*d^3 + (\\
& -144*I*b*c*d^2 + 144*I*a*d^3) * (b*x + a)) * \sin(4*b*x + 4*a)) * \operatorname{dilog}(e^{(I*b*x + \\
& I*a)}) + (-8*I*(b*x + a)^3*d^3 - 9*I*b*c*d^2 + 9*I*a*d^3 + (-18*I*b*c*d^2 + \\
& 18*I*a*d^3) * (b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - \\
& 9*I)*d^3) * (b*x + a) + (-8*I*(b*x + a)^3*d^3 - 9*I*b*c*d^2 + 9*I*a*d^3 + (\\
& -18*I*b*c*d^2 + 18*I*a*d^3) * (b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 \\
& + (-18*I*a^2 - 9*I)*d^3) * (b*x + a)) * \cos(8*b*x + 8*a) + (16*I*(b*x + a)^3*d^
\end{aligned}$$

$$\begin{aligned}
& 3 + 18I* b*c*d^2 - 18I*a*d^3 + (36I*b*c*d^2 - 36I*a*d^3)*(b*x + a)^2 + (\\
& 36I*b^2*c^2*d - 72I*a*b*c*d^2 + (36I*a^2 + 18I)*d^3)*(b*x + a))*\cos(4*b \\
& *x + 4*a) + (8*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 18*(b*c*d^2 - a*d^3) \\
& *(b*x + a)^2 + 9*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*\sin \\
& (8*b*x + 8*a) - 2*(8*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 18*(b*c*d^2 \\
& - a*d^3)*(b*x + a)^2 + 9*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x \\
& + a))*\sin(4*b*x + 4*a))*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos \\
& (2*b*x + 2*a) + 1) + (6I*(b*x + a)^3*d^3 + 9I*b*c*d^2 - 9I*a*d^3 + (18I \\
& *b*c*d^2 - 18I*a*d^3)*(b*x + a)^2 + (18I*b^2*c^2*d - 36I*a*b*c*d^2 + (1 \\
& 8I*a^2 + 9I)*d^3)*(b*x + a) + (6I*(b*x + a)^3*d^3 + 9I*b*c*d^2 - 9I*a*d \\
& ^3 + (18I*b*c*d^2 - 18I*a*d^3)*(b*x + a)^2 + (18I*b^2*c^2*d - 36I*a*b*c \\
& *d^2 + (18I*a^2 + 9I)*d^3)*(b*x + a))*\cos(8*b*x + 8*a) + (-12I*(b*x + a \\
&)^3*d^3 - 18I*b*c*d^2 + 18I*a*d^3 + (-36I*b*c*d^2 + 36I*a*d^3)*(b*x + a \\
&)^2 + (-36I*b^2*c^2*d + 72I*a*b*c*d^2 + (-36I*a^2 - 18I)*d^3)*(b*x + a) \\
&)*\cos(4*b*x + 4*a) - 3*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*(b*c*d^ \\
& 2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b \\
& *x + a))*\sin(8*b*x + 8*a) + 6*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6* \\
& (b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)* \\
& d^3)*(b*x + a))*\sin(4*b*x + 4*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos \\
& (b*x + a) + 1) + (6I*(b*x + a)^3*d^3 + 9I*b*c*d^2 - 9I*a*d^3 + (18I*b \\
& *c*d^2 - 18I*a*d^3)*(b*x + a)^2 + (18I*b^2*c^2*d - 36I*a*b*c*d^2 + (18I \\
& *a^2 + 9I)*d^3)*(b*x + a) + (6I*(b*x + a)^3*d^3 + 9I*b*c*d^2 - 9I*a*d^3 \\
& + (18I*b*c*d^2 - 18I*a*d^3)*(b*x + a)^2 + (18I*b^2*c^2*d - 36I*a*b*c*d \\
& ^2 + (18I*a^2 + 9I)*d^3)*(b*x + a))*\cos(8*b*x + 8*a) + (-12I*(b*x + a)^3 \\
& *d^3 - 18I*b*c*d^2 + 18I*a*d^3 + (-36I*b*c*d^2 + 36I*a*d^3)*(b*x + a)^2 \\
& + (-36I*b^2*c^2*d + 72I*a*b*c*d^2 + (-36I*a^2 - 18I)*d^3)*(b*x + a))*\cos \\
& (4*b*x + 4*a) - 3*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*(b*c*d^2 - \\
& a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x \\
& + a))*\sin(8*b*x + 8*a) + 6*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*(b \\
& *c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3 \\
&)*(b*x + a))*\sin(4*b*x + 4*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos \\
& (b*x + a) + 1) + (12*d^3*\cos(8*b*x + 8*a) - 24*d^3*\cos(4*b*x + 4*a) + 12*I*d \\
& ^3*\sin(8*b*x + 8*a) - 24*I*d^3*\sin(4*b*x + 4*a) + 12*d^3)*\text{polylog}(4, -e^{(2* \\
& I*b*x + 2*I*a)}) - (72*d^3*\cos(8*b*x + 8*a) - 144*d^3*\cos(4*b*x + 4*a) + 72* \\
& I*d^3*\sin(8*b*x + 8*a) - 144*I*d^3*\sin(4*b*x + 4*a) + 72*d^3)*\text{polylog}(4, -e \\
& ^{(I*b*x + I*a)}) - (72*d^3*\cos(8*b*x + 8*a) - 144*d^3*\cos(4*b*x + 4*a) + 72* \\
& I*d^3*\sin(8*b*x + 8*a) - 144*I*d^3*\sin(4*b*x + 4*a) + 72*d^3)*\text{polylog}(4, e^{ \\
& (I*b*x + I*a)}) + (-18I*b*c*d^2 - 24I*(b*x + a)*d^3 + 18I*a*d^3 + (-18I*b \\
& *c*d^2 - 24I*(b*x + a)*d^3 + 18I*a*d^3))*\cos(8*b*x + 8*a) + (36I*b*c*d^2 \\
& + 48I*(b*x + a)*d^3 - 36I*a*d^3))*\cos(4*b*x + 4*a) + 6*(3*b*c*d^2 + 4*(b*x \\
& + a)*d^3 - 3*a*d^3))*\sin(8*b*x + 8*a) - 12*(3*b*c*d^2 + 4*(b*x + a)*d^3 - \\
& 3*a*d^3))*\sin(4*b*x + 4*a))*\text{polylog}(3, -e^{(2*I*b*x + 2*I*a)}) + (72I*b*c*d^2 \\
& + 72I*(b*x + a)*d^3 - 72I*a*d^3 + (72I*b*c*d^2 + 72I*(b*x + a)*d^3 - 7 \\
& 2I*a*d^3))*\cos(8*b*x + 8*a) + (-144I*b*c*d^2 - 144I*(b*x + a)*d^3 + 144I \\
& *a*d^3))*\cos(4*b*x + 4*a) - 72*(b*c*d^2 + (b*x + a)*d^3 - a*d^3))*\sin(8*b*x + \\
& 8*a) + 144*(b*c*d^2 + (b*x + a)*d^3 - a*d^3))*\sin(4*b*x + 4*a))*\text{polylog}(3, \\
& -e^{(I*b*x + I*a)}) + (72I*b*c*d^2 + 72I*(b*x + a)*d^3 - 72I*a*d^3 + (72I \\
& *b*c*d^2 + 72I*(b*x + a)*d^3 - 72I*a*d^3))*\cos(8*b*x + 8*a) + (-144I*b*c \\
& *d^2 - 144I*(b*x + a)*d^3 + 144I*a*d^3))*\cos(4*b*x + 4*a) - 72*(b*c*d^2 + (\\
& b*x + a)*d^3 - a*d^3))*\sin(8*b*x + 8*a) + 144*(b*c*d^2 + (b*x + a)*d^3 - a*d \\
& ^3))*\sin(4*b*x + 4*a))*\text{polylog}(3, e^{(I*b*x + I*a)}) - (24*(b*x + a)^3*d^3 - 3 \\
& 6I*b^2*c^2*d + 72I*a*b*c*d^2 - 36I*a^2*d^3 + (72*b*c*d^2 - (72*a + 36I) \\
& *d^3)*(b*x + a)^2 + (72*b^2*c^2*d - (144*a + 72I)*b*c*d^2 + 72*(a^2 + I*a) \\
& *d^3)*(b*x + a))*\sin(6*b*x + 6*a) - (24*(b*x + a)^3*d^3 + 36I*b^2*c^2*d - \\
& 72I*a*b*c*d^2 + 36I*a^2*d^3 + (72*b*c*d^2 - (72*a - 36I)*d^3)*(b*x + a)^ \\
& 2 + (72*b^2*c^2*d - (144*a - 72I)*b*c*d^2 + 72*(a^2 - I*a)*d^3)*(b*x + a) \\
&)*\sin(2*b*x + 2*a))/(-6I*b^3*\cos(8*b*x + 8*a) + 12I*b^3*\cos(4*b*x + 4*a) + \\
& 6*b^3*\sin(8*b*x + 8*a) - 12*b^3*\sin(4*b*x + 4*a) - 6I*b^3))/b
\end{aligned}$$

$$\begin{aligned}
& 2*b^3*c^2*d + b*d^3)*x)*\cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + \\
& 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3) \\
&)*x)*\cos(b*x + a)^2*\log(-I*\cos(b*x + a) - \sin(b*x + a) + 1) - ((2*b^3*c^3 \\
& - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*\cos(b*x + a)^4 \\
& - (2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)* \\
& \cos(b*x + a)^2*\log(-1/2*\cos(b*x + a) + 1/2*I*\sin(b*x + a) + 1/2) - ((2*b^3 \\
& *c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*\cos(b*x + \\
& a)^4 - (2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)* \\
& d^3)*\cos(b*x + a)^2*\log(-1/2*\cos(b*x + a) - 1/2*I*\sin(b*x + a) + 1/2) - ((\\
& 2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + \\
& 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*\cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b \\
& ^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3* \\
& c^2*d + b*d^3)*x)*\cos(b*x + a)^2*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1) \\
& + ((2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)* \\
& \cos(b*x + a)^4 - (2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^ \\
& 3 + 3*a)*d^3)*\cos(b*x + a)^2*\log(-\cos(b*x + a) + I*\sin(b*x + a) + I) - ((2 \\
& *b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3 \\
& *a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*\cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^ \\
& ^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3* \\
& c^2*d + b*d^3)*x)*\cos(b*x + a)^2*\log(-\cos(b*x + a) - I*\sin(b*x + a) + 1) + \\
& ((2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*c \\
& \cos(b*x + a)^4 - (2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 \\
& + 3*a)*d^3)*\cos(b*x + a)^2*\log(-\cos(b*x + a) - I*\sin(b*x + a) + I) - (12* \\
& I*d^3*\cos(b*x + a)^4 - 12*I*d^3*\cos(b*x + a)^2)*\text{polylog}(4, \cos(b*x + a) + I \\
& *\sin(b*x + a)) - (-12*I*d^3*\cos(b*x + a)^4 + 12*I*d^3*\cos(b*x + a)^2)*\text{polyl} \\
& \text{og}(4, \cos(b*x + a) - I*\sin(b*x + a)) - (12*I*d^3*\cos(b*x + a)^4 - 12*I*d^3* \\
& \cos(b*x + a)^2)*\text{polylog}(4, I*\cos(b*x + a) + \sin(b*x + a)) - (-12*I*d^3*\cos(\\
& b*x + a)^4 + 12*I*d^3*\cos(b*x + a)^2)*\text{polylog}(4, I*\cos(b*x + a) - \sin(b*x + \\
& a)) - (-12*I*d^3*\cos(b*x + a)^4 + 12*I*d^3*\cos(b*x + a)^2)*\text{polylog}(4, -I*c \\
& \cos(b*x + a) + \sin(b*x + a)) - (12*I*d^3*\cos(b*x + a)^4 - 12*I*d^3*\cos(b*x + \\
& a)^2)*\text{polylog}(4, -I*\cos(b*x + a) - \sin(b*x + a)) - (-12*I*d^3*\cos(b*x + a) \\
& ^4 + 12*I*d^3*\cos(b*x + a)^2)*\text{polylog}(4, -\cos(b*x + a) + I*\sin(b*x + a)) - \\
& (12*I*d^3*\cos(b*x + a)^4 - 12*I*d^3*\cos(b*x + a)^2)*\text{polylog}(4, -\cos(b*x + a) \\
&) - I*\sin(b*x + a)) - 12*((b*d^3*x + b*c*d^2)*\cos(b*x + a)^4 - (b*d^3*x + b \\
& *c*d^2)*\cos(b*x + a)^2)*\text{polylog}(3, \cos(b*x + a) + I*\sin(b*x + a)) - 12*((b* \\
& d^3*x + b*c*d^2)*\cos(b*x + a)^4 - (b*d^3*x + b*c*d^2)*\cos(b*x + a)^2)*\text{polyl} \\
& \text{og}(3, \cos(b*x + a) - I*\sin(b*x + a)) + 12*((b*d^3*x + b*c*d^2)*\cos(b*x + a) \\
& ^4 - (b*d^3*x + b*c*d^2)*\cos(b*x + a)^2)*\text{polylog}(3, I*\cos(b*x + a) + \sin(b* \\
& x + a)) + 12*((b*d^3*x + b*c*d^2)*\cos(b*x + a)^4 - (b*d^3*x + b*c*d^2)*\cos(\\
& b*x + a)^2)*\text{polylog}(3, I*\cos(b*x + a) - \sin(b*x + a)) + 12*((b*d^3*x + b*c* \\
& d^2)*\cos(b*x + a)^4 - (b*d^3*x + b*c*d^2)*\cos(b*x + a)^2)*\text{polylog}(3, -I*\cos \\
& (b*x + a) + \sin(b*x + a)) + 12*((b*d^3*x + b*c*d^2)*\cos(b*x + a)^4 - (b*d^3 \\
& *x + b*c*d^2)*\cos(b*x + a)^2)*\text{polylog}(3, -I*\cos(b*x + a) - \sin(b*x + a)) - \\
& 12*((b*d^3*x + b*c*d^2)*\cos(b*x + a)^4 - (b*d^3*x + b*c*d^2)*\cos(b*x + a)^2) \\
&)*\text{polylog}(3, -\cos(b*x + a) + I*\sin(b*x + a)) - 12*((b*d^3*x + b*c*d^2)*\cos(\\
& b*x + a)^4 - (b*d^3*x + b*c*d^2)*\cos(b*x + a)^2)*\text{polylog}(3, -\cos(b*x + a) - \\
& I*\sin(b*x + a)))/(b^4*\cos(b*x + a)^4 - b^4*\cos(b*x + a)^2)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*csc(b*x+a)**3*sec(b*x+a)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \csc(bx + a)^3 \sec(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

3.324 $\int (c + dx)^2 \csc^3(a + bx) \sec^3(a + bx) dx$

Optimal. Leaf size=190

$$\frac{2id(c + dx)\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^2} - \frac{2id(c + dx)\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^2} - \frac{d^2\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{b^3} + \frac{d^2\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{b^3}$$

[Out] $(-4*(c + d*x)^2*\text{ArcTanh}[E^{((2*I)*(a + b*x))}])/b - (d^2*\text{ArcTanh}[\text{Cos}[2*a + 2*b*x]])/b^3 - (2*d*(c + d*x)*\text{Csc}[2*a + 2*b*x])/b^2 - (2*(c + d*x)^2*\text{Cot}[2*a + 2*b*x]*\text{Csc}[2*a + 2*b*x])/b + ((2*I)*d*(c + d*x)*\text{PolyLog}[2, -E^{((2*I)*(a + b*x))}])/b^2 - ((2*I)*d*(c + d*x)*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^2 - (d^2*\text{PolyLog}[3, -E^{((2*I)*(a + b*x))}])/b^3 + (d^2*\text{PolyLog}[3, E^{((2*I)*(a + b*x))}])/b^3$

Rubi [A] time = 0.212836, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4419, 4186, 3770, 4183, 2531, 2282, 6589}

$$\frac{2id(c + dx)\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^2} - \frac{2id(c + dx)\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^2} - \frac{d^2\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{b^3} + \frac{d^2\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^3, x]

[Out] $(-4*(c + d*x)^2*\text{ArcTanh}[E^{((2*I)*(a + b*x))}])/b - (d^2*\text{ArcTanh}[\text{Cos}[2*a + 2*b*x]])/b^3 - (2*d*(c + d*x)*\text{Csc}[2*a + 2*b*x])/b^2 - (2*(c + d*x)^2*\text{Cot}[2*a + 2*b*x]*\text{Csc}[2*a + 2*b*x])/b + ((2*I)*d*(c + d*x)*\text{PolyLog}[2, -E^{((2*I)*(a + b*x))}])/b^2 - ((2*I)*d*(c + d*x)*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^2 - (d^2*\text{PolyLog}[3, -E^{((2*I)*(a + b*x))}])/b^3 + (d^2*\text{PolyLog}[3, E^{((2*I)*(a + b*x))}])/b^3$

Rule 4419

Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]

Rule 4186

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4183

Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d

```
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \csc^3(a + bx) \sec^3(a + bx) dx &= 8 \int (c + dx)^2 \csc^3(2a + 2bx) dx \\ &= -\frac{2d(c + dx) \csc(2a + 2bx)}{b^2} - \frac{2(c + dx)^2 \cot(2a + 2bx) \csc(2a + 2bx)}{b} + 4 \int (c + dx) \csc^3(2a + 2bx) dx \\ &= -\frac{4(c + dx)^2 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{d^2 \tanh^{-1}(\cos(2a + 2bx))}{b^3} - \frac{2d(c + dx) \csc(2a + 2bx)}{b^2} \\ &= -\frac{4(c + dx)^2 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{d^2 \tanh^{-1}(\cos(2a + 2bx))}{b^3} - \frac{2d(c + dx) \csc(2a + 2bx)}{b^2} \\ &= -\frac{4(c + dx)^2 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{d^2 \tanh^{-1}(\cos(2a + 2bx))}{b^3} - \frac{2d(c + dx) \csc(2a + 2bx)}{b^2} \\ &= -\frac{4(c + dx)^2 \tanh^{-1}(e^{2i(a+bx)})}{b} - \frac{d^2 \tanh^{-1}(\cos(2a + 2bx))}{b^3} - \frac{2d(c + dx) \csc(2a + 2bx)}{b^2} \end{aligned}$$

Mathematica [B] time = 8.16677, size = 381, normalized size = 2.01

$$8 \left(-\frac{-2ibd(c + dx) \text{PolyLog}(2, -e^{2i(a+bx)}) + 2ibd(c + dx) \text{PolyLog}(2, e^{2i(a+bx)}) + d^2 \text{PolyLog}(3, -e^{2i(a+bx)}) - d^2 \text{PolyLog}(3, e^{2i(a+bx)})}{b^3} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] 8*(-(d*(c + d*x)*Csc[2*a])/(4*b^2) + ((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a + b*x]^2)/(16*b) - (4*b^2*c^2*ArcTanh[E^((2*I)*(a + b*x))]) + 2*d^2*ArcTanh[E^((2*I)*(a + b*x))] - 4*b^2*c*d*x*Log[1 - E^((2*I)*(a + b*x))] - 2*b^2*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] + 4*b^2*c*d*x*Log[1 + E^((2*I)*(a + b*x))] +
```

$$2*b^2*d^2*x^2*\text{Log}[1 + E^{((2*I)*(a + b*x))}] - (2*I)*b*d*(c + d*x)*\text{PolyLog}[2, -E^{((2*I)*(a + b*x))}] + (2*I)*b*d*(c + d*x)*\text{PolyLog}[2, E^{((2*I)*(a + b*x))}] + d^2*\text{PolyLog}[3, -E^{((2*I)*(a + b*x))}] - d^2*\text{PolyLog}[3, E^{((2*I)*(a + b*x))}]]/(8*b^3) + ((c^2 + 2*c*d*x + d^2*x^2)*\text{Sec}[a + b*x]^2)/(16*b) + (\text{Sec}[a]*\text{Sec}[a + b*x]*(-(c*d*\text{Sin}[b*x]) - d^2*x*\text{Sin}[b*x]))/(8*b^2) + (\text{Csc}[a]*\text{Csc}[a + b*x]*(c*d*\text{Sin}[b*x] + d^2*x*\text{Sin}[b*x]))/(8*b^2))$$

Maple [B] time = 0.239, size = 716, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x)

[Out] $-d^2*\text{polylog}(3, -\exp(2*I*(b*x+a)))/b^3 + 4/b^2/(\exp(2*I*(b*x+a))+1)^2/(\exp(2*I*(b*x+a))-1)^2*(d^2*x^2*b*\exp(6*I*(b*x+a))+2*c*d*x*b*\exp(6*I*(b*x+a))+c^2*b*\exp(6*I*(b*x+a))-I*d^2*x*\exp(6*I*(b*x+a))+b*d^2*x^2*\exp(2*I*(b*x+a))-I*d*c*\exp(6*I*(b*x+a))+2*b*c*d*x*\exp(2*I*(b*x+a))+b*c^2*\exp(2*I*(b*x+a))+I*d^2*x*\exp(2*I*(b*x+a))+I*d*c*\exp(2*I*(b*x+a)))-2/b*d^2*\ln(\exp(2*I*(b*x+a))+1)*x^2-2/b^3*d^2*\ln(1-\exp(I*(b*x+a)))*a^2+2/b*d^2*\ln(\exp(I*(b*x+a))+1)*x^2+2/b*d^2*\ln(1-\exp(I*(b*x+a)))*x^2-2/b*c^2*\ln(\exp(2*I*(b*x+a))+1)+1/b^3*d^2*\ln(\exp(I*(b*x+a))+1)+1/b^3*d^2*\ln(\exp(I*(b*x+a))-1)+4/b*c*d*\ln(\exp(I*(b*x+a))+1)*x+2/b*c^2*\ln(\exp(I*(b*x+a))+1)+2/b*c^2*\ln(\exp(I*(b*x+a))-1)-4*I/b^2*\text{polylog}(2, \exp(I*(b*x+a)))*d^2*x-4*I/b^2*c*d*\text{polylog}(2, -\exp(I*(b*x+a)))+2*I/b^2*d^2*\text{polylog}(2, -\exp(2*I*(b*x+a)))*x+2*I/b^2*c*d*\text{polylog}(2, -\exp(2*I*(b*x+a)))-4*I/b^2*c*d*\text{polylog}(2, \exp(I*(b*x+a)))+4*d^2*\text{polylog}(3, -\exp(I*(b*x+a)))/b^3+4*d^2*\text{polylog}(3, \exp(I*(b*x+a)))/b^3-1/b^3*d^2*\ln(\exp(2*I*(b*x+a))+1)-4/b*c*d*\ln(\exp(2*I*(b*x+a))+1)*x+4/b*c*d*\ln(1-\exp(I*(b*x+a)))*x+4/b^2*c*d*\ln(1-\exp(I*(b*x+a)))*a+2/b^3*d^2*a^2*\ln(\exp(I*(b*x+a))-1)-4/b^2*c*d*a*\ln(\exp(I*(b*x+a))-1)$

Maxima [B] time = 3.66754, size = 3676, normalized size = 19.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")

[Out] $-1/2*(c^2*((2*\sin(b*x + a))^2 - 1)/(\sin(b*x + a)^4 - \sin(b*x + a)^2) + 2*\log(\sin(b*x + a)^2 - 1) - 2*\log(\sin(b*x + a)^2)) - 2*a*c*d*((2*\sin(b*x + a))^2 - 1)/(\sin(b*x + a)^4 - \sin(b*x + a)^2) + 2*\log(\sin(b*x + a)^2 - 1) - 2*\log(\sin(b*x + a)^2))/b + a^2*d^2*((2*\sin(b*x + a))^2 - 1)/(\sin(b*x + a)^4 - \sin(b*x + a)^2) + 2*\log(\sin(b*x + a)^2 - 1) - 2*\log(\sin(b*x + a)^2))/b^2 + 2*((4*(b*x + a)^2*d^2 + 8*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + 2*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*\cos(8*b*x + 8*a) - 4*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*\cos(4*b*x + 4*a) + (4*I*(b*x + a))^2*d^2 + (8*I*b*c*d - 8*I*a*d^2)*(b*x + a) + 2*I*d^2)*\sin(8*b*x + 8*a) + (-8*I*(b*x + a)^2*d^2 + (-16*I*b*c*d + 16*I*a*d^2)*(b*x + a) - 4*I*d^2)*\sin(4*b*x + 4*a))*\arctan2(\sin(2*b*x + 2*a), \cos(2*b*x + 2*a) + 1) - (4*(b*x + a)^2*d^2 + 8*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + 2*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*\cos(8*b*x + 8*a) - 4*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*\cos(4*b*x + 4*a) - (-4*I*(b*x + a)^2*d^2 +$

$$\begin{aligned}
& (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) - 2*I*d^2*\sin(8*b*x + 8*a) - (8*I*(b*x \\
& + a)^2*d^2 + (16*I*b*c*d - 16*I*a*d^2)*(b*x + a) + 4*I*d^2)*\sin(4*b*x + 4* \\
& a))*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) - (2*d^2*\cos(8*b*x + 8*a) - 4*d \\
& ^2*\cos(4*b*x + 4*a) + 2*I*d^2*\sin(8*b*x + 8*a) - 4*I*d^2*\sin(4*b*x + 4*a) + \\
& 2*d^2)*\arctan2(\sin(b*x + a), \cos(b*x + a) - 1) + (4*(b*x + a)^2*d^2 + 8*(b \\
& *c*d - a*d^2)*(b*x + a) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a)) \\
& *\cos(8*b*x + 8*a) - 8*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*\cos(4 \\
& *b*x + 4*a) + (4*I*(b*x + a)^2*d^2 + (8*I*b*c*d - 8*I*a*d^2)*(b*x + a))*\sin \\
& (8*b*x + 8*a) + (-8*I*(b*x + a)^2*d^2 + (-16*I*b*c*d + 16*I*a*d^2)*(b*x + a \\
&))*\sin(4*b*x + 4*a))*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) + (8*I*(b*x + \\
& a)^2*d^2 + 8*b*c*d - 8*a*d^2 + (16*I*b*c*d - 8*(2*I*a - 1)*d^2)*(b*x + a)) \\
& *\cos(6*b*x + 6*a) + (8*I*(b*x + a)^2*d^2 - 8*b*c*d + 8*a*d^2 + (16*I*b*c*d \\
& - 8*(2*I*a + 1)*d^2)*(b*x + a))*\cos(2*b*x + 2*a) - (4*b*c*d + 4*(b*x + a)*d \\
& ^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2))*\cos(8*b*x + 8*a) - 8*(b*c*d \\
& + (b*x + a)*d^2 - a*d^2))*\cos(4*b*x + 4*a) - (-4*I*b*c*d - 4*I*(b*x + a)*d \\
& ^2 + 4*I*a*d^2)*\sin(8*b*x + 8*a) - (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d \\
& ^2)*\sin(4*b*x + 4*a))*\operatorname{dilog}(-e^{(2*I*b*x + 2*I*a)}) + (8*b*c*d + 8*(b*x + a)* \\
& d^2 - 8*a*d^2 + 8*(b*c*d + (b*x + a)*d^2 - a*d^2))*\cos(8*b*x + 8*a) - 16*(b* \\
& c*d + (b*x + a)*d^2 - a*d^2))*\cos(4*b*x + 4*a) + (8*I*b*c*d + 8*I*(b*x + a)* \\
& d^2 - 8*I*a*d^2)*\sin(8*b*x + 8*a) + (-16*I*b*c*d - 16*I*(b*x + a)*d^2 + 16* \\
& I*a*d^2)*\sin(4*b*x + 4*a))*\operatorname{dilog}(-e^{(I*b*x + I*a)}) + (8*b*c*d + 8*(b*x + a) \\
& *d^2 - 8*a*d^2 + 8*(b*c*d + (b*x + a)*d^2 - a*d^2))*\cos(8*b*x + 8*a) - 16*(b \\
& *c*d + (b*x + a)*d^2 - a*d^2))*\cos(4*b*x + 4*a) + (8*I*b*c*d + 8*I*(b*x + a) \\
& *d^2 - 8*I*a*d^2)*\sin(8*b*x + 8*a) + (-16*I*b*c*d - 16*I*(b*x + a)*d^2 + 16 \\
& *I*a*d^2)*\sin(4*b*x + 4*a))*\operatorname{dilog}(e^{(I*b*x + I*a)}) + (-2*I*(b*x + a)^2*d^2 \\
& + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a) - I*d^2 + (-2*I*(b*x + a)^2*d^2 + (-4* \\
& I*b*c*d + 4*I*a*d^2)*(b*x + a) - I*d^2))*\cos(8*b*x + 8*a) + (4*I*(b*x + a)^2 \\
& *d^2 + (8*I*b*c*d - 8*I*a*d^2)*(b*x + a) + 2*I*d^2))*\cos(4*b*x + 4*a) + (2*(\\
& b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2))*\sin(8*b*x + 8*a) - 2*(2 \\
& *(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2))*\sin(4*b*x + 4*a))*\log \\
& (\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) + (2*I*(\\
& b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a) + I*d^2 + (2*I*(b*x + a) \\
& ^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a) + I*d^2))*\cos(8*b*x + 8*a) + (-4* \\
& I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) - 2*I*d^2))*\cos(4*b*x \\
& + 4*a) - (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2))*\sin(8*b*x \\
& + 8*a) + 2*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2))*\sin(4*b \\
& *x + 4*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) + (2*I \\
& *(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a) + I*d^2 + (2*I*(b*x + \\
& a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a) + I*d^2))*\cos(8*b*x + 8*a) + (- \\
& 4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) - 2*I*d^2))*\cos(4*b \\
& *x + 4*a) - (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2))*\sin(8*b \\
& *x + 8*a) + 2*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2))*\sin(4 \\
& *b*x + 4*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) + (- \\
& 2*I*d^2*\cos(8*b*x + 8*a) + 4*I*d^2*\cos(4*b*x + 4*a) + 2*d^2*\sin(8*b*x + 8*a \\
&) - 4*d^2*\sin(4*b*x + 4*a) - 2*I*d^2)*\operatorname{polylog}(3, -e^{(2*I*b*x + 2*I*a)}) + (8 \\
& *I*d^2*\cos(8*b*x + 8*a) - 16*I*d^2*\cos(4*b*x + 4*a) - 8*d^2*\sin(8*b*x + 8*a \\
&) + 16*d^2*\sin(4*b*x + 4*a) + 8*I*d^2)*\operatorname{polylog}(3, -e^{(I*b*x + I*a)}) + (8*I \\
& d^2*\cos(8*b*x + 8*a) - 16*I*d^2*\cos(4*b*x + 4*a) - 8*d^2*\sin(8*b*x + 8*a) + \\
& 16*d^2*\sin(4*b*x + 4*a) + 8*I*d^2)*\operatorname{polylog}(3, e^{(I*b*x + I*a)}) - (8*(b*x + \\
& a)^2*d^2 - 8*I*b*c*d + 8*I*a*d^2 + (16*b*c*d - (16*a + 8*I)*d^2)*(b*x + a) \\
&)*\sin(6*b*x + 6*a) - (8*(b*x + a)^2*d^2 + 8*I*b*c*d - 8*I*a*d^2 + (16*b*c*d \\
& - (16*a - 8*I)*d^2)*(b*x + a))*\sin(2*b*x + 2*a))/(-2*I*b^2*\cos(8*b*x + 8*a \\
&) + 4*I*b^2*\cos(4*b*x + 4*a) + 2*b^2*\sin(8*b*x + 8*a) - 4*b^2*\sin(4*b*x + 4 \\
& *a) - 2*I*b^2))/b
\end{aligned}$$

Fricas [C] time = 1.11671, size = 5801, normalized size = 30.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + \\ & b^2*c^2)*\cos(b*x + a)^2 - 2*(b*d^2*x + b*c*d)*\cos(b*x + a)*\sin(b*x + a) - (\\ & (-4*I*b*d^2*x - 4*I*b*c*d)*\cos(b*x + a)^4 + (4*I*b*d^2*x + 4*I*b*c*d)*\cos(b \\ & *x + a)^2)*\operatorname{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) - ((4*I*b*d^2*x + 4*I*b*c*d) \\ &)*\cos(b*x + a)^4 + (-4*I*b*d^2*x - 4*I*b*c*d)*\cos(b*x + a)^2)*\operatorname{dilog}(\cos(b*x \\ & + a) - I*\sin(b*x + a)) - ((-4*I*b*d^2*x - 4*I*b*c*d)*\cos(b*x + a)^4 + (4*I \\ & *b*d^2*x + 4*I*b*c*d)*\cos(b*x + a)^2)*\operatorname{dilog}(I*\cos(b*x + a) + \sin(b*x + a)) \\ & - ((4*I*b*d^2*x + 4*I*b*c*d)*\cos(b*x + a)^4 + (-4*I*b*d^2*x - 4*I*b*c*d)*\cos \\ & (b*x + a)^2)*\operatorname{dilog}(I*\cos(b*x + a) - \sin(b*x + a)) - ((4*I*b*d^2*x + 4*I*b* \\ & c*d)*\cos(b*x + a)^4 + (-4*I*b*d^2*x - 4*I*b*c*d)*\cos(b*x + a)^2)*\operatorname{dilog}(-I*\cos \\ & (b*x + a) + \sin(b*x + a)) - ((-4*I*b*d^2*x - 4*I*b*c*d)*\cos(b*x + a)^4 + \\ & (4*I*b*d^2*x + 4*I*b*c*d)*\cos(b*x + a)^2)*\operatorname{dilog}(-I*\cos(b*x + a) - \sin(b*x + \\ & a)) - ((4*I*b*d^2*x + 4*I*b*c*d)*\cos(b*x + a)^4 + (-4*I*b*d^2*x - 4*I*b*c* \\ & d)*\cos(b*x + a)^2)*\operatorname{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) - ((-4*I*b*d^2*x - \\ & 4*I*b*c*d)*\cos(b*x + a)^4 + (4*I*b*d^2*x + 4*I*b*c*d)*\cos(b*x + a)^2)*\operatorname{dilo} \\ & g(-\cos(b*x + a) - I*\sin(b*x + a)) - ((2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c \\ & ^2 + d^2)*\cos(b*x + a)^4 - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)* \\ & \cos(b*x + a)^2)*\log(\cos(b*x + a) + I*\sin(b*x + a) + 1) + ((2*b^2*c^2 - 4*a* \\ & b*c*d + (2*a^2 + 1)*d^2)*\cos(b*x + a)^4 - (2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + \\ & 1)*d^2)*\cos(b*x + a)^2)*\log(\cos(b*x + a) + I*\sin(b*x + a) + I) - ((2*b^2*d \\ & ^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*\cos(b*x + a)^4 - (2*b^2*d^2*x^2 + 4 \\ & *b^2*c*d*x + 2*b^2*c^2 + d^2)*\cos(b*x + a)^2)*\log(\cos(b*x + a) - I*\sin(b*x \\ & + a) + 1) + ((2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*\cos(b*x + a)^4 - (2* \\ & b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*\cos(b*x + a)^2)*\log(\cos(b*x + a) - I \\ & *\sin(b*x + a) + I) + 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\cos \\ & (b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\cos(b*x + \\ & a)^2)*\log(I*\cos(b*x + a) + \sin(b*x + a) + 1) + 2*((b^2*d^2*x^2 + 2*b^2*c*d \\ & *x + 2*a*b*c*d - a^2*d^2)*\cos(b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a \\ & *b*c*d - a^2*d^2)*\cos(b*x + a)^2)*\log(I*\cos(b*x + a) - \sin(b*x + a) + 1) + \\ & 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\cos(b*x + a)^4 - (b^2*d \\ & ^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\cos(b*x + a)^2)*\log(-I*\cos(b*x \\ & + a) + \sin(b*x + a) + 1) + 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2 \\ & *d^2)*\cos(b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\cos \\ & (b*x + a)^2)*\log(-I*\cos(b*x + a) - \sin(b*x + a) + 1) - ((2*b^2*c^2 - 4*a*b \\ & *c*d + (2*a^2 + 1)*d^2)*\cos(b*x + a)^4 - (2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + \\ & 1)*d^2)*\cos(b*x + a)^2)*\log(-1/2*\cos(b*x + a) + 1/2*I*\sin(b*x + a) + 1/2) - \\ & ((2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*\cos(b*x + a)^4 - (2*b^2*c^2 - 4 \\ & *a*b*c*d + (2*a^2 + 1)*d^2)*\cos(b*x + a)^2)*\log(-1/2*\cos(b*x + a) - 1/2*I*s \\ & \sin(b*x + a) + 1/2) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\cos \\ & (b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\cos(b*x + \\ & a)^2)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + 1) + ((2*b^2*c^2 - 4*a*b*c*d + \\ & (2*a^2 + 1)*d^2)*\cos(b*x + a)^4 - (2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2) \\ & *\cos(b*x + a)^2)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + I) - 2*((b^2*d^2*x^2 \\ & + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\cos(b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*c \\ & *d*x + 2*a*b*c*d - a^2*d^2)*\cos(b*x + a)^2)*\log(-\cos(b*x + a) - I*\sin(b*x \\ & + a) + 1) + ((2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*\cos(b*x + a)^4 - (2* \\ & b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*\cos(b*x + a)^2)*\log(-\cos(b*x + a) - \\ & I*\sin(b*x + a) + I) - 4*(d^2*\cos(b*x + a)^4 - d^2*\cos(b*x + a)^2)*\operatorname{polylog}(3 \\ & , \cos(b*x + a) + I*\sin(b*x + a)) - 4*(d^2*\cos(b*x + a)^4 - d^2*\cos(b*x + a) \\ & ^2)*\operatorname{polylog}(3, \cos(b*x + a) - I*\sin(b*x + a)) + 4*(d^2*\cos(b*x + a)^4 - d^2 \\ & *\cos(b*x + a)^2)*\operatorname{polylog}(3, I*\cos(b*x + a) + \sin(b*x + a)) + 4*(d^2*\cos(b*x \\ & + a)^4 - d^2*\cos(b*x + a)^2)*\operatorname{polylog}(3, I*\cos(b*x + a) - \sin(b*x + a)) + 4 \\ & *(d^2*\cos(b*x + a)^4 - d^2*\cos(b*x + a)^2)*\operatorname{polylog}(3, -I*\cos(b*x + a) + \sin \\ & (b*x + a)) + 4*(d^2*\cos(b*x + a)^4 - d^2*\cos(b*x + a)^2)*\operatorname{polylog}(3, -I*\cos \\ & (b*x + a) - \sin(b*x + a)) - 4*(d^2*\cos(b*x + a)^4 - d^2*\cos(b*x + a)^2)*\operatorname{poly} \end{aligned}$$

```
log(3, -cos(b*x + a) + I*sin(b*x + a)) - 4*(d^2*cos(b*x + a)^4 - d^2*cos(b*
x + a)^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a))/(b^3*cos(b*x + a)^4 -
b^3*cos(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**3*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \csc(bx + a)^3 \sec(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^3*sec(b*x + a)^3, x)
```


3.325 $\int (c + dx) \csc^3(a + bx) \sec^3(a + bx) dx$

Optimal. Leaf size=110

$$\frac{\operatorname{idPolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^2} - \frac{\operatorname{idPolyLog}\left(2, e^{2i(a+bx)}\right)}{b^2} - \frac{d \csc(2a + 2bx)}{b^2} - \frac{4(c + dx) \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{2(c + dx) \cot(a + bx)}{b}$$

[Out] $(-4*(c + d*x)*\operatorname{ArcTanh}[E^{((2*I)*(a + b*x))}])/b - (d*\operatorname{Csc}[2*a + 2*b*x])/b^2 - (2*(c + d*x)*\operatorname{Cot}[2*a + 2*b*x]*\operatorname{Csc}[2*a + 2*b*x])/b + (I*d*\operatorname{PolyLog}[2, -E^{((2*I)*(a + b*x))}])/b^2 - (I*d*\operatorname{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^2$

Rubi [A] time = 0.106173, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4419, 4185, 4183, 2279, 2391}

$$\frac{\operatorname{idPolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^2} - \frac{\operatorname{idPolyLog}\left(2, e^{2i(a+bx)}\right)}{b^2} - \frac{d \csc(2a + 2bx)}{b^2} - \frac{4(c + dx) \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{2(c + dx) \cot(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + d*x)*\operatorname{Csc}[a + b*x]^3*\operatorname{Sec}[a + b*x]^3, x]$

[Out] $(-4*(c + d*x)*\operatorname{ArcTanh}[E^{((2*I)*(a + b*x))}])/b - (d*\operatorname{Csc}[2*a + 2*b*x])/b^2 - (2*(c + d*x)*\operatorname{Cot}[2*a + 2*b*x]*\operatorname{Csc}[2*a + 2*b*x])/b + (I*d*\operatorname{PolyLog}[2, -E^{((2*I)*(a + b*x))}])/b^2 - (I*d*\operatorname{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^2$

Rule 4419

$\operatorname{Int}[\operatorname{Csc}[(a_.) + (b_.)*(x_.)]^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\operatorname{Sec}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] := \operatorname{Dist}[2^n, \operatorname{Int}[(c + d*x)^m*\operatorname{Csc}[2*a + 2*b*x]^n, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, m\}, x$ && $\operatorname{IntegerQ}[n]$ && $\operatorname{RationalQ}[m]$

Rule 4185

$\operatorname{Int}[(\operatorname{csc}[e_.] + (f_.)*(x_.))*(b_.))^{(n_.)}*((c_.) + (d_.)*(x_.)), x_Symbol] := -\operatorname{Simp}[b^2*(c + d*x)*\operatorname{Cot}[e + f*x]*(b*\operatorname{Csc}[e + f*x])^{(n-2)} / (f*(n-1)), x] + (\operatorname{Dist}[(b^2*(n-2)) / (n-1), \operatorname{Int}[(c + d*x)*(b*\operatorname{Csc}[e + f*x])^{(n-2)}, x], x] - \operatorname{Simp}[b^2*d*(b*\operatorname{Csc}[e + f*x])^{(n-2)} / (f^2*(n-1)*(n-2)), x]) /;$ $\operatorname{FreeQ}\{b, c, d, e, f\}, x$ && $\operatorname{GtQ}[n, 1]$ && $\operatorname{NeQ}[n, 2]$

Rule 4183

$\operatorname{Int}[\operatorname{csc}[(e_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] := \operatorname{Simp}[(-2*(c + d*x)^m*\operatorname{ArcTanh}[E^{(I*(e + f*x))}])/f, x] + (-\operatorname{Dist}[(d*m)/f, \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 - E^{(I*(e + f*x))}], x], x] + \operatorname{Dist}[(d*m)/f, \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 + E^{(I*(e + f*x))}], x], x]) /;$ $\operatorname{FreeQ}\{c, d, e, f\}, x$ && $\operatorname{IGtQ}[m, 0]$

Rule 2279

$\operatorname{Int}[\operatorname{Log}[(a_.) + (b_.)*((F_)^{((e_.)*((c_.) + (d_.)*(x_.)))})^{(n_.)}], x_Symbol] := \operatorname{Dist}[1/(d*e^n*\operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /;$ $\operatorname{FreeQ}\{F, a, b, c, d, e, n\}, x$ && $\operatorname{GtQ}[a, 0]$

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned} \int (c + dx) \csc^3(a + bx) \sec^3(a + bx) dx &= 8 \int (c + dx) \csc^3(2a + 2bx) dx \\ &= -\frac{d \csc(2a + 2bx)}{b^2} - \frac{2(c + dx) \cot(2a + 2bx) \csc(2a + 2bx)}{b} + 4 \int (c + dx) \csc(2a + 2bx) dx \\ &= -\frac{4(c + dx) \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d \csc(2a + 2bx)}{b^2} - \frac{2(c + dx) \cot(2a + 2bx) \csc(2a + 2bx)}{b} \\ &= -\frac{4(c + dx) \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d \csc(2a + 2bx)}{b^2} - \frac{2(c + dx) \cot(2a + 2bx) \csc(2a + 2bx)}{b} \\ &= -\frac{4(c + dx) \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d \csc(2a + 2bx)}{b^2} - \frac{2(c + dx) \cot(2a + 2bx) \csc(2a + 2bx)}{b} \end{aligned}$$

Mathematica [B] time = 2.21126, size = 236, normalized size = 2.15

$$\frac{d \left(i \left(\text{PolyLog} \left(2, -e^{2i(a+bx)} \right) - \text{PolyLog} \left(2, e^{2i(a+bx)} \right) \right) + 2(a + bx) \left(\log \left(1 - e^{2i(a+bx)} \right) - \log \left(1 + e^{2i(a+bx)} \right) \right) \right)}{b^2} - \frac{d \tan(a + bx)}{2b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^3,x]

[Out] -(d*Cot[a + b*x])/(2*b^2) - (c*Csc[a + b*x]^2)/(2*b) + (d*(2*a - 2*(a + b*x))*Csc[a + b*x]^2)/(4*b^2) - (2*c*Log[Cos[a + b*x]])/b + (2*c*Log[Sin[a + b*x]])/b - (2*a*d*Log[Tan[a + b*x]])/b^2 + (d*(2*(a + b*x)*(Log[1 - E^((2*I)*(a + b*x))] - Log[1 + E^((2*I)*(a + b*x))]) + I*(PolyLog[2, -E^((2*I)*(a + b*x))] - PolyLog[2, E^((2*I)*(a + b*x))])))/b^2 + (c*Sec[a + b*x]^2)/(2*b) + (d*(-2*a + 2*(a + b*x))*Sec[a + b*x]^2)/(4*b^2) - (d*Tan[a + b*x])/(2*b^2)

Maple [B] time = 0.209, size = 325, normalized size = 3.

$$2 \frac{2 dx b e^{6i(bx+a)} + 2 b c e^{6i(bx+a)} - i d e^{6i(bx+a)} + 2 b d x e^{2i(bx+a)} + 2 b c e^{2i(bx+a)} + i d e^{2i(bx+a)}}{b^2 \left(e^{2i(bx+a)} - 1 \right)^2 \left(e^{2i(bx+a)} + 1 \right)^2} - 2 \frac{c \ln \left(e^{2i(bx+a)} + 1 \right)}{b} + 2 \frac{c \ln \left(e^{i(bx+a)} - 1 \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x)

[Out] 2/b^2/(exp(2*I*(b*x+a))-1)^2/(exp(2*I*(b*x+a))+1)^2*(2*d*x*b*exp(6*I*(b*x+a))+2*b*c*exp(6*I*(b*x+a))-I*d*exp(6*I*(b*x+a))+2*b*d*x*exp(2*I*(b*x+a))+2*b*c*exp(2*I*(b*x+a))+I*d*exp(2*I*(b*x+a)))-2/b*c*ln(exp(2*I*(b*x+a))+1)+2/b*c*ln(exp(I*(b*x+a))-1)+2/b*c*ln(exp(I*(b*x+a))+1)+2/b*d*ln(1-exp(I*(b*x+a)))*x+2/b^2*d*ln(1-exp(I*(b*x+a)))*a-2*I/b^2*d*polylog(2,exp(I*(b*x+a)))-2/b*d*ln(exp(2*I*(b*x+a))+1)*x+I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+2/b*d*ln(exp(I*(b*x+a))+1)*x-2*I*d*polylog(2,-exp(I*(b*x+a)))/b^2-2/b^2*d*a*ln(exp(I*(b*x+a))-1)

Maxima [B] time = 2.46755, size = 1455, normalized size = 13.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -((2*b*d*x + 2*b*c + 2*(b*d*x + b*c))*\cos(8*b*x + 8*a) - 4*(b*d*x + b*c)*\cos(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*\sin(8*b*x + 8*a) + (-4*I*b*d*x - 4*I*b*c)*\sin(4*b*x + 4*a))*\arctan2(\sin(2*b*x + 2*a), \cos(2*b*x + 2*a) + 1) - (2*b*d*x + 2*b*c + 2*(b*d*x + b*c))*\cos(8*b*x + 8*a) - 4*(b*d*x + b*c)*\cos(4*b*x + 4*a) - (-2*I*b*d*x - 2*I*b*c)*\sin(8*b*x + 8*a) - (4*I*b*d*x + 4*I*b*c)*\sin(4*b*x + 4*a))*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) - (2*b*c*\cos(8*b*x + 8*a) - 4*b*c*\cos(4*b*x + 4*a) + 2*I*b*c*\sin(8*b*x + 8*a) - 4*I*b*c*\sin(4*b*x + 4*a) + 2*b*c)*\arctan2(\sin(b*x + a), \cos(b*x + a) - 1) + (2*b*d*x*\cos(8*b*x + 8*a) - 4*b*d*x*\cos(4*b*x + 4*a) + 2*I*b*d*x*\sin(8*b*x + 8*a) - 4*I*b*d*x*\sin(4*b*x + 4*a) + 2*b*d*x)*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) + (4*I*b*d*x + 4*I*b*c + 2*d)*\cos(6*b*x + 6*a) + (4*I*b*d*x + 4*I*b*c - 2*d)*\cos(2*b*x + 2*a) - (d*\cos(8*b*x + 8*a) - 2*d*\cos(4*b*x + 4*a) + I*d*\sin(8*b*x + 8*a) - 2*I*d*\sin(4*b*x + 4*a) + d)*\operatorname{dilog}(-e^{(2*I*b*x + 2*I*a)}) + (2*d*\cos(8*b*x + 8*a) - 4*d*\cos(4*b*x + 4*a) + 2*I*d*\sin(8*b*x + 8*a) - 4*I*d*\sin(4*b*x + 4*a) + 2*d)*\operatorname{dilog}(-e^{(I*b*x + I*a)}) + (2*d*\cos(8*b*x + 8*a) - 4*d*\cos(4*b*x + 4*a) + 2*I*d*\sin(8*b*x + 8*a) - 4*I*d*\sin(4*b*x + 4*a) + 2*d)*\operatorname{dilog}(e^{(I*b*x + I*a)}) + (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*\cos(8*b*x + 8*a) + (2*I*b*d*x + 2*I*b*c)*\cos(4*b*x + 4*a) + (b*d*x + b*c)*\sin(8*b*x + 8*a) - 2*(b*d*x + b*c)*\sin(4*b*x + 4*a))*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) + (I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*\cos(8*b*x + 8*a) + (-2*I*b*d*x - 2*I*b*c)*\cos(4*b*x + 4*a) - (b*d*x + b*c)*\sin(8*b*x + 8*a) + 2*(b*d*x + b*c)*\sin(4*b*x + 4*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) + (I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*\cos(8*b*x + 8*a) + (-2*I*b*d*x - 2*I*b*c)*\cos(4*b*x + 4*a) - (b*d*x + b*c)*\sin(8*b*x + 8*a) + 2*(b*d*x + b*c)*\sin(4*b*x + 4*a))*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) - 2*(2*b*d*x + 2*b*c - I*d)*\sin(6*b*x + 6*a) - 2*(2*b*d*x + 2*b*c + I*d)*\sin(2*b*x + 2*a))/(-I*b^2*\cos(8*b*x + 8*a) + 2*I*b^2*\cos(4*b*x + 4*a) + b^2*\sin(8*b*x + 8*a) - 2*b^2*\sin(4*b*x + 4*a) - I*b^2) \end{aligned}$$

Fricas [B] time = 0.797302, size = 3087, normalized size = 28.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/2*(b*d*x - 2*(b*d*x + b*c))*\cos(b*x + a)^2 - d*\cos(b*x + a)*\sin(b*x + a) + b*c - (-2*I*d*\cos(b*x + a)^4 + 2*I*d*\cos(b*x + a)^2)*\operatorname{dilog}(\cos(b*x + a) + I*\sin(b*x + a)) - (2*I*d*\cos(b*x + a)^4 - 2*I*d*\cos(b*x + a)^2)*\operatorname{dilog}(\cos(b*x + a) - I*\sin(b*x + a)) - (-2*I*d*\cos(b*x + a)^4 + 2*I*d*\cos(b*x + a)^2)*\operatorname{dilog}(I*\cos(b*x + a) + \sin(b*x + a)) - (2*I*d*\cos(b*x + a)^4 - 2*I*d*\cos(b*x + a)^2)*\operatorname{dilog}(I*\cos(b*x + a) - \sin(b*x + a)) - (2*I*d*\cos(b*x + a)^4 - 2*I*d*\cos(b*x + a)^2)*\operatorname{dilog}(-I*\cos(b*x + a) + \sin(b*x + a)) - (-2*I*d*\cos(b*x + a)^4 + 2*I*d*\cos(b*x + a)^2)*\operatorname{dilog}(-I*\cos(b*x + a) - \sin(b*x + a)) - (2*I*d*\cos(b*x + a)^4 - 2*I*d*\cos(b*x + a)^2)*\operatorname{dilog}(-\cos(b*x + a) + I*\sin(b*x + a)) - (-2*I*d*\cos(b*x + a)^4 + 2*I*d*\cos(b*x + a)^2)*\operatorname{dilog}(-\cos(b*x + a) - I*\sin(b*x + a)) - 2*((b*d*x + b*c)*\cos(b*x + a)^4 - (b*d*x + b*c)*\cos(b* \end{aligned}$$

```

x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + 2*((b*c - a*d)*cos(b*x +
a)^4 - (b*c - a*d)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + I)
- 2*((b*d*x + b*c)*cos(b*x + a)^4 - (b*d*x + b*c)*cos(b*x + a)^2)*log(cos(b
*x + a) - I*sin(b*x + a) + 1) + 2*((b*c - a*d)*cos(b*x + a)^4 - (b*c - a*d)
*cos(b*x + a)^2)*log(cos(b*x + a) - I*sin(b*x + a) + I) + 2*((b*d*x + a*d)*
*cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)^2)*log(I*cos(b*x + a) + sin(b*x
+ a) + 1) + 2*((b*d*x + a*d)*cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)^2
)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + 2*((b*d*x + a*d)*cos(b*x + a)^4
- (b*d*x + a*d)*cos(b*x + a)^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 2
*((b*d*x + a*d)*cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)^2)*log(-I*cos(b
*x + a) - sin(b*x + a) + 1) - 2*((b*c - a*d)*cos(b*x + a)^4 - (b*c - a*d)*c
os(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*((b*c
- a*d)*cos(b*x + a)^4 - (b*c - a*d)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) -
1/2*I*sin(b*x + a) + 1/2) - 2*((b*d*x + a*d)*cos(b*x + a)^4 - (b*d*x + a*d
)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 2*((b*c - a*d)*
*cos(b*x + a)^4 - (b*c - a*d)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x
+ a) + I) - 2*((b*d*x + a*d)*cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)^2
)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 2*((b*c - a*d)*cos(b*x + a)^4 -
(b*c - a*d)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I))/(b^2*c
os(b*x + a)^4 - b^2*cos(b*x + a)^2)

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)**3*sec(b*x+a)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \csc(bx + a)^3 \sec(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")

[Out] integrate((d*x + c)*csc(b*x + a)^3*sec(b*x + a)^3, x)

$$3.326 \quad \int \frac{\csc^3(a+bx) \sec^3(a+bx)}{c+dx} dx$$

Optimal. Leaf size=23

$$8\text{Unintegrable}\left(\frac{\csc^3(2a+2bx)}{c+dx}, x\right)$$

[Out] 8*Unintegrable[Csc[2*a + 2*b*x]^3/(c + d*x), x]

Rubi [A] time = 0.0871065, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^3(a+bx) \sec^3(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x), x]

[Out] 8*Defer[Int][Csc[2*a + 2*b*x]^3/(c + d*x), x]

Rubi steps

$$\int \frac{\csc^3(a+bx) \sec^3(a+bx)}{c+dx} dx = 8 \int \frac{\csc^3(2a+2bx)}{c+dx} dx$$

Mathematica [A] time = 29.3312, size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx) \sec^3(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x), x]

[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x), x]

Maple [A] time = 2.741, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx+a))^3 (\sec(bx+a))^3}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c), x)

[Out] int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c), x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)^3 \sec(bx+a)^3}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x, algorithm="fricas")

[Out] integral(csc(b*x + a)^3*sec(b*x + a)^3/(d*x + c), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)**3*sec(b*x+a)**3/(d*x+c),x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^3 \sec(bx+a)^3}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x, algorithm="giac")

[Out] integrate(csc(b*x + a)^3*sec(b*x + a)^3/(d*x + c), x)

$$3.327 \quad \int \frac{\csc^3(a+bx) \sec^3(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=23

$$8\text{Unintegrable}\left(\frac{\csc^3(2a+2bx)}{(c+dx)^2}, x\right)$$

[Out] 8*Unintegrable[Csc[2*a + 2*b*x]^3/(c + d*x)^2, x]

Rubi [A] time = 0.088088, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^3(a+bx) \sec^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x)^2, x]

[Out] 8*Defer[Int][Csc[2*a + 2*b*x]^3/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\csc^3(a+bx) \sec^3(a+bx)}{(c+dx)^2} dx = 8 \int \frac{\csc^3(2a+2bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 33.482, size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx) \sec^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x)^2, x]

[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x)^2, x]

Maple [A] time = 4.84, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx+a))^3 (\sec(bx+a))^3}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2, x)

[Out] int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2, x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)^3 \sec(bx+a)^3}{d^2x^2 + 2cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(csc(b*x + a)^3*sec(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)**3*sec(b*x+a)**3/(d*x+c)**2,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^3 \sec(bx+a)^3}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(csc(b*x + a)^3*sec(b*x + a)^3/(d*x + c)^2, x)

3.328 $\int x \cos^{\frac{5}{2}}(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=83

$$\frac{20\text{EllipticF}\left(\frac{1}{2}(a + bx), 2\right)}{147b^2} + \frac{4 \sin(a + bx) \cos^{\frac{5}{2}}(a + bx)}{49b^2} + \frac{20 \sin(a + bx) \sqrt{\cos(a + bx)}}{147b^2} - \frac{2x \cos^{\frac{7}{2}}(a + bx)}{7b}$$

[Out] $(-2*x*\text{Cos}[a + b*x]^{(7/2)})/(7*b) + (20*\text{EllipticF}[(a + b*x)/2, 2])/(147*b^2) + (20*\text{Sqrt}[\text{Cos}[a + b*x]]*\text{Sin}[a + b*x])/(147*b^2) + (4*\text{Cos}[a + b*x]^{(5/2)}*\text{Sin}[a + b*x])/(49*b^2)$

Rubi [A] time = 0.0535435, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3444, 2635, 2641}

$$\frac{20F\left(\frac{1}{2}(a + bx) \middle| 2\right)}{147b^2} + \frac{4 \sin(a + bx) \cos^{\frac{5}{2}}(a + bx)}{49b^2} + \frac{20 \sin(a + bx) \sqrt{\cos(a + bx)}}{147b^2} - \frac{2x \cos^{\frac{7}{2}}(a + bx)}{7b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{Cos}[a + b*x]^{(5/2)}*\text{Sin}[a + b*x], x]$

[Out] $(-2*x*\text{Cos}[a + b*x]^{(7/2)})/(7*b) + (20*\text{EllipticF}[(a + b*x)/2, 2])/(147*b^2) + (20*\text{Sqrt}[\text{Cos}[a + b*x]]*\text{Sin}[a + b*x])/(147*b^2) + (4*\text{Cos}[a + b*x]^{(5/2)}*\text{Sin}[a + b*x])/(49*b^2)$

Rule 3444

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_)^{(n_.)}]^{(p_.)}*(x_)^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_)^{(n_.)}], x_Symbol] := -\text{Simp}[(x^{(m - n + 1)}*\text{Cos}[a + b*x^n]^{(p + 1)})/(b*n*(p + 1)), x] + \text{Dist}[(m - n + 1)/(b*n*(p + 1)), \text{Int}[x^{(m - n)}*\text{Cos}[a + b*x^n]^{(p + 1)}, x], x] /; \text{FreeQ}\{a, b, p, x\} \&\& \text{LtQ}[0, n, m + 1] \&\& \text{NeQ}[p, -1]$

Rule 2635

$\text{Int}[(b_.)*\text{sin}[(c_.) + (d_.)*(x_)]^{(n_.)}, x_Symbol] := -\text{Simp}[(b*\text{Cos}[c + d*x]*\text{Sin}[c + d*x]^{(n - 1)})/(d*n), x] + \text{Dist}[(b^2*(n - 1))/n, \text{Int}[(b*\text{Sin}[c + d*x]^{(n - 2)})], x] /; \text{FreeQ}\{b, c, d, x\} \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\text{sin}[(c_.) + (d_.)*(x_)]], x_Symbol] := \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d, x\}$

Rubi steps

$$\begin{aligned}
\int x \cos^{\frac{5}{2}}(a+bx) \sin(a+bx) dx &= -\frac{2x \cos^{\frac{7}{2}}(a+bx)}{7b} + \frac{2 \int \cos^{\frac{7}{2}}(a+bx) dx}{7b} \\
&= -\frac{2x \cos^{\frac{7}{2}}(a+bx)}{7b} + \frac{4 \cos^{\frac{5}{2}}(a+bx) \sin(a+bx)}{49b^2} + \frac{10 \int \cos^{\frac{3}{2}}(a+bx) dx}{49b} \\
&= -\frac{2x \cos^{\frac{7}{2}}(a+bx)}{7b} + \frac{20\sqrt{\cos(a+bx)} \sin(a+bx)}{147b^2} + \frac{4 \cos^{\frac{5}{2}}(a+bx) \sin(a+bx)}{49b^2} + \frac{10 \int \cos^{\frac{1}{2}}(a+bx) dx}{49b} \\
&= -\frac{2x \cos^{\frac{7}{2}}(a+bx)}{7b} + \frac{20F\left(\frac{1}{2}(a+bx) \middle| 2\right)}{147b^2} + \frac{20\sqrt{\cos(a+bx)} \sin(a+bx)}{147b^2} + \frac{4 \cos^{\frac{5}{2}}(a+bx) \sin(a+bx)}{49b^2} + \frac{10 \int \cos^{\frac{1}{2}}(a+bx) dx}{49b}
\end{aligned}$$

Mathematica [A] time = 0.33476, size = 73, normalized size = 0.88

$$\frac{40\text{EllipticF}\left(\frac{1}{2}(a+bx), 2\right) + \sqrt{\cos(a+bx)}(46 \sin(a+bx) + 6 \sin(3(a+bx)) - 63bx \cos(a+bx) - 21bx \cos(3(a+bx)))}{294b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[a + b*x]^(5/2)*Sin[a + b*x], x]

[Out] (40*EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*(-63*b*x*Cos[a + b*x] - 21*b*x*Cos[3*(a + b*x)] + 46*Sin[a + b*x] + 6*Sin[3*(a + b*x)]))/(294*b^2)

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int x (\cos(bx+a))^{\frac{5}{2}} \sin(bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(b*x+a)^(5/2)*sin(b*x+a), x)

[Out] int(x*cos(b*x+a)^(5/2)*sin(b*x+a), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(bx+a)^{\frac{5}{2}} \sin(bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)^(5/2)*sin(b*x+a), x, algorithm="maxima")

[Out] integrate(x*cos(b*x + a)^(5/2)*sin(b*x + a), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)**(5/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.329 $\int x \cos^{\frac{3}{2}}(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=60

$$\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2} + \frac{4\sin(a+bx)\cos^{\frac{3}{2}}(a+bx)}{25b^2} - \frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}$$

[Out] $(-2*x*\text{Cos}[a + b*x]^{(5/2)})/(5*b) + (12*\text{EllipticE}[(a + b*x)/2, 2])/(25*b^2) + (4*\text{Cos}[a + b*x]^{(3/2)}*\text{Sin}[a + b*x])/(25*b^2)$

Rubi [A] time = 0.0397231, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3444, 2635, 2639}

$$\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2} + \frac{4\sin(a+bx)\cos^{\frac{3}{2}}(a+bx)}{25b^2} - \frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{Cos}[a + b*x]^{(3/2)}*\text{Sin}[a + b*x], x]$

[Out] $(-2*x*\text{Cos}[a + b*x]^{(5/2)})/(5*b) + (12*\text{EllipticE}[(a + b*x)/2, 2])/(25*b^2) + (4*\text{Cos}[a + b*x]^{(3/2)}*\text{Sin}[a + b*x])/(25*b^2)$

Rule 3444

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)^{(n_.)}]^{(p_.)}*(x_.)^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)^{(n_.)}], x_Symbol] \rightarrow -\text{Simp}[(x^{(m-n+1)}*\text{Cos}[a + b*x^n]^{(p+1)})/(b*n*(p+1)), x] + \text{Dist}[(m-n+1)/(b*n*(p+1)), \text{Int}[x^{(m-n)}*\text{Cos}[a + b*x^n]^{(p+1)}, x], x] /;$ $\text{FreeQ}\{a, b, p\}, x \ \&\& \ \text{LtQ}[0, n, m+1] \ \&\& \ \text{NeQ}[p, -1]$

Rule 2635

$\text{Int}[(b_.)*\text{sin}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x] * (b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] /;$ $\text{FreeQ}\{b, c, d\}, x \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 2639

$\text{Int}[\text{Sqrt}[\text{sin}[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /;$ $\text{FreeQ}\{c, d\}, x$

Rubi steps

$$\begin{aligned} \int x \cos^{\frac{3}{2}}(a + bx) \sin(a + bx) dx &= -\frac{2x \cos^{\frac{5}{2}}(a + bx)}{5b} + \frac{2 \int \cos^{\frac{5}{2}}(a + bx) dx}{5b} \\ &= -\frac{2x \cos^{\frac{5}{2}}(a + bx)}{5b} + \frac{4 \cos^{\frac{3}{2}}(a + bx) \sin(a + bx)}{25b^2} + \frac{6 \int \sqrt{\cos(a + bx)} dx}{25b} \\ &= -\frac{2x \cos^{\frac{5}{2}}(a + bx)}{5b} + \frac{12E\left(\frac{1}{2}(a + bx)\middle|2\right)}{25b^2} + \frac{4 \cos^{\frac{3}{2}}(a + bx) \sin(a + bx)}{25b^2} \end{aligned}$$

Mathematica [A] time = 0.390156, size = 51, normalized size = 0.85

$$\frac{2 \left(\cos^2(a + bx) (5bx \cos(a + bx) - 2 \sin(a + bx)) - 6E \left(\frac{1}{2}(a + bx) \middle| 2 \right) \right)}{25b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[a + b*x]^(3/2)*Sin[a + b*x], x]

[Out] (-2*(-6*EllipticE[(a + b*x)/2, 2] + Cos[a + b*x]^(3/2)*(5*b*x*Cos[a + b*x] - 2*Sin[a + b*x])))/(25*b^2)

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int x (\cos(bx + a))^{\frac{3}{2}} \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(b*x+a)^(3/2)*sin(b*x+a), x)

[Out] int(x*cos(b*x+a)^(3/2)*sin(b*x+a), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(bx + a)^{\frac{3}{2}} \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)^(3/2)*sin(b*x+a), x, algorithm="maxima")

[Out] integrate(x*cos(b*x + a)^(3/2)*sin(b*x + a), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)^(3/2)*sin(b*x+a), x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)**(3/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(bx + a)^{\frac{3}{2}} \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)^(3/2)*sin(b*x + a), x)
```

3.330 $\int x\sqrt{\cos(a+bx)}\sin(a+bx)dx$

Optimal. Leaf size=60

$$\frac{4\text{EllipticF}\left(\frac{1}{2}(a+bx), 2\right)}{9b^2} + \frac{4\sin(a+bx)\sqrt{\cos(a+bx)}}{9b^2} - \frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}$$

[Out] $(-2*x*\text{Cos}[a + b*x]^{(3/2)})/(3*b) + (4*\text{EllipticF}[(a + b*x)/2, 2])/(9*b^2) + (4*\text{Sqrt}[\text{Cos}[a + b*x]]*\text{Sin}[a + b*x])/(9*b^2)$

Rubi [A] time = 0.0388699, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3444, 2635, 2641}

$$\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{9b^2} + \frac{4\sin(a+bx)\sqrt{\cos(a+bx)}}{9b^2} - \frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{Sqrt}[\text{Cos}[a + b*x]]*\text{Sin}[a + b*x], x]$

[Out] $(-2*x*\text{Cos}[a + b*x]^{(3/2)})/(3*b) + (4*\text{EllipticF}[(a + b*x)/2, 2])/(9*b^2) + (4*\text{Sqrt}[\text{Cos}[a + b*x]]*\text{Sin}[a + b*x])/(9*b^2)$

Rule 3444

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_)^{(n_.)}]^{(p_.)}*(x_)^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_)^{(n_.)}], x_Symbol] := -\text{Simp}[(x^{(m-n+1)}*\text{Cos}[a + b*x^n]^{(p+1)})/(b*n*(p+1)), x] + \text{Dist}[(m-n+1)/(b*n*(p+1)), \text{Int}[x^{(m-n)}*\text{Cos}[a + b*x^n]^{(p+1)}, x], x] /; \text{FreeQ}\{a, b, p\}, x \&\& \text{LtQ}[0, n, m+1] \&\& \text{NeQ}[p, -1]$

Rule 2635

$\text{Int}[(b_.)*\text{sin}[(c_.) + (d_.)*(x_)]^{(n_.)}, x_Symbol] := -\text{Simp}[(b*\text{Cos}[c + d*x]*\text{Sin}[c + d*x]^{(n-1)})/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x]^{(n-2)})], x], x] /; \text{FreeQ}\{b, c, d\}, x \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\text{sin}[(c_.) + (d_.)*(x_)]], x_Symbol] := \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \int x\sqrt{\cos(a+bx)}\sin(a+bx)dx &= -\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b} + \frac{2\int\cos^{\frac{3}{2}}(a+bx)dx}{3b} \\ &= -\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b} + \frac{4\sqrt{\cos(a+bx)}\sin(a+bx)}{9b^2} + \frac{2\int\frac{1}{\sqrt{\cos(a+bx)}}dx}{9b} \\ &= -\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b} + \frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{9b^2} + \frac{4\sqrt{\cos(a+bx)}\sin(a+bx)}{9b^2} \end{aligned}$$

Mathematica [A] time = 0.152188, size = 52, normalized size = 0.87

$$\frac{4\text{EllipticF}\left(\frac{1}{2}(a + bx), 2\right) + 2\sqrt{\cos(a + bx)}(2\sin(a + bx) - 3bx\cos(a + bx))}{9b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[Cos[a + b*x]]*Sin[a + b*x],x]

[Out] (4*EllipticF[(a + b*x)/2, 2] + 2*Sqrt[Cos[a + b*x]]*(-3*b*x*Cos[a + b*x] + 2*Sin[a + b*x]))/(9*b^2)

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int x \sin(bx + a) \sqrt{\cos(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sin(b*x+a)*cos(b*x+a)^(1/2),x)

[Out] int(x*sin(b*x+a)*cos(b*x+a)^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \sqrt{\cos(bx + a)} \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)*cos(b*x+a)^(1/2),x, algorithm="maxima")

[Out] integrate(x*sqrt(cos(b*x + a))*sin(b*x + a), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)*cos(b*x+a)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(x*sin(b*x+a)*cos(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\cos(bx+a)}\sin(bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(cos(b*x + a))*sin(b*x + a), x)
```

$$3.331 \quad \int \frac{x \sin(a+bx)}{\sqrt{\cos(a+bx)}} dx$$

Optimal. Leaf size=33

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2} - \frac{2x\sqrt{\cos(a+bx)}}{b}$$

[Out] $(-2*x*\text{Sqrt}[\text{Cos}[a + b*x]])/b + (4*\text{EllipticE}[(a + b*x)/2, 2])/b^2$

Rubi [A] time = 0.0262149, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3444, 2639}

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2} - \frac{2x\sqrt{\cos(a+bx)}}{b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x*\text{Sin}[a + b*x])/ \text{Sqrt}[\text{Cos}[a + b*x]], x]$

[Out] $(-2*x*\text{Sqrt}[\text{Cos}[a + b*x]])/b + (4*\text{EllipticE}[(a + b*x)/2, 2])/b^2$

Rule 3444

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)^{(n_.)}]^{(p_.)}*(x_.)^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)^{(n_.)}], x_Symbol] :> -\text{Simp}[(x^{(m-n+1)}*\text{Cos}[a + b*x^n]^{(p+1)})/(b*n*(p+1)), x] + \text{Dist}[(m-n+1)/(b*n*(p+1)), \text{Int}[x^{(m-n)}*\text{Cos}[a + b*x^n]^{(p+1)}, x], x] /; \text{FreeQ}\{a, b, p\}, x] \&\& \text{LtQ}[0, n, m+1] \&\& \text{NeQ}[p, -1]$

Rule 2639

$\text{Int}[\text{Sqrt}[\text{sin}[(c_.) + (d_.)*(x_.)]], x_Symbol] :> \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \int \frac{x \sin(a+bx)}{\sqrt{\cos(a+bx)}} dx &= -\frac{2x\sqrt{\cos(a+bx)}}{b} + \frac{2 \int \sqrt{\cos(a+bx)} dx}{b} \\ &= -\frac{2x\sqrt{\cos(a+bx)}}{b} + \frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2} \end{aligned}$$

Mathematica [B] time = 1.7706, size = 181, normalized size = 5.48

$$4 \cos^2\left(\frac{1}{2}(a+bx)\right)^{3/2} \sqrt{\frac{\cos(a+bx)}{(\cos(a+bx)+1)^2}} \sqrt{\frac{1}{\cos(a+bx)+1}} \left(-2\sqrt{\sec^2\left(\frac{1}{2}(a+bx)\right)} \text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(a+bx)\right)\right), -1\right) + \left(2 \tan\left(\frac{1}{2}(a+bx)\right) \sqrt{\frac{\cos(a+bx)}{\cos(a+bx)+1}}\right)\right)$$

Warning: Unable to verify antiderivative.

[In] $\text{Integrate}[(x*\text{Sin}[a + b*x])/ \text{Sqrt}[\text{Cos}[a + b*x]], x]$

```
[Out] (4*(Cos[(a + b*x)/2]^2)^(3/2)*Sqrt[Cos[a + b*x]/(1 + Cos[a + b*x])^2]*Sqrt[(1 + Cos[a + b*x])^(-1)]*(2*EllipticE[ArcSin[Tan[(a + b*x)/2]], -1]*Sqrt[Sec[(a + b*x)/2]^2 - 2*EllipticF[ArcSin[Tan[(a + b*x)/2]], -1]*Sqrt[Sec[(a + b*x)/2]^2 + Sqrt[Cos[a + b*x]*Sec[(a + b*x)/2]^2*(-b*x) + 2*Tan[(a + b*x)/2]])))/(b^2*Sqrt[Cos[a + b*x]/(1 + Cos[a + b*x])])
```

Maple [C] time = 0.212, size = 310, normalized size = 9.4

$$-\frac{(bx + 2i)\left(\left(e^{i(bx+a)}\right)^2 + 1\right)\sqrt{2}}{b^2 e^{i(bx+a)}} \frac{1}{\sqrt{\frac{\left(e^{i(bx+a)}\right)^2 + 1}{e^{i(bx+a)}}}} - \frac{2i\sqrt{2}}{b^2 e^{i(bx+a)}} \left(-2 \frac{\left(e^{i(bx+a)}\right)^2 + 1}{\sqrt{\left(\left(e^{i(bx+a)}\right)^2 + 1\right) e^{i(bx+a)}}} + i\sqrt{2} \sqrt{-i\left(e^{i(bx+a)} + i\right)} \sqrt{i\left(e^{i(bx+a)} - i\right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/cos(b*x+a)^(1/2), x)
```

```
[Out] -(b*x+2*I)*(exp(I*(b*x+a))^2+1)/b^2*2^(1/2)/((exp(I*(b*x+a))^2+1)/exp(I*(b*x+a)))^(1/2)/exp(I*(b*x+a))-2*I/b^2*(-2*(exp(I*(b*x+a))^2+1)/((exp(I*(b*x+a))^2+1)*exp(I*(b*x+a)))^(1/2)+I*(-I*(exp(I*(b*x+a))+I))^(1/2)*2^(1/2)*(I*(exp(I*(b*x+a))-I))^(1/2)*(I*exp(I*(b*x+a)))^(1/2)/(exp(I*(b*x+a))^3+exp(I*(b*x+a)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(b*x+a))+I))^(1/2), 1/2*2^(1/2))+I*EllipticF((-I*(exp(I*(b*x+a))+I))^(1/2), 1/2*2^(1/2))))*2^(1/2)/((exp(I*(b*x+a))^2+1)/exp(I*(b*x+a)))^(1/2)*((exp(I*(b*x+a))^2+1)*exp(I*(b*x+a)))^(1/2)/exp(I*(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(bx + a)}{\sqrt{\cos(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(1/2), x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/sqrt(cos(b*x + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(1/2), x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin (bx + a)}{\sqrt{\cos (bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/sqrt(cos(b*x + a)), x)
```

$$3.332 \quad \int \frac{x \sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)} dx$$

Optimal. Leaf size=33

$$\frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{4\text{EllipticF}\left(\frac{1}{2}(a+bx), 2\right)}{b^2}$$

[Out] (2*x)/(b*Sqrt[Cos[a + b*x]]) - (4*EllipticF[(a + b*x)/2, 2])/b^2

Rubi [A] time = 0.0256239, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3444, 2641}

$$\frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{4F\left(\frac{1}{2}(a+bx) \middle| 2\right)}{b^2}$$

Antiderivative was successfully verified.

[In] Int[(x*Sin[a + b*x])/Cos[a + b*x]^(3/2), x]

[Out] (2*x)/(b*Sqrt[Cos[a + b*x]]) - (4*EllipticF[(a + b*x)/2, 2])/b^2

Rule 3444

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p + 1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{x \sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)} dx &= \frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{2 \int \frac{1}{\sqrt{\cos(a+bx)}} dx}{b} \\ &= \frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{4F\left(\frac{1}{2}(a+bx) \middle| 2\right)}{b^2} \end{aligned}$$

Mathematica [A] time = 0.164701, size = 33, normalized size = 1.

$$\frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{4\text{EllipticF}\left(\frac{1}{2}(a+bx), 2\right)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(3/2),x]

[Out] (2*x)/(b*Sqrt[Cos[a + b*x]]) - (4*EllipticF[(a + b*x)/2, 2])/b^2

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int x \sin(bx + a) (\cos(bx + a))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sin(b*x+a)/cos(b*x+a)^(3/2),x)

[Out] int(x*sin(b*x+a)/cos(b*x+a)^(3/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(bx + a)}{\cos(bx + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(3/2),x, algorithm="maxima")

[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(3/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin (bx + a)}{\cos (bx + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(3/2), x)
```

$$3.333 \quad \int \frac{x \sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)} dx$$

Optimal. Leaf size=60

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b^2} - \frac{4\sin(a+bx)}{3b^2\sqrt{\cos(a+bx)}} + \frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)}$$

[Out] (2*x)/(3*b*Cos[a + b*x]^(3/2)) + (4*EllipticE[(a + b*x)/2, 2])/(3*b^2) - (4*Sin[a + b*x])/(3*b^2*sqrt[Cos[a + b*x]])

Rubi [A] time = 0.0360545, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3444, 2636, 2639}

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b^2} - \frac{4\sin(a+bx)}{3b^2\sqrt{\cos(a+bx)}} + \frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(x*Sin[a + b*x])/Cos[a + b*x]^(5/2),x]

[Out] (2*x)/(3*b*Cos[a + b*x]^(3/2)) + (4*EllipticE[(a + b*x)/2, 2])/(3*b^2) - (4*Sin[a + b*x])/(3*b^2*sqrt[Cos[a + b*x]])

Rule 3444

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)], x_Symbol] := -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p + 1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]

Rule 2636

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x \sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)} dx &= \frac{2x}{3b \cos^{\frac{3}{2}}(a+bx)} - \frac{2 \int \frac{1}{\cos^{\frac{3}{2}}(a+bx)} dx}{3b} \\
&= \frac{2x}{3b \cos^{\frac{3}{2}}(a+bx)} - \frac{4 \sin(a+bx)}{3b^2 \sqrt{\cos(a+bx)}} + \frac{2 \int \sqrt{\cos(a+bx)} dx}{3b} \\
&= \frac{2x}{3b \cos^{\frac{3}{2}}(a+bx)} + \frac{4E\left(\frac{1}{2}(a+bx) \middle| 2\right)}{3b^2} - \frac{4 \sin(a+bx)}{3b^2 \sqrt{\cos(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.198494, size = 54, normalized size = 0.9

$$\frac{2 \left(-\sin(2(a+bx)) + 2 \cos^{\frac{3}{2}}(a+bx) E\left(\frac{1}{2}(a+bx) \middle| 2\right) + bx \right)}{3b^2 \cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(5/2), x]

[Out] (2*(b*x + 2*Cos[a + b*x]^(3/2)*EllipticE[(a + b*x)/2, 2] - Sin[2*(a + b*x)])/(3*b^2*Cos[a + b*x]^(3/2))

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int x \sin(bx+a) (\cos(bx+a))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sin(b*x+a)/cos(b*x+a)^(5/2), x)

[Out] int(x*sin(b*x+a)/cos(b*x+a)^(5/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(bx+a)}{\cos^{\frac{5}{2}}(bx+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(5/2), x, algorithm="maxima")

[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(5/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(b*x+a)/cos(b*x+a)^(5/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(b*x+a)/cos(b*x+a)**(5/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin (bx + a)}{\cos (bx + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(b*x+a)/cos(b*x+a)^(5/2),x, algorithm="giac")`

[Out] `integrate(x*sin(b*x + a)/cos(b*x + a)^(5/2), x)`

$$3.334 \quad \int \frac{x \sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)} dx$$

Optimal. Leaf size=60

$$-\frac{4\text{EllipticF}\left(\frac{1}{2}(a+bx), 2\right)}{15b^2} - \frac{4 \sin(a+bx)}{15b^2 \cos^{\frac{3}{2}}(a+bx)} + \frac{2x}{5b \cos^{\frac{5}{2}}(a+bx)}$$

[Out] (2*x)/(5*b*Cos[a + b*x]^(5/2)) - (4*EllipticF[(a + b*x)/2, 2])/(15*b^2) - (4*Sin[a + b*x])/(15*b^2*Cos[a + b*x]^(3/2))

Rubi [A] time = 0.0379388, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3444, 2636, 2641}

$$-\frac{4F\left(\frac{1}{2}(a+bx) \middle| 2\right)}{15b^2} - \frac{4 \sin(a+bx)}{15b^2 \cos^{\frac{3}{2}}(a+bx)} + \frac{2x}{5b \cos^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(x*Sin[a + b*x])/Cos[a + b*x]^(7/2), x]

[Out] (2*x)/(5*b*Cos[a + b*x]^(5/2)) - (4*EllipticF[(a + b*x)/2, 2])/(15*b^2) - (4*Sin[a + b*x])/(15*b^2*Cos[a + b*x]^(3/2))

Rule 3444

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)], x_Symbol] := -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p + 1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]

Rule 2636

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x \sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)} dx &= \frac{2x}{5b \cos^{\frac{5}{2}}(a+bx)} - \frac{2 \int \frac{1}{\cos^{\frac{5}{2}}(a+bx)} dx}{5b} \\
&= \frac{2x}{5b \cos^{\frac{5}{2}}(a+bx)} - \frac{4 \sin(a+bx)}{15b^2 \cos^{\frac{3}{2}}(a+bx)} - \frac{2 \int \frac{1}{\sqrt{\cos(a+bx)}} dx}{15b} \\
&= \frac{2x}{5b \cos^{\frac{5}{2}}(a+bx)} - \frac{4F\left(\frac{1}{2}(a+bx) \middle| 2\right)}{15b^2} - \frac{4 \sin(a+bx)}{15b^2 \cos^{\frac{3}{2}}(a+bx)}
\end{aligned}$$

Mathematica [A] time = 0.239905, size = 53, normalized size = 0.88

$$\frac{2 \left(2 \cos^{\frac{5}{2}}(a+bx) \text{EllipticF}\left(\frac{1}{2}(a+bx), 2\right) + \sin(2(a+bx)) - 3bx \right)}{15b^2 \cos^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(7/2), x]

[Out] (-2*(-3*b*x + 2*Cos[a + b*x]^(5/2))*EllipticF[(a + b*x)/2, 2] + Sin[2*(a + b*x)])/(15*b^2*Cos[a + b*x]^(5/2))

Maple [F] time = 0.09, size = 0, normalized size = 0.

$$\int x \sin(bx+a) (\cos(bx+a))^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sin(b*x+a)/cos(b*x+a)^(7/2), x)

[Out] int(x*sin(b*x+a)/cos(b*x+a)^(7/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(bx+a)}{\cos(bx+a)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(7/2), x, algorithm="maxima")

[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(7/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(bx + a)}{\cos(bx + a)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(7/2), x)
```

$$3.335 \quad \int \frac{x \sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)} dx$$

Optimal. Leaf size=83

$$\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2} - \frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)} - \frac{12\sin(a+bx)}{35b^2\sqrt{\cos(a+bx)}} + \frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)}$$

[Out] (2*x)/(7*b*Cos[a + b*x]^(7/2)) + (12*EllipticE[(a + b*x)/2, 2])/(35*b^2) - (4*Sin[a + b*x])/(35*b^2*Cos[a + b*x]^(5/2)) - (12*Sin[a + b*x])/(35*b^2*sqrt[Cos[a + b*x]])

Rubi [A] time = 0.0484516, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3444, 2636, 2639}

$$\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2} - \frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)} - \frac{12\sin(a+bx)}{35b^2\sqrt{\cos(a+bx)}} + \frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(x*Sin[a + b*x])/Cos[a + b*x]^(9/2),x]

[Out] (2*x)/(7*b*Cos[a + b*x]^(7/2)) + (12*EllipticE[(a + b*x)/2, 2])/(35*b^2) - (4*Sin[a + b*x])/(35*b^2*Cos[a + b*x]^(5/2)) - (12*Sin[a + b*x])/(35*b^2*sqrt[Cos[a + b*x]])

Rule 3444

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p + 1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]

Rule 2636

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x \sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)} dx &= \frac{2x}{7b \cos^{\frac{7}{2}}(a+bx)} - \frac{2 \int \frac{1}{\cos^{\frac{7}{2}}(a+bx)} dx}{7b} \\
&= \frac{2x}{7b \cos^{\frac{7}{2}}(a+bx)} - \frac{4 \sin(a+bx)}{35b^2 \cos^{\frac{5}{2}}(a+bx)} - \frac{6 \int \frac{1}{\cos^{\frac{3}{2}}(a+bx)} dx}{35b} \\
&= \frac{2x}{7b \cos^{\frac{7}{2}}(a+bx)} - \frac{4 \sin(a+bx)}{35b^2 \cos^{\frac{5}{2}}(a+bx)} - \frac{12 \sin(a+bx)}{35b^2 \sqrt{\cos(a+bx)}} + \frac{6 \int \sqrt{\cos(a+bx)} dx}{35b} \\
&= \frac{2x}{7b \cos^{\frac{7}{2}}(a+bx)} + \frac{12E\left(\frac{1}{2}(a+bx) \middle| 2\right)}{35b^2} - \frac{4 \sin(a+bx)}{35b^2 \cos^{\frac{5}{2}}(a+bx)} - \frac{12 \sin(a+bx)}{35b^2 \sqrt{\cos(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.275733, size = 65, normalized size = 0.78

$$\frac{-10 \sin(2(a+bx)) - 3 \sin(4(a+bx)) + 24 \cos^{\frac{7}{2}}(a+bx) E\left(\frac{1}{2}(a+bx) \middle| 2\right) + 20bx}{70b^2 \cos^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(9/2), x]

[Out] (20*b*x + 24*Cos[a + b*x]^(7/2)*EllipticE[(a + b*x)/2, 2] - 10*Sin[2*(a + b*x)] - 3*Sin[4*(a + b*x)])/(70*b^2*Cos[a + b*x]^(7/2))

Maple [F] time = 0.09, size = 0, normalized size = 0.

$$\int x \sin(bx+a) (\cos(bx+a))^{-\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sin(b*x+a)/cos(b*x+a)^(9/2), x)

[Out] int(x*sin(b*x+a)/cos(b*x+a)^(9/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(bx+a)}{\cos(bx+a)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(9/2), x, algorithm="maxima")

[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(9/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(9/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(9/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin (bx + a)}{\cos (bx + a)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(9/2),x, algorithm="giac")

[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(9/2), x)

3.336 $\int x \sec^{\frac{9}{2}}(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=103

$$\frac{4 \sin(a + bx) \sec^{\frac{5}{2}}(a + bx)}{35b^2} - \frac{12 \sin(a + bx) \sqrt{\sec(a + bx)}}{35b^2} + \frac{12 \sqrt{\cos(a + bx)} \sqrt{\sec(a + bx)} E\left(\frac{1}{2}(a + bx) \middle| 2\right)}{35b^2} + \frac{2x \sec^{\frac{7}{2}}(a + bx)}{7b}$$

[Out] (12*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(35*b^2) + (2*x*Sec[a + b*x]^(7/2))/(7*b) - (12*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/(35*b^2) - (4*Sec[a + b*x]^(5/2)*Sin[a + b*x])/(35*b^2)

Rubi [A] time = 0.0620826, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4212, 3768, 3771, 2639}

$$\frac{4 \sin(a + bx) \sec^{\frac{5}{2}}(a + bx)}{35b^2} - \frac{12 \sin(a + bx) \sqrt{\sec(a + bx)}}{35b^2} + \frac{12 \sqrt{\cos(a + bx)} \sqrt{\sec(a + bx)} E\left(\frac{1}{2}(a + bx) \middle| 2\right)}{35b^2} + \frac{2x \sec^{\frac{7}{2}}(a + bx)}{7b}$$

Antiderivative was successfully verified.

[In] Int[x*Sec[a + b*x]^(9/2)*Sin[a + b*x], x]

[Out] (12*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(35*b^2) + (2*x*Sec[a + b*x]^(7/2))/(7*b) - (12*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/(35*b^2) - (4*Sec[a + b*x]^(5/2)*Sin[a + b*x])/(35*b^2)

Rule 4212

Int[(x_)^(m_)*Sec[(a_) + (b_)*(x_)^(n_)]^(p_)*Sin[(a_) + (b_)*(x_)^(n_)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p, 1]

Rule 3768

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] :> -Simp[(b*Csc[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2639

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int x \sec^{\frac{9}{2}}(a+bx) \sin(a+bx) dx &= \frac{2x \sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{2 \int \sec^{\frac{7}{2}}(a+bx) dx}{7b} \\
&= \frac{2x \sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{4 \sec^{\frac{5}{2}}(a+bx) \sin(a+bx)}{35b^2} - \frac{6 \int \sec^{\frac{3}{2}}(a+bx) dx}{35b} \\
&= \frac{2x \sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{12\sqrt{\sec(a+bx)} \sin(a+bx)}{35b^2} - \frac{4 \sec^{\frac{5}{2}}(a+bx) \sin(a+bx)}{35b^2} + \frac{6 \int \sqrt{\sec(a+bx)} dx}{35b} \\
&= \frac{2x \sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{12\sqrt{\sec(a+bx)} \sin(a+bx)}{35b^2} - \frac{4 \sec^{\frac{5}{2}}(a+bx) \sin(a+bx)}{35b^2} + \frac{(6\sqrt{\cos(a+bx)}) E\left(\frac{1}{2}(a+bx) \middle| 2\right)}{35b} \\
&= \frac{12\sqrt{\cos(a+bx)} E\left(\frac{1}{2}(a+bx) \middle| 2\right) \sqrt{\sec(a+bx)}}{35b^2} + \frac{2x \sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{12\sqrt{\sec(a+bx)} \sin(a+bx)}{35b^2}
\end{aligned}$$

Mathematica [A] time = 0.286994, size = 65, normalized size = 0.63

$$\frac{\sec^{\frac{7}{2}}(a+bx) \left(-10 \sin(2(a+bx)) - 3 \sin(4(a+bx)) + 24 \cos^{\frac{7}{2}}(a+bx) E\left(\frac{1}{2}(a+bx) \middle| 2\right) + 20bx \right)}{70b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sec[a + b*x]^(9/2)*Sin[a + b*x],x]

[Out] (Sec[a + b*x]^(7/2)*(20*b*x + 24*Cos[a + b*x]^(7/2)*EllipticE[(a + b*x)/2, 2] - 10*Sin[2*(a + b*x)] - 3*Sin[4*(a + b*x)]))/(70*b^2)

Maple [F] time = 0.101, size = 0, normalized size = 0.

$$\int x (\sec(bx+a))^{\frac{9}{2}} \sin(bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sec(b*x+a)^(9/2)*sin(b*x+a),x)

[Out] int(x*sec(b*x+a)^(9/2)*sin(b*x+a),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \sec(bx+a)^{\frac{9}{2}} \sin(bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(b*x+a)^(9/2)*sin(b*x+a),x, algorithm="maxima")

[Out] integrate(x*sec(b*x + a)^(9/2)*sin(b*x + a), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(9/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)**(9/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \sec (bx + a)^{\frac{9}{2}} \sin (bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(9/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sec(b*x + a)^(9/2)*sin(b*x + a), x)
```

3.337 $\int x \sec^{\frac{7}{2}}(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=80

$$\frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx), 2\right)}{15b^2} - \frac{4\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{15b^2} + \frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}$$

[Out] (-4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(15*b^2) + (2*x*Sec[a + b*x]^(5/2))/(5*b) - (4*Sec[a + b*x]^(3/2)*Sin[a + b*x])/(15*b^2)

Rubi [A] time = 0.0488525, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4212, 3768, 3771, 2641}

$$-\frac{4\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{15b^2} - \frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b^2} + \frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

[In] Int[x*Sec[a + b*x]^(7/2)*Sin[a + b*x], x]

[Out] (-4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(15*b^2) + (2*x*Sec[a + b*x]^(5/2))/(5*b) - (4*Sec[a + b*x]^(3/2)*Sin[a + b*x])/(15*b^2)

Rule 4212

Int[(x_)^(m_)*Sec[(a_) + (b_)*(x_)^(n_)]^(p_)*Sin[(a_) + (b_)*(x_)^(n_)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p, 1]

Rule 3768

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2641

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int x \sec^{\frac{7}{2}}(a+bx) \sin(a+bx) dx &= \frac{2x \sec^{\frac{5}{2}}(a+bx)}{5b} - \frac{2 \int \sec^{\frac{5}{2}}(a+bx) dx}{5b} \\
&= \frac{2x \sec^{\frac{5}{2}}(a+bx)}{5b} - \frac{4 \sec^{\frac{3}{2}}(a+bx) \sin(a+bx)}{15b^2} - \frac{2 \int \sqrt{\sec(a+bx)} dx}{15b} \\
&= \frac{2x \sec^{\frac{5}{2}}(a+bx)}{5b} - \frac{4 \sec^{\frac{3}{2}}(a+bx) \sin(a+bx)}{15b^2} - \frac{(2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}) \int \frac{1}{\sqrt{\cos(a+bx)}} dx}{15b} \\
&= -\frac{4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx) \middle| 2\right)\sqrt{\sec(a+bx)}}{15b^2} + \frac{2x \sec^{\frac{5}{2}}(a+bx)}{5b} - \frac{4 \sec^{\frac{3}{2}}(a+bx)}{15b}
\end{aligned}$$

Mathematica [A] time = 0.242782, size = 61, normalized size = 0.76

$$\frac{2\sqrt{\sec(a+bx)}\left(-2\sqrt{\cos(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx), 2\right) - 2\tan(a+bx) + 3bx \sec^2(a+bx)\right)}{15b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sec[a + b*x]^(7/2)*Sin[a + b*x], x]

[Out] (2*sqrt[Sec[a + b*x]]*(-2*sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 3*b*x*Sec[a + b*x]^2 - 2*Tan[a + b*x]))/(15*b^2)

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int x (\sec(bx+a))^{\frac{7}{2}} \sin(bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sec(b*x+a)^(7/2)*sin(b*x+a), x)

[Out] int(x*sec(b*x+a)^(7/2)*sin(b*x+a), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \sec(bx+a)^{\frac{7}{2}} \sin(bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(b*x+a)^(7/2)*sin(b*x+a), x, algorithm="maxima")

[Out] integrate(x*sec(b*x + a)^(7/2)*sin(b*x + a), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(7/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)**(7/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \sec (bx + a)^{\frac{7}{2}} \sin (bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(7/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sec(b*x + a)^(7/2)*sin(b*x + a), x)
```

3.338 $\int x \sec^{\frac{5}{2}}(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=80

$$-\frac{4 \sin(a + bx) \sqrt{\sec(a + bx)}}{3b^2} + \frac{4 \sqrt{\cos(a + bx)} \sqrt{\sec(a + bx)} E\left(\frac{1}{2}(a + bx) \middle| 2\right)}{3b^2} + \frac{2x \sec^{\frac{3}{2}}(a + bx)}{3b}$$

[Out] (4*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(3*b^2) + (2*x*Sec[a + b*x]^(3/2))/(3*b) - (4*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/(3*b^2)

Rubi [A] time = 0.0525987, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4212, 3768, 3771, 2639}

$$-\frac{4 \sin(a + bx) \sqrt{\sec(a + bx)}}{3b^2} + \frac{4 \sqrt{\cos(a + bx)} \sqrt{\sec(a + bx)} E\left(\frac{1}{2}(a + bx) \middle| 2\right)}{3b^2} + \frac{2x \sec^{\frac{3}{2}}(a + bx)}{3b}$$

Antiderivative was successfully verified.

[In] Int[x*Sec[a + b*x]^(5/2)*Sin[a + b*x], x]

[Out] (4*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(3*b^2) + (2*x*Sec[a + b*x]^(3/2))/(3*b) - (4*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/(3*b^2)

Rule 4212

Int[(x_)^(m_)*Sec[(a_) + (b_)*(x_)^(n_)]^(p_)*Sin[(a_) + (b_)*(x_)^(n_)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p, 1]

Rule 3768

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x] * (b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x])^n * Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2639

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int x \sec^{\frac{5}{2}}(a+bx) \sin(a+bx) dx &= \frac{2x \sec^{\frac{3}{2}}(a+bx)}{3b} - \frac{2 \int \sec^{\frac{3}{2}}(a+bx) dx}{3b} \\
&= \frac{2x \sec^{\frac{3}{2}}(a+bx)}{3b} - \frac{4\sqrt{\sec(a+bx)} \sin(a+bx)}{3b^2} + \frac{2 \int \frac{1}{\sqrt{\sec(a+bx)}} dx}{3b} \\
&= \frac{2x \sec^{\frac{3}{2}}(a+bx)}{3b} - \frac{4\sqrt{\sec(a+bx)} \sin(a+bx)}{3b^2} + \frac{(2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}) \int \sqrt{\cos}}{3b} \\
&= \frac{4\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{3b^2} + \frac{2x \sec^{\frac{3}{2}}(a+bx)}{3b} - \frac{4\sqrt{\sec(a+bx)} \sin(a+bx)}{3b^2}
\end{aligned}$$

Mathematica [A] time = 0.195177, size = 54, normalized size = 0.68

$$\frac{2 \sec^{\frac{3}{2}}(a+bx) \left(-\sin(2(a+bx)) + 2 \cos^{\frac{3}{2}}(a+bx) E\left(\frac{1}{2}(a+bx)\middle|2\right) + bx \right)}{3b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sec[a + b*x]^(5/2)*Sin[a + b*x],x]

[Out] (2*Sec[a + b*x]^(3/2)*(b*x + 2*Cos[a + b*x]^(3/2)*EllipticE[(a + b*x)/2, 2] - Sin[2*(a + b*x)])/(3*b^2)

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int x (\sec (bx + a))^{\frac{5}{2}} \sin (bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sec(b*x+a)^(5/2)*sin(b*x+a),x)

[Out] int(x*sec(b*x+a)^(5/2)*sin(b*x+a),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \sec (bx + a)^{\frac{5}{2}} \sin (bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="maxima")

[Out] integrate(x*sec(b*x + a)^(5/2)*sin(b*x + a), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)**(5/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \sec (bx + a)^{\frac{5}{2}} \sin (bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sec(b*x + a)^(5/2)*sin(b*x + a), x)
```

3.339 $\int x \sec^{\frac{3}{2}}(a + bx) \sin(a + bx) dx$

Optimal. Leaf size=53

$$\frac{2x\sqrt{\sec(a+bx)}}{b} - \frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx), 2\right)}{b^2}$$

[Out] (2*x*Sqrt[Sec[a + b*x]])/b - (4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/b^2

Rubi [A] time = 0.0358781, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4212, 3771, 2641}

$$\frac{2x\sqrt{\sec(a+bx)}}{b} - \frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

[In] Int[x*Sec[a + b*x]^(3/2)*Sin[a + b*x], x]

[Out] (2*x*Sqrt[Sec[a + b*x]])/b - (4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/b^2

Rule 4212

Int[(x_)^(m_)*Sec[(a_) + (b_)*(x_)^(n_)]^(p_)*Sin[(a_) + (b_)*(x_)^(n_)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p, 1]

Rule 3771

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2641

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int x \sec^{\frac{3}{2}}(a + bx) \sin(a + bx) dx &= \frac{2x\sqrt{\sec(a+bx)}}{b} - \frac{2 \int \sqrt{\sec(a+bx)} dx}{b} \\ &= \frac{2x\sqrt{\sec(a+bx)}}{b} - \frac{(2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}) \int \frac{1}{\sqrt{\cos(a+bx)}} dx}{b} \\ &= \frac{2x\sqrt{\sec(a+bx)}}{b} - \frac{4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b^2} \end{aligned}$$

Mathematica [A] time = 0.140606, size = 42, normalized size = 0.79

$$\frac{2\sqrt{\sec(a+bx)}\left(bx - 2\sqrt{\cos(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx), 2\right)\right)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sec[a + b*x]^(3/2)*Sin[a + b*x], x]

[Out] (2*(b*x - 2*Sqrt[Cos[a + b*x]])*EllipticF[(a + b*x)/2, 2])*Sqrt[Sec[a + b*x]]/b^2

Maple [F] time = 0.101, size = 0, normalized size = 0.

$$\int x (\sec(bx + a))^{\frac{3}{2}} \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sec(b*x+a)^(3/2)*sin(b*x+a), x)

[Out] int(x*sec(b*x+a)^(3/2)*sin(b*x+a), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \sec(bx + a)^{\frac{3}{2}} \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(b*x+a)^(3/2)*sin(b*x+a), x, algorithm="maxima")

[Out] integrate(x*sec(b*x + a)^(3/2)*sin(b*x + a), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(b*x+a)^(3/2)*sin(b*x+a), x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)**(3/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \sec (bx + a)^{\frac{3}{2}} \sin (bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sec(b*x + a)^(3/2)*sin(b*x + a), x)
```

3.340 $\int x\sqrt{\sec(a+bx)}\sin(a+bx)dx$

Optimal. Leaf size=53

$$\frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2} - \frac{2x}{b\sqrt{\sec(a+bx)}}$$

[Out] $(-2*x)/(b*\text{Sqrt}[\text{Sec}[a + b*x]]) + (4*\text{Sqrt}[\text{Cos}[a + b*x]]*\text{EllipticE}[(a + b*x)/2, 2]*\text{Sqrt}[\text{Sec}[a + b*x]])/b^2$

Rubi [A] time = 0.0366521, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4212, 3771, 2639}

$$\frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2} - \frac{2x}{b\sqrt{\sec(a+bx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{Sqrt}[\text{Sec}[a + b*x]]*\text{Sin}[a + b*x], x]$

[Out] $(-2*x)/(b*\text{Sqrt}[\text{Sec}[a + b*x]]) + (4*\text{Sqrt}[\text{Cos}[a + b*x]]*\text{EllipticE}[(a + b*x)/2, 2]*\text{Sqrt}[\text{Sec}[a + b*x]])/b^2$

Rule 4212

$\text{Int}[(x_)^{(m_.)}*\text{Sec}[(a_.) + (b_.)*(x_)^{(n_.)}]^{(p_.)}*\text{Sin}[(a_.) + (b_.)*(x_)^{(n_.)}], x_Symbol] :> \text{Simp}[(x^{(m-n+1)}*\text{Sec}[a + b*x^n]^{(p-1)})/(b*n*(p-1)), x] - \text{Dist}[(m-n+1)/(b*n*(p-1)), \text{Int}[x^{(m-n)}*\text{Sec}[a + b*x^n]^{(p-1)}, x], x] /; \text{FreeQ}\{a, b, p\}, x \&\& \text{IntegerQ}[n] \&\& \text{GeQ}[m-n, 0] \&\& \text{NeQ}[p, 1]$

Rule 3771

$\text{Int}[(\text{csc}[c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] :> \text{Dist}[(b*\text{Csc}[c + d*x])^n*\text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] /; \text{FreeQ}\{b, c, d\}, x \&\& \text{EqQ}[n^2, 1/4]$

Rule 2639

$\text{Int}[\text{Sqrt}[\text{sin}[(c_.) + (d_.)*(x_.)]], x_Symbol] :> \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \int x\sqrt{\sec(a+bx)}\sin(a+bx)dx &= -\frac{2x}{b\sqrt{\sec(a+bx)}} + \frac{2\int\frac{1}{\sqrt{\sec(a+bx)}}dx}{b} \\ &= -\frac{2x}{b\sqrt{\sec(a+bx)}} + \frac{(2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)})\int\sqrt{\cos(a+bx)}dx}{b} \\ &= -\frac{2x}{b\sqrt{\sec(a+bx)}} + \frac{4\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b^2} \end{aligned}$$

Mathematica [B] time = 2.21082, size = 132, normalized size = 2.49

$$2 \frac{\left(\frac{2 \sec^2\left(\frac{1}{2}(a+bx)\right) \text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(a+bx)\right)\right), -1\right)}{\sqrt{\cos(a+bx) \sec^4\left(\frac{1}{2}(a+bx)\right)}} + 2 \tan\left(\frac{1}{2}(a+bx)\right) + \frac{2 \sec^2\left(\frac{1}{2}(a+bx)\right) E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)\right) - 1}{\sqrt{\cos(a+bx) \sec^4\left(\frac{1}{2}(a+bx)\right)}} - bx \right)}{b^2 \sqrt{\sec(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[Sec[a + b*x]]*Sin[a + b*x], x]

[Out] (2*(-(b*x) + (2*EllipticE[ArcSin[Tan[(a + b*x)/2]]], -1)*Sec[(a + b*x)/2]^2)/Sqrt[Cos[a + b*x]*Sec[(a + b*x)/2]^4] - (2*EllipticF[ArcSin[Tan[(a + b*x)/2]]], -1)*Sec[(a + b*x)/2]^2/Sqrt[Cos[a + b*x]*Sec[(a + b*x)/2]^4] + 2*Tan[(a + b*x)/2])/(b^2*Sqrt[Sec[a + b*x]])

Maple [C] time = 0.184, size = 310, normalized size = 5.9

$$-\frac{(bx + 2i) \left((e^{i(bx+a)})^2 + 1 \right) \sqrt{2}}{b^2 e^{i(bx+a)}} \sqrt{\frac{e^{i(bx+a)}}{(e^{i(bx+a)})^2 + 1}} - \frac{2i\sqrt{2}}{b^2 e^{i(bx+a)}} \left(-2 \frac{(e^{i(bx+a)})^2 + 1}{\sqrt{\left((e^{i(bx+a)})^2 + 1 \right) e^{i(bx+a)}}} + i\sqrt{2} \sqrt{-i(e^{i(bx+a)} + i)} \sqrt{i(e^{i(bx+a)} - i)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sin(b*x+a)*sec(b*x+a)^(1/2), x)

[Out] -(b*x+2*I)*(exp(I*(b*x+a))^2+1)/b^2*2^(1/2)*(exp(I*(b*x+a))/(exp(I*(b*x+a))^2+1))^(1/2)/exp(I*(b*x+a))-2*I/b^2*(-2*(exp(I*(b*x+a))^2+1)/((exp(I*(b*x+a))^2+1)*exp(I*(b*x+a)))^(1/2)+I*(-I*(exp(I*(b*x+a))+I))^(1/2)*2^(1/2)*(I*(exp(I*(b*x+a))-I))^(1/2)*(I*exp(I*(b*x+a)))^(1/2)/(exp(I*(b*x+a))^3+exp(I*(b*x+a)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(b*x+a))+I))^(1/2), 1/2*2^(1/2))+I*EllipticF((-I*(exp(I*(b*x+a))+I))^(1/2), 1/2*2^(1/2))))*2^(1/2)*(exp(I*(b*x+a))/(exp(I*(b*x+a))^2+1))^(1/2)*((exp(I*(b*x+a))^2+1)*exp(I*(b*x+a)))^(1/2)/exp(I*(b*x+a))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \sqrt{\sec(bx + a)} \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)*sec(b*x+a)^(1/2), x, algorithm="maxima")

[Out] integrate(x*sqrt(sec(b*x + a))*sin(b*x + a), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*sec(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \sin(a + bx) \sqrt{\sec(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*sec(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sin(a + b*x)*sqrt(sec(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \sqrt{\sec(bx + a)} \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*sec(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(sec(b*x + a))*sin(b*x + a), x)
```

$$3.341 \quad \int \frac{x \sin(a+bx)}{\sqrt{\sec(a+bx)}} dx$$

Optimal. Leaf size=80

$$\frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx), 2\right)}{9b^2} + \frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}} - \frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

[Out] (-2*x)/(3*b*Sec[a + b*x]^(3/2)) + (4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(9*b^2) + (4*Sin[a + b*x])/(9*b^2*Sqrt[Sec[a + b*x]])

Rubi [A] time = 0.0456498, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4212, 3769, 3771, 2641}

$$\frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}} + \frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{9b^2} - \frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(x*Sin[a + b*x])/Sqrt[Sec[a + b*x]], x]

[Out] (-2*x)/(3*b*Sec[a + b*x]^(3/2)) + (4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(9*b^2) + (4*Sin[a + b*x])/(9*b^2*Sqrt[Sec[a + b*x]])

Rule 4212

Int[(x_)^(m_)*Sec[(a_) + (b_)*(x_)^(n_)]^(p_)*Sin[(a_) + (b_)*(x_)^(n_)], x_Symbol] := Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p, 1]

Rule 3769

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2641

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x \sin(a + bx)}{\sqrt{\sec(a + bx)}} dx &= -\frac{2x}{3b \sec^{\frac{3}{2}}(a + bx)} + \frac{2 \int \frac{1}{\sec^{\frac{3}{2}}(a + bx)} dx}{3b} \\
&= -\frac{2x}{3b \sec^{\frac{3}{2}}(a + bx)} + \frac{4 \sin(a + bx)}{9b^2 \sqrt{\sec(a + bx)}} + \frac{2 \int \sqrt{\sec(a + bx)} dx}{9b} \\
&= -\frac{2x}{3b \sec^{\frac{3}{2}}(a + bx)} + \frac{4 \sin(a + bx)}{9b^2 \sqrt{\sec(a + bx)}} + \frac{(2\sqrt{\cos(a + bx)}\sqrt{\sec(a + bx)}) \int \frac{1}{\sqrt{\cos(a + bx)}} dx}{9b} \\
&= -\frac{2x}{3b \sec^{\frac{3}{2}}(a + bx)} + \frac{4\sqrt{\cos(a + bx)}F\left(\frac{1}{2}(a + bx) \middle| 2\right) \sqrt{\sec(a + bx)}}{9b^2} + \frac{4 \sin(a + bx)}{9b^2 \sqrt{\sec(a + bx)}}
\end{aligned}$$

Mathematica [A] time = 0.247597, size = 63, normalized size = 0.79

$$\frac{\sqrt{\sec(a + bx)} \left(4\sqrt{\cos(a + bx)} \text{EllipticF}\left(\frac{1}{2}(a + bx), 2\right) + 2 \sin(2(a + bx)) - 6bx \cos^2(a + bx) \right)}{9b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Sin[a + b*x])/Sqrt[Sec[a + b*x]], x]

[Out] (Sqrt[Sec[a + b*x]]*(-6*b*x*Cos[a + b*x]^2 + 4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 2*Sin[2*(a + b*x)]))/(9*b^2)

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int x \sin(bx + a) \frac{1}{\sqrt{\sec(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sin(b*x+a)/sec(b*x+a)^(1/2), x)

[Out] int(x*sin(b*x+a)/sec(b*x+a)^(1/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(bx + a)}{\sqrt{\sec(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(1/2), x, algorithm="maxima")

[Out] integrate(x*sin(b*x + a)/sqrt(sec(b*x + a)), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(a + bx)}{\sqrt{\sec(a + bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)/sec(b*x+a)**(1/2),x)

[Out] Integral(x*sin(a + b*x)/sqrt(sec(a + b*x)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(bx + a)}{\sqrt{\sec(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(1/2),x, algorithm="giac")

[Out] integrate(x*sin(b*x + a)/sqrt(sec(b*x + a)), x)

$$3.342 \quad \int \frac{x \sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)} dx$$

Optimal. Leaf size=80

$$\frac{4 \sin(a+bx)}{25b^2 \sec^{\frac{3}{2}}(a+bx)} + \frac{12\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2} - \frac{2x}{5b \sec^{\frac{5}{2}}(a+bx)}$$

[Out] $(-2*x)/(5*b*Sec[a + b*x]^{(5/2)}) + (12*sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*sqrt[Sec[a + b*x]])/(25*b^2) + (4*Sin[a + b*x])/(25*b^2*Sec[a + b*x]^{(3/2)})$

Rubi [A] time = 0.0465426, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4212, 3769, 3771, 2639}

$$\frac{4 \sin(a+bx)}{25b^2 \sec^{\frac{3}{2}}(a+bx)} + \frac{12\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2} - \frac{2x}{5b \sec^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x*\text{Sin}[a + b*x])/Sec[a + b*x]^{(3/2)}, x]$

[Out] $(-2*x)/(5*b*Sec[a + b*x]^{(5/2)}) + (12*sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*sqrt[Sec[a + b*x]])/(25*b^2) + (4*Sin[a + b*x])/(25*b^2*Sec[a + b*x]^{(3/2)})$

Rule 4212

$\text{Int}[(x_)^{(m_*)}*\text{Sec}[(a_*) + (b_*)*(x_)^{(n_*)}]^{(p_*)}*\text{Sin}[(a_*) + (b_*)*(x_)^{(n_*)}], x_Symbol] \rightarrow \text{Simp}[(x^{(m-n+1)}*\text{Sec}[a + b*x^n]^{(p-1)})/(b*n*(p-1)), x] - \text{Dist}[(m-n+1)/(b*n*(p-1)), \text{Int}[x^{(m-n)}*\text{Sec}[a + b*x^n]^{(p-1)}, x], x] /;$ FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m-n, 0] && NeQ[p, 1]

Rule 3769

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_*)]*(b_*)^{(n_*)}), x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Csc}[c + d*x])^{(n+1)})/(b*d*n), x] + \text{Dist}[(n+1)/(b^2*n), \text{Int}[(b*\text{Csc}[c + d*x])^{(n+2)}, x], x] /;$ FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3771

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_*)]*(b_*)^{(n_*)}), x_Symbol] \rightarrow \text{Dist}[(b*\text{Csc}[c + d*x])^n*\text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] /;$ FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2639

$\text{Int}[\text{Sqrt}[\text{sin}[(c_*) + (d_*)*(x_*)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x \sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)} dx &= -\frac{2x}{5b \sec^{\frac{5}{2}}(a+bx)} + \frac{2 \int \frac{1}{\sec^{\frac{5}{2}}(a+bx)} dx}{5b} \\
&= -\frac{2x}{5b \sec^{\frac{5}{2}}(a+bx)} + \frac{4 \sin(a+bx)}{25b^2 \sec^{\frac{3}{2}}(a+bx)} + \frac{6 \int \frac{1}{\sqrt{\sec(a+bx)}} dx}{25b} \\
&= -\frac{2x}{5b \sec^{\frac{5}{2}}(a+bx)} + \frac{4 \sin(a+bx)}{25b^2 \sec^{\frac{3}{2}}(a+bx)} + \frac{(6\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}) \int \sqrt{\cos(a+bx)} dx}{25b} \\
&= -\frac{2x}{5b \sec^{\frac{5}{2}}(a+bx)} + \frac{12\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{25b^2} + \frac{4 \sin(a+bx)}{25b^2 \sec^{\frac{3}{2}}(a+bx)}
\end{aligned}$$

Mathematica [B] time = 8.09831, size = 212, normalized size = 2.65

$$\frac{\cos^2\left(\frac{1}{2}(a+bx)\right)\sqrt{\sec(a+bx)}\left(-12\sqrt{\cos(a+bx)}\sec^4\left(\frac{1}{2}(a+bx)\right)\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(a+bx)\right)\right),-1\right)+\left(5(a+bx)-25b\right)\right)}{25b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Sin[a + b*x])/Sec[a + b*x]^(3/2),x]

[Out] (Sqrt[Sec[a + b*x]]*(-(x*Cos[a + b*x])/10 - (x*Cos[3*(a + b*x)]/10 + Sin[a + b*x]/(25*b) + Sin[3*(a + b*x)]/(25*b)))/b + (Cos[(a + b*x)/2]^2*Sqrt[Sec[a + b*x]]*(12*EllipticE[ArcSin[Tan[(a + b*x)/2]]], -1)*Sqrt[Cos[a + b*x]*Sec[(a + b*x)/2]^4] - 12*EllipticF[ArcSin[Tan[(a + b*x)/2]]], -1)*Sqrt[Cos[a + b*x]*Sec[(a + b*x)/2]^4] + (-5*a + 5*(a + b*x) - 12*Tan[(a + b*x)/2])*(-1 + Tan[(a + b*x)/2]^2))/(25*b^2)

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int x \sin (bx+a) (\sec (bx+a))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sin(b*x+a)/sec(b*x+a)^(3/2),x)

[Out] int(x*sin(b*x+a)/sec(b*x+a)^(3/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin (bx+a)}{\sec (bx+a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(3/2),x, algorithm="maxima")

[Out] integrate(x*sin(b*x + a)/sec(b*x + a)^(3/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(a + bx)}{\sec^2(a + bx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)/sec(b*x+a)**(3/2),x)

[Out] Integral(x*sin(a + b*x)/sec(a + b*x)**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(bx + a)}{\sec(bx + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(3/2),x, algorithm="giac")

[Out] integrate(x*sin(b*x + a)/sec(b*x + a)^(3/2), x)

$$3.343 \quad \int \frac{x \sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)} dx$$

Optimal. Leaf size=103

$$\frac{20\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx), 2\right)}{147b^2} + \frac{4 \sin(a+bx)}{49b^2 \sec^{\frac{5}{2}}(a+bx)} + \frac{20 \sin(a+bx)}{147b^2 \sqrt{\sec(a+bx)}} - \frac{2x}{7b \sec^{\frac{7}{2}}(a+bx)}$$

[Out] $(-2*x)/(7*b*\text{Sec}[a + b*x]^{(7/2)}) + (20*\text{Sqrt}[\text{Cos}[a + b*x]]*\text{EllipticF}[(a + b*x)/2, 2]*\text{Sqrt}[\text{Sec}[a + b*x]])/(147*b^2) + (4*\text{Sin}[a + b*x])/(49*b^2*\text{Sec}[a + b*x]^{(5/2)}) + (20*\text{Sin}[a + b*x])/(147*b^2*\text{Sqrt}[\text{Sec}[a + b*x]])$

Rubi [A] time = 0.0600873, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4212, 3769, 3771, 2641}

$$\frac{4 \sin(a+bx)}{49b^2 \sec^{\frac{5}{2}}(a+bx)} + \frac{20 \sin(a+bx)}{147b^2 \sqrt{\sec(a+bx)}} + \frac{20\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}F\left(\frac{1}{2}(a+bx) \middle| 2\right)}{147b^2} - \frac{2x}{7b \sec^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x*\text{Sin}[a + b*x])/ \text{Sec}[a + b*x]^{(5/2)}, x]$

[Out] $(-2*x)/(7*b*\text{Sec}[a + b*x]^{(7/2)}) + (20*\text{Sqrt}[\text{Cos}[a + b*x]]*\text{EllipticF}[(a + b*x)/2, 2]*\text{Sqrt}[\text{Sec}[a + b*x]])/(147*b^2) + (4*\text{Sin}[a + b*x])/(49*b^2*\text{Sec}[a + b*x]^{(5/2)}) + (20*\text{Sin}[a + b*x])/(147*b^2*\text{Sqrt}[\text{Sec}[a + b*x]])$

Rule 4212

$\text{Int}[(x_)^{(m_*)}*\text{Sec}[(a_*) + (b_*)*(x_)^{(n_*)}]^{(p_*)}*\text{Sin}[(a_*) + (b_*)*(x_)^{(n_*)}], x_Symbol] \rightarrow \text{Simp}[(x^{(m-n+1)}*\text{Sec}[a + b*x^n]^{(p-1)})/(b*n*(p-1)), x] - \text{Dist}[(m-n+1)/(b*n*(p-1)), \text{Int}[x^{(m-n)}*\text{Sec}[a + b*x^n]^{(p-1)}, x], x] /;$ FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m-n, 0] && NeQ[p, 1]

Rule 3769

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_*)]^{(n_*)}*(b_*))^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Csc}[c + d*x]^{(n+1)}))/(b*d*n), x] + \text{Dist}[(n+1)/(b^2*n), \text{Int}[(b*\text{Csc}[c + d*x]^{(n+2)}), x], x] /;$ FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3771

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_*)]^{(n_*)}*(b_*))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[(b*\text{Csc}[c + d*x]^{(n+1)})*\text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] /;$ FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2641

$\text{Int}[1/\text{Sqrt}[\text{sin}[(c_*) + (d_*)*(x_*)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x \sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)} dx &= -\frac{2x}{7b \sec^{\frac{7}{2}}(a+bx)} + \frac{2 \int \frac{1}{\sec^{\frac{7}{2}}(a+bx)} dx}{7b} \\
&= -\frac{2x}{7b \sec^{\frac{7}{2}}(a+bx)} + \frac{4 \sin(a+bx)}{49b^2 \sec^{\frac{5}{2}}(a+bx)} + \frac{10 \int \frac{1}{\sec^{\frac{3}{2}}(a+bx)} dx}{49b} \\
&= -\frac{2x}{7b \sec^{\frac{7}{2}}(a+bx)} + \frac{4 \sin(a+bx)}{49b^2 \sec^{\frac{5}{2}}(a+bx)} + \frac{20 \sin(a+bx)}{147b^2 \sqrt{\sec(a+bx)}} + \frac{10 \int \sqrt{\sec(a+bx)} dx}{147b} \\
&= -\frac{2x}{7b \sec^{\frac{7}{2}}(a+bx)} + \frac{4 \sin(a+bx)}{49b^2 \sec^{\frac{5}{2}}(a+bx)} + \frac{20 \sin(a+bx)}{147b^2 \sqrt{\sec(a+bx)}} + \frac{(10 \sqrt{\cos(a+bx)} \sqrt{\sec(a+bx)})}{147b} \\
&= -\frac{2x}{7b \sec^{\frac{7}{2}}(a+bx)} + \frac{20 \sqrt{\cos(a+bx)} F\left(\frac{1}{2}(a+bx) \middle| 2\right) \sqrt{\sec(a+bx)}}{147b^2} + \frac{4 \sin(a+bx)}{49b^2 \sec^{\frac{5}{2}}(a+bx)} + \frac{20}{147b}
\end{aligned}$$

Mathematica [A] time = 0.339256, size = 89, normalized size = 0.86

$$\frac{\sqrt{\sec(a+bx)} \left(80 \sqrt{\cos(a+bx)} \text{EllipticF}\left(\frac{1}{2}(a+bx), 2\right) + 52 \sin(2(a+bx)) + 6 \sin(4(a+bx)) - 84bx \cos(2(a+bx)) \right)}{588b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Sin[a + b*x])/Sec[a + b*x]^(5/2), x]

[Out] (Sqrt[Sec[a + b*x]]*(-63*b*x - 84*b*x*Cos[2*(a + b*x)] - 21*b*x*Cos[4*(a + b*x)] + 80*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 52*Sin[2*(a + b*x)] + 6*Sin[4*(a + b*x)]))/(588*b^2)

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int x \sin(bx+a) (\sec(bx+a))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sin(b*x+a)/sec(b*x+a)^(5/2), x)

[Out] int(x*sin(b*x+a)/sec(b*x+a)^(5/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(bx+a)}{\sec(bx+a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(5/2), x, algorithm="maxima")

[Out] integrate(x*sin(b*x + a)/sec(b*x + a)^(5/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)/sec(b*x+a)**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin (bx + a)}{\sec (bx + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(5/2),x, algorithm="giac")

[Out] integrate(x*sin(b*x + a)/sec(b*x + a)^(5/2), x)

3.344 $\int x \cos(a + bx) \sin^{\frac{5}{2}}(a + bx) dx$

Optimal. Leaf size=88

$$-\frac{20\text{EllipticF}\left(\frac{1}{2}\left(a + bx - \frac{\pi}{2}\right), 2\right)}{147b^2} + \frac{4\sin^{\frac{5}{2}}(a + bx)\cos(a + bx)}{49b^2} + \frac{20\sqrt{\sin(a + bx)}\cos(a + bx)}{147b^2} + \frac{2x\sin^{\frac{7}{2}}(a + bx)}{7b}$$

[Out] (-20*EllipticF[(a - Pi/2 + b*x)/2, 2])/(147*b^2) + (20*Cos[a + b*x]*Sqrt[Sin[a + b*x]])/(147*b^2) + (4*Cos[a + b*x]*Sin[a + b*x]^(5/2))/(49*b^2) + (2*x*Sin[a + b*x]^(7/2))/(7*b)

Rubi [A] time = 0.0442303, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3443, 2635, 2641}

$$-\frac{20F\left(\frac{1}{2}\left(a + bx - \frac{\pi}{2}\right)\middle|2\right)}{147b^2} + \frac{4\sin^{\frac{5}{2}}(a + bx)\cos(a + bx)}{49b^2} + \frac{20\sqrt{\sin(a + bx)}\cos(a + bx)}{147b^2} + \frac{2x\sin^{\frac{7}{2}}(a + bx)}{7b}$$

Antiderivative was successfully verified.

[In] Int[x*Cos[a + b*x]*Sin[a + b*x]^(5/2), x]

[Out] (-20*EllipticF[(a - Pi/2 + b*x)/2, 2])/(147*b^2) + (20*Cos[a + b*x]*Sqrt[Sin[a + b*x]])/(147*b^2) + (4*Cos[a + b*x]*Sin[a + b*x]^(5/2))/(49*b^2) + (2*x*Sin[a + b*x]^(7/2))/(7*b)

Rule 3443

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^(p_.), x_Symbol] := Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int x \cos(a + bx) \sin^{\frac{5}{2}}(a + bx) dx &= \frac{2x \sin^{\frac{7}{2}}(a + bx)}{7b} - \frac{2 \int \sin^{\frac{7}{2}}(a + bx) dx}{7b} \\
&= \frac{4 \cos(a + bx) \sin^{\frac{5}{2}}(a + bx)}{49b^2} + \frac{2x \sin^{\frac{7}{2}}(a + bx)}{7b} - \frac{10 \int \sin^{\frac{3}{2}}(a + bx) dx}{49b} \\
&= \frac{20 \cos(a + bx) \sqrt{\sin(a + bx)}}{147b^2} + \frac{4 \cos(a + bx) \sin^{\frac{5}{2}}(a + bx)}{49b^2} + \frac{2x \sin^{\frac{7}{2}}(a + bx)}{7b} - \frac{10 \int \frac{1}{\sqrt{\sin(a + bx)}} dx}{49b} \\
&= -\frac{20F\left(\frac{1}{2}\left(a - \frac{\pi}{2} + bx\right) \middle| 2\right)}{147b^2} + \frac{20 \cos(a + bx) \sqrt{\sin(a + bx)}}{147b^2} + \frac{4 \cos(a + bx) \sin^{\frac{5}{2}}(a + bx)}{49b^2}
\end{aligned}$$

Mathematica [A] time = 0.54583, size = 67, normalized size = 0.76

$$\frac{40\text{EllipticF}\left(\frac{1}{4}(-2a - 2bx + \pi), 2\right) + \sqrt{\sin(a + bx)}(84bx \sin^3(a + bx) + 46 \cos(a + bx) - 6 \cos(3(a + bx)))}{294b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[a + b*x]*Sin[a + b*x]^(5/2), x]

[Out] (40*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + Sqrt[Sin[a + b*x]]*(46*Cos[a + b*x] - 6*Cos[3*(a + b*x)] + 84*b*x*Sin[a + b*x]^3))/(294*b^2)

Maple [F] time = 0.104, size = 0, normalized size = 0.

$$\int x \cos(bx + a) (\sin(bx + a))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(b*x+a)*sin(b*x+a)^(5/2), x)

[Out] int(x*cos(b*x+a)*sin(b*x+a)^(5/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(bx + a) \sin(bx + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(5/2), x, algorithm="maxima")

[Out] integrate(x*cos(b*x + a)*sin(b*x + a)^(5/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(bx + a) \sin(bx + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*sin(b*x + a)^(5/2), x)
```

3.345 $\int x \cos(a + bx) \sin^{\frac{3}{2}}(a + bx) dx$

Optimal. Leaf size=65

$$-\frac{12E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2} + \frac{4\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{25b^2} + \frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}$$

[Out] $(-12*\text{EllipticE}[(a - \text{Pi}/2 + b*x)/2, 2])/(25*b^2) + (4*\text{Cos}[a + b*x]*\text{Sin}[a + b*x]^{(3/2)})/(25*b^2) + (2*x*\text{Sin}[a + b*x]^{(5/2)})/(5*b)$

Rubi [A] time = 0.0323581, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3443, 2635, 2639}

$$-\frac{12E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2} + \frac{4\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{25b^2} + \frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{Cos}[a + b*x]*\text{Sin}[a + b*x]^{(3/2)}, x]$

[Out] $(-12*\text{EllipticE}[(a - \text{Pi}/2 + b*x)/2, 2])/(25*b^2) + (4*\text{Cos}[a + b*x]*\text{Sin}[a + b*x]^{(3/2)})/(25*b^2) + (2*x*\text{Sin}[a + b*x]^{(5/2)})/(5*b)$

Rule 3443

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)^{(n_.)}]*(x_.)^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)^{(n_.)}]^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(x^{(m-n+1)}*\text{Sin}[a + b*x^n]^{(p+1)})/(b*n*(p+1)), x] - \text{Dist}[(m-n+1)/(b*n*(p+1)), \text{Int}[x^{(m-n)}*\text{Sin}[a + b*x^n]^{(p+1)}, x], x] /; \text{FreeQ}\{a, b, p\}, x] \&\& \text{LtQ}[0, n, m+1] \&\& \text{NeQ}[p, -1]$

Rule 2635

$\text{Int}[(b_.)*\text{sin}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

Rule 2639

$\text{Int}[\text{Sqrt}[\text{sin}[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \int x \cos(a + bx) \sin^{\frac{3}{2}}(a + bx) dx &= \frac{2x \sin^{\frac{5}{2}}(a + bx)}{5b} - \frac{2 \int \sin^{\frac{5}{2}}(a + bx) dx}{5b} \\ &= \frac{4 \cos(a + bx) \sin^{\frac{3}{2}}(a + bx)}{25b^2} + \frac{2x \sin^{\frac{5}{2}}(a + bx)}{5b} - \frac{6 \int \sqrt{\sin(a + bx)} dx}{25b} \\ &= -\frac{12E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{25b^2} + \frac{4 \cos(a + bx) \sin^{\frac{3}{2}}(a + bx)}{25b^2} + \frac{2x \sin^{\frac{5}{2}}(a + bx)}{5b} \end{aligned}$$

Mathematica [C] time = 0.925856, size = 108, normalized size = 1.66

$$\frac{\sqrt{\sin(a+bx)} \left(4 \tan\left(\frac{1}{2}(a+bx)\right) \sqrt{\sec^2\left(\frac{1}{2}(a+bx)\right)} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -\tan^2\left(\frac{1}{2}(a+bx)\right)\right) + 2 \sin(2(a+bx)) \right)}{25b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[a + b*x]*Sin[a + b*x]^(3/2), x]

[Out] (Sqrt[Sin[a + b*x]]*(5*b*x - 5*b*x*Cos[2*(a + b*x)] + 2*Sin[2*(a + b*x)] - 12*Tan[(a + b*x)/2] + 4*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[(a + b*x)/2]^2]*Sqrt[Sec[(a + b*x)/2]^2]*Tan[(a + b*x)/2]))/(25*b^2)

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int x \cos(bx + a) (\sin(bx + a))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(b*x+a)*sin(b*x+a)^(3/2), x)

[Out] int(x*cos(b*x+a)*sin(b*x+a)^(3/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(bx + a) \sin(bx + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(3/2), x, algorithm="maxima")

[Out] integrate(x*cos(b*x + a)*sin(b*x + a)^(3/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(3/2), x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(bx + a) \sin(bx + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*sin(b*x + a)^(3/2), x)
```

3.346 $\int x \cos(a + bx) \sqrt{\sin(a + bx)} dx$

Optimal. Leaf size=65

$$-\frac{4\text{EllipticF}\left(\frac{1}{2}\left(a + bx - \frac{\pi}{2}\right), 2\right)}{9b^2} + \frac{4\sqrt{\sin(a + bx)} \cos(a + bx)}{9b^2} + \frac{2x \sin^{\frac{3}{2}}(a + bx)}{3b}$$

[Out] $(-4\text{EllipticF}[(a - \text{Pi}/2 + b*x)/2, 2])/(9*b^2) + (4*\text{Cos}[a + b*x]*\text{Sqrt}[\text{Sin}[a + b*x]])/(9*b^2) + (2*x*\text{Sin}[a + b*x]^{(3/2)})/(3*b)$

Rubi [A] time = 0.0315743, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3443, 2635, 2641}

$$-\frac{4F\left(\frac{1}{2}\left(a + bx - \frac{\pi}{2}\right)\middle|2\right)}{9b^2} + \frac{4\sqrt{\sin(a + bx)} \cos(a + bx)}{9b^2} + \frac{2x \sin^{\frac{3}{2}}(a + bx)}{3b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{Cos}[a + b*x]*\text{Sqrt}[\text{Sin}[a + b*x]], x]$

[Out] $(-4\text{EllipticF}[(a - \text{Pi}/2 + b*x)/2, 2])/(9*b^2) + (4*\text{Cos}[a + b*x]*\text{Sqrt}[\text{Sin}[a + b*x]])/(9*b^2) + (2*x*\text{Sin}[a + b*x]^{(3/2)})/(3*b)$

Rule 3443

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_)^{(n_.)}]*(x_)^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_)^{(n_.)}]^{(p_.)}, x_Symbol] := \text{Simp}[(x^{(m - n + 1)}*\text{Sin}[a + b*x^n]^{(p + 1)})/(b*n*(p + 1)), x] - \text{Dist}[(m - n + 1)/(b*n*(p + 1)), \text{Int}[x^{(m - n)}*\text{Sin}[a + b*x^n]^{(p + 1)}, x], x] /;$ $\text{FreeQ}\{a, b, p, x\} \ \&\& \ \text{LtQ}[0, n, m + 1] \ \&\& \ \text{NeQ}[p, -1]$

Rule 2635

$\text{Int}[(b_.)*\text{sin}[(c_.) + (d_.)*(x_)]^{(n_.)}, x_Symbol] := -\text{Simp}[(b*\text{Cos}[c + d*x] * (b*\text{Sin}[c + d*x]^{(n - 1)}))/(d*n), x] + \text{Dist}[(b^2*(n - 1))/n, \text{Int}[(b*\text{Sin}[c + d*x]^{(n - 2)}), x], x] /;$ $\text{FreeQ}\{b, c, d, x\} \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\text{sin}[(c_.) + (d_.)*(x_)]], x_Symbol] := \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /;$ $\text{FreeQ}\{c, d, x\}$

Rubi steps

$$\begin{aligned} \int x \cos(a + bx) \sqrt{\sin(a + bx)} dx &= \frac{2x \sin^{\frac{3}{2}}(a + bx)}{3b} - \frac{2 \int \sin^{\frac{3}{2}}(a + bx) dx}{3b} \\ &= \frac{4 \cos(a + bx) \sqrt{\sin(a + bx)}}{9b^2} + \frac{2x \sin^{\frac{3}{2}}(a + bx)}{3b} - \frac{2 \int \frac{1}{\sqrt{\sin(a + bx)}} dx}{9b} \\ &= -\frac{4F\left(\frac{1}{2}\left(a - \frac{\pi}{2} + bx\right)\middle|2\right)}{9b^2} + \frac{4 \cos(a + bx) \sqrt{\sin(a + bx)}}{9b^2} + \frac{2x \sin^{\frac{3}{2}}(a + bx)}{3b} \end{aligned}$$

Mathematica [A] time = 0.188767, size = 56, normalized size = 0.86

$$\frac{4\text{EllipticF}\left(\frac{1}{4}(-2a - 2bx + \pi), 2\right) + 2\sqrt{\sin(a + bx)}(3bx \sin(a + bx) + 2 \cos(a + bx))}{9b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[a + b*x]*Sqrt[Sin[a + b*x]],x]

[Out] (4*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + 2*Sqrt[Sin[a + b*x]]*(2*Cos[a + b*x] + 3*b*x*Sin[a + b*x]))/(9*b^2)

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int x \cos(bx + a) \sqrt{\sin(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(b*x+a)*sin(b*x+a)^(1/2),x)

[Out] int(x*cos(b*x+a)*sin(b*x+a)^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(bx + a) \sqrt{\sin(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(1/2),x, algorithm="maxima")

[Out] integrate(x*cos(b*x + a)*sqrt(sin(b*x + a)), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \sqrt{\sin(a + bx)} \cos(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(sin(a + b*x))*cos(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(bx + a) \sqrt{\sin(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*sqrt(sin(b*x + a)), x)
```

$$3.347 \quad \int \frac{x \cos(a+bx)}{\sqrt{\sin(a+bx)}} dx$$

Optimal. Leaf size=38

$$\frac{2x\sqrt{\sin(a+bx)}}{b} - \frac{4E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

[Out] $(-4*\text{EllipticE}[(a - \text{Pi}/2 + b*x)/2, 2])/b^2 + (2*x*\text{Sqrt}[\text{Sin}[a + b*x]])/b$

Rubi [A] time = 0.0222333, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3443, 2639}

$$\frac{2x\sqrt{\sin(a+bx)}}{b} - \frac{4E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x*\text{Cos}[a + b*x])/ \text{Sqrt}[\text{Sin}[a + b*x]], x]$

[Out] $(-4*\text{EllipticE}[(a - \text{Pi}/2 + b*x)/2, 2])/b^2 + (2*x*\text{Sqrt}[\text{Sin}[a + b*x]])/b$

Rule 3443

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)^{(n_.)}]*(x_.)^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_.)^{(n_.)}]^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(x^{(m-n+1)}*\text{Sin}[a + b*x^n]^{(p+1)})/(b*n*(p+1)), x] - \text{Dist}[(m-n+1)/(b*n*(p+1)), \text{Int}[x^{(m-n)}*\text{Sin}[a + b*x^n]^{(p+1)}, x], x] /;$ $\text{FreeQ}\{a, b, p\}, x\} \ \&\& \ \text{LtQ}[0, n, m+1] \ \&\& \ \text{NeQ}[p, -1]$

Rule 2639

$\text{Int}[\text{Sqrt}[\text{sin}[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /;$ $\text{FreeQ}\{c, d\}, x\}$

Rubi steps

$$\begin{aligned} \int \frac{x \cos(a+bx)}{\sqrt{\sin(a+bx)}} dx &= \frac{2x\sqrt{\sin(a+bx)}}{b} - \frac{2 \int \sqrt{\sin(a+bx)} dx}{b} \\ &= -\frac{4E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{b^2} + \frac{2x\sqrt{\sin(a+bx)}}{b} \end{aligned}$$

Mathematica [C] time = 1.18553, size = 86, normalized size = 2.26

$$\frac{2\sqrt{\sin(a+bx)}\left(2 \tan\left(\frac{1}{2}(a+bx)\right)\sqrt{\sec^2\left(\frac{1}{2}(a+bx)\right)}\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -\tan^2\left(\frac{1}{2}(a+bx)\right)\right) - 6 \tan\left(\frac{1}{2}(a+bx)\right)\right)}{3b^2}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(x*\text{Cos}[a + b*x])/ \text{Sqrt}[\text{Sin}[a + b*x]], x]$

[Out] $(2\sqrt{\sin[a + bx]})(3bx - 6\tan[(a + bx)/2] + 2\text{Hypergeometric2F1}[1/2, 3/4, 7/4, -\tan[(a + bx)/2]^2]\sqrt{\sec[(a + bx)/2]^2}\tan[(a + bx)/2]) / (3b^2)$

Maple [C] time = 0.204, size = 308, normalized size = 8.1

$$\frac{-i(bx + 2i)\left(\left(e^{i(bx+a)}\right)^2 - 1\right)\sqrt{2}}{b^2 e^{i(bx+a)}} \frac{1}{\sqrt{\frac{-i\left(\left(e^{i(bx+a)}\right)^2 - 1\right)}{e^{i(bx+a)}}}} - 2 \frac{\sqrt{2}\sqrt{-i\left(\left(e^{i(bx+a)}\right)^2 - 1\right)}e^{i(bx+a)}}{b^2 e^{i(bx+a)}} \left(\frac{2i\left(i - i\left(e^{i(bx+a)}\right)^2\right)}{\sqrt{e^{i(bx+a)}\left(i - i\left(e^{i(bx+a)}\right)^2\right)}} - \sqrt{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cos(b*x+a)/sin(b*x+a)^(1/2),x)`

[Out] $-I*(b*x+2*I)*(exp(I*(b*x+a))^2-1)/b^2*2^(1/2)/(-I*(exp(I*(b*x+a))^2-1)/exp(I*(b*x+a)))^(1/2)/exp(I*(b*x+a))-2/b^2*(2*I*(I-I*exp(I*(b*x+a))^2)/(exp(I*(b*x+a))*(I-I*exp(I*(b*x+a))^2))^(1/2)-(exp(I*(b*x+a))+1)^(1/2)*(-2*exp(I*(b*x+a))+2)^(1/2)*(-exp(I*(b*x+a)))^(1/2)/(-I*exp(I*(b*x+a))^3+I*exp(I*(b*x+a))))^(1/2)*(-2*EllipticE((exp(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))+EllipticF((exp(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))))*2^(1/2)/(-I*(exp(I*(b*x+a))^2-1)/exp(I*(b*x+a)))^(1/2)*(-I*(exp(I*(b*x+a))^2-1)*exp(I*(b*x+a)))^(1/2)/exp(I*(b*x+a))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cos(bx + a)}{\sqrt{\sin(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(b*x+a)/sin(b*x+a)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x*cos(b*x + a)/sqrt(sin(b*x + a)), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(b*x+a)/sin(b*x+a)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cos(a + bx)}{\sqrt{\sin(a + bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(1/2),x)

[Out] Integral(x*cos(a + b*x)/sqrt(sin(a + b*x)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cos(bx + a)}{\sqrt{\sin(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(1/2),x, algorithm="giac")

[Out] integrate(x*cos(b*x + a)/sqrt(sin(b*x + a)), x)

$$3.348 \quad \int \frac{x \cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)} dx$$

Optimal. Leaf size=38

$$\frac{4\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right), 2\right)}{b^2} - \frac{2x}{b\sqrt{\sin(a+bx)}}$$

[Out] (4*EllipticF[(a - Pi/2 + b*x)/2, 2])/b^2 - (2*x)/(b*Sqrt[Sin[a + b*x]])

Rubi [A] time = 0.0229804, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3443, 2641}

$$\frac{4F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2} - \frac{2x}{b\sqrt{\sin(a+bx)}}$$

Antiderivative was successfully verified.

[In] Int[(x*Cos[a + b*x])/Sin[a + b*x]^(3/2), x]

[Out] (4*EllipticF[(a - Pi/2 + b*x)/2, 2])/b^2 - (2*x)/(b*Sqrt[Sin[a + b*x]])

Rule 3443

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{x \cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)} dx &= -\frac{2x}{b\sqrt{\sin(a+bx)}} + \frac{2 \int \frac{1}{\sqrt{\sin(a+bx)}} dx}{b} \\ &= \frac{4F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{b^2} - \frac{2x}{b\sqrt{\sin(a+bx)}} \end{aligned}$$

Mathematica [A] time = 0.181046, size = 37, normalized size = 0.97

$$\frac{2\left(-2\text{EllipticF}\left(\frac{1}{4}(-2a-2bx+\pi), 2\right) - \frac{bx}{\sqrt{\sin(a+bx)}}\right)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x*cos[a + b*x])/Sin[a + b*x]^(3/2),x]

[Out] (2*(-2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] - (b*x)/Sqrt[Sin[a + b*x]]))/b^2

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int x \cos (bx + a) (\sin (bx + a))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(b*x+a)/sin(b*x+a)^(3/2),x)

[Out] int(x*cos(b*x+a)/sin(b*x+a)^(3/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cos (bx + a)}{\sin (bx + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(3/2),x, algorithm="maxima")

[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(3/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cos (a + bx)}{\sin ^{\frac{3}{2}}(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(3/2),x)

[Out] Integral(x*cos(a + b*x)/sin(a + b*x)**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cos(bx + a)}{\sin(bx + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(3/2), x)
```

$$3.349 \quad \int \frac{x \cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)} dx$$

Optimal. Leaf size=65

$$-\frac{4E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2} - \frac{4\cos(a+bx)}{3b^2\sqrt{\sin(a+bx)}} - \frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}$$

[Out] (-4*EllipticE[(a - Pi/2 + b*x)/2, 2])/(3*b^2) - (2*x)/(3*b*Sin[a + b*x]^(3/2)) - (4*Cos[a + b*x])/(3*b^2*Sqrt[Sin[a + b*x]])

Rubi [A] time = 0.0319102, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3443, 2636, 2639}

$$-\frac{4E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2} - \frac{4\cos(a+bx)}{3b^2\sqrt{\sin(a+bx)}} - \frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(x*Cos[a + b*x])/Sin[a + b*x]^(5/2),x]

[Out] (-4*EllipticE[(a - Pi/2 + b*x)/2, 2])/(3*b^2) - (2*x)/(3*b*Sin[a + b*x]^(3/2)) - (4*Cos[a + b*x])/(3*b^2*Sqrt[Sin[a + b*x]])

Rule 3443

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]

Rule 2636

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x \cos(a + bx)}{\sin^{\frac{5}{2}}(a + bx)} dx &= -\frac{2x}{3b \sin^{\frac{3}{2}}(a + bx)} + \frac{2 \int \frac{1}{\sin^{\frac{3}{2}}(a + bx)} dx}{3b} \\
&= -\frac{2x}{3b \sin^{\frac{3}{2}}(a + bx)} - \frac{4 \cos(a + bx)}{3b^2 \sqrt{\sin(a + bx)}} - \frac{2 \int \sqrt{\sin(a + bx)} dx}{3b} \\
&= -\frac{4E\left(\frac{1}{2}\left(a - \frac{\pi}{2} + bx\right) \middle| 2\right)}{3b^2} - \frac{2x}{3b \sin^{\frac{3}{2}}(a + bx)} - \frac{4 \cos(a + bx)}{3b^2 \sqrt{\sin(a + bx)}}
\end{aligned}$$

Mathematica [A] time = 0.184205, size = 56, normalized size = 0.86

$$\frac{2\left(\sin(2(a + bx)) - 2 \sin^{\frac{3}{2}}(a + bx)E\left(\frac{1}{4}(-2a - 2bx + \pi) \middle| 2\right) + bx\right)}{3b^2 \sin^{\frac{3}{2}}(a + bx)}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(5/2), x]

[Out] (-2*(b*x - 2*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(3/2) + Sin[2*(a + b*x)])/(3*b^2*Sin[a + b*x]^(3/2))

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int x \cos(bx + a) (\sin(bx + a))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(b*x+a)/sin(b*x+a)^(5/2), x)

[Out] int(x*cos(b*x+a)/sin(b*x+a)^(5/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cos(bx + a)}{\sin(bx + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(5/2), x, algorithm="maxima")

[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(5/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cos(bx + a)}{\sin(bx + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(5/2), x)
```

$$3.350 \quad \int \frac{x \cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)} dx$$

Optimal. Leaf size=65

$$\frac{4\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right), 2\right)}{15b^2} - \frac{4\cos(a+bx)}{15b^2\sin^{\frac{3}{2}}(a+bx)} - \frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)}$$

[Out] (4*EllipticF[(a - Pi/2 + b*x)/2, 2])/(15*b^2) - (2*x)/(5*b*Sin[a + b*x]^(5/2)) - (4*Cos[a + b*x])/(15*b^2*Sin[a + b*x]^(3/2))

Rubi [A] time = 0.0339171, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3443, 2636, 2641}

$$\frac{4F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{15b^2} - \frac{4\cos(a+bx)}{15b^2\sin^{\frac{3}{2}}(a+bx)} - \frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(x*Cos[a + b*x])/Sin[a + b*x]^(7/2), x]

[Out] (4*EllipticF[(a - Pi/2 + b*x)/2, 2])/(15*b^2) - (2*x)/(5*b*Sin[a + b*x]^(5/2)) - (4*Cos[a + b*x])/(15*b^2*Sin[a + b*x]^(3/2))

Rule 3443

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^(p_.), x_Symbol] := Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]

Rule 2636

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x \cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)} dx &= -\frac{2x}{5b \sin^{\frac{5}{2}}(a+bx)} + \frac{2 \int \frac{1}{\sin^{\frac{5}{2}}(a+bx)} dx}{5b} \\
&= -\frac{2x}{5b \sin^{\frac{5}{2}}(a+bx)} - \frac{4 \cos(a+bx)}{15b^2 \sin^{\frac{3}{2}}(a+bx)} + \frac{2 \int \frac{1}{\sqrt{\sin(a+bx)}} dx}{15b} \\
&= \frac{4F\left(\frac{1}{2}\left(a - \frac{\pi}{2} + bx\right) \middle| 2\right)}{15b^2} - \frac{2x}{5b \sin^{\frac{5}{2}}(a+bx)} - \frac{4 \cos(a+bx)}{15b^2 \sin^{\frac{3}{2}}(a+bx)}
\end{aligned}$$

Mathematica [A] time = 0.232829, size = 57, normalized size = 0.88

$$-\frac{2\left(2 \sin^{\frac{5}{2}}(a+bx) \operatorname{EllipticF}\left(\frac{1}{4}(-2a-2bx+\pi), 2\right) + \sin(2(a+bx)) + 3bx\right)}{15b^2 \sin^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(7/2), x]

[Out] (-2*(3*b*x + 2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(5/2) + Sin[2*(a + b*x)])/(15*b^2*Sin[a + b*x]^(5/2))

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int x \cos(bx+a) (\sin(bx+a))^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(b*x+a)/sin(b*x+a)^(7/2), x)

[Out] int(x*cos(b*x+a)/sin(b*x+a)^(7/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cos(bx+a)}{\sin(bx+a)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(7/2), x, algorithm="maxima")

[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(7/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cos(bx + a)}{\sin(bx + a)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(7/2), x)
```

$$3.351 \quad \int \frac{x \cos(a+bx)}{\sin^2(a+bx)} dx$$

Optimal. Leaf size=88

$$-\frac{12E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2} - \frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)} - \frac{12\cos(a+bx)}{35b^2\sqrt{\sin(a+bx)}} - \frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

[Out] (-12*EllipticE[(a - Pi/2 + b*x)/2, 2])/(35*b^2) - (2*x)/(7*b*Sin[a + b*x]^(7/2)) - (4*Cos[a + b*x])/(35*b^2*Sin[a + b*x]^(5/2)) - (12*Cos[a + b*x])/(35*b^2*Sqrt[Sin[a + b*x]])

Rubi [A] time = 0.0443534, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3443, 2636, 2639}

$$-\frac{12E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2} - \frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)} - \frac{12\cos(a+bx)}{35b^2\sqrt{\sin(a+bx)}} - \frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(x*Cos[a + b*x])/Sin[a + b*x]^(9/2),x]

[Out] (-12*EllipticE[(a - Pi/2 + b*x)/2, 2])/(35*b^2) - (2*x)/(7*b*Sin[a + b*x]^(7/2)) - (4*Cos[a + b*x])/(35*b^2*Sin[a + b*x]^(5/2)) - (12*Cos[a + b*x])/(35*b^2*Sqrt[Sin[a + b*x]])

Rule 3443

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]

Rule 2636

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x \cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)} dx &= -\frac{2x}{7b \sin^{\frac{7}{2}}(a+bx)} + \frac{2 \int \frac{1}{\sin^{\frac{7}{2}}(a+bx)} dx}{7b} \\
&= -\frac{2x}{7b \sin^{\frac{7}{2}}(a+bx)} - \frac{4 \cos(a+bx)}{35b^2 \sin^{\frac{5}{2}}(a+bx)} + \frac{6 \int \frac{1}{\sin^{\frac{3}{2}}(a+bx)} dx}{35b} \\
&= -\frac{2x}{7b \sin^{\frac{7}{2}}(a+bx)} - \frac{4 \cos(a+bx)}{35b^2 \sin^{\frac{5}{2}}(a+bx)} - \frac{12 \cos(a+bx)}{35b^2 \sqrt{\sin(a+bx)}} - \frac{6 \int \sqrt{\sin(a+bx)} dx}{35b} \\
&= -\frac{12E\left(\frac{1}{2}\left(a - \frac{\pi}{2} + bx\right)\middle|2\right)}{35b^2} - \frac{2x}{7b \sin^{\frac{7}{2}}(a+bx)} - \frac{4 \cos(a+bx)}{35b^2 \sin^{\frac{5}{2}}(a+bx)} - \frac{12 \cos(a+bx)}{35b^2 \sqrt{\sin(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.299497, size = 73, normalized size = 0.83

$$\frac{2 \left(\sin(2(a+bx)) + 6 \sin^3(a+bx) \cos(a+bx) - 6 \sin^{\frac{7}{2}}(a+bx) E\left(\frac{1}{4}(-2a - 2bx + \pi)\middle|2\right) + 5bx \right)}{35b^2 \sin^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(9/2),x]

[Out] (-2*(5*b*x + 6*Cos[a + b*x]*Sin[a + b*x]^3 - 6*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(7/2) + Sin[2*(a + b*x)])/(35*b^2*Sin[a + b*x]^(7/2))

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int x \cos(bx+a) (\sin(bx+a))^{-\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(b*x+a)/sin(b*x+a)^(9/2),x)

[Out] int(x*cos(b*x+a)/sin(b*x+a)^(9/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cos(bx+a)}{\sin(bx+a)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(9/2),x, algorithm="maxima")

[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(9/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(9/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(9/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cos(bx + a)}{\sin(bx + a)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(9/2),x, algorithm="giac")

[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(9/2), x)

3.352 $\int x \cos(a + bx) \csc^2(a + bx) dx$

Optimal. Leaf size=108

$$\frac{4 \cos(a + bx) \csc^{\frac{5}{2}}(a + bx)}{35b^2} - \frac{12 \cos(a + bx) \sqrt{\csc(a + bx)}}{35b^2} - \frac{12 \sqrt{\sin(a + bx)} \sqrt{\csc(a + bx)} E\left(\frac{1}{2} \left(a + bx - \frac{\pi}{2}\right) \middle| 2\right)}{35b^2} - 2x$$

[Out] (-12*Cos[a + b*x]*Sqrt[Csc[a + b*x]])/(35*b^2) - (4*Cos[a + b*x]*Csc[a + b*x]^(5/2))/(35*b^2) - (2*x*Csc[a + b*x]^(7/2))/(7*b) - (12*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(35*b^2)

Rubi [A] time = 0.0576411, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4213, 3768, 3771, 2639}

$$\frac{4 \cos(a + bx) \csc^{\frac{5}{2}}(a + bx)}{35b^2} - \frac{12 \cos(a + bx) \sqrt{\csc(a + bx)}}{35b^2} - \frac{12 \sqrt{\sin(a + bx)} \sqrt{\csc(a + bx)} E\left(\frac{1}{2} \left(a + bx - \frac{\pi}{2}\right) \middle| 2\right)}{35b^2} - 2x$$

Antiderivative was successfully verified.

[In] Int[x*Cos[a + b*x]*Csc[a + b*x]^(9/2), x]

[Out] (-12*Cos[a + b*x]*Sqrt[Csc[a + b*x]])/(35*b^2) - (4*Cos[a + b*x]*Csc[a + b*x]^(5/2))/(35*b^2) - (2*x*Csc[a + b*x]^(7/2))/(7*b) - (12*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(35*b^2)

Rule 4213

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.), x_Symbol] := -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p, 1]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Csc[c + d*x]^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int x \cos(a + bx) \csc^{\frac{9}{2}}(a + bx) dx &= -\frac{2x \csc^{\frac{7}{2}}(a + bx)}{7b} + \frac{2 \int \csc^{\frac{7}{2}}(a + bx) dx}{7b} \\
&= -\frac{4 \cos(a + bx) \csc^{\frac{5}{2}}(a + bx)}{35b^2} - \frac{2x \csc^{\frac{7}{2}}(a + bx)}{7b} + \frac{6 \int \csc^{\frac{3}{2}}(a + bx) dx}{35b} \\
&= -\frac{12 \cos(a + bx) \sqrt{\csc(a + bx)}}{35b^2} - \frac{4 \cos(a + bx) \csc^{\frac{5}{2}}(a + bx)}{35b^2} - \frac{2x \csc^{\frac{7}{2}}(a + bx)}{7b} - \frac{6 \int \sqrt{\csc(a + bx)}}{35b} \\
&= -\frac{12 \cos(a + bx) \sqrt{\csc(a + bx)}}{35b^2} - \frac{4 \cos(a + bx) \csc^{\frac{5}{2}}(a + bx)}{35b^2} - \frac{2x \csc^{\frac{7}{2}}(a + bx)}{7b} - \frac{(6 \int \sqrt{\csc(a + bx)}}{35b} \\
&= -\frac{12 \cos(a + bx) \sqrt{\csc(a + bx)}}{35b^2} - \frac{4 \cos(a + bx) \csc^{\frac{5}{2}}(a + bx)}{35b^2} - \frac{2x \csc^{\frac{7}{2}}(a + bx)}{7b} - \frac{12 \int \sqrt{\csc(a + bx)}}{35b}
\end{aligned}$$

Mathematica [A] time = 0.272145, size = 73, normalized size = 0.68

$$\frac{2 \csc^{\frac{7}{2}}(a + bx) \left(\sin(2(a + bx)) + 6 \sin^3(a + bx) \cos(a + bx) - 6 \sin^{\frac{7}{2}}(a + bx) E \left(\frac{1}{4}(-2a - 2bx + \pi) \middle| 2 \right) + 5bx \right)}{35b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(9/2),x]

[Out] (-2*Csc[a + b*x]^(7/2)*(5*b*x + 6*Cos[a + b*x]*Sin[a + b*x]^3 - 6*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(7/2) + Sin[2*(a + b*x)]))/(35*b^2)

Maple [F] time = 0.116, size = 0, normalized size = 0.

$$\int x \cos(bx + a) (\csc(bx + a))^{\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(b*x+a)*csc(b*x+a)^(9/2),x)

[Out] int(x*cos(b*x+a)*csc(b*x+a)^(9/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(bx + a) \csc(bx + a)^{\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(9/2),x, algorithm="maxima")

[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(9/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(bx + a) \csc(bx + a)^{\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(9/2), x)
```

3.353 $\int x \cos(a + bx) \csc^{\frac{7}{2}}(a + bx) dx$

Optimal. Leaf size=85

$$\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right), 2\right)}{15b^2} - \frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2} - \frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

[Out] (-4*Cos[a + b*x]*Csc[a + b*x]^(3/2))/(15*b^2) - (2*x*Csc[a + b*x]^(5/2))/(5*b) + (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(15*b^2)

Rubi [A] time = 0.0443634, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4213, 3768, 3771, 2641}

$$-\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2} + \frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{15b^2} - \frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

[In] Int[x*Cos[a + b*x]*Csc[a + b*x]^(7/2), x]

[Out] (-4*Cos[a + b*x]*Csc[a + b*x]^(3/2))/(15*b^2) - (2*x*Csc[a + b*x]^(5/2))/(5*b) + (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(15*b^2)

Rule 4213

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.), x_Symbol] := -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p, 1]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Csc[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int x \cos(a + bx) \csc^{\frac{7}{2}}(a + bx) dx &= -\frac{2x \csc^{\frac{5}{2}}(a + bx)}{5b} + \frac{2 \int \csc^{\frac{5}{2}}(a + bx) dx}{5b} \\
&= -\frac{4 \cos(a + bx) \csc^{\frac{3}{2}}(a + bx)}{15b^2} - \frac{2x \csc^{\frac{5}{2}}(a + bx)}{5b} + \frac{2 \int \sqrt{\csc(a + bx)} dx}{15b} \\
&= -\frac{4 \cos(a + bx) \csc^{\frac{3}{2}}(a + bx)}{15b^2} - \frac{2x \csc^{\frac{5}{2}}(a + bx)}{5b} + \frac{(2\sqrt{\csc(a + bx)}\sqrt{\sin(a + bx)})}{15b} \\
&= -\frac{4 \cos(a + bx) \csc^{\frac{3}{2}}(a + bx)}{15b^2} - \frac{2x \csc^{\frac{5}{2}}(a + bx)}{5b} + \frac{4\sqrt{\csc(a + bx)}F\left(\frac{1}{2}\left(a - \frac{\pi}{2} + bx\right)\right)}{15b^2}
\end{aligned}$$

Mathematica [A] time = 0.297869, size = 65, normalized size = 0.76

$$\frac{2\sqrt{\csc(a + bx)}\left(2\sqrt{\sin(a + bx)}\text{EllipticF}\left(\frac{1}{4}(-2a - 2bx + \pi), 2\right) + 2 \cot(a + bx) + 3bx \csc^2(a + bx)\right)}{15b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(7/2), x]

[Out] (-2*Sqrt[Csc[a + b*x]]*(2*Cot[a + b*x] + 3*b*x*Csc[a + b*x]^2 + 2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]]))/(15*b^2)

Maple [F] time = 0.11, size = 0, normalized size = 0.

$$\int x \cos(bx + a) (\csc(bx + a))^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(b*x+a)*csc(b*x+a)^(7/2), x)

[Out] int(x*cos(b*x+a)*csc(b*x+a)^(7/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(bx + a) \csc(bx + a)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(7/2), x, algorithm="maxima")

[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(7/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos (bx + a) \csc (bx + a)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(7/2), x)
```

3.354 $\int x \cos(a + bx) \csc^2(a + bx) dx$

Optimal. Leaf size=85

$$\frac{4 \cos(a + bx) \sqrt{\csc(a + bx)}}{3b^2} - \frac{4 \sqrt{\sin(a + bx)} \sqrt{\csc(a + bx)} E\left(\frac{1}{2}\left(a + bx - \frac{\pi}{2}\right) \middle| 2\right)}{3b^2} - \frac{2x \csc^{\frac{3}{2}}(a + bx)}{3b}$$

[Out] (-4*Cos[a + b*x]*Sqrt[Csc[a + b*x]])/(3*b^2) - (2*x*Csc[a + b*x]^(3/2))/(3*b) - (4*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(3*b^2)

Rubi [A] time = 0.0401547, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4213, 3768, 3771, 2639}

$$\frac{4 \cos(a + bx) \sqrt{\csc(a + bx)}}{3b^2} - \frac{4 \sqrt{\sin(a + bx)} \sqrt{\csc(a + bx)} E\left(\frac{1}{2}\left(a + bx - \frac{\pi}{2}\right) \middle| 2\right)}{3b^2} - \frac{2x \csc^{\frac{3}{2}}(a + bx)}{3b}$$

Antiderivative was successfully verified.

[In] Int[x*Cos[a + b*x]*Csc[a + b*x]^(5/2), x]

[Out] (-4*Cos[a + b*x]*Sqrt[Csc[a + b*x]])/(3*b^2) - (2*x*Csc[a + b*x]^(3/2))/(3*b) - (4*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(3*b^2)

Rule 4213

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p, 1]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Csc[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int x \cos(a + bx) \csc^{\frac{5}{2}}(a + bx) dx &= -\frac{2x \csc^{\frac{3}{2}}(a + bx)}{3b} + \frac{2 \int \csc^{\frac{3}{2}}(a + bx) dx}{3b} \\
&= -\frac{4 \cos(a + bx) \sqrt{\csc(a + bx)}}{3b^2} - \frac{2x \csc^{\frac{3}{2}}(a + bx)}{3b} - \frac{2 \int \frac{1}{\sqrt{\csc(a + bx)}} dx}{3b} \\
&= -\frac{4 \cos(a + bx) \sqrt{\csc(a + bx)}}{3b^2} - \frac{2x \csc^{\frac{3}{2}}(a + bx)}{3b} - \frac{(2\sqrt{\csc(a + bx)}\sqrt{\sin(a + bx)}) \int \sqrt{\sin(a + bx)} dx}{3b} \\
&= -\frac{4 \cos(a + bx) \sqrt{\csc(a + bx)}}{3b^2} - \frac{2x \csc^{\frac{3}{2}}(a + bx)}{3b} - \frac{4\sqrt{\csc(a + bx)}E\left(\frac{1}{2}\left(a - \frac{\pi}{2} + bx\right)\middle|2\right)}{3b^2}
\end{aligned}$$

Mathematica [A] time = 0.184442, size = 56, normalized size = 0.66

$$\frac{2 \csc^{\frac{3}{2}}(a + bx) \left(\sin(2(a + bx)) - 2 \sin^{\frac{3}{2}}(a + bx) E\left(\frac{1}{4}(-2a - 2bx + \pi)\middle|2\right) + bx \right)}{3b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(5/2),x]

[Out] (-2*Csc[a + b*x]^(3/2)*(b*x - 2*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(3/2) + Sin[2*(a + b*x)])/(3*b^2)

Maple [F] time = 0.104, size = 0, normalized size = 0.

$$\int x \cos(bx + a) (\csc(bx + a))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(b*x+a)*csc(b*x+a)^(5/2),x)

[Out] int(x*cos(b*x+a)*csc(b*x+a)^(5/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(bx + a) \csc(bx + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(5/2),x, algorithm="maxima")

[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(5/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(bx + a) \csc(bx + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(5/2), x)
```

3.355 $\int x \cos(a + bx) \csc^{\frac{3}{2}}(a + bx) dx$

Optimal. Leaf size=58

$$\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{b^2} - \frac{2x\sqrt{\csc(a+bx)}}{b}$$

[Out] (-2*x*Sqrt[Csc[a + b*x]])/b + (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b^2

Rubi [A] time = 0.0320025, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4213, 3771, 2641}

$$\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2} - \frac{2x\sqrt{\csc(a+bx)}}{b}$$

Antiderivative was successfully verified.

[In] Int[x*Cos[a + b*x]*Csc[a + b*x]^(3/2), x]

[Out] (-2*x*Sqrt[Csc[a + b*x]])/b + (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b^2

Rule 4213

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p, 1]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int x \cos(a + bx) \csc^{\frac{3}{2}}(a + bx) dx &= -\frac{2x\sqrt{\csc(a + bx)}}{b} + \frac{2 \int \sqrt{\csc(a + bx)} dx}{b} \\ &= -\frac{2x\sqrt{\csc(a + bx)}}{b} + \frac{(2\sqrt{\csc(a + bx)}\sqrt{\sin(a + bx)}) \int \frac{1}{\sqrt{\sin(a + bx)}} dx}{b} \\ &= -\frac{2x\sqrt{\csc(a + bx)}}{b} + \frac{4\sqrt{\csc(a + bx)}F\left(\frac{1}{2}\left(a - \frac{\pi}{2} + bx\right)\middle|2\right)\sqrt{\sin(a + bx)}}{b^2} \end{aligned}$$

Mathematica [A] time = 0.148038, size = 46, normalized size = 0.79

$$\frac{2\sqrt{\csc(a+bx)}\left(2\sqrt{\sin(a+bx)}\operatorname{EllipticF}\left(\frac{1}{4}(-2a-2bx+\pi),2\right)+bx\right)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(3/2),x]

[Out] (-2*Sqrt[Csc[a + b*x]]*(b*x + 2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]]))/b^2

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int x \cos(bx + a) (\csc(bx + a))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(b*x+a)*csc(b*x+a)^(3/2),x)

[Out] int(x*cos(b*x+a)*csc(b*x+a)^(3/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(bx + a) \csc(bx + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(3/2),x, algorithm="maxima")

[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(3/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos (bx + a) \csc (bx + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(3/2), x)
```

3.356 $\int x \cos(a + bx) \sqrt{\csc(a + bx)} dx$

Optimal. Leaf size=58

$$\frac{2x}{b\sqrt{\csc(a + bx)}} - \frac{4\sqrt{\sin(a + bx)}\sqrt{\csc(a + bx)}E\left(\frac{1}{2}\left(a + bx - \frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

[Out] (2*x)/(b*Sqrt[Csc[a + b*x]]) - (4*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b^2

Rubi [A] time = 0.0319711, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4213, 3771, 2639}

$$\frac{2x}{b\sqrt{\csc(a + bx)}} - \frac{4\sqrt{\sin(a + bx)}\sqrt{\csc(a + bx)}E\left(\frac{1}{2}\left(a + bx - \frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

[In] Int[x*Cos[a + b*x]*Sqrt[Csc[a + b*x]], x]

[Out] (2*x)/(b*Sqrt[Csc[a + b*x]]) - (4*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b^2

Rule 4213

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p, 1]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int x \cos(a + bx) \sqrt{\csc(a + bx)} dx &= \frac{2x}{b\sqrt{\csc(a + bx)}} - \frac{2 \int \frac{1}{\sqrt{\csc(a + bx)}} dx}{b} \\ &= \frac{2x}{b\sqrt{\csc(a + bx)}} - \frac{(2\sqrt{\csc(a + bx)}\sqrt{\sin(a + bx)}) \int \sqrt{\sin(a + bx)} dx}{b} \\ &= \frac{2x}{b\sqrt{\csc(a + bx)}} - \frac{4\sqrt{\csc(a + bx)}E\left(\frac{1}{2}\left(a - \frac{\pi}{2} + bx\right)\middle|2\right)\sqrt{\sin(a + bx)}}{b^2} \end{aligned}$$

Mathematica [C] time = 0.729526, size = 106, normalized size = 1.83

$$\frac{4 \sin\left(\frac{1}{2}(a + bx)\right) \cos\left(\frac{1}{2}(a + bx)\right) \sqrt{\csc(a + bx)} \left(2 \tan\left(\frac{1}{2}(a + bx)\right) \sqrt{\sec^2\left(\frac{1}{2}(a + bx)\right)} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -\tan\left(\frac{1}{2}(a + bx)\right)\right)\right)}{3b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[a + b*x]*Sqrt[Csc[a + b*x]], x]

[Out] (4*Cos[(a + b*x)/2]*Sqrt[Csc[a + b*x]]*Sin[(a + b*x)/2]*(3*b*x - 6*Tan[(a + b*x)/2] + 2*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[(a + b*x)/2]^2]*Sqrt[Sec[(a + b*x)/2]^2]*Tan[(a + b*x)/2]))/(3*b^2)

Maple [C] time = 0.212, size = 308, normalized size = 5.3

$$\frac{-i(bx + 2i)\left(\left(e^{i(bx+a)}\right)^2 - 1\right)\sqrt{2}}{b^2 e^{i(bx+a)}} \sqrt{\frac{i e^{i(bx+a)}}{\left(e^{i(bx+a)}\right)^2 - 1}} - 2 \frac{\sqrt{2} \sqrt{i\left(\left(e^{i(bx+a)}\right)^2 - 1\right)} e^{i(bx+a)}}{b^2 e^{i(bx+a)}} \left(\frac{-2i\left(-i + i\left(e^{i(bx+a)}\right)^2\right)}{\sqrt{e^{i(bx+a)}\left(-i + i\left(e^{i(bx+a)}\right)^2\right)}} - \frac{\sqrt{e^{i(bx+a)}}}{\sqrt{e^{i(bx+a)}\left(-i + i\left(e^{i(bx+a)}\right)^2\right)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(b*x+a)*csc(b*x+a)^(1/2), x)

[Out] -I*(b*x+2*I)*(exp(I*(b*x+a))^2-1)/b^2*2^(1/2)*(I*exp(I*(b*x+a)))/(exp(I*(b*x+a))^2-1)^(1/2)/exp(I*(b*x+a))-2/b^2*(-2*I*(-I+I*exp(I*(b*x+a))^2)/(exp(I*(b*x+a))*(-I+I*exp(I*(b*x+a))^2))^(1/2)-(exp(I*(b*x+a))+1)^(1/2)*(-2*exp(I*(b*x+a))+2)^(1/2)*(-exp(I*(b*x+a)))^(1/2)/(I*exp(I*(b*x+a))^3-I*exp(I*(b*x+a))))^(1/2)*(-2*EllipticE((exp(I*(b*x+a))+1)^(1/2), 1/2*2^(1/2))+EllipticF((exp(I*(b*x+a))+1)^(1/2), 1/2*2^(1/2))))*2^(1/2)*(I*exp(I*(b*x+a)))/(exp(I*(b*x+a))^2-1)^(1/2)*(I*(exp(I*(b*x+a))^2-1)*exp(I*(b*x+a)))^(1/2)/exp(I*(b*x+a))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(bx + a) \sqrt{\csc(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(1/2), x, algorithm="maxima")

[Out] integrate(x*cos(b*x + a)*sqrt(csc(b*x + a)), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(1/2), x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(a + bx) \sqrt{\csc(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(1/2),x)

[Out] Integral(x*cos(a + b*x)*sqrt(csc(a + b*x)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(bx + a) \sqrt{\csc(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="giac")

[Out] integrate(x*cos(b*x + a)*sqrt(csc(b*x + a)), x)

$$3.357 \quad \int \frac{x \cos(a+bx)}{\sqrt{\csc(a+bx)}} dx$$

Optimal. Leaf size=85

$$\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right), 2\right)}{9b^2} + \frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}} + \frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)}$$

[Out] (2*x)/(3*b*Csc[a + b*x]^(3/2)) + (4*Cos[a + b*x])/(9*b^2*Sqrt[Csc[a + b*x]]) - (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(9*b^2)

Rubi [A] time = 0.0442773, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4213, 3769, 3771, 2641}

$$\frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}} - \frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{9b^2} + \frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(x*Cos[a + b*x])/Sqrt[Csc[a + b*x]], x]

[Out] (2*x)/(3*b*Csc[a + b*x]^(3/2)) + (4*Cos[a + b*x])/(9*b^2*Sqrt[Csc[a + b*x]]) - (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(9*b^2)

Rule 4213

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(m_.), x_Symbol] := -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p, 1]

Rule 3769

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x \cos(a + bx)}{\sqrt{\csc(a + bx)}} dx &= \frac{2x}{3b \csc^{\frac{3}{2}}(a + bx)} - \frac{2 \int \frac{1}{\csc^{\frac{3}{2}}(a + bx)} dx}{3b} \\
&= \frac{2x}{3b \csc^{\frac{3}{2}}(a + bx)} + \frac{4 \cos(a + bx)}{9b^2 \sqrt{\csc(a + bx)}} - \frac{2 \int \sqrt{\csc(a + bx)} dx}{9b} \\
&= \frac{2x}{3b \csc^{\frac{3}{2}}(a + bx)} + \frac{4 \cos(a + bx)}{9b^2 \sqrt{\csc(a + bx)}} - \frac{(2\sqrt{\csc(a + bx)}\sqrt{\sin(a + bx)}) \int \frac{1}{\sqrt{\sin(a + bx)}} dx}{9b} \\
&= \frac{2x}{3b \csc^{\frac{3}{2}}(a + bx)} + \frac{4 \cos(a + bx)}{9b^2 \sqrt{\csc(a + bx)}} - \frac{4\sqrt{\csc(a + bx)}F\left(\frac{1}{2}\left(a - \frac{\pi}{2} + bx\right)\middle|2\right)\sqrt{\sin(a + bx)}}{9b^2}
\end{aligned}$$

Mathematica [A] time = 0.244726, size = 65, normalized size = 0.76

$$\frac{2\sqrt{\csc(a + bx)}\left(2\sqrt{\sin(a + bx)}\text{EllipticF}\left(\frac{1}{4}(-2a - 2bx + \pi), 2\right) + 3bx \sin^2(a + bx) + \sin(2(a + bx))\right)}{9b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Cos[a + b*x])/Sqrt[Csc[a + b*x]], x]

[Out] (2*Sqrt[Csc[a + b*x]]*(2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]] + 3*b*x*Sin[a + b*x]^2 + Sin[2*(a + b*x)]))/(9*b^2)

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int x \cos(bx + a) \frac{1}{\sqrt{\csc(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(b*x+a)/csc(b*x+a)^(1/2), x)

[Out] int(x*cos(b*x+a)/csc(b*x+a)^(1/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cos(bx + a)}{\sqrt{\csc(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(1/2), x, algorithm="maxima")

[Out] integrate(x*cos(b*x + a)/sqrt(csc(b*x + a)), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cos(a + bx)}{\sqrt{\csc(a + bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/csc(b*x+a)**(1/2),x)

[Out] Integral(x*cos(a + b*x)/sqrt(csc(a + b*x)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cos(bx + a)}{\sqrt{\csc(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="giac")

[Out] integrate(x*cos(b*x + a)/sqrt(csc(b*x + a)), x)

$$3.358 \quad \int \frac{x \cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)} dx$$

Optimal. Leaf size=85

$$\frac{4 \cos(a+bx)}{25b^2 \csc^{\frac{3}{2}}(a+bx)} - \frac{12\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2} + \frac{2x}{5b \csc^{\frac{5}{2}}(a+bx)}$$

[Out] (2*x)/(5*b*Csc[a + b*x]^(5/2)) + (4*Cos[a + b*x])/(25*b^2*Csc[a + b*x]^(3/2)) - (12*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(25*b^2)

Rubi [A] time = 0.0437454, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4213, 3769, 3771, 2639}

$$\frac{4 \cos(a+bx)}{25b^2 \csc^{\frac{3}{2}}(a+bx)} - \frac{12\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2} + \frac{2x}{5b \csc^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(x*Cos[a + b*x])/Csc[a + b*x]^(3/2), x]

[Out] (2*x)/(5*b*Csc[a + b*x]^(5/2)) + (4*Cos[a + b*x])/(25*b^2*Csc[a + b*x]^(3/2)) - (12*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(25*b^2)

Rule 4213

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p, 1]

Rule 3769

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x \cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)} dx &= \frac{2x}{5b \csc^{\frac{5}{2}}(a+bx)} - \frac{2 \int \frac{1}{\csc^{\frac{5}{2}}(a+bx)} dx}{5b} \\
&= \frac{2x}{5b \csc^{\frac{5}{2}}(a+bx)} + \frac{4 \cos(a+bx)}{25b^2 \csc^{\frac{3}{2}}(a+bx)} - \frac{6 \int \frac{1}{\sqrt{\csc(a+bx)}} dx}{25b} \\
&= \frac{2x}{5b \csc^{\frac{5}{2}}(a+bx)} + \frac{4 \cos(a+bx)}{25b^2 \csc^{\frac{3}{2}}(a+bx)} - \frac{(6\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}) \int \sqrt{\sin(a+bx)} dx}{25b} \\
&= \frac{2x}{5b \csc^{\frac{5}{2}}(a+bx)} + \frac{4 \cos(a+bx)}{25b^2 \csc^{\frac{3}{2}}(a+bx)} - \frac{12\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a - \frac{\pi}{2} + bx\right)\right) \sqrt{\sin(a+bx)}}{25b^2}
\end{aligned}$$

Mathematica [C] time = 0.997851, size = 114, normalized size = 1.34

$$\frac{\tan\left(\frac{1}{2}(a+bx)\right) \left(4\sqrt{2}\sqrt{\frac{1}{\cos(a+bx)+1}} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -\tan^2\left(\frac{1}{2}(a+bx)\right)\right) + 10bx \sin(a+bx) + 5bx \sin(2(a+bx))\right)}{25b^2 \sqrt{\csc(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Cos[a + b*x])/Csc[a + b*x]^(3/2), x]

[Out] ((-10 + 4*Cos[a + b*x] + 2*Cos[2*(a + b*x)] + 4*Sqrt[2]*Sqrt[(1 + Cos[a + b*x])^(-1)]*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[(a + b*x)/2]^2] + 10*b*x*Sin[a + b*x] + 5*b*x*Sin[2*(a + b*x)])*Tan[(a + b*x)/2])/(25*b^2*Sqrt[Csc[a + b*x]])

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int x \cos(bx+a) (\csc(bx+a))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(b*x+a)/csc(b*x+a)^(3/2), x)

[Out] int(x*cos(b*x+a)/csc(b*x+a)^(3/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cos(bx+a)}{\csc^{\frac{3}{2}}(bx+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(3/2), x, algorithm="maxima")

[Out] integrate(x*cos(b*x + a)/csc(b*x + a)^(3/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cos(a + bx)}{\csc^2(a + bx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/csc(b*x+a)**(3/2),x)

[Out] Integral(x*cos(a + b*x)/csc(a + b*x)**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cos(bx + a)}{\csc(bx + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(3/2),x, algorithm="giac")

[Out] integrate(x*cos(b*x + a)/csc(b*x + a)^(3/2), x)

$$3.359 \quad \int \frac{x \cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)} dx$$

Optimal. Leaf size=108

$$-\frac{20\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right), 2\right)}{147b^2} + \frac{4\cos(a+bx)}{49b^2 \csc^{\frac{5}{2}}(a+bx)} + \frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}} + \frac{2x}{7b \csc^{\frac{7}{2}}(a+bx)}$$

[Out] (2*x)/(7*b*Csc[a + b*x]^(7/2)) + (4*Cos[a + b*x])/(49*b^2*Csc[a + b*x]^(5/2)) + (20*Cos[a + b*x])/(147*b^2*Sqrt[Csc[a + b*x]]) - (20*Sqrt[Csc[a + b*x]])*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]]/(147*b^2)

Rubi [A] time = 0.0596252, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4213, 3769, 3771, 2641}

$$\frac{4\cos(a+bx)}{49b^2 \csc^{\frac{5}{2}}(a+bx)} + \frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}} - \frac{20\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{147b^2} + \frac{2x}{7b \csc^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(x*Cos[a + b*x])/Csc[a + b*x]^(5/2), x]

[Out] (2*x)/(7*b*Csc[a + b*x]^(7/2)) + (4*Cos[a + b*x])/(49*b^2*Csc[a + b*x]^(5/2)) + (20*Cos[a + b*x])/(147*b^2*Sqrt[Csc[a + b*x]]) - (20*Sqrt[Csc[a + b*x]])*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]]/(147*b^2)

Rule 4213

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.), x_Symbol] := -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p, 1]

Rule 3769

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x \cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)} dx &= \frac{2x}{7b \csc^{\frac{7}{2}}(a+bx)} - \frac{2 \int \frac{1}{\csc^{\frac{7}{2}}(a+bx)} dx}{7b} \\
&= \frac{2x}{7b \csc^{\frac{7}{2}}(a+bx)} + \frac{4 \cos(a+bx)}{49b^2 \csc^{\frac{5}{2}}(a+bx)} - \frac{10 \int \frac{1}{\csc^{\frac{3}{2}}(a+bx)} dx}{49b} \\
&= \frac{2x}{7b \csc^{\frac{7}{2}}(a+bx)} + \frac{4 \cos(a+bx)}{49b^2 \csc^{\frac{5}{2}}(a+bx)} + \frac{20 \cos(a+bx)}{147b^2 \sqrt{\csc(a+bx)}} - \frac{10 \int \sqrt{\csc(a+bx)} dx}{147b} \\
&= \frac{2x}{7b \csc^{\frac{7}{2}}(a+bx)} + \frac{4 \cos(a+bx)}{49b^2 \csc^{\frac{5}{2}}(a+bx)} + \frac{20 \cos(a+bx)}{147b^2 \sqrt{\csc(a+bx)}} - \frac{(10 \sqrt{\csc(a+bx)} \sqrt{\sin(a+bx)})}{147b} \\
&= \frac{2x}{7b \csc^{\frac{7}{2}}(a+bx)} + \frac{4 \cos(a+bx)}{49b^2 \csc^{\frac{5}{2}}(a+bx)} + \frac{20 \cos(a+bx)}{147b^2 \sqrt{\csc(a+bx)}} - \frac{20 \sqrt{\csc(a+bx)} F\left(\frac{1}{2}\left(a - \frac{\pi}{2} + bx\right)\right)}{147b^2}
\end{aligned}$$

Mathematica [A] time = 0.420339, size = 93, normalized size = 0.86

$$\frac{\sqrt{\csc(a+bx)} \left(80 \sqrt{\sin(a+bx)} \text{EllipticF}\left(\frac{1}{4}(-2a-2bx+\pi), 2\right) + 52 \sin(2(a+bx)) - 6 \sin(4(a+bx)) - 84bx \cos(2(a+bx)) \right)}{588b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Cos[a + b*x])/Csc[a + b*x]^(5/2), x]

[Out] (Sqrt[Csc[a + b*x]]*(63*b*x - 84*b*x*Cos[2*(a + b*x)] + 21*b*x*Cos[4*(a + b*x)] + 80*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]] + 52*Sin[2*(a + b*x)] - 6*Sin[4*(a + b*x)]))/(588*b^2)

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int x \cos(bx+a) (\csc(bx+a))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(b*x+a)/csc(b*x+a)^(5/2), x)

[Out] int(x*cos(b*x+a)/csc(b*x+a)^(5/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cos(bx+a)}{\csc(bx+a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(5/2), x, algorithm="maxima")

[Out] integrate(x*cos(b*x + a)/csc(b*x + a)^(5/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/csc(b*x+a)**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cos(bx + a)}{\csc(bx + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(5/2),x, algorithm="giac")

[Out] integrate(x*cos(b*x + a)/csc(b*x + a)^(5/2), x)

3.360 $\int x \csc(x) \sin(3x) dx$

Optimal. Leaf size=31

$$\frac{x^2}{2} - \frac{\sin^2(x)}{4} + \frac{3 \cos^2(x)}{4} + 2x \sin(x) \cos(x)$$

[Out] $x^2/2 + (3*\text{Cos}[x]^2)/4 + 2*x*\text{Cos}[x]*\text{Sin}[x] - \text{Sin}[x]^2/4$

Rubi [A] time = 0.0412594, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4431, 3310, 30}

$$\frac{x^2}{2} - \frac{\sin^2(x)}{4} + \frac{3 \cos^2(x)}{4} + 2x \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{Csc}[x]*\text{Sin}[3*x], x]$

[Out] $x^2/2 + (3*\text{Cos}[x]^2)/4 + 2*x*\text{Cos}[x]*\text{Sin}[x] - \text{Sin}[x]^2/4$

Rule 4431

$\text{Int}[(e_{.}) + (f_{.})*(x_{.})^{(m_{.})}*(F_{.})[(a_{.}) + (b_{.})*(x_{.})]^{(p_{.})}*(G_{.})[(c_{.}) + (d_{.})*(x_{.})]^{(q_{.})}, x_{\text{Symbol}}] := \text{Int}[\text{ExpandTrigExpand}[(e + f*x)^m * G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, m\}, x\}$ && $\text{MemberQ}\{\text{Sin}, \text{Cos}\}, F\}$ && $\text{MemberQ}\{\text{Sec}, \text{Csc}\}, G\}$ && $\text{IGtQ}[p, 0]$ && $\text{IGtQ}[q, 0]$ && $\text{EQ}[b*c - a*d, 0]$ && $\text{IGtQ}[b/d, 1]$

Rule 3310

$\text{Int}[(c_{.}) + (d_{.})*(x_{.})*((b_{.})*\text{sin}[(e_{.}) + (f_{.})*(x_{.})])^{(n_{.})}, x_{\text{Symbol}}] := \text{Simp}[(d*(b*\text{Sin}[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n - 1))/n, \text{Int}[(c + d*x)*(b*\text{Sin}[e + f*x])^{(n - 2)}, x], x] - \text{Simp}[(b*(c + d*x)*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(n - 1)})/(f*n), x]) /;$ $\text{FreeQ}\{b, c, d, e, f\}, x\}$ && $\text{GtQ}[n, 1]$

Rule 30

$\text{Int}[(x_{.})^{(m_{.})}, x_{\text{Symbol}}] := \text{Simp}[x^{(m + 1)}/(m + 1), x] /;$ $\text{FreeQ}[m, x]$ && $\text{NeQ}[m, -1]$

Rubi steps

$$\begin{aligned} \int x \csc(x) \sin(3x) dx &= \int (3x \cos^2(x) - x \sin^2(x)) dx \\ &= 3 \int x \cos^2(x) dx - \int x \sin^2(x) dx \\ &= \frac{3 \cos^2(x)}{4} + 2x \cos(x) \sin(x) - \frac{\sin^2(x)}{4} - \frac{\int x dx}{2} + \frac{3 \int x dx}{2} \\ &= \frac{x^2}{2} + \frac{3 \cos^2(x)}{4} + 2x \cos(x) \sin(x) - \frac{\sin^2(x)}{4} \end{aligned}$$

Mathematica [A] time = 0.0152247, size = 22, normalized size = 0.71

$$\frac{x^2}{2} + x \sin(2x) + \frac{1}{2} \cos(2x)$$

Antiderivative was successfully verified.

[In] Integrate[x*Csc[x]*Sin[3*x],x]

[Out] x^2/2 + Cos[2*x]/2 + x*Sin[2*x]

Maple [A] time = 0.048, size = 26, normalized size = 0.8

$$4x(1/2 \cos(x) \sin(x) + x/2) - \frac{3x^2}{2} - (\sin(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*csc(x)*sin(3*x),x)

[Out] 4*x*(1/2*cos(x)*sin(x)+1/2*x)-3/2*x^2-sin(x)^2

Maxima [A] time = 0.990269, size = 24, normalized size = 0.77

$$\frac{1}{2} x^2 + x \sin(2x) + \frac{1}{2} \cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)*sin(3*x),x, algorithm="maxima")

[Out] 1/2*x^2 + x*sin(2*x) + 1/2*cos(2*x)

Fricas [A] time = 0.518457, size = 54, normalized size = 1.74

$$2x \cos(x) \sin(x) + \frac{1}{2} x^2 + \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)*sin(3*x),x, algorithm="fricas")

[Out] 2*x*cos(x)*sin(x) + 1/2*x^2 + cos(x)^2

Sympy [A] time = 6.2343, size = 37, normalized size = 1.19

$$-x^2 \sin^2(x) - x^2 \cos^2(x) + \frac{3x^2}{2} + 2x \sin(x) \cos(x) - \sin^2(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sin(3*x),x)
```

```
[Out] -x**2*sin(x)**2 - x**2*cos(x)**2 + 3*x**2/2 + 2*x*sin(x)*cos(x) - sin(x)**2
```

Giac [A] time = 1.14296, size = 24, normalized size = 0.77

$$\frac{1}{2}x^2 + x \sin(2x) + \frac{1}{2} \cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + x*sin(2*x) + 1/2*cos(2*x)
```

3.361 $\int (c + dx)^4 \csc(x) \sin(3x) dx$

Optimal. Leaf size=131

$$\frac{3}{2}d^3 \sin^2(x)(c + dx) - \frac{9}{2}d^3 \cos^2(x)(c + dx) - 6d^2 \sin(x) \cos(x)(c + dx)^2 + \frac{(c + dx)^5}{5d} - d(c + dx)^3 - d \sin^2(x)(c + dx)^3 + 3d$$

[Out] (3*d^4*x)/2 - d*(c + d*x)^3 + (c + d*x)^5/(5*d) - (9*d^3*(c + d*x)*Cos[x]^2)/2 + 3*d*(c + d*x)^3*Cos[x]^2 + 3*d^4*Cos[x]*Sin[x] - 6*d^2*(c + d*x)^2*Cos[x]*Sin[x] + 2*(c + d*x)^4*Cos[x]*Sin[x] + (3*d^3*(c + d*x)*Sin[x]^2)/2 - d*(c + d*x)^3*Ssin[x]^2

Rubi [A] time = 0.188099, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4431, 3311, 32, 2635, 8}

$$\frac{3}{2}d^3 \sin^2(x)(c + dx) - \frac{9}{2}d^3 \cos^2(x)(c + dx) - 6d^2 \sin(x) \cos(x)(c + dx)^2 + \frac{(c + dx)^5}{5d} - d(c + dx)^3 - d \sin^2(x)(c + dx)^3 + 3d$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^4*Csc[x]*Sin[3*x],x]

[Out] (3*d^4*x)/2 - d*(c + d*x)^3 + (c + d*x)^5/(5*d) - (9*d^3*(c + d*x)*Cos[x]^2)/2 + 3*d*(c + d*x)^3*Cos[x]^2 + 3*d^4*Cos[x]*Sin[x] - 6*d^2*(c + d*x)^2*Cos[x]*Sin[x] + 2*(c + d*x)^4*Cos[x]*Sin[x] + (3*d^3*(c + d*x)*Sin[x]^2)/2 - d*(c + d*x)^3*Ssin[x]^2

Rule 4431

Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol] := Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && MemberQ[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && EQ[b*c - a*d, 0] && IGtQ[b/d, 1]

Rule 3311

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(d*m*(c + d*x)^(m - 1)*(b*Ssin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Ssin[e + f*x])^(n - 2), x], x] - Dist[(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Ssin[e + f*x])^n, x], x] - Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Ssin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rubi steps

$$\begin{aligned}
 \int (c + dx)^4 \csc(x) \sin(3x) dx &= \int (3(c + dx)^4 \cos^2(x) - (c + dx)^4 \sin^2(x)) dx \\
 &= 3 \int (c + dx)^4 \cos^2(x) dx - \int (c + dx)^4 \sin^2(x) dx \\
 &= 3d(c + dx)^3 \cos^2(x) + 2(c + dx)^4 \cos(x) \sin(x) - d(c + dx)^3 \sin^2(x) - \frac{1}{2} \int (c + dx)^4 dx \\
 &= \frac{(c + dx)^5}{5d} - \frac{9}{2} d^3 (c + dx) \cos^2(x) + 3d(c + dx)^3 \cos^2(x) - 6d^2 (c + dx)^2 \cos(x) \sin(x) + 2 \\
 &= -d(c + dx)^3 + \frac{(c + dx)^5}{5d} - \frac{9}{2} d^3 (c + dx) \cos^2(x) + 3d(c + dx)^3 \cos^2(x) + 3d^4 \cos(x) \sin(x) \\
 &= \frac{3d^4 x}{2} - d(c + dx)^3 + \frac{(c + dx)^5}{5d} - \frac{9}{2} d^3 (c + dx) \cos^2(x) + 3d(c + dx)^3 \cos^2(x) + 3d^4 \cos(x) \sin(x)
 \end{aligned}$$

Mathematica [A] time = 0.218675, size = 154, normalized size = 1.18

$$2c^2 d^2 x^3 + \frac{1}{2} \sin(2x) (6c^2 d^2 (2x^2 - 1) + 8c^3 dx + 2c^4 + 4cd^3 x (2x^2 - 3) + d^4 (2x^4 - 6x^2 + 3)) + d \cos(2x) (6c^2 dx + 2c^3$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^4*Csc[x]*Sin[3*x],x]

[Out] c^4*x + 2*c^3*d*x^2 + 2*c^2*d^2*x^3 + c*d^3*x^4 + (d^4*x^5)/5 + d*(2*c^3 + 6*c^2*d*x + d^3*x*(-3 + 2*x^2) + 3*c*d^2*(-1 + 2*x^2))*Cos[2*x] + ((2*c^4 + 8*c^3*d*x + 4*c*d^3*x*(-3 + 2*x^2) + 6*c^2*d^2*(-1 + 2*x^2) + d^4*(3 - 6*x^2 + 2*x^4))*Sin[2*x])/2

Maple [B] time = 0.066, size = 260, normalized size = 2.

$$4d^4 \left(x^4 \left(\frac{1}{2} \cos(x) \sin(x) + x/2 \right) + x^3 (\cos(x))^2 - 3x^2 \left(\frac{1}{2} \cos(x) \sin(x) + x/2 \right) - 3/2 x (\cos(x))^2 + 3/4 \cos(x) \sin(x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^4*csc(x)*sin(3*x),x)

[Out] 4*d^4*(x^4*(1/2*cos(x)*sin(x)+1/2*x)+x^3*cos(x)^2-3*x^2*(1/2*cos(x)*sin(x)+1/2*x)-3/2*x*cos(x)^2+3/4*cos(x)*sin(x)+3/4*x+x^3-2/5*x^5)+16*c*d^3*(x^3*(1/2*cos(x)*sin(x)+1/2*x)+3/4*x^2*cos(x)^2-3/2*x*(1/2*cos(x)*sin(x)+1/2*x)+3/8*x^2+3/8*sin(x)^2-3/8*x^4)+24*c^2*d^2*(x^2*(1/2*cos(x)*sin(x)+1/2*x)+1/2*x*cos(x)^2-1/4*cos(x)*sin(x)-1/4*x-1/3*x^3)-1/5*d^4*x^5+16*c^3*d*(x*(1/2*cos(x)*sin(x)+1/2*x)-1/4*x^2-1/4*sin(x)^2)-c*d^3*x^4+4*c^4*(1/2*cos(x)*sin(x)+1/2*x)-2*c^2*d^2*x^3-2*c^3*d*x^2-c^4*x

Maxima [A] time = 1.04852, size = 197, normalized size = 1.5

$$2(x^2 + 2x \sin(2x) + \cos(2x))c^3d + (2x^3 + 6x \cos(2x) + 3(2x^2 - 1) \sin(2x))c^2d^2 + (x^4 + 3(2x^2 - 1) \cos(2x) + 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*csc(x)*sin(3*x),x, algorithm="maxima")

[Out] 2*(x^2 + 2*x*sin(2*x) + cos(2*x))*c^3*d + (2*x^3 + 6*x*cos(2*x) + 3*(2*x^2 - 1)*sin(2*x))*c^2*d^2 + (x^4 + 3*(2*x^2 - 1)*cos(2*x) + 2*(2*x^3 - 3*x)*sin(2*x))*c*d^3 + 1/10*(2*x^5 + 10*(2*x^3 - 3*x)*cos(2*x) + 5*(2*x^4 - 6*x^2 + 3)*sin(2*x))*d^4 + c^4*(x + sin(2*x))

Fricas [A] time = 0.515347, size = 419, normalized size = 3.2

$$\frac{1}{5}d^4x^5 + cd^3x^4 + 2(c^2d^2 - d^4)x^3 + 2(c^3d - 3cd^3)x^2 + 2(2d^4x^3 + 6cd^3x^2 + 2c^3d - 3cd^3 + 3(2c^2d^2 - d^4)x)\cos(x)^2 + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*csc(x)*sin(3*x),x, algorithm="fricas")

[Out] 1/5*d^4*x^5 + c*d^3*x^4 + 2*(c^2*d^2 - d^4)*x^3 + 2*(c^3*d - 3*c*d^3)*x^2 + 2*(2*d^4*x^3 + 6*c*d^3*x^2 + 2*c^3*d - 3*c*d^3 + 3*(2*c^2*d^2 - d^4)*x)*cos(x)^2 + (2*d^4*x^4 + 8*c*d^3*x^3 + 2*c^4 - 6*c^2*d^2 + 3*d^4 + 6*(2*c^2*d^2 - d^4)*x^2 + 4*(2*c^3*d - 3*c*d^3)*x)*cos(x)*sin(x) + (c^4 - 6*c^2*d^2 + 3*d^4)*x

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**4*csc(x)*sin(3*x),x)

[Out] Timed out

Giac [A] time = 1.13595, size = 225, normalized size = 1.72

$$\frac{1}{5}d^4x^5 + cd^3x^4 + 2c^2d^2x^3 + 2c^3dx^2 + c^4x + (2d^4x^3 + 6cd^3x^2 + 6c^2d^2x - 3d^4x + 2c^3d - 3cd^3)\cos(2x) + \frac{1}{2}(2d^4x^4 + 8cd^3x^3 + \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*csc(x)*sin(3*x),x, algorithm="giac")

[Out] 1/5*d^4*x^5 + c*d^3*x^4 + 2*c^2*d^2*x^3 + 2*c^3*d*x^2 + c^4*x + (2*d^4*x^3 + 6*c*d^3*x^2 + 6*c^2*d^2*x - 3*d^4*x + 2*c^3*d - 3*c*d^3)*cos(2*x) + 1/2*(2*d^4*x^4 + 8*c*d^3*x^3 + 12*c^2*d^2*x^2 - 6*d^4*x^2 + 8*c^3*d*x - 12*c*d^3*x + 2*c^4 - 6*c^2*d^2 + 3*d^4)*sin(2*x)

3.362 $\int (c + dx)^3 \csc(x) \sin(3x) dx$

Optimal. Leaf size=115

$$-\frac{3}{2}cd^2x - 3d^2 \sin(x) \cos(x)(c + dx) + \frac{(c + dx)^4}{4d} - \frac{3}{4}d \sin^2(x)(c + dx)^2 + \frac{9}{4}d \cos^2(x)(c + dx)^2 + 2 \sin(x) \cos(x)(c + dx)$$

[Out] $(-3*c*d^2*x)/2 - (3*d^3*x^2)/4 + (c + d*x)^4/(4*d) - (9*d^3*\text{Cos}[x]^2)/8 + (9*d*(c + d*x)^2*\text{Cos}[x]^2)/4 - 3*d^2*(c + d*x)*\text{Cos}[x]*\text{Sin}[x] + 2*(c + d*x)^3*\text{Cos}[x]*\text{Sin}[x] + (3*d^3*\text{Sin}[x]^2)/8 - (3*d*(c + d*x)^2*\text{Sin}[x]^2)/4$

Rubi [A] time = 0.141352, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4431, 3311, 32, 3310}

$$-\frac{3}{2}cd^2x - 3d^2 \sin(x) \cos(x)(c + dx) + \frac{(c + dx)^4}{4d} - \frac{3}{4}d \sin^2(x)(c + dx)^2 + \frac{9}{4}d \cos^2(x)(c + dx)^2 + 2 \sin(x) \cos(x)(c + dx)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^3*\text{Csc}[x]*\text{Sin}[3*x], x]$

[Out] $(-3*c*d^2*x)/2 - (3*d^3*x^2)/4 + (c + d*x)^4/(4*d) - (9*d^3*\text{Cos}[x]^2)/8 + (9*d*(c + d*x)^2*\text{Cos}[x]^2)/4 - 3*d^2*(c + d*x)*\text{Cos}[x]*\text{Sin}[x] + 2*(c + d*x)^3*\text{Cos}[x]*\text{Sin}[x] + (3*d^3*\text{Sin}[x]^2)/8 - (3*d*(c + d*x)^2*\text{Sin}[x]^2)/4$

Rule 4431

$\text{Int}[(e + f*x)^m*(F)[(a + b*x)^p*(G)[(c + d*x)^q], x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigExpand}[(e + f*x)^m*G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /;$ FreeQ[{a, b, c, d, e, f, m}, x] && MemberQ[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && EQ[b*c - a*d, 0] && IGtQ[b/d, 1]

Rule 3311

$\text{Int}[(c + d*x)^m*((b*\text{sin}[e + f*x])^n), x_Symbol] \rightarrow \text{Simp}[(d*m*(c + d*x)^{m-1}*(b*\text{Sin}[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n-1))/n, \text{Int}[(c + d*x)^m*(b*\text{Sin}[e + f*x])^{n-2}, x], x] - \text{Dist}[(d^2*m*(m-1))/(f^2*n^2), \text{Int}[(c + d*x)^{m-2}*(b*\text{Sin}[e + f*x])^n, x], x] - \text{Simp}[(b*(c + d*x)^m*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{n-1})/(f*n), x]) /;$ FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rule 32

$\text{Int}[(a + b*x)^m, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{m+1}/(b*(m+1)), x] /;$ FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 3310

$\text{Int}[(c + d*x)^m*((b*\text{sin}[e + f*x])^n), x_Symbol] \rightarrow \text{Simp}[(d*(b*\text{Sin}[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n-1))/n, \text{Int}[(c + d*x)*(b*\text{Sin}[e + f*x])^{n-2}, x], x] - \text{Simp}[(b*(c + d*x)*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{n-1})/(f*n), x]) /;$ FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 \csc(x) \sin(3x) dx &= \int (3(c + dx)^3 \cos^2(x) - (c + dx)^3 \sin^2(x)) dx \\
&= 3 \int (c + dx)^3 \cos^2(x) dx - \int (c + dx)^3 \sin^2(x) dx \\
&= \frac{9}{4}d(c + dx)^2 \cos^2(x) + 2(c + dx)^3 \cos(x) \sin(x) - \frac{3}{4}d(c + dx)^2 \sin^2(x) - \frac{1}{2} \int (c + dx)^3 dx + \dots \\
&= \frac{(c + dx)^4}{4d} - \frac{9}{8}d^3 \cos^2(x) + \frac{9}{4}d(c + dx)^2 \cos^2(x) - 3d^2(c + dx) \cos(x) \sin(x) + 2(c + dx)^3 \cos(x) \sin(x) \\
&= -\frac{3}{2}cd^2x - \frac{3d^3x^2}{4} + \frac{(c + dx)^4}{4d} - \frac{9}{8}d^3 \cos^2(x) + \frac{9}{4}d(c + dx)^2 \cos^2(x) - 3d^2(c + dx) \cos(x) \sin(x) + \dots
\end{aligned}$$

Mathematica [A] time = 0.157289, size = 109, normalized size = 0.95

$$\frac{1}{4} \left(x(6c^2dx + 4c^3 + 4cd^2x^2 + d^3x^3) + 2\sin(2x)(6c^2dx + 2c^3 + 3cd^2(2x^2 - 1) + d^3x(2x^2 - 3)) + 3d\cos(2x)(2c^2 + 4cdx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3*Csc[x]*Sin[3*x],x]

[Out] (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 3*d*(2*c^2 + 4*c*d*x + d^2*(-1 + 2*x^2))*Cos[2*x] + 2*(2*c^3 + 6*c^2*d*x + d^3*x*(-3 + 2*x^2) + 3*c*d^2*(-1 + 2*x^2))*Sin[2*x])/4

Maple [A] time = 0.051, size = 179, normalized size = 1.6

$$4d^3 \left(x^3 \left(\frac{1}{2} \cos(x) \sin(x) + \frac{x}{2} \right) + \frac{3}{4} x^2 (\cos(x))^2 - \frac{3}{2} x \left(\frac{1}{2} \cos(x) \sin(x) + \frac{x}{2} \right) + \frac{3}{8} x^2 + \frac{3}{8} (\sin(x))^2 - \frac{3}{8} x^4 \right) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*csc(x)*sin(3*x),x)

[Out] 4*d^3*(x^3*(1/2*cos(x)*sin(x)+1/2*x)+3/4*x^2*cos(x)^2-3/2*x*(1/2*cos(x)*sin(x)+1/2*x)+3/8*x^2+3/8*sin(x)^2-3/8*x^4)+12*d^2*c*(x^2*(1/2*cos(x)*sin(x)+1/2*x)+1/2*x*cos(x)^2-1/4*cos(x)*sin(x)-1/4*x-1/3*x^3)+12*c^2*d*(x*(1/2*cos(x)*sin(x)+1/2*x)-1/4*x^2-1/4*sin(x)^2)-1/4*d^3*x^4+4*c^3*(1/2*cos(x)*sin(x)+1/2*x)-c*d^2*x^3-3/2*c^2*d*x^2-c^3*x

Maxima [A] time = 1.02836, size = 136, normalized size = 1.18

$$\frac{3}{2} \left(x^2 + 2x \sin(2x) + \cos(2x) \right) c^2 d + \frac{1}{2} \left(2x^3 + 6x \cos(2x) + 3(2x^2 - 1) \sin(2x) \right) cd^2 + \frac{1}{4} \left(x^4 + 3(2x^2 - 1) \cos(2x) + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(x)*sin(3*x),x, algorithm="maxima")

[Out] 3/2*(x^2 + 2*x*sin(2*x) + cos(2*x))*c^2*d + 1/2*(2*x^3 + 6*x*cos(2*x) + 3*(2*x^2 - 1)*sin(2*x))*c*d^2 + 1/4*(x^4 + 3*(2*x^2 - 1)*cos(2*x) + 2*(2*x^3 - 3*x)*sin(2*x))*d^3 + c^3*(x + sin(2*x))

Fricas [A] time = 0.496439, size = 278, normalized size = 2.42

$$\frac{1}{4}d^3x^4 + cd^2x^3 + \frac{3}{2}(c^2d - d^3)x^2 + \frac{3}{2}(2d^3x^2 + 4cd^2x + 2c^2d - d^3)\cos(x)^2 + (2d^3x^3 + 6cd^2x^2 + 2c^3 - 3cd^2 + 3(2c^2d - d^3)x)\cos(x)\sin(x) + (c^3 - 3cd^2)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(x)*sin(3*x),x, algorithm="fricas")

[Out] 1/4*d^3*x^4 + c*d^2*x^3 + 3/2*(c^2*d - d^3)*x^2 + 3/2*(2*d^3*x^2 + 4*c*d^2*x + 2*c^2*d - d^3)*cos(x)^2 + (2*d^3*x^3 + 6*c*d^2*x^2 + 2*c^3 - 3*c*d^2 + 3*(2*c^2*d - d^3)*x)*cos(x)*sin(x) + (c^3 - 3*c*d^2)*x

Sympy [B] time = 146.651, size = 289, normalized size = 2.51

$$c^3x + c^3\sin(2x) - 3c^2dx^2\sin^2(x) - 3c^2dx^2\cos^2(x) + \frac{9c^2dx^2}{2} + 6c^2dx\sin(x)\cos(x) - 3c^2d\sin^2(x) - 2cd^2x^3\sin^2(x) - 2cd^2x^3\cos^2(x) + 3cd^2x^3\sin(x)\cos(x) + (c^3 - 3cd^2)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*csc(x)*sin(3*x),x)

[Out] c**3*x + c**3*sin(2*x) - 3*c**2*d*x**2*sin(x)**2 - 3*c**2*d*x**2*cos(x)**2 + 9*c**2*d*x**2/2 + 6*c**2*d*x*sin(x)*cos(x) - 3*c**2*d*sin(x)**2 - 2*c*d**2*x**3*sin(x)**2 - 2*c*d**2*x**3*cos(x)**2 + 3*c*d**2*x**3 + 6*c*d**2*x**2*sin(x)*cos(x) - 3*c*d**2*x*sin(x)**2 + 3*c*d**2*x*cos(x)**2 - 3*c*d**2*sin(x)*cos(x) - d**3*x**4*sin(x)**2/2 - d**3*x**4*cos(x)**2/2 + 3*d**3*x**4/4 + 2*d**3*x**3*sin(x)*cos(x) - 3*d**3*x**2*sin(x)**2/2 + 3*d**3*x**2*cos(x)**2/2 - 3*d**3*x*sin(x)*cos(x) + 3*d**3*sin(x)**2/2

Giac [A] time = 1.14367, size = 151, normalized size = 1.31

$$\frac{1}{4}d^3x^4 + cd^2x^3 + \frac{3}{2}c^2dx^2 + c^3x + \frac{3}{4}(2d^3x^2 + 4cd^2x + 2c^2d - d^3)\cos(2x) + \frac{1}{2}(2d^3x^3 + 6cd^2x^2 + 6c^2dx - 3d^3x + 2c^3 - 3cd^2)x\sin(2x) + (c^3 - 3cd^2)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(x)*sin(3*x),x, algorithm="giac")

[Out] 1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + c^3*x + 3/4*(2*d^3*x^2 + 4*c*d^2*x + 2*c^2*d - d^3)*cos(2*x) + 1/2*(2*d^3*x^3 + 6*c*d^2*x^2 + 6*c^2*d*x - 3*d^3*x + 2*c^3 - 3*c*d^2)*sin(2*x) + (c^3 - 3*c*d^2)x

3.363 $\int (c + dx)^2 \csc(x) \sin(3x) dx$

Optimal. Leaf size=73

$$\frac{(c + dx)^3}{3d} - \frac{1}{2}d \sin^2(x)(c + dx) + \frac{3}{2}d \cos^2(x)(c + dx) + 2 \sin(x) \cos(x)(c + dx)^2 - \frac{d^2x}{2} - d^2 \sin(x) \cos(x)$$

[Out] $-(d^2*x)/2 + (c + d*x)^3/(3*d) + (3*d*(c + d*x)*\text{Cos}[x]^2)/2 - d^2*\text{Cos}[x]*\text{Sin}[x] + 2*(c + d*x)^2*\text{Cos}[x]*\text{Sin}[x] - (d*(c + d*x)*\text{Sin}[x]^2)/2$

Rubi [A] time = 0.102692, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4431, 3311, 32, 2635, 8}

$$\frac{(c + dx)^3}{3d} - \frac{1}{2}d \sin^2(x)(c + dx) + \frac{3}{2}d \cos^2(x)(c + dx) + 2 \sin(x) \cos(x)(c + dx)^2 - \frac{d^2x}{2} - d^2 \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^2*\text{Csc}[x]*\text{Sin}[3*x], x]$

[Out] $-(d^2*x)/2 + (c + d*x)^3/(3*d) + (3*d*(c + d*x)*\text{Cos}[x]^2)/2 - d^2*\text{Cos}[x]*\text{Sin}[x] + 2*(c + d*x)^2*\text{Cos}[x]*\text{Sin}[x] - (d*(c + d*x)*\text{Sin}[x]^2)/2$

Rule 4431

$\text{Int}[(e + f*x)^m*(F)[(a + b*x)^p*(G)[(c + d*x)^q], x_Symbol] := \text{Int}[\text{ExpandTrigExpand}[(e + f*x)^m*G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /;$ FreeQ[{a, b, c, d, e, f, m}, x] && MemberQ[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && EQ[b*c - a*d, 0] && IGtQ[b/d, 1]

Rule 3311

$\text{Int}[(c + d*x)^m*((b*\sin[e + f*x])^n), x_Symbol] := \text{Simp}[(d*m*(c + d*x)^{m-1}*(b*\sin[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n-1))/n, \text{Int}[(c + d*x)^m*(b*\sin[e + f*x])^{n-2}, x], x] - \text{Dist}[(d^2*m*(m-1))/(f^2*n^2), \text{Int}[(c + d*x)^{m-2}*(b*\sin[e + f*x])^n, x], x] - \text{Simp}[(b*(c + d*x)^m*\text{Cos}[e + f*x]*(b*\sin[e + f*x])^{n-1})/(f*n), x]) /;$ FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rule 32

$\text{Int}[(a + b*x)^m, x_Symbol] := \text{Simp}[(a + b*x)^{m+1}/(b*(m+1)), x] /;$ FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2635

$\text{Int}[(b*\sin[c + d*x])^n, x_Symbol] := -\text{Simp}[(b*\text{Cos}[c + d*x]*(b*\sin[c + d*x])^{n-1})/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\sin[c + d*x])^{n-2}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

$\text{Int}[a, x_Symbol] := \text{Simp}[a*x, x] /;$ FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int (c + dx)^2 \csc(x) \sin(3x) dx &= \int (3(c + dx)^2 \cos^2(x) - (c + dx)^2 \sin^2(x)) dx \\
&= 3 \int (c + dx)^2 \cos^2(x) dx - \int (c + dx)^2 \sin^2(x) dx \\
&= \frac{3}{2} d(c + dx) \cos^2(x) + 2(c + dx)^2 \cos(x) \sin(x) - \frac{1}{2} d(c + dx) \sin^2(x) - \frac{1}{2} \int (c + dx)^2 dx \\
&= \frac{(c + dx)^3}{3d} + \frac{3}{2} d(c + dx) \cos^2(x) - d^2 \cos(x) \sin(x) + 2(c + dx)^2 \cos(x) \sin(x) - \frac{1}{2} d(c + dx) \sin^2(x) \\
&= -\frac{d^2 x}{2} + \frac{(c + dx)^3}{3d} + \frac{3}{2} d(c + dx) \cos^2(x) - d^2 \cos(x) \sin(x) + 2(c + dx)^2 \cos(x) \sin(x) - \frac{1}{2} d(c + dx) \sin^2(x)
\end{aligned}$$

Mathematica [A] time = 0.110072, size = 60, normalized size = 0.82

$$\sin(x) \cos(x) (2c^2 + 4cdx + d^2 (2x^2 - 1)) + c^2 x + cdx^2 + d \cos(2x)(c + dx) + \frac{d^2 x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2*Csc[x]*Sin[3*x], x]

[Out] c^2*x + c*d*x^2 + (d^2*x^3)/3 + d*(c + d*x)*Cos[2*x] + (2*c^2 + 4*c*d*x + d^2*(-1 + 2*x^2))*Cos[x]*Sin[x]

Maple [A] time = 0.054, size = 107, normalized size = 1.5

$$4d^2 \left(x^2 \left(\frac{1}{2} \cos(x) \sin(x) + \frac{x}{2} \right) + \frac{1}{2} x (\cos(x))^2 - \frac{1}{4} \cos(x) \sin(x) - \frac{x}{4} - \frac{1}{3} x^3 \right) + 8cd \left(x \left(\frac{1}{2} \cos(x) \sin(x) + \frac{x}{2} \right) + \frac{1}{2} x (\cos(x))^2 - \frac{1}{4} \cos(x) \sin(x) - \frac{x}{4} - \frac{1}{3} x^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*csc(x)*sin(3*x), x)

[Out] 4*d^2*(x^2*(1/2*cos(x)*sin(x)+1/2*x)+1/2*x*cos(x)^2-1/4*cos(x)*sin(x)-1/4*x-1/3*x^3)+8*c*d*(x*(1/2*cos(x)*sin(x)+1/2*x)-1/4*x^2-1/4*sin(x)^2)+4*c^2*(1/2*cos(x)*sin(x)+1/2*x)-1/3*d^2*x^3-c*d*x^2-c^2*x

Maxima [A] time = 0.995402, size = 81, normalized size = 1.11

$$(x^2 + 2x \sin(2x) + \cos(2x))cd + \frac{1}{6} (2x^3 + 6x \cos(2x) + 3(2x^2 - 1) \sin(2x))d^2 + c^2(x + \sin(2x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(x)*sin(3*x), x, algorithm="maxima")

[Out] (x^2 + 2*x*sin(2*x) + cos(2*x))*c*d + 1/6*(2*x^3 + 6*x*cos(2*x) + 3*(2*x^2 - 1)*sin(2*x))*d^2 + c^2*(x + sin(2*x))

Fricas [A] time = 0.512624, size = 159, normalized size = 2.18

$$\frac{1}{3}d^2x^3 + cdx^2 + 2(d^2x + cd)\cos(x)^2 + (2d^2x^2 + 4cdx + 2c^2 - d^2)\cos(x)\sin(x) + (c^2 - d^2)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(x)*sin(3*x),x, algorithm="fricas")

[Out] 1/3*d^2*x^3 + c*d*x^2 + 2*(d^2*x + c*d)*cos(x)^2 + (2*d^2*x^2 + 4*c*d*x + 2*c^2 - d^2)*cos(x)*sin(x) + (c^2 - d^2)*x

Sympy [B] time = 21.0139, size = 155, normalized size = 2.12

$$c^2x + c^2 \sin(2x) - 2cdx^2 \sin^2(x) - 2cdx^2 \cos^2(x) + 3cdx^2 + 4cdx \sin(x) \cos(x) - 2cd \sin^2(x) - \frac{2d^2x^3 \sin^2(x)}{3} - \frac{2d^2x^3 \cos^2(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*csc(x)*sin(3*x),x)

[Out] c**2*x + c**2*sin(2*x) - 2*c*d*x**2*sin(x)**2 - 2*c*d*x**2*cos(x)**2 + 3*c*d*x**2 + 4*c*d*x*sin(x)*cos(x) - 2*c*d*sin(x)**2 - 2*d**2*x**3*sin(x)**2/3 - 2*d**2*x**3*cos(x)**2/3 + d**2*x**3 + 2*d**2*x**2*sin(x)*cos(x) - d**2*x*sin(x)**2 + d**2*x*cos(x)**2 - d**2*sin(x)*cos(x)

Giac [A] time = 1.12762, size = 86, normalized size = 1.18

$$\frac{1}{3}d^2x^3 + cdx^2 + c^2x + (d^2x + cd)\cos(2x) + \frac{1}{2}(2d^2x^2 + 4cdx + 2c^2 - d^2)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(x)*sin(3*x),x, algorithm="giac")

[Out] 1/3*d^2*x^3 + c*d*x^2 + c^2*x + (d^2*x + c*d)*cos(2*x) + 1/2*(2*d^2*x^2 + 4*c*d*x + 2*c^2 - d^2)*sin(2*x)

3.364 $\int (c + dx) \csc(x) \sin(3x) dx$

Optimal. Leaf size=41

$$2 \sin(x) \cos(x)(c + dx) + cx + \frac{dx^2}{2} - \frac{1}{4}d \sin^2(x) + \frac{3}{4}d \cos^2(x)$$

[Out] $c*x + (d*x^2)/2 + (3*d*\text{Cos}[x]^2)/4 + 2*(c + d*x)*\text{Cos}[x]*\text{Sin}[x] - (d*\text{Sin}[x]^2)/4$

Rubi [A] time = 0.0563706, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4431, 3310}

$$2 \sin(x) \cos(x)(c + dx) + cx + \frac{dx^2}{2} - \frac{1}{4}d \sin^2(x) + \frac{3}{4}d \cos^2(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)*\text{Csc}[x]*\text{Sin}[3*x], x]$

[Out] $c*x + (d*x^2)/2 + (3*d*\text{Cos}[x]^2)/4 + 2*(c + d*x)*\text{Cos}[x]*\text{Sin}[x] - (d*\text{Sin}[x]^2)/4$

Rule 4431

$\text{Int}[(e + f*x)^m * (a + b*x)^p * (c + d*x)^q, x_Symbol] := \text{Int}[\text{ExpandTrigExpand}[(e + f*x)^m * G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /;$ FreeQ[{a, b, c, d, e, f, m}, x] && MemberQ[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && EQ[b*c - a*d, 0] && IGtQ[b/d, 1]

Rule 3310

$\text{Int}[(c + d*x) * (b * \sin[e + f*x])^n, x_Symbol] := \text{Simp}[(d * (b * \sin[e + f*x])^n) / (f^2 * n^2), x] + (\text{Dist}[(b^2 * (n - 1)) / n, \text{Int}[(c + d*x) * (b * \sin[e + f*x])^{n - 2}, x], x] - \text{Simp}[(b * (c + d*x) * \cos[e + f*x] * (b * \sin[e + f*x])^{n - 1}) / (f * n), x]) /;$ FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int (c + dx) \csc(x) \sin(3x) dx &= \int (3(c + dx) \cos^2(x) - (c + dx) \sin^2(x)) dx \\ &= 3 \int (c + dx) \cos^2(x) dx - \int (c + dx) \sin^2(x) dx \\ &= \frac{3}{4}d \cos^2(x) + 2(c + dx) \cos(x) \sin(x) - \frac{1}{4}d \sin^2(x) - \frac{1}{2} \int (c + dx) dx + \frac{3}{2} \int (c + dx) dx \\ &= cx + \frac{dx^2}{2} + \frac{3}{4}d \cos^2(x) + 2(c + dx) \cos(x) \sin(x) - \frac{1}{4}d \sin^2(x) \end{aligned}$$

Mathematica [A] time = 0.0198871, size = 34, normalized size = 0.83

$$cx + c \sin(2x) + \frac{dx^2}{2} + dx \sin(2x) + \frac{1}{2}d \cos(2x)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Csc[x]*Sin[3*x],x]

[Out] c*x + (d*x^2)/2 + (d*Cos[2*x])/2 + c*Sin[2*x] + d*x*Sin[2*x]

Maple [A] time = 0.048, size = 52, normalized size = 1.3

$$4d \left(x \left(\frac{1}{2} \cos(x) \sin(x) + x/2 \right) - \frac{1}{4} x^2 - \frac{1}{4} (\sin(x))^2 \right) + 4c \left(\frac{1}{2} \cos(x) \sin(x) + x/2 \right) - \frac{dx^2}{2} - cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*csc(x)*sin(3*x),x)

[Out] 4*d*(x*(1/2*cos(x)*sin(x)+1/2*x)-1/4*x^2-1/4*sin(x)^2)+4*c*(1/2*cos(x)*sin(x)+1/2*x)-1/2*d*x^2-c*x

Maxima [A] time = 1.02177, size = 36, normalized size = 0.88

$$\frac{1}{2} \left(x^2 + 2x \sin(2x) + \cos(2x) \right) d + c(x + \sin(2x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(x)*sin(3*x),x, algorithm="maxima")

[Out] 1/2*(x^2 + 2*x*sin(2*x) + cos(2*x))*d + c*(x + sin(2*x))

Fricas [A] time = 0.492165, size = 78, normalized size = 1.9

$$\frac{1}{2} dx^2 + d \cos(x)^2 + 2(dx + c) \cos(x) \sin(x) + cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(x)*sin(3*x),x, algorithm="fricas")

[Out] 1/2*d*x^2 + d*cos(x)^2 + 2*(d*x + c)*cos(x)*sin(x) + c*x

Sympy [A] time = 14.3026, size = 56, normalized size = 1.37

$$cx + c \sin(2x) - dx^2 \sin^2(x) - dx^2 \cos^2(x) + \frac{3dx^2}{2} + 2dx \sin(x) \cos(x) - d \sin^2(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(x)*sin(3*x),x)

```
[Out] c*x + c*sin(2*x) - d*x**2*sin(x)**2 - d*x**2*cos(x)**2 + 3*d*x**2/2 + 2*d*x
*sin(x)*cos(x) - d*sin(x)**2
```

Giac [A] time = 1.0992, size = 36, normalized size = 0.88

$$\frac{1}{2} dx^2 + cx + \frac{1}{2} d \cos(2x) + (dx + c) \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] 1/2*d*x^2 + c*x + 1/2*d*cos(2*x) + (d*x + c)*sin(2*x)
```

$$3.365 \quad \int \frac{\csc(x) \sin(3x)}{c+dx} dx$$

Optimal. Leaf size=57

$$\frac{2 \cos\left(\frac{2c}{d}\right) \text{CosIntegral}\left(\frac{2c}{d} + 2x\right)}{d} + \frac{2 \sin\left(\frac{2c}{d}\right) \text{Si}\left(\frac{2c}{d} + 2x\right)}{d} + \frac{\log(c + dx)}{d}$$

[Out] (2*Cos[(2*c)/d]*CosIntegral[(2*c)/d + 2*x])/d + Log[c + d*x]/d + (2*Sin[(2*c)/d]*SinIntegral[(2*c)/d + 2*x])/d

Rubi [A] time = 0.251852, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4431, 3312, 3303, 3299, 3302}

$$\frac{2 \cos\left(\frac{2c}{d}\right) \text{CosIntegral}\left(\frac{2c}{d} + 2x\right)}{d} + \frac{2 \sin\left(\frac{2c}{d}\right) \text{Si}\left(\frac{2c}{d} + 2x\right)}{d} + \frac{\log(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(Csc[x]*Sin[3*x])/(c + d*x), x]

[Out] (2*Cos[(2*c)/d]*CosIntegral[(2*c)/d + 2*x])/d + Log[c + d*x]/d + (2*Sin[(2*c)/d]*SinIntegral[(2*c)/d + 2*x])/d

Rule 4431

Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol] := Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && MemberQ[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]

Rule 3312

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\csc(x) \sin(3x)}{c + dx} dx &= \int \left(\frac{3 \cos^2(x)}{c + dx} - \frac{\sin^2(x)}{c + dx} \right) dx \\
&= 3 \int \frac{\cos^2(x)}{c + dx} dx - \int \frac{\sin^2(x)}{c + dx} dx \\
&= 3 \int \left(\frac{1}{2(c + dx)} + \frac{\cos(2x)}{2(c + dx)} \right) dx - \int \left(\frac{1}{2(c + dx)} - \frac{\cos(2x)}{2(c + dx)} \right) dx \\
&= \frac{\log(c + dx)}{d} + \frac{1}{2} \int \frac{\cos(2x)}{c + dx} dx + \frac{3}{2} \int \frac{\cos(2x)}{c + dx} dx \\
&= \frac{\log(c + dx)}{d} + \frac{1}{2} \cos\left(\frac{2c}{d}\right) \int \frac{\cos\left(\frac{2c}{d} + 2x\right)}{c + dx} dx + \frac{1}{2} \left(3 \cos\left(\frac{2c}{d}\right) \right) \int \frac{\cos\left(\frac{2c}{d} + 2x\right)}{c + dx} dx + \frac{1}{2} \sin\left(\frac{2c}{d}\right) \int \frac{\sin\left(\frac{2c}{d} + 2x\right)}{c + dx} dx \\
&= \frac{2 \cos\left(\frac{2c}{d}\right) \text{Ci}\left(\frac{2c}{d} + 2x\right)}{d} + \frac{\log(c + dx)}{d} + \frac{2 \sin\left(\frac{2c}{d}\right) \text{Si}\left(\frac{2c}{d} + 2x\right)}{d}
\end{aligned}$$

Mathematica [A] time = 0.0631687, size = 49, normalized size = 0.86

$$\frac{2 \cos\left(\frac{2c}{d}\right) \text{CosIntegral}\left(2\left(\frac{c}{d} + x\right)\right) + 2 \sin\left(\frac{2c}{d}\right) \text{Si}\left(2\left(\frac{c}{d} + x\right)\right) + \log(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x]*Sin[3*x])/(c + d*x), x]

[Out] (2*Cos[(2*c)/d]*CosIntegral[2*(c/d + x)] + Log[c + d*x] + 2*Sin[(2*c)/d]*SinIntegral[2*(c/d + x)])/d

Maple [A] time = 0.051, size = 58, normalized size = 1.

$$2 \frac{1}{d} \text{Ci}\left(2 \frac{c}{d} + 2x\right) \cos\left(2 \frac{c}{d}\right) + \frac{\ln(dx + c)}{d} + 2 \frac{1}{d} \text{Si}\left(2 \frac{c}{d} + 2x\right) \sin\left(2 \frac{c}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)*sin(3*x)/(d*x+c), x)

[Out] 2*Ci(2*c/d+2*x)*cos(2*c/d)/d+ln(d*x+c)/d+2*Si(2*c/d+2*x)*sin(2*c/d)/d

Maxima [C] time = 1.23435, size = 128, normalized size = 2.25

$$\frac{\left(E_1\left(\frac{2idx+2ic}{d}\right) + E_1\left(-\frac{2idx+2ic}{d}\right)\right) \cos\left(\frac{2c}{d}\right) - \left(-iE_1\left(\frac{2idx+2ic}{d}\right) + iE_1\left(-\frac{2idx+2ic}{d}\right)\right) \sin\left(\frac{2c}{d}\right) - \log(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)*sin(3*x)/(d*x+c), x, algorithm="maxima")

[Out] -((exp_integral_e(1, (2*I*d*x + 2*I*c)/d) + exp_integral_e(1, -(2*I*d*x + 2*I*c)/d))*cos(2*c/d) - (-I*exp_integral_e(1, (2*I*d*x + 2*I*c)/d) + I*exp_i

`ntegral_e(1, -(2*I*d*x + 2*I*c)/d)*sin(2*c/d) - log(d*x + c))/d`

Fricas [A] time = 0.504552, size = 182, normalized size = 3.19

$$\frac{\left(\operatorname{Ci}\left(\frac{2(dx+c)}{d}\right) + \operatorname{Ci}\left(-\frac{2(dx+c)}{d}\right)\right) \cos\left(\frac{2c}{d}\right) + 2 \sin\left(\frac{2c}{d}\right) \operatorname{Si}\left(\frac{2(dx+c)}{d}\right) + \log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)*sin(3*x)/(d*x+c),x, algorithm="fricas")`

[Out] `((cos_integral(2*(d*x + c)/d) + cos_integral(-2*(d*x + c)/d))*cos(2*c/d) + 2*sin(2*c/d)*sin_integral(2*(d*x + c)/d) + log(d*x + c))/d`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(3x) \csc(x)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)*sin(3*x)/(d*x+c),x)`

[Out] `Integral(sin(3*x)*csc(x)/(c + d*x), x)`

Giac [A] time = 1.12076, size = 69, normalized size = 1.21

$$\frac{2 \cos\left(\frac{2c}{d}\right) \operatorname{Ci}\left(\frac{2(dx+c)}{d}\right) + 2 \sin\left(\frac{2c}{d}\right) \operatorname{Si}\left(\frac{2(dx+c)}{d}\right) + \log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)*sin(3*x)/(d*x+c),x, algorithm="giac")`

[Out] `(2*cos(2*c/d)*cos_integral(2*(d*x + c)/d) + 2*sin(2*c/d)*sin_integral(2*(d*x + c)/d) + log(d*x + c))/d`

$$3.366 \quad \int \frac{\csc(x) \sin(3x)}{(c+dx)^2} dx$$

Optimal. Leaf size=78

$$\frac{4 \sin\left(\frac{2c}{d}\right) \text{CosIntegral}\left(\frac{2c}{d} + 2x\right)}{d^2} - \frac{4 \cos\left(\frac{2c}{d}\right) \text{Si}\left(\frac{2c}{d} + 2x\right)}{d^2} + \frac{\sin^2(x)}{d(c+dx)} - \frac{3 \cos^2(x)}{d(c+dx)}$$

[Out] (-3*Cos[x]^2)/(d*(c + d*x)) + (4*CosIntegral[(2*c)/d + 2*x]*Sin[(2*c)/d])/d^2 + Sin[x]^2/(d*(c + d*x)) - (4*Cos[(2*c)/d]*SinIntegral[(2*c)/d + 2*x])/d^2

Rubi [A] time = 0.237926, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4431, 3313, 12, 3303, 3299, 3302}

$$\frac{4 \sin\left(\frac{2c}{d}\right) \text{CosIntegral}\left(\frac{2c}{d} + 2x\right)}{d^2} - \frac{4 \cos\left(\frac{2c}{d}\right) \text{Si}\left(\frac{2c}{d} + 2x\right)}{d^2} + \frac{\sin^2(x)}{d(c+dx)} - \frac{3 \cos^2(x)}{d(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(Csc[x]*Sin[3*x])/(c + d*x)^2,x]

[Out] (-3*Cos[x]^2)/(d*(c + d*x)) + (4*CosIntegral[(2*c)/d + 2*x]*Sin[(2*c)/d])/d^2 + Sin[x]^2/(d*(c + d*x)) - (4*Cos[(2*c)/d]*SinIntegral[(2*c)/d + 2*x])/d^2

Rule 4431

Int[((e_.) + (f_.)*(x_.))^(m_.)*(F_)[(a_.) + (b_.)*(x_.)]^(p_.)*(G_)[(c_.) + (d_.)*(x_.)]^(q_.), x_Symbol] := Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && MemberQ[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]

Rule 3313

Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m + 1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n - 1), x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\csc(x) \sin(3x)}{(c+dx)^2} dx &= \int \left(\frac{3 \cos^2(x)}{(c+dx)^2} - \frac{\sin^2(x)}{(c+dx)^2} \right) dx \\
 &= 3 \int \frac{\cos^2(x)}{(c+dx)^2} dx - \int \frac{\sin^2(x)}{(c+dx)^2} dx \\
 &= -\frac{3 \cos^2(x)}{d(c+dx)} + \frac{\sin^2(x)}{d(c+dx)} - \frac{2 \int \frac{\sin(2x)}{2(c+dx)} dx}{d} + \frac{6 \int -\frac{\sin(2x)}{2(c+dx)} dx}{d} \\
 &= -\frac{3 \cos^2(x)}{d(c+dx)} + \frac{\sin^2(x)}{d(c+dx)} - \frac{\int \frac{\sin(2x)}{c+dx} dx}{d} - \frac{3 \int \frac{\sin(2x)}{c+dx} dx}{d} \\
 &= -\frac{3 \cos^2(x)}{d(c+dx)} + \frac{\sin^2(x)}{d(c+dx)} - \frac{\cos\left(\frac{2c}{d}\right) \int \frac{\sin\left(\frac{2c}{d}+2x\right)}{c+dx} dx}{d} - \frac{\left(3 \cos\left(\frac{2c}{d}\right)\right) \int \frac{\sin\left(\frac{2c}{d}+2x\right)}{c+dx} dx}{d} + \frac{\sin\left(\frac{2c}{d}\right) \int \frac{\cos\left(\frac{2c}{d}+2x\right)}{c+dx} dx}{d} \\
 &= -\frac{3 \cos^2(x)}{d(c+dx)} + \frac{4 \operatorname{Ci}\left(\frac{2c}{d}+2x\right) \sin\left(\frac{2c}{d}\right)}{d^2} + \frac{\sin^2(x)}{d(c+dx)} - \frac{4 \cos\left(\frac{2c}{d}\right) \operatorname{Si}\left(\frac{2c}{d}+2x\right)}{d^2}
 \end{aligned}$$

Mathematica [A] time = 0.138473, size = 61, normalized size = 0.78

$$\frac{4 \sin\left(\frac{2c}{d}\right) \operatorname{CosIntegral}\left(2\left(\frac{c}{d}+x\right)\right) - 4 \cos\left(\frac{2c}{d}\right) \operatorname{Si}\left(2\left(\frac{c}{d}+x\right)\right) - \frac{d(2 \cos(2x)+1)}{c+dx}}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x]*Sin[3*x])/(c + d*x)^2, x]

[Out] (-((d*(1 + 2*Cos[2*x]))/(c + d*x)) + 4*CosIntegral[2*(c/d + x)]*Sin[(2*c)/d] - 4*Cos[(2*c)/d]*SinIntegral[2*(c/d + x)]/d^2

Maple [A] time = 0.054, size = 82, normalized size = 1.1

$$-2 \frac{\cos(2x)}{(dx+c)d} - 2 \frac{1}{d} \left(2 \frac{1}{d} \operatorname{Si}\left(2 \frac{c}{d} + 2x\right) \cos\left(2 \frac{c}{d}\right) - 2 \frac{1}{d} \operatorname{Ci}\left(2 \frac{c}{d} + 2x\right) \sin\left(2 \frac{c}{d}\right) \right) - \frac{1}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)*sin(3*x)/(d*x+c)^2, x)

[Out] -2*cos(2*x)/(d*x+c)/d - 2*(2*Si(2*c/d+2*x)*cos(2*c/d)/d - 2*Ci(2*c/d+2*x)*sin(2*c/d)/d)/d - 1/d/(d*x+c)

Maxima [C] time = 1.22548, size = 437, normalized size = 5.6

$$\frac{\left(E_2\left(\frac{2idx+2ic}{d}\right) + E_2\left(-\frac{2idx+2ic}{d}\right)\right) \cos\left(\frac{2c}{d}\right)^3 + \left(iE_2\left(\frac{2idx+2ic}{d}\right) - iE_2\left(-\frac{2idx+2ic}{d}\right)\right) \sin\left(\frac{2c}{d}\right)^3 + \left(\left(E_2\left(\frac{2idx+2ic}{d}\right) + E_2\left(-\frac{2idx+2ic}{d}\right)\right) \cos\left(\frac{2c}{d}\right)^2 + \left(iE_2\left(\frac{2idx+2ic}{d}\right) - iE_2\left(-\frac{2idx+2ic}{d}\right)\right) \sin\left(\frac{2c}{d}\right)^2\right) d}{d^3x + cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)*sin(3*x)/(d*x+c)^2,x, algorithm="maxima")

[Out]
$$\frac{-1/2 * ((\exp_integral_e(2, (2*I*d*x + 2*I*c)/d) + \exp_integral_e(2, -(2*I*d*x + 2*I*c)/d)) * \cos(2*c/d)^3 + (I * \exp_integral_e(2, (2*I*d*x + 2*I*c)/d) - I * \exp_integral_e(2, -(2*I*d*x + 2*I*c)/d)) * \sin(2*c/d)^3 + ((\exp_integral_e(2, (2*I*d*x + 2*I*c)/d) + \exp_integral_e(2, -(2*I*d*x + 2*I*c)/d)) * \cos(2*c/d)^2 + (\exp_integral_e(2, (2*I*d*x + 2*I*c)/d) + \exp_integral_e(2, -(2*I*d*x + 2*I*c)/d)) * \cos(2*c/d) + 2 * \cos(2*c/d)^2 + ((I * \exp_integral_e(2, (2*I*d*x + 2*I*c)/d) - I * \exp_integral_e(2, -(2*I*d*x + 2*I*c)/d)) * \cos(2*c/d)^2 + I * \exp_integral_e(2, (2*I*d*x + 2*I*c)/d) - I * \exp_integral_e(2, -(2*I*d*x + 2*I*c)/d)) * \sin(2*c/d)) / ((\cos(2*c/d)^2 + \sin(2*c/d)^2) * d^2 * x + (c * \cos(2*c/d)^2 + c * \sin(2*c/d)^2) * d)}$$

Fricas [A] time = 0.514176, size = 251, normalized size = 3.22

$$\frac{4d \cos(x)^2 + 4(dx+c) \cos\left(\frac{2c}{d}\right) \text{Si}\left(\frac{2(dx+c)}{d}\right) - 2\left((dx+c) \text{Ci}\left(\frac{2(dx+c)}{d}\right) + (dx+c) \text{Ci}\left(-\frac{2(dx+c)}{d}\right)\right) \sin\left(\frac{2c}{d}\right) - d}{d^3x + cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)*sin(3*x)/(d*x+c)^2,x, algorithm="fricas")

[Out]
$$\frac{-(4*d*\cos(x)^2 + 4*(d*x + c)*\cos(2*c/d)*\sin_integral(2*(d*x + c)/d) - 2*((d*x + c)*\cos_integral(2*(d*x + c)/d) + (d*x + c)*\cos_integral(-2*(d*x + c)/d))*\sin(2*c/d) - d}{(d^3*x + c*d^2)}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)*sin(3*x)/(d*x+c)**2,x)

[Out] Timed out

Giac [A] time = 1.11966, size = 150, normalized size = 1.92

$$\frac{4dx \text{Ci}\left(\frac{2(dx+c)}{d}\right) \sin\left(\frac{2c}{d}\right) - 4dx \cos\left(\frac{2c}{d}\right) \text{Si}\left(\frac{2(dx+c)}{d}\right) + 4c \text{Ci}\left(\frac{2(dx+c)}{d}\right) \sin\left(\frac{2c}{d}\right) - 4c \cos\left(\frac{2c}{d}\right) \text{Si}\left(\frac{2(dx+c)}{d}\right) - 2d \cos(2c/d)}{d^3x + cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (4*d*x*cos_integral(2*(d*x + c)/d)*sin(2*c/d) - 4*d*x*cos(2*c/d)*sin_integr  
al(2*(d*x + c)/d) + 4*c*cos_integral(2*(d*x + c)/d)*sin(2*c/d) - 4*c*cos(2*  
c/d)*sin_integral(2*(d*x + c)/d) - 2*d*cos(2*x) - d)/(d^3*x + c*d^2)
```

$$3.367 \quad \int \frac{\csc(x) \sin(3x)}{(c+dx)^3} dx$$

Optimal. Leaf size=99

$$-\frac{4 \cos\left(\frac{2c}{d}\right) \text{CosIntegral}\left(\frac{2c}{d} + 2x\right)}{d^3} - \frac{4 \sin\left(\frac{2c}{d}\right) \text{Si}\left(\frac{2c}{d} + 2x\right)}{d^3} + \frac{4 \sin(x) \cos(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} - \frac{3 \cos^2(x)}{2d(c+dx)^2}$$

[Out] $(-3*\text{Cos}[x]^2)/(2*d*(c+d*x)^2) - (4*\text{Cos}[(2*c)/d]*\text{CosIntegral}[(2*c)/d + 2*x])/d^3 + (4*\text{Cos}[x]*\text{Sin}[x])/(d^2*(c+d*x)) + \text{Sin}[x]^2/(2*d*(c+d*x)^2) - (4*\text{Sin}[(2*c)/d]*\text{SinIntegral}[(2*c)/d + 2*x])/d^3$

Rubi [A] time = 0.328565, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {4431, 3314, 31, 3312, 3303, 3299, 3302}

$$-\frac{4 \cos\left(\frac{2c}{d}\right) \text{CosIntegral}\left(\frac{2c}{d} + 2x\right)}{d^3} - \frac{4 \sin\left(\frac{2c}{d}\right) \text{Si}\left(\frac{2c}{d} + 2x\right)}{d^3} + \frac{4 \sin(x) \cos(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} - \frac{3 \cos^2(x)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Csc}[x]*\text{Sin}[3*x])/(c+d*x)^3, x]$

[Out] $(-3*\text{Cos}[x]^2)/(2*d*(c+d*x)^2) - (4*\text{Cos}[(2*c)/d]*\text{CosIntegral}[(2*c)/d + 2*x])/d^3 + (4*\text{Cos}[x]*\text{Sin}[x])/(d^2*(c+d*x)) + \text{Sin}[x]^2/(2*d*(c+d*x)^2) - (4*\text{Sin}[(2*c)/d]*\text{SinIntegral}[(2*c)/d + 2*x])/d^3$

Rule 4431

$\text{Int}[(e_{.}) + (f_{.})*(x_{.})^{(m_{.})}*(F_{.})[(a_{.}) + (b_{.})*(x_{.})]^{(p_{.})}*(G_{.})[(c_{.}) + (d_{.})*(x_{.})]^{(q_{.})}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigExpand}[(e + f*x)^m * G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \text{MemberQ}\{\{\text{Sin}, \text{Cos}\}, F\} \&\& \text{MemberQ}\{\{\text{Sec}, \text{Csc}\}, G\} \&\& \text{IGtQ}[p, 0] \&\& \text{IGtQ}[q, 0] \&\& \text{EQ}[b*c - a*d, 0] \&\& \text{IGtQ}[b/d, 1]$

Rule 3314

$\text{Int}[(c_{.}) + (d_{.})*(x_{.})^{(m_{.})}*((b_{.})*\text{sin}[(e_{.}) + (f_{.})*(x_{.})])^{(n_{.})}, x_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m+1)}*(b*\text{Sin}[e + f*x])^n / (d*(m+1)), x] + (\text{Dist}[(b^2*f^2*n*(n-1)) / (d^2*(m+1)*(m+2)), \text{Int}[(c + d*x)^{(m+2)}*(b*\text{Sin}[e + f*x])^{(n-2)}, x], x] - \text{Dist}[(f^2*n^2) / (d^2*(m+1)*(m+2)), \text{Int}[(c + d*x)^{(m+2)}*(b*\text{Sin}[e + f*x])^n, x], x] - \text{Simp}[(b*f*n*(c + d*x)^{(m+2)}*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(n-1)}) / (d^2*(m+1)*(m+2)), x]) /; \text{FreeQ}\{b, c, d, e, f\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{LtQ}[m, -2]$

Rule 31

$\text{Int}[(a_{.}) + (b_{.})*(x_{.})^{(-1)}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$

Rule 3312

$\text{Int}[(c_{.}) + (d_{.})*(x_{.})^{(m_{.})}*\text{sin}[(e_{.}) + (f_{.})*(x_{.})]^{(n_{.})}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[e + f*x]^n, x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x] \&\& \text{IGtQ}[n, 1] \&\& (!\text{RationalQ}[m] || (\text{GeQ}[m, -1] \&\& \text{LtQ}[m, 1]))$

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(x) \sin(3x)}{(c+dx)^3} dx &= \int \left(\frac{3 \cos^2(x)}{(c+dx)^3} - \frac{\sin^2(x)}{(c+dx)^3} \right) dx \\
&= 3 \int \frac{\cos^2(x)}{(c+dx)^3} dx - \int \frac{\sin^2(x)}{(c+dx)^3} dx \\
&= -\frac{3 \cos^2(x)}{2d(c+dx)^2} + \frac{4 \cos(x) \sin(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} - \frac{\int \frac{1}{c+dx} dx}{d^2} + \frac{2 \int \frac{\sin^2(x)}{c+dx} dx}{d^2} + \frac{3 \int \frac{1}{c+dx} dx}{d^2} - \frac{6 \int \frac{\cos^2(x)}{c+dx} dx}{d^2} \\
&= -\frac{3 \cos^2(x)}{2d(c+dx)^2} + \frac{2 \log(c+dx)}{d^3} + \frac{4 \cos(x) \sin(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} + \frac{2 \int \left(\frac{1}{2(c+dx)} - \frac{\cos(2x)}{2(c+dx)} \right) dx}{d^2} - \frac{6 \int \frac{\cos^2(x)}{c+dx} dx}{d^2} \\
&= -\frac{3 \cos^2(x)}{2d(c+dx)^2} + \frac{4 \cos(x) \sin(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} - \frac{\int \frac{\cos(2x)}{c+dx} dx}{d^2} - \frac{3 \int \frac{\cos(2x)}{c+dx} dx}{d^2} \\
&= -\frac{3 \cos^2(x)}{2d(c+dx)^2} + \frac{4 \cos(x) \sin(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} - \frac{\cos\left(\frac{2c}{d}\right) \int \frac{\cos\left(\frac{2c}{d}+2x\right)}{c+dx} dx}{d^2} - \frac{\left(3 \cos\left(\frac{2c}{d}\right)\right) \int \frac{\cos\left(\frac{2c}{d}+2x\right)}{c+dx} dx}{d^2} \\
&= -\frac{3 \cos^2(x)}{2d(c+dx)^2} - \frac{4 \cos\left(\frac{2c}{d}\right) \text{Ci}\left(\frac{2c}{d}+2x\right)}{d^3} + \frac{4 \cos(x) \sin(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} - \frac{4 \sin\left(\frac{2c}{d}\right) \text{Si}\left(\frac{2c}{d}+2x\right)}{d^3}
\end{aligned}$$

Mathematica [A] time = 0.233797, size = 77, normalized size = 0.78

$$\frac{-8 \cos\left(\frac{2c}{d}\right) \text{CosIntegral}\left(2\left(\frac{c}{d}+x\right)\right) - 8 \sin\left(\frac{2c}{d}\right) \text{Si}\left(2\left(\frac{c}{d}+x\right)\right) + \frac{d(4 \sin(2x)(c+dx) - 2d \cos(2x) - d)}{(c+dx)^2}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]*Sin[3*x])/(c + d*x)^3, x]
```

```
[Out] (-8*Cos[(2*c)/d]*CosIntegral[2*(c/d + x)] + (d*(-d - 2*d*Cos[2*x] + 4*(c +
d*x)*Sin[2*x]))/(c + d*x)^2 - 8*Sin[(2*c)/d]*SinIntegral[2*(c/d + x)]/(2*d
^3)
```


Maple [A] time = 0.053, size = 104, normalized size = 1.1

$$-\frac{\cos(2x)}{(dx+c)^2 d} - \frac{1}{d} \left(-2 \frac{\sin(2x)}{(dx+c)d} + 2 \frac{1}{d} \left(2 \frac{1}{d} \operatorname{Si} \left(2 \frac{c}{d} + 2x \right) \sin \left(2 \frac{c}{d} \right) + 2 \frac{1}{d} \operatorname{Ci} \left(2 \frac{c}{d} + 2x \right) \cos \left(2 \frac{c}{d} \right) \right) \right) - \frac{1}{2(dx+c)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)*sin(3*x)/(d*x+c)^3,x)

[Out] $-\cos(2*x)/(d*x+c)^2/d - (-2*\sin(2*x)/(d*x+c)/d + 2*(2*\operatorname{Si}(2*c/d+2*x)*\sin(2*c/d)/d + 2*\operatorname{Ci}(2*c/d+2*x)*\cos(2*c/d)/d)/d - 1/2/d/(d*x+c)^2$

Maxima [C] time = 1.37194, size = 489, normalized size = 4.94

$$\frac{2 \left(E_3 \left(\frac{2i dx + 2ic}{d} \right) + E_3 \left(-\frac{2i dx + 2ic}{d} \right) \right) \cos \left(\frac{2c}{d} \right)^3 + \left(2i E_3 \left(\frac{2i dx + 2ic}{d} \right) - 2i E_3 \left(-\frac{2i dx + 2ic}{d} \right) \right) \sin \left(\frac{2c}{d} \right)^3 + 2 \left(\left(E_3 \left(\frac{2i dx + 2ic}{d} \right) + E_3 \left(-\frac{2i dx + 2ic}{d} \right) \right) \cos \left(\frac{2c}{d} \right) + \left(2i E_3 \left(\frac{2i dx + 2ic}{d} \right) - 2i E_3 \left(-\frac{2i dx + 2ic}{d} \right) \right) \sin \left(\frac{2c}{d} \right) \right)^2}{4 \left(\cos \left(\frac{2c}{d} \right) + \sin \left(\frac{2c}{d} \right) \right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)*sin(3*x)/(d*x+c)^3,x, algorithm="maxima")

[Out] $-1/4*(2*(\exp_integral_e(3, (2*I*d*x + 2*I*c)/d) + \exp_integral_e(3, -(2*I*d*x + 2*I*c)/d))*\cos(2*c/d)^3 + (2*I*\exp_integral_e(3, (2*I*d*x + 2*I*c)/d) - 2*I*\exp_integral_e(3, -(2*I*d*x + 2*I*c)/d))*\sin(2*c/d)^3 + 2*((\exp_integral_e(3, (2*I*d*x + 2*I*c)/d) + \exp_integral_e(3, -(2*I*d*x + 2*I*c)/d))*\cos(2*c/d) + 1)*\sin(2*c/d)^2 + 2*(\exp_integral_e(3, (2*I*d*x + 2*I*c)/d) + \exp_integral_e(3, -(2*I*d*x + 2*I*c)/d))*\cos(2*c/d) + 2*\cos(2*c/d)^2 + ((2*I*\exp_integral_e(3, (2*I*d*x + 2*I*c)/d) - 2*I*\exp_integral_e(3, -(2*I*d*x + 2*I*c)/d))*\cos(2*c/d)^2 + 2*I*\exp_integral_e(3, (2*I*d*x + 2*I*c)/d) - 2*I*\exp_integral_e(3, -(2*I*d*x + 2*I*c)/d))*\sin(2*c/d)/((\cos(2*c/d)^2 + \sin(2*c/d)^2)*d^3*x^2 + 2*(c*\cos(2*c/d)^2 + c*\sin(2*c/d)^2)*d^2*x + (c^2*\cos(2*c/d)^2 + c^2*\sin(2*c/d)^2)*d)$

Fricas [A] time = 0.529877, size = 392, normalized size = 3.96

$$\frac{4d^2 \cos(x)^2 - 8(d^2x + cd) \cos(x) \sin(x) + 8(d^2x^2 + 2cdx + c^2) \sin\left(\frac{2c}{d}\right) \operatorname{Si}\left(\frac{2(dx+c)}{d}\right) - d^2 + 4\left((d^2x^2 + 2cdx + c^2) \cos\left(\frac{2c}{d}\right) + (d^2x^2 + 2cdx + c^2) \sin\left(\frac{2c}{d}\right)\right)}{2(d^5x^2 + 2cd^4x + c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)*sin(3*x)/(d*x+c)^3,x, algorithm="fricas")

[Out] $-1/2*(4*d^2*\cos(x)^2 - 8*(d^2*x + c*d)*\cos(x)*\sin(x) + 8*(d^2*x^2 + 2*c*d*x + c^2)*\sin(2*c/d)*\sin_integral(2*(d*x + c)/d) - d^2 + 4*((d^2*x^2 + 2*c*d*x + c^2)*\cos_integral(2*(d*x + c)/d) + (d^2*x^2 + 2*c*d*x + c^2)*\cos_integral(-2*(d*x + c)/d))*\cos(2*c/d)/((d^5*x^2 + 2*c*d^4*x + c^2*d^3)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)*sin(3*x)/(d*x+c)**3,x)

[Out] Timed out

Giac [B] time = 1.11449, size = 271, normalized size = 2.74

$$\frac{8d^2x^2 \cos\left(\frac{2c}{d}\right) \text{Ci}\left(\frac{2(dx+c)}{d}\right) + 8d^2x^2 \sin\left(\frac{2c}{d}\right) \text{Si}\left(\frac{2(dx+c)}{d}\right) + 16cdx \cos\left(\frac{2c}{d}\right) \text{Ci}\left(\frac{2(dx+c)}{d}\right) + 16cdx \sin\left(\frac{2c}{d}\right) \text{Si}\left(\frac{2(dx+c)}{d}\right) + \dots}{2(d^5x^2 + 2cd^4x + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)*sin(3*x)/(d*x+c)^3,x, algorithm="giac")

[Out]
$$-1/2*(8*d^2*x^2*\cos(2*c/d)*\cos_integral(2*(d*x + c)/d) + 8*d^2*x^2*\sin(2*c/d)*\sin_integral(2*(d*x + c)/d) + 16*c*d*x*\cos(2*c/d)*\cos_integral(2*(d*x + c)/d) + 16*c*d*x*\sin(2*c/d)*\sin_integral(2*(d*x + c)/d) + 8*c^2*\cos(2*c/d)*\cos_integral(2*(d*x + c)/d) - 4*d^2*x*\sin(2*x) + 8*c^2*\sin(2*c/d)*\sin_integral(2*(d*x + c)/d) + 2*d^2*\cos(2*x) - 4*c*d*\sin(2*x) + d^2)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)$$

3.368 $\int (c + dx)^4 \csc(a + bx) \sin(3a + 3bx) dx$

Optimal. Leaf size=198

$$\frac{3d^3(c + dx) \sin^2(a + bx)}{2b^4} - \frac{9d^3(c + dx) \cos^2(a + bx)}{2b^4} - \frac{6d^2(c + dx)^2 \sin(a + bx) \cos(a + bx)}{b^3} - \frac{d(c + dx)^3 \sin^2(a + bx)}{b^2}$$

[Out] $(3*d^4*x)/(2*b^4) - (d*(c + d*x)^3)/b^2 + (c + d*x)^5/(5*d) - (9*d^3*(c + d*x)*\text{Cos}[a + b*x]^2)/(2*b^4) + (3*d*(c + d*x)^3*\text{Cos}[a + b*x]^2)/b^2 + (3*d^4*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b^5 - (6*d^2*(c + d*x)^2*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b^3 + (2*(c + d*x)^4*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b + (3*d^3*(c + d*x)*\text{Sin}[a + b*x]^2)/(2*b^4) - (d*(c + d*x)^3*\text{Sin}[a + b*x]^2)/b^2$

Rubi [A] time = 0.251829, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4431, 3311, 32, 2635, 8}

$$\frac{3d^3(c + dx) \sin^2(a + bx)}{2b^4} - \frac{9d^3(c + dx) \cos^2(a + bx)}{2b^4} - \frac{6d^2(c + dx)^2 \sin(a + bx) \cos(a + bx)}{b^3} - \frac{d(c + dx)^3 \sin^2(a + bx)}{b^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^4*\text{Csc}[a + b*x]*\text{Sin}[3*a + 3*b*x],x]$

[Out] $(3*d^4*x)/(2*b^4) - (d*(c + d*x)^3)/b^2 + (c + d*x)^5/(5*d) - (9*d^3*(c + d*x)*\text{Cos}[a + b*x]^2)/(2*b^4) + (3*d*(c + d*x)^3*\text{Cos}[a + b*x]^2)/b^2 + (3*d^4*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b^5 - (6*d^2*(c + d*x)^2*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b^3 + (2*(c + d*x)^4*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b + (3*d^3*(c + d*x)*\text{Sin}[a + b*x]^2)/(2*b^4) - (d*(c + d*x)^3*\text{Sin}[a + b*x]^2)/b^2$

Rule 4431

$\text{Int}[(e_{.}) + (f_{.})*(x_{.})]^{(m_{.})}*(F_{.})[(a_{.}) + (b_{.})*(x_{.})]^{(p_{.})}*(G_{.})[(c_{.}) + (d_{.})*(x_{.})]^{(q_{.})}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigExpand}[(e + f*x)^m*G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x\} \&\& \text{MemberQ}\{\{\text{Sin}, \text{Cos}\}, F\} \&\& \text{MemberQ}\{\{\text{Sec}, \text{Csc}\}, G\} \&\& \text{IGtQ}\{p, 0\} \&\& \text{IGtQ}\{q, 0\} \&\& \text{EQ}\{b*c - a*d, 0\} \&\& \text{IGtQ}\{b/d, 1\}$

Rule 3311

$\text{Int}[(c_{.}) + (d_{.})*(x_{.})]^{(m_{.})}*((b_{.})*\text{sin}[(e_{.}) + (f_{.})*(x_{.})])^{(n_{.})}, x_Symbol] \rightarrow \text{Simp}[(d*m*(c + d*x)^{(m-1)}*(b*\text{Sin}[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n-1))/n, \text{Int}[(c + d*x)^m*(b*\text{Sin}[e + f*x])^{(n-2)}, x], x] - \text{Dist}[d^2*m*(m-1)/(f^2*n^2), \text{Int}[(c + d*x)^{(m-2)}*(b*\text{Sin}[e + f*x])^n, x], x] - \text{Simp}[(b*(c + d*x)^m*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(n-1)})/(f*n), x]) /; \text{FreeQ}\{b, c, d, e, f\}, x\} \&\& \text{GtQ}\{n, 1\} \&\& \text{GtQ}\{m, 1\}$

Rule 32

$\text{Int}[(a_{.}) + (b_{.})*(x_{.})]^{(m_{.})}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m+1)}/(b*(m+1)), x] /; \text{FreeQ}\{a, b, m\}, x\} \&\& \text{NeQ}\{m, -1\}$

Rule 2635

$\text{Int}[(b_{.})*\text{sin}[(c_{.}) + (d_{.})*(x_{.})])^{(n_{.})}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x\} \&\& \text{GtQ}\{n, 1\} \&\& \text{IntegerQ}\{2*n\}$

]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int (c+dx)^4 \csc(a+bx) \sin(3a+3bx) dx &= \int (3(c+dx)^4 \cos^2(a+bx) - (c+dx)^4 \sin^2(a+bx)) dx \\
&= 3 \int (c+dx)^4 \cos^2(a+bx) dx - \int (c+dx)^4 \sin^2(a+bx) dx \\
&= \frac{3d(c+dx)^3 \cos^2(a+bx)}{b^2} + \frac{2(c+dx)^4 \cos(a+bx) \sin(a+bx)}{b} - \frac{d(c+dx)^3 \sin^2(a+bx)}{b^2} \\
&= \frac{(c+dx)^5}{5d} - \frac{9d^3(c+dx) \cos^2(a+bx)}{2b^4} + \frac{3d(c+dx)^3 \cos^2(a+bx)}{b^2} - \frac{6d^2(c+dx)^2 \sin^2(a+bx)}{b^2} \\
&= -\frac{d(c+dx)^3}{b^2} + \frac{(c+dx)^5}{5d} - \frac{9d^3(c+dx) \cos^2(a+bx)}{2b^4} + \frac{3d(c+dx)^3 \cos^2(a+bx)}{b^2} \\
&= \frac{3d^4x}{2b^4} - \frac{d(c+dx)^3}{b^2} + \frac{(c+dx)^5}{5d} - \frac{9d^3(c+dx) \cos^2(a+bx)}{2b^4} + \frac{3d(c+dx)^3 \cos^2(a+bx)}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.692733, size = 128, normalized size = 0.65

$$\frac{\sin(2(a+bx))(-6b^2d^2(c+dx)^2 + 2b^4(c+dx)^4 + 3d^4)}{2b^5} + \frac{d(c+dx) \cos(2(a+bx))(2b^2(c+dx)^2 - 3d^2)}{b^4} + 2c^2d^2x^3 + 2c^3d^2x^2$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^4*Csc[a + b*x]*Sin[3*a + 3*b*x], x]

[Out] c^4*x + 2*c^3*d*x^2 + 2*c^2*d^2*x^3 + c*d^3*x^4 + (d^4*x^5)/5 + (d*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])/b^4 + ((3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Sin[2*(a + b*x)])/(2*b^5)

Maple [B] time = 0.049, size = 1000, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a), x)

[Out] -c^4*x-1/5*d^4*x^5+4*c^4/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-2*c^3*d*x^2-2*c^2*d^2*x^3-c*d^3*x^4+4*d^4/b^5*((b*x+a)^4*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+(b*x+a)^3*cos(b*x+a)^2-3*(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/2*(b*x+a)*cos(b*x+a)^2+3/4*cos(b*x+a)*sin(b*x+a)+3/4*b*x+3/4*a+(b*x+a)^3-2/5*(b*x+a)^5-4*a*((b*x+a)^3*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)^2*cos(b*x+a)^2-3/2*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/8*(b*x+a)^2+3/8*sin(b*x+a)^2-3/8*(b*x+a)^4)+6*a^2*((b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3)-4*a^3*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+a^4*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))+16*c^3*d/b^2*((b*x+a)*(1/2*cos(b

$x+a) \sin(bx+a) + 1/2 * bx + 1/2 * a) - 1/4 * (bx+a)^2 - 1/4 * \sin(bx+a)^2 - a * (1/2 * \cos(bx+a) \sin(bx+a) + 1/2 * bx + 1/2 * a) + 24 * c^2 * d^2 / b^3 * ((bx+a)^2 * (1/2 * \cos(bx+a) \sin(bx+a) + 1/2 * bx + 1/2 * a) + 1/2 * (bx+a) * \cos(bx+a)^2 - 1/4 * \cos(bx+a) * \sin(bx+a) - 1/4 * bx - 1/4 * a - 1/3 * (bx+a)^3 - 2 * a * ((bx+a) * (1/2 * \cos(bx+a) \sin(bx+a) + 1/2 * bx + 1/2 * a) - 1/4 * (bx+a)^2 - 1/4 * \sin(bx+a)^2) + a^2 * (1/2 * \cos(bx+a) \sin(bx+a) + 1/2 * bx + 1/2 * a) + 16 * c * d^3 / b^4 * ((bx+a)^3 * (1/2 * \cos(bx+a) \sin(bx+a) + 1/2 * bx + 1/2 * a) + 3/4 * (bx+a)^2 * \cos(bx+a)^2 - 3/2 * (bx+a) * (1/2 * \cos(bx+a) \sin(bx+a) + 1/2 * bx + 1/2 * a) + 3/8 * (bx+a)^2 + 3/8 * \sin(bx+a)^2 - 3/8 * (bx+a)^4 - 3 * a * ((bx+a)^2 * (1/2 * \cos(bx+a) \sin(bx+a) + 1/2 * bx + 1/2 * a) + 1/2 * (bx+a) * \cos(bx+a)^2 - 1/4 * \cos(bx+a) * \sin(bx+a) - 1/4 * bx - 1/4 * a - 1/3 * (bx+a)^3) + 3 * a^2 * ((bx+a) * (1/2 * \cos(bx+a) \sin(bx+a) + 1/2 * bx + 1/2 * a) - 1/4 * (bx+a)^2 - 1/4 * \sin(bx+a)^2) - a^3 * (1/2 * \cos(bx+a) \sin(bx+a) + 1/2 * bx + 1/2 * a))$

Maxima [A] time = 1.19957, size = 329, normalized size = 1.66

$$\frac{(bx + \sin(2bx + 2a))c^4}{b} + \frac{2(b^2x^2 + 2bx \sin(2bx + 2a) + \cos(2bx + 2a))c^3d}{b^2} + \frac{(2b^3x^3 + 6bx \cos(2bx + 2a) + 3 \dots)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")

[Out] $(bx + \sin(2bx + 2a)) * c^4 / b + 2 * (b^2 * x^2 + 2 * bx * \sin(2bx + 2a) + \cos(2bx + 2a)) * c^3 * d / b^2 + (2 * b^3 * x^3 + 6 * bx * \cos(2bx + 2a) + 3 * (2 * b^2 * x^2 - 1) * \sin(2bx + 2a)) * c^2 * d^2 / b^3 + (b^4 * x^4 + 3 * (2 * b^2 * x^2 - 1) * \cos(2bx + 2a) + 2 * (2 * b^3 * x^3 - 3 * bx) * \sin(2bx + 2a)) * c * d^3 / b^4 + 1/10 * (2 * b^5 * x^5 + 10 * (2 * b^3 * x^3 - 3 * bx) * \cos(2bx + 2a) + 5 * (2 * b^4 * x^4 - 6 * b^2 * x^2 + 3) * \sin(2bx + 2a)) * d^4 / b^5$

Fricas [A] time = 0.564808, size = 579, normalized size = 2.92

$$b^5 d^4 x^5 + 5 b^5 c d^3 x^4 + 10 (b^5 c^2 d^2 - b^3 d^4) x^3 + 10 (b^5 c^3 d - 3 b^3 c d^3) x^2 + 10 (2 b^3 d^4 x^3 + 6 b^3 c d^3 x^2 + 2 b^3 c^3 d - 3 b c d^3 + 3 \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")

[Out] $1/5 * (b^5 * d^4 * x^5 + 5 * b^5 * c * d^3 * x^4 + 10 * (b^5 * c^2 * d^2 - b^3 * d^4) * x^3 + 10 * (b^5 * c^3 * d - 3 * b^3 * c * d^3) * x^2 + 10 * (2 * b^3 * d^4 * x^3 + 6 * b^3 * c * d^3 * x^2 + 2 * b^3 * c^3 * d - 3 * b * c * d^3 + 3 * (2 * b^3 * c^2 * d^2 - b * d^4) * x) * \cos(bx + a)^2 + 5 * (2 * b^4 * d^4 * x^4 + 8 * b^4 * c * d^3 * x^3 + 2 * b^4 * c^2 * d^2 - 6 * b^2 * c^2 * d^2 + 3 * d^4 + 6 * (2 * b^4 * c^2 * d^2 - b^2 * d^4) * x^2 + 4 * (2 * b^4 * c^3 * d - 3 * b^2 * c * d^3) * x) * \cos(bx + a) * \sin(bx + a) + 5 * (b^5 * c^4 - 6 * b^3 * c^2 * d^2 + 3 * b * d^4) * x) / b^5$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**4*csc(b*x+a)*sin(3*b*x+3*a),x)

[Out] Timed out

Giac [B] time = 1.7428, size = 6323, normalized size = 31.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")

[Out]
$$\begin{aligned} & \frac{1}{5} \cdot (b^5 d^4 x^5 \tan(1/2 b x)^4 \tan(1/2 a)^4 + 5 b^5 c d^3 x^4 \tan(1/2 b x)^4 \tan(1/2 a)^4 + 2 b^5 d^4 x^5 \tan(1/2 b x)^4 \tan(1/2 a)^2 + 2 b^5 d^4 x^5 \tan(1/2 b x)^2 \tan(1/2 a)^4 + 10 b^5 c^2 d^2 x^3 \tan(1/2 b x)^4 \tan(1/2 a)^4 + 10 b^5 c d^3 x^4 \tan(1/2 b x)^4 \tan(1/2 a)^2 - 20 b^4 d^4 x^4 \tan(1/2 b x)^4 \tan(1/2 a)^3 + 10 b^5 c d^3 x^4 \tan(1/2 b x)^2 \tan(1/2 a)^4 - 20 b^4 d^4 x^4 \tan(1/2 b x)^3 \tan(1/2 a)^4 + 10 b^5 c^3 d x^2 \tan(1/2 b x)^4 \tan(1/2 a)^4 + b^5 d^4 x^5 \tan(1/2 b x)^4 + 4 b^5 d^4 x^5 \tan(1/2 b x)^2 \tan(1/2 a)^2 + 20 b^5 c^2 d^2 x^3 \tan(1/2 b x)^4 \tan(1/2 a)^2 - 80 b^4 c d^3 x^3 \tan(1/2 b x)^4 \tan(1/2 a)^3 + b^5 d^4 x^5 \tan(1/2 a)^4 + 20 b^5 c^2 d^2 x^3 \tan(1/2 b x)^2 \tan(1/2 a)^4 - 80 b^4 c d^3 x^3 \tan(1/2 b x)^3 \tan(1/2 a)^4 + 5 b^5 c^4 x \tan(1/2 b x)^4 \tan(1/2 a)^4 + 10 b^3 d^4 x^3 \tan(1/2 b x)^4 \tan(1/2 a)^4 + 5 b^5 c d^3 x^4 \tan(1/2 b x)^4 + 20 b^4 d^4 x^4 \tan(1/2 b x)^4 \tan(1/2 a) + 20 b^5 c d^3 x^4 \tan(1/2 b x)^2 \tan(1/2 a)^2 + 120 b^4 d^4 x^4 \tan(1/2 b x)^3 \tan(1/2 a)^2 + 20 b^5 c^3 d x^2 \tan(1/2 b x)^4 \tan(1/2 a)^2 + 120 b^4 d^4 x^4 \tan(1/2 b x)^2 \tan(1/2 a)^3 - 120 b^4 c^2 d^2 x^2 \tan(1/2 b x)^4 \tan(1/2 a)^3 + 5 b^5 c d^3 x^4 \tan(1/2 a)^4 + 20 b^4 d^4 x^4 \tan(1/2 b x) \tan(1/2 a)^4 + 20 b^5 c^3 d x^2 \tan(1/2 b x)^2 \tan(1/2 a)^4 - 120 b^4 c^2 d^2 x^2 \tan(1/2 b x)^3 \tan(1/2 a)^4 + 30 b^3 c d^3 x^2 \tan(1/2 b x)^4 \tan(1/2 a)^4 + 2 b^5 d^4 x^5 \tan(1/2 b x)^2 + 10 b^5 c^2 d^2 x^3 \tan(1/2 b x)^4 + 80 b^4 c d^3 x^3 \tan(1/2 b x)^4 \tan(1/2 a) + 2 b^5 d^4 x^5 \tan(1/2 a)^2 + 40 b^5 c^2 d^2 x^3 \tan(1/2 b x)^2 \tan(1/2 a)^2 + 480 b^4 c d^3 x^3 \tan(1/2 b x)^3 \tan(1/2 a)^2 + 10 b^5 c^4 x \tan(1/2 b x)^4 \tan(1/2 a)^2 - 60 b^3 d^4 x^3 \tan(1/2 b x)^4 \tan(1/2 a)^2 + 480 b^4 c d^3 x^3 \tan(1/2 b x)^2 \tan(1/2 a)^3 - 160 b^3 d^4 x^3 \tan(1/2 b x)^3 \tan(1/2 a)^3 - 80 b^4 c^3 d x \tan(1/2 b x)^4 \tan(1/2 a)^3 + 10 b^5 c^2 d^2 x^3 \tan(1/2 a)^4 + 80 b^4 c d^3 x^3 \tan(1/2 b x) \tan(1/2 a)^4 + 10 b^5 c^4 x \tan(1/2 b x)^2 \tan(1/2 a)^4 - 60 b^3 d^4 x^3 \tan(1/2 b x)^2 \tan(1/2 a)^4 - 80 b^4 c^3 d x \tan(1/2 b x)^3 \tan(1/2 a)^4 + 30 b^3 c^2 d^2 x \tan(1/2 b x)^4 \tan(1/2 a)^4 + 10 b^5 c d^3 x^4 \tan(1/2 b x)^2 - 20 b^4 d^4 x^4 \tan(1/2 b x)^3 + 10 b^5 c^3 d x^2 \tan(1/2 b x)^4 - 120 b^4 d^4 x^4 \tan(1/2 b x)^2 \tan(1/2 a) + 120 b^4 c^2 d^2 x^2 \tan(1/2 b x)^4 \tan(1/2 a) + 10 b^5 c d^3 x^4 \tan(1/2 a)^2 - 120 b^4 d^4 x^4 \tan(1/2 b x) \tan(1/2 a)^2 + 40 b^5 c^3 d x^2 \tan(1/2 b x)^2 \tan(1/2 a)^2 + 720 b^4 c^2 d^2 x^2 \tan(1/2 b x)^3 \tan(1/2 a)^2 - 180 b^3 c d^3 x^2 \tan(1/2 b x)^4 \tan(1/2 a)^2 - 20 b^4 d^4 x^4 \tan(1/2 a)^3 + 720 b^4 c^2 d^2 x^2 \tan(1/2 b x)^2 \tan(1/2 a)^3 - 480 b^3 c d^3 x^2 \tan(1/2 b x)^3 \tan(1/2 a)^3 - 20 b^4 c^4 \tan(1/2 b x)^4 \tan(1/2 a)^3 + 60 b^2 d^4 x^2 \tan(1/2 b x)^4 \tan(1/2 a)^3 + 10 b^5 c^3 d x^2 \tan(1/2 a)^4 + 120 b^4 c^2 d^2 x^2 \tan(1/2 b x) \tan(1/2 a)^4 - 180 b^3 c d^3 x^2 \tan(1/2 b x)^2 \tan(1/2 a)^4 - 20 b^4 c^4 \tan(1/2 b x)^3 \tan(1/2 a)^4 + 60 b^2 d^4 x^2 \tan(1/2 b x)^3 \tan(1/2 a)^4 + 10 b^3 c^3 d \tan(1/2 b x)^4 \tan(1/2 a)^4 + b^5 d^4 x^5 + 20 b^5 c^2 d^2 x^3 \tan(1/2 b x)^2 - 80 b^4 c d^3 x^3 \tan(1/2 b x)^3 + 5 b^5 c^4 x \tan(1/2 b x)^4 + 10 b^3 d^4 x^3 \tan(1/2 b x)^4 - 480 b^4 c d^3 x^3 \tan(1/2 b x)^2 \tan(1/2 a) + 160 b^3 d^4 x^3 \tan(1/2 b x)^3 \tan(1/2 a) + 80 b^4 c^3 d x \tan(1/2 b x)^4 \tan(1/2 a) + 20 b^5 c^2 d^2 x^3 \tan(1/2 a)^2 - 480 b^4 c d^3 x^3 \tan(1/2 b x) \tan(1/2 a)^2 + 20 b^5 c^4 x \tan(1/2 b x)^2 \tan(1/2 a)^2 + 360 b^3 d^4 x^3 \tan(1/2 b x)^2 \tan(1/2 a)^2 + 480 b^4 c^3 d x \tan(1/2 b x)^3 \tan(1/2 a)^2 - 180 b^3 c^2 d^2 x \tan(1/2 b x)^4 \tan(1/2 a)^2 - 80 b^4 \end{aligned}$$

$$\begin{aligned}
& *c*d^3*x^3*\tan(1/2*a)^3 + 160*b^3*d^4*x^3*\tan(1/2*b*x)*\tan(1/2*a)^3 + 480*b^4*c^3*d*x*\tan(1/2*b*x)^2*\tan(1/2*a)^3 - 480*b^3*c^2*d^2*x*\tan(1/2*b*x)^3*\tan(1/2*a)^3 + 120*b^2*c*d^3*x*\tan(1/2*b*x)^4*\tan(1/2*a)^3 + 5*b^5*c^4*x*\tan(1/2*a)^4 + 10*b^3*d^4*x^3*\tan(1/2*a)^4 + 80*b^4*c^3*d*x*\tan(1/2*b*x)*\tan(1/2*a)^4 - 180*b^3*c^2*d^2*x*\tan(1/2*b*x)^2*\tan(1/2*a)^4 + 120*b^2*c*d^3*x*\tan(1/2*b*x)^3*\tan(1/2*a)^4 - 15*b*d^4*x*\tan(1/2*b*x)^4*\tan(1/2*a)^4 + 5*b^5*c*d^3*x^4 + 20*b^4*d^4*x^4*\tan(1/2*b*x) + 20*b^5*c^3*d*x^2*\tan(1/2*b*x)^2 - 120*b^4*c^2*d^2*x^2*\tan(1/2*b*x)^3 + 30*b^3*c*d^3*x^2*\tan(1/2*b*x)^4 + 20*b^4*d^4*x^4*\tan(1/2*a) - 720*b^4*c^2*d^2*x^2*\tan(1/2*b*x)^2*\tan(1/2*a) + 480*b^3*c*d^3*x^2*\tan(1/2*b*x)^3*\tan(1/2*a) + 20*b^4*c^4*\tan(1/2*b*x)^4*\tan(1/2*a) - 60*b^2*d^4*x^2*\tan(1/2*b*x)^4*\tan(1/2*a) + 20*b^5*c^3*d*x^2*\tan(1/2*a)^2 - 720*b^4*c^2*d^2*x^2*\tan(1/2*b*x)*\tan(1/2*a)^2 + 1080*b^3*c*d^3*x^2*\tan(1/2*b*x)^2*\tan(1/2*a)^2 + 120*b^4*c^4*\tan(1/2*b*x)^3*\tan(1/2*a)^2 - 360*b^2*d^4*x^2*\tan(1/2*b*x)^3*\tan(1/2*a)^2 - 60*b^3*c^3*d*\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 120*b^4*c^2*d^2*x^2*\tan(1/2*a)^3 + 480*b^3*c*d^3*x^2*\tan(1/2*b*x)*\tan(1/2*a)^3 + 120*b^4*c^4*\tan(1/2*b*x)^2*\tan(1/2*a)^3 - 360*b^2*d^4*x^2*\tan(1/2*b*x)^2*\tan(1/2*a)^3 - 160*b^3*c^3*d*\tan(1/2*b*x)^3*\tan(1/2*a)^3 + 60*b^2*c^2*d^2*\tan(1/2*b*x)^4*\tan(1/2*a)^3 + 30*b^3*c*d^3*x^2*\tan(1/2*a)^4 + 20*b^4*c^4*\tan(1/2*b*x)*\tan(1/2*a)^4 - 60*b^2*d^4*x^2*\tan(1/2*b*x)*\tan(1/2*a)^4 - 60*b^3*c^3*d*\tan(1/2*b*x)^2*\tan(1/2*a)^4 + 60*b^2*c^2*d^2*\tan(1/2*b*x)^3*\tan(1/2*a)^4 - 15*b*c*d^3*\tan(1/2*b*x)^4*\tan(1/2*a)^4 + 10*b^5*c^2*d^2*x^3 + 80*b^4*c*d^3*x^3*\tan(1/2*b*x) + 10*b^5*c^4*x*\tan(1/2*b*x)^2 - 60*b^3*d^4*x^3*\tan(1/2*b*x)^2 - 80*b^4*c^3*d*x*\tan(1/2*b*x)^3 + 30*b^3*c^2*d^2*x*\tan(1/2*b*x)^4 + 80*b^4*c*d^3*x^3*\tan(1/2*a) - 160*b^3*d^4*x^3*\tan(1/2*b*x)*\tan(1/2*a) - 480*b^4*c^3*d*x*\tan(1/2*b*x)^2*\tan(1/2*a) + 480*b^3*c^2*d^2*x*\tan(1/2*b*x)^3*\tan(1/2*a) - 120*b^2*c*d^3*x*\tan(1/2*b*x)^4*\tan(1/2*a) + 10*b^5*c^4*x*\tan(1/2*a)^2 - 60*b^3*d^4*x^3*\tan(1/2*a)^2 - 480*b^4*c^3*d*x*\tan(1/2*b*x)*\tan(1/2*a)^2 + 1080*b^3*c^2*d^2*x*\tan(1/2*b*x)^2*\tan(1/2*a)^2 - 720*b^2*c*d^3*x*\tan(1/2*b*x)^3*\tan(1/2*a)^2 + 90*b*d^4*x*\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 80*b^4*c^3*d*x*\tan(1/2*a)^3 + 480*b^3*c^2*d^2*x*\tan(1/2*b*x)*\tan(1/2*a)^3 - 720*b^2*c*d^3*x*\tan(1/2*b*x)^2*\tan(1/2*a)^3 + 240*b*d^4*x*\tan(1/2*b*x)^3*\tan(1/2*a)^3 + 30*b^3*c^2*d^2*x*\tan(1/2*a)^4 - 120*b^2*c*d^3*x*\tan(1/2*b*x)*\tan(1/2*a)^4 + 90*b*d^4*x*\tan(1/2*b*x)^2*\tan(1/2*a)^4 + 10*b^5*c^3*d*x^2 + 120*b^4*c^2*d^2*x^2*\tan(1/2*b*x) - 180*b^3*c*d^3*x^2*\tan(1/2*b*x)^2 - 20*b^4*c^4*\tan(1/2*b*x)^3 + 60*b^2*d^4*x^2*\tan(1/2*b*x)^3 + 10*b^3*c^3*d*\tan(1/2*b*x)^4 + 120*b^4*c^2*d^2*x^2*\tan(1/2*a) - 480*b^3*c*d^3*x^2*\tan(1/2*b*x)*\tan(1/2*a) - 120*b^4*c^4*\tan(1/2*b*x)^2*\tan(1/2*a) + 360*b^2*d^4*x^2*\tan(1/2*b*x)^2*\tan(1/2*a) + 160*b^3*c^3*d*\tan(1/2*b*x)^3*\tan(1/2*a) - 60*b^2*c^2*d^2*\tan(1/2*b*x)^4*\tan(1/2*a) - 180*b^3*c*d^3*x^2*\tan(1/2*a)^2 - 120*b^4*c^4*\tan(1/2*b*x)*\tan(1/2*a)^2 + 360*b^2*d^4*x^2*\tan(1/2*b*x)*\tan(1/2*a)^2 + 360*b^3*c^3*d*\tan(1/2*b*x)^2*\tan(1/2*a)^2 - 360*b^2*c^2*d^2*\tan(1/2*b*x)^3*\tan(1/2*a)^2 + 90*b*c*d^3*\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 20*b^4*c^4*\tan(1/2*a)^3 + 60*b^2*d^4*x^2*\tan(1/2*a)^3 + 160*b^3*c^3*d*\tan(1/2*b*x)*\tan(1/2*a)^3 - 360*b^2*c^2*d^2*\tan(1/2*b*x)^2*\tan(1/2*a)^3 + 240*b*c*d^3*\tan(1/2*b*x)^3*\tan(1/2*a)^3 - 30*d^4*\tan(1/2*b*x)^4*\tan(1/2*a)^3 + 10*b^3*c^3*d*\tan(1/2*a)^4 - 60*b^2*c^2*d^2*\tan(1/2*b*x)*\tan(1/2*a)^4 + 90*b*c*d^3*\tan(1/2*b*x)^2*\tan(1/2*a)^4 - 30*d^4*\tan(1/2*b*x)^3*\tan(1/2*a)^4 + 5*b^5*c^4*x + 10*b^3*d^4*x^3 + 80*b^4*c^3*d*x*\tan(1/2*b*x) - 180*b^3*c^2*d^2*x*\tan(1/2*b*x)^2 + 120*b^2*c*d^3*x*\tan(1/2*b*x)^3 - 15*b*d^4*x*\tan(1/2*b*x)^4 + 80*b^4*c^3*d*x*\tan(1/2*a) - 480*b^3*c^2*d^2*x*\tan(1/2*b*x)*\tan(1/2*a) + 720*b^2*c*d^3*x*\tan(1/2*b*x)^2*\tan(1/2*a) - 240*b*d^4*x*\tan(1/2*b*x)^3*\tan(1/2*a) - 180*b^3*c^2*d^2*x*\tan(1/2*a)^2 + 720*b^2*c*d^3*x*\tan(1/2*b*x)*\tan(1/2*a)^2 - 540*b*d^4*x*\tan(1/2*b*x)^2*\tan(1/2*a)^2 + 120*b^2*c*d^3*x*\tan(1/2*a)^3 - 240*b*d^4*x*\tan(1/2*b*x)*\tan(1/2*a)^3 - 15*b*d^4*x*\tan(1/2*a)^4 + 30*b^3*c*d^3*x^2 + 20*b^4*c^4*\tan(1/2*b*x) - 60*b^2*d^4*x^2*\tan(1/2*b*x) - 60*b^3*c^3*d*\tan(1/2*b*x)^2 + 60*b^2*c^2*d^2*\tan(1/2*b*x)^3 - 15*b*c*d^3*\tan(1/2*b*x)^4 + 20*b^4*c^4*\tan(1/2*a) - 60*b^2*d^4*x^2*\tan(1/2*a) - 160*b^3*c^3*d*\tan(1/2*b*x)*\tan(1/2*a) + 360*b^2*c^2*d^2*\tan(1/2*b*x)^2*\tan(1/2*a) - 240*b*c*d^3*\tan(1/2*b*x)^3*\tan(1/2*a) + 30*d^4*\tan(1/2*b*x)^4*\tan(1/2*a) - 60*b^3
\end{aligned}$$

$$\begin{aligned}
& *c^3*d*\tan(1/2*a)^2 + 360*b^2*c^2*d^2*\tan(1/2*b*x)*\tan(1/2*a)^2 - 540*b*c*d \\
& ^3*\tan(1/2*b*x)^2*\tan(1/2*a)^2 + 180*d^4*\tan(1/2*b*x)^3*\tan(1/2*a)^2 + 60*b \\
& ^2*c^2*d^2*\tan(1/2*a)^3 - 240*b*c*d^3*\tan(1/2*b*x)*\tan(1/2*a)^3 + 180*d^4*t \\
& \tan(1/2*b*x)^2*\tan(1/2*a)^3 - 15*b*c*d^3*\tan(1/2*a)^4 + 30*d^4*\tan(1/2*b*x)* \\
& \tan(1/2*a)^4 + 30*b^3*c^2*d^2*x - 120*b^2*c*d^3*x*\tan(1/2*b*x) + 90*b*d^4*x \\
& *\tan(1/2*b*x)^2 - 120*b^2*c*d^3*x*\tan(1/2*a) + 240*b*d^4*x*\tan(1/2*b*x)*\tan \\
& (1/2*a) + 90*b*d^4*x*\tan(1/2*a)^2 + 10*b^3*c^3*d - 60*b^2*c^2*d^2*\tan(1/2*b \\
& *x) + 90*b*c*d^3*\tan(1/2*b*x)^2 - 30*d^4*\tan(1/2*b*x)^3 - 60*b^2*c^2*d^2*ta \\
& n(1/2*a) + 240*b*c*d^3*\tan(1/2*b*x)*\tan(1/2*a) - 180*d^4*\tan(1/2*b*x)^2*\tan \\
& (1/2*a) + 90*b*c*d^3*\tan(1/2*a)^2 - 180*d^4*\tan(1/2*b*x)*\tan(1/2*a)^2 - 30* \\
& d^4*\tan(1/2*a)^3 - 15*b*d^4*x - 15*b*c*d^3 + 30*d^4*\tan(1/2*b*x) + 30*d^4*t \\
& \tan(1/2*a))/(b^5*\tan(1/2*b*x)^4*\tan(1/2*a)^4 + 2*b^5*\tan(1/2*b*x)^4*\tan(1/2* \\
& a)^2 + 2*b^5*\tan(1/2*b*x)^2*\tan(1/2*a)^4 + b^5*\tan(1/2*b*x)^4 + 4*b^5*\tan(1 \\
& /2*b*x)^2*\tan(1/2*a)^2 + b^5*\tan(1/2*a)^4 + 2*b^5*\tan(1/2*b*x)^2 + 2*b^5*ta \\
& n(1/2*a)^2 + b^5)
\end{aligned}$$

3.369 $\int (c + dx)^3 \csc(a + bx) \sin(3a + 3bx) dx$

Optimal. Leaf size=171

$$\frac{3d^2(c + dx) \sin(a + bx) \cos(a + bx)}{b^3} - \frac{3d(c + dx)^2 \sin^2(a + bx)}{4b^2} + \frac{9d(c + dx)^2 \cos^2(a + bx)}{4b^2} + \frac{3d^3 \sin^2(a + bx)}{8b^4} - \frac{9d^3}{8b^4}$$

[Out] $(-3*c*d^2*x)/(2*b^2) - (3*d^3*x^2)/(4*b^2) + (c + d*x)^4/(4*d) - (9*d^3*\text{Cos}[a + b*x]^2)/(8*b^4) + (9*d*(c + d*x)^2*\text{Cos}[a + b*x]^2)/(4*b^2) - (3*d^2*(c + d*x)*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b^3 + (2*(c + d*x)^3*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b + (3*d^3*\text{Sin}[a + b*x]^2)/(8*b^4) - (3*d*(c + d*x)^2*\text{Sin}[a + b*x]^2)/(4*b^2)$

Rubi [A] time = 0.183662, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4431, 3311, 32, 3310}

$$\frac{3d^2(c + dx) \sin(a + bx) \cos(a + bx)}{b^3} - \frac{3d(c + dx)^2 \sin^2(a + bx)}{4b^2} + \frac{9d(c + dx)^2 \cos^2(a + bx)}{4b^2} + \frac{3d^3 \sin^2(a + bx)}{8b^4} - \frac{9d^3}{8b^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^3*\text{Csc}[a + b*x]*\text{Sin}[3*a + 3*b*x],x]$

[Out] $(-3*c*d^2*x)/(2*b^2) - (3*d^3*x^2)/(4*b^2) + (c + d*x)^4/(4*d) - (9*d^3*\text{Cos}[a + b*x]^2)/(8*b^4) + (9*d*(c + d*x)^2*\text{Cos}[a + b*x]^2)/(4*b^2) - (3*d^2*(c + d*x)*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b^3 + (2*(c + d*x)^3*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b + (3*d^3*\text{Sin}[a + b*x]^2)/(8*b^4) - (3*d*(c + d*x)^2*\text{Sin}[a + b*x]^2)/(4*b^2)$

Rule 4431

$\text{Int}[(e_.) + (f_.)*(x_)]^{(m_.)}*(F_)[(a_.) + (b_.)*(x_)]^{(p_.)}*(G_)[(c_.) + (d_.)*(x_)]^{(q_.)}, x_Symbol] := \text{Int}[\text{ExpandTrigExpand}[(e + f*x)^m*G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /;$ FreeQ[{a, b, c, d, e, f, m}, x] && MemberQ[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]

Rule 3311

$\text{Int}[(c_.) + (d_.)*(x_)]^{(m_.)}*((b_.)*\text{sin}[(e_.) + (f_.)*(x_)])^{(n_.)}, x_Symbol] := \text{Simp}[(d*m*(c + d*x)^{(m-1)}*(b*\text{Sin}[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n-1))/n, \text{Int}[(c + d*x)^m*(b*\text{Sin}[e + f*x])^{(n-2)}, x], x] - \text{Dist}[(d^2*m*(m-1))/(f^2*n^2), \text{Int}[(c + d*x)^{(m-2)}*(b*\text{Sin}[e + f*x])^n, x], x] - \text{Simp}[(b*(c + d*x)^m*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(n-1)})/(f*n), x]) /;$ FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rule 32

$\text{Int}[(a_.) + (b_.)*(x_)]^{(m_.)}, x_Symbol] := \text{Simp}[(a + b*x)^{(m+1)}/(b*(m+1)), x] /;$ FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 3310

$\text{Int}[(c_.) + (d_.)*(x_)]*((b_.)*\text{sin}[(e_.) + (f_.)*(x_)])^{(n_.)}, x_Symbol] := \text{Simp}[(d*(b*\text{Sin}[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n-1))/n, \text{Int}[(c + d*x)*(b*\text{Sin}[e + f*x])^{(n-2)}, x], x] - \text{Simp}[(b*(c + d*x)*\text{Cos}[e + f*x]*(b$

`*Sin[e + f*x]]^(n - 1))/(f*n), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]`

Rubi steps

$$\begin{aligned}
 \int (c + dx)^3 \csc(a + bx) \sin(3a + 3bx) dx &= \int (3(c + dx)^3 \cos^2(a + bx) - (c + dx)^3 \sin^2(a + bx)) dx \\
 &= 3 \int (c + dx)^3 \cos^2(a + bx) dx - \int (c + dx)^3 \sin^2(a + bx) dx \\
 &= \frac{9d(c + dx)^2 \cos^2(a + bx)}{4b^2} + \frac{2(c + dx)^3 \cos(a + bx) \sin(a + bx)}{b} - \frac{3d(c + dx)^2 \sin^2(a + bx)}{4b} \\
 &= \frac{(c + dx)^4}{4d} - \frac{9d^3 \cos^2(a + bx)}{8b^4} + \frac{9d(c + dx)^2 \cos^2(a + bx)}{4b^2} - \frac{3d^2(c + dx) \cos(a + bx) \sin(a + bx)}{b} \\
 &= -\frac{3cd^2x}{2b^2} - \frac{3d^3x^2}{4b^2} + \frac{(c + dx)^4}{4d} - \frac{9d^3 \cos^2(a + bx)}{8b^4} + \frac{9d(c + dx)^2 \cos^2(a + bx)}{4b^2}
 \end{aligned}$$

Mathematica [A] time = 0.414968, size = 105, normalized size = 0.61

$$\frac{2b(c + dx) \sin(2(a + bx)) (2b^2(c + dx)^2 - 3d^2) + 3d \cos(2(a + bx)) (2b^2(c + dx)^2 - d^2) + b^4x (6c^2dx + 4c^3 + 4cd^2x^2 + d^3x^3)}{4b^4}$$

Antiderivative was successfully verified.

[In] `Integrate[(c + d*x)^3*Csc[a + b*x]*Sin[3*a + 3*b*x], x]`

[Out] `(b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 3*d*(-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 2*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(4*b^4)`

Maple [B] time = 0.043, size = 580, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a), x)`

[Out] `-c^3*x-1/4*d^3*x^4+4*c^3/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/2*c^2*d*x^2-c*d^2*x^3+4*d^3/b^4*((b*x+a)^3*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)^2*cos(b*x+a)^2-3/2*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/8*(b*x+a)^2+3/8*sin(b*x+a)^2-3/8*(b*x+a)^4-3*a*((b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3)+3*a^2*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)-a^3*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))+12*c^2*d/b^2*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)-a*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))+12*d^2*c/b^3*((b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3-2*a*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+a^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))`

Maxima [A] time = 1.12931, size = 234, normalized size = 1.37

$$\frac{(bx + \sin(2bx + 2a))c^3}{b} + \frac{3(b^2x^2 + 2bx \sin(2bx + 2a) + \cos(2bx + 2a))c^2d}{2b^2} + \frac{(2b^3x^3 + 6bx \cos(2bx + 2a) + 3 \cos(2bx + 2a))c^2d}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")

[Out] (b*x + sin(2*b*x + 2*a))*c^3/b + 3/2*(b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*c^2*d/b^2 + 1/2*(2*b^3*x^3 + 6*b*x*cos(2*b*x + 2*a) + 3*(2*b^2*x^2 - 1)*sin(2*b*x + 2*a))*c*d^2/b^3 + 1/4*(b^4*x^4 + 3*(2*b^2*x^2 - 1)*cos(2*b*x + 2*a) + 2*(2*b^3*x^3 - 3*b*x)*sin(2*b*x + 2*a))*d^3/b^4

Fricas [A] time = 0.504972, size = 389, normalized size = 2.27

$$\frac{b^4d^3x^4 + 4b^4cd^2x^3 + 6(b^4c^2d - b^2d^3)x^2 + 6(2b^2d^3x^2 + 4b^2cd^2x + 2b^2c^2d - d^3)\cos(bx + a)^2 + 4(2b^3d^3x^3 + 6b^3cd^2x^2 + 6b^3c^2d^2x + 2b^3c^3 - 3b^2cd^2 + 3(2b^3c^2d - b^2d^3)x)\sin(bx + a) + 4(b^4c^3 - 3b^2cd^2)x}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")

[Out] 1/4*(b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 6*(b^4*c^2*d - b^2*d^3)*x^2 + 6*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 4*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^2*d - b*d^3)*x)*sin(b*x + a) + 4*(b^4*c^3 - 3*b^2*c*d^2)*x/b^4

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*csc(b*x+a)*sin(3*b*x+3*a),x)

[Out] Timed out

Giac [B] time = 1.46399, size = 4238, normalized size = 24.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")

[Out] 1/4*(b^4*d^3*x^4*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b^4*c*d^2*x^3*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^4*d^3*x^4*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^4*d^3*x^4*tan(1/2*b*x)^2*tan(1/2*a)^4 + 6*b^4*c^2*d*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*b^4*c*d^2*x^3*tan(1/2*b*x)^4*tan(1/2*a)^2 - 16*b^3*d^3*x^3*tan(1/2*b*x)^4*tan(1/2*a)^3 + 8*b^4*c*d^2*x^3*tan(1/2*b*x)^2*tan(1/2*a)^4 - 16*b^3*d^3*x^3*tan(1/2*b*x)^2*tan(1/2*a)^3)

$$\begin{aligned}
& x^3 \tan(1/2 * b * x)^3 \tan(1/2 * a)^4 + 4 * b^4 * c^3 * x * \tan(1/2 * b * x)^4 \tan(1/2 * a)^4 + \\
& b^4 * d^3 * x^4 \tan(1/2 * b * x)^4 + 4 * b^4 * d^3 * x^4 \tan(1/2 * b * x)^2 \tan(1/2 * a)^2 + 1 \\
& 2 * b^4 * c^2 * d * x^2 \tan(1/2 * b * x)^4 \tan(1/2 * a)^2 - 48 * b^3 * c * d^2 * x^2 \tan(1/2 * b * x) \\
& ^4 \tan(1/2 * a)^3 + b^4 * d^3 * x^4 \tan(1/2 * a)^4 + 12 * b^4 * c^2 * d * x^2 \tan(1/2 * b * x)^ \\
& 2 \tan(1/2 * a)^4 - 48 * b^3 * c * d^2 * x^2 \tan(1/2 * b * x)^3 \tan(1/2 * a)^4 + 6 * b^2 * d^3 * x \\
& ^2 \tan(1/2 * b * x)^4 \tan(1/2 * a)^4 + 4 * b^4 * c * d^2 * x^3 \tan(1/2 * b * x)^4 + 16 * b^3 * d^ \\
& 3 * x^3 \tan(1/2 * b * x)^4 \tan(1/2 * a) + 16 * b^4 * c * d^2 * x^3 \tan(1/2 * b * x)^2 \tan(1/2 * a \\
&)^2 + 96 * b^3 * d^3 * x^3 \tan(1/2 * b * x)^3 \tan(1/2 * a)^2 + 8 * b^4 * c^3 * x * \tan(1/2 * b * x) \\
& ^4 \tan(1/2 * a)^2 + 96 * b^3 * d^3 * x^3 \tan(1/2 * b * x)^2 \tan(1/2 * a)^3 - 48 * b^3 * c^2 * d \\
& * x * \tan(1/2 * b * x)^4 \tan(1/2 * a)^3 + 4 * b^4 * c * d^2 * x^3 \tan(1/2 * a)^4 + 16 * b^3 * d^3 * \\
& x^3 \tan(1/2 * b * x) * \tan(1/2 * a)^4 + 8 * b^4 * c^3 * x * \tan(1/2 * b * x)^2 \tan(1/2 * a)^4 - 4 \\
& 8 * b^3 * c^2 * d * x * \tan(1/2 * b * x)^3 \tan(1/2 * a)^4 + 12 * b^2 * c * d^2 * x * \tan(1/2 * b * x)^4 * t \\
& an(1/2 * a)^4 + 2 * b^4 * d^3 * x^4 \tan(1/2 * b * x)^2 + 6 * b^4 * c^2 * d * x^2 \tan(1/2 * b * x)^4 \\
& + 48 * b^3 * c * d^2 * x^2 \tan(1/2 * b * x)^4 \tan(1/2 * a) + 2 * b^4 * d^3 * x^4 \tan(1/2 * a)^2 \\
& + 24 * b^4 * c^2 * d * x^2 \tan(1/2 * b * x)^2 \tan(1/2 * a)^2 + 288 * b^3 * c * d^2 * x^2 \tan(1/2 * \\
& b * x)^3 \tan(1/2 * a)^2 - 36 * b^2 * d^3 * x^2 \tan(1/2 * b * x)^4 \tan(1/2 * a)^2 + 288 * b^3 * c \\
& * d^2 * x^2 \tan(1/2 * b * x)^2 \tan(1/2 * a)^3 - 96 * b^2 * d^3 * x^2 \tan(1/2 * b * x)^3 \tan(1 \\
& /2 * a)^3 - 16 * b^3 * c^3 * \tan(1/2 * b * x)^4 \tan(1/2 * a)^3 + 6 * b^4 * c^2 * d * x^2 \tan(1/2 * \\
& a)^4 + 48 * b^3 * c * d^2 * x^2 \tan(1/2 * b * x) * \tan(1/2 * a)^4 - 36 * b^2 * d^3 * x^2 \tan(1/2 * \\
& b * x)^2 \tan(1/2 * a)^4 - 16 * b^3 * c^3 * \tan(1/2 * b * x)^3 \tan(1/2 * a)^4 + 6 * b^2 * c^2 * d * \\
& \tan(1/2 * b * x)^4 \tan(1/2 * a)^4 + 8 * b^4 * c * d^2 * x^3 \tan(1/2 * b * x)^2 - 16 * b^3 * d^3 * x \\
& ^3 \tan(1/2 * b * x)^3 + 4 * b^4 * c^3 * x * \tan(1/2 * b * x)^4 - 96 * b^3 * d^3 * x^3 \tan(1/2 * b * x \\
&)^2 \tan(1/2 * a) + 48 * b^3 * c^2 * d * x * \tan(1/2 * b * x)^4 \tan(1/2 * a) + 8 * b^4 * c * d^2 * x^3 \\
& * \tan(1/2 * a)^2 - 96 * b^3 * d^3 * x^3 \tan(1/2 * b * x) * \tan(1/2 * a)^2 + 16 * b^4 * c^3 * x * \tan \\
& (1/2 * b * x)^2 \tan(1/2 * a)^2 + 288 * b^3 * c^2 * d * x * \tan(1/2 * b * x)^3 \tan(1/2 * a)^2 - 72 \\
& * b^2 * c * d^2 * x * \tan(1/2 * b * x)^4 \tan(1/2 * a)^2 - 16 * b^3 * d^3 * x^3 \tan(1/2 * a)^3 + 28 \\
& 8 * b^3 * c^2 * d * x * \tan(1/2 * b * x)^2 \tan(1/2 * a)^3 - 192 * b^2 * c * d^2 * x * \tan(1/2 * b * x)^3 * \\
& \tan(1/2 * a)^3 + 24 * b * d^3 * x * \tan(1/2 * b * x)^4 \tan(1/2 * a)^3 + 4 * b^4 * c^3 * x * \tan(1/2 \\
& * a)^4 + 48 * b^3 * c^2 * d * x * \tan(1/2 * b * x) * \tan(1/2 * a)^4 - 72 * b^2 * c * d^2 * x * \tan(1/2 * b \\
& * x)^2 \tan(1/2 * a)^4 + 24 * b * d^3 * x * \tan(1/2 * b * x)^3 \tan(1/2 * a)^4 + b^4 * d^3 * x^4 + \\
& 12 * b^4 * c^2 * d * x^2 \tan(1/2 * b * x)^2 - 48 * b^3 * c * d^2 * x^2 \tan(1/2 * b * x)^3 + 6 * b^2 * \\
& d^3 * x^2 \tan(1/2 * b * x)^4 - 288 * b^3 * c * d^2 * x^2 \tan(1/2 * b * x)^2 \tan(1/2 * a) + 96 * b \\
& ^2 * d^3 * x^2 \tan(1/2 * b * x)^3 \tan(1/2 * a) + 16 * b^3 * c^3 * \tan(1/2 * b * x)^4 \tan(1/2 * a) \\
& + 12 * b^4 * c^2 * d * x^2 \tan(1/2 * a)^2 - 288 * b^3 * c * d^2 * x^2 \tan(1/2 * b * x) * \tan(1/2 * a \\
&)^2 + 216 * b^2 * d^3 * x^2 \tan(1/2 * b * x)^2 \tan(1/2 * a)^2 + 96 * b^3 * c^3 * \tan(1/2 * b * x) \\
& ^3 \tan(1/2 * a)^2 - 36 * b^2 * c^2 * d * \tan(1/2 * b * x)^4 \tan(1/2 * a)^2 - 48 * b^3 * c * d^2 * x \\
& ^2 \tan(1/2 * a)^3 + 96 * b^2 * d^3 * x^2 \tan(1/2 * b * x) * \tan(1/2 * a)^3 + 96 * b^3 * c^3 * \tan \\
& (1/2 * b * x)^2 \tan(1/2 * a)^3 - 96 * b^2 * c^2 * d * \tan(1/2 * b * x)^3 \tan(1/2 * a)^3 + 24 * b * \\
& c * d^2 * \tan(1/2 * b * x)^4 \tan(1/2 * a)^3 + 6 * b^2 * d^3 * x^2 \tan(1/2 * a)^4 + 16 * b^3 * c^3 \\
& * \tan(1/2 * b * x) * \tan(1/2 * a)^4 - 36 * b^2 * c^2 * d * \tan(1/2 * b * x)^2 \tan(1/2 * a)^4 + 24 * \\
& b * c * d^2 * \tan(1/2 * b * x)^3 \tan(1/2 * a)^4 - 3 * d^3 * \tan(1/2 * b * x)^4 \tan(1/2 * a)^4 + 4 \\
& * b^4 * c * d^2 * x^3 + 16 * b^3 * d^3 * x^3 \tan(1/2 * b * x) + 8 * b^4 * c^3 * x * \tan(1/2 * b * x)^2 - \\
& 48 * b^3 * c^2 * d * x * \tan(1/2 * b * x)^3 + 12 * b^2 * c * d^2 * x * \tan(1/2 * b * x)^4 + 16 * b^3 * d^3 \\
& * x^3 \tan(1/2 * a) - 288 * b^3 * c^2 * d * x * \tan(1/2 * b * x)^2 \tan(1/2 * a) + 192 * b^2 * c * d^2 \\
& * x * \tan(1/2 * b * x)^3 \tan(1/2 * a) - 24 * b * d^3 * x * \tan(1/2 * b * x)^4 \tan(1/2 * a) + 8 * b^4 \\
& * c^3 * x * \tan(1/2 * a)^2 - 288 * b^3 * c^2 * d * x * \tan(1/2 * b * x) * \tan(1/2 * a)^2 + 432 * b^2 * c \\
& * d^2 * x * \tan(1/2 * b * x)^2 \tan(1/2 * a)^2 - 144 * b * d^3 * x * \tan(1/2 * b * x)^3 \tan(1/2 * a)^ \\
& 2 - 48 * b^3 * c^2 * d * x * \tan(1/2 * a)^3 + 192 * b^2 * c * d^2 * x * \tan(1/2 * b * x) * \tan(1/2 * a)^3 \\
& - 144 * b * d^3 * x * \tan(1/2 * b * x)^2 \tan(1/2 * a)^3 + 12 * b^2 * c * d^2 * x * \tan(1/2 * a)^4 - \\
& 24 * b * d^3 * x * \tan(1/2 * b * x) * \tan(1/2 * a)^4 + 6 * b^4 * c^2 * d * x^2 + 48 * b^3 * c * d^2 * x^2 * t \\
& an(1/2 * b * x) - 36 * b^2 * d^3 * x^2 \tan(1/2 * b * x)^2 - 16 * b^3 * c^3 * \tan(1/2 * b * x)^3 + 6 \\
& * b^2 * c^2 * d * \tan(1/2 * b * x)^4 + 48 * b^3 * c * d^2 * x^2 \tan(1/2 * a) - 96 * b^2 * d^3 * x^2 * ta \\
& n(1/2 * b * x) * \tan(1/2 * a) - 96 * b^3 * c^3 * \tan(1/2 * b * x)^2 \tan(1/2 * a) + 96 * b^2 * c^2 * d \\
& * \tan(1/2 * b * x)^3 \tan(1/2 * a) - 24 * b * c * d^2 * \tan(1/2 * b * x)^4 \tan(1/2 * a) - 36 * b^2 * \\
& d^3 * x^2 \tan(1/2 * a)^2 - 96 * b^3 * c^3 * \tan(1/2 * b * x) * \tan(1/2 * a)^2 + 216 * b^2 * c^2 * d \\
& * \tan(1/2 * b * x)^2 \tan(1/2 * a)^2 - 144 * b * c * d^2 * \tan(1/2 * b * x)^3 \tan(1/2 * a)^2 + 18 \\
& * d^3 * \tan(1/2 * b * x)^4 \tan(1/2 * a)^2 - 16 * b^3 * c^3 * \tan(1/2 * a)^3 + 96 * b^2 * c^2 * d * t \\
& an(1/2 * b * x) * \tan(1/2 * a)^3 - 144 * b * c * d^2 * \tan(1/2 * b * x)^2 \tan(1/2 * a)^3 + 48 * d^3 \\
& * \tan(1/2 * b * x)^3 \tan(1/2 * a)^3 + 6 * b^2 * c^2 * d * \tan(1/2 * a)^4 - 24 * b * c * d^2 * \tan(1/
\end{aligned}$$

$$\begin{aligned}
& 2*b*x)*\tan(1/2*a)^4 + 18*d^3*\tan(1/2*b*x)^2*\tan(1/2*a)^4 + 4*b^4*c^3*x + 48 \\
& *b^3*c^2*d*x*\tan(1/2*b*x) - 72*b^2*c*d^2*x*\tan(1/2*b*x)^2 + 24*b*d^3*x*\tan(\\
& 1/2*b*x)^3 + 48*b^3*c^2*d*x*\tan(1/2*a) - 192*b^2*c*d^2*x*\tan(1/2*b*x)*\tan(1 \\
& /2*a) + 144*b*d^3*x*\tan(1/2*b*x)^2*\tan(1/2*a) - 72*b^2*c*d^2*x*\tan(1/2*a)^2 \\
& + 144*b*d^3*x*\tan(1/2*b*x)*\tan(1/2*a)^2 + 24*b*d^3*x*\tan(1/2*a)^3 + 6*b^2* \\
& d^3*x^2 + 16*b^3*c^3*\tan(1/2*b*x) - 36*b^2*c^2*d*\tan(1/2*b*x)^2 + 24*b*c*d^ \\
& 2*\tan(1/2*b*x)^3 - 3*d^3*\tan(1/2*b*x)^4 + 16*b^3*c^3*\tan(1/2*a) - 96*b^2*c^ \\
& 2*d*\tan(1/2*b*x)*\tan(1/2*a) + 144*b*c*d^2*\tan(1/2*b*x)^2*\tan(1/2*a) - 48*d^ \\
& 3*\tan(1/2*b*x)^3*\tan(1/2*a) - 36*b^2*c^2*d*\tan(1/2*a)^2 + 144*b*c*d^2*\tan(1 \\
& /2*b*x)*\tan(1/2*a)^2 - 108*d^3*\tan(1/2*b*x)^2*\tan(1/2*a)^2 + 24*b*c*d^2*\tan \\
& (1/2*a)^3 - 48*d^3*\tan(1/2*b*x)*\tan(1/2*a)^3 - 3*d^3*\tan(1/2*a)^4 + 12*b^2* \\
& c*d^2*x - 24*b*d^3*x*\tan(1/2*b*x) - 24*b*d^3*x*\tan(1/2*a) + 6*b^2*c^2*d - 2 \\
& 4*b*c*d^2*\tan(1/2*b*x) + 18*d^3*\tan(1/2*b*x)^2 - 24*b*c*d^2*\tan(1/2*a) + 48 \\
& *d^3*\tan(1/2*b*x)*\tan(1/2*a) + 18*d^3*\tan(1/2*a)^2 - 3*d^3)/(b^4*\tan(1/2*b* \\
& x)^4*\tan(1/2*a)^4 + 2*b^4*\tan(1/2*b*x)^4*\tan(1/2*a)^2 + 2*b^4*\tan(1/2*b*x)^ \\
& 2*\tan(1/2*a)^4 + b^4*\tan(1/2*b*x)^4 + 4*b^4*\tan(1/2*b*x)^2*\tan(1/2*a)^2 + b \\
& ^4*\tan(1/2*a)^4 + 2*b^4*\tan(1/2*b*x)^2 + 2*b^4*\tan(1/2*a)^2 + b^4)
\end{aligned}$$

3.370 $\int (c + dx)^2 \csc(a + bx) \sin(3a + 3bx) dx$

Optimal. Leaf size=112

$$-\frac{d(c + dx) \sin^2(a + bx)}{2b^2} + \frac{3d(c + dx) \cos^2(a + bx)}{2b^2} - \frac{d^2 \sin(a + bx) \cos(a + bx)}{b^3} + \frac{2(c + dx)^2 \sin(a + bx) \cos(a + bx)}{b} - \frac{d}{2}$$

[Out] $-(d^2*x)/(2*b^2) + (c + d*x)^3/(3*d) + (3*d*(c + d*x)*\text{Cos}[a + b*x]^2)/(2*b^2) - (d^2*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b^3 + (2*(c + d*x)^2*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b - (d*(c + d*x)*\text{Sin}[a + b*x]^2)/(2*b^2)$

Rubi [A] time = 0.13595, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4431, 3311, 32, 2635, 8}

$$-\frac{d(c + dx) \sin^2(a + bx)}{2b^2} + \frac{3d(c + dx) \cos^2(a + bx)}{2b^2} - \frac{d^2 \sin(a + bx) \cos(a + bx)}{b^3} + \frac{2(c + dx)^2 \sin(a + bx) \cos(a + bx)}{b} - \frac{d}{2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^2*\text{Csc}[a + b*x]*\text{Sin}[3*a + 3*b*x],x]$

[Out] $-(d^2*x)/(2*b^2) + (c + d*x)^3/(3*d) + (3*d*(c + d*x)*\text{Cos}[a + b*x]^2)/(2*b^2) - (d^2*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b^3 + (2*(c + d*x)^2*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b - (d*(c + d*x)*\text{Sin}[a + b*x]^2)/(2*b^2)$

Rule 4431

$\text{Int}[(e + f*x)^m*(a + b*x)^p*(c + d*x)^q, x] \text{ :> } \text{Int}[\text{ExpandTrigExpand}[(e + f*x)^m*(a + b*x)^p*(c + d*x)^q, x], x] \text{ ; FreeQ}\{a, b, c, d, e, f, m, p, q\}, x \text{ \&\& MemberQ}\{\text{Sin}, \text{Cos}\}, F \text{ \&\& MemberQ}\{\text{Sec}, \text{Csc}\}, G \text{ \&\& IGtQ}\{p, 0\} \text{ \&\& IGtQ}\{q, 0\} \text{ \&\& EQ}\{b*c - a*d, 0\} \text{ \&\& IGtQ}\{b/d, 1\}$

Rule 3311

$\text{Int}[(c + d*x)^m*(b*\text{Sin}[e + f*x])^n, x] \text{ :> } \text{Simp}[(d*m*(c + d*x)^{m-1}*(b*\text{Sin}[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n-1))/n, \text{Int}[(c + d*x)^m*(b*\text{Sin}[e + f*x])^{n-2}, x], x] - \text{Dist}[(d^2*m*(m-1))/(f^2*n^2), \text{Int}[(c + d*x)^{m-2}*(b*\text{Sin}[e + f*x])^n, x], x] - \text{Simp}[(b*(c + d*x)^m*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{n-1})/(f*n), x]) \text{ ; FreeQ}\{b, c, d, e, f\}, x \text{ \&\& GtQ}\{n, 1\} \text{ \&\& GtQ}\{m, 1\}$

Rule 32

$\text{Int}[(a + b*x)^m, x] \text{ :> } \text{Simp}[(a + b*x)^{m+1}/(b*(m+1)), x] \text{ ; FreeQ}\{a, b, m\}, x \text{ \&\& NeQ}\{m, -1\}$

Rule 2635

$\text{Int}[(b*\text{Sin}[c + d*x])^n, x] \text{ :> } -\text{Simp}[(b*\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{n-1})/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{n-2}, x], x] \text{ ; FreeQ}\{b, c, d\}, x \text{ \&\& GtQ}\{n, 1\} \text{ \&\& IntegerQ}\{2*n\}$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
 \int (c + dx)^2 \csc(a + bx) \sin(3a + 3bx) dx &= \int (3(c + dx)^2 \cos^2(a + bx) - (c + dx)^2 \sin^2(a + bx)) dx \\
 &= 3 \int (c + dx)^2 \cos^2(a + bx) dx - \int (c + dx)^2 \sin^2(a + bx) dx \\
 &= \frac{3d(c + dx) \cos^2(a + bx)}{2b^2} + \frac{2(c + dx)^2 \cos(a + bx) \sin(a + bx)}{b} - \frac{d(c + dx) \sin^2(a + bx)}{2b} \\
 &= \frac{(c + dx)^3}{3d} + \frac{3d(c + dx) \cos^2(a + bx)}{2b^2} - \frac{d^2 \cos(a + bx) \sin(a + bx)}{b^3} + \frac{2(c + dx) \sin^2(a + bx)}{2b} \\
 &= -\frac{d^2 x}{2b^2} + \frac{(c + dx)^3}{3d} + \frac{3d(c + dx) \cos^2(a + bx)}{2b^2} - \frac{d^2 \cos(a + bx) \sin(a + bx)}{b^3}
 \end{aligned}$$

Mathematica [A] time = 0.394428, size = 73, normalized size = 0.65

$$\frac{\sin(2(a + bx)) (2b^2(c + dx)^2 - d^2)}{2b^3} + \frac{d(c + dx) \cos(2(a + bx))}{b^2} + c^2 x + cd x^2 + \frac{d^2 x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2*Csc[a + b*x]*Sin[3*a + 3*b*x], x]

[Out] c^2*x + c*d*x^2 + (d^2*x^3)/3 + (d*(c + d*x)*Cos[2*(a + b*x)])/b^2 + ((-d^2 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(2*b^3)

Maple [B] time = 0.041, size = 294, normalized size = 2.6

$$-c^2 x - \frac{d^2 x^3}{3} + 4 \frac{c^2 (1/2 \cos(bx + a) \sin(bx + a) + 1/2 bx + a/2)}{b} - cd x^2 + 4 \frac{d^2 ((bx + a)^2 (1/2 \cos(bx + a) \sin(bx + a) + 1/2 bx + a/2))}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a), x)

[Out] -c^2*x-1/3*d^2*x^3+4*c^2/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-c*d*x^2+4*d^2/b^3*((b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3-2*a*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+a^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))+8*c*d/b^2*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2-a*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))

Maxima [A] time = 1.07337, size = 146, normalized size = 1.3

$$\frac{(bx + \sin(2bx + 2a))c^2}{b} + \frac{(b^2x^2 + 2bx \sin(2bx + 2a) + \cos(2bx + 2a))cd}{b^2} + \frac{(2b^3x^3 + 6bx \cos(2bx + 2a) + 3(2bx + a) \sin(2bx + 2a))d^2}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")

[Out] (b*x + sin(2*b*x + 2*a))*c^2/b + (b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*c*d/b^2 + 1/6*(2*b^3*x^3 + 6*b*x*cos(2*b*x + 2*a) + 3*(2*b^2*x^2 - 1)*sin(2*b*x + 2*a))*d^2/b^3

Fricas [A] time = 0.497992, size = 240, normalized size = 2.14

$$\frac{b^3 d^2 x^3 + 3 b^3 c d x^2 + 6 (b d^2 x + b c d) \cos(b x + a)^2 + 3 (2 b^2 d^2 x^2 + 4 b^2 c d x + 2 b^2 c^2 - d^2) \cos(b x + a) \sin(b x + a) + 3 (b^3 c^2 - 3 b^3)}{3 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")

[Out] 1/3*(b^3*d^2*x^3 + 3*b^3*c*d*x^2 + 6*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(b*x + a)*sin(b*x + a) + 3*(b^3*c^2 - b*d^2)*x)/b^3

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*csc(b*x+a)*sin(3*b*x+3*a),x)

[Out] Timed out

Giac [B] time = 1.3095, size = 2538, normalized size = 22.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")

[Out] 1/3*(b^3*d^2*x^3*tan(1/2*b*x)^4*tan(1/2*a)^4 + 3*b^3*c*d*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^3*d^2*x^3*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^3*d^2*x^3*tan(1/2*b*x)^2*tan(1/2*a)^4 + 3*b^3*c^2*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 6*b^3*c*d*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 - 12*b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^3 + 6*b^3*c*d*x^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 12*b^2*d^2*x^2*tan(1/2*b*x)^3*tan(1/2*a)^4 + b^3*d^2*x^3*tan(1/2*b*x)^4 + 4*b^3*d^2*x^3*tan(1/2*b*x)^2*tan(1/2*a)^2 + 6*b^3*c^2*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 24*b^2*c*d*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + b^3*d^2*x^3*tan(1/2*a)^4 + 6*b^3*c^2*x*tan(1/2*b*x)^2*tan(1/2*a)^4 - 24*b^2*c*d*x*tan(1/2*b*x)^3*tan(1/2*a)^4 + 3*b*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 3*b^3*c*d*x^2*tan(1/2*b*x)^4 + 12*b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a) + 12*b^3*c*d*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 72*b^2*d^2*x^2*tan(1/2*b*x)^3*tan(1/2*a)^2 + 72*b^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a)^3 - 12*b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^3 + 3*b^3*c*d*x^2*tan(1/2*a)^4 + 12*b^2*d^2*x^2*tan(1/2*b*x)*tan(1/2*a)^4 - 12*b^2*c^2*tan(1/2*

$$\begin{aligned}
& b^3 x^3 \tan(1/2 a)^4 + 3 b^2 c d \tan(1/2 b x)^4 \tan(1/2 a)^4 + 2 b^3 d^2 x^3 \tan(1/2 b x)^2 + 3 b^3 c^2 x \tan(1/2 b x)^4 + 24 b^2 c d x \tan(1/2 b x)^4 \tan(1/2 a) + 2 b^3 d^2 x^3 \tan(1/2 a)^2 + 12 b^3 c^2 x \tan(1/2 b x)^2 \tan(1/2 a)^2 + 144 b^2 c d x \tan(1/2 b x)^3 \tan(1/2 a)^2 - 18 b d^2 x \tan(1/2 b x)^4 \tan(1/2 a)^2 + 144 b^2 c d x \tan(1/2 b x)^2 \tan(1/2 a)^3 - 48 b d^2 x \tan(1/2 b x)^3 \tan(1/2 a)^3 + 3 b^3 c^2 x \tan(1/2 a)^4 + 24 b^2 c d x \tan(1/2 b x) \tan(1/2 a)^4 - 18 b d^2 x \tan(1/2 b x)^2 \tan(1/2 a)^4 + 6 b^3 c d x^2 \tan(1/2 b x)^2 - 12 b^2 d^2 x^2 \tan(1/2 b x)^3 - 72 b^2 d^2 x^2 \tan(1/2 b x)^2 \tan(1/2 a) + 12 b^2 c^2 \tan(1/2 b x)^4 \tan(1/2 a) + 6 b^3 c d x^2 \tan(1/2 a)^2 - 72 b^2 d^2 x^2 \tan(1/2 b x) \tan(1/2 a)^2 + 72 b^2 c^2 \tan(1/2 b x)^3 \tan(1/2 a)^2 - 18 b c d \tan(1/2 b x)^4 \tan(1/2 a)^2 - 12 b^2 d^2 x^2 \tan(1/2 a)^3 + 72 b^2 c^2 \tan(1/2 b x)^2 \tan(1/2 a)^3 - 48 b c d \tan(1/2 b x)^3 \tan(1/2 a)^3 + 6 d^2 \tan(1/2 b x)^4 \tan(1/2 a)^3 + 12 b^2 c^2 \tan(1/2 b x) \tan(1/2 a)^4 - 18 b c d \tan(1/2 b x)^2 \tan(1/2 a)^4 + 6 d^2 \tan(1/2 b x)^3 \tan(1/2 a)^4 + b^3 d^2 x^3 + 6 b^3 c^2 x \tan(1/2 b x)^2 - 24 b^2 c d x \tan(1/2 b x)^3 + 3 b d^2 x \tan(1/2 b x)^4 - 144 b^2 c d x \tan(1/2 b x)^2 \tan(1/2 a) + 48 b d^2 x \tan(1/2 b x)^3 \tan(1/2 a) + 6 b^3 c^2 x \tan(1/2 a)^2 - 144 b^2 c d x \tan(1/2 b x) \tan(1/2 a)^2 + 108 b d^2 x \tan(1/2 b x)^2 \tan(1/2 a)^2 - 24 b^2 c d x \tan(1/2 a)^3 + 48 b d^2 x \tan(1/2 b x) \tan(1/2 a)^3 + 3 b d^2 x \tan(1/2 a)^4 + 3 b^3 c d x^2 + 12 b^2 d^2 x^2 \tan(1/2 b x) - 12 b^2 c^2 \tan(1/2 b x)^3 + 3 b c d \tan(1/2 b x)^4 + 12 b^2 d^2 x^2 \tan(1/2 a) - 72 b^2 c^2 \tan(1/2 b x)^2 \tan(1/2 a) + 48 b c d \tan(1/2 b x)^3 \tan(1/2 a) - 6 d^2 \tan(1/2 b x)^4 \tan(1/2 a) - 72 b^2 c^2 \tan(1/2 b x) \tan(1/2 a)^2 + 108 b c d \tan(1/2 b x)^2 \tan(1/2 a)^2 - 36 d^2 \tan(1/2 b x)^3 \tan(1/2 a)^2 - 12 b^2 c^2 \tan(1/2 a)^3 + 48 b c d \tan(1/2 b x) \tan(1/2 a)^3 - 36 d^2 \tan(1/2 b x)^2 \tan(1/2 a)^3 + 3 b c d \tan(1/2 a)^4 - 6 d^2 \tan(1/2 b x) \tan(1/2 a)^4 + 3 b^3 c^2 x + 24 b^2 c d x \tan(1/2 b x) - 18 b d^2 x \tan(1/2 b x)^2 + 24 b^2 c d x \tan(1/2 a) - 48 b d^2 x \tan(1/2 b x) \tan(1/2 a) - 18 b d^2 x \tan(1/2 a)^2 + 12 b^2 c^2 \tan(1/2 b x) - 18 b c d \tan(1/2 b x)^2 + 6 d^2 \tan(1/2 b x)^3 + 12 b^2 c^2 \tan(1/2 a) - 48 b c d \tan(1/2 b x) \tan(1/2 a) + 36 d^2 \tan(1/2 b x)^2 \tan(1/2 a) - 18 b c d \tan(1/2 a)^2 + 36 d^2 \tan(1/2 b x) \tan(1/2 a)^2 + 6 d^2 \tan(1/2 a)^3 + 3 b d^2 x + 3 b c d - 6 d^2 \tan(1/2 b x) - 6 d^2 \tan(1/2 a)) / (b^3 \tan(1/2 b x)^4 \tan(1/2 a)^4 + 2 b^3 \tan(1/2 b x)^4 \tan(1/2 a)^2 + 2 b^3 \tan(1/2 b x)^2 \tan(1/2 a)^4 + b^3 \tan(1/2 b x)^4 + 4 b^3 \tan(1/2 b x)^2 \tan(1/2 a)^2 + b^3 \tan(1/2 a)^4 + 2 b^3 \tan(1/2 b x)^2 + 2 b^3 \tan(1/2 a)^2 + b^3)
\end{aligned}$$

3.371 $\int (c + dx) \csc(a + bx) \sin(3a + 3bx) dx$

Optimal. Leaf size=66

$$-\frac{d \sin^2(a + bx)}{4b^2} + \frac{3d \cos^2(a + bx)}{4b^2} + \frac{2(c + dx) \sin(a + bx) \cos(a + bx)}{b} + cx + \frac{dx^2}{2}$$

[Out] c*x + (d*x^2)/2 + (3*d*Cos[a + b*x]^2)/(4*b^2) + (2*(c + d*x)*Cos[a + b*x]*Sin[a + b*x])/b - (d*Sin[a + b*x]^2)/(4*b^2)

Rubi [A] time = 0.0682019, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4431, 3310}

$$-\frac{d \sin^2(a + bx)}{4b^2} + \frac{3d \cos^2(a + bx)}{4b^2} + \frac{2(c + dx) \sin(a + bx) \cos(a + bx)}{b} + cx + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Csc[a + b*x]*Sin[3*a + 3*b*x], x]

[Out] c*x + (d*x^2)/2 + (3*d*Cos[a + b*x]^2)/(4*b^2) + (2*(c + d*x)*Cos[a + b*x]*Sin[a + b*x])/b - (d*Sin[a + b*x]^2)/(4*b^2)

Rule 4431

Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol] := Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && MemberQ[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && EQ[b*c - a*d, 0] && IGtQ[b/d, 1]

Rule 3310

Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] := Simp[(d*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)*(b*Sine[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b*Sine[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int (c + dx) \csc(a + bx) \sin(3a + 3bx) dx &= \int (3(c + dx) \cos^2(a + bx) - (c + dx) \sin^2(a + bx)) dx \\ &= 3 \int (c + dx) \cos^2(a + bx) dx - \int (c + dx) \sin^2(a + bx) dx \\ &= \frac{3d \cos^2(a + bx)}{4b^2} + \frac{2(c + dx) \cos(a + bx) \sin(a + bx)}{b} - \frac{d \sin^2(a + bx)}{4b^2} - \frac{1}{2} \int (c + dx) \sin^2(a + bx) dx \\ &= cx + \frac{dx^2}{2} + \frac{3d \cos^2(a + bx)}{4b^2} + \frac{2(c + dx) \cos(a + bx) \sin(a + bx)}{b} - \frac{d \sin^2(a + bx)}{4b^2} \end{aligned}$$

Mathematica [A] time = 0.135017, size = 46, normalized size = 0.7

$$\frac{b(2(c + dx) \sin(2(a + bx)) + bx(2c + dx)) + d \cos(2(a + bx))}{2b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Csc[a + b*x]*Sin[3*a + 3*b*x], x]

[Out] (d*cos[2*(a + b*x)] + b*(b*x*(2*c + d*x) + 2*(c + d*x)*Sin[2*(a + b*x)]))/(2*b^2)

Maple [A] time = 0.037, size = 119, normalized size = 1.8

$$-cx - \frac{dx^2}{2} + 4 \frac{c(1/2 \cos(bx + a) \sin(bx + a) + 1/2 bx + a/2)}{b} + 4 \frac{d((bx + a)(1/2 \cos(bx + a) \sin(bx + a) + 1/2 bx + a/2))}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a), x)

[Out] -c*x-1/2*d*x^2+4*c/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+4*d/b^2*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2-a*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))

Maxima [A] time = 1.06442, size = 74, normalized size = 1.12

$$\frac{(bx + \sin(2bx + 2a))c}{b} + \frac{(b^2x^2 + 2bx \sin(2bx + 2a) + \cos(2bx + 2a))d}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a), x, algorithm="maxima")

[Out] (b*x + sin(2*b*x + 2*a))*c/b + 1/2*(b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*d/b^2

Fricas [A] time = 0.484193, size = 132, normalized size = 2.

$$\frac{b^2dx^2 + 2b^2cx + 2d \cos(bx + a)^2 + 4(bdx + bc) \cos(bx + a) \sin(bx + a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a), x, algorithm="fricas")

[Out] 1/2*(b^2*d*x^2 + 2*b^2*c*x + 2*d*cos(b*x + a)^2 + 4*(b*d*x + b*c)*cos(b*x + a)*sin(b*x + a))/b^2

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20591, size = 1242, normalized size = 18.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*c*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*d*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*d*x^2*tan(1/2*b*x)^2*tan(1/2*a)^4 + 4*b^2*c*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 8*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + 4*b^2*c*x*tan(1/2*b*x)^2*tan(1/2*a)^4 - 8*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^4 + b^2*d*x^2*tan(1/2*b*x)^4 + 4*b^2*d*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 8*b*c*tan(1/2*b*x)^4*tan(1/2*a)^3 + b^2*d*x^2*tan(1/2*a)^4 - 8*b*c*tan(1/2*b*x)^3*tan(1/2*a)^4 + d*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*c*x*tan(1/2*b*x)^4 + 8*b*d*x*tan(1/2*b*x)^4*tan(1/2*a) + 8*b^2*c*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 48*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^2 + 48*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^3 + 2*b^2*c*x*tan(1/2*a)^4 + 8*b*d*x*tan(1/2*b*x)*tan(1/2*a)^4 + 2*b^2*d*x^2*tan(1/2*b*x)^2 + 8*b*c*tan(1/2*b*x)^4*tan(1/2*a) + 2*b^2*d*x^2*tan(1/2*a)^2 + 48*b*c*tan(1/2*b*x)^3*tan(1/2*a)^2 - 6*d*tan(1/2*b*x)^4*tan(1/2*a)^2 + 48*b*c*tan(1/2*b*x)^2*tan(1/2*a)^3 - 16*d*tan(1/2*b*x)^3*tan(1/2*a)^3 + 8*b*c*tan(1/2*b*x)*tan(1/2*a)^4 - 6*d*tan(1/2*b*x)^2*tan(1/2*a)^4 + 4*b^2*c*x*tan(1/2*b*x)^2 - 8*b*d*x*tan(1/2*b*x)^3 - 48*b*d*x*tan(1/2*b*x)^2*tan(1/2*a) + 4*b^2*c*x*tan(1/2*a)^2 - 48*b*d*x*tan(1/2*b*x)*tan(1/2*a)^2 - 8*b*d*x*tan(1/2*a)^3 + b^2*d*x^2 - 8*b*c*tan(1/2*b*x)^3 + d*tan(1/2*b*x)^4 - 48*b*c*tan(1/2*b*x)^2*tan(1/2*a) + 16*d*tan(1/2*b*x)^3*tan(1/2*a) - 48*b*c*tan(1/2*b*x)*tan(1/2*a)^2 + 36*d*tan(1/2*b*x)^2*tan(1/2*a)^2 - 8*b*c*tan(1/2*a)^3 + 16*d*tan(1/2*b*x)*tan(1/2*a)^3 + d*tan(1/2*a)^4 + 2*b^2*c*x + 8*b*d*x*tan(1/2*b*x) + 8*b*d*x*tan(1/2*a) + 8*b*c*tan(1/2*b*x) - 6*d*tan(1/2*b*x)^2 + 8*b*c*tan(1/2*a) - 16*d*tan(1/2*b*x)*tan(1/2*a) - 6*d*tan(1/2*a)^2 + d)/(b^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*tan(1/2*b*x)^2*tan(1/2*a)^4 + b^2*tan(1/2*b*x)^4 + 4*b^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + b^2*tan(1/2*a)^4 + 2*b^2*tan(1/2*b*x)^2 + 2*b^2*tan(1/2*a)^2 + b^2)
```

$$3.372 \quad \int \frac{\csc(a+bx) \sin(3a+3bx)}{c+dx} dx$$

Optimal. Leaf size=71

$$\frac{2 \cos\left(2a - \frac{2bc}{d}\right) \operatorname{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{d} - \frac{2 \sin\left(2a - \frac{2bc}{d}\right) \operatorname{Si}\left(\frac{2bc}{d} + 2bx\right)}{d} + \frac{\log(c+dx)}{d}$$

[Out] (2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d + Log[c + d*x]/d - (2*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d

Rubi [A] time = 0.281018, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4431, 3312, 3303, 3299, 3302}

$$\frac{2 \cos\left(2a - \frac{2bc}{d}\right) \operatorname{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{d} - \frac{2 \sin\left(2a - \frac{2bc}{d}\right) \operatorname{Si}\left(\frac{2bc}{d} + 2bx\right)}{d} + \frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]

[Out] (2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d + Log[c + d*x]/d - (2*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d

Rule 4431

Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol] := Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && MemberQ[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]

Rule 3312

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx} dx &= \int \left(\frac{3\cos^2(a+bx)}{c+dx} - \frac{\sin^2(a+bx)}{c+dx} \right) dx \\
&= 3 \int \frac{\cos^2(a+bx)}{c+dx} dx - \int \frac{\sin^2(a+bx)}{c+dx} dx \\
&= 3 \int \left(\frac{1}{2(c+dx)} + \frac{\cos(2a+2bx)}{2(c+dx)} \right) dx - \int \left(\frac{1}{2(c+dx)} - \frac{\cos(2a+2bx)}{2(c+dx)} \right) dx \\
&= \frac{\log(c+dx)}{d} + \frac{1}{2} \int \frac{\cos(2a+2bx)}{c+dx} dx + \frac{3}{2} \int \frac{\cos(2a+2bx)}{c+dx} dx \\
&= \frac{\log(c+dx)}{d} + \frac{1}{2} \cos\left(2a - \frac{2bc}{d}\right) \int \frac{\cos\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx + \frac{1}{2} \left(3 \cos\left(2a - \frac{2bc}{d}\right)\right) \int \frac{\cos\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx \\
&= \frac{2 \cos\left(2a - \frac{2bc}{d}\right) \text{Ci}\left(\frac{2bc}{d} + 2bx\right)}{d} + \frac{\log(c+dx)}{d} - \frac{2 \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{d}
\end{aligned}$$

Mathematica [A] time = 0.15281, size = 63, normalized size = 0.89

$$\frac{2 \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) - 2 \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2b(c+dx)}{d}\right) + \log(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x), x]

[Out] (2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + Log[c + d*x] - 2*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d

Maple [A] time = 0.037, size = 116, normalized size = 1.6

$$-\frac{\ln(dx+c)}{d} + 2 \frac{1}{d} \text{Si}\left(2bx + 2a + 2 \frac{-ad+bc}{d}\right) \sin\left(2 \frac{-ad+bc}{d}\right) + 2 \frac{1}{d} \text{Ci}\left(2bx + 2a + 2 \frac{-ad+bc}{d}\right) \cos\left(2 \frac{-ad+bc}{d}\right) + 2 \frac{1}{d} \text{Si}\left(2bx + 2a + 2 \frac{-ad+bc}{d}\right) \cos\left(2 \frac{-ad+bc}{d}\right) + 2 \frac{1}{d} \text{Ci}\left(2bx + 2a + 2 \frac{-ad+bc}{d}\right) \sin\left(2 \frac{-ad+bc}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c), x)

[Out] -ln(d*x+c)/d+2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d+2*ln((b*x+a)*d-a*d+b*c)/d

Maxima [C] time = 1.2865, size = 158, normalized size = 2.23

$$\frac{\left(E_1\left(\frac{2ibdx+2ibc}{d}\right) + E_1\left(-\frac{2ibdx+2ibc}{d}\right)\right) \cos\left(-\frac{2(bc-ad)}{d}\right) - \left(i E_1\left(\frac{2ibdx+2ibc}{d}\right) - i E_1\left(-\frac{2ibdx+2ibc}{d}\right)\right) \sin\left(-\frac{2(bc-ad)}{d}\right) - \log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c), x, algorithm="maxima")

```
[Out] -((exp_integral_e(1, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(1, -(2*I*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) - (I*exp_integral_e(1, (2*I*b*d*x + 2*I*b*c)/d) - I*exp_integral_e(1, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c - a*d)/d) - log(d*x + c))/d
```

Fricas [A] time = 0.489241, size = 228, normalized size = 3.21

$$\frac{\left(\operatorname{Ci}\left(\frac{2(bdx+bc)}{d}\right) + \operatorname{Ci}\left(-\frac{2(bdx+bc)}{d}\right) \right) \cos\left(-\frac{2(bc-ad)}{d}\right) - 2 \sin\left(-\frac{2(bc-ad)}{d}\right) \operatorname{Si}\left(\frac{2(bdx+bc)}{d}\right) + \log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] ((cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d) - 2*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) + log(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.29773, size = 1509, normalized size = 21.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
```

```
[Out] (log(abs(d*x + c))*tan(1/2*a)^4*tan(b*c/d)^2 - real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 - real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d) - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d) + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^4*tan(b*c/d) - 4*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 + 4*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 - 8*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^3*tan(b*c/d)^2 + log(abs(d*x + c))*tan(1/2*a)^4 + real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4 + real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4 - 8*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) - 8*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) + 2*log(abs(d*x + c))*tan(1/2*a)^2*tan(b*c/d)^2 + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d)^2 + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d)^2 + 4*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3 - 4
```

```

*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3 + 8*sin_integral(2*
(b*d*x + b*c)/d)*tan(1/2*a)^3 - 12*imag_part(cos_integral(2*b*x + 2*b*c/d))
*tan(1/2*a)^2*tan(b*c/d) + 12*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(1/2*a)^2*tan(b*c/d) - 24*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(
b*c/d) + 4*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2
- 4*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 + 8*
sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)*tan(b*c/d)^2 + 2*log(abs(d*x + c
))*tan(1/2*a)^2 - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2 -
6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2 + 8*real_part(cos
_integral(2*b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d) + 8*real_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/d) + log(abs(d*x + c))*tan(b*c/d)^2
- real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - real_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 4*imag_part(cos_integral(2*b*x + 2
*b*c/d))*tan(1/2*a) + 4*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a
) - 8*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a) + 2*imag_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))
*tan(b*c/d) + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) + log(abs(d*x +
c)) + real_part(cos_integral(2*b*x + 2*b*c/d)) + real_part(cos_integral(-2*
b*x - 2*b*c/d)))/(d*tan(1/2*a)^4*tan(b*c/d)^2 + d*tan(1/2*a)^4 + 2*d*tan(1/
2*a)^2*tan(b*c/d)^2 + 2*d*tan(1/2*a)^2 + d*tan(b*c/d)^2 + d)

```


$$3.373 \quad \int \frac{\csc(a+bx) \sin(3a+3bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=102

$$\frac{4b \sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{d^2} - \frac{4b \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^2} + \frac{\sin^2(a+bx)}{d(c+dx)} - \frac{3 \cos^2(a+bx)}{d(c+dx)}$$

[Out] (-3*Cos[a + b*x]^2)/(d*(c + d*x)) - (4*b*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/d^2 + Sin[a + b*x]^2/(d*(c + d*x)) - (4*b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^2

Rubi [A] time = 0.276183, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4431, 3313, 12, 3303, 3299, 3302}

$$\frac{4b \sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{d^2} - \frac{4b \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^2} + \frac{\sin^2(a+bx)}{d(c+dx)} - \frac{3 \cos^2(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]

[Out] (-3*Cos[a + b*x]^2)/(d*(c + d*x)) - (4*b*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/d^2 + Sin[a + b*x]^2/(d*(c + d*x)) - (4*b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^2

Rule 4431

Int[((e_.) + (f_.)*(x_.))^(m_.)*(F_)[(a_.) + (b_.)*(x_.)]^(p_.)*(G_)[(c_.) + (d_.)*(x_.)]^(q_.), x_Symbol] := Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && MemberQ[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]

Rule 3313

Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := Simp[(c + d*x)^(m + 1)*Sin[e + f*x]^n/(d*(m + 1)), x] - Dist[(f*n)/(d*(m + 1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n - 1), x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\csc(a + bx) \sin(3a + 3bx)}{(c + dx)^2} dx &= \int \left(\frac{3 \cos^2(a + bx)}{(c + dx)^2} - \frac{\sin^2(a + bx)}{(c + dx)^2} \right) dx \\
 &= 3 \int \frac{\cos^2(a + bx)}{(c + dx)^2} dx - \int \frac{\sin^2(a + bx)}{(c + dx)^2} dx \\
 &= -\frac{3 \cos^2(a + bx)}{d(c + dx)} + \frac{\sin^2(a + bx)}{d(c + dx)} - \frac{(2b) \int \frac{\sin(2a+2bx)}{2(c+dx)} dx}{d} + \frac{(6b) \int -\frac{\sin(2a+2bx)}{2(c+dx)} dx}{d} \\
 &= -\frac{3 \cos^2(a + bx)}{d(c + dx)} + \frac{\sin^2(a + bx)}{d(c + dx)} - \frac{b \int \frac{\sin(2a+2bx)}{c+dx} dx}{d} - \frac{(3b) \int \frac{\sin(2a+2bx)}{c+dx} dx}{d} \\
 &= -\frac{3 \cos^2(a + bx)}{d(c + dx)} + \frac{\sin^2(a + bx)}{d(c + dx)} - \frac{\left(b \cos\left(2a - \frac{2bc}{d}\right)\right) \int \frac{\sin\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx}{d} - \frac{\left(3b \cos\left(2a - \frac{2bc}{d}\right)\right) \int \frac{\sin\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx}{d} \\
 &= -\frac{3 \cos^2(a + bx)}{d(c + dx)} - \frac{4b \operatorname{Ci}\left(\frac{2bc}{d} + 2bx\right) \sin\left(2a - \frac{2bc}{d}\right)}{d^2} + \frac{\sin^2(a + bx)}{d(c + dx)} - \frac{4b \cos\left(2a - \frac{2bc}{d}\right)}{d^2}
 \end{aligned}$$

Mathematica [A] time = 0.54519, size = 81, normalized size = 0.79

$$\frac{4b \sin\left(2a - \frac{2bc}{d}\right) \operatorname{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) + 4b \cos\left(2a - \frac{2bc}{d}\right) \operatorname{Si}\left(\frac{2b(c+dx)}{d}\right) + \frac{d(2 \cos(2(a+bx))+1)}{c+dx}}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2, x]

[Out] -(((d*(1 + 2*Cos[2*(a + b*x)])))/(c + d*x) + 4*b*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + 4*b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d^2)

Maple [A] time = 0.041, size = 169, normalized size = 1.7

$$\frac{1}{d(dx+c)} + 4 \frac{1}{b} \left(\frac{1}{4} b^2 \left(-2 \frac{\cos(2bx+2a)}{((bx+a)d-ad+bc)d} - 2 \frac{1}{d} \left(2 \frac{1}{d} \operatorname{Si}\left(2bx+2a+2 \frac{-ad+bc}{d}\right) \cos\left(2 \frac{-ad+bc}{d}\right) - 2 \frac{1}{d} \operatorname{Ci}\left(2bx+2a+2 \frac{-ad+bc}{d}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2, x)

[Out] 1/d/(d*x+c)+4/b*(1/4*b^2*(-2*cos(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d))

$d) * \sin(2 * (-a * d + b * c) / d) / d) / d) - 1 / 2 * b^2 / ((b * x + a) * d - a * d + b * c) / d)$

Maxima [C] time = 1.32883, size = 159, normalized size = 1.56

$$\frac{\left(E_2\left(\frac{2i b d x + 2i b c}{d}\right) + E_2\left(-\frac{2i b d x + 2i b c}{d}\right)\right) \cos\left(-\frac{2(b c - a d)}{d}\right) - \left(i E_2\left(\frac{2i b d x + 2i b c}{d}\right) - i E_2\left(-\frac{2i b d x + 2i b c}{d}\right)\right) \sin\left(-\frac{2(b c - a d)}{d}\right) + 1}{d^2 x + c d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")

[Out] -((exp_integral_e(2, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(2, -(2*I*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) - (I*exp_integral_e(2, (2*I*b*d*x + 2*I*b*c)/d) - I*exp_integral_e(2, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c - a*d)/d) + 1)/(d^2*x + c*d)

Fricas [A] time = 0.518525, size = 321, normalized size = 3.15

$$\frac{4 d \cos(b x + a)^2 + 4 (b d x + b c) \cos\left(-\frac{2(b c - a d)}{d}\right) \operatorname{Si}\left(\frac{2(b d x + b c)}{d}\right) + 2\left((b d x + b c) \operatorname{Ci}\left(\frac{2(b d x + b c)}{d}\right) + (b d x + b c) \operatorname{Ci}\left(-\frac{2(b d x + b c)}{d}\right)\right)}{d^3 x + c d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")

[Out] -(4*d*cos(b*x + a)^2 + 4*(b*d*x + b*c)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) + 2*((b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d) - d)/(d^3*x + c*d^2)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx + a) \sin(3bx + 3a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(csc(b*x + a)*sin(3*b*x + 3*a)/(d*x + c)^2, x)

$$3.374 \quad \int \frac{\csc(a+bx) \sin(3a+3bx)}{(c+dx)^3} dx$$

Optimal. Leaf size=136

$$-\frac{4b^2 \cos\left(2a - \frac{2bc}{d}\right) \operatorname{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{d^3} + \frac{4b^2 \sin\left(2a - \frac{2bc}{d}\right) \operatorname{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^3} + \frac{4b \sin(a+bx) \cos(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+bx)}{2d(c+dx)}$$

[Out] $(-3*\operatorname{Cos}[a + b*x]^2)/(2*d*(c + d*x)^2) - (4*b^2*\operatorname{Cos}[2*a - (2*b*c)/d]*\operatorname{CosIntegral}[(2*b*c)/d + 2*b*x])/d^3 + (4*b*\operatorname{Cos}[a + b*x]*\operatorname{Sin}[a + b*x])/(d^2*(c + d*x)) + \operatorname{Sin}[a + b*x]^2/(2*d*(c + d*x)^2) + (4*b^2*\operatorname{Sin}[2*a - (2*b*c)/d]*\operatorname{SinIntegral}[(2*b*c)/d + 2*b*x])/d^3$

Rubi [A] time = 0.374217, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4431, 3314, 31, 3312, 3303, 3299, 3302}

$$-\frac{4b^2 \cos\left(2a - \frac{2bc}{d}\right) \operatorname{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{d^3} + \frac{4b^2 \sin\left(2a - \frac{2bc}{d}\right) \operatorname{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^3} + \frac{4b \sin(a+bx) \cos(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+bx)}{2d(c+dx)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Csc}[a + b*x]*\operatorname{Sin}[3*a + 3*b*x])/(c + d*x)^3, x]$

[Out] $(-3*\operatorname{Cos}[a + b*x]^2)/(2*d*(c + d*x)^2) - (4*b^2*\operatorname{Cos}[2*a - (2*b*c)/d]*\operatorname{CosIntegral}[(2*b*c)/d + 2*b*x])/d^3 + (4*b*\operatorname{Cos}[a + b*x]*\operatorname{Sin}[a + b*x])/(d^2*(c + d*x)) + \operatorname{Sin}[a + b*x]^2/(2*d*(c + d*x)^2) + (4*b^2*\operatorname{Sin}[2*a - (2*b*c)/d]*\operatorname{SinIntegral}[(2*b*c)/d + 2*b*x])/d^3$

Rule 4431

$\operatorname{Int}[(e + f*x)^m*(a + b*x)^p*(c + d*x)^q, x] \rightarrow \operatorname{Int}[\operatorname{ExpandTrigExpand}[(e + f*x)^m*(a + b*x)^p*(c + d*x)^q, x], x] /;$ FreeQ[{a, b, c, d, e, f, m}, x] && MemberQ[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]

Rule 3314

$\operatorname{Int}[(c + d*x)^m*(b*\sin[e + f*x])^n, x] \rightarrow \operatorname{Simp}[(c + d*x)^{m+1}*(b*\sin[e + f*x])^n/(d*(m+1)), x] + (\operatorname{Dist}[(b^2*f^2*n*(n-1))/(d^2*(m+1)*(m+2)), \operatorname{Int}[(c + d*x)^{m+2}*(b*\sin[e + f*x])^{n-2}, x], x] - \operatorname{Dist}[(f^2*n^2)/(d^2*(m+1)*(m+2)), \operatorname{Int}[(c + d*x)^{m+2}*(b*\sin[e + f*x])^n, x], x] - \operatorname{Simp}[(b*f*n*(c + d*x)^{m+2}*\cos[e + f*x]*(b*\sin[e + f*x])^{n-1})/(d^2*(m+1)*(m+2)), x]) /;$ FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]

Rule 31

$\operatorname{Int}[(a + b*x)^{-1}, x] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /;$ FreeQ[{a, b}, x]

Rule 3312

$\operatorname{Int}[(c + d*x)^m*\sin[e + f*x]^n, x] \rightarrow \operatorname{Int}[\operatorname{ExpandTrigReduce}[(c + d*x)^m*\sin[e + f*x]^n, x], x] /;$ FreeQ[{c, d, e, f}, x]

, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rule 3303

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\csc(a+bx) \sin(3a+3bx)}{(c+dx)^3} dx &= \int \left(\frac{3 \cos^2(a+bx)}{(c+dx)^3} - \frac{\sin^2(a+bx)}{(c+dx)^3} \right) dx \\
 &= 3 \int \frac{\cos^2(a+bx)}{(c+dx)^3} dx - \int \frac{\sin^2(a+bx)}{(c+dx)^3} dx \\
 &= -\frac{3 \cos^2(a+bx)}{2d(c+dx)^2} + \frac{4b \cos(a+bx) \sin(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+bx)}{2d(c+dx)^2} - \frac{b^2 \int \frac{1}{c+dx} dx}{d^2} + \frac{(2b \int \frac{1}{c+dx} dx)}{d^2} \\
 &= -\frac{3 \cos^2(a+bx)}{2d(c+dx)^2} + \frac{2b^2 \log(c+dx)}{d^3} + \frac{4b \cos(a+bx) \sin(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+bx)}{2d(c+dx)^2} + \frac{2b^2 \int \frac{1}{c+dx} dx}{d^2} \\
 &= -\frac{3 \cos^2(a+bx)}{2d(c+dx)^2} + \frac{4b \cos(a+bx) \sin(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+bx)}{2d(c+dx)^2} - \frac{b^2 \int \frac{\cos(2a+2bx)}{c+dx} dx}{d^2} \\
 &= -\frac{3 \cos^2(a+bx)}{2d(c+dx)^2} + \frac{4b \cos(a+bx) \sin(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+bx)}{2d(c+dx)^2} - \frac{\left(b^2 \cos\left(2a - \frac{2bc}{d}\right) \right)}{d^2} \\
 &= -\frac{3 \cos^2(a+bx)}{2d(c+dx)^2} - \frac{4b^2 \cos\left(2a - \frac{2bc}{d}\right) \text{Ci}\left(\frac{2bc}{d} + 2bx\right)}{d^3} + \frac{4b \cos(a+bx) \sin(a+bx)}{d^2(c+dx)}
 \end{aligned}$$

Mathematica [A] time = 0.998589, size = 104, normalized size = 0.76

$$\frac{8b^2 \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) - 8b^2 \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2b(c+dx)}{d}\right) + \frac{d(-4b(c+dx) \sin(2(a+bx)) + 2d \cos(2(a+bx)) + d)}{(c+dx)^2}}{2d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]

[Out] -(8*b^2*cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + (d*(d + 2*d*cos[2*(a + b*x)] - 4*b*(c + d*x)*Sin[2*(a + b*x)]))/(c + d*x)^2 - 8*b^2*sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/(2*d^3)

Maple [A] time = 0.038, size = 207, normalized size = 1.5

$$\frac{1}{2d(dx+c)^2} + 4\frac{1}{b}\left(\frac{1}{4}b^3\left(-\frac{\cos(2bx+2a)}{((bx+a)d-ad+bc)^2d} - \frac{1}{d}\left(-2\frac{\sin(2bx+2a)}{((bx+a)d-ad+bc)d} + 2\frac{1}{d}\left(2\frac{1}{d}\text{Si}\left(2bx+2a+2\frac{-ad+bc}{d}\right)\right)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x)

[Out] 1/2/d/(d*x+c)^2+4/b*(1/4*b^3*(-cos(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)^2/d-(-2*sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)/d+2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)-1/4*b^3/((b*x+a)*d-a*d+b*c)^2/d)

Maxima [C] time = 1.4501, size = 176, normalized size = 1.29

$$\frac{2\left(E_3\left(\frac{2ibdx+2ibc}{d}\right) + E_3\left(-\frac{2ibdx+2ibc}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right) - \left(2iE_3\left(\frac{2ibdx+2ibc}{d}\right) - 2iE_3\left(-\frac{2ibdx+2ibc}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right) + 1}{2\left(d^3x^2 + 2cd^2x + c^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")

[Out] -1/2*(2*(exp_integral_e(3, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(3, -(2*I*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) - (2*I*exp_integral_e(3, (2*I*b*d*x + 2*I*b*c)/d) - 2*I*exp_integral_e(3, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c - a*d)/d) + 1)/(d^3*x^2 + 2*c*d^2*x + c^2*d)

Fricas [A] time = 0.530617, size = 516, normalized size = 3.79

$$\frac{4d^2\cos(bx+a)^2 - 8(bd^2x + bcd)\cos(bx+a)\sin(bx+a) - 8(b^2d^2x^2 + 2b^2cdx + b^2c^2)\sin\left(-\frac{2(bc-ad)}{d}\right)\text{Si}\left(\frac{2(bdx+bc)}{d}\right) - 8(b^2d^2x^2 + 2b^2cdx + b^2c^2)\cos\left(-\frac{2(bc-ad)}{d}\right)\text{Ci}\left(\frac{2(bdx+bc)}{d}\right)}{2(d^5x^2 + 2cd^4x + c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")

[Out] -1/2*(4*d^2*cos(b*x + a)^2 - 8*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) - 8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - d^2 + 4*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned}
& x^2 \sin_integral(2*(b*d*x + b*c)/d) * \tan(1/2*a)^4 * \tan(b*c/d) - 8*b^2*c^2 * \text{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(b*x)^2 * \tan(1/2*a)^4 * \tan(b*c/d) + \\
& 8*b^2*c^2 * \text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(b*x)^2 * \tan(1/2*a)^4 * \tan(b*c/d) - 16*b^2*c^2 * \sin_integral(2*(b*d*x + b*c)/d) * \tan(b*x)^2 * \tan(1/2*a)^4 * \tan(b*c/d) - \\
& 16*b^2*d^2*x^2 * \text{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(b*x)^2 * \tan(1/2*a) * \tan(b*c/d)^2 + 16*b^2*d^2*x^2 * \text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(b*x)^2 * \tan(1/2*a) * \tan(b*c/d)^2 - \\
& 32*b^2*d^2*x^2 * \sin_integral(2*(b*d*x + b*c)/d) * \tan(b*x)^2 * \tan(1/2*a) * \tan(b*c/d)^2 - 48*b^2*c*d * x * \text{real_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(b*x)^2 * \tan(1/2*a)^2 * \tan(b*c/d)^2 - \\
& 48*b^2*c*d * x * \text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(b*x)^2 * \tan(1/2*a)^2 * \tan(b*c/d)^2 + 16*b^2*d^2*x^2 * \text{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(1/2*a)^3 * \tan(b*c/d)^2 - \\
& 16*b^2*d^2*x^2 * \text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(1/2*a)^3 * \tan(b*c/d)^2 + 32*b^2*d^2*x^2 * \sin_integral(2*(b*d*x + b*c)/d) * \tan(1/2*a)^3 * \tan(b*c/d)^2 + \\
& 16*b^2*c^2 * \text{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(b*x)^2 * \tan(1/2*a)^3 * \tan(b*c/d)^2 - 16*b^2*c^2 * \text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(b*x)^2 * \tan(1/2*a)^3 * \tan(b*c/d)^2 + \\
& 32*b^2*c^2 * \sin_integral(2*(b*d*x + b*c)/d) * \tan(b*x)^2 * \tan(1/2*a)^3 * \tan(b*c/d)^2 + 8*b^2*c*d * x * \text{real_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(1/2*a)^4 * \tan(b*c/d)^2 + \\
& 8*b^2*c*d * x * \text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(1/2*a)^4 * \tan(b*c/d)^2 + 24*b^2*d^2*x^2 * \text{real_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(b*x)^2 * \tan(1/2*a)^2 + \\
& 24*b^2*d^2*x^2 * \text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(b*x)^2 * \tan(1/2*a)^2 - 32*b^2*c*d * x * \text{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(b*x)^2 * \tan(1/2*a)^3 + \\
& 32*b^2*c*d * x * \text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(b*x)^2 * \tan(1/2*a)^3 - 64*b^2*c*d * x * \sin_integral(2*(b*d*x + b*c)/d) * \tan(b*x)^2 * \tan(1/2*a)^3 - \\
& 4*b^2*d^2*x^2 * \text{real_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(1/2*a)^4 - 4*b^2*d^2*x^2 * \text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(1/2*a)^4 - 4*b^2*c^2 * \text{real_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(b*x)^2 * \tan(1/2*a)^4 - \\
& 4*b^2*c^2 * \text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(b*x)^2 * \tan(1/2*a)^4 - 32*b^2*d^2*x^2 * \text{real_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(b*x)^2 * \tan(1/2*a) * \tan(b*c/d) - \\
& 32*b^2*d^2*x^2 * \text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(b*x)^2 * \tan(1/2*a) * \tan(b*c/d) + 96*b^2*c*d * x * \text{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(b*x)^2 * \tan(1/2*a)^2 * \tan(b*c/d) - \\
& 96*b^2*c*d * x * \text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(b*x)^2 * \tan(1/2*a)^2 * \tan(b*c/d) + 192*b^2*c*d * x * \sin_integral(2*(b*d*x + b*c)/d) * \tan(b*x)^2 * \tan(1/2*a)^2 * \tan(b*c/d) + \\
& 32*b^2*d^2*x^2 * \text{real_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(1/2*a)^3 * \tan(b*c/d) + 32*b^2*d^2*x^2 * \text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(1/2*a)^3 * \tan(b*c/d) + \\
& 32*b^2*c^2 * \text{real_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(b*x)^2 * \tan(1/2*a)^3 * \tan(b*c/d) + 32*b^2*c^2 * \text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(b*x)^2 * \tan(1/2*a)^3 * \tan(b*c/d) - \\
& 16*b^2*c*d * x * \text{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(1/2*a)^4 * \tan(b*c/d) + 16*b^2*c*d * x * \text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(1/2*a)^4 * \tan(b*c/d) - \\
& 32*b^2*c*d * x * \sin_integral(2*(b*d*x + b*c)/d) * \tan(1/2*a)^4 * \tan(b*c/d) + 4*b^2*d^2*x^2 * \text{real_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(b*x)^2 * \tan(b*c/d)^2 + 4*b^2*d^2*x^2 * \text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(b*x)^2 * \tan(b*c/d)^2 - \\
& 32*b^2*c*d * x * \text{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(1/2*a) * \tan(b*c/d)^2 + 32*b^2*c*d * x * \text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(1/2*a) * \tan(b*c/d)^2 - \\
& 64*b^2*c*d * x * \sin_integral(2*(b*d*x + b*c)/d) * \tan(b*x)^2 * \tan(1/2*a) * \tan(b*c/d)^2 - 24*b^2*d^2*x^2 * \text{real_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(1/2*a)^2 * \tan(b*c/d)^2 - \\
& 24*b^2*d^2*x^2 * \text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(1/2*a)^2 * \tan(b*c/d)^2 - 24*b^2*c^2 * \text{real_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(b*x)^2 * \tan(1/2*a)^2 * \tan(b*c/d)^2 - \\
& 24*b^2*c^2 * \text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(b*x)^2 * \tan(1/2*a)^2 * \tan(b*c/d)^2 + 32*b^2*c*d * x * \text{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(1/2*a)^3 * \tan(b*c/d)^2 - \\
& 32*b^2*c*d * x * \text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(1/2*a)^3 * \tan(b*c/d)^2 + 64*b^2*c*d * x * \sin_integral(2*(b*d*x + b*c)/d) * \tan(1/2*a)^3 * \tan(b*c/d)^2 + \\
& 16*b*d^2*x * \tan(b*x)^2 * \tan(1/2*a)^3 * \tan(b*c/d)^2 + 4*b^2*c^2 * \text{real_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(1/2*a)^4 * \tan(b*c/d)^2 + 4*b^2*c^2 * \text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(1/2*a)^4 * \tan(b*c/d)^2 + \\
& 8*b*d^2*x * \tan(b*x) * \tan(1/2*a)^4
\end{aligned}$$

$$\begin{aligned}
& * \tan(b*c/d)^2 + 16*b^2*d^2*x^2*imag_part(\cos_integral(2*b*x + 2*b*c/d))*\tan \\
& (b*x)^2*\tan(1/2*a) - 16*b^2*d^2*x^2*imag_part(\cos_integral(-2*b*x - 2*b*c/d \\
&))*\tan(b*x)^2*\tan(1/2*a) + 32*b^2*d^2*x^2*\sin_integral(2*(b*d*x + b*c)/d)*\tan \\
& (b*x)^2*\tan(1/2*a) + 48*b^2*c*d*x*real_part(\cos_integral(2*b*x + 2*b*c/d) \\
&)*\tan(b*x)^2*\tan(1/2*a)^2 + 48*b^2*c*d*x*real_part(\cos_integral(-2*b*x - 2* \\
& b*c/d))*\tan(b*x)^2*\tan(1/2*a)^2 - 16*b^2*d^2*x^2*imag_part(\cos_integral(2*b \\
& *x + 2*b*c/d))*\tan(1/2*a)^3 + 16*b^2*d^2*x^2*imag_part(\cos_integral(-2*b*x \\
& - 2*b*c/d))*\tan(1/2*a)^3 - 32*b^2*d^2*x^2*\sin_integral(2*(b*d*x + b*c)/d)*\tan \\
& (1/2*a)^3 - 16*b^2*c^2*imag_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(b*x)^ \\
& 2*\tan(1/2*a)^3 + 16*b^2*c^2*imag_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(b \\
& *x)^2*\tan(1/2*a)^3 - 32*b^2*c^2*\sin_integral(2*(b*d*x + b*c)/d)*\tan(b*x)^2* \\
& \tan(1/2*a)^3 - 8*b^2*c*d*x*real_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(1/2 \\
& *a)^4 - 8*b^2*c*d*x*real_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(1/2*a)^4 \\
& - 8*b^2*d^2*x^2*imag_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(b*c \\
& /d) + 8*b^2*d^2*x^2*imag_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan \\
& (b*c/d) - 16*b^2*d^2*x^2*\sin_integral(2*(b*d*x + b*c)/d)*\tan(b*x)^2*\tan(b* \\
& c/d) - 64*b^2*c*d*x*real_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan \\
& (1/2*a)*\tan(b*c/d) - 64*b^2*c*d*x*real_part(\cos_integral(-2*b*x - 2*b*c/d) \\
&)*\tan(b*x)^2*\tan(1/2*a)*\tan(b*c/d) + 48*b^2*d^2*x^2*imag_part(\cos_integral(2 \\
& *b*x + 2*b*c/d))*\tan(1/2*a)^2*\tan(b*c/d) - 48*b^2*d^2*x^2*imag_part(\cos_int \\
& egral(-2*b*x - 2*b*c/d))*\tan(1/2*a)^2*\tan(b*c/d) + 96*b^2*d^2*x^2*\sin_integ \\
& ral(2*(b*d*x + b*c)/d)*\tan(1/2*a)^2*\tan(b*c/d) + 48*b^2*c^2*imag_part(\cos_i \\
& ntegral(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(1/2*a)^2*\tan(b*c/d) - 48*b^2*c^2*i \\
& mag_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(1/2*a)^2*\tan(b*c/d) \\
& + 96*b^2*c^2*\sin_integral(2*(b*d*x + b*c)/d)*\tan(b*x)^2*\tan(1/2*a)^2*\tan(b \\
& *c/d) + 64*b^2*c*d*x*real_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(1/2*a)^3* \\
& \tan(b*c/d) + 64*b^2*c*d*x*real_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(1/2 \\
& *a)^3*\tan(b*c/d) - 8*b^2*c^2*imag_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(1 \\
& /2*a)^4*\tan(b*c/d) + 8*b^2*c^2*imag_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan \\
& (1/2*a)^4*\tan(b*c/d) - 16*b^2*c^2*\sin_integral(2*(b*d*x + b*c)/d)*\tan(1/2* \\
& a)^4*\tan(b*c/d) + 8*b^2*c*d*x*real_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(\\
& b*x)^2*\tan(b*c/d)^2 + 8*b^2*c*d*x*real_part(\cos_integral(-2*b*x - 2*b*c/d) \\
&)*\tan(b*x)^2*\tan(b*c/d)^2 - 16*b^2*d^2*x^2*imag_part(\cos_integral(2*b*x + 2* \\
& b*c/d))*\tan(1/2*a)*\tan(b*c/d)^2 + 16*b^2*d^2*x^2*imag_part(\cos_integral(-2* \\
& b*x - 2*b*c/d))*\tan(1/2*a)*\tan(b*c/d)^2 - 32*b^2*d^2*x^2*\sin_integral(2*(b* \\
& d*x + b*c)/d)*\tan(1/2*a)*\tan(b*c/d)^2 - 16*b^2*c^2*imag_part(\cos_integral(2 \\
& *b*x + 2*b*c/d))*\tan(b*x)^2*\tan(1/2*a)*\tan(b*c/d)^2 + 16*b^2*c^2*imag_part(\\
& \cos_integral(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(1/2*a)*\tan(b*c/d)^2 - 32*b^2 \\
& *c^2*\sin_integral(2*(b*d*x + b*c)/d)*\tan(b*x)^2*\tan(1/2*a)*\tan(b*c/d)^2 - 4 \\
& 8*b^2*c*d*x*real_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(1/2*a)^2*\tan(b*c/d \\
&)^2 - 48*b^2*c*d*x*real_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(1/2*a)^2* \\
& \tan(b*c/d)^2 + 16*b^2*c^2*imag_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(1/2*a \\
&)^3*\tan(b*c/d)^2 - 16*b^2*c^2*imag_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan \\
& (1/2*a)^3*\tan(b*c/d)^2 + 32*b^2*c^2*\sin_integral(2*(b*d*x + b*c)/d)*\tan(1/2 \\
& *a)^3*\tan(b*c/d)^2 + 16*b*c*d*\tan(b*x)^2*\tan(1/2*a)^3*\tan(b*c/d)^2 + 8*b*c* \\
& d*\tan(b*x)*\tan(1/2*a)^4*\tan(b*c/d)^2 + d^2*\tan(b*x)^2*\tan(1/2*a)^4*\tan(b*c/ \\
& d)^2 - 4*b^2*d^2*x^2*real_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(b*x)^2 - \\
& 4*b^2*d^2*x^2*real_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(b*x)^2 + 32*b^2 \\
& *c*d*x*imag_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(1/2*a) - 32* \\
& b^2*c*d*x*imag_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(1/2*a) + \\
& 64*b^2*c*d*x*\sin_integral(2*(b*d*x + b*c)/d)*\tan(b*x)^2*\tan(1/2*a) + 24*b^ \\
& 2*d^2*x^2*real_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(1/2*a)^2 + 24*b^2*d^ \\
& 2*x^2*real_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(1/2*a)^2 + 24*b^2*c^2*r \\
& eal_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(b*x)^2*\tan(1/2*a)^2 + 24*b^2*c^ \\
& 2*real_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(b*x)^2*\tan(1/2*a)^2 - 32*b^ \\
& 2*c*d*x*imag_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(1/2*a)^3 + 32*b^2*c*d* \\
& x*imag_part(\cos_integral(-2*b*x - 2*b*c/d))*\tan(1/2*a)^3 - 64*b^2*c*d*x*\sin \\
& _integral(2*(b*d*x + b*c)/d)*\tan(1/2*a)^3 + 16*b*d^2*x*\tan(b*x)^2*\tan(1/2*a \\
&)^3 - 4*b^2*c^2*real_part(\cos_integral(2*b*x + 2*b*c/d))*\tan(1/2*a)^4 - 4*b
\end{aligned}$$

$$\begin{aligned}
& ^2c^2\text{real_part}(\cos_integral(-2bx - 2bc/d))\tan(1/2a)^4 + 8bd^2x\text{tan}(bx)\tan(1/2a)^4 - 16b^2cdx\text{imag_part}(\cos_integral(2bx + 2bc/d))\tan(bx)^2\tan(bc/d) + 16b^2cdx\text{imag_part}(\cos_integral(-2bx - 2bc/d))\tan(bx)^2\tan(bc/d) - 32b^2cdx\sin_integral(2(bdx + bc)/d)\tan(bx)^2\tan(bc/d) - 32b^2d^2x^2\text{real_part}(\cos_integral(2bx + 2bc/d))\tan(1/2a)\tan(bc/d) - 32b^2d^2x^2\text{real_part}(\cos_integral(-2bx - 2bc/d))\tan(1/2a)\tan(bc/d) - 32b^2c^2\text{real_part}(\cos_integral(2bx + 2bc/d))\tan(bx)^2\tan(1/2a)\tan(bc/d) - 32b^2c^2\text{real_part}(\cos_integral(-2bx - 2bc/d))\tan(bx)^2\tan(1/2a)\tan(bc/d) + 96b^2cdx\text{imag_part}(\cos_integral(2bx + 2bc/d))\tan(1/2a)^2\tan(bc/d) - 96b^2cdx\text{imag_part}(\cos_integral(-2bx - 2bc/d))\tan(1/2a)^2\tan(bc/d) + 192b^2cdx\sin_integral(2(bdx + bc)/d)\tan(1/2a)^2\tan(bc/d) + 32b^2c^2\text{real_part}(\cos_integral(2bx + 2bc/d))\tan(1/2a)^3\tan(bc/d) + 32b^2c^2\text{real_part}(\cos_integral(-2bx - 2bc/d))\tan(1/2a)^3\tan(bc/d) + 4b^2d^2x^2\text{real_part}(\cos_integral(2bx + 2bc/d))\tan(bc/d)^2 + 4b^2d^2x^2\text{real_part}(\cos_integral(-2bx - 2bc/d))\tan(bc/d)^2 + 4b^2c^2\text{real_part}(\cos_integral(2bx + 2bc/d))\tan(bx)^2\tan(bc/d)^2 + 4b^2c^2\text{real_part}(\cos_integral(-2bx - 2bc/d))\tan(bx)^2\tan(bc/d)^2 - 32b^2cdx\text{imag_part}(\cos_integral(2bx + 2bc/d))\tan(1/2a)\tan(bc/d)^2 + 32b^2cdx\text{imag_part}(\cos_integral(-2bx - 2bc/d))\tan(1/2a)\tan(bc/d)^2 - 64b^2cdx\sin_integral(2(bdx + bc)/d)\tan(1/2a)\tan(bc/d)^2 - 16bd^2x\tan(bx)^2\tan(1/2a)\tan(bc/d)^2 - 24b^2c^2\text{real_part}(\cos_integral(2bx + 2bc/d))\tan(1/2a)^2\tan(bc/d)^2 - 24b^2c^2\text{real_part}(\cos_integral(-2bx - 2bc/d))\tan(1/2a)^2\tan(bc/d)^2 - 48bd^2x\tan(bx)\tan(1/2a)^2\tan(bc/d)^2 - 16bd^2x\tan(1/2a)^3\tan(bc/d)^2 - 8b^2cdx\text{real_part}(\cos_integral(2bx + 2bc/d))\tan(bx)^2 - 8b^2cdx\text{real_part}(\cos_integral(-2bx - 2bc/d))\tan(bx)^2 + 16b^2d^2x^2\text{imag_part}(\cos_integral(2bx + 2bc/d))\tan(1/2a) - 16b^2d^2x^2\text{imag_part}(\cos_integral(-2bx - 2bc/d))\tan(1/2a) + 32b^2d^2x^2\sin_integral(2(bdx + bc)/d)\tan(1/2a) + 16b^2c^2\text{imag_part}(\cos_integral(2bx + 2bc/d))\tan(bx)^2\tan(1/2a) - 16b^2c^2\text{imag_part}(\cos_integral(-2bx - 2bc/d))\tan(bx)^2\tan(1/2a) + 32b^2c^2\sin_integral(2(bdx + bc)/d)\tan(bx)^2\tan(1/2a) + 48b^2cdx\text{real_part}(\cos_integral(2bx + 2bc/d))\tan(1/2a)^2 + 48b^2cdx\text{real_part}(\cos_integral(-2bx - 2bc/d))\tan(1/2a)^2 - 16b^2c^2\text{imag_part}(\cos_integral(2bx + 2bc/d))\tan(1/2a)^3 + 16b^2c^2\text{imag_part}(\cos_integral(-2bx - 2bc/d))\tan(1/2a)^3 - 32b^2c^2\sin_integral(2(bdx + bc)/d)\tan(1/2a)^3 + 16b^2cd\tan(bx)^2\tan(1/2a)^3 + 8b^2cd\tan(bx)\tan(1/2a)^4 + d^2\tan(bx)^2\tan(1/2a)^4 - 8b^2d^2x^2\text{imag_part}(\cos_integral(2bx + 2bc/d))\tan(bc/d) + 8b^2d^2x^2\text{imag_part}(\cos_integral(-2bx - 2bc/d))\tan(bc/d) - 16b^2d^2x^2\sin_integral(2(bdx + bc)/d)\tan(bc/d) - 8b^2c^2\text{imag_part}(\cos_integral(2bx + 2bc/d))\tan(bx)^2\tan(bc/d) + 8b^2c^2\text{imag_part}(\cos_integral(-2bx - 2bc/d))\tan(bx)^2\tan(bc/d) - 16b^2c^2\sin_integral(2(bdx + bc)/d)\tan(bx)^2\tan(bc/d) - 64b^2cdx\text{real_part}(\cos_integral(2bx + 2bc/d))\tan(1/2a)\tan(bc/d) - 64b^2cdx\text{real_part}(\cos_integral(-2bx - 2bc/d))\tan(1/2a)\tan(bc/d) + 48b^2c^2\text{imag_part}(\cos_integral(2bx + 2bc/d))\tan(1/2a)^2\tan(bc/d) - 48b^2c^2\text{imag_part}(\cos_integral(-2bx - 2bc/d))\tan(1/2a)^2\tan(bc/d) + 96b^2c^2\sin_integral(2(bdx + bc)/d)\tan(1/2a)^2\tan(bc/d) + 8b^2cdx\text{real_part}(\cos_integral(2bx + 2bc/d))\tan(bc/d)^2 + 8b^2cdx\text{real_part}(\cos_integral(-2bx - 2bc/d))\tan(bc/d)^2 - 16b^2c^2\text{imag_part}(\cos_integral(2bx + 2bc/d))\tan(1/2a)\tan(bc/d)^2 + 16b^2c^2\text{imag_part}(\cos_integral(-2bx - 2bc/d))\tan(1/2a)\tan(bc/d)^2 - 32b^2c^2\sin_integral(2(bdx + bc)/d)\tan(1/2a)\tan(bc/d)^2 - 16b^2cd\tan(bx)^2\tan(1/2a)\tan(bc/d)^2 - 48b^2cd\tan(bx)\tan(1/2a)^2\tan(bc/d)^2 - 14d^2\tan(bx)^2\tan(1/2a)^2\tan(bc/d)^2 - 16b^2cd\tan(1/2a)^3\tan(bc/d)^2 - 16d^2\tan(bx)\tan(1/2a)^3\tan(bc/d)^2 - 3d^2\tan(1/2a)^4\tan(bc/d)^2 - 4b^2d^2x^2\text{real_part}(\cos_integral(2bx + 2bc/d)) - 4b^2d^2x^2\text{real_part}(\cos_integral(-2bx - 2bc/d)) - 4b^2c^2\text{real_part}(\cos_integral(2bx +
\end{aligned}$$

$$\begin{aligned}
& 2*b*c/d)) * \tan(b*x)^2 - 4*b^2*c^2 * \text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \\
& \tan(b*x)^2 + 32*b^2*c*d*x * \text{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(1/2*a) - 32*b^2*c*d*x * \\
& \text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(1/2*a) + 64*b^2*c*d*x * \sin_integral(2*(b*d*x + b*c)/d) * \tan(1/2*a) - 16*b*d^2*x * \tan(b*x) \\
& ^2 * \tan(1/2*a) + 24*b^2*c^2 * \text{real_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(1/2*a)^2 + 24*b^2*c^2 * \text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(1/2*a)^2 - \\
& 48*b*d^2*x * \tan(b*x) * \tan(1/2*a)^2 - 16*b*d^2*x * \tan(1/2*a)^3 - 16*b^2*c*d*x * \text{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(b*c/d) + 16*b^2*c*d*x * \text{imag_part} \\
& (\cos_integral(-2*b*x - 2*b*c/d)) * \tan(b*c/d) - 32*b^2*c*d*x * \sin_integral(2*(b*d*x + b*c)/d) * \tan(b*c/d) - 32*b^2*c^2 * \text{real_part}(\cos_integral(2*b*x + 2*b \\
& *c/d)) * \tan(1/2*a) * \tan(b*c/d) - 32*b^2*c^2 * \text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(1/2*a) * \tan(b*c/d) + 4*b^2*c^2 * \text{real_part}(\cos_integral(2*b*x + 2 \\
& *b*c/d)) * \tan(b*c/d)^2 + 4*b^2*c^2 * \text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(b*c/d)^2 + 8*b*d^2*x * \tan(b*x) * \tan(b*c/d)^2 + 16*b*d^2*x * \tan(1/2*a) * \tan \\
& (b*c/d)^2 - 8*b^2*c*d*x * \text{real_part}(\cos_integral(2*b*x + 2*b*c/d)) - 8*b^2*c*d*x * \text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) + 16*b^2*c^2 * \text{imag_part}(\cos_in \\
& tegral(2*b*x + 2*b*c/d)) * \tan(1/2*a) - 16*b^2*c^2 * \text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d)) * \tan(1/2*a) + 32*b^2*c^2 * \sin_integral(2*(b*d*x + b*c)/d) * \tan \\
& (1/2*a) - 16*b*c*d * \tan(b*x)^2 * \tan(1/2*a) - 48*b*c*d * \tan(b*x) * \tan(1/2*a)^2 - 14*d^2 * \tan(b*x)^2 * \tan(1/2*a)^2 - 16*b*c*d * \tan(1/2*a)^3 - 16*d^2 * \tan(b*x) * \tan \\
& (1/2*a)^3 - 3*d^2 * \tan(1/2*a)^4 - 8*b^2*c^2 * \text{imag_part}(\cos_integral(2*b*x + 2*b*c/d)) * \tan(b*c/d) + 8*b^2*c^2 * \text{imag_part}(\cos_integral(-2*b*x - 2*b*c/d)) \\
& * \tan(b*c/d) - 16*b^2*c^2 * \sin_integral(2*(b*d*x + b*c)/d) * \tan(b*c/d) + 8*b*c*d * \tan(b*x) * \tan(b*c/d)^2 + d^2 * \tan(b*x)^2 * \tan(b*c/d)^2 + 16*b*c*d * \tan(1/2*a) \\
& * \tan(b*c/d)^2 + 16*d^2 * \tan(b*x) * \tan(1/2*a) * \tan(b*c/d)^2 + 10*d^2 * \tan(1/2*a)^2 * \tan(b*c/d)^2 - 4*b^2*c^2 * \text{real_part}(\cos_integral(2*b*x + 2*b*c/d)) - 4*b \\
& ^2*c^2 * \text{real_part}(\cos_integral(-2*b*x - 2*b*c/d)) + 8*b*d^2*x * \tan(b*x) + 16*b*d^2*x * \tan(1/2*a) + 8*b*c*d * \tan(b*x) + d^2 * \tan(b*x)^2 + 16*b*c*d * \tan(1/2*a) \\
& + 16*d^2 * \tan(b*x) * \tan(1/2*a) + 10*d^2 * \tan(1/2*a)^2 - 3*d^2 * \tan(b*c/d)^2 - 3*d^2) / (d^5*x^2 * \tan(b*x)^2 * \tan(1/2*a)^4 * \tan(b*c/d)^2 + 2*c*d^4*x * \tan(b*x)^2 * \tan(1/2*a)^4 * \tan(b*c/d)^2 + d^5*x^2 * \tan(b*x)^2 * \tan(1/2*a)^4 + 2*d^5*x^2 * \tan(b*x)^2 * \tan(1/2*a)^2 * \tan(b*c/d)^2 + d^5*x^2 * \tan(1/2*a)^4 * \tan(b*c/d)^2 + c^2*d^3 * \tan(b*x)^2 * \tan(1/2*a)^4 * \tan(b*c/d)^2 + 2*c*d^4*x * \tan(b*x)^2 * \tan(1/2*a)^4 + 4*c*d^4*x * \tan(b*x)^2 * \tan(1/2*a)^2 * \tan(b*c/d)^2 + 2*c*d^4*x * \tan(1/2*a)^4 * \tan(b*c/d)^2 + 2*d^5*x^2 * \tan(b*x)^2 * \tan(1/2*a)^2 + d^5*x^2 * \tan(1/2*a)^4 + c^2*d^3 * \tan(b*x)^2 * \tan(1/2*a)^4 + d^5*x^2 * \tan(b*x)^2 * \tan(b*c/d)^2 + 2*d^5*x^2 * \tan(1/2*a)^2 * \tan(b*c/d)^2 + 2*c^2*d^3 * \tan(b*x)^2 * \tan(1/2*a)^2 * \tan(b*c/d)^2 + c^2*d^3 * \tan(1/2*a)^4 * \tan(b*c/d)^2 + 4*c*d^4*x * \tan(b*x)^2 * \tan(1/2*a)^2 + 2*c*d^4*x * \tan(1/2*a)^4 + 2*c*d^4*x * \tan(b*x)^2 * \tan(b*c/d)^2 + 4*c*d^4*x * \tan(1/2*a)^2 * \tan(b*c/d)^2 + d^5*x^2 * \tan(b*x)^2 + 2*d^5*x^2 * \tan(1/2*a)^2 + 2*c^2*d^3 * \tan(b*x)^2 * \tan(1/2*a)^2 + c^2*d^3 * \tan(1/2*a)^4 + d^5*x^2 * \tan(b*c/d)^2 + c^2*d^3 * \tan(b*x)^2 * \tan(b*c/d)^2 + 2*c^2*d^3 * \tan(1/2*a)^2 * \tan(b*c/d)^2 + 2*c*d^4*x * \tan(b*x)^2 + 4*c*d^4*x * \tan(1/2*a)^2 + 2*c*d^4*x * \tan(b*c/d)^2 + d^5*x^2 + c^2*d^3 * \tan(b*x)^2 + 2*c^2*d^3 * \tan(1/2*a)^2 + c^2*d^3 * \tan(b*c/d)^2 + 2*c*d^4*x + c^2*d^3)
\end{aligned}$$

$$3.375 \quad \int \frac{\csc(a+bx) \sin(3a+3bx)}{(c+dx)^4} dx$$

Optimal. Leaf size=205

$$\frac{8b^3 \sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} + \frac{8b^3 \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} - \frac{2b^2 \sin^2(a+bx)}{3d^3(c+dx)} + \frac{2b^2 \cos^2(a+bx)}{d^3(c+dx)} +$$

[Out] $(-2*b^2)/(3*d^3*(c+d*x)) - \text{Cos}[a+b*x]^2/(d*(c+d*x)^3) + (2*b^2*\text{Cos}[a+b*x]^2)/(d^3*(c+d*x)) + (8*b^3*\text{CosIntegral}[(2*b*c)/d+2*b*x]*\text{Sin}[2*a-(2*b*c)/d])/(3*d^4) + (4*b*\text{Cos}[a+b*x]*\text{Sin}[a+b*x])/(3*d^2*(c+d*x)^2) + \text{Sin}[a+b*x]^2/(3*d*(c+d*x)^3) - (2*b^2*\text{Sin}[a+b*x]^2)/(3*d^3*(c+d*x)) + (8*b^3*\text{Cos}[2*a-(2*b*c)/d]*\text{SinIntegral}[(2*b*c)/d+2*b*x])/(3*d^4)$

Rubi [A] time = 0.379893, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4431, 3314, 32, 3313, 12, 3303, 3299, 3302}

$$\frac{8b^3 \sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} + \frac{8b^3 \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} - \frac{2b^2 \sin^2(a+bx)}{3d^3(c+dx)} + \frac{2b^2 \cos^2(a+bx)}{d^3(c+dx)} +$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Csc}[a+b*x]*\text{Sin}[3*a+3*b*x])/(c+d*x)^4, x]$

[Out] $(-2*b^2)/(3*d^3*(c+d*x)) - \text{Cos}[a+b*x]^2/(d*(c+d*x)^3) + (2*b^2*\text{Cos}[a+b*x]^2)/(d^3*(c+d*x)) + (8*b^3*\text{CosIntegral}[(2*b*c)/d+2*b*x]*\text{Sin}[2*a-(2*b*c)/d])/(3*d^4) + (4*b*\text{Cos}[a+b*x]*\text{Sin}[a+b*x])/(3*d^2*(c+d*x)^2) + \text{Sin}[a+b*x]^2/(3*d*(c+d*x)^3) - (2*b^2*\text{Sin}[a+b*x]^2)/(3*d^3*(c+d*x)) + (8*b^3*\text{Cos}[2*a-(2*b*c)/d]*\text{SinIntegral}[(2*b*c)/d+2*b*x])/(3*d^4)$

Rule 4431

$\text{Int}[(e_.) + (f_.)*(x_)]^{(m_.)}*(F_)[(a_.) + (b_.)*(x_)]^{(p_.)}*(G_)[(c_.) + (d_.)*(x_)]^{(q_.)}, x_Symbol] := \text{Int}[\text{ExpandTrigExpand}[(e+f*x)^m*G[c+d*x]^q, F, c+d*x, p, b/d, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \text{MemberQ}\{\{\text{Sin}, \text{Cos}\}, F\} \&\& \text{MemberQ}\{\{\text{Sec}, \text{Csc}\}, G\} \&\& \text{IGtQ}[p, 0] \&\& \text{IGtQ}[q, 0] \&\& \text{E} \text{qQ}[b*c - a*d, 0] \&\& \text{IGtQ}[b/d, 1]$

Rule 3314

$\text{Int}[(c_.) + (d_.)*(x_)]^{(m_.)}*((b_.)*\text{sin}[(e_.) + (f_.)*(x_)]^{(n_.)}, x_Symbol] := \text{Simp}[(c+d*x)^{(m+1)}*(b*\text{Sin}[e+f*x])^n/(d*(m+1)), x] + (\text{Dist}[(b^2*f^2*n*(n-1))/(d^2*(m+1)*(m+2)), \text{Int}[(c+d*x)^{(m+2)}*(b*\text{Sin}[e+f*x])^{(n-2)}, x], x] - \text{Dist}[(f^2*n^2)/(d^2*(m+1)*(m+2)), \text{Int}[(c+d*x)^{(m+2)}*(b*\text{Sin}[e+f*x])^n, x], x] - \text{Simp}[(b*f*n*(c+d*x)^{(m+2)}*\text{Cos}[e+f*x]*(b*\text{Sin}[e+f*x])^{(n-1)})/(d^2*(m+1)*(m+2)), x]) /; \text{FreeQ}\{b, c, d, e, f\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{LtQ}[m, -2]$

Rule 32

$\text{Int}[(a_.) + (b_.)*(x_)]^{(m_.)}, x_Symbol] := \text{Simp}[(a+b*x)^{(m+1)}/(b*(m+1)), x] /; \text{FreeQ}\{a, b, m\}, x] \&\& \text{NeQ}[m, -1]$

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
1), x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(a + bx) \sin(3a + 3bx)}{(c + dx)^4} dx &= \int \left(\frac{3 \cos^2(a + bx)}{(c + dx)^4} - \frac{\sin^2(a + bx)}{(c + dx)^4} \right) dx \\
&= 3 \int \frac{\cos^2(a + bx)}{(c + dx)^4} dx - \int \frac{\sin^2(a + bx)}{(c + dx)^4} dx \\
&= -\frac{\cos^2(a + bx)}{d(c + dx)^3} + \frac{4b \cos(a + bx) \sin(a + bx)}{3d^2(c + dx)^2} + \frac{\sin^2(a + bx)}{3d(c + dx)^3} - \frac{b^2 \int \frac{1}{(c+dx)^2} dx}{3d^2} + \frac{(2b^3 \operatorname{Ci}\left(\frac{2bc}{d} + 2bx\right) \sin\left(2a - \frac{2bc}{d}\right))}{3d^4} \\
&= -\frac{2b^2}{3d^3(c + dx)} - \frac{\cos^2(a + bx)}{d(c + dx)^3} + \frac{2b^2 \cos^2(a + bx)}{d^3(c + dx)} + \frac{4b \cos(a + bx) \sin(a + bx)}{3d^2(c + dx)^2} + \frac{\sin^2(a + bx)}{3d(c + dx)^3} \\
&= -\frac{2b^2}{3d^3(c + dx)} - \frac{\cos^2(a + bx)}{d(c + dx)^3} + \frac{2b^2 \cos^2(a + bx)}{d^3(c + dx)} + \frac{4b \cos(a + bx) \sin(a + bx)}{3d^2(c + dx)^2} + \frac{\sin^2(a + bx)}{3d(c + dx)^3} \\
&= -\frac{2b^2}{3d^3(c + dx)} - \frac{\cos^2(a + bx)}{d(c + dx)^3} + \frac{2b^2 \cos^2(a + bx)}{d^3(c + dx)} + \frac{4b \cos(a + bx) \sin(a + bx)}{3d^2(c + dx)^2} + \frac{\sin^2(a + bx)}{3d(c + dx)^3} \\
&= -\frac{2b^2}{3d^3(c + dx)} - \frac{\cos^2(a + bx)}{d(c + dx)^3} + \frac{2b^2 \cos^2(a + bx)}{d^3(c + dx)} + \frac{8b^3 \operatorname{Ci}\left(\frac{2bc}{d} + 2bx\right) \sin\left(2a - \frac{2bc}{d}\right)}{3d^4}
\end{aligned}$$

Mathematica [A] time = 1.12737, size = 125, normalized size = 0.61

$$\frac{8b^3 \sin\left(2a - \frac{2bc}{d}\right) \operatorname{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) + \frac{d(\cos(2(a+bx))(4b^2(c+dx)^2 - 2d^2) + d(2b(c+dx) \sin(2(a+bx)) - d))}{(c+dx)^3} + 8b^3 \cos\left(2a - \frac{2bc}{d}\right) \operatorname{Si}\left(\frac{2b(c+dx)}{d}\right)}{3d^4}$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^4,x]

[Out] (8*b^3*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*((-2*d^2 + 4*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + d*(-d + 2*b*(c + d*x)*Sin[2*(a + b*x)])))/(c + d*x)^3 + 8*b^3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d]/(3*d^4)

Maple [A] time = 0.039, size = 243, normalized size = 1.2

$$\frac{1}{3d(dx+c)^3} + 4\frac{1}{b}\left(\frac{1}{4}b^4\left(-\frac{2}{3}\frac{\cos(2bx+2a)}{((bx+a)d-ad+bc)^3d} - \frac{2}{3}\frac{1}{d}\left(-\frac{\sin(2bx+2a)}{((bx+a)d-ad+bc)^2d} + \frac{1}{d}\left(-2\frac{\cos(2bx+2a)}{((bx+a)d-ad+bc)}\right)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x)

[Out] 1/3/d/(d*x+c)^3+4/b*(1/4*b^4*(-2/3*cos(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)^3/d-2/3*(-sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)^2/d+(-2*cos(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)-1/6*b^4/((b*x+a)*d-a*d+b*c)^3/d)

Maxima [C] time = 1.51092, size = 190, normalized size = 0.93

$$\frac{3\left(E_4\left(\frac{2ibdx+2ibc}{d}\right) + E_4\left(-\frac{2ibdx+2ibc}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right) - \left(3iE_4\left(\frac{2ibdx+2ibc}{d}\right) - 3iE_4\left(-\frac{2ibdx+2ibc}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right) + 1}{3(d^4x^3 + 3cd^3x^2 + 3c^2d^2x + c^3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x, algorithm="maxima")

[Out] -1/3*(3*(exp_integral_e(4, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(4, -(2*I*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) - (3*I*exp_integral_e(4, (2*I*b*d*x + 2*I*b*c)/d) - 3*I*exp_integral_e(4, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c - a*d)/d + 1)/(d^4*x^3 + 3*c*d^3*x^2 + 3*c^2*d^2*x + c^3*d)

Fricas [A] time = 0.556165, size = 748, normalized size = 3.65

$$4b^2d^3x^2 + 8b^2cd^2x + 4b^2c^2d - d^3 - 4(2b^2d^3x^2 + 4b^2cd^2x + 2b^2c^2d - d^3)\cos(bx+a)^2 - 4(bd^3x + bcd^2)\cos(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x, algorithm="fricas")

[Out] -1/3*(4*b^2*d^3*x^2 + 8*b^2*c*d^2*x + 4*b^2*c^2*d - d^3 - 4*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 - 4*(b*d^3*x + b*c*d^2)*cos(b*x + a)*sin(b*x + a) - 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - 4*((b

$$\frac{(b^3 d^3 x^3 + 3 b^3 c d^2 x^2 + 3 b^3 c^2 d x + b^3 c^3) \cos_{\text{integral}}(2(b d x + b c)/d) + (b^3 d^3 x^3 + 3 b^3 c d^2 x^2 + 3 b^3 c^2 d x + b^3 c^3) \cos_{\text{integral}}(-2(b d x + b c)/d) \sin(-2(b c - a d)/d)}{(d^7 x^3 + 3 c d^6 x^2 + 3 c^2 d^5 x + c^3 d^4)}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**4,x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x, algorithm="giac")

[Out] Timed out

3.376 $\int (c + dx)^3 \csc^2(a + bx) \sin(3a + 3bx) dx$

Optimal. Leaf size=255

$$-\frac{18d^2(c + dx)\text{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^3} + \frac{18d^2(c + dx)\text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3} + \frac{9id(c + dx)^2\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^2} - \frac{9id(c + dx)^2\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^2}$$

```
[Out] (-6*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b - (24*d^2*(c + d*x)*Cos[a + b*x])/b^3 + (4*(c + d*x)^3*Cos[a + b*x])/b + ((9*I)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((9*I)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^2 - (18*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (18*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((18*I)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((18*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4 + (24*d^3*Sin[a + b*x])/b^4 - (12*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rubi [A] time = 0.34706, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.36$, Rules used = {4431, 4408, 3296, 2637, 4183, 2531, 6609, 2282, 6589}

$$-\frac{18d^2(c + dx)\text{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^3} + \frac{18d^2(c + dx)\text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3} + \frac{9id(c + dx)^2\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^2} - \frac{9id(c + dx)^2\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^2*Sin[3*a + 3*b*x], x]
```

```
[Out] (-6*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b - (24*d^2*(c + d*x)*Cos[a + b*x])/b^3 + (4*(c + d*x)^3*Cos[a + b*x])/b + ((9*I)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((9*I)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^2 - (18*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (18*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((18*I)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((18*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4 + (24*d^3*Sin[a + b*x])/b^4 - (12*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol] := Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && MemberQ[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && EQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637


```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(
m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 \csc^2(a + bx) \sin(3a + 3bx) dx &= \int \left(3(c + dx)^3 \cos(a + bx) \cot(a + bx) - (c + dx)^3 \sin(a + bx) \right) dx \\
&= 3 \int (c + dx)^3 \cos(a + bx) \cot(a + bx) dx - \int (c + dx)^3 \sin(a + bx) dx \\
&= \frac{(c + dx)^3 \cos(a + bx)}{b} + 3 \int (c + dx)^3 \csc(a + bx) dx - 3 \int (c + dx)^3 \sin(a + bx) dx \\
&= -\frac{6(c + dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4(c + dx)^3 \cos(a + bx)}{b} - \frac{3d(c + dx)^2 \sin(a + bx)}{b^2} \\
&= -\frac{6(c + dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d^2(c + dx) \cos(a + bx)}{b^3} + \frac{4(c + dx)^3 \cos(a + bx)}{b} \\
&= -\frac{6(c + dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{24d^2(c + dx) \cos(a + bx)}{b^3} + \frac{4(c + dx)^3 \cos(a + bx)}{b} \\
&= -\frac{6(c + dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{24d^2(c + dx) \cos(a + bx)}{b^3} + \frac{4(c + dx)^3 \cos(a + bx)}{b} \\
&= -\frac{6(c + dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{24d^2(c + dx) \cos(a + bx)}{b^3} + \frac{4(c + dx)^3 \cos(a + bx)}{b}
\end{aligned}$$

Mathematica [A] time = 1.57214, size = 459, normalized size = 1.8

$$3 \left(3id \left(b^2(c + dx)^2 \text{PolyLog}(2, -\cos(a + bx) - i \sin(a + bx)) + 2ibd(c + dx) \text{PolyLog}(3, -\cos(a + bx) - i \sin(a + bx)) \right) - 2 \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]

[Out] (3*(-2*b^3*(c + d*x)^3*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]] + (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] - 2*d^2*PolyLog[4, -Cos[a + b*x] - I*Sin[a + b*x]]) - (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, Cos[a + b*x] + I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, Cos[a + b*x] + I*Sin[a + b*x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Sin[a + b*x]]))/b^4 + (4*Cos[b*x]*(b^3*c^3*Cos[a] - 6*b*c*d^2*Cos[a] + 3*b^3*c^2*d*x*Cos[a] - 6*b*d^3*x*Cos[a] + 3*b^3*c*d^2*x^2*Cos[a] + b^3*d^3*x^3*Cos[a] - 3*b^2*c^2*d*Sin[a] + 6*d^3*Sin[a] - 6*b^2*c*d^2*x*Sin[a] - 3*b^2*d^3*x^2*Sin[a]))/b^4 - (4*(3*b^2*c^2*d*Cos[a] - 6*d^3*Cos[a] + 6*b^2*c*d^2*x*Cos[a] + 3*b^2*d^3*x^2*Cos[a] + b^3*c^3*Sin[a] - 6*b*c*d^2*Sin[a] + 3*b^3*c^2*d*x*Sin[a] - 6*b*d^3*x*Sin[a] + 3*b^3*c*d^2*x^2*Sin[a] + b^3*d^3*x^3*Sin[a])*Sin[b*x])/b^4

Maple [B] time = 0.196, size = 849, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x)

[Out] 2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))-9/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+9/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2+18*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4-9*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2+9*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-9*I/b^2*c^2*d*polylog(2,exp(I*(b

$$\begin{aligned} & x+a)))+6/b^4*d^3*a^3*\operatorname{arctanh}(\exp(I*(b*x+a)))-3/b^4*d^3*\ln(\exp(I*(b*x+a))+1) \\ & *a^3+9/b^3*c*d^2*a^2*\ln(\exp(I*(b*x+a))+1)+9*I/b^2*c^2*d*\operatorname{polylog}(2,-\exp(I*(b \\ & *x+a)))+2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^ \\ & 2-6*b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*\exp(I*(b*x \\ & +a))+18/b^2*c^2*d*a*\operatorname{arctanh}(\exp(I*(b*x+a)))-18/b^3*c*d^2*a^2*\operatorname{arctanh}(\exp(I* \\ & (b*x+a)))-9/b^2*c^2*d*\ln(\exp(I*(b*x+a))+1)*a-18*I/b^2*c*d^2*\operatorname{polylog}(2,\exp(I \\ & *(b*x+a)))*x+18*I/b^2*c*d^2*\operatorname{polylog}(2,-\exp(I*(b*x+a)))*x+3/b*d^3*\ln(1-\exp(I \\ & *(b*x+a)))*x^3+3/b^4*d^3*\ln(1-\exp(I*(b*x+a)))*a^3-3/b*d^3*\ln(\exp(I*(b*x+a)) \\ & +1)*x^3-9/b^3*c*d^2*a^2*\ln(1-\exp(I*(b*x+a)))-9/b*c^2*d*\ln(\exp(I*(b*x+a))+1) \\ & *x+9/b*c^2*d*\ln(1-\exp(I*(b*x+a)))*x+9/b^2*c^2*d*\ln(1-\exp(I*(b*x+a)))*a-6/b* \\ & c^3*\operatorname{arctanh}(\exp(I*(b*x+a)))-18*I*d^3*\operatorname{polylog}(4,-\exp(I*(b*x+a)))/b^4+18/b^3* \\ & d^3*\operatorname{polylog}(3,\exp(I*(b*x+a)))*x-18/b^3*d^3*\operatorname{polylog}(3,-\exp(I*(b*x+a)))*x+18/ \\ & b^3*c*d^2*\operatorname{polylog}(3,\exp(I*(b*x+a)))-18/b^3*c*d^2*\operatorname{polylog}(3,-\exp(I*(b*x+a))) \end{aligned}$$

Maxima [B] time = 1.80947, size = 813, normalized size = 3.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")

[Out] $\frac{1}{2}c^3(8\cos(bx+a) - 3\log(\cos(bx)^2 + 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 - 2\sin(bx)\sin(a) + \sin(a)^2) + 3\log(\cos(bx)^2 - 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 + 2\sin(bx)\sin(a) + \sin(a)^2))/b - \frac{1}{2}(36I^3d^3\operatorname{polylog}(4, -e^{(Ibx+Ia)}) - 36I^3d^3\operatorname{polylog}(4, e^{(Ibx+Ia)}) + (6I^3b^3d^3x^3 + 18I^3b^3cd^2x^2 + 18I^3b^3c^2d^2x)*\operatorname{arctan}2(\sin(bx+a), \cos(bx+a)+1) + (6I^3b^3d^3x^3 + 18I^3b^3cd^2x^2 + 18I^3b^3c^2d^2x)*\operatorname{arctan}2(\sin(bx+a), -\cos(bx+a)+1) - 8(b^3d^3x^3 + 3b^3cd^2x^2 - 6b^3cd^2 + 3(b^3c^2d - 2bd^3)x)*\cos(bx+a) + (-18I^3b^2d^3x^2 - 36I^3b^2cd^2x - 18I^3b^2c^2d)*\operatorname{dilog}(-e^{(Ibx+Ia)}) + (18I^3b^2d^3x^2 + 36I^3b^2cd^2x + 18I^3b^2c^2d)*\operatorname{dilog}(e^{(Ibx+Ia)}) + 3(b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2d^2x)*\log(\cos(bx+a)^2 + \sin(bx+a)^2 + 2\cos(bx+a) + 1) - 3(b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2d^2x)*\log(\cos(bx+a)^2 + \sin(bx+a)^2 - 2\cos(bx+a) + 1) + 36(b^3d^3x + b^3cd^2)*\operatorname{polylog}(3, -e^{(Ibx+Ia)}) - 36(b^3d^3x + b^3cd^2)*\operatorname{polylog}(3, e^{(Ibx+Ia)}) + 24(b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3)*\sin(bx+a))/b^4$

Fricas [C] time = 0.75591, size = 2314, normalized size = 9.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")

[Out] $\frac{1}{2}(18I^3d^3\operatorname{polylog}(4, \cos(bx+a) + I\sin(bx+a)) - 18I^3d^3\operatorname{polylog}(4, \cos(bx+a) - I\sin(bx+a)) + 18I^3d^3\operatorname{polylog}(4, -\cos(bx+a) + I\sin(bx+a)) - 18I^3d^3\operatorname{polylog}(4, -\cos(bx+a) - I\sin(bx+a)) + 8(b^3d^3x^3 + 3b^3cd^2x^2 + b^3c^3 - 6b^3cd^2 + 3(b^3c^2d - 2bd^3)x)*\cos(bx+a) + (-9I^3b^2d^3x^2 - 18I^3b^2cd^2x - 9I^3b^2c^2d)*\operatorname{dilog}(\cos(bx+a) + I\sin(bx+a)) + (9I^3b^2d^3x^2 + 18I^3b^2cd^2x + 9I^3b^2c^2d)*\operatorname{dilog}(\cos(bx+a) - I\sin(bx+a)) + (-9I^3b^2d^3x^2 - 18I^3b^2cd^2x - 9I^3b^2c^2d)*\operatorname{dilog}(-\cos(bx+a) + I\sin(bx+a)) + (9*$

```
I*b^2*d^3*x^2 + 18*I*b^2*c*d^2*x + 9*I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 18*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 18*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 18*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 18*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) - 24*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \csc(bx + a)^2 \sin(3bx + 3a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^2*sin(3*b*x + 3*a), x)
```

3.377 $\int (c + dx)^2 \csc^2(a + bx) \sin(3a + 3bx) dx$

Optimal. Leaf size=172

$$\frac{6id(c + dx)\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^2} - \frac{6id(c + dx)\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^2} - \frac{6d^2\text{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^3} + \frac{6d^2\text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3}$$

```
[Out] (-6*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (8*d^2*Cos[a + b*x])/b^3 + (4
*(c + d*x)^2*Cos[a + b*x])/b + ((6*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x
))])/b^2 - ((6*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (6*d^2*Pol
yLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^2*PolyLog[3, E^(I*(a + b*x))])/b^3 -
(8*d*(c + d*x)*Sin[a + b*x])/b^2
```

Rubi [A] time = 0.228793, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$, Rules used = {4431, 4408, 3296, 2638, 4183, 2531, 2282, 6589}

$$\frac{6id(c + dx)\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^2} - \frac{6id(c + dx)\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^2} - \frac{6d^2\text{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^3} + \frac{6d^2\text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^2*Sin[3*a + 3*b*x], x]
```

```
[Out] (-6*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (8*d^2*Cos[a + b*x])/b^3 + (4
*(c + d*x)^2*Cos[a + b*x])/b + ((6*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x
))])/b^2 - ((6*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (6*d^2*Pol
yLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^2*PolyLog[3, E^(I*(a + b*x))])/b^3 -
(8*d*(c + d*x)*Sin[a + b*x])/b^2
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] := Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] := -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/((b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^2 \csc^2(a + bx) \sin(3a + 3bx) dx &= \int (3(c + dx)^2 \cos(a + bx) \cot(a + bx) - (c + dx)^2 \sin(a + bx)) dx \\
&= 3 \int (c + dx)^2 \cos(a + bx) \cot(a + bx) dx - \int (c + dx)^2 \sin(a + bx) dx \\
&= \frac{(c + dx)^2 \cos(a + bx)}{b} + 3 \int (c + dx)^2 \csc(a + bx) dx - 3 \int (c + dx)^2 \sin(a + bx) dx \\
&= -\frac{6(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{4(c + dx)^2 \cos(a + bx)}{b} - \frac{2d(c + dx) \sin(a + bx)}{b^2} \\
&= -\frac{6(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{2d^2 \cos(a + bx)}{b^3} + \frac{4(c + dx)^2 \cos(a + bx)}{b} + \frac{6d(c + dx) \sin(a + bx)}{b^2} \\
&= -\frac{6(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{8d^2 \cos(a + bx)}{b^3} + \frac{4(c + dx)^2 \cos(a + bx)}{b} + \frac{6d(c + dx) \sin(a + bx)}{b^2} \\
&= -\frac{6(c + dx)^2 \tanh^{-1}(e^{i(a+bx)})}{b} - \frac{8d^2 \cos(a + bx)}{b^3} + \frac{4(c + dx)^2 \cos(a + bx)}{b} + \frac{6d(c + dx) \sin(a + bx)}{b^2}
\end{aligned}$$

Mathematica [A] time = 1.15642, size = 223, normalized size = 1.3

$$\frac{6ibd(c + dx)\text{PolyLog}\left(2, -e^{i(a+bx)}\right) - 6ibd(c + dx)\text{PolyLog}\left(2, e^{i(a+bx)}\right) - 6d^2\text{PolyLog}\left(3, -e^{i(a+bx)}\right) + 6d^2\text{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sin[3*a + 3*b*x], x]

```
[Out] (3*b^2*(c + d*x)^2*Log[1 - E^(I*(a + b*x))] - 3*b^2*(c + d*x)^2*Log[1 + E^(I*(a + b*x))] + (6*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] - (6*I)*b*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] - 6*d^2*PolyLog[3, -E^(I*(a + b*x))] + 6*d^2*PolyLog[3, E^(I*(a + b*x))] + 4*Cos[b*x]*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] - 2*b*d*(c + d*x)*Sin[a]) - 4*(2*b*d*(c + d*x)*Cos[a] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x])/b^3
```

Maple [B] time = 0.259, size = 481, normalized size = 2.8

$$2 \frac{(d^2 x^2 b^2 + 2 b^2 c d x + b^2 c^2 + 2 i b d^2 x - 2 d^2 + 2 i b c d) e^{i(bx+a)}}{b^3} + 2 \frac{(d^2 x^2 b^2 + 2 b^2 c d x + b^2 c^2 - 2 i b d^2 x - 2 d^2 - 2 i b c d) e^{-i(bx+a)}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x)
```

```
[Out] 2*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(I*(b*x+a))+2*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)/b^3*exp(-I*(b*x+a))-6/b^3*d^2*a^2*arctanh(exp(I*(b*x+a)))-6*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+6/b^3*c*d*ln(1-exp(I*(b*x+a)))*x+6/b^2*c*d*ln(1-exp(I*(b*x+a)))*a-6/b^2*c*d*ln(exp(I*(b*x+a))+1)*x-6/b^2*c*d*ln(exp(I*(b*x+a))+1)*a+12/b^2*c*d*a*arctanh(exp(I*(b*x+a)))-6*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x+6*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+3/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-3/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+6*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-3/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b^3*d^2*ln(exp(I*(b*x+a))+1)*a^2-6*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))+6*d^2*polylog(3,exp(I*(b*x+a)))/b^3-6/b^3*c^2*arctanh(exp(I*(b*x+a)))
```

Maxima [B] time = 1.56023, size = 552, normalized size = 3.21

$$\frac{c^2(8 \cos(bx + a) - 3 \log(\cos(bx)^2 + 2 \cos(bx) \cos(a) + \cos(a)^2 + \sin(bx)^2 - 2 \sin(bx) \sin(a) + \sin(a)^2) + 3 \log(2b))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*(8*cos(b*x + a) - 3*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b - 1/2*(12*d^2*polylog(3, -e^(I*b*x + I*a)) - 12*d^2*polylog(3, e^(I*b*x + I*a)) + (6*I*b^2*d^2*x^2 + 12*I*b^2*c*d*x)*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (6*I*b^2*d^2*x^2 + 12*I*b^2*c*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 8*(b^2*d^2*x^2 + 2*b^2*c*d*x - 2*d^2)*cos(b*x + a) + (-12*I*b*d^2*x - 12*I*b*c*d)*dilog(-e^(I*b*x + I*a)) + (12*I*b*d^2*x + 12*I*b*c*d)*dilog(e^(I*b*x + I*a)) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 16*(b*d^2*x + b*c*d)*sin(b*x + a))/b^3
```

Fricas [C] time = 0.637698, size = 1480, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*d^2*polylog(3, cos
(b*x + a) - I*sin(b*x + a)) - 6*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x +
a)) - 6*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 8*(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a) + (-6*I*b*d^2*x - 6*I*b*c*d)*dil
og(cos(b*x + a) + I*sin(b*x + a)) + (6*I*b*d^2*x + 6*I*b*c*d)*dilog(cos(b*x
+ a) - I*sin(b*x + a)) + (-6*I*b*d^2*x - 6*I*b*c*d)*dilog(-cos(b*x + a) +
I*sin(b*x + a)) + (6*I*b*d^2*x + 6*I*b*c*d)*dilog(-cos(b*x + a) - I*sin(b*x
+ a)) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) + I*sin(b
*x + a) + 1) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) - I
*sin(b*x + a) + 1) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x + a
) + 1/2*I*sin(b*x + a) + 1/2) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*
cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2
*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*(b^2*d^2*x^
2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) +
1) - 16*(b*d^2*x + b*c*d)*sin(b*x + a))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \csc(bx + a)^2 \sin(3bx + 3a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^2*sin(3*b*x + 3*a), x)
```


3.378 $\int (c + dx) \csc^2(a + bx) \sin(3a + 3bx) dx$

Optimal. Leaf size=95

$$\frac{3id\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^2} - \frac{3id\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^2} - \frac{4d \sin(a + bx)}{b^2} + \frac{4(c + dx) \cos(a + bx)}{b} - \frac{6(c + dx) \tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

[Out] $(-6*(c + d*x)*\text{ArcTanh}[E^{(I*(a + b*x))}])/b + (4*(c + d*x)*\text{Cos}[a + b*x])/b + ((3*I)*d*\text{PolyLog}[2, -E^{(I*(a + b*x))}])/b^2 - ((3*I)*d*\text{PolyLog}[2, E^{(I*(a + b*x))}])/b^2 - (4*d*\text{Sin}[a + b*x])/b^2$

Rubi [A] time = 0.108855, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4431, 4408, 3296, 2637, 4183, 2279, 2391}

$$\frac{3id\text{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^2} - \frac{3id\text{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^2} - \frac{4d \sin(a + bx)}{b^2} + \frac{4(c + dx) \cos(a + bx)}{b} - \frac{6(c + dx) \tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)*\text{Csc}[a + b*x]^2*\text{Sin}[3*a + 3*b*x], x]$

[Out] $(-6*(c + d*x)*\text{ArcTanh}[E^{(I*(a + b*x))}])/b + (4*(c + d*x)*\text{Cos}[a + b*x])/b + ((3*I)*d*\text{PolyLog}[2, -E^{(I*(a + b*x))}])/b^2 - ((3*I)*d*\text{PolyLog}[2, E^{(I*(a + b*x))}])/b^2 - (4*d*\text{Sin}[a + b*x])/b^2$

Rule 4431

$\text{Int}[(e_{.}) + (f_{.})*(x_{.})^{(m_{.})}*(F_{.})[(a_{.}) + (b_{.})*(x_{.})]^{(p_{.})}*(G_{.})[(c_{.}) + (d_{.})*(x_{.})]^{(q_{.})}, x_Symbol] := \text{Int}[\text{ExpandTrigExpand}[(e + f*x)^m*G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /;$ FreeQ[{a, b, c, d, e, f, m}, x] && MemberQ[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && EQ[b*c - a*d, 0] && IGtQ[b/d, 1]

Rule 4408

$\text{Int}[\text{Cos}[(a_{.}) + (b_{.})*(x_{.})]^{(n_{.})}*\text{Cot}[(a_{.}) + (b_{.})*(x_{.})]^{(p_{.})}*((c_{.}) + (d_{.})*(x_{.}))^{(m_{.})}, x_Symbol] := -\text{Int}[(c + d*x)^m*\text{Cos}[a + b*x]^n*\text{Cot}[a + b*x]^{(p - 2)}, x] + \text{Int}[(c + d*x)^m*\text{Cos}[a + b*x]^{(n - 2)}*\text{Cot}[a + b*x]^p, x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

$\text{Int}[(c_{.}) + (d_{.})*(x_{.})]^{(m_{.})}*\text{sin}[(e_{.}) + (f_{.})*(x_{.})], x_Symbol] := -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m - 1)}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

$\text{Int}[\text{sin}[\text{Pi}/2 + (c_{.}) + (d_{.})*(x_{.})], x_Symbol] := \text{Simp}[\text{Sin}[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 4183

$\text{Int}[\text{csc}[(e_{.}) + (f_{.})*(x_{.})]*((c_{.}) + (d_{.})*(x_{.}))^{(m_{.})}, x_Symbol] := \text{Simp}[(c + d*x)^m*\text{ArcTanh}[E^{(I*(e + f*x))}])/f, x] + (-\text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 - E^{(I*(e + f*x))}], x], x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + E^{(I*(e + f*x))}], x], x]$

```
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
 \int (c + dx) \csc^2(a + bx) \sin(3a + 3bx) dx &= \int (3(c + dx) \cos(a + bx) \cot(a + bx) - (c + dx) \sin(a + bx)) dx \\
 &= 3 \int (c + dx) \cos(a + bx) \cot(a + bx) dx - \int (c + dx) \sin(a + bx) dx \\
 &= \frac{(c + dx) \cos(a + bx)}{b} + 3 \int (c + dx) \csc(a + bx) dx - 3 \int (c + dx) \sin(a + bx) dx \\
 &= -\frac{6(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{4(c + dx) \cos(a + bx)}{b} - \frac{d \sin(a + bx)}{b^2} - \frac{(3d) \int (c + dx) \csc(a + bx) dx}{b^2} \\
 &= -\frac{6(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{4(c + dx) \cos(a + bx)}{b} - \frac{4d \sin(a + bx)}{b^2} + \frac{(3id) \int (c + dx) \csc(a + bx) dx}{b^2} \\
 &= -\frac{6(c + dx) \tanh^{-1}(e^{i(a+bx)})}{b} + \frac{4(c + dx) \cos(a + bx)}{b} + \frac{3id \operatorname{Li}_2(-e^{i(a+bx)})}{b^2} - \frac{3d \int (c + dx) \csc(a + bx) dx}{b^2}
 \end{aligned}$$

Mathematica [A] time = 0.341125, size = 171, normalized size = 1.8

$$\frac{3d \left(i \left(\operatorname{PolyLog} \left(2, -e^{i(a+bx)} \right) - \operatorname{PolyLog} \left(2, e^{i(a+bx)} \right) \right) + (a + bx) \left(\log \left(1 - e^{i(a+bx)} \right) - \log \left(1 + e^{i(a+bx)} \right) \right) \right)}{b^2} - \frac{4d \sin(a + bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^2*Sin[3*a + 3*b*x], x]
```

```
[Out] (4*c*Cos[a + b*x])/b + (4*d*x*Cos[a + b*x])/b - (3*c*Log[Cos[(a + b*x)/2]])
/b + (3*c*Log[Sin[(a + b*x)/2]])/b - (3*a*d*Log[Tan[(a + b*x)/2]])/b^2 + (3
*d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*(Po
lyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a + b*x))]))/b^2 - (4*d*Sin[
a + b*x])/b^2
```

Maple [B] time = 0.249, size = 205, normalized size = 2.2

$$2 \frac{(dxb + bc + id) e^{i(bx+a)}}{b^2} + 2 \frac{(dxb + bc - id) e^{-i(bx+a)}}{b^2} - 6 \frac{c \operatorname{Artanh}(e^{i(bx+a)})}{b} + 3 \frac{d \ln(1 - e^{i(bx+a)}) x}{b} + 3 \frac{d \ln(1 - e^{i(bx+a)})}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a), x)
```

```
[Out] 2*(d*x*b+b*c+I*d)/b^2*exp(I*(b*x+a))+2*(d*x*b+b*c-I*d)/b^2*exp(-I*(b*x+a))-
6/b*c*arctanh(exp(I*(b*x+a)))+3/b*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*d*ln(1-exp
(I*(b*x+a)))*a-3*I*d*polylog(2,exp(I*(b*x+a)))/b^2-3/b*d*ln(exp(I*(b*x+a))+
1)*x-3/b^2*d*ln(exp(I*(b*x+a))+1)*a+3*I*d*polylog(2,-exp(I*(b*x+a)))/b^2+6/
b^2*d*a*arctanh(exp(I*(b*x+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{c(8 \cos(bx + a) - 3 \log(\cos(bx)^2 + 2 \cos(bx) \cos(a) + \cos(a)^2 + \sin(bx)^2 - 2 \sin(bx) \sin(a) + \sin(a)^2) + 3 \log(\cos(bx)^2 - 2 \cos(bx) \cos(a) + \cos(a)^2 + \sin(bx)^2 - 2 \sin(bx) \sin(a) + \sin(a)^2))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] 1/2*c*(8*cos(b*x + a) - 3*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + s
in(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*log(cos(b*x)^2 - 2*cos(b*x)*c
os(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b + (4*b*x*c
os(b*x + a) + 3*b^2*integrate(x*sin(b*x + a)/(cos(b*x + a)^2 + sin(b*x + a)
^2 + 2*cos(b*x + a) + 1), x) + 3*b^2*integrate(x*sin(b*x + a)/(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1), x) - 4*sin(b*x + a))*d/b^2
```

Fricas [B] time = 0.58448, size = 814, normalized size = 8.57

$$8(bdx + bc) \cos(bx + a) - 3i d \operatorname{Li}_2(\cos(bx + a) + i \sin(bx + a)) + 3i d \operatorname{Li}_2(\cos(bx + a) - i \sin(bx + a)) - 3i d \operatorname{Li}_2(-\cos(bx + a) + i \sin(bx + a)) + 3i d \operatorname{Li}_2(-\cos(bx + a) - i \sin(bx + a))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(8*(b*d*x + b*c)*cos(b*x + a) - 3*I*d*dilog(cos(b*x + a) + I*sin(b*x +
a)) + 3*I*d*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*I*d*dilog(-cos(b*x + a
) + I*sin(b*x + a)) + 3*I*d*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*(b*d*x
+ b*c)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b*d*x + b*c)*log(cos(b
*x + a) - I*sin(b*x + a) + 1) + 3*(b*c - a*d)*log(-1/2*cos(b*x + a) + 1/2*I
*sin(b*x + a) + 1/2) + 3*(b*c - a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x
+ a) + 1/2) + 3*(b*d*x + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*(
b*d*x + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 8*d*sin(b*x + a))/b^
2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \csc (bx + a)^2 \sin (3bx + 3a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^2*sin(3*b*x + 3*a), x)
```

$$3.379 \quad \int \frac{\csc^2(a+bx) \sin(3a+3bx)}{c+dx} dx$$

Optimal. Leaf size=71

$$3\text{Unintegrable}\left(\frac{\csc(a+bx)}{c+dx}, x\right) - \frac{4 \sin\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{d} - \frac{4 \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d}$$

[Out] (-4*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d - (4*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d + 3*Unintegrable[Csc[a + b*x]/(c + d*x), x]

Rubi [A] time = 0.2104, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^2(a+bx) \sin(3a+3bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x), x]

[Out] (-4*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d - (4*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d + 3*Defer[Int][Csc[a + b*x]/(c + d*x), x]

Rubi steps

$$\begin{aligned} \int \frac{\csc^2(a+bx) \sin(3a+3bx)}{c+dx} dx &= \int \left(\frac{3 \cos(a+bx) \cot(a+bx)}{c+dx} - \frac{\sin(a+bx)}{c+dx} \right) dx \\ &= 3 \int \frac{\cos(a+bx) \cot(a+bx)}{c+dx} dx - \int \frac{\sin(a+bx)}{c+dx} dx \\ &= 3 \int \frac{\csc(a+bx)}{c+dx} dx - 3 \int \frac{\sin(a+bx)}{c+dx} dx - \cos\left(a - \frac{bc}{d}\right) \int \frac{\sin\left(\frac{bc}{d} + bx\right)}{c+dx} dx - \sin\left(a - \frac{bc}{d}\right) \int \frac{\cos\left(\frac{bc}{d} + bx\right)}{c+dx} dx \\ &= -\frac{\text{Ci}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{d} - \frac{\cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d} + 3 \int \frac{\csc(a+bx)}{c+dx} dx - \left(3 \int \frac{\sin(a+bx)}{c+dx} dx\right) \\ &= -\frac{4 \text{Ci}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{d} - \frac{4 \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d} + 3 \int \frac{\csc(a+bx)}{c+dx} dx \end{aligned}$$

Mathematica [A] time = 6.23243, size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx) \sin(3a+3bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x), x]

[Out] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x), x]

Maple [A] time = 0.349, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx + a))^2 \sin(3bx + 3a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)

[Out] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(2i E_1\left(\frac{ibdx+ibc}{d}\right) - 2i E_1\left(-\frac{ibdx+ibc}{d}\right)\right) \cos\left(-\frac{bc-ad}{d}\right) + 3d \int \frac{\sin(bx+a)}{(dx+c)(\cos(bx+a)^2 + \sin(bx+a)^2 + 2\cos(bx+a)+1)} dx + 3d \int \frac{1}{(dx+c)(\cos(bx+a)^2 + \sin(bx+a)^2 + 2\cos(bx+a)+1)} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")

[Out] ((2*I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - 2*I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + 3*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)*cos(b*x + a) + c), x) + 3*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) + 2*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d)/d

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx + a)^2 \sin(3bx + 3a)}{dx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")

[Out] integral(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c),x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx + a)^2 \sin(3bx + 3a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

$$3.380 \quad \int \frac{\csc^2(a+bx) \sin(3a+3bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=91

$$3\text{Unintegrable}\left(\frac{\csc(a+bx)}{(c+dx)^2}, x\right) - \frac{4b \cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{d^2} + \frac{4b \sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d^2} + \frac{4 \sin(a+bx)}{d(c+dx)}$$

[Out] (-4*b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2 + (4*Sin[a + b*x])/(d*(c + d*x)) + (4*b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + 3*Unintegrable[Csc[a + b*x]/(c + d*x)^2, x]

Rubi [A] time = 0.283036, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^2(a+bx) \sin(3a+3bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]

[Out] (-4*b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2 + (4*Sin[a + b*x])/(d*(c + d*x)) + (4*b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + 3*Derivative[Int][Csc[a + b*x]/(c + d*x)^2, x]

Rubi steps

$$\begin{aligned} \int \frac{\csc^2(a+bx) \sin(3a+3bx)}{(c+dx)^2} dx &= \int \left(\frac{3 \cos(a+bx) \cot(a+bx)}{(c+dx)^2} - \frac{\sin(a+bx)}{(c+dx)^2} \right) dx \\ &= 3 \int \frac{\cos(a+bx) \cot(a+bx)}{(c+dx)^2} dx - \int \frac{\sin(a+bx)}{(c+dx)^2} dx \\ &= \frac{\sin(a+bx)}{d(c+dx)} + 3 \int \frac{\csc(a+bx)}{(c+dx)^2} dx - 3 \int \frac{\sin(a+bx)}{(c+dx)^2} dx - \frac{b \int \frac{\cos(a+bx)}{c+dx} dx}{d} \\ &= \frac{4 \sin(a+bx)}{d(c+dx)} + 3 \int \frac{\csc(a+bx)}{(c+dx)^2} dx - \frac{(3b) \int \frac{\cos(a+bx)}{c+dx} dx}{d} - \frac{\left(b \cos\left(a - \frac{bc}{d}\right)\right) \int \frac{\cos\left(\frac{bc}{d} + bx\right)}{c+dx} dx}{d} \\ &= -\frac{b \cos\left(a - \frac{bc}{d}\right) \text{Ci}\left(\frac{bc}{d} + bx\right)}{d^2} + \frac{4 \sin(a+bx)}{d(c+dx)} + \frac{b \sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d^2} + 3 \int \frac{\csc(a+bx)}{(c+dx)^2} dx \\ &= -\frac{4b \cos\left(a - \frac{bc}{d}\right) \text{Ci}\left(\frac{bc}{d} + bx\right)}{d^2} + \frac{4 \sin(a+bx)}{d(c+dx)} + \frac{4b \sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d^2} + 3 \int \frac{\csc(a+bx)}{(c+dx)^2} dx \end{aligned}$$

Mathematica [A] time = 6.78663, size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx) \sin(3a+3bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]

[Out] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2, x]

Maple [A] time = 0.7, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx + a))^2 \sin(3bx + 3a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)

[Out] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx + a)^2 \sin(3bx + 3a)}{d^2x^2 + 2cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**2,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx + a)^2 \sin(3bx + 3a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^2, x)
```

$$3.381 \quad \int \frac{\csc^2(a+bx) \sin(3a+3bx)}{(c+dx)^3} dx$$

Optimal. Leaf size=114

$$3\text{Unintegrable}\left(\frac{\csc(a+bx)}{(c+dx)^3}, x\right) + \frac{2b^2 \sin\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{d^3} + \frac{2b^2 \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d^3} + \frac{2b \cos(a)}{d^2(c+dx)}$$

[Out] (2*b*Cos[a + b*x])/(d^2*(c + d*x)) + (2*b^2*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d^3 + (2*Sin[a + b*x])/(d*(c + d*x)^2) + (2*b^2*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^3 + 3*Unintegrable[Csc[a + b*x]/(c + d*x)^3, x]

Rubi [A] time = 0.328324, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^2(a+bx) \sin(3a+3bx)}{(c+dx)^3} dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3, x]

[Out] (2*b*Cos[a + b*x])/(d^2*(c + d*x)) + (2*b^2*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d^3 + (2*Sin[a + b*x])/(d*(c + d*x)^2) + (2*b^2*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^3 + 3*Defer[Int][Csc[a + b*x]/(c + d*x)^3, x]

Rubi steps

$$\begin{aligned} \int \frac{\csc^2(a+bx) \sin(3a+3bx)}{(c+dx)^3} dx &= \int \left(\frac{3 \cos(a+bx) \cot(a+bx)}{(c+dx)^3} - \frac{\sin(a+bx)}{(c+dx)^3} \right) dx \\ &= 3 \int \frac{\cos(a+bx) \cot(a+bx)}{(c+dx)^3} dx - \int \frac{\sin(a+bx)}{(c+dx)^3} dx \\ &= \frac{\sin(a+bx)}{2d(c+dx)^2} + 3 \int \frac{\csc(a+bx)}{(c+dx)^3} dx - 3 \int \frac{\sin(a+bx)}{(c+dx)^3} dx - \frac{b \int \frac{\cos(a+bx)}{(c+dx)^2} dx}{2d} \\ &= \frac{b \cos(a+bx)}{2d^2(c+dx)} + \frac{2 \sin(a+bx)}{d(c+dx)^2} + 3 \int \frac{\csc(a+bx)}{(c+dx)^3} dx + \frac{b^2 \int \frac{\sin(a+bx)}{c+dx} dx}{2d^2} - \frac{(3b) \int \frac{\cos(a+bx)}{c+dx} dx}{2d} \\ &= \frac{2b \cos(a+bx)}{d^2(c+dx)} + \frac{2 \sin(a+bx)}{d(c+dx)^2} + 3 \int \frac{\csc(a+bx)}{(c+dx)^3} dx + \frac{(3b^2) \int \frac{\sin(a+bx)}{c+dx} dx}{2d^2} + \frac{(3b) \int \frac{\cos(a+bx)}{c+dx} dx}{2d} \\ &= \frac{2b \cos(a+bx)}{d^2(c+dx)} + \frac{b^2 \text{Ci}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{2d^3} + \frac{2 \sin(a+bx)}{d(c+dx)^2} + \frac{b^2 \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{2d^3} \\ &= \frac{2b \cos(a+bx)}{d^2(c+dx)} + \frac{2b^2 \text{Ci}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{d^3} + \frac{2 \sin(a+bx)}{d(c+dx)^2} + \frac{2b^2 \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d^3} \end{aligned}$$

Mathematica [A] time = 7.24875, size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx) \sin(3a+3bx)}{(c+dx)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]

[Out] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3, x]

Maple [A] time = 0.71, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx + a))^2 \sin(3bx + 3a)}{(dx + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)

[Out] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx + a)^2 \sin(3bx + 3a)}{d^3x^3 + 3cd^2x^2 + 3c^2dx + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")

[Out] integral(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**3,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx + a)^2 \sin(3bx + 3a)}{(dx + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^3, x)
```

3.382 $\int (c + dx)^4 \sec(a + bx) \sin(3a + 3bx) dx$

Optimal. Leaf size=299

$$\frac{3d^2(c + dx)^2 \text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{b^3} + \frac{3id^3(c + dx) \text{PolyLog}\left(4, -e^{2i(a+bx)}\right)}{b^4} - \frac{2id(c + dx)^3 \text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^2} - \frac{3d^4 P}{b^2}$$

[Out] $(6*c*d^3*x)/b^3 + (3*d^4*x^2)/b^3 - (c + d*x)^4/b - ((I/5)*(c + d*x)^5)/d + ((c + d*x)^4*\text{Log}[1 + E^((2*I)*(a + b*x))])/b - ((2*I)*d*(c + d*x)^3*\text{PolyLog}[2, -E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)^2*\text{PolyLog}[3, -E^((2*I)*(a + b*x))])/b^3 + ((3*I)*d^3*(c + d*x)*\text{PolyLog}[4, -E^((2*I)*(a + b*x))])/b^4 - (3*d^4*\text{PolyLog}[5, -E^((2*I)*(a + b*x))])/(2*b^5) - (6*d^3*(c + d*x)*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b^4 + (4*d*(c + d*x)^3*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b^2 + (3*d^4*\text{Sin}[a + b*x]^2)/b^5 - (6*d^2*(c + d*x)^2*\text{Sin}[a + b*x]^2)/b^3 + (2*(c + d*x)^4*\text{Sin}[a + b*x]^2)/b$

Rubi [A] time = 0.503125, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {4431, 4404, 3311, 32, 3310, 4407, 3719, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c + dx)^2 \text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{b^3} + \frac{3id^3(c + dx) \text{PolyLog}\left(4, -e^{2i(a+bx)}\right)}{b^4} - \frac{2id(c + dx)^3 \text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^2} - \frac{3d^4 P}{b^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^4*\text{Sec}[a + b*x]*\text{Sin}[3*a + 3*b*x], x]$

[Out] $(6*c*d^3*x)/b^3 + (3*d^4*x^2)/b^3 - (c + d*x)^4/b - ((I/5)*(c + d*x)^5)/d + ((c + d*x)^4*\text{Log}[1 + E^((2*I)*(a + b*x))])/b - ((2*I)*d*(c + d*x)^3*\text{PolyLog}[2, -E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)^2*\text{PolyLog}[3, -E^((2*I)*(a + b*x))])/b^3 + ((3*I)*d^3*(c + d*x)*\text{PolyLog}[4, -E^((2*I)*(a + b*x))])/b^4 - (3*d^4*\text{PolyLog}[5, -E^((2*I)*(a + b*x))])/(2*b^5) - (6*d^3*(c + d*x)*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b^4 + (4*d*(c + d*x)^3*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b^2 + (3*d^4*\text{Sin}[a + b*x]^2)/b^5 - (6*d^2*(c + d*x)^2*\text{Sin}[a + b*x]^2)/b^3 + (2*(c + d*x)^4*\text{Sin}[a + b*x]^2)/b$

Rule 4431

$\text{Int}[(e_. + (f_.)*(x_.))^(m_.)*(F_.)[(a_. + (b_.)*(x_.))^(p_.)*(G_.)[(c_. + (d_.)*(x_.))^(q_.), x_Symbol] :> \text{Int}[\text{ExpandTrigExpand}[(e + f*x)^m*G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x\} \&\& \text{MemberQ}\{\{\text{Sin}, \text{Cos}\}, F\} \&\& \text{MemberQ}\{\{\text{Sec}, \text{Csc}\}, G\} \&\& \text{IGtQ}[p, 0] \&\& \text{IGtQ}[q, 0] \&\& \text{EQ}[b*c - a*d, 0] \&\& \text{IGtQ}[b/d, 1]$

Rule 4404

$\text{Int}[\text{Cos}[(a_. + (b_.)*(x_.))*((c_. + (d_.)*(x_.))^(m_.)*\text{Sin}[(a_. + (b_.)*(x_.))^(n_.), x_Symbol] :> \text{Simp}[(c + d*x)^m*\text{Sin}[a + b*x]^(n + 1)]/(b*(n + 1)), x] - \text{Dist}[(d*m)/(b*(n + 1)), \text{Int}[(c + d*x)^(m - 1)*\text{Sin}[a + b*x]^(n + 1), x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x\} \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[n, -1]$

Rule 3311

$\text{Int}[(c_. + (d_.)*(x_.))^(m_.)*((b_.)*\text{sin}[(e_. + (f_.)*(x_.))^(n_.), x_Symbol] :> \text{Simp}[(d*m*(c + d*x)^(m - 1)*(b*\text{Sin}[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n - 1))/n, \text{Int}[(c + d*x)^m*(b*\text{Sin}[e + f*x])^(n - 2), x], x] - \text{Dist}[(d^2*m*(m - 1))/(f^2*n^2), \text{Int}[(c + d*x)^(m - 2)*(b*\text{Sin}[e + f*x])^n, x], x]$

- Simp[(b*(c + d*x)^m*cos[e + f*x]*(b*sin[e + f*x])^(n - 1))/(f*n), x] /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 3310

Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
Simp[(d*(b*sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)*(b*sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*cos[e + f*x]*(b*sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rule 4407

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(a_.) + (b_.)*(x_)]^(n_.)*tan[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := -Int[(c + d*x)^m*sin[a + b*x]^n*tan[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*sin[a + b*x]^(n - 2)*tan[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3719

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[(c + d*x)^m*E^(2*I*(e + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2190

Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] := Simp[(c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.))]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[(f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 6609

Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(p_.)], x_Symbol] := Simp[(e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[

{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned} \int (c + dx)^4 \sec(a + bx) \sin(3a + 3bx) dx &= \int (3(c + dx)^4 \cos(a + bx) \sin(a + bx) - (c + dx)^4 \sin^2(a + bx) \tan(a + bx)) dx \\ &= 3 \int (c + dx)^4 \cos(a + bx) \sin(a + bx) dx - \int (c + dx)^4 \sin^2(a + bx) \tan(a + bx) dx \\ &= \frac{3(c + dx)^4 \sin^2(a + bx)}{2b} - \frac{(6d) \int (c + dx)^3 \sin^2(a + bx) dx}{b} + \int (c + dx)^4 \cos(a + bx) dx \\ &= -\frac{i(c + dx)^5}{5d} + \frac{3d(c + dx)^3 \cos(a + bx) \sin(a + bx)}{b^2} - \frac{9d^2(c + dx)^2 \sin^2(a + bx)}{2b^3} \\ &= -\frac{3(c + dx)^4}{4b} - \frac{i(c + dx)^5}{5d} + \frac{(c + dx)^4 \log(1 + e^{2i(a+bx)})}{b} - \frac{9d^3(c + dx) \cos(a + bx)}{2b^4} \\ &= \frac{9cd^3x}{2b^3} + \frac{9d^4x^2}{4b^3} - \frac{(c + dx)^4}{b} - \frac{i(c + dx)^5}{5d} + \frac{(c + dx)^4 \log(1 + e^{2i(a+bx)})}{b} - \frac{2id(c + dx)^4}{b^4} \\ &= \frac{6cd^3x}{b^3} + \frac{3d^4x^2}{b^3} - \frac{(c + dx)^4}{b} - \frac{i(c + dx)^5}{5d} + \frac{(c + dx)^4 \log(1 + e^{2i(a+bx)})}{b} - \frac{2id(c + dx)^4}{b^4} \\ &= \frac{6cd^3x}{b^3} + \frac{3d^4x^2}{b^3} - \frac{(c + dx)^4}{b} - \frac{i(c + dx)^5}{5d} + \frac{(c + dx)^4 \log(1 + e^{2i(a+bx)})}{b} - \frac{2id(c + dx)^4}{b^4} \\ &= \frac{6cd^3x}{b^3} + \frac{3d^4x^2}{b^3} - \frac{(c + dx)^4}{b} - \frac{i(c + dx)^5}{5d} + \frac{(c + dx)^4 \log(1 + e^{2i(a+bx)})}{b} - \frac{2id(c + dx)^4}{b^4} \\ &= \frac{6cd^3x}{b^3} + \frac{3d^4x^2}{b^3} - \frac{(c + dx)^4}{b} - \frac{i(c + dx)^5}{5d} + \frac{(c + dx)^4 \log(1 + e^{2i(a+bx)})}{b} - \frac{2id(c + dx)^4}{b^4} \end{aligned}$$

Mathematica [B] time = 6.74213, size = 2482, normalized size = 8.3

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^4*Sec[a + b*x]*Sin[3*a + 3*b*x],x]

[Out] ((I/2)*c^2*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))]*Sec[a])/(b^3*E^(I*a)) + (I/2)*c*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a) - ((4*I)*(1 + E^((-2*I)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1 + E^((2*I)*a))*(2*b^2*x^2*PolyLog[2, -E^((-2*I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-2*I)*(a + b*x))] - PolyLog[4, -E^((-2*I)*(a + b*x))]))/(b^4*E^((2*I)*a))*Sec[a] - (d^4*((-4*I)*x^5 - (10*(1 + E^((2*I)*a))*x^4*Log[1 + E^((-2*I)*(a + b*x))])/b + (5*(1 + E^((2*I)*a))*((-4*I)*b^3*x^3*PolyLog[2, -E^((-2*I)*(a + b*x))] - 6*b^2*x^2*PolyLog[3, -E^((-2*I)*(a + b*x))] + (6*I)*b*x*PolyLog[4, -E^((-2*I)*(a + b*x))] + 3*PolyLog[5, -E^((-2*I)*(a + b*x))]))/b^5*Sec[a])/(20*E^(I*a)) + (c^4*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (2*c^3*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]])) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]])) - Pi*Log[1 + E^((-2*I)*b*x

$$\begin{aligned}
&)] - 2*(b*x - \text{ArcTan}[\text{Cot}[a]])*\text{Log}[1 - E^{((2*I)*(b*x - \text{ArcTan}[\text{Cot}[a]]))}] + P \\
&i*\text{Log}[\text{Cos}[b*x]] - 2*\text{ArcTan}[\text{Cot}[a]]*\text{Log}[\text{Sin}[b*x - \text{ArcTan}[\text{Cot}[a]]]] + I*\text{PolyL} \\
&\text{og}[2, E^{((2*I)*(b*x - \text{ArcTan}[\text{Cot}[a]]))}]/\text{Sqrt}[1 + \text{Cot}[a]^2]*\text{Sec}[a]/(b^2* \\
&\text{Sqrt}[\text{Csc}[a]^2*(\text{Cos}[a]^2 + \text{Sin}[a]^2)]) + \text{Sec}[a]*(\text{Cos}[2*a + 2*b*x]/(40*b^5) - \\
&((I/40)*\text{Sin}[2*a + 2*b*x])/b^5)*(-20*b^4*c^4*\text{Cos}[a] + (40*I)*b^3*c^3*d*\text{Cos}[\\
&a] + 60*b^2*c^2*d^2*\text{Cos}[a] - (60*I)*b*c*d^3*\text{Cos}[a] - 30*d^4*\text{Cos}[a] - 80*b^4 \\
&*c^3*d*x*\text{Cos}[a] + (120*I)*b^3*c^2*d^2*x*\text{Cos}[a] + 120*b^2*c*d^3*x*\text{Cos}[a] - (\\
&60*I)*b*d^4*x*\text{Cos}[a] - 120*b^4*c^2*d^2*x^2*\text{Cos}[a] + (120*I)*b^3*c*d^3*x^2*\text{C} \\
&\text{os}[a] + 60*b^2*d^4*x^2*\text{Cos}[a] - 80*b^4*c*d^3*x^3*\text{Cos}[a] + (40*I)*b^3*d^4*x^ \\
&3*\text{Cos}[a] - 20*b^4*d^4*x^4*\text{Cos}[a] - (20*I)*b^5*c^4*x*\text{Cos}[a + 2*b*x] - (40*I) \\
&*b^5*c^3*d*x^2*\text{Cos}[a + 2*b*x] - (40*I)*b^5*c^2*d^2*x^3*\text{Cos}[a + 2*b*x] - (20 \\
&*I)*b^5*c*d^3*x^4*\text{Cos}[a + 2*b*x] - (4*I)*b^5*d^4*x^5*\text{Cos}[a + 2*b*x] + (20*I) \\
&)*b^5*c^4*x*\text{Cos}[3*a + 2*b*x] + (40*I)*b^5*c^3*d*x^2*\text{Cos}[3*a + 2*b*x] + (40* \\
&I)*b^5*c^2*d^2*x^3*\text{Cos}[3*a + 2*b*x] + (20*I)*b^5*c*d^3*x^4*\text{Cos}[3*a + 2*b*x] \\
&+ (4*I)*b^5*d^4*x^5*\text{Cos}[3*a + 2*b*x] - 10*b^4*c^4*\text{Cos}[3*a + 4*b*x] - (20*I) \\
&)*b^3*c^3*d*\text{Cos}[3*a + 4*b*x] + 30*b^2*c^2*d^2*\text{Cos}[3*a + 4*b*x] + (30*I)*b*c \\
&*d^3*\text{Cos}[3*a + 4*b*x] - 15*d^4*\text{Cos}[3*a + 4*b*x] - 40*b^4*c^3*d*x*\text{Cos}[3*a + \\
&4*b*x] - (60*I)*b^3*c^2*d^2*x*\text{Cos}[3*a + 4*b*x] + 60*b^2*c*d^3*x*\text{Cos}[3*a + 4 \\
&*b*x] + (30*I)*b*d^4*x*\text{Cos}[3*a + 4*b*x] - 60*b^4*c^2*d^2*x^2*\text{Cos}[3*a + 4*b* \\
&x] - (60*I)*b^3*c*d^3*x^2*\text{Cos}[3*a + 4*b*x] + 30*b^2*d^4*x^2*\text{Cos}[3*a + 4*b*x] \\
&- 40*b^4*c*d^3*x^3*\text{Cos}[3*a + 4*b*x] - (20*I)*b^3*d^4*x^3*\text{Cos}[3*a + 4*b*x] \\
&- 10*b^4*d^4*x^4*\text{Cos}[3*a + 4*b*x] - 10*b^4*c^4*\text{Cos}[5*a + 4*b*x] - (20*I)*b \\
&^3*c^3*d*\text{Cos}[5*a + 4*b*x] + 30*b^2*c^2*d^2*\text{Cos}[5*a + 4*b*x] + (30*I)*b*c*d^ \\
&3*\text{Cos}[5*a + 4*b*x] - 15*d^4*\text{Cos}[5*a + 4*b*x] - 40*b^4*c^3*d*x*\text{Cos}[5*a + 4*b \\
&*x] - (60*I)*b^3*c^2*d^2*x*\text{Cos}[5*a + 4*b*x] + 60*b^2*c*d^3*x*\text{Cos}[5*a + 4*b* \\
&x] + (30*I)*b*d^4*x*\text{Cos}[5*a + 4*b*x] - 60*b^4*c^2*d^2*x^2*\text{Cos}[5*a + 4*b*x] \\
&- (60*I)*b^3*c*d^3*x^2*\text{Cos}[5*a + 4*b*x] + 30*b^2*d^4*x^2*\text{Cos}[5*a + 4*b*x] - \\
&40*b^4*c*d^3*x^3*\text{Cos}[5*a + 4*b*x] - (20*I)*b^3*d^4*x^3*\text{Cos}[5*a + 4*b*x] - \\
&10*b^4*d^4*x^4*\text{Cos}[5*a + 4*b*x] + 20*b^5*c^4*x*\text{Sin}[a + 2*b*x] + 40*b^5*c^3* \\
&d*x^2*\text{Sin}[a + 2*b*x] + 40*b^5*c^2*d^2*x^3*\text{Sin}[a + 2*b*x] + 20*b^5*c*d^3*x^4 \\
&)*\text{Sin}[a + 2*b*x] + 4*b^5*d^4*x^5*\text{Sin}[a + 2*b*x] - 20*b^5*c^4*x*\text{Sin}[3*a + 2*b \\
&]*x] - 40*b^5*c^3*d*x^2*\text{Sin}[3*a + 2*b*x] - 40*b^5*c^2*d^2*x^3*\text{Sin}[3*a + 2*b* \\
&x] - 20*b^5*c*d^3*x^4*\text{Sin}[3*a + 2*b*x] - 4*b^5*d^4*x^5*\text{Sin}[3*a + 2*b*x] - (\\
&10*I)*b^4*c^4*\text{Sin}[3*a + 4*b*x] + 20*b^3*c^3*d*\text{Sin}[3*a + 4*b*x] + (30*I)*b^2 \\
&)*c^2*d^2*\text{Sin}[3*a + 4*b*x] - 30*b*c*d^3*\text{Sin}[3*a + 4*b*x] - (15*I)*d^4*\text{Sin}[3* \\
&a + 4*b*x] - (40*I)*b^4*c^3*d*x*\text{Sin}[3*a + 4*b*x] + 60*b^3*c^2*d^2*x*\text{Sin}[3*a \\
&+ 4*b*x] + (60*I)*b^2*c*d^3*x*\text{Sin}[3*a + 4*b*x] - 30*b*d^4*x*\text{Sin}[3*a + 4*b* \\
&x] - (60*I)*b^4*c^2*d^2*x^2*\text{Sin}[3*a + 4*b*x] + 60*b^3*c*d^3*x^2*\text{Sin}[3*a + 4 \\
&*b*x] + (30*I)*b^2*d^4*x^2*\text{Sin}[3*a + 4*b*x] - (40*I)*b^4*c*d^3*x^3*\text{Sin}[3*a \\
&+ 4*b*x] + 20*b^3*d^4*x^3*\text{Sin}[3*a + 4*b*x] - (10*I)*b^4*d^4*x^4*\text{Sin}[3*a + 4 \\
&*b*x] - (10*I)*b^4*c^4*\text{Sin}[5*a + 4*b*x] + 20*b^3*c^3*d*\text{Sin}[5*a + 4*b*x] + (\\
&30*I)*b^2*c^2*d^2*\text{Sin}[5*a + 4*b*x] - 30*b*c*d^3*\text{Sin}[5*a + 4*b*x] - (15*I)*d \\
&^4*\text{Sin}[5*a + 4*b*x] - (40*I)*b^4*c^3*d*x*\text{Sin}[5*a + 4*b*x] + 60*b^3*c^2*d^2* \\
&x*\text{Sin}[5*a + 4*b*x] + (60*I)*b^2*c*d^3*x*\text{Sin}[5*a + 4*b*x] - 30*b*d^4*x*\text{Sin}[5 \\
&*a + 4*b*x] - (60*I)*b^4*c^2*d^2*x^2*\text{Sin}[5*a + 4*b*x] + 60*b^3*c*d^3*x^2*\text{Si} \\
&\text{n}[5*a + 4*b*x] + (30*I)*b^2*d^4*x^2*\text{Sin}[5*a + 4*b*x] - (40*I)*b^4*c*d^3*x^3 \\
&)*\text{Sin}[5*a + 4*b*x] + 20*b^3*d^4*x^3*\text{Sin}[5*a + 4*b*x] - (10*I)*b^4*d^4*x^4*\text{Si} \\
&\text{n}[5*a + 4*b*x])
\end{aligned}$$

Maple [B] time = 0.255, size = 956, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x)

```
[Out] -3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5+6/b^3*c*d^3*polylog(3,-exp(2*I*(b*x+a)))*x+1/b*d^4*ln(exp(2*I*(b*x+a))+1)*x^4+I*c^4*x-1/4*(2*d^4*x^4*b^4+4*I*b^3*d^4*x^3+8*b^4*c*d^3*x^3+12*I*b^3*c*d^3*x^2+12*b^4*c^2*d^2*x^2+12*I*b^3*c^2*d^2*x+8*b^4*c^3*d*x+4*I*b^3*c^3*d+2*b^4*c^4-6*b^2*d^4*x^2-6*I*b*d^4*x-12*b^2*c*d^3*x-6*I*b*c*d^3-6*c^2*d^2*b^2+3*d^4)/b^5*exp(2*I*(b*x+a))+6/b*c^2*d^2*ln(exp(2*I*(b*x+a))+1)*x^2+4/b*c^3*d*ln(exp(2*I*(b*x+a))+1)*x+8/5*I/b^5*d^4*a^5-I*c*d^3*x^4-2*I*c^2*d^2*x^3-2*I*c^3*d*x^2+4/b*c*d^3*ln(exp(2*I*(b*x+a))+1)*x^3-6*I/b^2*c^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-6*I/b^2*c*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2+3*I/b^4*c*d^3*polylog(4,-exp(2*I*(b*x+a)))-2*I/b^2*c^3*d*polylog(2,-exp(2*I*(b*x+a)))-2*I/b^2*d^4*polylog(2,-exp(2*I*(b*x+a)))*x^3+3/b^3*d^4*polylog(3,-exp(2*I*(b*x+a)))*x^2+3/b^3*c^2*d^2*polylog(3,-exp(2*I*(b*x+a)))-2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))+1/b*c^4*ln(exp(2*I*(b*x+a))+1)-1/5*I*d^4*x^5+8/b^2*c^3*d*a*ln(exp(I*(b*x+a)))-12/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a)))+8/b^4*c*d^3*a^3*ln(exp(I*(b*x+a)))+2*I/b^4*d^4*a^4*x+8*I/b^3*c^2*d^2*a^3-4*I/b^2*c^3*d*a^2-6*I/b^4*c*d^3*a^4-8*I/b*c^3*d*a*x+12*I/b^2*c^2*d^2*a^2*x-2/b*c^4*ln(exp(I*(b*x+a)))-1/4*(2*d^4*x^4*b^4-4*I*b^3*d^4*x^3+8*b^4*c*d^3*x^3-12*I*b^3*c*d^3*x^2+12*b^4*c^2*d^2*x^2-12*I*b^3*c^2*d^2*x+8*b^4*c^3*d*x-4*I*b^3*c^3*d+2*b^4*c^4-6*b^2*d^4*x^2+6*I*b*d^4*x-12*b^2*c*d^3*x+6*I*b*c*d^3-6*c^2*d^2*b^2+3*d^4)/b^5*exp(-2*I*(b*x+a))-8*I/b^3*c*d^3*a^3*x+3*I/b^4*d^4*polylog(4,-exp(2*I*(b*x+a)))*x
```

Maxima [B] time = 1.6788, size = 819, normalized size = 2.74

$$\frac{c^4(2 \cos(2bx + 2a) - \log(\cos(2bx)^2 + 2 \cos(2bx) \cos(2a) + \cos(2a)^2 + \sin(2bx)^2 - 2 \sin(2bx) \sin(2a) + \sin(2a)^2))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -1/2*c^4*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + c*cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b + 1/30*(-6*I*b^5*d^4*x^5 - 30*I*b^5*c*d^3*x^4 - 60*I*b^5*c^2*d^2*x^3 - 60*I*b^5*c^3*d*x^2 - 90*d^4*polylog(5, -e^(2*I*b*x + 2*I*a)) + (60*I*b^4*d^4*x^4 + 160*I*b^4*c*d^3*x^3 + 180*I*b^4*c^2*d^2*x^2 + 120*I*b^4*c^3*d*x)*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 15*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 - b^2*d^4))*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(2*b*x + 2*a) + (-120*I*b^3*d^4*x^3 - 240*I*b^3*c*d^3*x^2 - 180*I*b^3*c^2*d^2*x - 60*I*b^3*c^3*d)*dilog(-e^(2*I*b*x + 2*I*a)) + 10*(3*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 9*b^4*c^2*d^2*x^2 + 6*b^4*c^3*d*x)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (180*I*b*d^4*x + 120*I*b*c*d^3)*polylog(4, -e^(2*I*b*x + 2*I*a)) + 30*(6*b^2*d^4*x^2 + 8*b^2*c*d^3*x + 3*b^2*c^2*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) + 30*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*x)*sin(2*b*x + 2*a))/b^5
```

Fricas [C] time = 1.00211, size = 3866, normalized size = 12.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 - 24*d^4*polylog(5, I*cos(b*x + a) + sin(b*x + a)) - 24*d^4*polylog(5, I*cos(b*x + a) - sin(b*x + a)) - 24*d^4*polylog(5, -I*cos(b*x + a) + sin(b*x + a)) - 24*d^4*polylog(5, -I*cos(b*x + a) - sin(b*x + a)) + 6*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 - 2*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)*sin(b*x + a) + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(cos(b*x + a) - I*sin(b*x + a) + I) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, I*cos(b*x + a) + sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)))/b^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^4 \sec(bx + a) \sin(3bx + 3a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

3.383 $\int (c + dx)^3 \sec(a + bx) \sin(3a + 3bx) dx$

Optimal. Leaf size=242

$$\frac{3d^2(c + dx)\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} - \frac{3id(c + dx)^2\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^2} + \frac{3id^3\text{PolyLog}\left(4, -e^{2i(a+bx)}\right)}{4b^4} - \frac{3d^2(c + dx)}{2b^3}$$

```
[Out] (3*d^3*x)/(2*b^3) - (c + d*x)^3/b - ((I/4)*(c + d*x)^4)/d + ((c + d*x)^3*Log[1 + E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 - (3*d^3*Cos[a + b*x]*Sin[a + b*x])/(2*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/b^2 - (3*d^2*(c + d*x)*Sin[a + b*x]^2)/b^3 + (2*(c + d*x)^3*Sin[a + b*x]^2)/b
```

Rubi [A] time = 0.445514, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {4431, 4404, 3311, 32, 2635, 8, 4407, 3719, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c + dx)\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} - \frac{3id(c + dx)^2\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^2} + \frac{3id^3\text{PolyLog}\left(4, -e^{2i(a+bx)}\right)}{4b^4} - \frac{3d^2(c + dx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]*Sin[3*a + 3*b*x], x]
```

```
[Out] (3*d^3*x)/(2*b^3) - (c + d*x)^3/b - ((I/4)*(c + d*x)^4)/d + ((c + d*x)^3*Log[1 + E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 - (3*d^3*Cos[a + b*x]*Sin[a + b*x])/(2*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/b^2 - (3*d^2*(c + d*x)*Sin[a + b*x]^2)/b^3 + (2*(c + d*x)^3*Sin[a + b*x]^2)/b
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol] := Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && MemberQ[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
```

FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 4407

Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := -Int[(c + d*x)^m*SIN[a + b*x]^n*TAN[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*SIN[a + b*x]^(n - 2)*TAN[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3719

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[(c + d*x)^m*E^(2*I*(e + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 6609

Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^3 \sec(a + bx) \sin(3a + 3bx) dx &= \int (3(c + dx)^3 \cos(a + bx) \sin(a + bx) - (c + dx)^3 \sin^2(a + bx) \tan(a + bx)) dx \\ &= 3 \int (c + dx)^3 \cos(a + bx) \sin(a + bx) dx - \int (c + dx)^3 \sin^2(a + bx) \tan(a + bx) dx \\ &= \frac{3(c + dx)^3 \sin^2(a + bx)}{2b} - \frac{(9d) \int (c + dx)^2 \sin^2(a + bx) dx}{2b} + \int (c + dx)^3 \cos(a + bx) \sin(a + bx) dx \\ &= -\frac{i(c + dx)^4}{4d} + \frac{9d(c + dx)^2 \cos(a + bx) \sin(a + bx)}{4b^2} - \frac{9d^2(c + dx) \sin^2(a + bx)}{4b^3} \\ &= -\frac{3(c + dx)^3}{4b} - \frac{i(c + dx)^4}{4d} + \frac{(c + dx)^3 \log(1 + e^{2i(a+bx)})}{b} - \frac{9d^3 \cos(a + bx) \sin^2(a + bx)}{8b^4} \\ &= \frac{9d^3 x}{8b^3} - \frac{(c + dx)^3}{b} - \frac{i(c + dx)^4}{4d} + \frac{(c + dx)^3 \log(1 + e^{2i(a+bx)})}{b} - \frac{3id(c + dx)^3}{8b^4} \\ &= \frac{3d^3 x}{2b^3} - \frac{(c + dx)^3}{b} - \frac{i(c + dx)^4}{4d} + \frac{(c + dx)^3 \log(1 + e^{2i(a+bx)})}{b} - \frac{3id(c + dx)^3}{8b^4} \\ &= \frac{3d^3 x}{2b^3} - \frac{(c + dx)^3}{b} - \frac{i(c + dx)^4}{4d} + \frac{(c + dx)^3 \log(1 + e^{2i(a+bx)})}{b} - \frac{3id(c + dx)^3}{8b^4} \\ &= \frac{3d^3 x}{2b^3} - \frac{(c + dx)^3}{b} - \frac{i(c + dx)^4}{4d} + \frac{(c + dx)^3 \log(1 + e^{2i(a+bx)})}{b} - \frac{3id(c + dx)^3}{8b^4} \end{aligned}$$

Mathematica [B] time = 6.60769, size = 1719, normalized size = 7.1

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] ((I/4)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))]) - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))]*Sec[a])/(b^3*E^(I*a)) + (I/8)*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a) - ((4*I)*(1 + E^((-2*I)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1 + E^((2*I)*a))*(2*b^2*x^2*PolyLog[2, -E^((-2*I)*(a + b*x))]) - (2*I)*b*x*PolyLog[3, -E^((-2*I)*(a + b*x))]) - PolyLog[4, -E^((-2*I)*(a + b*x))])/b^4*E^((2*I)*a))*Sec[a] + (c^3*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x]) - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]])])]) + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]])]) + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]])])))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^2
```

$$\begin{aligned} & *(\cos[a]^2 + \sin[a]^2)) + \sec[a] * (\cos[2*a + 2*b*x] / (16*b^4) - ((I/16)*\sin[\\ & 2*a + 2*b*x]) / b^4) * (-8*b^3*c^3*\cos[a] + (12*I)*b^2*c^2*d*\cos[a] + 12*b*c*d^ \\ & 2*\cos[a] - (6*I)*d^3*\cos[a] - 24*b^3*c^2*d*x*\cos[a] + (24*I)*b^2*c*d^2*x*\cos \\ & [a] + 12*b*d^3*x*\cos[a] - 24*b^3*c*d^2*x^2*\cos[a] + (12*I)*b^2*d^3*x^2*\cos \\ & [a] - 8*b^3*d^3*x^3*\cos[a] - (8*I)*b^4*c^3*x*\cos[a + 2*b*x] - (12*I)*b^4*c^ \\ & 2*d*x^2*\cos[a + 2*b*x] - (8*I)*b^4*c*d^2*x^3*\cos[a + 2*b*x] - (2*I)*b^4*d^3 \\ & *x^4*\cos[a + 2*b*x] + (8*I)*b^4*c^3*x*\cos[3*a + 2*b*x] + (12*I)*b^4*c^2*d*x \\ & ^2*\cos[3*a + 2*b*x] + (8*I)*b^4*c*d^2*x^3*\cos[3*a + 2*b*x] + (2*I)*b^4*d^3*x \\ & ^4*\cos[3*a + 2*b*x] - 4*b^3*c^3*\cos[3*a + 4*b*x] - (6*I)*b^2*c^2*d*\cos[3*a \\ & + 4*b*x] + 6*b*c*d^2*\cos[3*a + 4*b*x] + (3*I)*d^3*\cos[3*a + 4*b*x] - 12*b^ \\ & 3*c^2*d*x*\cos[3*a + 4*b*x] - (12*I)*b^2*c*d^2*x*\cos[3*a + 4*b*x] + 6*b*d^3*x \\ & *\cos[3*a + 4*b*x] - 12*b^3*c*d^2*x^2*\cos[3*a + 4*b*x] - (6*I)*b^2*d^3*x^2* \\ & \cos[3*a + 4*b*x] - 4*b^3*d^3*x^3*\cos[3*a + 4*b*x] - 4*b^3*c^3*\cos[5*a + 4*b \\ & *x] - (6*I)*b^2*c^2*d*\cos[5*a + 4*b*x] + 6*b*c*d^2*\cos[5*a + 4*b*x] + (3*I) \\ & *d^3*\cos[5*a + 4*b*x] - 12*b^3*c^2*d*x*\cos[5*a + 4*b*x] - (12*I)*b^2*c*d^2*x \\ & *\cos[5*a + 4*b*x] + 6*b*d^3*x*\cos[5*a + 4*b*x] - 12*b^3*c*d^2*x^2*\cos[5*a \\ & + 4*b*x] - (6*I)*b^2*d^3*x^2*\cos[5*a + 4*b*x] - 4*b^3*d^3*x^3*\cos[5*a + 4*b \\ & *x] + 8*b^4*c^3*x*\sin[a + 2*b*x] + 12*b^4*c^2*d*x^2*\sin[a + 2*b*x] + 8*b^4*c \\ & *d^2*x^3*\sin[a + 2*b*x] + 2*b^4*d^3*x^4*\sin[a + 2*b*x] - 8*b^4*c^3*x*\sin[3 \\ & *a + 2*b*x] - 12*b^4*c^2*d*x^2*\sin[3*a + 2*b*x] - 8*b^4*c*d^2*x^3*\sin[3*a + \\ & 2*b*x] - 2*b^4*d^3*x^4*\sin[3*a + 2*b*x] - (4*I)*b^3*c^3*\sin[3*a + 4*b*x] + \\ & 6*b^2*c^2*d*\sin[3*a + 4*b*x] + (6*I)*b*c*d^2*\sin[3*a + 4*b*x] - 3*d^3*\sin[\\ & 3*a + 4*b*x] - (12*I)*b^3*c^2*d*x*\sin[3*a + 4*b*x] + 12*b^2*c*d^2*x*\sin[3*a \\ & + 4*b*x] + (6*I)*b*d^3*x*\sin[3*a + 4*b*x] - (12*I)*b^3*c*d^2*x^2*\sin[3*a + \\ & 4*b*x] + 6*b^2*d^3*x^2*\sin[3*a + 4*b*x] - (4*I)*b^3*d^3*x^3*\sin[3*a + 4*b* \\ & x] - (4*I)*b^3*c^3*\sin[5*a + 4*b*x] + 6*b^2*c^2*d*\sin[5*a + 4*b*x] + (6*I)* \\ & b*c*d^2*\sin[5*a + 4*b*x] - 3*d^3*\sin[5*a + 4*b*x] - (12*I)*b^3*c^2*d*x*\sin[\\ & 5*a + 4*b*x] + 12*b^2*c*d^2*x*\sin[5*a + 4*b*x] + (6*I)*b*d^3*x*\sin[5*a + 4* \\ & b*x] - (12*I)*b^3*c*d^2*x^2*\sin[5*a + 4*b*x] + 6*b^2*d^3*x^2*\sin[5*a + 4*b* \\ & x] - (4*I)*b^3*d^3*x^3*\sin[5*a + 4*b*x]) \end{aligned}$$

Maple [B] time = 0.214, size = 639, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d*x+c)^3*\sec(b*x+a)*\sin(3*b*x+3*a),x)$

[Out] $\frac{3}{4}I*d^3*\text{polylog}(4,-\exp(2*I*(b*x+a)))/b^4+1/b*c^3*\ln(\exp(2*I*(b*x+a))+1)+3/2/b^3*c*d^2*\text{polylog}(3,-\exp(2*I*(b*x+a)))+3/2/b^3*d^3*\text{polylog}(3,-\exp(2*I*(b*x+a)))*x-3/2I/b^4*d^3*a^4-3/2I/b^2*c^2*d*\text{polylog}(2,-\exp(2*I*(b*x+a)))-3/2I/b^2*d^3*\text{polylog}(2,-\exp(2*I*(b*x+a)))*x^2+4I/b^3*a^3*c*d^2-3I/b^2*a^2*c^2*d+3/b*c^2*d*\ln(\exp(2*I*(b*x+a))+1)*x+3/b*c*d^2*\ln(\exp(2*I*(b*x+a))+1)*x^2+1/b*d^3*\ln(\exp(2*I*(b*x+a))+1)*x^3-1/8*(4*d^3*x^3*b^3+6*I*b^2*d^3*x^2+12*b^3*c*d^2*x^2+12*I*b^2*c*d^2*x+12*b^3*c^2*d*x+6*I*b^2*c^2*d+4*b^3*c^3-6*b*d^3*x-3*I*d^3-6*c*d^2*b)/b^4*\exp(2*I*(b*x+a))-2I/b^3*d^3*a^3*x-6/b^3*c*d^2*a^2*\ln(\exp(I*(b*x+a)))+6/b^2*c^2*d*a*\ln(\exp(I*(b*x+a)))+I*c^3*x-1/8*(4*d^3*x^3*b^3-6*I*b^2*d^3*x^2+12*b^3*c*d^2*x^2-12*I*b^2*c*d^2*x+12*b^3*c^2*d*x-6*I*b^2*c^2*d+4*b^3*c^3-6*b*d^3*x+3*I*d^3-6*c*d^2*b)/b^4*\exp(-2*I*(b*x+a))-I*c*d^2*x^3-3/2I*c^2*d*x^2+2/b^4*d^3*a^3*\ln(\exp(I*(b*x+a)))-3I/b^2*\text{polylog}(2,-\exp(2*I*(b*x+a)))*c*d^2*x-6I/b*a*c^2*d*x+6I/b^2*a^2*c*d^2*x-2/b*c^3*\ln(\exp(I*(b*x+a)))-1/4I*d^3*x^4$

Maxima [B] time = 1.50017, size = 597, normalized size = 2.47

$$\frac{c^3(2 \cos(2bx + 2a) - \log(\cos(2bx)^2 + 2 \cos(2bx) \cos(2a) + \cos(2a)^2 + \sin(2bx)^2 - 2 \sin(2bx) \sin(2a) + \sin(2a)^2) - \log(\cos(2bx)^2 + 2 \cos(2bx) \cos(2a) + \cos(2a)^2 + \sin(2bx)^2 - 2 \sin(2bx) \sin(2a) + \sin(2a)^2)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")

[Out] $-1/2*c^3*(2*\cos(2*b*x + 2*a) - \log(\cos(2*b*x)^2 + 2*\cos(2*b*x)*\cos(2*a) + \cos(2*a)^2 + \sin(2*b*x)^2 - 2*\sin(2*b*x)*\sin(2*a) + \sin(2*a)^2))/b + 1/12*(-3*I*b^4*d^3*x^4 - 12*I*b^4*c*d^2*x^3 - 18*I*b^4*c^2*d*x^2 + 12*I*d^3*\text{polylog}(4, -e^{(2*I*b*x + 2*I*a)}) + (16*I*b^3*d^3*x^3 + 36*I*b^3*c*d^2*x^2 + 36*I*b^3*c^2*d*x)*\arctan2(\sin(2*b*x + 2*a), \cos(2*b*x + 2*a) + 1) - 6*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*\cos(2*b*x + 2*a) + (-24*I*b^2*d^3*x^2 - 36*I*b^2*c*d^2*x - 18*I*b^2*c^2*d)*\text{dilog}(-e^{(2*I*b*x + 2*I*a)}) + 2*(4*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x)*\log(\cos(2*b*x + 2*a)^2 + \sin(2*b*x + 2*a)^2 + 2*\cos(2*b*x + 2*a) + 1) + 6*(4*b*d^3*x + 3*b*c*d^2)*\text{polylog}(3, -e^{(2*I*b*x + 2*I*a)}) + 9*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*\sin(2*b*x + 2*a))/b^4$

Fricas [C] time = 0.865917, size = 2703, normalized size = 11.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")

[Out] $1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 6*I*d^3*\text{polylog}(4, I*\cos(b*x + a) + \sin(b*x + a)) + 6*I*d^3*\text{polylog}(4, I*\cos(b*x + a) - \sin(b*x + a)) + 6*I*d^3*\text{polylog}(4, -I*\cos(b*x + a) + \sin(b*x + a)) - 6*I*d^3*\text{polylog}(4, -I*\cos(b*x + a) - \sin(b*x + a)) - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*\cos(b*x + a)^2 + 3*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*\cos(b*x + a)*\sin(b*x + a) + 3*(2*b^3*c^2*d - b*d^3)*x + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*\text{dilog}(I*\cos(b*x + a) + \sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*\text{dilog}(I*\cos(b*x + a) - \sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*\text{dilog}(-I*\cos(b*x + a) + \sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*\text{dilog}(-I*\cos(b*x + a) - \sin(b*x + a)) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\log(\cos(b*x + a) + I*\sin(b*x + a) + I) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\log(\cos(b*x + a) - I*\sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*\log(I*\cos(b*x + a) + \sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*\log(-I*\cos(b*x + a) + \sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*\log(-I*\cos(b*x + a) - \sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\log(-\cos(b*x + a) + I*\sin(b*x + a) + I) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\log(-\cos(b*x + a) - I*\sin(b*x + a) + I) + 6*(b*d^3*x + b*c*d^2)*\text{polylog}(3, I*\cos(b*x + a) + \sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*\text{polylog}(3, -I*\cos(b*x + a) + \sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*\text{polylog}(3, -I*\cos(b*x + a) - \sin(b*x + a)))/b^4$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*sec(b*x+a)*sin(3*b*x+3*a),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \sec(bx + a) \sin(3bx + 3a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")

[Out] integrate((d*x + c)^3*sec(b*x + a)*sin(3*b*x + 3*a), x)

3.384 $\int (c + dx)^2 \sec(a + bx) \sin(3a + 3bx) dx$

Optimal. Leaf size=173

$$\frac{id(c + dx)\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^2} + \frac{d^2\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} + \frac{2d(c + dx)\sin(a + bx)\cos(a + bx)}{b^2} - \frac{d^2\sin^2(a + bx)}{b^3}$$

[Out] $(-2*c*d*x)/b - (d^2*x^2)/b - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*\text{Log}[1 + E^{((2*I)*(a + b*x))}])/b - (I*d*(c + d*x)*\text{PolyLog}[2, -E^{((2*I)*(a + b*x))}])/b^2 + (d^2*\text{PolyLog}[3, -E^{((2*I)*(a + b*x))}])/(2*b^3) + (2*d*(c + d*x)*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b^2 - (d^2*\text{Sin}[a + b*x]^2)/b^3 + (2*(c + d*x)^2*\text{Sin}[a + b*x]^2)/b$

Rubi [A] time = 0.328273, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {4431, 4404, 3310, 4407, 3719, 2190, 2531, 2282, 6589}

$$\frac{id(c + dx)\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^2} + \frac{d^2\text{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{2b^3} + \frac{2d(c + dx)\sin(a + bx)\cos(a + bx)}{b^2} - \frac{d^2\sin^2(a + bx)}{b^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^2*\text{Sec}[a + b*x]*\text{Sin}[3*a + 3*b*x], x]$

[Out] $(-2*c*d*x)/b - (d^2*x^2)/b - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*\text{Log}[1 + E^{((2*I)*(a + b*x))}])/b - (I*d*(c + d*x)*\text{PolyLog}[2, -E^{((2*I)*(a + b*x))}])/b^2 + (d^2*\text{PolyLog}[3, -E^{((2*I)*(a + b*x))}])/(2*b^3) + (2*d*(c + d*x)*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b^2 - (d^2*\text{Sin}[a + b*x]^2)/b^3 + (2*(c + d*x)^2*\text{Sin}[a + b*x]^2)/b$

Rule 4431

$\text{Int}[(e_{.}) + (f_{.})*(x_{.})^{(m_{.})}*(F_{.})[(a_{.}) + (b_{.})*(x_{.})^{(p_{.})}*(G_{.})[(c_{.}) + (d_{.})*(x_{.})^{(q_{.})}], x_{\text{Symbol}}] := \text{Int}[\text{ExpandTrigExpand}[(e + f*x)^m*G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /;$ FreeQ[{a, b, c, d, e, f, m}, x] && MemberQ[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]

Rule 4404

$\text{Int}[\text{Cos}[(a_{.}) + (b_{.})*(x_{.})^{(n_{.})}]*((c_{.}) + (d_{.})*(x_{.})^{(m_{.})})*\text{Sin}[(a_{.}) + (b_{.})*(x_{.})^{(n_{.})}], x_{\text{Symbol}}] := \text{Simp}[(c + d*x)^m*\text{Sin}[a + b*x]^{(n + 1)}/(b*(n + 1)), x] - \text{Dist}[(d*m)/(b*(n + 1)), \text{Int}[(c + d*x)^{(m - 1)}*\text{Sin}[a + b*x]^{(n + 1)}, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 3310

$\text{Int}[(c_{.}) + (d_{.})*(x_{.})^{(n_{.})}]*((b_{.})*\text{sin}[(e_{.}) + (f_{.})*(x_{.})^{(n_{.})}], x_{\text{Symbol}}] := \text{Simp}[(d*(b*\text{Sin}[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n - 1))/n, \text{Int}[(c + d*x)*(b*\text{Sin}[e + f*x])^{(n - 2)}, x], x] - \text{Simp}[(b*(c + d*x)*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(n - 1)})/(f*n), x]) /;$ FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rule 4407

$\text{Int}[(c_{.}) + (d_{.})*(x_{.})^{(m_{.})}*\text{Sin}[(a_{.}) + (b_{.})*(x_{.})^{(n_{.})}]*\text{Tan}[(a_{.}) + (b_{.})*(x_{.})^{(p_{.})}], x_{\text{Symbol}}] := -\text{Int}[(c + d*x)^m*\text{Sin}[a + b*x]^n*\text{Tan}[a + b*x]^$

$(p - 2), x] + \text{Int}[(c + d*x)^m * \text{Sin}[a + b*x]^{(n - 2)} * \text{Tan}[a + b*x]^p, x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3719

$\text{Int}[(c + d*x)^m * \text{tan}[(e + f*x)], x_Symbol] := \text{Simp}[(I*(c + d*x)^{(m + 1)})/(d*(m + 1)), x] - \text{Dist}[2*I, \text{Int}[(c + d*x)^m * E^{(2*I*(e + f*x))}/(1 + E^{(2*I*(e + f*x))}), x], x] /;$ FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2190

$\text{Int}[(F)^{(g*(e + f*x))^{(n)}} * (c + d*x)^m / ((a + b*(F)^{(g*(e + f*x))^{(n)}})), x_Symbol] := \text{Simp}[(c + d*x)^m * \text{Log}[1 + (b*(F)^{(g*(e + f*x))^{(n)}})/a] / (b*f*g*n * \text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n * \text{Log}[F]), \text{Int}[(c + d*x)^{(m - 1)} * \text{Log}[1 + (b*(F)^{(g*(e + f*x))^{(n)}})/a], x], x] /;$ FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

$\text{Int}[\text{Log}[1 + (e + f*x)^{(c*(a + b*x))^{(n)}}] * (f + g*x)^m, x_Symbol] := -\text{Simp}[(f + g*x)^m * \text{PolyLog}[2, -(e*(F)^{(c*(a + b*x))^{(n)}})] / (b*c*n * \text{Log}[F]), x] + \text{Dist}[(g*m)/(b*c*n * \text{Log}[F]), \text{Int}[(f + g*x)^{(m - 1)} * \text{PolyLog}[2, -(e*(F)^{(c*(a + b*x))^{(n)}})], x], x] /;$ FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 2282

$\text{Int}[u, x_Symbol] := \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /;$ FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^{(c_)*((a_)+(b_)*x)}*(F_)[v_]] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]

Rule 6589

$\text{Int}[\text{PolyLog}[n, (c + d*x)^m * (a + b*x)^p] / ((d + e*x)), x_Symbol] := \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p] / (e*p), x] /;$ FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \sec(a + bx) \sin(3a + 3bx) dx &= \int (3(c + dx)^2 \cos(a + bx) \sin(a + bx) - (c + dx)^2 \sin^2(a + bx) \tan(a + bx)) dx \\ &= 3 \int (c + dx)^2 \cos(a + bx) \sin(a + bx) dx - \int (c + dx)^2 \sin^2(a + bx) \tan(a + bx) dx \\ &= \frac{3(c + dx)^2 \sin^2(a + bx)}{2b} - \frac{(3d) \int (c + dx) \sin^2(a + bx) dx}{b} + \int (c + dx)^2 \cos(a + bx) dx \\ &= -\frac{i(c + dx)^3}{3d} + \frac{3d(c + dx) \cos(a + bx) \sin(a + bx)}{2b^2} - \frac{3d^2 \sin^2(a + bx)}{4b^3} + \frac{2d(c + dx) \cos(a + bx)}{b} \\ &= -\frac{3cdx}{2b} - \frac{3d^2x^2}{4b} - \frac{i(c + dx)^3}{3d} + \frac{(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b} + \frac{2d(c + dx) \cos(a + bx)}{b} \\ &= -\frac{2cdx}{b} - \frac{d^2x^2}{b} - \frac{i(c + dx)^3}{3d} + \frac{(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b} - \frac{id(c + dx) \operatorname{Li}_2(-e^{2i(a+bx)})}{b^2} \\ &= -\frac{2cdx}{b} - \frac{d^2x^2}{b} - \frac{i(c + dx)^3}{3d} + \frac{(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b} - \frac{id(c + dx) \operatorname{Li}_2(-e^{2i(a+bx)})}{b^2} \\ &= -\frac{2cdx}{b} - \frac{d^2x^2}{b} - \frac{i(c + dx)^3}{3d} + \frac{(c + dx)^2 \log(1 + e^{2i(a+bx)})}{b} - \frac{id(c + dx) \operatorname{Li}_2(-e^{2i(a+bx)})}{b^2} \end{aligned}$$

Mathematica [B] time = 6.54109, size = 516, normalized size = 2.98

$$cd \operatorname{csc}(a) \sec(a) \left(b^2 x^2 e^{-i \tan^{-1}(\cot(a))} - \frac{\cot(a) \left(i \operatorname{PolyLog}\left(2, e^{2i(bx - \tan^{-1}(\cot(a)))} \right) \right) + ibx(-2 \tan^{-1}(\cot(a)) - \pi) - 2(bx - \tan^{-1}(\cot(a))) \log\left(1 - e^{2i(bx - \tan^{-1}(\cot(a)))} \right)}{\sqrt{\cot^2(a) + 1}} \right)}{b^2 \sqrt{\operatorname{csc}^2(a) (\sin^2(a) + \cos^2(a))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]*Sin[3*a + 3*b*x], x]
```

```
[Out] ((I/12)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a]/(b^3*E^(I*a) + (c^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (c*d*Csc[a]*(b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x]) - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]])])]) + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]]) + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]])]))/Sqrt[1 + Cot[a]^2])*Sec[a]/(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)) - (Cos[2*b*x]*(2*b^2*c^2*Cos[2*a] - d^2*Cos[2*a] + 4*b^2*c*d*x*Cos[2*a] + 2*b^2*d^2*x^2*Cos[2*a] - 2*b*c*d*Sin[2*a] - 2*b*d^2*x*Sin[2*a]))/(2*b^3) + ((2*b*c*d*Cos[2*a] + 2*b*d^2*x*Cos[2*a] + 2*b^2*c^2*Sin[2*a] - d^2*Sin[2*a] + 4*b^2*c*d*x*Sin[2*a] + 2*b^2*d^2*x^2*Sin[2*a])*Sin[2*b*x])/(2*b^3) - (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Tan[a])/3
```

Maple [B] time = 0.243, size = 377, normalized size = 2.2

$$-icdx^2 - \frac{4iacdx}{b} + ic^2x - \frac{(2d^2x^2b^2 + 2ibd^2x + 4b^2cdx + 2ibcd + 2b^2c^2 - d^2)e^{2i(bx+a)}}{4b^3} - \frac{(2d^2x^2b^2 - 2ibd^2x + 4b^2cdx + 2ibcd + 2b^2c^2 - d^2)e^{2i(bx+a)}}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a), x)
```

```
[Out] -I*c*d*x^2-4*I/b*a*c*d*x+I*c^2*x-1/4*(2*d^2*x^2*b^2+2*I*b*d^2*x+4*b^2*c*d*x
+2*I*b*c*d+2*b^2*c^2-d^2)/b^3*exp(2*I*(b*x+a))-1/4*(2*d^2*x^2*b^2-2*I*b*d^2
*x+4*b^2*c*d*x-2*I*b*c*d+2*b^2*c^2-d^2)/b^3*exp(-2*I*(b*x+a))+1/b*c^2*ln(ex
p(2*I*(b*x+a))+1)-2/b*c^2*ln(exp(I*(b*x+a)))-2/b^3*d^2*a^2*ln(exp(I*(b*x+a)
))+2/b*c*d*ln(exp(2*I*(b*x+a))+1)*x-I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*
x-1/3*I*d^2*x^3+1/b*d^2*ln(exp(2*I*(b*x+a))+1)*x^2-I/b^2*c*d*polylog(2,-exp
(2*I*(b*x+a)))+1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+4/b^2*c*d*a*ln(exp(
I*(b*x+a)))-2*I/b^2*a^2*c*d+2*I/b^2*a^2*d^2*x+4/3*I/b^3*a^3*d^2
```

Maxima [A] time = 1.39856, size = 406, normalized size = 2.35

$$\frac{c^2(2 \cos(2bx + 2a) - \log(\cos(2bx)^2 + 2 \cos(2bx) \cos(2a) + \cos(2a)^2 + \sin(2bx)^2 - 2 \sin(2bx) \sin(2a) + \sin(2a)^2))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -1/2*c^2*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + c
os(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b + 1/6*(-2
*I*b^3*d^2*x^3 - 6*I*b^3*c*d*x^2 + 3*d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) +
(6*I*b^2*d^2*x^2 + 12*I*b^2*c*d*x)*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2
*a) + 1) - 3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x - d^2)*cos(2*b*x + 2*a) + (-6*I*b
*d^2*x - 6*I*b*c*d)*dilog(-e^(2*I*b*x + 2*I*a)) + 3*(b^2*d^2*x^2 + 2*b^2*c*
d*x)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)
+ 6*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a))/b^3
```

Fricas [C] time = 0.718819, size = 1728, normalized size = 9.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x - 2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c
^2 - d^2)*cos(b*x + a)^2 + 2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a))
+ 2*d^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*polylog(3, -I*cos
(b*x + a) + sin(b*x + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)
) + 4*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) + (2*I*b*d^2*x + 2*I*b*c*
d)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(
I*cos(b*x + a) - sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(-I*cos(b
x + a) + sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-I*cos(b*x + a) -
sin(b*x + a)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*sin(b*
x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*sin(b*x
+ a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x
+ a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^
2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*
a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 +
2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)
+ (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I)
+ (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I))/
b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*sec(b*x+a)*sin(3*b*x+3*a),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \sec(bx + a) \sin(3bx + 3a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")

[Out] integrate((d*x + c)^2*sec(b*x + a)*sin(3*b*x + 3*a), x)

3.385 $\int (c + dx) \sec(a + bx) \sin(3a + 3bx) dx$

Optimal. Leaf size=107

$$-\frac{id\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^2} + \frac{d \sin(a + bx) \cos(a + bx)}{b^2} + \frac{(c + dx) \log\left(1 + e^{2i(a+bx)}\right)}{b} + \frac{2(c + dx) \sin^2(a + bx)}{b} - \frac{dx}{b} - \frac{i(c}{b}$$

[Out] $-\left(\frac{d*x}{b}\right) - \left(\frac{I}{2}\right)*(c + d*x)^2/d + \left(\frac{(c + d*x)*\text{Log}\left[1 + E^{\left(\frac{2*I}{1}\right)*(a + b*x)}\right]}{b} - \left(\frac{I}{2}\right)*d*\text{PolyLog}\left[2, -E^{\left(\frac{2*I}{1}\right)*(a + b*x)}\right]\right)/b^2 + (d*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b^2 + (2*(c + d*x)*\text{Sin}[a + b*x]^2)/b$

Rubi [A] time = 0.180171, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4431, 4404, 2635, 8, 4407, 3719, 2190, 2279, 2391}

$$-\frac{id\text{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^2} + \frac{d \sin(a + bx) \cos(a + bx)}{b^2} + \frac{(c + dx) \log\left(1 + e^{2i(a+bx)}\right)}{b} + \frac{2(c + dx) \sin^2(a + bx)}{b} - \frac{dx}{b} - \frac{i(c}{b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)*\text{Sec}[a + b*x]*\text{Sin}[3*a + 3*b*x], x]$

[Out] $-\left(\frac{d*x}{b}\right) - \left(\frac{I}{2}\right)*(c + d*x)^2/d + \left(\frac{(c + d*x)*\text{Log}\left[1 + E^{\left(\frac{2*I}{1}\right)*(a + b*x)}\right]}{b} - \left(\frac{I}{2}\right)*d*\text{PolyLog}\left[2, -E^{\left(\frac{2*I}{1}\right)*(a + b*x)}\right]\right)/b^2 + (d*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/b^2 + (2*(c + d*x)*\text{Sin}[a + b*x]^2)/b$

Rule 4431

$\text{Int}[\left((e_{.}) + (f_{.})*(x_{.})\right)^{(m_{.})}*(F_{.})\left[\left(a_{.}\right) + \left(b_{.}\right)*(x_{.})\right]^{(p_{.})}*(G_{.})\left[\left(c_{.}\right) + \left(d_{.}\right)*(x_{.})\right]^{(q_{.})}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigExpand}[(e + f*x)^m*G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \&\& \text{MemberQ}[\{\text{Sin}, \text{Cos}\}, F] \&\& \text{MemberQ}[\{\text{Sec}, \text{Csc}\}, G] \&\& \text{IGtQ}[p, 0] \&\& \text{IGtQ}[q, 0] \&\& \text{EQ}[b*c - a*d, 0] \&\& \text{IGtQ}[b/d, 1]$

Rule 4404

$\text{Int}[\text{Cos}[\left(a_{.}\right) + \left(b_{.}\right)*(x_{.})]*\left(\left(c_{.}\right) + \left(d_{.}\right)*(x_{.})\right)^{(m_{.})}*\text{Sin}[\left(a_{.}\right) + \left(b_{.}\right)*(x_{.})]^{(n_{.})}, x_Symbol] \rightarrow \text{Simp}[\left(\frac{(c + d*x)^m*\text{Sin}[a + b*x]^{(n + 1)}}{b*(n + 1)}\right), x] - \text{Dist}[\left(\frac{d*m}{b*(n + 1)}\right), \text{Int}[(c + d*x)^{(m - 1)}*\text{Sin}[a + b*x]^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[n, -1]$

Rule 2635

$\text{Int}[\left(\left(b_{.}\right)*\text{sin}[\left(c_{.}\right) + \left(d_{.}\right)*(x_{.})]\right)^{(n_{.})}, x_Symbol] \rightarrow -\text{Simp}[\left(\frac{b*\text{Cos}[c + d*x]*\left(b*\text{Sin}[c + d*x]\right)^{(n - 1)}}{d*n}\right), x] + \text{Dist}[\left(\frac{b^2*(n - 1)}{n}\right), \text{Int}[\left(b*\text{Sin}[c + d*x]\right)^{(n - 2)}, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

Rule 8

$\text{Int}[a_{.}, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 4407

$\text{Int}[\left(\left(c_{.}\right) + \left(d_{.}\right)*(x_{.})\right)^{(m_{.})}*\text{Sin}[\left(a_{.}\right) + \left(b_{.}\right)*(x_{.})]^{(n_{.})}*\text{Tan}[\left(a_{.}\right) + \left(b_{.}\right)*(x_{.})]^{(p_{.})}, x_Symbol] \rightarrow -\text{Int}[(c + d*x)^m*\text{Sin}[a + b*x]^{(n)}*\text{Tan}[a + b*x]^{(p - 2)}, x] + \text{Int}[(c + d*x)^m*\text{Sin}[a + b*x]^{(n - 2)}*\text{Tan}[a + b*x]^p, x] /; \text{Fr}$

eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3719

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1)/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned} \int (c + dx) \sec(a + bx) \sin(3a + 3bx) dx &= \int (3(c + dx) \cos(a + bx) \sin(a + bx) - (c + dx) \sin^2(a + bx) \tan(a + bx)) dx \\ &= 3 \int (c + dx) \cos(a + bx) \sin(a + bx) dx - \int (c + dx) \sin^2(a + bx) \tan(a + bx) dx \\ &= \frac{3(c + dx) \sin^2(a + bx)}{2b} - \frac{(3d) \int \sin^2(a + bx) dx}{2b} + \int (c + dx) \cos(a + bx) \sin(a + bx) dx \\ &= -\frac{i(c + dx)^2}{2d} + \frac{3d \cos(a + bx) \sin(a + bx)}{4b^2} + \frac{2(c + dx) \sin^2(a + bx)}{b} + 2i \int \frac{e^{i(a+bx)}}{1 + e^{2i(a+bx)}} dx \\ &= -\frac{3dx}{4b} - \frac{i(c + dx)^2}{2d} + \frac{(c + dx) \log(1 + e^{2i(a+bx)})}{b} + \frac{d \cos(a + bx) \sin(a + bx)}{b^2} \\ &= -\frac{dx}{b} - \frac{i(c + dx)^2}{2d} + \frac{(c + dx) \log(1 + e^{2i(a+bx)})}{b} + \frac{d \cos(a + bx) \sin(a + bx)}{b^2} \\ &= -\frac{dx}{b} - \frac{i(c + dx)^2}{2d} + \frac{(c + dx) \log(1 + e^{2i(a+bx)})}{b} - \frac{id \operatorname{Li}_2(-e^{2i(a+bx)})}{2b^2} + \frac{d \cos(a + bx) \sin(a + bx)}{b^2} \end{aligned}$$

Mathematica [B] time = 5.82378, size = 254, normalized size = 2.37

$$d \operatorname{csc}(a) \sec(a) \left(b^2 x^2 e^{-i \tan^{-1}(\cot(a))} - \frac{\cot(a) \left(i \operatorname{PolyLog} \left(2, e^{2i(bx - \tan^{-1}(\cot(a)))} \right) \right) + ibx(-2 \tan^{-1}(\cot(a)) - \pi) - 2(bx - \tan^{-1}(\cot(a))) \log \left(1 - e^{2i(bx - \tan^{-1}(\cot(a)))} \right)}{\sqrt{\cot^2(a) + 1}} \right)}{2b^2 \sqrt{\operatorname{csc}^2(a) (\sin^2(a) + \cos^2(a))}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)*Sec[a + b*x]*Sin[3*a + 3*b*x],x]

[Out] (d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]])) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]])) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]])]) + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]]) + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]])])]) / Sqrt[1 + Cot[a]^2]*Sec[a]) / (2*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (d*Cos[2*b*x]*(2*b*x*Cos[2*a] - Sin[2*a])) / (2*b^2) + (d*(Cos[2*a] + 2*b*x*Sin[2*a])*Sin[2*b*x]) / (2*b^2) + (c*(Log[Cos[a + b*x]] + 2*Sin[a + b*x]^2)) / b - (d*x^2*Tan[a]) / 2

Maple [A] time = 0.324, size = 177, normalized size = 1.7

$$-\frac{i}{2}dx^2 + icx - \frac{(2dxb + id + 2bc)e^{2i(bx+a)}}{4b^2} - \frac{(2dxb - id + 2bc)e^{-2i(bx+a)}}{4b^2} - 2\frac{c \ln(e^{i(bx+a)})}{b} + \frac{c \ln(e^{2i(bx+a)} + 1)}{b} - \frac{2idax}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x)

[Out] -1/2*I*d*x^2+I*c*x-1/4*(2*d*x*b+I*d+2*b*c)/b^2*exp(2*I*(b*x+a))-1/4*(2*d*x*b-I*d+2*b*c)/b^2*exp(-2*I*(b*x+a))-2/b*c*ln(exp(I*(b*x+a)))+1/b*c*ln(exp(2*I*(b*x+a))+1)-2*I/b*d*a*x-I/b^2*d*a^2+1/b*d*ln(exp(2*I*(b*x+a))+1)*x-1/2*I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+2/b^2*d*a*ln(exp(I*(b*x+a)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{c(2 \cos(2bx + 2a) - \log(\cos(2bx)^2 + 2 \cos(2bx) \cos(2a) + \cos(2a)^2 + \sin(2bx)^2 - 2 \sin(2bx) \sin(2a) + \sin(2a)^2))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")

[Out] -1/2*c*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b - 1/2*(2*b*x*cos(2*b*x + 2*a) + 4*b^2*integrate(x*sin(2*b*x + 2*a)/(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1), x) - sin(2*b*x + 2*a))*d/b^2

Fricas [B] time = 0.621951, size = 937, normalized size = 8.76

$$\frac{2bdx - 4(bdx + bc) \cos(bx + a)^2 + 2d \cos(bx + a) \sin(bx + a) + i d \operatorname{Li}_2(i \cos(bx + a) + \sin(bx + a)) - i d \operatorname{Li}_2(i \cos(bx + a) - \sin(bx + a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")

[Out] 1/2*(2*b*d*x - 4*(b*d*x + b*c)*cos(b*x + a)^2 + 2*d*cos(b*x + a)*sin(b*x + a) + I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) - I*d*dilog(I*cos(b*x + a) - sin(b*x + a))

```

sin(b*x + a)) - I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) + (b*d*x + a*d)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x + a*d)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)*log(-cos(b*x + a) - I*sin(b*x + a) + I))/b^2

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \sec(bx + a) \sin(3bx + 3a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

$$3.386 \quad \int \frac{\sec(a+bx) \sin(3a+3bx)}{c+dx} dx$$

Optimal. Leaf size=79

$$-\text{Unintegrable}\left(\frac{\tan(a+bx)}{c+dx}, x\right) + \frac{2 \sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{d} + \frac{2 \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{d}$$

[Out] (2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/d + (2*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d - Unintegrable[Tan[a + b*x]/(c + d*x), x]

Rubi [A] time = 0.299071, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sec(a+bx) \sin(3a+3bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]

[Out] (2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/d + (2*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d - Defer[Int][Tan[a + b*x]/(c + d*x), x]

Rubi steps

$$\begin{aligned} \int \frac{\sec(a+bx) \sin(3a+3bx)}{c+dx} dx &= \int \left(\frac{3 \cos(a+bx) \sin(a+bx)}{c+dx} - \frac{\sin^2(a+bx) \tan(a+bx)}{c+dx} \right) dx \\ &= 3 \int \frac{\cos(a+bx) \sin(a+bx)}{c+dx} dx - \int \frac{\sin^2(a+bx) \tan(a+bx)}{c+dx} dx \\ &= 3 \int \frac{\sin(2a+2bx)}{2(c+dx)} dx + \int \frac{\cos(a+bx) \sin(a+bx)}{c+dx} dx - \int \frac{\tan(a+bx)}{c+dx} dx \\ &= \frac{3}{2} \int \frac{\sin(2a+2bx)}{c+dx} dx + \int \frac{\sin(2a+2bx)}{2(c+dx)} dx - \int \frac{\tan(a+bx)}{c+dx} dx \\ &= \frac{1}{2} \int \frac{\sin(2a+2bx)}{c+dx} dx + \frac{1}{2} \left(3 \cos\left(2a - \frac{2bc}{d}\right) \right) \int \frac{\sin\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx + \frac{1}{2} \left(3 \sin\left(2a - \frac{2bc}{d}\right) \right) \int \frac{\cos\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx \\ &= \frac{3 \text{Ci}\left(\frac{2bc}{d} + 2bx\right) \sin\left(2a - \frac{2bc}{d}\right)}{2d} + \frac{3 \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{2d} + \frac{1}{2} \cos\left(2a - \frac{2bc}{d}\right) \int \frac{\cos\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx \\ &= \frac{2 \text{Ci}\left(\frac{2bc}{d} + 2bx\right) \sin\left(2a - \frac{2bc}{d}\right)}{d} + \frac{2 \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{d} - \int \frac{\tan(a+bx)}{c+dx} dx \end{aligned}$$

Mathematica [A] time = 3.15739, size = 0, normalized size = 0.

$$\int \frac{\sec(a+bx) \sin(3a+3bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]

[Out] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x), x]

Maple [A] time = 0.371, size = 0, normalized size = 0.

$$\int \frac{\sec(bx + a) \sin(3bx + 3a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c), x)

[Out] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(i E_1 \left(\frac{2i b d x + 2i b c}{d} \right) - i E_1 \left(-\frac{2i b d x + 2i b c}{d} \right) \right) \cos \left(-\frac{2(b c - a d)}{d} \right) + 2 d \int \frac{\sin(2 b x + 2 a)}{(d x + c) (\cos(2 b x + 2 a)^2 + \sin(2 b x + 2 a)^2 + 2 \cos(2 b x + 2 a) + 1)} dx + \left(E_1 \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c), x, algorithm="maxima")

[Out] -((I*exp_integral_e(1, (2*I*b*d*x + 2*I*b*c)/d) - I*exp_integral_e(1, -(2*I*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 2*d*integrate(sin(2*b*x + 2*a)/((d*x + c)*cos(2*b*x + 2*a)^2 + (d*x + c)*sin(2*b*x + 2*a)^2 + d*x + 2*(d*x + c)*cos(2*b*x + 2*a) + c), x) + (exp_integral_e(1, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(1, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c - a*d)/d))/d

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sec(bx + a) \sin(3bx + 3a)}{dx + c}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c), x, algorithm="fricas")

[Out] integral(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c), x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx + a) \sin(3bx + 3a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")`

[Out] `integrate(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c), x)`

$$3.387 \quad \int \frac{\sec(a+bx) \sin(3a+3bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=102

$$-\text{Unintegrable}\left(\frac{\tan(a+bx)}{(c+dx)^2}, x\right) + \frac{4b \cos\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{d^2} - \frac{4b \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^2} - \frac{2 \tan(a+bx)}{(c+dx)^2}$$

[Out] (4*b*cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2 - (2*Sin[2*a + 2*b*x])/(d*(c + d*x)) - (4*b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^2 - Unintegrable[Tan[a + b*x]/(c + d*x)^2, x]

Rubi [A] time = 0.341935, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sec(a+bx) \sin(3a+3bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2, x]

[Out] (4*b*cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2 - (2*Sin[2*a + 2*b*x])/(d*(c + d*x)) - (4*b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^2 - Defer[Int][Tan[a + b*x]/(c + d*x)^2, x]

Rubi steps

$$\begin{aligned} \int \frac{\sec(a+bx) \sin(3a+3bx)}{(c+dx)^2} dx &= \int \left(\frac{3 \cos(a+bx) \sin(a+bx)}{(c+dx)^2} - \frac{\sin^2(a+bx) \tan(a+bx)}{(c+dx)^2} \right) dx \\ &= 3 \int \frac{\cos(a+bx) \sin(a+bx)}{(c+dx)^2} dx - \int \frac{\sin^2(a+bx) \tan(a+bx)}{(c+dx)^2} dx \\ &= 3 \int \frac{\sin(2a+2bx)}{2(c+dx)^2} dx + \int \frac{\cos(a+bx) \sin(a+bx)}{(c+dx)^2} dx - \int \frac{\tan(a+bx)}{(c+dx)^2} dx \\ &= \frac{3}{2} \int \frac{\sin(2a+2bx)}{(c+dx)^2} dx + \int \frac{\sin(2a+2bx)}{2(c+dx)^2} dx - \int \frac{\tan(a+bx)}{(c+dx)^2} dx \\ &= -\frac{3 \sin(2a+2bx)}{2d(c+dx)} + \frac{1}{2} \int \frac{\sin(2a+2bx)}{(c+dx)^2} dx + \frac{(3b) \int \frac{\cos(2a+2bx)}{c+dx} dx}{d} - \int \frac{\tan(a+bx)}{(c+dx)^2} dx \\ &= -\frac{2 \sin(2a+2bx)}{d(c+dx)} + \frac{b \int \frac{\cos(2a+2bx)}{c+dx} dx}{d} + \frac{\left(3b \cos\left(2a - \frac{2bc}{d}\right)\right) \int \frac{\cos\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx}{d} - \left(3b \int \frac{\tan(a+bx)}{(c+dx)^2} dx\right) \\ &= \frac{3b \cos\left(2a - \frac{2bc}{d}\right) \text{Ci}\left(\frac{2bc}{d} + 2bx\right)}{d^2} - \frac{2 \sin(2a+2bx)}{d(c+dx)} - \frac{3b \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^2} \\ &= \frac{4b \cos\left(2a - \frac{2bc}{d}\right) \text{Ci}\left(\frac{2bc}{d} + 2bx\right)}{d^2} - \frac{2 \sin(2a+2bx)}{d(c+dx)} - \frac{4b \sin\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^2} \end{aligned}$$

Mathematica [A] time = 4.25604, size = 0, normalized size = 0.

$$\int \frac{\sec(a+bx) \sin(3a+3bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]

[Out] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2, x]

Maple [A] time = 0.663, size = 0, normalized size = 0.

$$\int \frac{\sec(bx + a) \sin(3bx + 3a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x)

[Out] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\left(i E_2 \left(\frac{2i b d x + 2i b c}{d} \right) - i E_2 \left(-\frac{2i b d x + 2i b c}{d} \right) \right) \cos \left(-\frac{2(bc - ad)}{d} \right) + 2(d^2 x + cd) \int \frac{\sin(2bx + 2a)}{(dx + c)^2 (\cos(2bx + 2a)^2 + \sin(2bx + 2a)^2 + 2 \cos(2bx + 2a) + 1)} dx$$

$$d^2 x + cd$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")

[Out] -((I*exp_integral_e(2, (2*I*b*d*x + 2*I*b*c)/d) - I*exp_integral_e(2, -(2*I*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 2*(d^2*x + c*d)*integrate(sin(2*b*x + 2*a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x + 2*a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(2*b*x + 2*a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x + 2*a)), x) + (exp_integral_e(2, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(2, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c - a*d)/d)/(d^2*x + c*d)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sec(bx + a) \sin(3bx + 3a)}{d^2 x^2 + 2cdx + c^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(sec(b*x + a)*sin(3*b*x + 3*a)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx + a) \sin(3bx + 3a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c)^2, x)
```

$$3.388 \quad \int \frac{\sec(a+bx) \sin(3a+3bx)}{(c+dx)^3} dx$$

Optimal. Leaf size=128

$$-\text{Unintegrable}\left(\frac{\tan(a+bx)}{(c+dx)^3}, x\right) - \frac{4b^2 \sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{d^3} - \frac{4b^2 \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^3} - \frac{2b^2 \text{Ci}\left(\frac{2bc}{d} + 2bx\right)}{d^3}$$

[Out] $(-2*b*\text{Cos}[2*a + 2*b*x])/(d^2*(c + d*x)) - (4*b^2*\text{CosIntegral}[(2*b*c)/d + 2*b*x]*\text{Sin}[2*a - (2*b*c)/d])/d^3 - \text{Sin}[2*a + 2*b*x]/(d*(c + d*x)^2) - (4*b^2*\text{Cos}[2*a - (2*b*c)/d]*\text{SinIntegral}[(2*b*c)/d + 2*b*x])/d^3 - \text{Unintegrable}[\text{Tan}[a + b*x]/(c + d*x)^3, x]$

Rubi [A] time = 0.397779, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sec(a+bx) \sin(3a+3bx)}{(c+dx)^3} dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[(\text{Sec}[a + b*x]*\text{Sin}[3*a + 3*b*x])/(c + d*x)^3, x]$

[Out] $(-2*b*\text{Cos}[2*a + 2*b*x])/(d^2*(c + d*x)) - (4*b^2*\text{CosIntegral}[(2*b*c)/d + 2*b*x]*\text{Sin}[2*a - (2*b*c)/d])/d^3 - \text{Sin}[2*a + 2*b*x]/(d*(c + d*x)^2) - (4*b^2*\text{Cos}[2*a - (2*b*c)/d]*\text{SinIntegral}[(2*b*c)/d + 2*b*x])/d^3 - \text{Defer}[\text{Int}][\text{Tan}[a + b*x]/(c + d*x)^3, x]$

Rubi steps

$$\begin{aligned} \int \frac{\sec(a+bx) \sin(3a+3bx)}{(c+dx)^3} dx &= \int \left(\frac{3 \cos(a+bx) \sin(a+bx)}{(c+dx)^3} - \frac{\sin^2(a+bx) \tan(a+bx)}{(c+dx)^3} \right) dx \\ &= 3 \int \frac{\cos(a+bx) \sin(a+bx)}{(c+dx)^3} dx - \int \frac{\sin^2(a+bx) \tan(a+bx)}{(c+dx)^3} dx \\ &= 3 \int \frac{\sin(2a+2bx)}{2(c+dx)^3} dx + \int \frac{\cos(a+bx) \sin(a+bx)}{(c+dx)^3} dx - \int \frac{\tan(a+bx)}{(c+dx)^3} dx \\ &= \frac{3}{2} \int \frac{\sin(2a+2bx)}{(c+dx)^3} dx + \int \frac{\sin(2a+2bx)}{2(c+dx)^3} dx - \int \frac{\tan(a+bx)}{(c+dx)^3} dx \\ &= -\frac{3 \sin(2a+2bx)}{4d(c+dx)^2} + \frac{1}{2} \int \frac{\sin(2a+2bx)}{(c+dx)^3} dx + \frac{(3b) \int \frac{\cos(2a+2bx)}{(c+dx)^2} dx}{2d} - \int \frac{\tan(a+bx)}{(c+dx)^3} dx \\ &= -\frac{3b \cos(2a+2bx)}{2d^2(c+dx)} - \frac{\sin(2a+2bx)}{d(c+dx)^2} - \frac{(3b^2) \int \frac{\sin(2a+2bx)}{c+dx} dx}{d^2} + \frac{b \int \frac{\cos(2a+2bx)}{(c+dx)^2} dx}{2d} - \int \frac{\tan(a+bx)}{(c+dx)^3} dx \\ &= -\frac{2b \cos(2a+2bx)}{d^2(c+dx)} - \frac{\sin(2a+2bx)}{d(c+dx)^2} - \frac{b^2 \int \frac{\sin(2a+2bx)}{c+dx} dx}{d^2} - \frac{(3b^2 \cos(2a - \frac{2bc}{d})) \int \frac{\sin(a+bx)}{c+dx} dx}{d^2} \\ &= -\frac{2b \cos(2a+2bx)}{d^2(c+dx)} - \frac{3b^2 \text{Ci}\left(\frac{2bc}{d} + 2bx\right) \sin\left(2a - \frac{2bc}{d}\right)}{d^3} - \frac{\sin(2a+2bx)}{d(c+dx)^2} - \frac{3b^2 \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^3} \\ &= -\frac{2b \cos(2a+2bx)}{d^2(c+dx)} - \frac{4b^2 \text{Ci}\left(\frac{2bc}{d} + 2bx\right) \sin\left(2a - \frac{2bc}{d}\right)}{d^3} - \frac{\sin(2a+2bx)}{d(c+dx)^2} - \frac{4b^2 \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{d^3} \end{aligned}$$

Mathematica [A] time = 5.89447, size = 0, normalized size = 0.

$$\int \frac{\sec(a + bx) \sin(3a + 3bx)}{(c + dx)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]

[Out] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3, x]

Maple [A] time = 0.504, size = 0, normalized size = 0.

$$\int \frac{\sec(bx + a) \sin(3bx + 3a)}{(dx + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x)

[Out] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(bx + a) \sin(3bx + 3a)}{d^3x^3 + 3cd^2x^2 + 3c^2dx + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")

[Out] integral(sec(b*x + a)*sin(3*b*x + 3*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx + a) \sin(3bx + 3a)}{(dx + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c)^3, x)
```

3.389 $\int (c + dx)^3 \sec^2(a + bx) \sin(3a + 3bx) dx$

Optimal. Leaf size=230

$$\frac{6id^2(c + dx)\text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} - \frac{6id^2(c + dx)\text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3} - \frac{6d^3\text{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^4} + \frac{6d^3\text{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^4}$$

```
[Out] ((-6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 + (24*d^2*(c + d*x)*Cos[a + b*x])/b^3 - (4*(c + d*x)^3*Cos[a + b*x])/b + ((6*I)*d^2*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 - ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 + (6*d^3*PolyLog[3, I*E^(I*(a + b*x))])/b^4 - ((c + d*x)^3*Sec[a + b*x])/b - (24*d^3*Sin[a + b*x])/b^4 + (12*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rubi [A] time = 0.329964, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.36$, Rules used = {4431, 3296, 2637, 4407, 4409, 4181, 2531, 2282, 6589}

$$\frac{6id^2(c + dx)\text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} - \frac{6id^2(c + dx)\text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3} - \frac{6d^3\text{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^4} + \frac{6d^3\text{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]^2*Sin[3*a + 3*b*x], x]
```

```
[Out] ((-6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 + (24*d^2*(c + d*x)*Cos[a + b*x])/b^3 - (4*(c + d*x)^3*Cos[a + b*x])/b + ((6*I)*d^2*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 - ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 + (6*d^3*PolyLog[3, I*E^(I*(a + b*x))])/b^4 - ((c + d*x)^3*Sec[a + b*x])/b - (24*d^3*Sin[a + b*x])/b^4 + (12*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol] := Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && MemberQ[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
```

eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 4409

Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] - Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

Rule 4181

Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 \sec^2(a + bx) \sin(3a + 3bx) dx &= \int \left(3(c + dx)^3 \sin(a + bx) - (c + dx)^3 \sin(a + bx) \tan^2(a + bx) \right) dx \\
&= 3 \int (c + dx)^3 \sin(a + bx) dx - \int (c + dx)^3 \sin(a + bx) \tan^2(a + bx) dx \\
&= -\frac{3(c + dx)^3 \cos(a + bx)}{b} + \frac{(9d) \int (c + dx)^2 \cos(a + bx) dx}{b} + \int (c + dx)^3 \sin(a + bx) dx \\
&= -\frac{4(c + dx)^3 \cos(a + bx)}{b} - \frac{(c + dx)^3 \sec(a + bx)}{b} + \frac{9d(c + dx)^2 \sin(a + bx)}{b^2} \\
&= -\frac{6id(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{18d^2(c + dx) \cos(a + bx)}{b^3} - \frac{4(c + dx)^3 \cos(a + bx)}{b} \\
&= -\frac{6id(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{24d^2(c + dx) \cos(a + bx)}{b^3} - \frac{4(c + dx)^3 \cos(a + bx)}{b} \\
&= -\frac{6id(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{24d^2(c + dx) \cos(a + bx)}{b^3} - \frac{4(c + dx)^3 \cos(a + bx)}{b} \\
&= -\frac{6id(c + dx)^2 \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{24d^2(c + dx) \cos(a + bx)}{b^3} - \frac{4(c + dx)^3 \cos(a + bx)}{b}
\end{aligned}$$

Mathematica [B] time = 2.33488, size = 547, normalized size = 2.38

$$\sec(a + bx) \left(6ibcd^2 \cos(a + bx) \text{PolyLog}(2, -\sin(a + bx) + i \cos(a + bx)) - 6ibcd^2 \cos(a + bx) \text{PolyLog}(2, \sin(a + bx) + i \cos(a + bx)) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^3*Sec[a + b*x]^2*Sin[3*a + 3*b*x], x]

[Out] -((Sec[a + b*x]*(3*b^3*c^3 - 12*b*c*d^2 + 9*b^3*c^2*d*x - 12*b*d^3*x + 9*b^3*c*d^2*x^2 + 3*b^3*d^3*x^3 - (6*I)*b^2*d^3*x^2*ArcTan[Cos[a + b*x] - I*Sin[a + b*x]]*Cos[a + b*x] + (6*I)*b^2*c^2*d*ArcTan[Cos[a + b*x] + I*Sin[a + b*x]]*Cos[a + b*x] + (12*I)*b^2*c*d^2*x*ArcTan[Cos[a + b*x] + I*Sin[a + b*x]]*Cos[a + b*x] + 2*b^3*c^3*Cos[2*(a + b*x)] - 12*b*c*d^2*Cos[2*(a + b*x)] + 6*b^3*c^2*d*x*Cos[2*(a + b*x)] - 12*b*d^3*x*Cos[2*(a + b*x)] + 6*b^3*c*d^2*x^2*Cos[2*(a + b*x)] + 2*b^3*d^3*x^3*Cos[2*(a + b*x)] - (6*I)*b*d^3*x*Cos[a + b*x]*PolyLog[2, (-I)*Cos[a + b*x] - Sin[a + b*x]] + (6*I)*b*c*d^2*Cos[a + b*x]*PolyLog[2, I*Cos[a + b*x] - Sin[a + b*x]] - (6*I)*b*c*d^2*Cos[a + b*x]*PolyLog[2, (-I)*Cos[a + b*x] + Sin[a + b*x]] + (6*I)*b*d^3*x*Cos[a + b*x]*PolyLog[2, I*Cos[a + b*x] + Sin[a + b*x]] - 6*d^3*Cos[a + b*x]*PolyLog[3, (-I)*Cos[a + b*x] - Sin[a + b*x]] + 6*d^3*Cos[a + b*x]*PolyLog[3, I*Cos[a + b*x] + Sin[a + b*x]] - 6*b^2*c^2*d*Sin[2*(a + b*x)] + 12*d^3*Sin[2*(a + b*x)] - 12*b^2*c*d^2*x*Sin[2*(a + b*x)] - 6*b^2*d^3*x^2*Sin[2*(a + b*x)]))/b^4)

Maple [B] time = 0.302, size = 677, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a), x)

[Out] -2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))-2*

$$\begin{aligned} & (d^3x^3b^3+3b^3c^2d^2x^2+3b^3c^2d^2x+b^3c^3-3Ib^2d^3x^2-6bd^3x \\ & x-6Ib^2c^2d^2x-6c^2d^2b-3Ib^2c^2d+6Id^3)/b^4\exp(-I(b*x+a))-2(d \\ & ^3x^3+3c^2d^2x^2+3c^2d^2x+c^3)\exp(I(b*x+a))/b/(\exp(2I(b*x+a))+1)+12* \\ & Id^2/b^3c^2\arctan(\exp(I(b*x+a)))-6Id/b^2c^2\arctan(\exp(I(b*x+a)))-6 \\ & *Id^3/b^4a^2\arctan(\exp(I(b*x+a)))-6d^2/b^3c^2\ln(1+I\exp(I(b*x+a)))*a- \\ & 6d^2/b^2c^2\ln(1+I\exp(I(b*x+a)))*x-3d^3/b^2\ln(1+I\exp(I(b*x+a)))*x^2-6 \\ & *Id^3/b^3\text{polylog}(2,I\exp(I(b*x+a)))*x-3d^3/b^4a^2\ln(1-I\exp(I(b*x+a) \\ &))-6d^3\text{polylog}(3,-I\exp(I(b*x+a)))/b^4-6Id^2/b^3c^2\text{polylog}(2,I\exp(I(b*x+a) \\ &))+6d^2/b^3c^2\ln(1-I\exp(I(b*x+a)))*a+6d^2/b^2c^2\ln(1-I\exp(I(b*x+a) \\ &))*x+3d^3/b^2\ln(1-I\exp(I(b*x+a)))*x^2+6d^3\text{polylog}(3,I\exp(I(b*x+a) \\ &))/b^4+6Id^3/b^3\text{polylog}(2,-I\exp(I(b*x+a)))*x+6Id^2/b^3c^2\text{polylog}(2, \\ & -I\exp(I(b*x+a)))+3d^3/b^4a^2\ln(1+I\exp(I(b*x+a))) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")

[Out]
$$\begin{aligned} & -2*((\cos(3bx + 3a) + \cos(bx + a))\cos(4bx + 4a) + (3\cos(2bx + 2a) \\ & + 1)\cos(3bx + 3a) + 3\cos(2bx + 2a)\cos(bx + a) + (\sin(3bx + 3a) \\ & + \sin(bx + a))\sin(4bx + 4a) + 3\sin(3bx + 3a)\sin(2bx + 2a) + \\ & 3\sin(2bx + 2a)\sin(bx + a) + \cos(bx + a))c^3/(b\cos(3bx + 3a)^2 \\ & + 2b\cos(3bx + 3a)\cos(bx + a) + b\cos(bx + a)^2 + b\sin(3bx + 3a) \\ & ^2 + 2b\sin(3bx + 3a)\sin(bx + a) + b\sin(bx + a)^2) - 3/2*(4(\cos(a) \\ & ^2 + \sin(a)^2)*bx\cos(bx + a) + 12*(bx\cos(2bx + 3a)\cos(bx + 2a) + \\ & bx\cos(bx + 2a)\cos(a) + bx\sin(2bx + 3a)\sin(bx + 2a) + bx\sin(bx + 2a) \\ & \sin(a))\cos(3bx + 3a)^2 + 4*(bx\cos(bx + a) - \sin(bx + a)) \\ & *\cos(2bx + 3a)^2 + 12*(bx\cos(2bx + 3a)\cos(bx + 2a) + bx\cos(bx \\ & + 2a)\cos(a) + bx\sin(2bx + 3a)\sin(bx + 2a) + bx\sin(bx + 2a) \\ & \sin(a))\sin(3bx + 3a)^2 + 4*(bx\cos(bx + a) - \sin(bx + a))\sin(2bx + \\ & 3a)^2 + 4*((bx\cos(2bx + 3a) + bx\cos(a) + \sin(2bx + 3a) + \sin(a) \\ &)\cos(3bx + 3a)^2 + (bx\cos(a) + \sin(a))\cos(bx + a)^2 + (bx\cos(2bx \\ & + 3a) + bx\cos(a) + \sin(2bx + 3a) + \sin(a))\sin(3bx + 3a)^2 + (bx \\ & \cos(a) + \sin(a))\sin(bx + a)^2 + 2*(bx\cos(2bx + 3a)\cos(bx + a) + \\ & (bx\cos(a) + \sin(a))\cos(bx + a) + \cos(bx + a)\sin(2bx + 3a))\cos(3bx \\ & + 3a) + (bx\cos(bx + a)^2 + bx\sin(bx + a)^2)\cos(2bx + 3a) + 2* \\ & (bx\cos(2bx + 3a)\sin(bx + a) + (bx\cos(a) + \sin(a))\sin(bx + a) + \sin(2bx \\ & + 3a)\sin(bx + a))\sin(3bx + 3a) + (\cos(bx + a)^2 + \sin(bx \\ & + a)^2)\sin(2bx + 3a)\cos(3bx + 4a) + 4*(6bx\cos(bx + 2a)\cos(bx \\ & + a)\cos(a) + 6bx\cos(bx + a)\sin(bx + 2a)\sin(a) + bx\cos(2bx + \\ & 3a)^2 + bx\sin(2bx + 3a)^2 + (\cos(a)^2 + \sin(a)^2)*bx + 2*(3bx\cos(bx \\ & + 2a)\cos(bx + a) + bx\cos(a))\cos(2bx + 3a) + 2*(3bx\cos(bx + \\ & a)\sin(bx + 2a) + bx\sin(a))\sin(2bx + 3a)\cos(3bx + 3a) + 4*(2* \\ & bx\cos(bx + a)\cos(a) + 3*(bx\cos(bx + a)^2 + bx\sin(bx + a)^2)\cos(bx \\ & + 2a) - 2\cos(a)\sin(bx + a)\cos(2bx + 3a) + 12*(bx\cos(bx + a)^2 \\ & *2\cos(a) + bx\cos(a)\sin(bx + a)^2)\cos(bx + 2a) - ((\cos(2bx + 3a)^2 \\ & + 2\cos(2bx + 3a)\cos(a) + \cos(a)^2 + \sin(2bx + 3a)^2 + 2\sin(2bx \\ & + 3a)\sin(a) + \sin(a)^2)\cos(3bx + 3a)^2 + (\cos(bx + a)^2 + \sin(bx + \\ & a)^2)\cos(2bx + 3a)^2 + (\cos(a)^2 + \sin(a)^2)\cos(bx + a)^2 + (\cos(2bx \\ & + 3a)^2 + 2\cos(2bx + 3a)\cos(a) + \cos(a)^2 + \sin(2bx + 3a)^2 + 2* \\ & \sin(2bx + 3a)\sin(a) + \sin(a)^2)\sin(3bx + 3a)^2 + (\cos(bx + a)^2 + \\ & \sin(bx + a)^2)\sin(2bx + 3a)^2 + (\cos(a)^2 + \sin(a)^2)\sin(bx + a)^2 + \\ & 2*(\cos(2bx + 3a)^2\cos(bx + a) + 2\cos(2bx + 3a)\cos(bx + a)\cos(a) \\ &) + \cos(bx + a)\sin(2bx + 3a)^2 + 2\cos(bx + a)\sin(2bx + 3a)\sin(a) \end{aligned}$$

$$\begin{aligned}
&) + (\cos(a)^2 + \sin(a)^2) \cos(b*x + a) \cos(3*b*x + 3*a) + 2 * (\cos(b*x + a)^2 \cos(a) + \cos(a) \sin(b*x + a)^2) \cos(2*b*x + 3*a) + 2 * (\cos(2*b*x + 3*a)^2 \sin(b*x + a) + 2 * \cos(2*b*x + 3*a) \cos(a) \sin(b*x + a) + \sin(2*b*x + 3*a)^2 \sin(b*x + a) + 2 * \sin(2*b*x + 3*a) \sin(b*x + a) \sin(a) + (\cos(a)^2 + \sin(a)^2) \sin(b*x + a) \sin(3*b*x + 3*a) + 2 * (\cos(b*x + a)^2 \sin(a) + \sin(b*x + a)^2 \sin(a)) \sin(2*b*x + 3*a) * \log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2 * \sin(b*x + a) + 1) + ((\cos(2*b*x + 3*a)^2 + 2 * \cos(2*b*x + 3*a) \cos(a) + \cos(a)^2 + \sin(2*b*x + 3*a)^2 + 2 * \sin(2*b*x + 3*a) \sin(a) + \sin(a)^2) \cos(3*b*x + 3*a))^2 + (\cos(b*x + a)^2 + \sin(b*x + a)^2) \cos(2*b*x + 3*a)^2 + (\cos(a)^2 + \sin(a)^2) \cos(b*x + a)^2 + (\cos(2*b*x + 3*a)^2 + 2 * \cos(2*b*x + 3*a) \cos(a) + \cos(a)^2 + \sin(2*b*x + 3*a)^2 + 2 * \sin(2*b*x + 3*a) \sin(a) + \sin(a)^2) \sin(3*b*x + 3*a)^2 + (\cos(b*x + a)^2 + \sin(b*x + a)^2) \sin(2*b*x + 3*a)^2 + (\cos(a)^2 + \sin(a)^2) \sin(b*x + a)^2 + 2 * (\cos(2*b*x + 3*a)^2 \cos(b*x + a) + 2 * \cos(2*b*x + 3*a) \cos(b*x + a) \cos(a) + \cos(b*x + a) \sin(2*b*x + 3*a)^2 + 2 * \cos(b*x + a) \sin(2*b*x + 3*a) \sin(a) + (\cos(a)^2 + \sin(a)^2) \cos(b*x + a)) \cos(3*b*x + 3*a) + 2 * (\cos(b*x + a)^2 \cos(a) + \cos(a) \sin(b*x + a)^2) \cos(2*b*x + 3*a) + 2 * (\cos(2*b*x + 3*a)^2 \sin(b*x + a) + 2 * \cos(2*b*x + 3*a) \cos(a) \sin(b*x + a) + \sin(2*b*x + 3*a)^2 \sin(b*x + a) + 2 * \sin(2*b*x + 3*a) \sin(b*x + a) \sin(a) + (\cos(a)^2 + \sin(a)^2) \sin(b*x + a)) \sin(3*b*x + 3*a) + 2 * (\cos(b*x + a)^2 \sin(a) + \sin(b*x + a)^2 \sin(a)) \sin(2*b*x + 3*a) * \log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2 * \sin(b*x + a) + 1) + 4 * ((b*x * \sin(2*b*x + 3*a) + b*x * \sin(a) - \cos(2*b*x + 3*a) - \cos(a)) \cos(3*b*x + 3*a)^2 + (b*x * \sin(a) - \cos(a)) \cos(b*x + a)^2 + (b*x * \sin(2*b*x + 3*a) + b*x * \sin(a) - \cos(2*b*x + 3*a) - \cos(a)) \sin(3*b*x + 3*a)^2 + (b*x * \sin(a) - \cos(a)) \sin(b*x + a)^2 + 2 * (b*x * \cos(b*x + a) \sin(2*b*x + 3*a) + (b*x * \sin(a) - \cos(a)) \cos(b*x + a) - \cos(2*b*x + 3*a) \cos(b*x + a)) \cos(3*b*x + 3*a) - (\cos(b*x + a)^2 + \sin(b*x + a)^2) \cos(2*b*x + 3*a) + 2 * (b*x * \sin(2*b*x + 3*a) \sin(b*x + a) + (b*x * \sin(a) - \cos(a)) \sin(b*x + a) - \cos(2*b*x + 3*a) \sin(b*x + a)) \sin(3*b*x + 3*a)) + (b*x * \cos(b*x + a)^2 + b*x * \sin(b*x + a)^2) \sin(2*b*x + 3*a) \sin(3*b*x + 4*a) + 4 * (6 * b*x * \cos(b*x + 2*a) \cos(a) \sin(b*x + a) + 6 * b*x * \sin(b*x + 2*a) \sin(b*x + a) \sin(a) + 2 * (3 * b*x * \cos(b*x + 2*a) \sin(b*x + a) - \cos(a)) \cos(2*b*x + 3*a) - \cos(2*b*x + 3*a)^2 - \cos(a)^2 + 2 * (3 * b*x * \sin(b*x + 2*a) \sin(b*x + a) - \sin(a)) \sin(2*b*x + 3*a) - \sin(2*b*x + 3*a)^2 - \sin(a)^2) \sin(3*b*x + 3*a) + 4 * (2 * b*x * \cos(b*x + a) \sin(a) + 3 * (b*x * \cos(b*x + a)^2 + b*x * \sin(b*x + a)^2) \sin(b*x + 2*a) - 2 * \sin(b*x + a) \sin(a)) \sin(2*b*x + 3*a) + 12 * (b*x * \cos(b*x + a)^2 \sin(a) + b*x * \sin(b*x + a)^2 \sin(a)) \sin(b*x + 2*a) - 4 * (\cos(a)^2 + \sin(a)^2) \sin(b*x + a) * c^2 * d / ((\cos(a)^2 + \sin(a)^2) * b^2 * \cos(b*x + a)^2 + (\cos(a)^2 + \sin(a)^2) * b^2 * \sin(b*x + a)^2 + (b^2 * \cos(2*b*x + 3*a)^2 + 2 * b^2 * \cos(2*b*x + 3*a) \cos(a) + b^2 * \sin(2*b*x + 3*a)^2 + 2 * b^2 * \sin(2*b*x + 3*a) \sin(a) + (\cos(a)^2 + \sin(a)^2) * b^2) \cos(3*b*x + 3*a)^2 + (b^2 * \cos(b*x + a)^2 + b^2 * \sin(b*x + a)^2) \cos(2*b*x + 3*a)^2 + (b^2 * \cos(2*b*x + 3*a)^2 + 2 * b^2 * \cos(2*b*x + 3*a) \cos(a) + b^2 * \sin(2*b*x + 3*a)^2 + 2 * b^2 * \sin(2*b*x + 3*a) \sin(a) + (\cos(a)^2 + \sin(a)^2) * b^2) \sin(3*b*x + 3*a)^2 + (b^2 * \cos(b*x + a)^2 + b^2 * \sin(b*x + a)^2) \sin(2*b*x + 3*a)^2 + 2 * (b^2 * \cos(2*b*x + 3*a)^2 \cos(b*x + a) + 2 * b^2 * \cos(2*b*x + 3*a) \cos(b*x + a) \cos(a) + b^2 * \cos(b*x + a) \sin(2*b*x + 3*a)^2 + 2 * b^2 * \cos(b*x + a) \sin(2*b*x + 3*a) \sin(a) + (\cos(a)^2 + \sin(a)^2) * b^2 * \cos(b*x + a)) \cos(3*b*x + 3*a) + 2 * (b^2 * \cos(b*x + a)^2 \cos(a) + b^2 * \cos(a) \sin(b*x + a)^2) \cos(2*b*x + 3*a) + 2 * (b^2 * \cos(2*b*x + 3*a)^2 \sin(b*x + a) + 2 * b^2 * \cos(2*b*x + 3*a) \cos(a) \sin(b*x + a) + b^2 * \sin(2*b*x + 3*a)^2 \sin(b*x + a) + 2 * b^2 * \sin(2*b*x + 3*a) \sin(b*x + a) \sin(a) + (\cos(a)^2 + \sin(a)^2) * b^2 * \sin(b*x + a)) \sin(3*b*x + 3*a) + 2 * (b^2 * \cos(b*x + a)^2 \sin(a) + b^2 * \sin(b*x + a)^2 \sin(a)) \sin(2*b*x + 3*a) - (6 * ((b^3 * d^3 * x^3 + 3 * b^3 * c * d^2 * x^2 - 4 * b * d^3 * x - 4 * b * c * d^2) \cos(2*b*x + 3*a) \cos(b*x + 2*a) + (b^3 * d^3 * x^3 + 3 * b^3 * c * d^2 * x^2 - 4 * b * d^3 * x - 4 * b * c * d^2) \sin(2*b*x + 3*a) \sin(b*x + 2*a) + (b^3 * d^3 * x^3 \cos(a) + 3 * b^3 * c * d^2 * x^2 \cos(a) - 4 * b * d^3 * x \cos(a) - 4 * b * c * d^2 \cos(a)) \cos(b*x + 2*a) + (b^3 * d^3 * x^3 \sin(a) + 3 * b^3 * c * d^2 * x^2 \sin(a) - 4 * b * d^3 * x \sin(a) - 4 * b * c * d^2 \sin(a)) \sin(b*x + 2*a)) * \cos(3*b*x + 3*a)^2 + 2 * ((b^3 * d^3 * x^3 + 3 * b^3 * c * d^2 * x^2 - 6 * b * d^3 * x - 6 * b * c * d^2) \cos(b*x + a) - 3 * (b^2 * d^3 * x^2 + 2 * b^2 * c * d^2 * x - 2 * d^3) \sin(b*x + a)) * c
\end{aligned}$$

$$\begin{aligned}
& x + a)^2 * \cos(2 * b * x + 3 * a)^2 + (b^4 * \cos(2 * b * x + 3 * a)^2 + 2 * b^4 * \cos(2 * b * x + \\
& 3 * a) * \cos(a) + b^4 * \sin(2 * b * x + 3 * a)^2 + 2 * b^4 * \sin(2 * b * x + 3 * a) * \sin(a) + (\cos \\
& (a)^2 + \sin(a)^2) * b^4) * \sin(3 * b * x + 3 * a)^2 + (b^4 * \cos(b * x + a)^2 + b^4 * \sin(b \\
& * x + a)^2) * \sin(2 * b * x + 3 * a)^2 + 2 * (b^4 * \cos(2 * b * x + 3 * a)^2 * \cos(b * x + a) + 2 * \\
& b^4 * \cos(2 * b * x + 3 * a) * \cos(b * x + a) * \cos(a) + b^4 * \cos(b * x + a) * \sin(2 * b * x + 3 * a \\
&)^2 + 2 * b^4 * \cos(b * x + a) * \sin(2 * b * x + 3 * a) * \sin(a) + (\cos(a)^2 + \sin(a)^2) * b^4 \\
& 4 * \cos(b * x + a)) * \cos(3 * b * x + 3 * a) + 2 * (b^4 * \cos(b * x + a)^2 * \cos(a) + b^4 * \cos(a) \\
&) * \sin(b * x + a)^2) * \cos(2 * b * x + 3 * a) + 2 * (b^4 * \cos(2 * b * x + 3 * a)^2 * \sin(b * x + a) \\
& + 2 * b^4 * \cos(2 * b * x + 3 * a) * \cos(a) * \sin(b * x + a) + b^4 * \sin(2 * b * x + 3 * a)^2 * \sin(\\
& b * x + a) + 2 * b^4 * \sin(2 * b * x + 3 * a) * \sin(b * x + a) * \sin(a) + (\cos(a)^2 + \sin(a)^2) * b^4 \\
& 2) * b^4 * \sin(b * x + a)) * \sin(3 * b * x + 3 * a) + 2 * (b^4 * \cos(b * x + a)^2 * \sin(a) + b^4 * \\
& \sin(b * x + a)^2 * \sin(a)) * \sin(2 * b * x + 3 * a)) * \int (6 * ((d^3 * x^2 + 2 * c * d^2 * x) \\
& * \cos(2 * b * x + 2 * a) * \cos(b * x + a) + (d^3 * x^2 + 2 * c * d^2 * x) * \sin(2 * b * x + 2 * a) * \sin \\
& (b * x + a) + (d^3 * x^2 + 2 * c * d^2 * x) * \cos(b * x + a)) / (b * \cos(2 * b * x + 2 * a)^2 + b * \sin \\
& (2 * b * x + 2 * a)^2 + 2 * b * \cos(2 * b * x + 2 * a) + b), x) + 2 * ((b^3 * d^3 * x^3 * \sin(a) \\
& - 6 * b * c * d^2 * \sin(a) + 6 * d^3 * \cos(a) + 3 * (b^3 * c * d^2 * \sin(a) - b^2 * d^3 * \cos(a)) * x \\
& ^2 - 6 * (b^2 * c * d^2 * \cos(a) + b * d^3 * \sin(a)) * x - 3 * (b^2 * d^3 * x^2 + 2 * b^2 * c * d^2 * x \\
& - 2 * d^3) * \cos(2 * b * x + 3 * a) + (b^3 * d^3 * x^3 + 3 * b^3 * c * d^2 * x^2 - 6 * b * d^3 * x - 6 \\
& * b * c * d^2) * \sin(2 * b * x + 3 * a)) * \cos(3 * b * x + 3 * a)^2 + (b^3 * d^3 * x^3 * \sin(a) - 6 * b * \\
& c * d^2 * \sin(a) + 6 * d^3 * \cos(a) + 3 * (b^3 * c * d^2 * \sin(a) - b^2 * d^3 * \cos(a)) * x^2 - 6 \\
& * (b^2 * c * d^2 * \cos(a) + b * d^3 * \sin(a)) * x) * \cos(b * x + a)^2 + (b^3 * d^3 * x^3 * \sin(a) \\
& - 6 * b * c * d^2 * \sin(a) + 6 * d^3 * \cos(a) + 3 * (b^3 * c * d^2 * \sin(a) - b^2 * d^3 * \cos(a)) * x \\
& ^2 - 6 * (b^2 * c * d^2 * \cos(a) + b * d^3 * \sin(a)) * x - 3 * (b^2 * d^3 * x^2 + 2 * b^2 * c * d^2 * x \\
& - 2 * d^3) * \cos(2 * b * x + 3 * a) + (b^3 * d^3 * x^3 + 3 * b^3 * c * d^2 * x^2 - 6 * b * d^3 * x - 6 \\
& * b * c * d^2) * \sin(2 * b * x + 3 * a)) * \sin(3 * b * x + 3 * a)^2 + (b^3 * d^3 * x^3 * \sin(a) - 6 * b * \\
& c * d^2 * \sin(a) + 6 * d^3 * \cos(a) + 3 * (b^3 * c * d^2 * \sin(a) - b^2 * d^3 * \cos(a)) * x^2 - 6 \\
& * (b^2 * c * d^2 * \cos(a) + b * d^3 * \sin(a)) * x) * \sin(b * x + a)^2 - 2 * (3 * (b^2 * d^3 * x^2 + \\
& 2 * b^2 * c * d^2 * x - 2 * d^3) * \cos(2 * b * x + 3 * a) * \cos(b * x + a) - (b^3 * d^3 * x^3 + 3 * b^3 \\
& * c * d^2 * x^2 - 6 * b * d^3 * x - 6 * b * c * d^2) * \cos(b * x + a) * \sin(2 * b * x + 3 * a) - (b^3 * d^3 \\
& * x^3 * \sin(a) - 6 * b * c * d^2 * \sin(a) + 6 * d^3 * \cos(a) + 3 * (b^3 * c * d^2 * \sin(a) - b^2 * \\
& d^3 * \cos(a)) * x^2 - 6 * (b^2 * c * d^2 * \cos(a) + b * d^3 * \sin(a)) * x) * \cos(b * x + a)) * \cos(\\
& 3 * b * x + 3 * a) - 3 * ((b^2 * d^3 * x^2 + 2 * b^2 * c * d^2 * x - 2 * d^3) * \cos(b * x + a)^2 + (b \\
& ^2 * d^3 * x^2 + 2 * b^2 * c * d^2 * x - 2 * d^3) * \sin(b * x + a)^2) * \cos(2 * b * x + 3 * a) - 2 * (3 \\
& * (b^2 * d^3 * x^2 + 2 * b^2 * c * d^2 * x - 2 * d^3) * \cos(2 * b * x + 3 * a) * \sin(b * x + a) - (b^3 \\
& * d^3 * x^3 + 3 * b^3 * c * d^2 * x^2 - 6 * b * d^3 * x - 6 * b * c * d^2) * \sin(2 * b * x + 3 * a) * \sin(b * \\
& x + a) - (b^3 * d^3 * x^3 * \sin(a) - 6 * b * c * d^2 * \sin(a) + 6 * d^3 * \cos(a) + 3 * (b^3 * c * d^2 \\
& ^2 * \sin(a) - b^2 * d^3 * \cos(a)) * x^2 - 6 * (b^2 * c * d^2 * \cos(a) + b * d^3 * \sin(a)) * x) * \sin \\
& (b * x + a)) * \sin(3 * b * x + 3 * a) + ((b^3 * d^3 * x^3 + 3 * b^3 * c * d^2 * x^2 - 6 * b * d^3 * x \\
& - 6 * b * c * d^2) * \cos(b * x + a)^2 + (b^3 * d^3 * x^3 + 3 * b^3 * c * d^2 * x^2 - 6 * b * d^3 * x - \\
& 6 * b * c * d^2) * \sin(b * x + a)^2) * \sin(2 * b * x + 3 * a)) * \sin(3 * b * x + 4 * a) - 6 * ((\cos(a)^2 \\
& + \sin(a)^2) * b^2 * d^3 * x^2 + 2 * (\cos(a)^2 + \sin(a)^2) * b^2 * c * d^2 * x - 2 * (\cos(a)^2 \\
& + \sin(a)^2) * d^3 + (b^2 * d^3 * x^2 + 2 * b^2 * c * d^2 * x - 2 * d^3) * \cos(2 * b * x + 3 * a) \\
& ^2 + (b^2 * d^3 * x^2 + 2 * b^2 * c * d^2 * x - 2 * d^3) * \sin(2 * b * x + 3 * a)^2 - 2 * (b^3 * d^3 * \\
& x^3 * \cos(a) + 3 * b^3 * c * d^2 * x^2 * \cos(a) - 4 * b * d^3 * x * \cos(a) - 4 * b * c * d^2 * \cos(a)) * \\
& \cos(b * x + 2 * a) * \sin(b * x + a) - 2 * (b^3 * d^3 * x^3 * \sin(a) + 3 * b^3 * c * d^2 * x^2 * \sin(a) \\
&) - 4 * b * d^3 * x * \sin(a) - 4 * b * c * d^2 * \sin(a)) * \sin(b * x + 2 * a) * \sin(b * x + a) + 2 * (b \\
& ^2 * d^3 * x^2 * \cos(a) + 2 * b^2 * c * d^2 * x * \cos(a) - 2 * d^3 * \cos(a) - (b^3 * d^3 * x^3 + 3 * \\
& b^3 * c * d^2 * x^2 - 4 * b * d^3 * x - 4 * b * c * d^2) * \cos(b * x + 2 * a) * \sin(b * x + a)) * \cos(2 * b \\
& * x + 3 * a) + 2 * (b^2 * d^3 * x^2 * \sin(a) + 2 * b^2 * c * d^2 * x * \sin(a) - 2 * d^3 * \sin(a) - (\\
& b^3 * d^3 * x^3 + 3 * b^3 * c * d^2 * x^2 - 4 * b * d^3 * x - 4 * b * c * d^2) * \sin(b * x + 2 * a) * \sin(b \\
& * x + a)) * \sin(2 * b * x + 3 * a)) * \sin(3 * b * x + 3 * a) + 2 * (2 * (b^3 * d^3 * x^3 * \sin(a) + 3 * \\
& b^3 * c * d^2 * x^2 * \sin(a) - 6 * b * d^3 * x * \sin(a) - 6 * b * c * d^2 * \sin(a)) * \cos(b * x + a) + \\
& 3 * ((b^3 * d^3 * x^3 + 3 * b^3 * c * d^2 * x^2 - 4 * b * d^3 * x - 4 * b * c * d^2) * \cos(b * x + a)^2 + \\
& (b^3 * d^3 * x^3 + 3 * b^3 * c * d^2 * x^2 - 4 * b * d^3 * x - 4 * b * c * d^2) * \sin(b * x + a)^2) * \sin \\
& (b * x + 2 * a) - 6 * (b^2 * d^3 * x^2 * \sin(a) + 2 * b^2 * c * d^2 * x * \sin(a) - 2 * d^3 * \sin(a)) \\
& * \sin(b * x + a)) * \sin(2 * b * x + 3 * a) + 6 * ((b^3 * d^3 * x^3 * \sin(a) + 3 * b^3 * c * d^2 * x^2 * \\
& \sin(a) - 4 * b * d^3 * x * \sin(a) - 4 * b * c * d^2 * \sin(a)) * \cos(b * x + a)^2 + (b^3 * d^3 * x^3 \\
& * \sin(a) + 3 * b^3 * c * d^2 * x^2 * \sin(a) - 4 * b * d^3 * x * \sin(a) - 4 * b * c * d^2 * \sin(a)) * \sin \\
& (b * x + a)^2) * \sin(b * x + 2 * a) - 6 * ((\cos(a)^2 + \sin(a)^2) * b^2 * d^3 * x^2 + 2 * (\cos
\end{aligned}$$

$$\frac{(a)^2 + \sin(a)^2 * b^2 * c * d^2 * x - 2 * (\cos(a)^2 + \sin(a)^2) * d^3 * \sin(b * x + a)}{((\cos(a)^2 + \sin(a)^2) * b^4 * \cos(b * x + a)^2 + (\cos(a)^2 + \sin(a)^2) * b^4 * \sin(b * x + a)^2 + (b^4 * \cos(2 * b * x + 3 * a)^2 + 2 * b^4 * \cos(2 * b * x + 3 * a) * \cos(a) + b^4 * \sin(2 * b * x + 3 * a)^2 + 2 * b^4 * \sin(2 * b * x + 3 * a) * \sin(a) + (\cos(a)^2 + \sin(a)^2) * b^4) * \cos(3 * b * x + 3 * a)^2 + (b^4 * \cos(b * x + a)^2 + b^4 * \sin(b * x + a)^2) * \cos(2 * b * x + 3 * a)^2 + (b^4 * \cos(2 * b * x + 3 * a)^2 + 2 * b^4 * \cos(2 * b * x + 3 * a) * \cos(a) + b^4 * \sin(2 * b * x + 3 * a)^2 + 2 * b^4 * \sin(2 * b * x + 3 * a) * \sin(a) + (\cos(a)^2 + \sin(a)^2) * b^4) * \sin(3 * b * x + 3 * a)^2 + (b^4 * \cos(b * x + a)^2 + b^4 * \sin(b * x + a)^2) * \sin(2 * b * x + 3 * a)^2 + 2 * (b^4 * \cos(2 * b * x + 3 * a)^2 * \cos(b * x + a) + 2 * b^4 * \cos(2 * b * x + 3 * a) * \cos(b * x + a) * \cos(a) + b^4 * \cos(b * x + a) * \sin(2 * b * x + 3 * a)^2 + 2 * b^4 * \cos(b * x + a) * \sin(2 * b * x + 3 * a) * \sin(a) + (\cos(a)^2 + \sin(a)^2) * b^4 * \cos(b * x + a)) * \cos(3 * b * x + 3 * a) + 2 * (b^4 * \cos(b * x + a)^2 * \cos(a) + b^4 * \cos(a) * \sin(b * x + a)^2) * \cos(2 * b * x + 3 * a) + 2 * (b^4 * \cos(2 * b * x + 3 * a)^2 * \sin(b * x + a) + 2 * b^4 * \cos(2 * b * x + 3 * a) * \cos(a) * \sin(b * x + a) + b^4 * \sin(2 * b * x + 3 * a)^2 * \sin(b * x + a) + 2 * b^4 * \sin(2 * b * x + 3 * a) * \sin(b * x + a) * \sin(a) + (\cos(a)^2 + \sin(a)^2) * b^4 * \sin(b * x + a)) * \sin(3 * b * x + 3 * a) + 2 * (b^4 * \cos(b * x + a)^2 * \sin(a) + b^4 * \sin(b * x + a)^2 * \sin(a)) * \sin(2 * b * x + 3 * a)}$$

Fricas [C] time = 0.755471, size = 2240, normalized size = 9.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")

[Out]
$$-1/2 * (2 * b^3 * d^3 * x^3 + 6 * b^3 * c * d^2 * x^2 + 6 * b^3 * c^2 * d * x + 2 * b^3 * c^3 + 6 * d^3 * c \cos(b * x + a) * \text{polylog}(3, I * \cos(b * x + a) + \sin(b * x + a)) - 6 * d^3 * \cos(b * x + a) * \text{polylog}(3, I * \cos(b * x + a) - \sin(b * x + a)) + 6 * d^3 * \cos(b * x + a) * \text{polylog}(3, -I * \cos(b * x + a) + \sin(b * x + a)) - 6 * d^3 * \cos(b * x + a) * \text{polylog}(3, -I * \cos(b * x + a) - \sin(b * x + a)) + 8 * (b^3 * d^3 * x^3 + 3 * b^3 * c * d^2 * x^2 + b^3 * c^3 - 6 * b * c * d^2 + 3 * (b^3 * c^2 * d - 2 * b * d^3) * x) * \cos(b * x + a)^2 - (-6 * I * b * d^3 * x - 6 * I * b * c * d^2) * \cos(b * x + a) * \text{dilog}(I * \cos(b * x + a) + \sin(b * x + a)) - (-6 * I * b * d^3 * x - 6 * I * b * c * d^2) * \cos(b * x + a) * \text{dilog}(I * \cos(b * x + a) - \sin(b * x + a)) - (6 * I * b * d^3 * x + 6 * I * b * c * d^2) * \cos(b * x + a) * \text{dilog}(-I * \cos(b * x + a) + \sin(b * x + a)) - (6 * I * b * d^3 * x + 6 * I * b * c * d^2) * \cos(b * x + a) * \text{dilog}(-I * \cos(b * x + a) - \sin(b * x + a)) - 3 * (b^2 * c^2 * d - 2 * a * b * c * d^2 + a^2 * d^3) * \cos(b * x + a) * \log(\cos(b * x + a) + I * \sin(b * x + a) + I) + 3 * (b^2 * c^2 * d - 2 * a * b * c * d^2 + a^2 * d^3) * \cos(b * x + a) * \log(\cos(b * x + a) - I * \sin(b * x + a) + I) - 3 * (b^2 * d^3 * x^2 + 2 * b^2 * c * d^2 * x + 2 * a * b * c * d^2 - a^2 * d^3) * \cos(b * x + a) * \log(I * \cos(b * x + a) + \sin(b * x + a) + 1) + 3 * (b^2 * d^3 * x^2 + 2 * b^2 * c * d^2 * x + 2 * a * b * c * d^2 - a^2 * d^3) * \cos(b * x + a) * \log(I * \cos(b * x + a) - \sin(b * x + a) + 1) - 3 * (b^2 * d^3 * x^2 + 2 * b^2 * c * d^2 * x + 2 * a * b * c * d^2 - a^2 * d^3) * \cos(b * x + a) * \log(-I * \cos(b * x + a) + \sin(b * x + a) + 1) + 3 * (b^2 * d^3 * x^2 + 2 * b^2 * c * d^2 * x + 2 * a * b * c * d^2 - a^2 * d^3) * \cos(b * x + a) * \log(-I * \cos(b * x + a) - \sin(b * x + a) + 1) - 3 * (b^2 * c^2 * d - 2 * a * b * c * d^2 + a^2 * d^3) * \cos(b * x + a) * \log(-\cos(b * x + a) + I * \sin(b * x + a) + I) + 3 * (b^2 * c^2 * d - 2 * a * b * c * d^2 + a^2 * d^3) * \cos(b * x + a) * \log(-\cos(b * x + a) - I * \sin(b * x + a) + I) - 24 * (b^2 * d^3 * x^2 + 2 * b^2 * c * d^2 * x + b^2 * c^2 * d - 2 * d^3) * \cos(b * x + a) * \sin(b * x + a)) / (b^4 * \cos(b * x + a))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \sec(bx + a)^2 \sin(3bx + 3a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)^2*sin(3*b*x + 3*a), x)
```

3.390 $\int (c + dx)^2 \sec^2(a + bx) \sin(3a + 3bx) dx$

Optimal. Leaf size=147

$$\frac{2id^2 \text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} - \frac{2id^2 \text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3} + \frac{8d(c+dx)\sin(a+bx)}{b^2} - \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{8d^2 \cos(a+bx)}{b^3}$$

```
[Out] ((-4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 + (8*d^2*Cos[a + b*x])/b^3
- (4*(c + d*x)^2*Cos[a + b*x])/b + ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a + b*
x))])/b^3 - ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((c + d*x)^2*Se
c[a + b*x])/b + (8*d*(c + d*x)*Sin[a + b*x])/b^2
```

Rubi [A] time = 0.212481, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$, Rules used = {4431, 3296, 2638, 4407, 4409, 4181, 2279, 2391}

$$\frac{2id^2 \text{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} - \frac{2id^2 \text{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3} + \frac{8d(c+dx)\sin(a+bx)}{b^2} - \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{8d^2 \cos(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]^2*Sin[3*a + 3*b*x], x]
```

```
[Out] ((-4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 + (8*d^2*Cos[a + b*x])/b^3
- (4*(c + d*x)^2*Cos[a + b*x])/b + ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a + b*
x))])/b^3 - ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((c + d*x)^2*Se
c[a + b*x])/b + (8*d*(c + d*x)*Sin[a + b*x])/b^2
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] := Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] := -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p-2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n-2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*Sec[(a_.) + (b_.)*(x_.)]^(n_.)*Tan[(a_.) + (b_.)*(x_.)]^(p_.), x_Symbol] := Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] - Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_.))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \sec^2(a + bx) \sin(3a + 3bx) dx &= \int (3(c + dx)^2 \sin(a + bx) - (c + dx)^2 \sin(a + bx) \tan^2(a + bx)) dx \\ &= 3 \int (c + dx)^2 \sin(a + bx) dx - \int (c + dx)^2 \sin(a + bx) \tan^2(a + bx) dx \\ &= -\frac{3(c + dx)^2 \cos(a + bx)}{b} + \frac{(6d) \int (c + dx) \cos(a + bx) dx}{b} + \int (c + dx)^2 \sin(a + bx) dx \\ &= -\frac{4(c + dx)^2 \cos(a + bx)}{b} - \frac{(c + dx)^2 \sec(a + bx)}{b} + \frac{6d(c + dx) \sin(a + bx)}{b^2} \\ &= -\frac{4id(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{6d^2 \cos(a + bx)}{b^3} - \frac{4(c + dx)^2 \cos(a + bx)}{b} \\ &= -\frac{4id(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{8d^2 \cos(a + bx)}{b^3} - \frac{4(c + dx)^2 \cos(a + bx)}{b} \\ &= -\frac{4id(c + dx) \tan^{-1}(e^{i(a+bx)})}{b^2} + \frac{8d^2 \cos(a + bx)}{b^3} - \frac{4(c + dx)^2 \cos(a + bx)}{b} \end{aligned}$$

Mathematica [B] time = 3.7371, size = 364, normalized size = 2.48

$$2d^2 \left(2 \tan^{-1}(\cot(a)) \tanh^{-1} \left(\cos(a) \tan \left(\frac{bx}{2} \right) + \sin(a) \right) - \frac{\csc(a) \left(i \operatorname{PolyLog} \left(2, -e^{i(bx - \tan^{-1}(\cot(a)))} \right) - i \operatorname{PolyLog} \left(2, e^{i(bx - \tan^{-1}(\cot(a)))} \right) \right) + (b \dots)}{\sqrt{\csc^2(a)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]^2*Sin[3*a + 3*b*x], x]
```

```
[Out] (4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + 2*d^2*(2*ArcTan[Cot[a]]*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - (Csc[a]*((b*x - ArcTan[Cot[a]])*(Log[1 - E^(I*(b*x - ArcTan[Cot[a]])]) - Log[1 + E^(I*(b*x - ArcTan[Cot[a]])])
```

$$+ I \cdot \text{PolyLog}[2, -E^{(I \cdot (b \cdot x - \text{ArcTan}[\text{Cot}[a]]))}] - I \cdot \text{PolyLog}[2, E^{(I \cdot (b \cdot x - \text{ArcTan}[\text{Cot}[a]]))}] / \sqrt{\text{Csc}[a]^2}] - b^2 \cdot (c + d \cdot x)^2 \cdot \text{Sec}[a] - 4 \cdot \text{Cos}[b \cdot x] \cdot ((-2 \cdot d^2 + b^2 \cdot (c + d \cdot x)^2) \cdot \text{Cos}[a] - 2 \cdot b \cdot d \cdot (c + d \cdot x) \cdot \text{Sin}[a]) + 4 \cdot (2 \cdot b \cdot d \cdot (c + d \cdot x) \cdot \text{Cos}[a] + (-2 \cdot d^2 + b^2 \cdot (c + d \cdot x)^2) \cdot \text{Sin}[a]) \cdot \text{Sin}[b \cdot x] - (b^2 \cdot (c + d \cdot x)^2 \cdot \text{Sin}[(b \cdot x)/2]) / ((\text{Cos}[a/2] - \text{Sin}[a/2]) \cdot (\text{Cos}[(a + b \cdot x)/2] - \text{Sin}[(a + b \cdot x)/2])) + (b^2 \cdot (c + d \cdot x)^2 \cdot \text{Sin}[(b \cdot x)/2]) / ((\text{Cos}[a/2] + \text{Sin}[a/2]) \cdot (\text{Cos}[(a + b \cdot x)/2] + \text{Sin}[(a + b \cdot x)/2])) / b^3$$

Maple [B] time = 0.308, size = 345, normalized size = 2.4

$$-2 \frac{(d^2 x^2 b^2 + 2 b^2 c d x + b^2 c^2 + 2 i b d^2 x - 2 d^2 + 2 i b c d) e^{i(bx+a)}}{b^3} - 2 \frac{(d^2 x^2 b^2 + 2 b^2 c d x + b^2 c^2 - 2 i b d^2 x - 2 d^2 - 2 i b c d) e^{-i(bx+a)}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x)

[Out] $-2 \cdot (d^2 \cdot x^2 \cdot b^2 + 2 \cdot b^2 \cdot c \cdot d \cdot x + b^2 \cdot c^2 + 2 \cdot I \cdot b \cdot d^2 \cdot x - 2 \cdot d^2 + 2 \cdot I \cdot b \cdot c \cdot d) / b^3 \cdot \exp(I \cdot (b \cdot x + a)) - 2 \cdot (d^2 \cdot x^2 \cdot b^2 + 2 \cdot b^2 \cdot c \cdot d \cdot x + b^2 \cdot c^2 - 2 \cdot I \cdot b \cdot d^2 \cdot x - 2 \cdot d^2 - 2 \cdot I \cdot b \cdot c \cdot d) / b^3 \cdot \exp(-I \cdot (b \cdot x + a)) - 2 \cdot \exp(I \cdot (b \cdot x + a)) \cdot (d^2 \cdot x^2 + 2 \cdot c \cdot d \cdot x + c^2) / b / (\exp(2 \cdot I \cdot (b \cdot x + a)) + 1) - 4 \cdot I \cdot d / b^2 \cdot c \cdot \arctan(\exp(I \cdot (b \cdot x + a))) - 2 \cdot d^2 / b^2 \cdot \ln(1 + I \cdot \exp(I \cdot (b \cdot x + a))) \cdot x - 2 \cdot d^2 / b^3 \cdot \ln(1 + I \cdot \exp(I \cdot (b \cdot x + a))) \cdot a + 2 \cdot d^2 / b^2 \cdot \ln(1 - I \cdot \exp(I \cdot (b \cdot x + a))) \cdot x + 2 \cdot d^2 / b^3 \cdot \ln(1 - I \cdot \exp(I \cdot (b \cdot x + a))) \cdot a + 2 \cdot I \cdot d^2 / b^3 \cdot \text{dilog}(1 + I \cdot \exp(I \cdot (b \cdot x + a))) - 2 \cdot I \cdot d^2 / b^3 \cdot \text{dilog}(1 - I \cdot \exp(I \cdot (b \cdot x + a))) + 4 \cdot I \cdot d^2 / b^3 \cdot a \cdot \arctan(\exp(I \cdot (b \cdot x + a)))$

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")

[Out] Timed out

Fricas [B] time = 0.638272, size = 1320, normalized size = 8.98

$$\frac{b^2 d^2 x^2 + 2 b^2 c d x + b^2 c^2 + i d^2 \cos(bx + a) \text{Li}_2(i \cos(bx + a) + \sin(bx + a)) + i d^2 \cos(bx + a) \text{Li}_2(i \cos(bx + a) - \sin(bx + a))}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")

[Out] $-(b^2 \cdot d^2 \cdot x^2 + 2 \cdot b^2 \cdot c \cdot d \cdot x + b^2 \cdot c^2 + I \cdot d^2 \cdot \cos(b \cdot x + a) \cdot \text{dilog}(I \cdot \cos(b \cdot x + a) + \sin(b \cdot x + a)) + I \cdot d^2 \cdot \cos(b \cdot x + a) \cdot \text{dilog}(I \cdot \cos(b \cdot x + a) - \sin(b \cdot x + a))) - I \cdot d^2 \cdot \cos(b \cdot x + a) \cdot \text{dilog}(-I \cdot \cos(b \cdot x + a) + \sin(b \cdot x + a)) - I \cdot d^2 \cdot \cos(b \cdot x + a) \cdot \text{dilog}(-I \cdot \cos(b \cdot x + a) - \sin(b \cdot x + a)) + 4 \cdot (b^2 \cdot d^2 \cdot x^2 + 2 \cdot b^2 \cdot c \cdot d \cdot x + b^2 \cdot c^2 - 2 \cdot d^2) \cdot \cos(b \cdot x + a)^2 - (b \cdot c \cdot d - a \cdot d^2) \cdot \cos(b \cdot x + a) \cdot \log(\cos(b \cdot x + a) + I \cdot \sin(b \cdot x + a) + I) + (b \cdot c \cdot d - a \cdot d^2) \cdot \cos(b \cdot x + a) \cdot \log(\cos(b \cdot x + a) - I \cdot \sin(b \cdot x + a) + I) - (b \cdot d^2 \cdot x + a \cdot d^2) \cdot \cos(b \cdot x + a) \cdot \log(I \cdot \cos(b \cdot x + a) + \sin(b \cdot x + a)) - (b \cdot d^2 \cdot x + a \cdot d^2) \cdot \cos(b \cdot x + a) \cdot \log(I \cdot \cos(b \cdot x + a) - \sin(b \cdot x + a))$


```

a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a)
- sin(b*x + a) + 1) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) +
sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - s
in(b*x + a) + 1) - (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b
*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x +
a) + I) - 8*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3*cos(b*x + a)
)

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \sec(bx + a)^2 \sin(3bx + 3a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)^2*sin(3*b*x + 3*a), x)
```

3.391 $\int (c + dx) \sec^2(a + bx) \sin(3a + 3bx) dx$

Optimal. Leaf size=57

$$\frac{4d \sin(a + bx)}{b^2} + \frac{d \tanh^{-1}(\sin(a + bx))}{b^2} - \frac{4(c + dx) \cos(a + bx)}{b} - \frac{(c + dx) \sec(a + bx)}{b}$$

[Out] (d*ArcTanh[Sin[a + b*x]])/b^2 - (4*(c + d*x)*Cos[a + b*x])/b - ((c + d*x)*Sec[a + b*x])/b + (4*d*Sin[a + b*x])/b^2

Rubi [A] time = 0.0914535, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4431, 3296, 2637, 4407, 4409, 3770}

$$\frac{4d \sin(a + bx)}{b^2} + \frac{d \tanh^{-1}(\sin(a + bx))}{b^2} - \frac{4(c + dx) \cos(a + bx)}{b} - \frac{(c + dx) \sec(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Sec[a + b*x]^2*Sin[3*a + 3*b*x], x]

[Out] (d*ArcTanh[Sin[a + b*x]])/b^2 - (4*(c + d*x)*Cos[a + b*x])/b - ((c + d*x)*Sec[a + b*x])/b + (4*d*Sin[a + b*x])/b^2

Rule 4431

Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && MemberQ[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4407

Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 4409

Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*Sec[a + b*x]^n/(b*n), x] - Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

Rule 3770

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned} \int (c + dx) \sec^2(a + bx) \sin(3a + 3bx) dx &= \int (3(c + dx) \sin(a + bx) - (c + dx) \sin(a + bx) \tan^2(a + bx)) dx \\ &= 3 \int (c + dx) \sin(a + bx) dx - \int (c + dx) \sin(a + bx) \tan^2(a + bx) dx \\ &= -\frac{3(c + dx) \cos(a + bx)}{b} + \frac{(3d) \int \cos(a + bx) dx}{b} + \int (c + dx) \sin(a + bx) dx \\ &= -\frac{4(c + dx) \cos(a + bx)}{b} - \frac{(c + dx) \sec(a + bx)}{b} + \frac{3d \sin(a + bx)}{b^2} + \frac{d \int \cos(a + bx) dx}{b} \\ &= \frac{d \tanh^{-1}(\sin(a + bx))}{b^2} - \frac{4(c + dx) \cos(a + bx)}{b} - \frac{(c + dx) \sec(a + bx)}{b} + \frac{4d \sin(a + bx)}{b^2} \end{aligned}$$

Mathematica [A] time = 0.453908, size = 105, normalized size = 1.84

$$\frac{\sec(a + bx) \left(2b(c + dx) \cos(2(a + bx)) - 2d \sin(2(a + bx)) + d \cos(a + bx) \left(\log \left(\cos \left(\frac{1}{2}(a + bx) \right) - \sin \left(\frac{1}{2}(a + bx) \right) \right) \right) \right)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Sec[a + b*x]^2*Sin[3*a + 3*b*x], x]

[Out] -((Sec[a + b*x]*(3*b*c + 3*b*d*x + 2*b*(c + d*x)*Cos[2*(a + b*x)] + d*Cos[a + b*x]*(Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]] - Log[Cos[(a + b*x)/2] + Sin[(a + b*x)/2]])) - 2*d*Sin[2*(a + b*x)]))/b^2

Maple [A] time = 0.046, size = 87, normalized size = 1.5

$$-4 \frac{c \cos(bx + a)}{b} - \frac{c}{b \cos(bx + a)} - 4 \frac{d \cos(bx + a)x}{b} + 4 \frac{d \sin(bx + a)}{b^2} - \frac{dx}{b \cos(bx + a)} + \frac{d \ln(\sec(bx + a) + \tan(bx + a))}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a), x)

[Out] -4*c/b*cos(b*x+a)-c/b/cos(b*x+a)-4*d/b*cos(b*x+a)*x+4*d*sin(b*x+a)/b^2-d/b/cos(b*x+a)*x+d/b^2*ln(sec(b*x+a)+tan(b*x+a))

Maxima [B] time = 1.85269, size = 4496, normalized size = 78.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a), x, algorithm="maxima")

```

[Out] -2*((cos(3*b*x + 3*a) + cos(b*x + a))*cos(4*b*x + 4*a) + (3*cos(2*b*x + 2*a)
) + 1)*cos(3*b*x + 3*a) + 3*cos(2*b*x + 2*a)*cos(b*x + a) + (sin(3*b*x + 3*
a) + sin(b*x + a))*sin(4*b*x + 4*a) + 3*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) +
3*sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))*c/(b*cos(3*b*x + 3*a)^2 +
2*b*cos(3*b*x + 3*a)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x + 3*a)^2
+ 2*b*sin(3*b*x + 3*a)*sin(b*x + a) + b*sin(b*x + a)^2) - 1/2*(4*(cos(a)^2
+ sin(a)^2)*b*x*cos(b*x + a) + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) + b
*x*cos(b*x + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(b*
x + 2*a)*sin(a))*cos(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))*c
os(2*b*x + 3*a)^2 + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) + b*x*cos(b*x +
2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(b*x + 2*a)*sin
(a))*sin(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))*sin(2*b*x + 3
*a)^2 + 4*((b*x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*
cos(3*b*x + 3*a)^2 + (b*x*cos(a) + sin(a))*cos(b*x + a)^2 + (b*x*cos(2*b*x
+ 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*sin(3*b*x + 3*a)^2 + (b*x*
cos(a) + sin(a))*sin(b*x + a)^2 + 2*(b*x*cos(2*b*x + 3*a)*cos(b*x + a) + (b
*x*cos(a) + sin(a))*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 3*a))*cos(3*b*x
+ 3*a) + (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(b
*x*cos(2*b*x + 3*a)*sin(b*x + a) + (b*x*cos(a) + sin(a))*sin(b*x + a) + sin
(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a) + (cos(b*x + a)^2 + sin(b*x +
a)^2)*sin(2*b*x + 3*a))*cos(3*b*x + 4*a) + 4*(6*b*x*cos(b*x + 2*a)*cos(b*x
+ a)*cos(a) + 6*b*x*cos(b*x + a)*sin(b*x + 2*a)*sin(a) + b*x*cos(2*b*x + 3*
a)^2 + b*x*sin(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*b*x + 2*(3*b*x*cos(b*
x + 2*a)*cos(b*x + a) + b*x*cos(a))*cos(2*b*x + 3*a) + 2*(3*b*x*cos(b*x + a
)*sin(b*x + 2*a) + b*x*sin(a))*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + 4*(2*b*
x*cos(b*x + a)*cos(a) + 3*(b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(b*x
+ 2*a) - 2*cos(a)*sin(b*x + a))*cos(2*b*x + 3*a) + 12*(b*x*cos(b*x + a)^2*
cos(a) + b*x*cos(a)*sin(b*x + a)^2)*cos(b*x + 2*a) - ((cos(2*b*x + 3*a)^2 +
2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(2*b*x +
3*a)*sin(a) + sin(a)^2)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)
^2)*cos(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*cos(b*x + a)^2 + (cos(2*b*x
+ 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*si
n(2*b*x + 3*a)*sin(a) + sin(a)^2)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + si
n(b*x + a)^2)*sin(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*sin(b*x + a)^2 + 2
*(cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*cos(2*b*x + 3*a)*cos(b*x + a)*cos(a)
+ cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*cos(b*x + a)*sin(2*b*x + 3*a)*sin(a)
+ (cos(a)^2 + sin(a)^2)*cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2*
cos(a) + cos(a)*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(cos(2*b*x + 3*a)^2*si
n(b*x + a) + 2*cos(2*b*x + 3*a)*cos(a)*sin(b*x + a) + sin(2*b*x + 3*a)^2*si
n(b*x + a) + 2*sin(2*b*x + 3*a)*sin(b*x + a)*sin(a) + (cos(a)^2 + sin(a)^2)
*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(cos(b*x + a)^2*sin(a) + sin(b*x + a)^2
*sin(a))*sin(2*b*x + 3*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x
+ a) + 1) + ((cos(2*b*x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + s
in(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*cos(3*b*x + 3*a)^
2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 3*a)^2 + (cos(a)^2 + sin(
a)^2)*cos(b*x + a)^2 + (cos(2*b*x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + co
s(a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*sin(3*b
*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 3*a)^2 + (cos(a
)^2 + sin(a)^2)*sin(b*x + a)^2 + 2*(cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*cos
(2*b*x + 3*a)*cos(b*x + a)*cos(a) + cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*cos
(b*x + a)*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*cos(b*x + a))*cos
(3*b*x + 3*a) + 2*(cos(b*x + a)^2*cos(a) + cos(a)*sin(b*x + a)^2)*cos(2*b*x
+ 3*a) + 2*(cos(2*b*x + 3*a)^2*sin(b*x + a) + 2*cos(2*b*x + 3*a)*cos(a)*si
n(b*x + a) + sin(2*b*x + 3*a)^2*sin(b*x + a) + 2*sin(2*b*x + 3*a)*sin(b*x +
a)*sin(a) + (cos(a)^2 + sin(a)^2)*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(cos(
b*x + a)^2*sin(a) + sin(b*x + a)^2*sin(a))*sin(2*b*x + 3*a))*log(cos(b*x +
a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + 4*((b*x*sin(2*b*x + 3*a) + b*
x*sin(a) - cos(2*b*x + 3*a) - cos(a))*cos(3*b*x + 3*a)^2 + (b*x*sin(a) - co
s(a))*cos(b*x + a)^2 + (b*x*sin(2*b*x + 3*a) + b*x*sin(a) - cos(2*b*x + 3*a

```

$$\begin{aligned} & - \cos(a) \sin(3bx + 3a)^2 + (bx \sin(a) - \cos(a)) \sin(bx + a)^2 + 2(bx \cos(bx + a) \sin(2bx + 3a) + (bx \sin(a) - \cos(a)) \cos(bx + a) - \cos(2bx + 3a) \cos(bx + a)) \cos(3bx + 3a) - (\cos(bx + a)^2 + \sin(bx + a)^2) \cos(2bx + 3a) + 2(bx \sin(2bx + 3a) \sin(bx + a) + (bx \sin(a) - \cos(a)) \sin(bx + a) - \cos(2bx + 3a) \sin(bx + a)) \sin(3bx + 3a) \\ & + (bx \cos(bx + a)^2 + bx \sin(bx + a)^2) \sin(2bx + 3a) \sin(3bx + 4a) + 4(6bx \cos(bx + 2a) \cos(a) \sin(bx + a) + 6bx \sin(bx + 2a) \sin(bx + a) \sin(a) + 2(3bx \cos(bx + 2a) \sin(bx + a) - \cos(a)) \cos(2bx + 3a) - \cos(2bx + 3a)^2 - \cos(a)^2 + 2(3bx \sin(bx + 2a) \sin(bx + a) - \sin(a)) \sin(2bx + 3a) - \sin(2bx + 3a)^2 - \sin(a)^2) \sin(3bx + 3a) \\ & + 4(2bx \cos(bx + a) \sin(a) + 3(bx \cos(bx + a)^2 + bx \sin(bx + a)^2) \sin(bx + 2a) - 2 \sin(bx + a) \sin(a)) \sin(2bx + 3a) + 12(bx \cos(bx + a)^2 \sin(a) + bx \sin(bx + a)^2 \sin(a)) \sin(bx + 2a) - 4(\cos(a)^2 + \sin(a)^2) \sin(bx + a) \cdot d / ((\cos(a)^2 + \sin(a)^2) b^2 \cos(bx + a)^2 + (\cos(a)^2 + \sin(a)^2) b^2 \sin(bx + a)^2 + (b^2 \cos(2bx + 3a)^2 + 2b^2 \cos(2bx + 3a) \cos(a) + b^2 \sin(2bx + 3a)^2 + 2b^2 \sin(2bx + 3a) \sin(a) + (\cos(a)^2 + \sin(a)^2) b^2) \cos(3bx + 3a)^2 + (b^2 \cos(bx + a)^2 + b^2 \sin(bx + a)^2) \cos(2bx + 3a)^2 + (b^2 \cos(2bx + 3a)^2 + 2b^2 \cos(2bx + 3a) \cos(a) + b^2 \sin(2bx + 3a)^2 + 2b^2 \sin(2bx + 3a) \sin(a) + (\cos(a)^2 + \sin(a)^2) b^2) \sin(3bx + 3a)^2 + (b^2 \cos(bx + a)^2 + b^2 \sin(bx + a)^2) \sin(2bx + 3a)^2 + 2(b^2 \cos(2bx + 3a)^2 \cos(bx + a) + 2b^2 \cos(2bx + 3a) \cos(bx + a) \cos(a) + b^2 \cos(bx + a) \sin(2bx + 3a)^2 + 2b^2 \cos(bx + a) \sin(2bx + 3a) \sin(a) + (\cos(a)^2 + \sin(a)^2) b^2 \cos(bx + a)) \cos(3bx + 3a) + 2(b^2 \cos(bx + a)^2 \cos(a) + b^2 \cos(a) \sin(bx + a)^2) \cos(2bx + 3a) + 2(b^2 \cos(2bx + 3a)^2 \sin(bx + a) + 2b^2 \cos(2bx + 3a) \cos(a) \sin(bx + a) + b^2 \sin(2bx + 3a)^2 \sin(bx + a) + 2b^2 \sin(2bx + 3a) \sin(bx + a) \sin(a) + (\cos(a)^2 + \sin(a)^2) b^2 \sin(bx + a)) \sin(3bx + 3a) + 2(b^2 \cos(bx + a)^2 \sin(a) + b^2 \sin(bx + a)^2 \sin(a)) \sin(2bx + 3a) \end{aligned}$$

Fricas [A] time = 0.513506, size = 252, normalized size = 4.42

$$\frac{2bdx + 8(bdx + bc) \cos(bx + a)^2 - d \cos(bx + a) \log(\sin(bx + a) + 1) + d \cos(bx + a) \log(-\sin(bx + a) + 1) - 8d \cos(bx + a) \sin(bx + a) + 2b^2 \cos(bx + a)}{2b^2 \cos(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")

[Out] -1/2*(2*b*d*x + 8*(b*d*x + b*c))*cos(b*x + a)^2 - d*cos(b*x + a)*log(sin(b*x + a) + 1) + d*cos(b*x + a)*log(-sin(b*x + a) + 1) - 8*d*cos(b*x + a)*sin(b*x + a) + 2*b*c)/(b^2*cos(b*x + a))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*sec(b*x+a)**2*sin(3*b*x+3*a),x)

[Out] Timed out

$$\begin{aligned}
& n(1/2*a)^2 + 2*\tan(1/2*b*x) + 2*\tan(1/2*a) + 1)) * \tan(1/2*b*x) * \tan(1/2*a)^3 \\
& - 96*d*\tan(1/2*b*x)^2*\tan(1/2*a)^3 + 10*b*c*\tan(1/2*a)^4 - d*\log(2*(\tan(1/2 \\
& *a)^2 + 1)/(\tan(1/2*b*x)^4*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^4*\tan(1/2*a) + 2*t \\
& \tan(1/2*b*x)^3*\tan(1/2*a)^2 + \tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^2*\tan(1/2*a)^2 \\
& - 2*\tan(1/2*b*x)^3 + 2*\tan(1/2*b*x)*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^2 + \tan(\\
& 1/2*a)^2 - 2*\tan(1/2*b*x) - 2*\tan(1/2*a) + 1)) * \tan(1/2*a)^4 + d*\log(2*(\tan(\\
& 1/2*a)^2 + 1)/(\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^4*\tan(1/2*a) - \\
& 2*\tan(1/2*b*x)^3*\tan(1/2*a)^2 + \tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^2*\tan(1/2*a \\
&)^2 + 2*\tan(1/2*b*x)^3 - 2*\tan(1/2*b*x)*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^2 + t \\
& \tan(1/2*a)^2 + 2*\tan(1/2*b*x) + 2*\tan(1/2*a) + 1)) * \tan(1/2*a)^4 - 16*d*\tan(1 \\
& /2*b*x)*\tan(1/2*a)^4 - 12*b*d*x*\tan(1/2*b*x)^2 - 64*b*d*x*\tan(1/2*b*x)*\tan(\\
& 1/2*a) - 12*b*d*x*\tan(1/2*a)^2 - 12*b*c*\tan(1/2*b*x)^2 + 16*d*\tan(1/2*b*x)^ \\
& 3 - 64*b*c*\tan(1/2*b*x)*\tan(1/2*a) - 4*d*\log(2*(\tan(1/2*a)^2 + 1)/(\tan(1/2* \\
& b*x)^4*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^4*\tan(1/2*a) + 2*\tan(1/2*b*x)^3*\tan(1/ \\
& 2*a)^2 + \tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^2*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^3 \\
& + 2*\tan(1/2*b*x)*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^2 + \tan(1/2*a)^2 - 2*\tan(1/2 \\
& *b*x) - 2*\tan(1/2*a) + 1)) * \tan(1/2*b*x)*\tan(1/2*a) + 4*d*\log(2*(\tan(1/2*a)^ \\
& 2 + 1)/(\tan(1/2*b*x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^4*\tan(1/2*a) - 2*\tan(1 \\
& /2*b*x)^3*\tan(1/2*a)^2 + \tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^2*\tan(1/2*a)^2 + 2 \\
& *\tan(1/2*b*x)^3 - 2*\tan(1/2*b*x)*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^2 + \tan(1/2* \\
& a)^2 + 2*\tan(1/2*b*x) + 2*\tan(1/2*a) + 1)) * \tan(1/2*b*x)*\tan(1/2*a) + 96*d*t \\
& \tan(1/2*b*x)^2*\tan(1/2*a) - 12*b*c*\tan(1/2*a)^2 + 96*d*\tan(1/2*b*x)*\tan(1/2* \\
& a)^2 + 16*d*\tan(1/2*a)^3 + 10*b*d*x + 10*b*c + d*\log(2*(\tan(1/2*a)^2 + 1)/ \\
& \tan(1/2*b*x)^4*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^4*\tan(1/2*a) + 2*\tan(1/2*b*x)^ \\
& 3*\tan(1/2*a)^2 + \tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^2*\tan(1/2*a)^2 - 2*\tan(1/2 \\
& *b*x)^3 + 2*\tan(1/2*b*x)*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^2 + \tan(1/2*a)^2 - 2 \\
& *\tan(1/2*b*x) - 2*\tan(1/2*a) + 1)) - d*\log(2*(\tan(1/2*a)^2 + 1)/(\tan(1/2*b* \\
& x)^4*\tan(1/2*a)^2 - 2*\tan(1/2*b*x)^4*\tan(1/2*a) - 2*\tan(1/2*b*x)^3*\tan(1/2* \\
& a)^2 + \tan(1/2*b*x)^4 + 2*\tan(1/2*b*x)^2*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^3 - \\
& 2*\tan(1/2*b*x)*\tan(1/2*a)^2 + 2*\tan(1/2*b*x)^2 + \tan(1/2*a)^2 + 2*\tan(1/2*b \\
& *x) + 2*\tan(1/2*a) + 1)) - 16*d*\tan(1/2*b*x) - 16*d*\tan(1/2*a))/(b^2*\tan(1/ \\
& 2*b*x)^4*\tan(1/2*a)^4 - 4*b^2*\tan(1/2*b*x)^3*\tan(1/2*a)^3 - b^2*\tan(1/2*b*x \\
&)^4 - 4*b^2*\tan(1/2*b*x)^3*\tan(1/2*a) - 4*b^2*\tan(1/2*b*x)*\tan(1/2*a)^3 - b \\
& ^2*\tan(1/2*a)^4 - 4*b^2*\tan(1/2*b*x)*\tan(1/2*a) + b^2)
\end{aligned}$$

$$3.392 \quad \int \frac{\sec^2(a+bx) \sin(3a+3bx)}{c+dx} dx$$

Optimal. Leaf size=77

$$-\text{CannotIntegrate}\left(\frac{\tan(a+bx) \sec(a+bx)}{c+dx}, x\right) + \frac{4 \sin\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{d} + \frac{4 \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d}$$

[Out] -CannotIntegrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x), x] + (4*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d + (4*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d

Rubi [A] time = 0.266208, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sec^2(a+bx) \sin(3a+3bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x), x]

[Out] (4*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d + (4*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d - Defer[Int] [(Sec[a + b*x]*Tan[a + b*x])/(c + d*x), x]

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(a+bx) \sin(3a+3bx)}{c+dx} dx &= \int \left(\frac{3 \sin(a+bx)}{c+dx} - \frac{\sin(a+bx) \tan^2(a+bx)}{c+dx} \right) dx \\ &= 3 \int \frac{\sin(a+bx)}{c+dx} dx - \int \frac{\sin(a+bx) \tan^2(a+bx)}{c+dx} dx \\ &= \left(3 \cos\left(a - \frac{bc}{d}\right) \right) \int \frac{\sin\left(\frac{bc}{d} + bx\right)}{c+dx} dx + \left(3 \sin\left(a - \frac{bc}{d}\right) \right) \int \frac{\cos\left(\frac{bc}{d} + bx\right)}{c+dx} dx + \int \frac{\sin\left(\frac{bc}{d} + bx\right)}{c+dx} dx \\ &= \frac{3 \text{Ci}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{d} + \frac{3 \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d} + \cos\left(a - \frac{bc}{d}\right) \int \frac{\sin\left(\frac{bc}{d} + bx\right)}{c+dx} dx \\ &= \frac{4 \text{Ci}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{d} + \frac{4 \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d} - \int \frac{\sec(a+bx) \tan(a+bx)}{c+dx} dx \end{aligned}$$

Mathematica [A] time = 13.5578, size = 0, normalized size = 0.

$$\int \frac{\sec^2(a+bx) \sin(3a+3bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x), x]

[Out] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x), x]

Maple [A] time = 0.388, size = 0, normalized size = 0.

$$\int \frac{(\sec(bx + a))^2 \sin(3bx + 3a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)

[Out] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(bx + a)^2 \sin(3bx + 3a)}{dx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")

[Out] integral(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c),x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx + a)^2 \sin(3bx + 3a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

$$3.393 \quad \int \frac{\sec^2(a+bx) \sin(3a+3bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=97

$$-\text{CannotIntegrate}\left(\frac{\tan(a+bx) \sec(a+bx)}{(c+dx)^2}, x\right) + \frac{4b \cos\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{d^2} - \frac{4b \sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d^2}$$

[Out] -CannotIntegrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x] + (4*b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2 - (4*Sin[a + b*x])/(d*(c + d*x)) - (4*b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2

Rubi [A] time = 0.344218, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0$, Rules used = {}

$$\int \frac{\sec^2(a+bx) \sin(3a+3bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]

[Out] (4*b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2 - (4*Sin[a + b*x])/(d*(c + d*x)) - (4*b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 - Defer [Int] [(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(a+bx) \sin(3a+3bx)}{(c+dx)^2} dx &= \int \left(\frac{3 \sin(a+bx)}{(c+dx)^2} - \frac{\sin(a+bx) \tan^2(a+bx)}{(c+dx)^2} \right) dx \\ &= 3 \int \frac{\sin(a+bx)}{(c+dx)^2} dx - \int \frac{\sin(a+bx) \tan^2(a+bx)}{(c+dx)^2} dx \\ &= -\frac{3 \sin(a+bx)}{d(c+dx)} + \frac{(3b) \int \frac{\cos(a+bx)}{c+dx} dx}{d} + \int \frac{\sin(a+bx)}{(c+dx)^2} dx - \int \frac{\sec(a+bx) \tan(a+bx)}{(c+dx)^2} dx \\ &= -\frac{4 \sin(a+bx)}{d(c+dx)} + \frac{b \int \frac{\cos(a+bx)}{c+dx} dx}{d} + \frac{\left(3b \cos\left(a - \frac{bc}{d}\right)\right) \int \frac{\cos\left(\frac{bc}{d} + bx\right)}{c+dx} dx}{d} - \frac{\left(3b \sin\left(a - \frac{bc}{d}\right)\right) \int \frac{\sin\left(\frac{bc}{d} + bx\right)}{c+dx} dx}{d} \\ &= \frac{3b \cos\left(a - \frac{bc}{d}\right) \text{Ci}\left(\frac{bc}{d} + bx\right)}{d^2} - \frac{4 \sin(a+bx)}{d(c+dx)} - \frac{3b \sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d^2} + \frac{\left(b \cos\left(a - \frac{bc}{d}\right)\right) \int \frac{\cos\left(\frac{bc}{d} + bx\right)}{c+dx} dx}{d} \\ &= \frac{4b \cos\left(a - \frac{bc}{d}\right) \text{Ci}\left(\frac{bc}{d} + bx\right)}{d^2} - \frac{4 \sin(a+bx)}{d(c+dx)} - \frac{4b \sin\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d^2} - \int \frac{\sec(a+bx) \tan(a+bx)}{(c+dx)^2} dx \end{aligned}$$

Mathematica [A] time = 16.5538, size = 0, normalized size = 0.

$$\int \frac{\sec^2(a+bx) \sin(3a+3bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]

[Out] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2, x]

Maple [A] time = 0.5, size = 0, normalized size = 0.

$$\int \frac{(\sec(bx + a))^2 \sin(3bx + 3a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)

[Out] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(bx + a)^2 \sin(3bx + 3a)}{d^2x^2 + 2cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**2,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (b x + a)^2 \sin (3 b x + 3 a)}{(d x + c)^2} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^2, x)
```

$$3.394 \quad \int \frac{\sec^2(a+bx) \sin(3a+3bx)}{(c+dx)^3} dx$$

Optimal. Leaf size=120

$$-\text{CannotIntegrate}\left(\frac{\tan(a+bx) \sec(a+bx)}{(c+dx)^3}, x\right) - \frac{2b^2 \sin\left(a - \frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d} + bx\right)}{d^3} - \frac{2b^2 \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d^3}$$

[Out] -CannotIntegrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^3, x] - (2*b*Cos[a + b*x])/(d^2*(c + d*x)) - (2*b^2*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d^3 - (2*Sin[a + b*x])/(d*(c + d*x)^2) - (2*b^2*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^3

Rubi [A] time = 0.436226, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sec^2(a+bx) \sin(3a+3bx)}{(c+dx)^3} dx$$

Verification is Not applicable to the result.

[In] Int[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3, x]

[Out] (-2*b*Cos[a + b*x])/(d^2*(c + d*x)) - (2*b^2*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d^3 - (2*Sin[a + b*x])/(d*(c + d*x)^2) - (2*b^2*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^3 - Defer[Int] [(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^3, x]

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(a+bx) \sin(3a+3bx)}{(c+dx)^3} dx &= \int \left(\frac{3 \sin(a+bx)}{(c+dx)^3} - \frac{\sin(a+bx) \tan^2(a+bx)}{(c+dx)^3} \right) dx \\ &= 3 \int \frac{\sin(a+bx)}{(c+dx)^3} dx - \int \frac{\sin(a+bx) \tan^2(a+bx)}{(c+dx)^3} dx \\ &= -\frac{3 \sin(a+bx)}{2d(c+dx)^2} + \frac{(3b) \int \frac{\cos(a+bx)}{(c+dx)^2} dx}{2d} + \int \frac{\sin(a+bx)}{(c+dx)^3} dx - \int \frac{\sec(a+bx) \tan(a+bx)}{(c+dx)^3} dx \\ &= -\frac{3b \cos(a+bx)}{2d^2(c+dx)} - \frac{2 \sin(a+bx)}{d(c+dx)^2} - \frac{(3b^2) \int \frac{\sin(a+bx)}{c+dx} dx}{2d^2} + \frac{b \int \frac{\cos(a+bx)}{(c+dx)^2} dx}{2d} - \int \frac{\sec(a+bx) \tan(a+bx)}{(c+dx)^3} dx \\ &= -\frac{2b \cos(a+bx)}{d^2(c+dx)} - \frac{2 \sin(a+bx)}{d(c+dx)^2} - \frac{b^2 \int \frac{\sin(a+bx)}{c+dx} dx}{2d^2} - \frac{\left(3b^2 \cos\left(a - \frac{bc}{d}\right)\right) \int \frac{\sin\left(\frac{bc}{d} + bx\right)}{c+dx} dx}{2d^2} \\ &= -\frac{2b \cos(a+bx)}{d^2(c+dx)} - \frac{3b^2 \text{Ci}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{2d^3} - \frac{2 \sin(a+bx)}{d(c+dx)^2} - \frac{3b^2 \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{2d^3} \\ &= -\frac{2b \cos(a+bx)}{d^2(c+dx)} - \frac{2b^2 \text{Ci}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{d^3} - \frac{2 \sin(a+bx)}{d(c+dx)^2} - \frac{2b^2 \cos\left(a - \frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d} + bx\right)}{d^3} \end{aligned}$$

Mathematica [A] time = 18.9277, size = 0, normalized size = 0.

$$\int \frac{\sec^2(a+bx) \sin(3a+3bx)}{(c+dx)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]

[Out] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3, x]

Maple [A] time = 0.996, size = 0, normalized size = 0.

$$\int \frac{(\sec(bx + a))^2 \sin(3bx + 3a)}{(dx + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)

[Out] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(bx + a)^2 \sin(3bx + 3a)}{d^3x^3 + 3cd^2x^2 + 3c^2dx + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")

[Out] integral(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**3,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx + a)^2 \sin(3bx + 3a)}{(dx + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")

[Out] integrate(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^3, x)

3.395 $\int x \cos(2x) \sec(x) dx$

Optimal. Leaf size=57

$$-i\text{PolyLog}(2, -ie^{ix}) + i\text{PolyLog}(2, ie^{ix}) + 2x \sin(x) + 2 \cos(x) + 2ix \tan^{-1}(e^{ix})$$

[Out] (2*I)*x*ArcTan[E^(I*x)] + 2*Cos[x] - I*PolyLog[2, (-I)*E^(I*x)] + I*PolyLog[2, I*E^(I*x)] + 2*x*Sin[x]

Rubi [A] time = 0.0674738, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {4431, 3296, 2638, 4407, 4181, 2279, 2391}

$$-i\text{PolyLog}(2, -ie^{ix}) + i\text{PolyLog}(2, ie^{ix}) + 2x \sin(x) + 2 \cos(x) + 2ix \tan^{-1}(e^{ix})$$

Antiderivative was successfully verified.

[In] Int[x*Cos[2*x]*Sec[x], x]

[Out] (2*I)*x*ArcTan[E^(I*x)] + 2*Cos[x] - I*PolyLog[2, (-I)*E^(I*x)] + I*PolyLog[2, I*E^(I*x)] + 2*x*Sin[x]

Rule 4431

Int[((e_.) + (f_.)*(x_.))^(m_.)*(F_)[(a_.) + (b_.)*(x_.)]^(p_.)*(G_)[(c_.) + (d_.)*(x_.)]^(q_.), x_Symbol] := Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && MemberQ[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]

Rule 3296

Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] := -Simp[(c + d*x)^m*cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4407

Int[((c_.) + (d_.)*(x_.))^(m_.)*Sin[(a_.) + (b_.)*(x_.)]^(n_.)*Tan[(a_.) + (b_.)*(x_.)]^(p_.), x_Symbol] := -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^(p-2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n-2)*Tan[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 4181

Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m-1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int x \cos(2x) \sec(x) dx &= \int (x \cos(x) - x \sin(x) \tan(x)) dx \\
&= \int x \cos(x) dx - \int x \sin(x) \tan(x) dx \\
&= x \sin(x) + \int x \cos(x) dx - \int x \sec(x) dx - \int \sin(x) dx \\
&= 2ix \tan^{-1}(e^{ix}) + \cos(x) + 2x \sin(x) + \int \log(1 - ie^{ix}) dx - \int \log(1 + ie^{ix}) dx - \int \sin(x) dx \\
&= 2ix \tan^{-1}(e^{ix}) + 2 \cos(x) + 2x \sin(x) - i \operatorname{Subst}\left(\int \frac{\log(1 - ix)}{x} dx, x, e^{ix}\right) + i \operatorname{Subst}\left(\int \frac{\log(1 + ix)}{x} dx, x, e^{ix}\right) \\
&= 2ix \tan^{-1}(e^{ix}) + 2 \cos(x) - i \operatorname{Li}_2(-ie^{ix}) + i \operatorname{Li}_2(ie^{ix}) + 2x \sin(x)
\end{aligned}$$

Mathematica [A] time = 0.031047, size = 77, normalized size = 1.35

$$-i \left(\operatorname{PolyLog}\left(2, -ie^{ix}\right) - \operatorname{PolyLog}\left(2, ie^{ix}\right) \right) - x \left(\log\left(1 - ie^{ix}\right) - \log\left(1 + ie^{ix}\right) \right) + 2x \sin(x) + 2 \cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[2*x]*Sec[x], x]
```

```
[Out] 2*Cos[x] - x*(Log[1 - I*E^(I*x)] - Log[1 + I*E^(I*x)]) - I*(PolyLog[2, (-I)*E^(I*x)] - PolyLog[2, I*E^(I*x)]) + 2*x*Sin[x]
```

Maple [A] time = 0.192, size = 81, normalized size = 1.4

$$-i(x+i)e^{ix} + i(x-i)e^{-ix} + x \ln(1 + ie^{ix}) - x \ln(1 - ie^{ix}) - i \operatorname{dilog}(1 + ie^{ix}) + i \operatorname{dilog}(1 - ie^{ix})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(2*x)*sec(x), x)
```

```
[Out] -I*(x+I)*exp(I*x)+I*(x-I)*exp(-I*x)+x*ln(1+I*exp(I*x))-x*ln(1-I*exp(I*x))-I*dilog(1+I*exp(I*x))+I*dilog(1-I*exp(I*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2x \sin(x) + 2 \cos(x) - 2 \int \frac{x \cos(2x) \cos(x) + x \sin(2x) \sin(x) + x \cos(x)}{\cos(2x)^2 + \sin(2x)^2 + 2 \cos(2x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(2*x)*sec(x),x, algorithm="maxima")

[Out] $2*x*\sin(x) + 2*\cos(x) - 2*\integrate((x*\cos(2*x)*\cos(x) + x*\sin(2*x)*\sin(x) + x*\cos(x))/(\cos(2*x)^2 + \sin(2*x)^2 + 2*\cos(2*x) + 1), x)$

Fricas [B] time = 0.529158, size = 405, normalized size = 7.11

$-\frac{1}{2}x \log(i \cos(x) + \sin(x) + 1) + \frac{1}{2}x \log(i \cos(x) - \sin(x) + 1) - \frac{1}{2}x \log(-i \cos(x) + \sin(x) + 1) + \frac{1}{2}x \log(-i \cos(x) - \sin(x) + 1)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(2*x)*sec(x),x, algorithm="fricas")

[Out] $-1/2*x*\log(I*\cos(x) + \sin(x) + 1) + 1/2*x*\log(I*\cos(x) - \sin(x) + 1) - 1/2*x*\log(-I*\cos(x) + \sin(x) + 1) + 1/2*x*\log(-I*\cos(x) - \sin(x) + 1) + 2*x*\sin(x) + 2*\cos(x) + 1/2*I*dilog(I*\cos(x) + \sin(x)) + 1/2*I*dilog(I*\cos(x) - \sin(x)) - 1/2*I*dilog(-I*\cos(x) + \sin(x)) - 1/2*I*dilog(-I*\cos(x) - \sin(x))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(2x) \sec(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(2*x)*sec(x),x)

[Out] Integral(x*cos(2*x)*sec(x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(2x) \sec(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(2*x)*sec(x),x, algorithm="giac")

[Out] integrate(x*cos(2*x)*sec(x), x)

3.396 $\int x \cos(2x) \sec^2(x) dx$

Optimal. Leaf size=14

$$x^2 - x \tan(x) - \log(\cos(x))$$

[Out] $x^2 - \text{Log}[\text{Cos}[x]] - x*\text{Tan}[x]$

Rubi [A] time = 0.0331439, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {4431, 3720, 3475, 30}

$$x^2 - x \tan(x) - \log(\cos(x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{Cos}[2*x]*\text{Sec}[x]^2, x]$

[Out] $x^2 - \text{Log}[\text{Cos}[x]] - x*\text{Tan}[x]$

Rule 4431

$\text{Int}[(e_.) + (f_.)*(x_.)]^{(m_.)}*(F_.)[(a_.) + (b_.)*(x_.)]^{(p_.)}*(G_.)[(c_.) + (d_.)*(x_.)]^{(q_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigExpand}[(e + f*x)^m * G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /;$ FreeQ[{a, b, c, d, e, f, m}, x] && MemberQ[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && EQ[b*c - a*d, 0] && IGtQ[b/d, 1]

Rule 3720

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*((b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(b*(c + d*x)^m*(b*\text{Tan}[e + f*x])^{(n-1)})/(f*(n-1)), x] + (-\text{Dist}[(b*d*m)/(f*(n-1)), \text{Int}[(c + d*x)^{(m-1)}*(b*\text{Tan}[e + f*x])^{(n-1)}, x], x] - \text{Dist}[b^2, \text{Int}[(c + d*x)^m*(b*\text{Tan}[e + f*x])^{(n-2)}, x], x]) /;$ FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 3475

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /;$ FreeQ[{c, d}, x]

Rule 30

$\text{Int}[(x_.)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[x^{(m+1)}/(m+1), x] /;$ FreeQ[m, x] && NEQ[m, -1]

Rubi steps

$$\begin{aligned} \int x \cos(2x) \sec^2(x) dx &= \int (x - x \tan^2(x)) dx \\ &= \frac{x^2}{2} - \int x \tan^2(x) dx \\ &= \frac{x^2}{2} - x \tan(x) + \int x dx + \int \tan(x) dx \\ &= x^2 - \log(\cos(x)) - x \tan(x) \end{aligned}$$

Mathematica [A] time = 0.0232502, size = 14, normalized size = 1.

$$x^2 - x \tan(x) - \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[2*x]*Sec[x]^2,x]

[Out] x^2 - Log[Cos[x]] - x*Tan[x]

Maple [A] time = 0.042, size = 15, normalized size = 1.1

$$x^2 - \ln(\cos(x)) - x \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(2*x)*sec(x)^2,x)

[Out] x^2-ln(cos(x))-x*tan(x)

Maxima [B] time = 1.56808, size = 150, normalized size = 10.71

$$\frac{2x^2 \cos(2x)^2 + 2x^2 \sin(2x)^2 + 4x^2 \cos(2x) + 2x^2 - (\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1) \log(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1)}{2(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(2*x)*sec(x)^2,x, algorithm="maxima")

[Out] 1/2*(2*x^2*cos(2*x)^2 + 2*x^2*sin(2*x)^2 + 4*x^2*cos(2*x) + 2*x^2 - (cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)

Fricas [A] time = 0.486144, size = 73, normalized size = 5.21

$$\frac{x^2 \cos(x) - \cos(x) \log(-\cos(x)) - x \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(2*x)*sec(x)^2,x, algorithm="fricas")

[Out] (x^2*cos(x) - cos(x)*log(-cos(x)) - x*sin(x))/cos(x)

Sympy [B] time = 53.0657, size = 144, normalized size = 10.29

$$x^2 + \frac{2x \tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right) - 1} - \frac{\log\left(\tan\left(\frac{x}{2}\right) - 1\right) \tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right) - 1} + \frac{\log\left(\tan\left(\frac{x}{2}\right) - 1\right)}{\tan^2\left(\frac{x}{2}\right) - 1} - \frac{\log\left(\tan\left(\frac{x}{2}\right) + 1\right) \tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right) - 1} + \frac{\log\left(\tan\left(\frac{x}{2}\right) + 1\right)}{\tan^2\left(\frac{x}{2}\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(2*x)*sec(x)**2,x)

[Out] x**2 + 2*x*tan(x/2)/(tan(x/2)**2 - 1) - log(tan(x/2) - 1)*tan(x/2)**2/(tan(x/2)**2 - 1) + log(tan(x/2) - 1)/(tan(x/2)**2 - 1) - log(tan(x/2) + 1)*tan(x/2)**2/(tan(x/2)**2 - 1) + log(tan(x/2) + 1)/(tan(x/2)**2 - 1) + log(tan(x/2)**2 + 1)*tan(x/2)**2/(tan(x/2)**2 - 1) - log(tan(x/2)**2 + 1)/(tan(x/2)**2 - 1)

Giac [B] time = 1.16055, size = 159, normalized size = 11.36

$$\frac{2x^2 \tan\left(\frac{1}{2}x\right)^2 - \log\left(\frac{4\left(\tan\left(\frac{1}{2}x\right)^4 - 2\tan\left(\frac{1}{2}x\right)^2 + 1\right)}{\tan\left(\frac{1}{2}x\right)^4 + 2\tan\left(\frac{1}{2}x\right)^2 + 1}\right) \tan\left(\frac{1}{2}x\right)^2 - 2x^2 + 4x \tan\left(\frac{1}{2}x\right) + \log\left(\frac{4\left(\tan\left(\frac{1}{2}x\right)^4 - 2\tan\left(\frac{1}{2}x\right)^2 + 1\right)}{\tan\left(\frac{1}{2}x\right)^4 + 2\tan\left(\frac{1}{2}x\right)^2 + 1}\right)}{2\left(\tan\left(\frac{1}{2}x\right)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(2*x)*sec(x)^2,x, algorithm="giac")

[Out] 1/2*(2*x^2*tan(1/2*x)^2 - log(4*(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - 2*x^2 + 4*x*tan(1/2*x) + log(4*(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)))/(tan(1/2*x)^2 - 1)

3.397 $\int x \cos(2x) \sec^3(x) dx$

Optimal. Leaf size=67

$$\frac{3}{2}i\text{PolyLog}(2, -ie^{ix}) - \frac{3}{2}i\text{PolyLog}(2, ie^{ix}) - 3ix \tan^{-1}(e^{ix}) + \frac{\sec(x)}{2} - \frac{1}{2}x \tan(x) \sec(x)$$

[Out] $(-3*I)*x*\text{ArcTan}[E^{(I*x)}] + ((3*I)/2)*\text{PolyLog}[2, (-I)*E^{(I*x)}] - ((3*I)/2)*\text{PolyLog}[2, I*E^{(I*x)}] + \text{Sec}[x]/2 - (x*\text{Sec}[x]*\text{Tan}[x])/2$

Rubi [A] time = 0.137247, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {4431, 4181, 2279, 2391, 4413, 4185}

$$\frac{3}{2}i\text{PolyLog}(2, -ie^{ix}) - \frac{3}{2}i\text{PolyLog}(2, ie^{ix}) - 3ix \tan^{-1}(e^{ix}) + \frac{\sec(x)}{2} - \frac{1}{2}x \tan(x) \sec(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{Cos}[2*x]*\text{Sec}[x]^3, x]$

[Out] $(-3*I)*x*\text{ArcTan}[E^{(I*x)}] + ((3*I)/2)*\text{PolyLog}[2, (-I)*E^{(I*x)}] - ((3*I)/2)*\text{PolyLog}[2, I*E^{(I*x)}] + \text{Sec}[x]/2 - (x*\text{Sec}[x]*\text{Tan}[x])/2$

Rule 4431

$\text{Int}[(e_{.}) + (f_{.})*(x_{.})^{(m_{.})}*(F_{.})[(a_{.}) + (b_{.})*(x_{.})]^{(p_{.})}*(G_{.})[(c_{.}) + (d_{.})*(x_{.})]^{(q_{.})}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigExpand}[(e + f*x)^m * G[c + d*x]^q, F, c + d*x, p, b/d, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x\} \&\& \text{MemberQ}\{\{\text{Sin}, \text{Cos}\}, F\} \&\& \text{MemberQ}\{\{\text{Sec}, \text{Csc}\}, G\} \&\& \text{IGtQ}\{p, 0\} \&\& \text{IGtQ}\{q, 0\} \&\& \text{EqQ}\{b*c - a*d, 0\} \&\& \text{IGtQ}\{b/d, 1\}$

Rule 4181

$\text{Int}[\text{csc}[(e_{.}) + \text{Pi}*(k_{.}) + (f_{.})*(x_{.})]*((c_{.}) + (d_{.})*(x_{.}))^{(m_{.})}, x_Symbol] \rightarrow \text{Simp}[-2*(c + d*x)^m * \text{ArcTanh}[E^{(I*k*Pi)}*E^{(I*(e + f*x))}]/f, x] + (-\text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)} * \text{Log}[1 - E^{(I*k*Pi)}*E^{(I*(e + f*x))}], x], x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)} * \text{Log}[1 + E^{(I*k*Pi)}*E^{(I*(e + f*x))}], x], x]) /; \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{IntegerQ}\{2*k\} \&\& \text{IGtQ}\{m, 0\}$

Rule 2279

$\text{Int}[\text{Log}[(a_{.}) + (b_{.})*((F_{.})^{((e_{.})*((c_{.}) + (d_{.})*(x_{.})))^{(n_{.})})], x_Symbol] \rightarrow \text{Dist}[1/(d*e^n * \text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \&\& \text{GtQ}\{a, 0\}$

Rule 2391

$\text{Int}[\text{Log}[(c_{.})*((d_{.}) + (e_{.})*(x_{.})^{(n_{.})})]/(x_{.}), x_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x\} \&\& \text{EqQ}\{c*d, 1\}$

Rule 4413

$\text{Int}[(c_{.}) + (d_{.})*(x_{.})^{(m_{.})} * \text{Sec}[(a_{.}) + (b_{.})*(x_{.})] * \text{Tan}[(a_{.}) + (b_{.})*(x_{.})]^{(p_{.})}, x_Symbol] \rightarrow -\text{Int}[(c + d*x)^m * \text{Sec}[a + b*x] * \text{Tan}[a + b*x]^{(p-2)}, x] + \text{Int}[(c + d*x)^m * \text{Sec}[a + b*x]^3 * \text{Tan}[a + b*x]^{(p-2)}, x] /; \text{FreeQ}\{a, b, c, d, m\}, x\} \&\& \text{IGtQ}\{p/2, 0\}$

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_.)), x_Symbol] :>
-Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\int x \cos(2x) \sec^3(x) dx &= \int (x \sec(x) - x \sec(x) \tan^2(x)) dx \\
&= \int x \sec(x) dx - \int x \sec(x) \tan^2(x) dx \\
&= -2ix \tan^{-1}(e^{ix}) - \int \log(1 - ie^{ix}) dx + \int \log(1 + ie^{ix}) dx + \int x \sec(x) dx - \int x \sec^3(x) dx \\
&= -4ix \tan^{-1}(e^{ix}) + \frac{\sec(x)}{2} - \frac{1}{2}x \sec(x) \tan(x) + i \operatorname{Subst}\left(\int \frac{\log(1 - ix)}{x} dx, x, e^{ix}\right) - i \operatorname{Subst}\left(\int \frac{\log(1 + ix)}{x} dx, x, e^{ix}\right) \\
&= -3ix \tan^{-1}(e^{ix}) + i \operatorname{Li}_2(-ie^{ix}) - i \operatorname{Li}_2(ie^{ix}) + \frac{\sec(x)}{2} - \frac{1}{2}x \sec(x) \tan(x) + i \operatorname{Subst}\left(\int \frac{\log(1 - ix)}{x} dx, x, e^{ix}\right) - i \operatorname{Subst}\left(\int \frac{\log(1 + ix)}{x} dx, x, e^{ix}\right) \\
&= -3ix \tan^{-1}(e^{ix}) + 2i \operatorname{Li}_2(-ie^{ix}) - 2i \operatorname{Li}_2(ie^{ix}) + \frac{\sec(x)}{2} - \frac{1}{2}x \sec(x) \tan(x) - \frac{1}{2}i \operatorname{Subst}\left(\int \frac{\log(1 - ix)}{x} dx, x, e^{ix}\right) + \frac{1}{2}i \operatorname{Subst}\left(\int \frac{\log(1 + ix)}{x} dx, x, e^{ix}\right) \\
&= -3ix \tan^{-1}(e^{ix}) + \frac{3}{2}i \operatorname{Li}_2(-ie^{ix}) - \frac{3}{2}i \operatorname{Li}_2(ie^{ix}) + \frac{\sec(x)}{2} - \frac{1}{2}x \sec(x) \tan(x)
\end{aligned}$$

Mathematica [B] time = 0.27406, size = 146, normalized size = 2.18

$$\frac{1}{4} \left(6i \operatorname{PolyLog}(2, -ie^{ix}) - 6i \operatorname{PolyLog}(2, ie^{ix}) + 6x \log(1 - ie^{ix}) - 6x \log(1 + ie^{ix}) + \frac{x}{\sin(x) - 1} + \frac{x}{\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)^2} + \dots \right)$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[2*x]*Sec[x]^3,x]

[Out] (6*x*Log[1 - I*E^(I*x)] - 6*x*Log[1 + I*E^(I*x)] + (6*I)*PolyLog[2, (-I)*E^(I*x)] - (6*I)*PolyLog[2, I*E^(I*x)] + (2*Sin[x/2])/(Cos[x/2] - Sin[x/2]) + x/(Cos[x/2] + Sin[x/2])^2 - (2*Sin[x/2])/(Cos[x/2] + Sin[x/2]) + x/(-1 + Sin[x]))/4

Maple [B] time = 0.223, size = 102, normalized size = 1.5

$$\frac{i(xe^{3ix} - xe^{ix} - ie^{3ix} - ie^{ix})}{(e^{2ix} + 1)^2} - \frac{3x \ln(1 + ie^{ix})}{2} + \frac{3x \ln(1 - ie^{ix})}{2} + \frac{3i}{2} \operatorname{dilog}(1 + ie^{ix}) - \frac{3i}{2} \operatorname{dilog}(1 - ie^{ix})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(2*x)*sec(x)^3,x)

[Out] I/(exp(2*I*x)+1)^2*(x*exp(3*I*x)-x*exp(I*x)-I*exp(3*I*x)-I*exp(I*x))-3/2*x*ln(1+I*exp(I*x))+3/2*x*ln(1-I*exp(I*x))+3/2*I*dilog(1+I*exp(I*x))-3/2*I*dilog(1-I*exp(I*x))

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(2*x)*sec(x)^3,x, algorithm="maxima")

[Out] Timed out

Fricas [B] time = 0.538735, size = 500, normalized size = 7.46

$$\frac{3x \cos(x)^2 \log(i \cos(x) + \sin(x) + 1) - 3x \cos(x)^2 \log(i \cos(x) - \sin(x) + 1) + 3x \cos(x)^2 \log(-i \cos(x) + \sin(x))}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(2*x)*sec(x)^3,x, algorithm="fricas")

[Out]
$$\frac{1}{4} * (3 * x * \cos(x)^2 * \log(I * \cos(x) + \sin(x) + 1) - 3 * x * \cos(x)^2 * \log(I * \cos(x) - \sin(x) + 1) + 3 * x * \cos(x)^2 * \log(-I * \cos(x) + \sin(x) + 1) - 3 * x * \cos(x)^2 * \log(-I * \cos(x) - \sin(x) + 1) - 3 * I * \cos(x)^2 * \operatorname{dilog}(I * \cos(x) + \sin(x)) - 3 * I * \cos(x)^2 * \operatorname{dilog}(I * \cos(x) - \sin(x)) + 3 * I * \cos(x)^2 * \operatorname{dilog}(-I * \cos(x) + \sin(x)) + 3 * I * \cos(x)^2 * \operatorname{dilog}(-I * \cos(x) - \sin(x)) - 2 * x * \sin(x) + 2 * \cos(x)) / \cos(x)^2$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(2*x)*sec(x)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(2x) \sec(x)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(2*x)*sec(x)^3,x, algorithm="giac")

[Out] integrate(x*cos(2*x)*sec(x)^3, x)

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*   is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*   antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
22       If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25       "C"],
26     If[FreeQ[result,Integrate] && FreeQ[result,Int],
27       "C",
28       "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
```



```

101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1},func]

```

4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 # if leaf size is "too large". Set at 500,000
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 # see problem 156, file Apostol_Problems
11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
14     debug:=false;
15
16     leaf_count_result:=leafcount(result);
17     #do NOT call ExpnType() if leaf size is too large. Recursion problem
18     if leaf_count_result > 500000 then
19         return "B";
20     fi;
21
22     leaf_count_optimal:=leafcount(optimal);
23
24     ExpnType_result:=ExpnType(result);
25     ExpnType_optimal:=ExpnType(optimal);
26
27     if debug then
28         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
29             ExpnType_optimal);
30     fi;
31
32 # If result and optimal are mathematical expressions,
33 # GradeAntiderivative[result,optimal] returns
34 # "F" if the result fails to integrate an expression that
35 # is integrable
36 # "C" if result involves higher level functions than necessary
37 # "B" if result is more than twice the size of the optimal
38 # antiderivative
39 # "A" if result can be considered optimal
40
41 #This check below actually is not needed, since I only
42 #call this grading only for passed integrals. i.e. I check
43 #for "F" before calling this. But no harm of keeping it here.
44 #just in case.
45
46 if not type(result,freeof('int')) then
47     return "F";
48 end if;
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then
56             if debug then

```

```

57         print("both result and optimal complex");
58         fi;
59         #both result and optimal complex
60         if leaf_count_result<=2*leaf_count_optimal then
61             return "A";
62         else
63             return "B";
64         end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do
not as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
102 is_contains_complex:= proc(expression)
103     return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)

```

```

119   if type(expn,'atomic') then
120       1
121   elif type(expn,'list') then
122       apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124       if type(op(1,expn),'rational') then
125           1
126       else
127           max(2,ExpnType(op(1,expn)))
128       end if
129   elif type(expn,'^^') then
130       if type(op(2,expn),'integer') then
131           ExpnType(op(1,expn))
132       elif type(op(2,expn),'rational') then
133           if type(op(1,expn),'rational') then
134               1
135           else
136               max(2,ExpnType(op(1,expn)))
137           end if
138       else
139           max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140       end if
141   elif type(expn,'+`') or type(expn,'*`') then
142       max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144       max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146       max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148       max(5,apply(max,map(ExpnType,[op(expn)])))
149   elif AppellFunctionQ(op(0,expn)) then
150       max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152       max(8,apply(max,map(ExpnType,[op(expn)]))) else
153       9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159     member(func,[
160         exp,log,ln,
161         sin,cos,tan,cot,sec,csc,
162         arcsin,arccos,arctan,arccot,arcsec,arccsc,
163         sinh,cosh,tanh,coth,sech,csch,
164         arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168     member(func,[
169         erf,erfc,erfi,
170         FresnelS,FresnelC,
171         Ei,Ei,Li,Si,Ci,Shi,Chi,
172         GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173         EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177     member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181     member(func,[AppellF1])

```

```

182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:
42         if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43             return True

```



```

44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,``^`)
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn
72 )))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,``+`) or
77 type(expn,``*`)
78     m1 = expnType(expn.args[0])
79     m2 = expnType(list(expn.args[1:]))
80     return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82     return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84     m1 = max(map(expnType, list(expn.args)))
85     return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88     m1 = max(map(expnType, list(expn.args)))
89     return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
90     elif is_appell_function(expn.func):
91     m1 = max(map(expnType, list(expn.args)))
92     return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
93     elif isinstance(expn,RootSum):
94     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
95 ,Apply[List,expn]],7]],
96     return max(7,m1)
97     elif str(expn).find("Integral") != -1:
98     m1 = max(map(expnType, list(expn.args)))
99     return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
100     else:
101     return 9
102
103 #main function
104 def grade_antiderivative(result,optimal):
105
106     leaf_count_result = leaf_count(result)

```

```

102 leaf_count_optimal = leaf_count(optimal)
103
104 expnType_result = expnType(result)
105 expnType_optimal = expnType(optimal)
106
107 if str(result).find("Integral") != -1:
108     return "F"
109
110 if expnType_result <= expnType_optimal:
111     if result.has(I):
112         if optimal.has(I): #both result and optimal complex
113             if leaf_count_result <= 2*leaf_count_optimal:
114                 return "A"
115             else:
116                 return "B"
117         else: #result contains complex but optimal is not
118             return "C"
119     else: # result do not contain complex, this assumes optimal do not as
well
120         if leaf_count_result <= 2*leaf_count_optimal:
121             return "A"
122         else:
123             return "B"
124 else:
125     return "C"

```

4.0.4 SageMath grading function

```

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by
2 #     Albert Rich to use with Sagemath. This is used to
3 #     grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #     'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
(flatten(tree(anti))))
33         return round(1.35*len(flatten(tree(anti)))) #fudge factor
34             #since this estimate of leaf count is bit lower than

```

```

35         #what it should be compared to Mathematica's
36
37 def is_sqrt(expr):
38     debug=False;
39     if expr.operator() == operator.pow: #isinstance(expr,Pow):
40         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
41             if debug: print ("expr is sqrt")
42             return True
43         else:
44             return False
45     else:
46         return False
47
48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func , " is special_function")
83         else:
84             print ("func ", func , " is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','
91     hypergeometric_U']
92
93 def is_appell_function(func):
94     return func.name() in ['hypergeometric'] #[appellf1] can't find this in
95     sagemath

```

```

95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
104             return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list: #isinstance(expn,list):
121         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
127     elif expn.operator() == operator.pow: #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
129             return expnType(expn.operands()[0]) #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
132                 return 1
133             else:
134                 return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137     elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138         m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139         m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
141     elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
142         return max(3,expnType(expn.operands()[0]))
143     elif is_special_function(expn.operator()): #is_special_function(expn.func)
144         m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))

```

```

145     return max(4,m1)    #max(4,m1)
146     elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
148         return max(5,m1)    #max(5,m1)
149     elif is_appell_function(expn.operator()):
150         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
151         return max(6,m1)    #max(6,m1)
152     elif str(expn).find("Integral") != -1: #this will never happen, since it
153         #is checked before calling the grading function that is passed.
154         #but kept it here.
155         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
156         return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
157     else:
158         return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)
176
177     if expnType_result <= expnType_optimal:
178         if result.has(I):
179             if optimal.has(I): #both result and optimal complex
180                 if leaf_count_result <= 2*leaf_count_optimal:
181                     return "A"
182                 else:
183                     return "B"
184             else: #result contains complex but optimal is not
185                 return "C"
186         else: # result do not contain complex, this assumes optimal do not as
well
187             if leaf_count_result <= 2*leaf_count_optimal:
188                 return "A"
189             else:
190                 return "B"
191     else:
192         return "C"

```